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# Alternative Methods for Failure Prediction in Twin-Cell Box-Girder Bridges

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*Intense research works on twin-cell box-girder bridges are limited when compared to single-cell box-girder bridges and hence, not many sources are available to study the simultaneous effect of bending and torsion in them. The estimation of ultimate load in a twin-cell box-girder bridge under different modes of failure using the two existing simplified methods—namely, the space truss analogy and collapse mechanism—demands more research attention. The primary objective of this paper is to develop simplified equations for twin-cell box-girder bridges using the principles of collapse mechanism. The second main objective is to check the suitability of using space truss analogy and collapse mechanism in different modes of failure. Experimental work for studying the effects of various structural actions due to an eccentric loading on a simply supported twin-cell concrete box-girder bridge is conducted and numerical analyses are presented to understand the effect of load positions and reinforcement ratios in the failure modes.*

**Keywords:** collapse mechanism; failure modes; space truss analogy; twin-cell box-girder bridges.

## INTRODUCTION

Box-girder bridges are considered to be one among the finest choices when designing long-span bridges. Due to their structural efficiency in handling torsion, as well as economic and aesthetic reasons, they have become very popular in the highway bridge design industry. These structures are thin-walled and hence have very peculiar stress and deformation patterns under the effects of torsion and distortion. To know more about the structural actions and reactions in box-girder bridges, various research is being conducted all over the world. As traffic congestion increases day by day, there comes high demand on larger carriageway width, which can be accomplished by using multi-cell box cross sections. As the number of cells increases, however, the risks involved in its construction also shoot up. Considering all these aspects, twin-cell box-girder bridges are considered an ingenious solution in the design of long-span bridges with larger carriage way width. Hence, studies on such structures are very essential. The complete behavior of a structure can be analyzed only by conducting nonlinear analysis. This can be achieved either by conducting experimental analysis or using three-dimensional (3-D) finite element analysis (FEA). Even though the results of these analyses are realistic and accurate to a certain extent, they are time consuming and expensive. As a bridge designer is always interested in the ultimate load of the structure, there is always a need to understand simplified methods used in estimating the ultimate load of a structure. A thorough knowledge on the existing simplified methods like the space truss analogy and the collapse mechanism may avoid the risks involved in conducting experimental and 3-D finite element studies. A brief review on the various investigations conducted in the area of simplified methods used in estimating the ultimate load of a structure is also included in this paper.

## RESEARCH SIGNIFANCE

Simplified methods are largely used in estimating the failure load to avoid the rigorous 3-D FEA. It is found that simplified equations based on collapse mechanism are not available in the case of twin-cell box-girder bridges. Hence, the availability of equations to find the capacity of twin-cell box-girder bridges will be a major breakthrough in this area. Moreover, studies are conducted on finding suitability of space truss analogy and collapse mechanism in different modes of failure. This helps with identifying the best method that can provide safe results while estimating the collapse load in different failure mechanisms.

### Brief literature review

The space truss analogy constitutes a landmark in the research on torsion in reinforced concrete structures. The truss theory was first postulated by Ritter (1899) with parallel tension and compression chords inclined at 45 degrees to depict the behavior of a simply supported prismatic reinforced concrete beam. Similar to that of shear, the truss theory for torsion was developed by Rausch (1929), where the reinforced concrete member is assumed to act like a tube and torsion is resisted by a circulatory shear flow in the walls of the tube. This thin-walled tube comprises the longitudinal and transverse reinforcement along with the surrounding layer of concrete, which becomes fully effective in the post-cracking phase. Evans and Sarkar (1965) assumed in their work that all the reinforcements passing through the failure surface reach their yield value. Lampert and Thürlimann (1968) and Hsu (1968) established the difference in the pre- and post-cracking stages in reinforced concrete members subjected to torsion. The test results proved that the cracking torque is less in hollow sections when compared to equivalent solid sections, thus establishing the contribution of concrete core in handling cracking torque. But this difference was not observed in ultimate torque. The reason behind this scenario

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was explained as owing to the elongation of stirrups in the reinforced concrete member. After cracking, the concrete core no longer contributes to the torsional forces; instead, a thin layer of concrete surrounding the reinforcement may remain active in resisting the torsional moment. The present form of the space truss model used to estimate the capacity of a box girder was proposed by Lampert and Thürlimann (1971). They proposed a failure model based on the plastic theory of concrete. By using the variable angle truss model, the combined effects of bending and torsion can be brought out more effectively in the truss model. This angle is based on the ratio of effective proportions of longitudinal and transverse reinforcements. The researchers Kuyt (1971) and Karlsson and Elfgren (1972) compared the results of space truss analogy with the existing theoretical and experimental results. This was done in case a reinforced concrete beam was subjected to torsion, thereby confirming the reliability of this method. This method is now the basis for many codes all over the world, including several European codes and the CEB-FIP model code. Strut-and-tie models used in the design of regions where there exist no standard design recommendations are more generalized versions of the truss analogy. It was Park and Paulay (1975) who extended many of the analytical and design concepts used in truss analogy to develop strut-and-tie models for the design of both B and D regions in a structure.

By the mid-twentieth century, extensive research works commenced in the field of box-girder bridges to understand the non-linear behavior of deformable reinforced concrete box section. Until then, the design of box-girder bridges was done using linear elastic analysis of simplified models. The post-cracking behavior of box-girder bridges was first studied by Spence and Morley (1975) to introduce certain theoretical formulations to obtain the ultimate load in the case of a simply supported box-girder bridge subjected to eccentric load. In this theory, collapse mechanism principles were used to estimate the collapse load in a structure. Two such mechanisms were established by Spence and Morley (1975) in the case of box girders. Collapse mechanism principles use the upper-bound theorem based on the plastic theory of structures. Here, the principle of virtual work is used to estimate the collapse load of a structure. As the work equations suggested by Spence and Morley (1975) did not accommodate the distortional deformability, the collapse load obtained from the work equations was erroneous. Danesi and Edwards (1983) conducted experimental studies and concluded that both thickness and reinforcement ratio influence the collapse load in box sections. Contrary to the assumption made by Spence and Morley (1975), Rasmussen and Baker (1999) found that the length of plastic hinge does not extend throughout the length of the box girder in case of distortion-bending collapse mechanism. They also suggested that the length of plastic hinge in a structure such as a box girder is greatly influenced by the ratio of longitudinal and transverse reinforcement. Kurian and Menon (2007) suggested a remedy to this situation by using modified plastic hinge length in the work equations based on the then-available experimental results of box-girder bridges. They suggested that the ratio of the area of reinforcement provided to that required in a unit length possess an empirical relationship with the ratio of modified plastic hinge length to the total length of girder. This modified plastic hinge length was used to revise the work equation for distortion bending collapse mechanism. With the modified plastic hinge length, the work equation for distortion bending collapse mechanism was revised and applied to get more accurate results. With the beginning of twentieth century, revolutionary developments had happened in the field of computer science, thus making 3-D FEA much simpler and less time consuming. However, even with this fast development in the computer industry, researchers like El-Sheikh (1996) suggested the need for development in approximate analysis methods, especially for preliminary designs and redesign. Hence, the development of simplified models for the analysis and design of various structures are inevitable in the field of structural engineering. Sennah and Kennedy (2002) suggested that, except for 3-D FEA, all the other simplified methods have limitations in their scope and applicability. Hence, more research work is required in this field to understand the suitability of these methods for various failure modes in case of twin-cell box-girder bridges.

## Present study

Knittel and Worsh (1965) resolved a concentrated load acting at the midspan of a box girder into its symmetric and asymmetric components. Kupfer (1969) proved that the asymmetric component can be again resolved into its torsion and distortion components. These components act in the horizontal and vertical directions of the plane of plate elements in a box-girder bridge. The effect of these force components differ in various situations based on the predominance of certain forces. Both individual action and the combination of these actions lead to different failure modes of the specimen. The two major failure patterns identified in this study in the case of a twin-cell box-girder bridge are:

1. Pure bending collapse mechanism
2. Distortion-bending collapse mechanism

In pure bending collapse mechanism, the symmetric component of loading (bending component) acts through the plane of web element. In case of distortion-bending collapse mechanism, the anti-symmetrical component of loading causes the cross section of the box to distort along with bending. The details of these two mechanisms are provided along with simplified equations to estimate the collapse load in the later sections of this manuscript.

From the available literatures, it was observed that works concentrating on this area of simplified methods are mainly on single-cell box-girder bridges. Hence, such works on twin-cell box-girder bridges are necessary. In this paper, a detailed study on the behavior of twin-cell box-girder bridges subjected to the combined action of bending and torsion is provided. This experimental study is conducted on a scaled down model, the results of which are used in the validation of numerical analysis. Simplified methods are used to predict the capacity of the structure in different failure modes and the results are compared with the experimental studies and numerical studies.

## EXPERIMENTAL INVESTIGATION

The experimental data aimed at understanding the realistic behavior of twin-cell box-girder bridges, available in literature, are found to be inadequate. Hence, in this paper, a detailed report on the experimental study conducted on a scaled-down model of a twin-cell box girder is presented. Due to the rising demand in decreasing the self-weight of bridges, thin-walled cross sections are essential while fixing the box dimensions. To make the cross section thin-walled, the dimensions are selected as per Vlasov's thin-walled criterion (Maisel and Roll 1974).

Here, a twin-cell box-girder bridge with end diaphragms and constant cross section simply supported at two ends was tested to collapse under eccentric loading. The dimensions of the twin-cell box section were fixed approximately to a scale of 1:10 in relation to a box-girder bridge prototype. The cross-sectional and longitudinal dimensions are provided in Fig. 1(a) and (b). The span-depth ratio adopted for the bridge model is 12, with a depth of 0.25 m (9.84 in.) and span of 3 m (118.11 in.), as the usual span-depth ratio adopted in box-girder bridges lies in the range of 12 to 30. A thickness of 60 mm (2.36 in.), which is the smallest possible thickness that can be adopted to accommodate two layers of 6 mm (0.236 in.) stirrup reinforcement, with 10 mm (0.394 in.) cover is used throughout the structure. A flange overhang length of 0.2 m (7.87 in.) is provided, as the usual practice is to provide a maximum of 0.45 times the distance between webs as the overhang length. The ratio of wall thickness to flange width and the ratio of depth to length of the specimen are 0.0984 and 0.083, confirming to Vlasov's criterion. End diaphragms of thickness 60 mm (2.36 in.) are provided at the two supports.

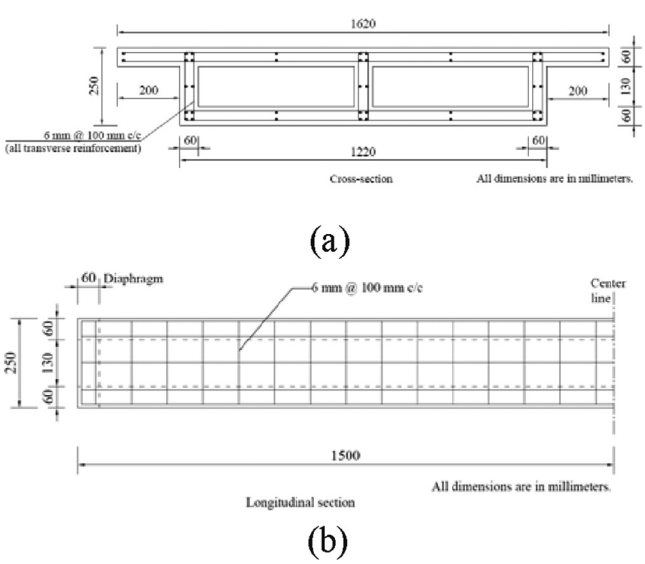



*Fig. 1—Reinforcement details of twin-cell box girder (half span).*

To prepare the reinforcement cage, steel rods of 6 mm (0.236 in.) diameter conforming to IS 1786 with yield stress 562 MPa (81.51 ksi) and ultimate stress 678 MPa (98.34 ksi) are used. The reinforcement details are clearly shown in Fig. 1(a) and (b). The mold used in casting, reinforcement cage, and the casting of the twin-cell are shown in Fig. 2(a), (b), and (c).

The fabrication of specimen, cement, fine aggregate, coarse aggregate, and water are mixed in the ratio 1:1.87:2.17:0.36 to prepare Grade M40 concrete. The mixture was achieved after doing mixture design as per IS 10262-2019. The results of the companion cube specimens cast to find the characteristic compressive strength is provided in Table 1. Portland pozzolana cement conforming to IS 1489 is used in the construction.

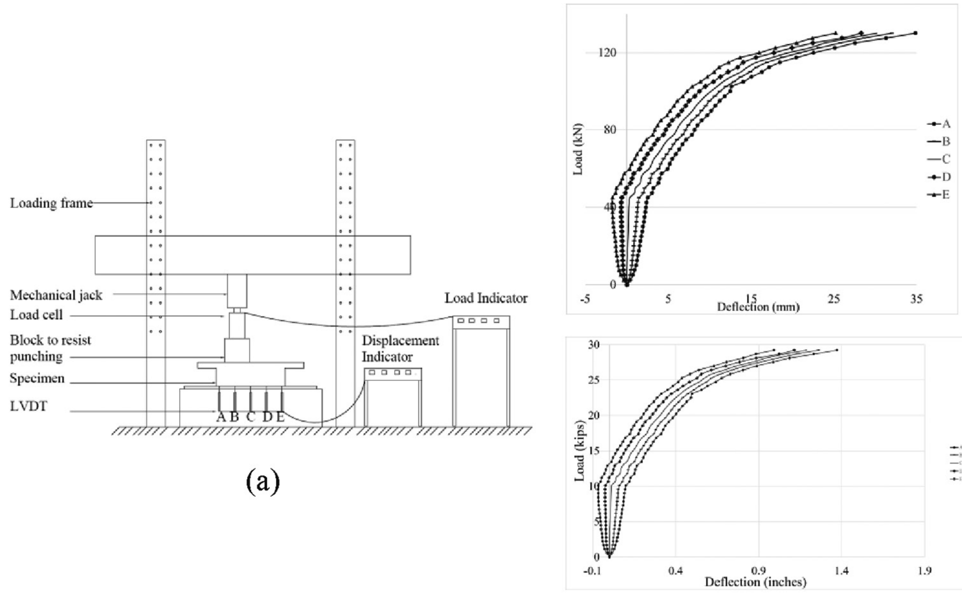
A loading frame with 40 ton capacity was used to apply the load, which was placed at an eccentricity of 290 mm (11.42 in.) to achieve a combined effect of bending and torsion in the bridge model. To avoid punching failure, the load was applied on an area of 660 x 260 mm (25.98 x 10.236 in.). The experimental test setup adopted in this work is shown in Fig. 2(d). The specimen was mounted on stiff pedestal supports at its two ends. The pedestal was placed on the floor, ensuring that it was rigidly fixed at its bottom. A steel rod was embedded on the top face of the pedestal on which the diaphragm rested, ensuring a simply supported support condition for the box-girder bridge specimen. The schematic diagram of the test setup adopted in the study is shown in Fig. 3(a). The first crack was observed at 47.5 kN (10.68 kip) on the midspan of exterior web where the load is applied. The crack was found at a distance of 50 mm

**Table 1—Compressive strength of companion cubes**

| Specimen No. | Mixture | Compressive strength, MPa (ksi) |
|---|---|---|
| 1 | 1:1.87:2.17:0.36 | 47.33 (6.86) |
| 2 | 1:1.87:2.17:0.36 | 51.22 (7.43) |
| 3 | 1:1.87:2.17:0.36 | 49.71 (7.21) |

(a) (b) (c) (d)

*Fig. 2—(a) Wooden mold; (b) reinforcement cage; (c) casting of box girder; and (d) experimental setup for twin-cell box-girder bridge.*




(a)

(b)

*Fig. 3—(a) Schematic diagram of experimental test setup; and (b) load-deflection curve from experimental study.*

(1.97 in.) from the center span. With the increase in load applied, more cracks appeared on both webs along with the widening of existing cracks. The second crack was seen at a load of 55 kN (12.36 kip), followed by the third crack at 57.5 kN (12.93 kip). The cracks were initially vertical and later inclined with the application of load. It was observed that the cracks formed at the web near the loading showcased more inclination when compared to the cracks in the web away from loading. The specimen finally collapsed at a load of 130 kN (29.23 kip). It was observed that the final deflected shape of the specimen showcased effects of distortion. Linear variable displacement transducers (LVDTs) were kept at five different locations at the bottom flange at points marked A, B, C, D, and E, as in Fig. 3(a), to find the deflection of the specimen. The load-deflection curves obtained from the experimental study are plotted in Fig. 3(b).

## NUMERICAL INVESTIGATION

To capture the actual behavior of the bridge model up to failure and to compare the results with those of the experiment, a 3-D FEA was carried out using the ANSYS software package. The concrete and steel in the reinforced concrete bridge was modeled using the SOLID 65 and LINK 180 elements using perfect elasto-plastic constitutive relations. The SOLID 65 element is an isoperimetric element with eight nodes and three degrees of freedom at each node—that is, the translation degree of freedom in the nodal *x*, *y*, and *z* directions. The SOLID 65 element is capable of crushing under compressive stress and cracking under tensile forces. It is also capable of depicting plastic deformations and creep. LINK 180, a 3-D spar element, is a uniaxial tension compression element with three degrees of freedom at each node. The twin-cell box-girder was modeled using 678,062 nodes and 618,884 elements. During meshing, the aspect ratio was kept constant and mesh convergence studies were conducted to find the most suitable mesh size. With an aspect ratio of 1, it was observed that 10 mm (0.39 in.) mesh size is the optimum. When the mesh size is made coarser, the results turned out to be less satisfactory, and with finer mesh size, the computational time taken for an analysis happens to be very high. The constitutive relation used to model the stress-strain relationship in concrete is shown in Fig. 4(a). A bilinear plot was used to model the stress-strain relationship of steel and is shown in Fig. 4(b). The properties of the materials used for the numerical study are the same as those used for the experimental study. Properties—namely, modulus of elasticity, stress-strain relations, Poisson's ratio, and compressive strength—are provided as input data for the concrete. The cross-sectional area, Young's modulus, Poisson's ratio, and a bilinear stress-strain relation are assigned to model reinforcement. The Young's modulus and Poisson's ratio of the structural steel are 200 GPa and 0.3, respectively, and that for concrete are 31.6 GPa and 0.2, respectively. The model created for the numerical analysis is shown in Fig. 5(a). Due to the eccentric load, the effects of torsion and distortion were clearly visible. These effects are prominent in the midspan under the load. The final distorted shape at the midspan and vertical deflection at various positions obtained from numerical study are shown in Fig. 5(b). The load-deflection curve obtained from the numerical study is compared with that of the experimental study, and the same is shown in Fig. 6. The crack pattern obtained from both the experimental and numerical study are provided in Fig. 7 and 8, respectively.

### Numerical analysis to study distortion bending in twin-cell box-girder bridges

To learn the distortion bending mechanism in twin-cell box-girder bridges, numerical analysis was conducted. In this study, the reinforcement ratio in the twin cell is varied by providing different stirrup dimensions. The various stirrup diameters used are 8, 7, 6, 5, and 4 mm (0.315, 0.276, 0.236, 0.197, and 0.157 in.) contributing to various reinforcement ratios ($\phi_0$) of 0.12, 0.09, 0.07, 0.05, and 0.03,

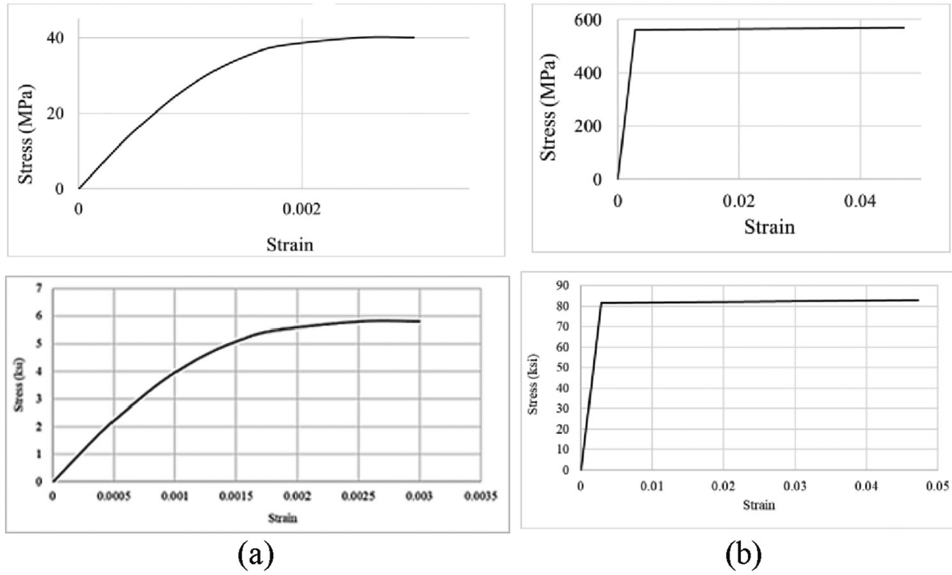



*Fig. 4—Stress-strain relationship: (a) concrete; and (b) steel.*

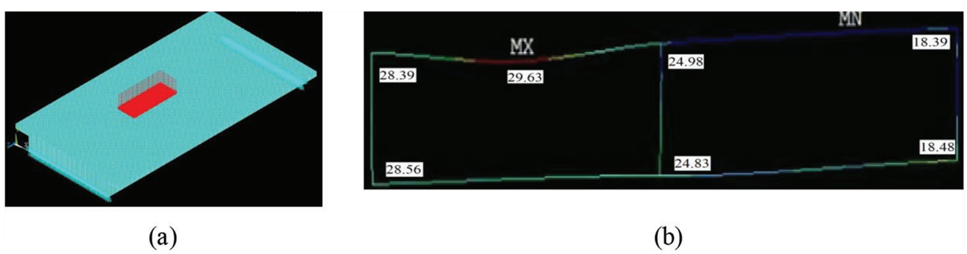



*Fig. 5—(a) Numerical model; and (b) distorted shape of cross section (vertical deflection, mm).*

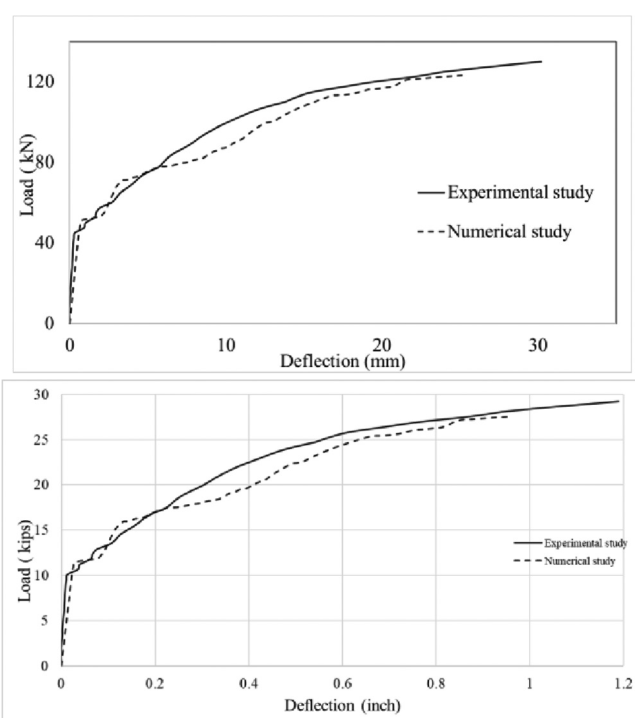



*Fig. 6—Load-deflection curve obtained from experimental and numerical study.*

respectively. Each of these cases may be mentioned as case 1, 2, 3, 4, and 5, respectively, where the load is applied on the web at midspan. The load is applied vertically on an area of 60 x 200 mm (2.36 x 7.87 in.) depicting a concentrated load on the specimen. Displacements at all three webs were noted at each increment of load. The displacements at each web are denoted as $S_1$, $S_2$, and $S_3$, as shown in Fig. 9(a), and the displacement increments are denoted as $dS_1$, $dS_2$, and $dS_3$. The displacement increment ratio of web 2 and 3 with respect to web 1 is denoted as $dS_2/dS_1$ and $dS_3/dS_1$, respectively. To understand the relative displacement of unloaded webs (webs 2 and 3) with respect to the loaded web (web 1), graphs are plotted showing the variation of displacement ratios with respect to the displacement of the loaded web as shown in Fig. 9(b), (c), (d), (e), and (f) for cases 1, 2, 3, 4, and 5, respectively.

From the study, it is observed that, at the final stage of loading, hinges are formed at four corners: two hinges at the loaded web and two in the middle web. The transverse reinforcement has yielded at four corners in addition to the longitudinal reinforcement. Hence, with the formation of hinges at corners directly below the load and at the adjacent web-flange junctions along with the yielding of longitudinal reinforcement, the structure has reached its collapse state by distortion bending mechanism. It is also observed from Fig. 9 that at the collapse stage, the relative displacement of web 3 with respect to web1 (loaded web) is negligible in cases 3, 4, and 5. To substantiate this finding, the longitudinal strain readings in webs (case 3) at midspan is provided in Fig. 10. It is observed that there is zero strain in the unloaded web. Hence, among the two cells in a twin-cell box-girder bridge,

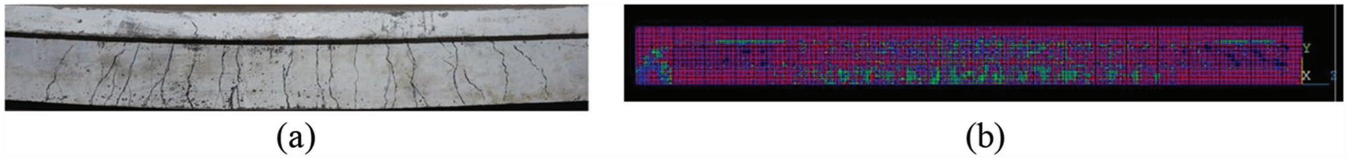



*Fig. 7—Cracks in exterior web near load: (a) experimental study; and (b) numerical study.*

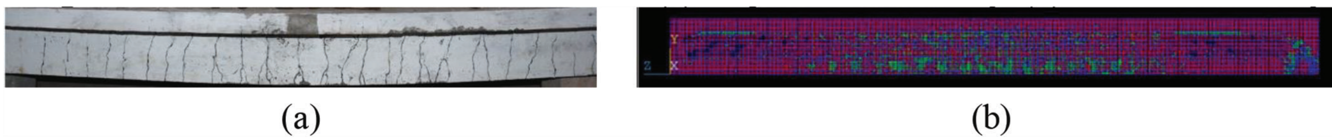



*Fig. 8—Cracks in exterior web away from load: (a) experimental study; and (b) numerical study.*

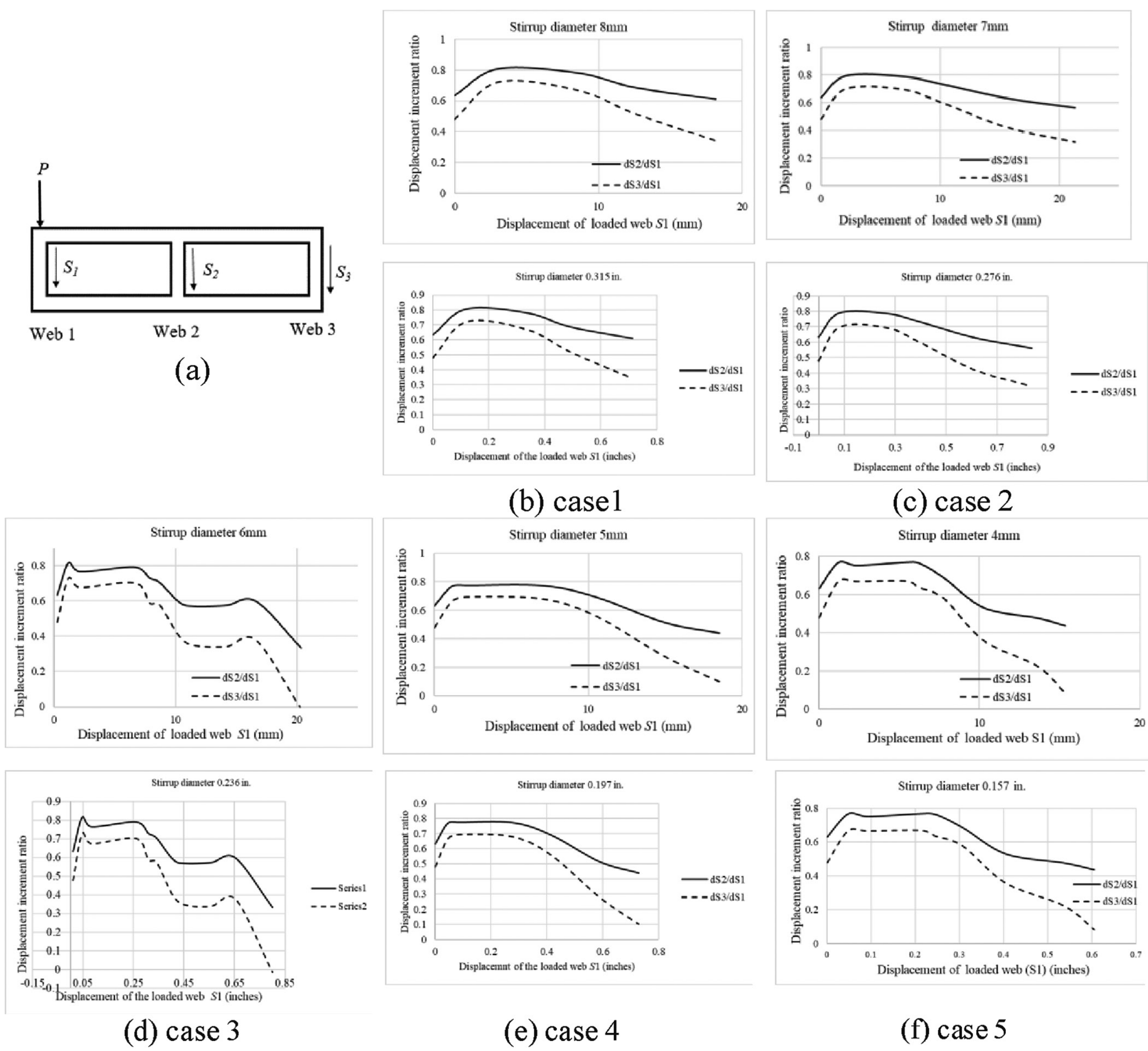



*Fig. 9—Displacement increment ratios versus displacement of loaded web.*

only the cell on which the load acts are actively contributing to the distortion bending mechanism. Hence, as the loaded cell reaches its ultimate capacity, the twin cell fails in distortion bending mechanism. Due to these reasons, to find the capacity of a twin-cell box girder in distortion-bending mechanism, the work equation used in the case of single-cell box girder can be used with slight modifications.

## COLLAPSE LOAD ESTIMATION USING SPACE TRUSS ANALOGY

The space truss analogy is based on the lower-bound theorem of plasticity. Space frame comprises of longitudinal bars at each corner known as stringers, accompanied by transverse ties depicting stirrup reinforcements which are interconnected by diagonal compression members representing concrete inclined at an angle θ, with six degrees of

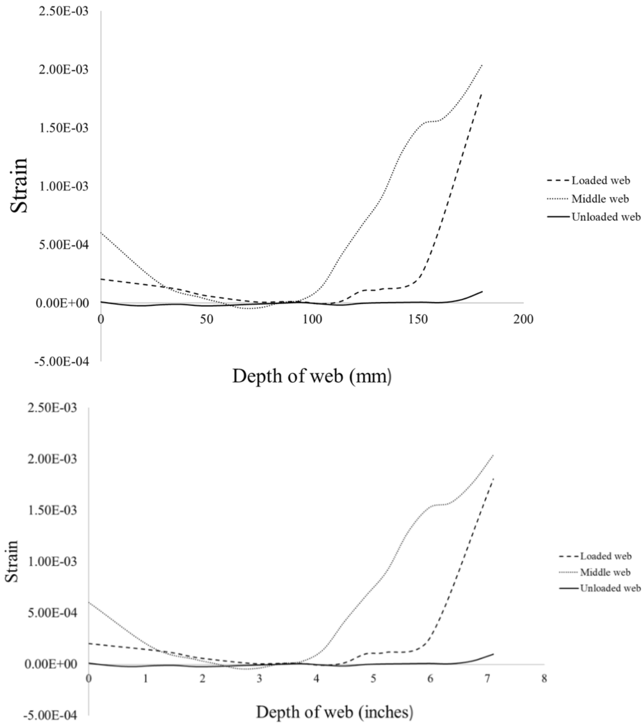



*Fig. 10—Longitudinal strain readings of webs at midspan.*

freedom at each joint. In the case of combined bending and torsion, Lampert and Thürlimann (1971) suggested a parabolic interaction formula depicting the relationship between the applied torsion ($T_u$) and bending moment ($M_u$) with that of the torsional capacity ($T_{ur}$) under pure torsion and bending capacity ($M_{ur}$) under pure bending cases of the specimen considered. These equations were originally formulated using the studies conducted on rectangular reinforced beams, but these can be used for hollow sections also as the failure mode is the same in both scenarios

$$\frac{{T_u}^2}{{T_{ur}}^2} = \frac{A_s\left(1 - \dfrac{M_u}{M_{ur}}\right)}{A_s'} \tag{1}$$

where $A_s$ and $A_s'$ are the area of steel in the tension and compression zones, respectively.

### Modeling of space truss for twin-cell box-girder bridge

There are always infinite options in creating a truss model for a particular case. Using the minimum strain energy approach, it is observed that when the struts are inclined at 45 degrees, the optimum model is obtained. These struts are provided in all four different planes in the space truss. The inclined struts throughout the structure represent concrete cracks when the box girder is subjected to pure torsion. The truss model created for the present study is provided in Fig. 11.

The space truss is created based on the geometry and amount of reinforcement present in the box girder. The dimensions of the diagonal members are kept based on ACI 318-08. As shown in Fig. 11(a), loads are applied as point loads, where the bending component is provided vertically downwards and the anti-symmetric component is provided as couple in horizontal and vertical directions. At a load of 126 kN (28.33 kip), the bottom stringer reaches its yield and the truss reached its ultimate capacity.

To further study the effects of distortion in estimating the ultimate capacity of box girder using space truss analogy, an eccentric load acting at one of the external webs is considered. For case 3, when the $\phi_0$ is 0.07, the ultimate load obtained from space truss analogy is 95 kN (21.36 kip) and the result obtained from the numerical study is 88.45 kN (19.88 kip). It is observed that in case of distortion, more overestimation of collapse load occurs leading to unsafe results. This shows the inability of space truss to accommodate distortion effects. Rasmussen and Baker (1999) suggested reducing the torsion capacity obtained from space truss analogy by 25% while designing a single-cell box-girder bridge subjected to extreme distortion, but similar studies had not happened in twin-cell box-girder bridges.

## COLLAPSE LOAD ESTIMATION USING PLANE TRUSS ANALOGY

The critical web in a twin-cell box-girder is selected for the plane truss analogy. The optimum model is obtained using the minimum strain energy principle when the strut angle is kept at 45 degrees. The dimensions of the diagonal members are kept based on ACI 318-08. The load is applied as a point load on the truss along with the self-weight of the truss. As the load is increased gradually, the bottom truss member is found to reach its yield value. The shear force in the vertical truss member is then compared with the force obtained from Knittel and Worsh's (1965) resolution of forces. Considering the shear force distribution in a simply supported beam with load at midspan, and applying the Bredt-Batho theory (Megson 2019), the total shear stress acting at the critical web due to the symmetric and antisymmetric components of loading is obtained. The ultimate load obtained from plane truss analogy is 102.17 kN (22.97 kip). Hence, it is found that in case of box-girder bridges subjected to eccentric loading, the space truss analogy results are more accurate than those of the plane truss.

## COLLAPSE LOAD ESTIMATION USING COLLAPSE MECHANISM

The major assumptions followed by Spence and Morley (1975) in developing the work equations for collapse mechanism are that the webs and flanges are sufficiently thin to develop thin-walled action, and large areas of webs and flange have rigid body motion in their planes with no shear strain. The tensile strength of concrete is neglected, and plastic energy dissipation occurs in steel. Even though these assumptions overestimate the strength in the structure, they are likely to give the true shape of collapse locus. Later, Kurian and Menon (2007) modified the collapse mechanism equations by introducing the concept of plastic hinge length ($L'$) to get more accurate results.

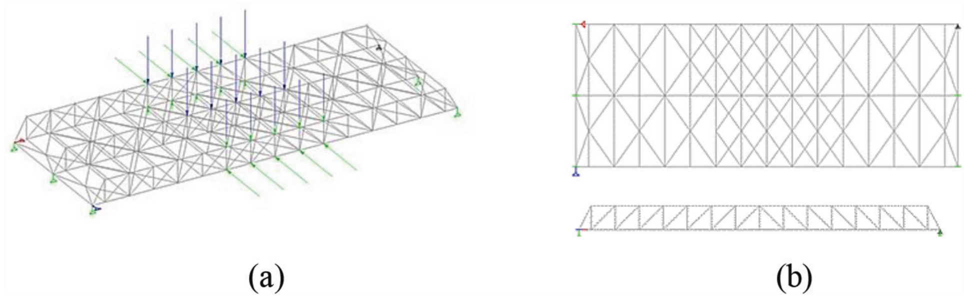



*Fig. 11—Space truss model: (a) isometric view; and (b) top view and elevation.*

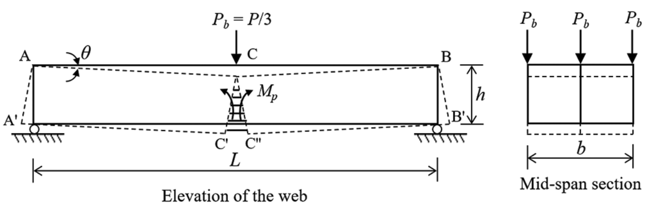



*Fig. 12—Pure bending collapse mechanism in twin-cell box girder.*

### Pure bending collapse mechanism in twin-cell box-girder bridges

In pure bending collapse mechanism, the box girder is assumed to divide into two and rotate about a transverse axis at the midspan. At this stage, all the longitudinal reinforcement bars yield in tension. Here, the failure is caused by the symmetrical loading component and the work done associated with the antisymmetric component is not considered. The symmetric component $P_b$ (= $P/3$) acting along each web is shown in Fig. 12. The equation for pure bending collapse mechanism can be formulated as in Eq. (2) with reference to Fig. 12. When a uniformly distributed load $w$ acts over a distance of $2a$ at the midspan, the equation for collapse load can be modified as in Eq. (3)

$$P = 4\frac{h}{L}\left(F_b + \frac{3}{2}F_w\right) \tag{2}$$

$$w = 2h\frac{\left(F_b + \frac{3}{2}F_w\right)}{a\,(L-a)} \tag{3}$$

where $M_p$ is the midspan plastic yield moment; $\theta$ is the hinge rotation angle in Fig. 11; and $F_b$ and $F_w$ are the total yield force contributed by the reinforcing bars present in the bottom flange and one web, respectively.

It was observed that the twin-cell box-girder bridge subjected to experimental investigations collapsed due to pure bending collapse mechanism. This mode of failure is expected due to the symmetrical component of loading. Substituting the values in Eq. (3), the collapse load is obtained as 108.55 kN (24.40 kip). Hence, it is found that the theoretical prediction of the failure mechanism and the collapse load are found to be comparable with respect to experimental results.

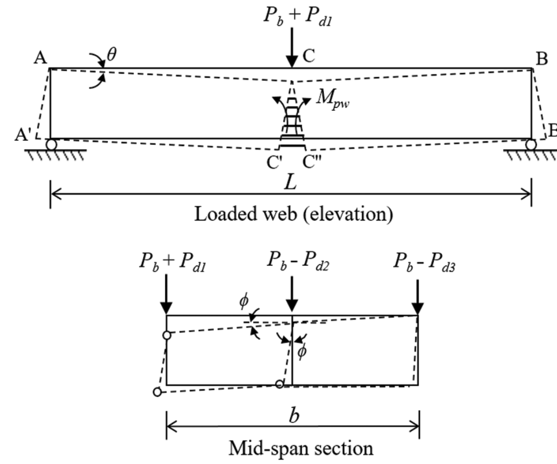



*Fig. 13—Distortion bending collapse mechanism in twin-cell box-girder bridge.*

### Distortion bending collapse mechanism in twin-cell box-girder bridge

To demonstrate distortion bending failure in twin-cell box-girder bridges, numerical studies are conducted and the results of the studies are provided in earlier sections of this manuscript. From the numerical study, it was observed that, in distortion bending failure mode, the transverse bending happens along with the bending and yielding of longitudinal reinforcing bars and four corner hinges are formed at the web-flange junctions of the loaded web and the junctions of adjacent web. It is assumed that the vertical bending of the lightly loaded webs is neglected and the heavily loaded web undergoes rigid body rotation about the horizontal plane, causing the longitudinal steel in the web to yield. The bottom flange is assumed to rotate about a vertical axis passing through the midspan causing the longitudinal steel to yield. The cross section distorts by the formation of four hinges at the web flange junctions directly under the heavily loaded web and at the adjacent web. The out-of-plane angles of web and flanges are equal at midspan denoted as $\phi$, as shown in Fig. 13. Here, $\phi$ is as provided in Eq. (4)

$$\phi = \theta L/b \tag{4}$$

As the diaphragms prevent cross-sectional distortion, the angle of distortion is considered as zero at diaphragms. The angle of distortion is assumed to vary from zero at the supports to $2\phi$ at the midspan and hence, the average angle of rotation at each corner is taken as $\phi$ at the midweb. The angle of distortion at the extreme unloaded web (web 3) is observed to be very small and hence neglected. From the numerical studies, it was also observed that with the distortion of the loaded cell, the entire twin cell reaches its collapse in case of distortion bending collapse mechanism. The amount of external work done by the load in a twin-cell box girder can be calculated using Eq. (5).

The external virtual work done by the load ($P_b + P_{d1}$) act on the heavily loaded web.

$$= (P_b + P_{d1})b\phi/2 \quad (5)$$

The internal virtual work is contributed by the yielding of longitudinal reinforcing bar in the bottom flange and at the heavily loaded web. In addition to yielding of reinforcement, the transverse bending at the four web-flange junctions also contribute to the internal virtual work.

Hence, the work equation for distortion bending collapse mechanism can be formulated as in Eq. (6)

$$\left(\frac{P}{3} + P_{d1}\right)b = \frac{2bh(F_b + F_w)}{L} + 4\,m_c\,L \quad (6)$$

From all these findings and considering the modified plastic hinge length concept suggested by Kurian and Menon (2007), the work equation for distortion bending collapse mechanism in case of twin-cell box-girder bridge can be formulated as provided in Eq. (7).

$$\left(\frac{P}{3} + P_d\right)b = \frac{2bh(F_b + F_w)}{L} + 8\,m_c\,L' \quad (7)$$

Using Eq. (7), the collapse load found for case 3 (mentioned earlier in the section "Numerical analysis to study distortion bending in twin-cell box girder bridges") is 81.43 kN (18.31 kip).

## RESULTS AND DISCUSSIONS

In this work, the ultimate load of a twin-cell concrete box-girder bridge exposed to the collective effects of bending and torsion are found experimentally and numerically. The results are compared with the results of truss analogy and the collapse mechanism. The results obtained from the various methods are tabulated in Table 2.

It is observed from Table 2 that all the different methods of analysis can satisfactorily predict the collapse load, but subjected to certain restrictions. In the case of eccentric loading ($e = b/4$), the numerical study was found to match well with the results of the experiment with a deviation of just 5% from the experimental result, which is negligible. The results of the space truss analogy found great agreement with both the experimental and numerical results with a small deviation of 4% and 2%, respectively, depicting the reliability of this method in analyzing a box girder subjected to the combined effects of bending and distortion. The plane truss analogy results are found to deviate from that of the experimental results by 21% due to its two-dimensional nature. The collapse mechanism also provided conservative results, leading to safe design of the structure.

**Table 2—Collapse load obtained for twin-cell box girder bridge**

| Sl. No. | Method of analysis | Collapse load, kN (kip) | |
|---|---|---|---|
| | | $e = b/4$ | $e = b/2$ (case 3) |
| 1 | Experimental study | 130.00 (29.23) | — |
| 2 | 3-D FEA | 123.13 (27.68) | 88.45 (19.88) |
| 3 | Space truss analogy | 126.00 (28.33) | 95.00 (21.36) |
| 4 | Plane truss analogy | 102.17 (22.97) | 75.18 (16.90) |
| 5 | Collapse mechanism | 108.55 (24.40) | 81.43 (18.31) |

When the twin-cell box girder is subjected to distortion ($e = b/2$), the space truss has provided an overestimated capacity leading to unsafe design of the structure. In this case, the collapse mechanism is found to give conservative results and can be safely used for design purposes. It is hence advisable to reduce the capacity of space truss analogy results in case the twin-cell box girder is subjected to extreme eccentric loads.

## CONCLUSIONS

This paper throws light on the different aspects concerning the design and analysis of a twin-cell concrete box-girder bridge. A numerical study is conducted to validate the experimental results using load-deflection curves and more parametric studies were conducted to comprehend the distortion effects in twin-cell box-girder bridges. An attempt is made to develop work equations based on the collapse mechanism principle for twin-cell box-girder bridges. Using truss analogy principles, two- and three-dimensional (3-D) truss models are created, and collapse load is found for different load cases. From the different analyses conducted, the following conclusions are drawn.

- The method of plane truss can be adopted when trial and error procedures are required in the early stages of design and construction, as the results are obtained in very short time. Due to the 3-D nature, a space truss can distribute any kind of difficult loading pattern in a more efficient and realistic way when compared to a plane truss. The space truss analogy is found to be reliable in estimating the capacity of a structure except in case of extreme torsion cases. In such cases of extreme eccentric loading, the results obtained from the truss analogy must be reduced by 25% for creating safe designs. As these kind of extreme distortion cases are unrealistic, space truss analogy can be used in the design of box-girder bridges. This drawback of space truss analogy creates serious issues in forensic studies associated with box-girder bridges.
- Collapse mechanism leads to safe design as it provides conservative results. Especially in cases of extreme distortion, collapse mechanism proves to be more reliable than any other existing simplified method used in establishing collapse load in box-girder bridges. A detailed understanding of the deflection profiles is required while using this method to estimate the collapse of a structure.

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