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Flexural Stiffness of Reinforced Concrete Columns and Beams: Experimental Verification

by Madhu Khuntia and S. K. Ghosh

*The current ACI code (ACI 318-02) provisions on effective stiffnesses of beams and columns have been reviewed in a companion paper, in which simple formulas have been proposed to determine the effective stiffnesses of reinforced concrete columns and beams, based on an analytical parametric study. Analytical axial load-bending moment diagrams of slender columns for a given initial eccentricity (*M/P *ratio), obtained using the proposed stiffness assumptions, are compared in this paper with numerous published test data and are found to be in good agreement. The proposed stiffness expressions are applicable for all levels of applied loading, including both service and ultimate loads. The analytical and experimental results show that the flexural stiffness assumption in the current ACI code procedure for design of slender columns using the moment magnifier method (Eq. (10-12) and Eq. (10-18)) is extremely conservative.*

Recommendations are made concerning stiffness assumptions in the analysis of reinforced concrete frames, including frames containing slender columns under lateral loads.

Keywords: beam; column; moment; slender column.

INTRODUCTION

In a companion paper, Khuntia and Ghosh (2004) have proposed that the effective EI of a column ($P_u/A_g f_c' \geq 0.10$) under short-term loading can be taken as

$$EI_e = E_c I_g (0.80 + 25\rho_g) \times \left(1 - \frac{e}{h} - 0.5\frac{P_u}{P_o}\right) \leq E_c I_g \geq E_c I_{beam} \quad (1)$$

In. Eq (1), I_g is the gross moment of inertia of column cross section, equal to $bh^3/12$ for a rectangular section with width b and total depth h. Gross reinforcement ratio ρ_g should be expressed as a decimal fraction. The elastic modulus of normal-weight concrete can be expressed as (ACI 318 Section 8.5.1)

$$E_c = w_c^{1.5} 33\sqrt{f_c'} \quad (2)$$

In Eq. (2), both E_c and f_c' are in psi, and w_c is concrete unit weight in lb/ft^3.

Instead of Eq. (1), Eq. 3(a) and (b), as follows, may be used, which are only valid near the design strength interaction diagrams. Note that near the design strength curves, the assumption of $e/h + P_u/P_o = 0.7$ is found to be quite reasonable. Therefore, Eq. (1) reduces to

$$EI_e = E_c I_g (0.80 + 25\rho_g) \times \left(0.65 - 0.5\frac{e}{h}\right) \leq E_c I_g \geq E_c I_{beam} \quad (3a)$$

Alternatively, the short-term EI of a column may be expressed as

$$EI_e = E_c I_g (0.80 + 25\rho_g) \times \left(0.30 + 0.5\frac{P_u}{P_o}\right) \leq E_c I_g \geq E_c I_{beam} \quad (3b)$$

For service load conditions and for better accuracy, it is recommended to use Eq. (1). For service loads, P_u becomes P and $e = M/P$ in the previous expressions.

The upper limit of $E_c I_g$ on the effective EI of a column is for purposes of conservatism; it can be higher for heavily reinforced columns with low e/h (= M/Ph) ratios. The lower limit for the effective EI of a column is taken to be the EI_e of an equivalent beam, that is, EI_e of the member when it can be treated as a beam rather than a column. This happens when a member is subjected to a very low axial load and a high e/h ratio ($e/h > 0.8$, for example). For calculating EI_e of an equivalent beam, ρ, the tensile steel ratio (not the gross steel ratio) must be used, which can be approximately taken as half of ρ_g for a column with symmetrical reinforcement. Mainly the reinforcement on the tension side contributes to the flexural stiffness of beams, whereas for columns, the reinforcement over the whole section is generally effective.

Based on an analytical parametric study, a simplified equation (Eq. (4)) was proposed (Khuntia and Ghosh 2004) for the effective EI of reinforced concrete beams of normal-strength concrete

$$EI_e = E_c I_g (0.10 + 25\rho)(1.2 - 0.2b/d) \leq 0.6 E_c I_g \quad (4)$$

where $(1.2 - 0.2b/d) \leq 1.0$.

For high-strength concrete beams, Eq. (4) can be modified to

$$EI_e = E_c I_g (0.10 + 25\rho)(1.2 - 0.2b/d) \times (1.15 - 4 \times 10^{-5} f_c') \leq 0.6 E_c I_g \quad (5)$$

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ACI member **Madhu Khuntia** *is a research structural engineer at DuKane Precast Inc., Naperville, Ill. He received his PhD from the University of Michigan, Ann Arbor, Mich. He is a member of ACI Committee 335, Composite and Hybrid Structures.*

S. K. Ghosh, *FACI, heads the consulting practice, S.K. Ghosh Associates Inc., Palatine, Ill. He is a member of ACI Committees 318, Structural Concrete Building Code; 371, Concrete Pedestal Water Towers; 435, Deflection of Concrete Building Structures; Joint ACI-ASCE-TMS Committee 530, Masonry Standards Joint Committee; and TAC Technology Transfer Committee.*

where $(1.2 - 0.2b/d) \leq 1.0$ and f_c' is in psi. It was suggested (Khuntia and Ghosh 2004) that Eq. (5) be used for $f_c' > 6000$ psi for better accuracy.

Axial load-bending moment histories of slender columns (for a given initial M/P ratio), based on the proposed and the ACI 318 stiffness assumptions, are compared in this paper with test results from the literature. Columns in both sway (under lateral loading) and nonsway (under gravity loading) frames are considered. The tests were conducted on both normal- and high-strength concrete columns with different reinforcement ratios, end eccentricity ratios, axial load ratios, and slenderness ratios.

A review of flexural stiffness recommendations of ACI 318-02 has been included in Khuntia and Ghosh (2004), and the limitations of ACI stiffness assumptions have been pointed out.

RESEARCH SIGNIFICANCE

This paper is related to the work of the Slender Column Task Group of ACI Committee 318, Structural Concrete Building Code. The Task Group is trying to formulate code provisions to streamline and, if possible, simplify the requirements of ACI 318 Sections 10.11 to 10.13 on slender column design. One of the major parameters in slender column design is a suitable assumption for the effective flexural stiffness EI_e of the column.

COMPARISON WITH TEST RESULTS

Columns in two types of frames are considered for comparison: a) nonsway frames; and b) sway frames. The variables included are: compressive strength of concrete f_c', eccentricity ratio e/h, axial load ratio P_u/P_o, effective slenderness ratio kl_u/r, and end moment ratio M_1/M_2. The columns in nonsway frames are divided into two categories: columns in single-curvature bending and columns under double-curvature bending.

Note that all the appropriate figures (to be shown later) for comparison show the following curves/lines:

1. Nominal strength P-M interaction diagram;
2. Design strength P-M interaction diagram;
3. Loading history considering short column behavior (ignoring slenderness effects);

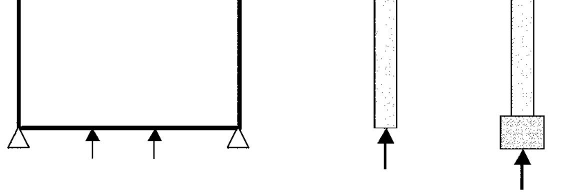

Fig. 1—Schematic diagram of loading arrangement by different investigators.

4. Loading history using the proposed method (using fixed EI of the Appendix, Section B, and Eq. (1) for EI_e);
5. Loading history using ACI Code method (using fixed EI of the Appendix, Section B, and ACI 318 Section 10.12.3 for EI_e); and
6. Experimentally derived loading history.

Columns in nonsway frames under single-curvature bending

Nine sets of test data from three investigations (Furlong and Ferguson 1966; Green 1966; Lloyd and Rangan 1996) are considered in this comparative study. It may be noted that, of the three investigations, only the tests by Furlong and Ferguson are frame tests and the rest are tests on hinged columns. Figure 1 gives the schematic diagrams of test setups in the three investigations previously mentioned. It may be noted that in all cases, columns are under single-curvature bending—a condition considered to be the worst case for causing maximum secondary moments (P-δ effects).

Table 1 gives the details of the variables in the previous investigations. The principal variables were: concrete strength f_c' (3240 to 8400 psi), eccentricity ratio e/h (0.086 to 0.42), and axial load ratio P_u/P_o (0.27 to 0.80). Other variables included the slenderness ratio kl_u/r (ranging between 31 and 61) and the longitudinal reinforcement ratio ρ_g (ranging between 1.80 and 2.15%).

The test results are compared with the axial load-bending moment histories computed using the proposed and the ACI 318 stiffness assumptions. The complete axial load-bending moment histories were analytically obtained by the procedure described in the Appendix. A more accurate (variable EI_e), as well as a more approximate (fixed EI_e) procedure, is described in the Appendix. The approximate procedure is similar to the more accurate method, except for the following difference: the accurate method considers a change in the initial eccentricity at the location of maximum moment with gradual increases in M and P. In other words, as P/P_o increases, the effective flexural stiffness of a column decreases per Eq. (1), leading to a reduction in P_c. This increases the magnitude of δ_{ns} (refer to the Appendix for formula for moment magnifier δ_{ns}) and the corresponding magnified moment. Thus, the initial eccentricity is magnified, yielding a larger e/h than initially assumed. The increase in e/h significantly affects the magnitude of EI_e of a column (refer to Eq. (1)).

If one considers only the P_u/P_o value associated with external loading (presumably close to the maximum allowable P_u/P_o value) to calculate the effective EI using Eq. (1), one would disregard the EI reduction due to gradual increases in the e/h value. It may be useful to point out that in any practical column design, a column section is chosen, for which the magnitude of the factored axial load P_u, the factored bending moment M_u, and the corresponding initial eccentricity e ($= M_u/P_u$) are known from analysis. With the known values of P_u/P_o and the initial e/h, and for a particular reinforcement ratio, the effective EI can be found using Eq. (1). The gradual increase in e/h, however, can not be considered in design using the formula for moment magnification: $\delta_{ns} = C_m/(1 - P_u/P_c)$. To account for this effect, a stiffness reduction factor needs to be included in the proposed approximate procedure. The factor is recommended to be 0.75, based on a comparison with test results and in conformity with ACI 318 provisions. It may be noted that the stiffness reduction factor of 0.75, which considers a change in e/h, is not strictly applicable to most columns under double-curvature bending or in cases where the moment magnification factor δ_{ns} is less than 1, as there is no chance of an increase in eccentricity e from the initial value. In other words, when $\delta_{ns} \leq 1.0$, the initial applied moment will not be amplified leading to a change in the initial e ($= M_u/P_u$). Experimental results from frame tests by Breen and Ferguson (1964) on columns in nonsway frames under double-curvature bending confirm this observation (shown later in Table 2).

Only the *P-M* curves (radial lines in Fig. 2, for example) obtained using the approximate (or fixed EI_e) procedure are considered for comparison with the test results and the results obtained following Section 10.12.3 of the ACI 318 slender column provisions. However, a comparison with the results of the more accurate (variable EI) failure analysis (Appendix, Section A) is shown for one set of test data (Fig. 2(c)). Note that the ACI Code procedure (Section 10.12.3) is essentially identical to the approximate procedure, except for the difference in effective EI.

Table 1 (Column 24) shows the effective EI for column sections of the test modules according to ACI 318-02. The larger value calculated using Eq. (10-11) and (10-12) was used for comparison. It is interesting to note that for all the nine columns considered, the ACI effective EI was found to be $0.4E_cI_g$. On the other hand, the predicted EI_e (refer to Column 16 of Table 1), calculated using Eq. (1) (with P_u in Eq. (1) = failure load from test results), was found to be significantly higher (0.6 to $0.9E_cI_g$). Note the eccentricity ratios for different tests (Column 10). Table 1 (Column 22) shows a C_m value of 1.0 for all the columns (single-curvature bending), which occurs in rare practical cases. In addition, δ_{ns} of more than 1.5 (that is, secondary moment larger than 50% of the primary moment) computed for some columns (refer to Column 23) would normally call for redesign.

A detailed discussion of each investigation is given as follows:

Furlong and Ferguson tests—The relevant data for the frames tested by Furlong and Ferguson (1966) are shown in Table 1. The beams of the frames were loaded (refer to Fig. 1) so that the columns would be in single curvature. The following points are noteworthy concerning this investigation:

1. The columns had slenderness ratios (kl_u/r) of 45 to 57. The effective EI by the proposed equation (Eq. (1)) was found to be 0.56 to $0.77E_cI_g$ (Table 1, Column 16) under the maximum test loads;

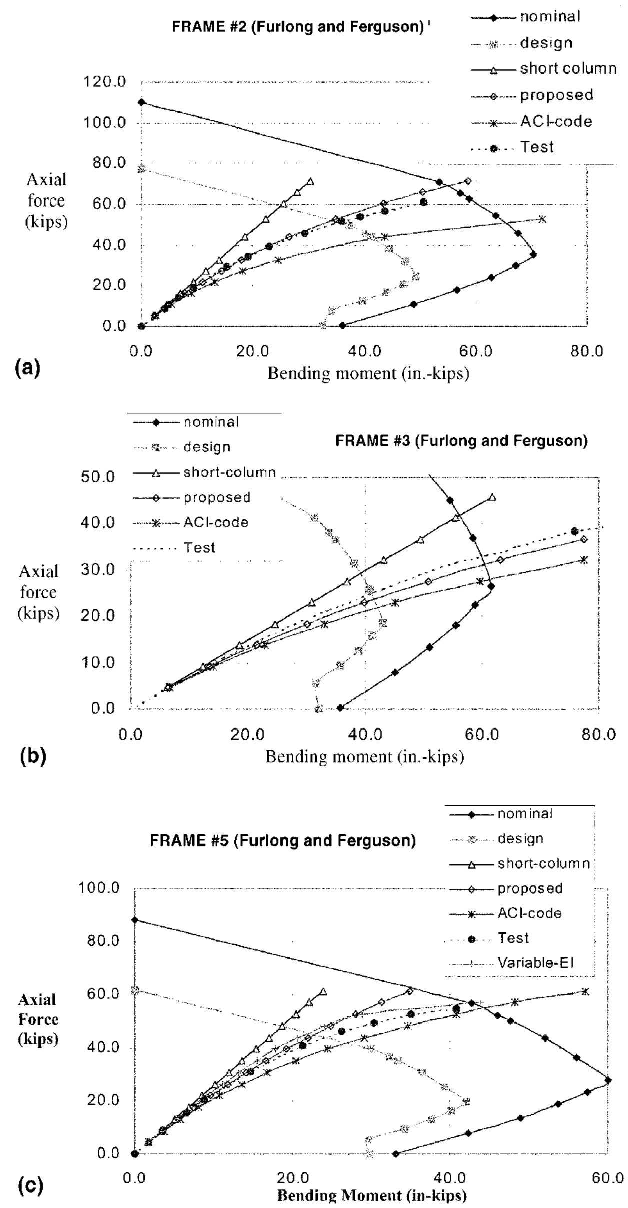

Fig. 2—Comparison of proposed and ACI code methods with test results (Furlong and Ferguson): (a) Frame No. 2; (b) Frame No. 3; and (c) Frame No. 5.

Table 1—Details of experimental program on columns in nonsway frames under single-curvature bending (including some analysis)

Investigator	Specimen no.	f'_c, ksi	f_y, ksi	ρ_g, %	b, in.	h, in.	l_u, in.	e, in.	e/h	P_o, kips	P_u, kips	P_u/P_o, kips	E_c, ksi	I_g, in.^{4}	EI_c/E_cI_g, Eq. (1)	k	P_{cns}, kips	P_u/P_{cns}	l_u/r	kl_u/r	C_m	δ_{ns}	EI_e/E_cI_g, ACI
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
Furlong	2	4.3	54.9	1.8	6	4	80	0.42	0.106	110	61.6	0.56	3738	32	0.77	0.85	196	0.31	67	57	1	1.72	0.4
Furlong	3	3.34	57.2	1.8	6	4	80	1.35	0.338	92	39.7	0.43	3294	32	0.56	0.82	135	0.29	67	55	1	1.65	0.4
Furlong	5	3.24	52.8	1.8	6	4	60	0.39	0.097	88	55.5	0.63	3244	32	0.73	0.9	257	0.22	50	45	1	1.40	0.4
Lloyd	IA	8.4	65	2.15	7	7	66	0.60	0.086	411	329.0	0.80	5224	200	0.69	1	1626	0.20	31	31	1	1.37	0.4
Lloyd	IB	8.4	65	2.15	7	7	66	1.97	0.281	411	185.0	0.45	5224	200	0.66	1	1561	0.12	31	31	1	1.19	0.4
Lloyd	IC	8.4	65	2.15	7	7	66	2.56	0.366	411	147.3	0.36	5224	200	0.61	1	1440	0.10	31	31	1	1.16	0.4
Green	S4	4	60	2	6	4.05	74	0.73	0.180	110	42.0	0.38	3605	33	0.82	1	176	0.24	61	61	1	1.47	0.4
Green	S5	4	60	2	6	4.05	74	0.43	0.106	110	41.5	0.38	3605	33	0.92	1	198	0.21	61	61	1	1.39	0.4
Green	S9	4	60	2	6	4.05	74	1.70	0.420	110	30.0	0.27	3605	33	0.58	1	124	0.24	61	61	1	1.47	0.4

Notes: 1. Column 16 shows value using Eq. (1); 2. Column 24 shows value using ACI Section 10.12.3; 3. Column 18 shows P_{cns} obtained using Eq. (A3); 4. Column 12 shows P_u = P_{test}; 5. Column 23 shows δ_{ns} obtained using Eq. (A7); and 6. Column 22 shows C_m obtained using Eq. (A2).

2. Figure 2 shows the comparison of test results with the *P-M* Curves obtained using the proposed (approximate) procedure (per the Appendix, Section B) and the ACI code procedure. As can be seen, the predictions by the proposed procedure are in good agreement with the test results. The prediction is more conservative for Frame No. 3 (Fig. 2(b)) which had a smaller P_u/P_o (0.43) and a larger e/h (0.338), compared with Frame No. 2 (Fig. 2(a)) and Frame No. 5 (Fig. 2(c)), which had P_u/P_o of more than 0.56 and a lower e/h (0.1). On the other hand, the prediction by the ACI code procedure (Section 10.12.3) is very conservative, mainly because of the smaller EI assumption of $0.4E_cI_g$;

3. The comparison of the variable EI_e procedure (refer to the Appendix, Section A) for Frame No. 5 (Fig. 2(c)) shows that the prediction matches test results more closely near the nominal strength curve. The method, however, is not suitable for use in design offices because of its complexity; and

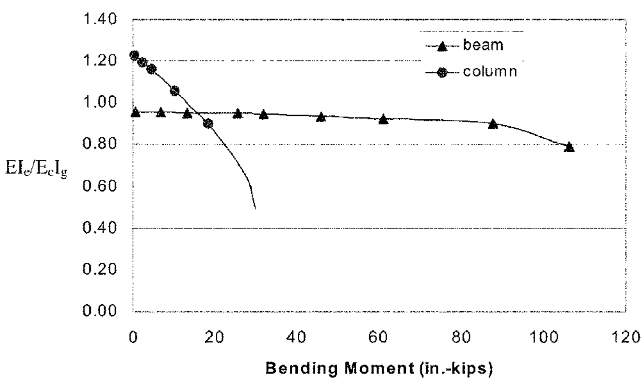

Fig. 3—Variation of effective I *of beams and columns in Frame No. 5 of Furlong and Ferguson.*

4. In the test series, identical I_e for both the beams and the columns was assumed by the investigators (Furlong and Ferguson 1966). This assumption is generally not valid for any practical frame. In addition, for the test series, it was found that the beams contained a very large percentage of tensile reinforcement (more than 4%), compared with the gross column reinforcement ratio of only 1.8%. Analysis of Frame No. 5 shows that the effective EI of the beam section stays approximately at $0.9E_cI_g$, whereas the effective EI of the column section gradually decreases with increasing bending moment (Fig. 3). This is mainly due to the higher strains in the concrete on the compression side, which typically exceed 0.0015, leading to a reduction in E_c (Khuntia and Ghosh 2004). This type of reduction typically is not expected in practical columns where compression strains in the concrete typically do not go beyond 0.0015 (Khuntia and Ghosh 2004). The initial assumption of identical EI_e for both beams and columns by the investigators, however, may be reasonable for Frame No. 5 (because of the very high percentage of reinforcement in beams). Analysis also shows that the frame has a strong beam-weak column configuration. Therefore, the column ends would yield before the beam ends reach their yield strengths. This would not be the case with any column designed in accordance with the strong column-weak beam concept. In other words, the behavior of this frame at higher loads may not be truly representative of practical design.

Green tests—The relevant data for the hinged columns tested by Green (1966) are shown in Table 1. The series of tests was conducted under sustained loading at a certain percentage of P_o. The effects of sustained loading are not

Table 2—Details of experimental program on columns in nonsway frames under double-curvature bending (including some analysis)

Investigator	Specimen no.	f'_c, ksi	f_y, ksi	ρ_g, %	b, in.	h, in.	l_u, in.	e, in.	e/h	P_o, kips	P_{TEST}, kips	$P_{ANALYSIS}$, kips	$P_{TEST}/P_{ANALYSIS}$	End conditions
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
MacGregor and Barter	A1	4.88	44	4	4.4	2.5	67	0.50	0.200	63	37.95	41.7	0.91	Hinged
MacGregor and Barter	A2	4.74	44	4	4.4	2.5	67	0.50	0.200	62	38.00	41.0	0.93	Hinged
MacGregor and Barter	C1	3.84	44	4	4.4	2.5	67	0.50	0.200	54	38.01	35.6	1.07	Restrained
MacGregor and Barter	C2	4.41	44	4	4.4	2.5	67	0.50	0.200	59	39.69	38.9	1.02	Restrained
Breen and Ferguson	F1	4.05	53	1.8	6	4	120	1.20	0.300	104	59.00	48.2	1.22	Restrained
Breen and Ferguson	F2	3.04	52.1	1.8	6	4	120	0.40	0.100	83	59.00	68.7	0.86	Restrained
Breen and Ferguson	F3	3.88	52.8	1.8	6	4	60	1.20	0.300	100	61.00	46.4	1.31	Restrained
Breen and Ferguson	F4	3.26	52.3	1.8	6	4	60	0.40	0.100	88	83.50	72.6	1.15	Restrained

Table 2 (cont.)—Details of experimental program on columns in nonsway frames under double-curvature bending (including some analysis)

Investigator	Specimen no.	E_c, ksi	I_g, in.^{4}	P_{TEST}/P_o	EI_e/E_cI_g, Eq. (1)	k_{ns}	p_{cns}, kips	P_{TEST}/P_{cns}, (proposed)	l_u/r	$k_{ns}l_u/r$	C_m	δ_{ns}	EI_e/E_cI_g, ACI	P_{TEST}/P_{cns}, ACI
		(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)
MacGregor and Barter	A1	3982	5.73	0.60	0.90	1	45	0.84	89	89	0.20	1.27	0.56	1.36
MacGregor and Barter	A2	3924	5.73	0.61	0.89	1	44	0.87	89	89	0.20	1.50	0.56	1.37
MacGregor and Barter	C1	3532	5.73	0.71	0.80	0.85	49	0.77	89	76	0.20	1.00	0.60	1.02
MacGregor and Barter	C2	3785	5.73	0.67	0.83	0.85	55	0.72	89	76	0.20	1.00	0.58	1.04
Breen and Ferguson	F1	3627	32.00	0.57	0.52	0.8	65	0.91	100	80	0.47	5.38	0.40	1.19
Breen and Ferguson	F2	3143	32.00	0.71	0.68	0.8	73	0.80	100	80	0.47	2.39	0.40	1.37
Breen and Ferguson	F3	3548	32.00	0.61	0.50	0.9	190	0.32	50	45	0.47	1.00	0.40	0.40
Breen and Ferguson	F4	3254	32.00	0.95	0.53	0.9	187	0.45	50	45	0.47	1.00	0.40	0.59

discussed here. Therefore, the loading history up to the time of application of sustained loading is considered for comparison. The following points are noteworthy concerning these test data:

1. The columns had a slenderness ratio (kl_u/r) of 61 and a ρ_g of approximately 2%. The effective EI by the proposed Eq. (1) was found to be 0.58 to $0.92E_cI_g$ under the applied sustained loading (Table 1, Column 16);

2. Figure 4 shows a comparison of Green's test results with the P-M curves obtained using the proposed and the ACI code procedures. As can be seen, the prediction by the proposed method up to the time of application of the sustained loading is in very good agreement with the test results. On the other hand, the prediction by the ACI code method is very conservative, mainly because of the smaller EI_e assumption; and

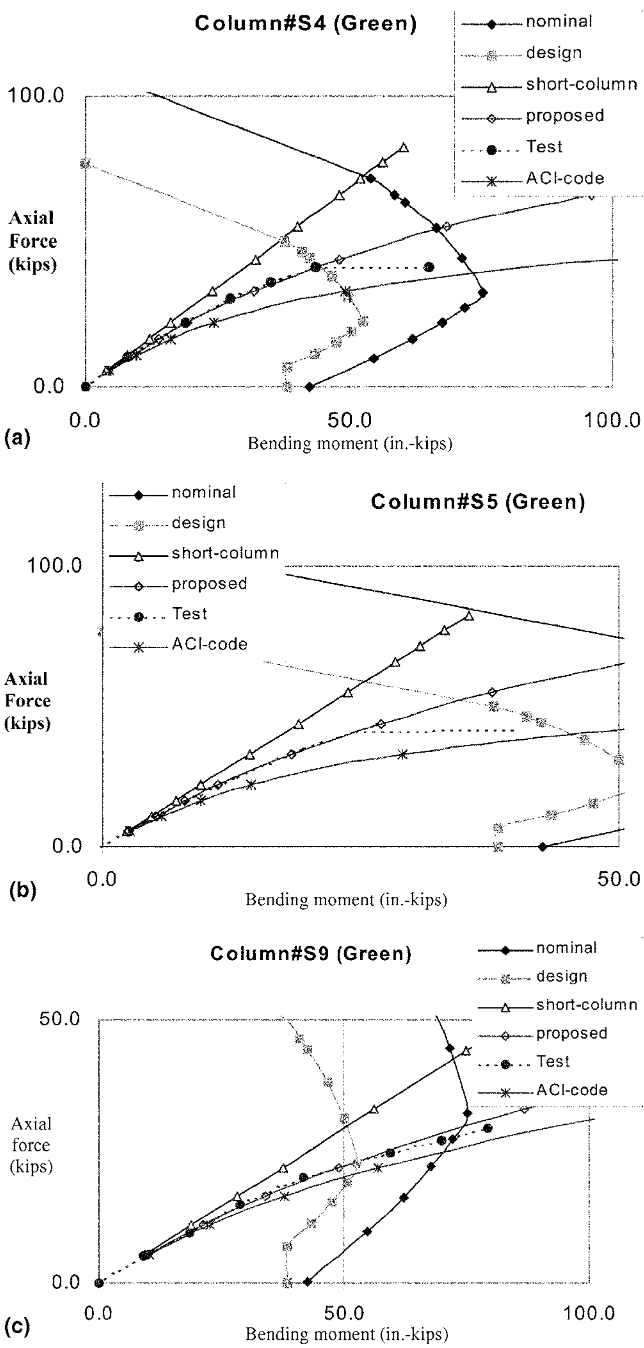

Fig. 4—Comparison of proposed and ACI code methods with Green's test: (a) Column S4; (b) Column S5; and (c) Column S9.

3. The columns had larger cross sections at the ends (for a length of about $l_u/7$ at each end). This might have resulted in slightly larger EI for the entire column. This did not, however, appreciably change the stress conditions at the midheight of the column and was ignored.

Lloyd and Rangan tests—The relevant data for the hinged columns tested by Lloyd and Rangan (1996) are shown in Table 1. The following points are noteworthy concerning these test data:

1. All the columns were made of high-strength concrete ($f_c' > 8000$ psi);

2. The columns had a slenderness ratio (kl_u/r) of 31 and a ρ_g of 2.15%. The effective EI by the proposed Eq. (1) was found to be 0.61 to $0.69E_cI_g$ (Table 1, Column 16) under the maximum test loads; and

3. Figure 5 shows the comparison of Lloyd and Rangan's test results with the P-M curves obtained using the proposed and the ACI code procedures. As can be seen, the predictions by the proposed procedure are in very good agreement with test results. On the other hand, the prediction by the ACI code procedure is very conservative, mainly because of the smaller EI_e assumption.

Columns in nonsway frames under double-curvature bending

When columns in nonsway frames are in double-curvature bending, the slenderness effect is rather insignificant unless the member is extremely slender (refer to Eq. (A8) for δ_{ns} in the Appendix). This is because the magnitude of C_m is quite low (less than 0.6) for columns under double-curvature bending. For a C_m of 0.6, the value of δ_{ns} will exceed 1.0 only in cases where P_u/P_{cns} exceeds 0.4, which may occur in rare instances. It may be emphasized that the stiffness reduction factor of 0.75 is applicable only for columns where δ_{ns} is greater than 1. Note that the effective EI is calculated using Eq. (1), which includes the factored axial load and the factored bending moment (and the corresponding M_u/P_uh or e/h). The e/h would increase when M_u is magnified, that is, when δ_{ns} is greater than 1.0. Therefore, for columns in nonsway frames under double-curvature bending, it is not necessary to include the stiffness reduction factor of 0.75 (when using Eq. (A8)) with P_{cns}, as the initial eccentricity is unlikely to increase (because $\delta_{ns} \leq 1.0$). In summary, the strength of a column under double-curvature bending is unlikely to decrease due to slenderness effects. It is interesting to note that most practical columns are under double-curvature bending. There are a few investigations available that include tests on columns under double-curvature bending. Two of these investigations (MacGregor and Barter 1966; Breen and Ferguson 1964) are considered here. Figure 6 shows the typical test setups for those two investigations. Table 2 shows relevant geometric and material properties of the test specimens, including important analytical results.

MacGregor and Barter tests—The relevant data for four columns (out of eight) tested by MacGregor and Barter (1966) are shown in Table 2. The remaining four columns have e/h ratios of 1.5, which is not encountered in any practical columns. As explained earlier, such a high value of e/h would allow a member to be treated as a beam.

The following points are noteworthy concerning these test data:

1. The columns had slenderness ratios (kl_u/r) of 76 to 89 and a ρ_g of 4%. The effective EI by the proposed equation (Eq. (1)) was found to range between 0.8 and $0.9E_cI_g$ under the maximum test loads (Table 2, Column 19);

2. The columns were in double curvature, with equal end eccentricities. Therefore, the end-moment ratio for these columns is equal to –1.0, giving C_m a value of 0.2 (Table 2) to be used in the computation of δ_{ns} (in the Appendix, Section B). Note that the C_m would be 0.4 per ACI 318 (the specified minimum); per AISC-LRFD (AISC 1993) and Eq. (A2), it would be 0.2;

3. Table 2 shows the comparison of test results with short-column strength. The analytical value of P ($P_{ANALYSIS}$) is obtained from the P-M nominal strength interaction diagram for short columns. As can be seen, there is a marginal reduction in strength for hinged columns and no reduction for restrained columns. A careful review of Table 2 (see Column 14 for $P_{TEST}/P_{ANALYSIS}$ and Column 26 for δ_{ns}) shows that the test results for hinged columns (Specimens A1 and A2, refer to Column 2) are significantly higher than the strengths predicted by using the moment magnifier δ_{ns}; and

4. The predictions by the proposed method are in good agreement with the test results. It may be noted that the computation of δ_{ns} was done without using the reduction factor of 0.75 (Eq. (A8) of the Appendix). On the other hand, the prediction by the ACI code is very conservative, mainly because of the smaller EI_e assumption. Note the magnitudes of P_{cns} calculated using the effective EI recommended in the ACI code (Column 27 of Table 2), which are substantially less than the corresponding values given by the proposed procedure (Column 19 of Table 2). The values are quite small in comparison with the test results. It should be emphasized that the theoretical value of P_u/P_{cns} may not exceed 1.0, as it would mean stability failure (refer to Eq. (A8) for δ_{ns} in the Appendix). As can be seen (Table 2, Column 28), based on ACI stiffness assumptions and the corresponding P_{cns}, the columns should have had instability failure long before reaching P_{TEST}.

Breen and Ferguson tests—The relevant data for four columns (out of six) tested by Breen and Ferguson (1964) are shown in Table 2. The columns were part of frames that were tested. The beams of the frames were loaded (Fig. 6) such that the columns were in double curvature, with end eccentricities at one end considerably smaller than those at the other end.

The following points are noteworthy concerning these test data:

1. The columns had slenderness ratios (kl_u/r) of 45 to 80 and ρ_g of 1.8%. The effective EI by the proposed Eq. (1) was found to be 0.50 to 0.68 E_cI_g (Table 2, Column 19) under the maximum test loads;

2. As the end-moment ratio for these columns is approximately equal to –1/3, the value of C_m to be used in the computation of δ_{ns} (Appendix, Section B) is 0.47 (Table 2, Column 25);

3. Table 2 shows the comparison of test results with short column strength. As can be seen (Column 14), there is a 14% reduction in strength for only one column (Specimen F2) and no reduction for the other columns; and

4. The predictions by the proposed method (using Eq. (A8) for δ_{ns}) are on the conservative side for all four tests. Compare the $P_{TEST}/P_{ANALYSIS}$ (Column 14) and δ_{ns} (Column 26) for the four frames in Table 2. A close look at these data for Frame F2 shows that the predicted moment magnification for the column is 2.39 (significantly higher than a practical value), whereas the reduction in strength is only 14%. On the other hand, the prediction by the ACI code procedure is extremely conservative, mainly because of the smaller EI_e assumption. Note that using ACI values would indicate stability failure for the columns of Frames F1 and F2 at $P_u = P_{TEST}$ (refer to Table 2, Column 28). When $P_u > P_{cns}$, stability

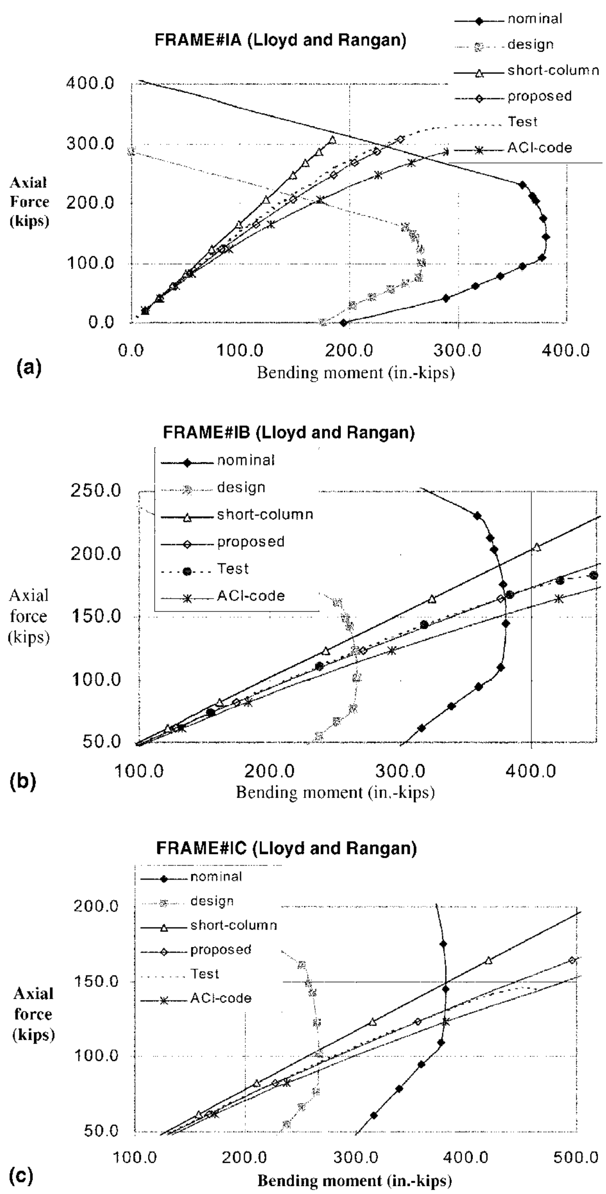

Fig. 5—Comparison of proposed and ACI code methods with test results (Lloyd and Rangan): (a) Column IA; (b) Column IB; and (c) Column IC.

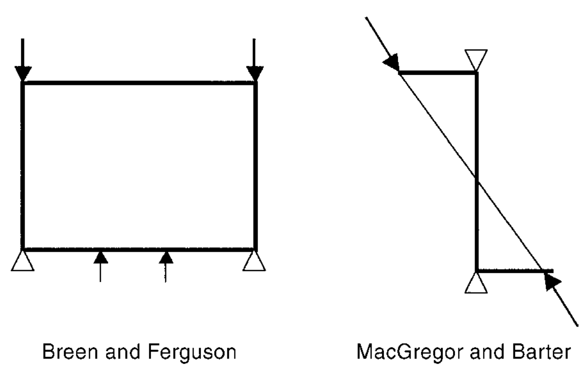

Fig. 6—Schematic diagram of loading arrangement by different investigators for columns in nonsway frames under double-curvature bending.

failure will occur or δ_{ns} will be negative (refer to Eq. (A7) or (A8) of the Appendix).

In general, it can be concluded that slenderness effects can be neglected for columns in nonsway frames (except only for extremely slender columns) under double-curvature bending.

Columns in sway frames or columns under lateral loading

In sway frames, the magnitude of bending moment at a column end and the corresponding *e*/*h* ratio are relatively high, compared with those in nonsway frames. Therefore, the effective *EI* of columns in sway frames is expected to be low. There is only one experimental investigation available on slender columns in sway frames. An additional investigation on columns under lateral seismic loading, however, is also considered for comparison, as their behavior is similar to that of columns in sway frames. Note that in the case of a sway frame, the moment magnification factor δ_s (not δ_{ns}, which was used for columns in nonsway frames) is given by

$$\delta_s = \frac{1}{1 - \Sigma P_u / 0.75 \Sigma P_c} \quad \text{(ACI 318 Section 10.13.4.3)}$$

Note that the factor δ_s accounts for P-Δ effects for an entire story, whereas δ_{ns} accounts for P-δ effects for individual columns. In addition, it is important to note that the stiffness assumption of ACI 318 Section 10.12.3 is considered for the computation of magnified moments in sway frames in Section 10.13.4.3, which is widely used in practice.

Ferguson-Breen tests—The relevant data for three frames (out of a total of seven) tested by Ferguson and Breen (1966) are shown in Table 3. Figure 7 shows a schematic diagram of the test setup. The columns were in double curvature under lateral loading.

The following points are noteworthy concerning these test data:

1. The columns had very high slenderness ratios (kl_u/r) of 89 to 104 and a gross reinforcement ratio ρ_g of 1.95%. The effective *EI* by the proposed Eq. (1) was found to be 0.76 to $0.94E_cI_g$ (Table 3, Column 18) under the maximum test loads;

2. Figure 8 shows a comparison of Ferguson and Breen's test results with the *P-M* curves obtained using the proposed and the ACI code procedures. As can be seen, the predictions by the proposed procedure are in good agreement with the test results and are generally on the conservative side. On the other hand, the predictions by the ACI code procedure are very conservative, mainly because of the smaller EI_e values assumed. In fact, Table 3 (Column 26) shows that the values of $0.75P_{cs}$ (using the ACI stiffness assumption) for columns in two of the three frames are less than the test results, indicating stability failure (refer to values of δ_s-ACI in Column 28 of Table 3, and also Eq. (A9)); and

3. The analyses also show that the frames have a strong beam-weak column configuration. Therefore, the column ends would yield even before the beam ends reach their yield strengths. This is not the case with any column designed in accordance with the strong column-weak beam concept.

Watson and Park tests—The relevant data for nine square columns tested by Watson and Park (1994) are shown in Table 3.

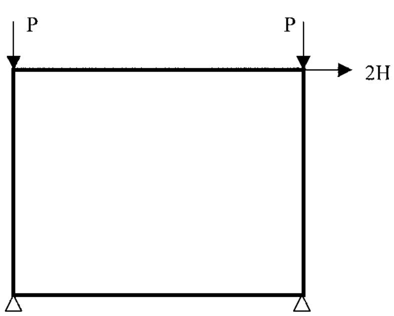

Fig. 7—Schematic representation of test frame (Ferguson-Breen 1966).

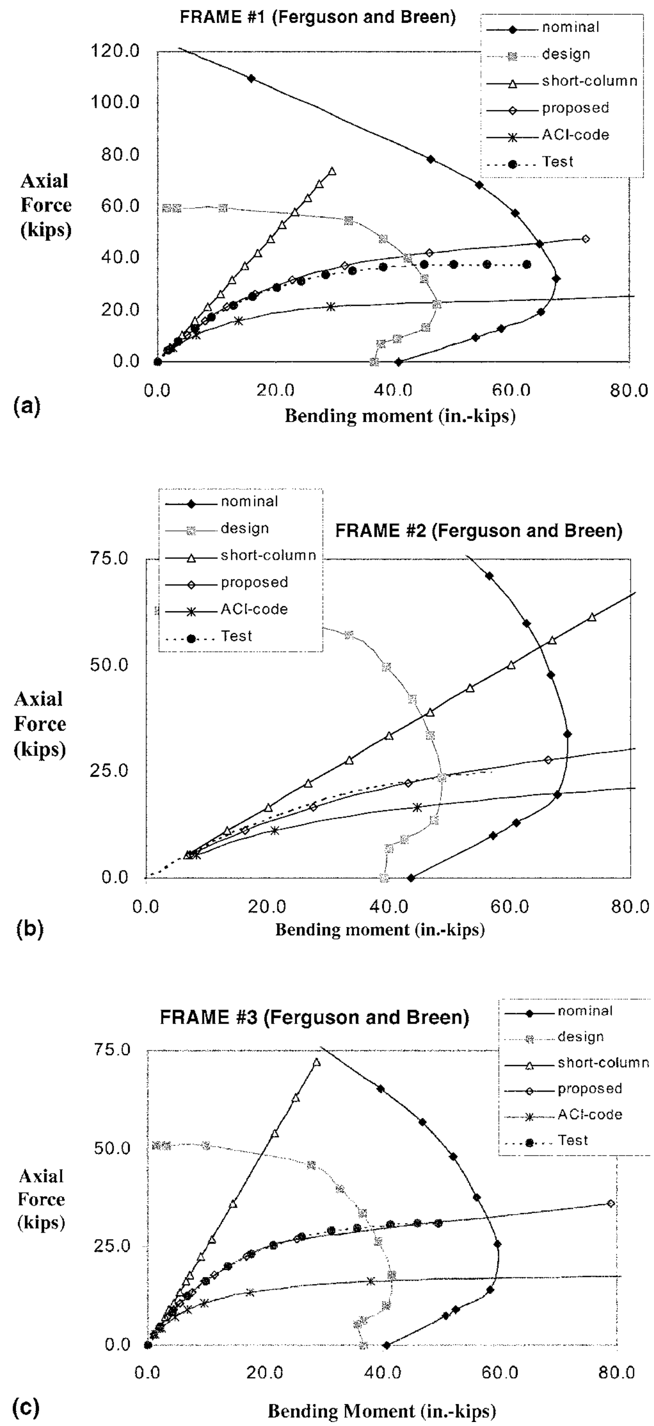

Fig. 8—Comparison of proposed and ACI code methods with test results for columns in sway frame (Ferguson and Breen): (a) Frame 1; (b) Frame 2; and (c) Frame 3.

Figure 9 shows a schematic diagram of the test setup. The columns were in single curvature under lateral loading.

The following points are noteworthy concerning these test data:

1. The columns had a slenderness ratio (kl_u/r) of about 32 and a gross reinforcement ratio ρ_g of 1.51%. The effective EI by the proposed Eq. (1) was found to be 0.29 to 0.59 E_cI_g (Table 3, Column 18) under the maximum test loads;

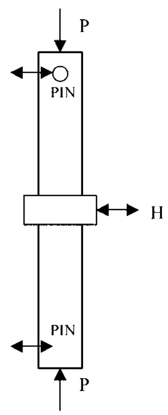

Fig. 9—Schematic representation of test frame (Watson and Park 1994).

2. Table 3 shows the comparison of Watson and Park's test results with values computed by the proposed (Appendix, Section B) and the ACI Code (Appendix, Section C) procedures. As can be seen, the predictions by the proposed procedure are in very good agreement with the test results. Note the ratios of $P_{TEST}/P_{ANALYSIS}$ in Table 3 (Column 15) ($P_{ANALYSIS}$ is obtained from the nominal strength diagrams for short columns). Also note the values of δ_s. If δ_s is more than 1.0, it indicates a decrease in load-carrying capacity compared with short column strength. Table 3 shows relatively lower moment magnification (1.07 to 1.25) when the EI_e is computed by the proposed procedure (Eq. (1)). On the other hand, the predictions by the ACI Code provisions are more conservative, mainly because of the smaller EI_e values assumed (Column 25). Note the δ_s values (Column 28) computed using EI_e of the ACI code;

3. The predicted EI_e is found to be on the conservative side of test results (Table 3, Column 18). The experimental EI_e values from Watson and Park's test results have also been reported by Mehanny, Kuramoto, and Deierlein (2001). The mean test/predicted EI_e by using Eq. (1) was calculated to be 1.54. In their paper, Mehanny, Kuramoto, and Deierlein (2001) suggested a simplified expression for the effective EI of beam-columns to be used in frame analysis. For the same sets of test data, the test/predicted ratios of EI_e by Mehanny,

Table 3—Details of experimental program and analysis for columns under lateral loading

Investigator	Specimen no.	f_c', ksi	f_y, ksi	ρ_g, %	b, in.	h, in.	l_u, in.	e, in.	e/h	P_o, kips	P_{TEST}, kips	P_{TEST}/P_o	$P_{ANALYSIS}$, kips	$P_{TEST}/P_{ANALYSIS}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Ferguson and Breen	1	4	55	1.95	6	4	80	0.4	0.10	106	37.5	0.35	Refer to Fig. 8	
Ferguson and Breen	2	4.2	59	1.95	6	4	80	1.2	0.30	112	25.0	0.22	Refer to Fig. 8	
Ferguson and Breen	3	3.2	56	1.95	6	4	80	0.4	0.10	90	31.0	0.34	Refer to Fig. 8	
Watson and Park	1	6.8	65	1.51	16	16	142	16.0	1.00	1711	169.1	0.10	160	1.06
Watson and Park	2	6.4	65	1.51	16	16	142	7.7	0.48	1618	474.8	0.29	450	1.06
Watson and Park	3	6.4	65	1.51	16	16	142	7.7	0.48	1618	474.8	0.29	450	1.06
Watson and Park	4	5.8	65	1.51	16	16	142	8.0	0.50	1493	431.7	0.29	405	1.07
Watson and Park	5	5.9	69	1.51	16	16	142	4.6	0.29	1540	737.4	0.48	745	0.99
Watson and Park	6	5.8	69	1.51	16	16	142	4.7	0.29	1509	719.4	0.48	735	0.98
Watson and Park	7	6.1	69	1.51	16	16	142	2.6	0.16	1571	1057.6	0.67	1120	0.94
Watson and Park	8	5.7	69	1.51	16	16	142	2.7	0.17	1478	982.0	0.66	1030	0.95
Watson and Park	9	5.8	69	1.51	16	16	142	2.7	0.17	1509	1007.2	0.67	1040	0.97

Table 3 (cont.)—Details of experimental program and analysis for columns under lateral loading

Investigator	Specimen no.	E_c, ksi	I_g, in.^{4}	EI_e/E_cI_g, Eq. (1)	k_s	P_{cs}, kips	P_{TEST}/P_{cs}	l_u/r	kl_u/r	δ_s	EI_e/E_cI_g, ACI	ACI-P_{cs}	H/P	δ_s-ACI*
		(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)
Ferguson and Breen	1	3605	32	0.93	1.34	92	0.41	67	89	2.19	0.4	40	0.01	–3.80
Ferguson and Breen	2	3694	32	0.76	1.34	77	0.33	67	89	1.77	0.4	41	0.03	5.61
Ferguson and Breen	3	3224	32	0.94	1.56	61	0.51	67	104	3.08	0.4	26	0.01	–1.72
Watson and Park	1	4706	5461	0.29	1	3629	0.05	30	30	1.07	0.4	5027	0.45	1.05
Watson and Park	2	4553	5461	0.44	1	5353	0.09	30	30	1.13	0.4	4864	0.22	1.15
Watson and Park	3	4553	5461	0.44	1	5336	0.09	30	30	1.13	0.4	4864	0.22	1.15
Watson and Park	4	4341	5461	0.42	1	4871	0.09	30	30	1.13	0.4	4637	0.22	1.14
Watson and Park	5	4395	5461	0.55	1	6499	0.11	30	30	1.18	0.4	4695	0.13	1.26
Watson and Park	6	4341	5461	0.55	1	6387	0.11	30	30	1.18	0.4	4637	0.13	1.26
Watson and Park	7	4449	5461	0.59	1	7021	0.15	30	30	1.25	0.4	4752	0.07	1.42
Watson and Park	8	4287	5461	0.59	1	6741	0.15	30	30	1.24	0.4	4579	0.08	1.40
Watson and Park	9	4341	5461	0.59	1	6826	0.15	30	30	1.24	0.4	4637	0.07	1.41

*Negative value indicates that $P_u > 0.75P_{cs}$ (using ACI Eq. (10-18)).

Kuramoto, and Deierlein, CEB (European model) and AIJ (Japanese model) are found to be 1.06, 1.37, and 2.31, respectively (Mehanny, Kuramoto, and Deierlein 2001). It may be noted that the expression proposed by Mehanny, Kuramoto, and Deierlein (2001) involves calculation of transformed moment of inertia (including the contribution of reinforcement) as well as the balanced axial load, instead of just the gross moment of inertia I_g and the axial load strength at zero eccentricity P_o, as proposed in this paper. Although a few test data are not sufficient for any definite conclusion regarding the accuracy of various proposals, the previous comparison shows that the proposed method (Eq. (1)) gives a conservative estimate of the EI_e of beam columns, but not an overly conservative value, as would result from recommendations by ACI 318 in Section 10.12.3.

RECOMMENDATION FOR FRAME ANALYSIS AND CONCLUSIONS

Based on results of the analytical study reported in a companion paper (Khuntia and Ghosh 2004) and their comparisons with the results of existing experimental research reported in this paper, the following recommendations are made concerning the effective EI of beams and columns to be used in the lateral analysis of frames in general and of frames including slender columns, in particular:

1. In frame analysis (both first order and second order elastic), it is recommended to initially assume beam $EI = 0.35E_cI_g$ (which occurs for a beam with ρ of 1% per Eq. (4)) and column $EI = 0.70\ E_cI_g$ (which occurs with ρ_g = 1.5%, e/h = 0.20, and P_u/P_o = 0.40 per Eq. (1)). On the completion of lateral analysis, however, the effective EI for beams and columns needs to be recalculated using Eq. (4) and (1), respectively. Note that depending on the magnitude of e/h (or M_u/P_uh), the EI_e value will change. If the final EI_e values are different from the initially assumed values by more than 15%, it is recommended to perform the analysis again using the revised EI_e;

2. After performing the analysis, the final moments at column ends need to be checked to see whether a column is under single-curvature bending (for nonsway frames only). If any of the columns is in single-curvature bending, the EI_e for that column must be reduced to 0.75 times that originally assumed and a new analysis carried out. This would yield appropriate moment magnification in second order elastic analysis. This would also give appropriate displacements in first order elastic analysis (to be used to calculate stability index Q, if needed, for Section 10.13.4.2 of ACI 318). For first order elastic analysis in compliance with Section 10.12.3 or 10.13.4.3, however, the bending moments need to be magnified by δ_{ns} or δ_s, using appropriate equations (including the 0.75 factor with P_c [P_{cns} or P_{cs}] for columns in single-curvature bending);

3. The factor of 0.75 (in the calculation of δ_{ns} per Eq. (A7)) is to be considered for columns in nonsway frames only in cases of single-curvature bending, as the initial eccentricity from elastic analysis ($e = M_u/P_u$) will gradually increase for those cases. Theoretically, e will increase only when δ_{ns} is more than 1.0, which generally occurs only for columns under single curvature. Note that for sway frames, the factor 0.75 must always be used with P_{cs}, as δ_s is always more than 1.0 (that is, the applied moment is always magnified); and

4. It has been found from analyses that an increase in e/h, although it reduces the effective EI, does not affect the magnification factor (δ_{ns} or δ_s) for slender column moments significantly. Note that

$$\delta_{ns} = \frac{C_m}{1 - P_u/0.75P_c}$$

per the ACI code. It can be shown that the magnitude of P_u/P_c (P_u can be P_{us} or P_{uns} and P_c can be P_{cns} or P_{cs}) is proportional to P_u/EI_e of the column. For a column with high e/h, EI_e decreases significantly. The magnitude of P_u, however, also decreases with an increase in e/h, although not proportionately (Khuntia and Ghosh 2004). Therefore, the increase in moment magnification factor due to a reduction in EI_e (caused by an increase in e/h) is arrested to a large extent by the corresponding reduction in the P_u/P_o ratio (refer to Eq. (1)). In addition, the columns with higher e/h ratios are least affected by sustained loading effects, which are most pronounced at large P_u/P_o ratios.

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NOTATION

A_g	=	gross cross-sectional area, in.^{2}
A_{st}	=	gross steel area in column, in.^{2}
b	=	width of member, in.
b_w	=	width of web of T-beams, in.
C_m	=	factor relating actual moment diagram to equivalent uniform moment diagram
	=	$0.6 + 0.4M_1/M_2$ (M_1/M_2 is positive under single curvature)
c	=	depth of neutral axis, in.
E_c	=	modulus of elasticity of concrete, ksi
EI	=	flexural stiffness of member or cross section, in.^{2}-lb
E_s	=	modulus of elasticity of reinforcing steel, ksi
e	=	eccentricity of axial load, in.
e/h	=	eccentricity ratio = M/Ph
f_c	=	compressive stress in concrete at a strain of ε_c, psi
f'_c	=	specified compressive strength of concrete, psi
f_y	=	yield strength of reinforcements, ksi
H	=	lateral load on sway frame, kips
h	=	overall depth of member, in.
I	=	moment of inertia of cross section, in.^{4}
I_{beam}	=	effective moment of inertia of flexural member, in.^{4}
I_{cr}	=	moment of inertia of cracked cross section of flexural member, calculated using transformed area concept
I_e	=	effective moment of inertia of cross section, in.^{4}
I_g	=	moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement, in.^{4}
I_s	=	moment of inertia of reinforcing steel about centroidal axis, in.^{4}
k	=	effective length factor for compression members
k_{ns}	=	effective length factor for compression members in non-sway frames
k_s	=	effective length factor for compression members in sway frames
l_c	=	length of compression member in a frame, measured from center-to-center of joints in frame, in.
l_u	=	unsupported length of compression member, in.
M	=	bending moment, in.-lb = M_u in the context of strength design
M_1	=	smaller factored non-sway end moment on compression member, in.-lb
M_2	=	larger factored non-sway end moment on compression member, in.-lb
M_{max}	=	maximum moment along length of compression member, in.-lb
M_n	=	nominal flexural strength, in.-lb
M_{ns}	=	moment due to loads that do not cause appreciable side sway (typically gravity loads), in.-lb
M_s	=	moment due to loads that cause appreciable side sway (lateral loads), in.-lb

M_u = factored moment or required moment strength at section, in.-lb
n = modular ratio = E_s/E_c
P = axial load, kips = P_u in context of strength design
$P_{ANALYSIS}$ = theoretical axial load at attainment of nominal strength, kips
P_c = critical load for compression member, kips
P_{cns} = critical load for compression member in nonsway frame, kips
P_{cs} = critical load for compression member in sway frame, kips
P_n = nominal axial load strength, kips
P_o = nominal axial load strength at zero eccentricity, kips
P_{TEST} = axial load at failure/crushing in experimental study, kips
P_u = factored axial load or required axial load strength, kips
P_u/P_o = axial load ratio
r = radius of gyration of cross section of compression member, in.
w_c = unit weight of concrete, lb/ft^3
Δ = story drift, in.
δ = deflection of compression member relative to chord joining ends of column in deflected frame, in.
δ_{ns} = moment magnification factor for columns in nonsway frames
δ_s = moment magnification factor for columns in sway frames
ε_c = compressive strain in concrete, in./in.
ε_{max} = maximum compressive strain in concrete, in./in.
ε_o = compressive strain in concrete at peak stress, in./in.
ε_s = tensile strain in steel, in./in.
ε_y = yield strain in steel, in./in.
ϕ = strength reduction factor
= curvature at section, rad/in.
ρ = tensile reinforcement ratio in flexural member, %
ρ_g = gross reinforcement ratio in compression member, %

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APPENDIX

A. More accurate or variable EI_e method for *P-M* curve of reinforced concrete column

The following steps are recommended:

1. Given the section (b, h), ρ_g, e, f_y, f_c', l_u
2. Calculate $P_o = 0.85f_c'(A_g - A_{st}) + f_y A_{st}$ (A1)
3. Calculate $C_m = 0.6 + 0.4M_1/M_2$ (M_1/M_2 is positive under single-curvature bending) (A2)
4. Assume any initial $P = 0.05P_o$ (for example)
5. Calculate primary bending moment $M = P \times e$
6. Calculate effective EI using Eq.(1) (with known e/h, P/P_o taken as P_u/P_o and ρ_g)
7. Calculate k using a spreadsheet or chart (such as ACI 318 Fig. R10.12.1)
8. Calculate

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} \tag{A3}$$

(Note that $P_c = P_{cns}$ for nonsway and = P_{cs} for sway frames)

9. Calculate moment magnification factor using

$$\delta_{ns} = \frac{C_m}{1 - P_u/P_{cns}}$$

(with P taken equal to P_u) or

$$\delta_s = \frac{1}{1 - P_u/P_s} \tag{A4}$$

10. Calculate magnified moment,
$M_{ns} = \delta_{ns} \times M$, or $M_s = \delta_s \times M$ (A5)
11. Calculate modified eccentricity $e = \delta_{ns}$ (or δ_s) $\times$ previous e (A6)
12. Assume next larger $P = 0.10P_o$ (for example) and repeat Steps 5 to 12 until the nominal strength curve is reached, that is, the column theoretically fails.

B. Approximate or fixed EI_e method for *P-M* curve of reinforced concrete column

The following steps are recommended:

1. Given the section (b, h), ρ_g, f_y, f_c', l_u
2. Calculate $P_o = 0.85 f_c'(A_g - A_{st}) + f_y A_{st}$ (A1)
3. Under any applied loading, the factored axial force on column P_u and the factored bending moment M_u are known. Calculate the eccentricity of axial load, $e = M_u/P_u$.
4. Calculate $C_m = 0.6 + 0.4M_1/M_2$ (M_1/M_2 is positive under single-curvature bending) (A2)
5. Calculate P_u/P_o
6. Calculate effective EI using Eq. (1) (with known e/h, P_u/P_o and ρ_g)
7. Calculate k using a spreadsheet or chart
8. Calculate

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} \tag{A3}$$

9. Assume any initial $P = 0.05P_o$ (for example)
10. Calculate primary moment $M = P \times e$
11. Calculate moment magnification factor as follows:

For columns in nonsway frames under single-curvature bending

$$\delta_{ns} = \frac{C_m}{1 - P/0.75P_{cns}} \tag{A7}$$

For columns in nonsway frames under double-curvature bending

$$\delta_{ns} = \frac{C_m}{1 - P/P_{cns}} \quad \text{(A8)}$$

For columns in sway frames

$$\delta_{ns} = \frac{C_m}{1 - \Sigma P/0.75\Sigma P_{cs}} \quad \text{(A9)}$$

12. Calculate the magnified moment, $M_{ns} = \delta_{ns} \times M$. For columns in sway frames, the magnified moment, $M_s = \delta_s \times M$

13. Assume next larger $P = 0.10P_o$ (for example) and repeat Steps 10 to 13 until the nominal strength curve is reached.

C. ACI code procedure

All the steps mentioned in the approximate procedure are to be followed, except that in Step 6, the effective *EI* is to be computed using ACI 318 Eq. (10-11) or (10-12). Under short-term loading, a β_d of 0 is to be used.

D. Test results

For columns under single-curvature bending, the maximum bending moment is computed as

$$M_{max} = P(e + \delta) \quad \text{(A10)}$$

where P and e are the axial load and the eccentricity at column end, respectively, and δ is the maximum deflection at the midheight of the column (Fig. A), as reported by the investigators. For the test results by Green (1966), the *P-M* values are directly read from the report, as appropriate load-deflection plots were not available.

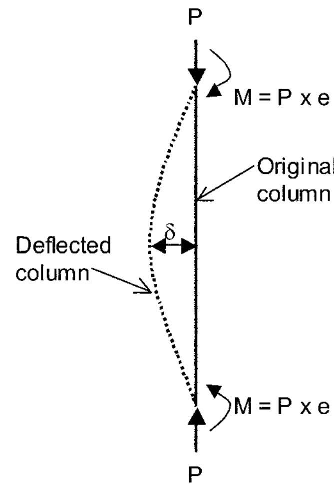

Fig. A—Calculation of maximum moment for column under single-curvature bending.