No provision for minimum amount of tensile steel appeared in previous design recommendations in this country. The provision is a precaution against abrupt flexural failure resulting from rupture of the prestressing steel when ultimate capacity is reached immediately after cracking. The usual member requires considerable additional load beyond cracking to reach ultimate capacity. Thus, considerable deflection warns that the ultimate capacity is being approached. However, if ultimate capacity occurs shortly after cracking the warning deflection may not occur.

2610—Shear*

These provisions are based upon a critical appraisal of 244 bonded prestressed beams which failed in shear, including both monolithic and composite sections up to 39 in. in width and 25-1/2 in. in depth. The equations in Section 2610 have also predicted satisfactorily the ultimate strength in shear of thirty five 9 $\times$ 18 in. prestressed I-beams with web reinforcement.

A very limited number of prestressed concrete slabs have been tested in shear. The provisions of Section 2610 are not intended for the design of slabs.

The shear provisions are adequate to include draped or depressed prestressing steel and partial prestressing.

Two equations are presented for proportioning web steel. Eq. (26-11) defines the minimum area of web steel unless members meet the conditions of Section 2610(c). Eq. (26-10) defines the amount of steel required by shear, and provides that stirrups, stressed to the yield point, must carry the shear above V_c.

The equation for minimum shear reinforcement was revised because in the equation of ACI-ASCE Committee 423 (1958) more steel was required as the web width b' increased. In members with multiple webs and subject to uniform loads, the term b' shall be taken as the sum of the widths of all webs of the member. Eq. (26-11) can be simplified by multiplying top and bottom of the right hand side by bd. If $f_s' = 250{,}000$ and $f_y = 40{,}000$, the expression for A_v becomes:

$$A_v = 0.08pbs\sqrt{\frac{d}{b'}}$$

Eq. (26-10) may be derived as follows: The ultimate shear that can be sustained by a beam at any cross-section is the sum of the amount of shear sustained by the concrete plus the amount sustained by the web reinforcement, i.e. $V_u = \phi\,(V_c + V')$, where V_u is the shear force due to specified ultimate load, V_c is the shear carried by concrete, V' is the shear carried by web reinforcement, and ϕ is the strength reduction coefficient $= 0.85$. V' is given empirically by the expression: $V' = A_v f_y\, d/s$ which represents the vertical force carried by the stirrups, $A_v d/s$, within a length d, at the yield stress f_y. Fig. 16 illustrates the agreement between this expression and experimental data. Then

$$V_u = \phi\left(V_c + \frac{A_v f_y d}{s}\right) \quad \text{or} \quad A_v = \frac{(V_u - \phi V_c)s}{\phi f_y d},$$

which is Eq. (26-10).

*See also: Mast, Paul, "Short Cuts for the Shear Analysis of Standard Prestressed Concrete Members," *Journal*, Prestressed Concrete Institute, V. 9, No. 5, Oct. 1964, pp. 15-47.

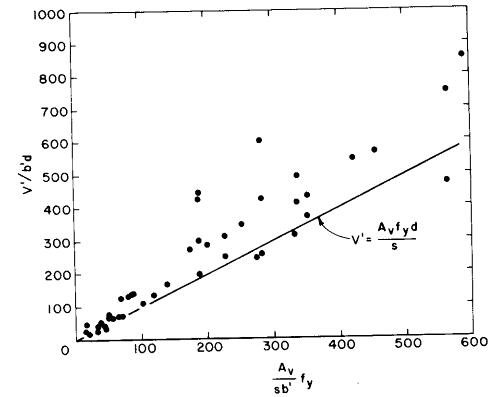

Fig. 16—Increase in shear strength of a prestressed member due to web reinforcement

To solve Eq. (26-10), a value for V_c must be determined. This requires an investigation into the two types of shear cracking that can occur: Type 1 is related to flexural cracking, Type 2 appears in portions of the member not flexurally cracked. Fig. 17 illustrates these two types of cracking and the general areas where they can occur.

Eq. (26-12) determines V_{ci} for Type 1 cracking. Eq. (26-13) determines V_{cw} for Type 2 cracking. The smaller governs the design and is used for V_c in Eq. (26-10).

To reduce the capacity of a beam, a diagonal crack must have a projection on the longitudinal axis of the beam equal to the depth of the beam. Consider a given Cross Section B-B (Fig. 18). A flexural crack distance d away (in the direction of decreasing moment) may lead to a diagonal crack which could be critical for Section B-B. The principal tensile stresses along the path of the incipient diagonal crack will be increased by flexural cracking within distance d. The principal tensile stress triggering the diagonal crack occurs near the centroid of the beam. A flexure crack occurring at a distance $d/2$ from Section B-B is a sign indicating the imminence of a Type 1 crack.

The terms

$$\frac{M_{cr}}{\frac{M}{V} - \frac{d}{2}} + V_d$$

in Eq. (26-12) are the shear corresponding to the formation of this crack.

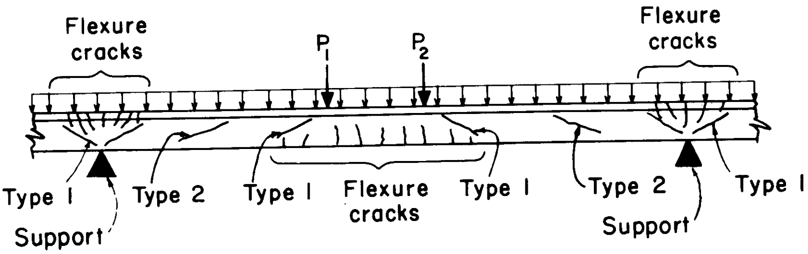

Fig. 17—Types of diagonal cracks occurring in a prestressed concrete member

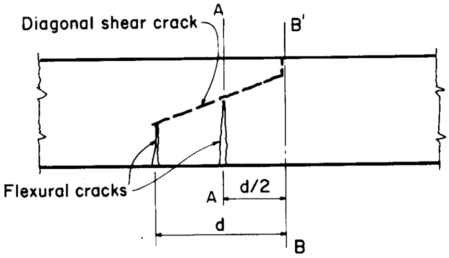

Fig. 18—Theory for Eq. (26-12) for V_{ci}

To derive Eq. (26-12) consider Section B-B (Fig. 18), where, due to externally applied loads, the moment* is M and the shear is V. The moment at Section A-A is M_{cr}, with the corresponding shear V_{cr}. Since the change in moment from one cross section to another is equal to the area of the shear diagram between the sections

$$M - M_{cr} = \frac{V + V_{cr}}{2}\frac{d}{2}$$

In a typical prestressed concrete member the difference between V and V_{cr} over the distance $d/2$ will be small. Hence:

$$M - M_{cr} = V\frac{d}{2} \quad \text{or} \quad \frac{M}{V} - \frac{M_{cr}}{V} = \frac{d}{2}$$

and,

$$V = \frac{M_{cr}}{\dfrac{M}{V} - \dfrac{d}{2}}$$

M_{cr} being the moment due to applied loads when flexural cracking occurs, the middle term of the equation for V_{ci} is the shear due to applied loads when flexural cracking occurs, even though expressed in terms of moment.

$$M_{cr} = \frac{I}{y}(6\sqrt{f_c'} + f_{pe} - f_d)$$

where $6\sqrt{f_c'}$ = modulus of rupture of the concrete, f_{pe} = compressive stress in concrete due to prestress, and f_d = stress due to dead load.

The total shear V_{cr}, due to both applied loads and dead loads when the critical flexural crack occurs is:

$$V_{cr} = \frac{M_{cr}}{M/V - d/2} + V_d$$

Dead load shear, V_d, is considered separately for two reasons:

1. Dead load is usually uniformly distributed, whereas live loads can have any distribution.

2. The dead load effect is always computed for the prestressed section alone. The live load effect is computed for the composite section in composite construction.

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*A footnote in the Code refers to M and V as caused by "external ultimate loads." Their ratio, M/V, however, is not affected by load factors.

The shear V_{ci} which causes the Type 1 diagonal crack has been found to be equal to the shear corresponding to the formation of the flexure crack at a distance $d/2$ from the section under consideration, plus a shear which appears to be a function of the dimensions of the cross section and the strength of the concrete. This increment of shear is accounted for by the term $0.6b'd\sqrt{f_c'}$.

The total shear at the formation of a Type 1 diagonal crack is therefore

$$V_{ci} = 0.6b'd\sqrt{f_c'} + \frac{M_{cr}}{\dfrac{M}{V} - \dfrac{d}{2}} + V_d$$

which is Eq. (26-12)

Fig. 19 shows the close agreement between Eq. (26-12) and available experimental data.

A lower limit of $1.7b'd\sqrt{f_c'}$ is proposed for V_{ci} because the only test beams in which V_{ci} fell below this limit were those having an abnormally low degree of prestress.

Type 2 diagonal cracking occurs when the maximum principal tensile stress becomes equal to the tensile strength of the concrete. The principal stresses can be calculated using the classical elastic theory equations. The expression for V_{cw} presented in the Code by Eq. (26-13) is a simplification of the classical principal stress formula.

V_{cw} is the shear in the nonflexurally cracked member at the time that diagonal cracking occurs in the beam web. The difference between V_{cw} and V_u is the shear

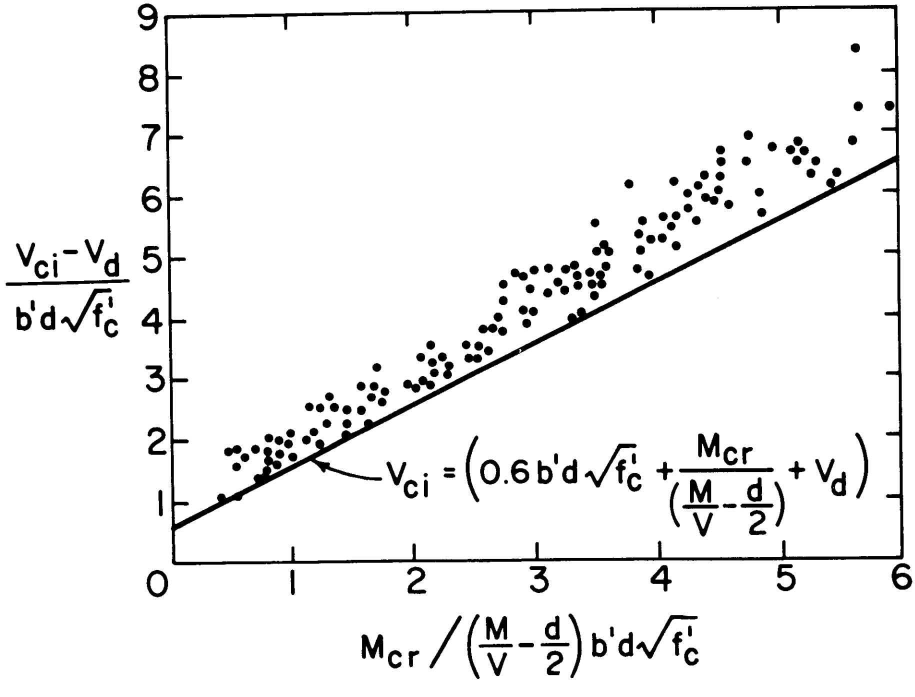

Fig. 19—Diagonal cracking in those regions of beams previously cracked in flexure

to be carried by stirrups. Tests have shown that the maximum principal tensile stress will usually occur near the centroid of the cross section.

The capacity of the member is reached if:

$$f_t = \sqrt{v_{cw}^2 + \left(\frac{f_{pc}}{2}\right)^2} - \frac{f_{pc}}{2}$$

where f_t = tensile strength of concrete, v_{cw} = shear stress, and f_{pc} = compressive stress due to prestress. This relation yields:

$$\left(f_t + \frac{f_{pc}}{2}\right)^2 = v_{cw}^2 + \left(\frac{f_{pc}}{2}\right)^2$$

or,

$$v_{cw} = f_t \sqrt{1 + \frac{f_{pc}}{f_t}}$$

A value for f_t of $4\sqrt{f_c'}$ appears to be substantiated by tests, but since v_{cw} is the nominal shear stress and not the actual maximal one, $f_t = 3.5\sqrt{f_c'}$ is used in the expression for v_{cw}:

$$v_{cw} = 3.5\sqrt{f_c'}\left(\sqrt{1 + \frac{f_{pc}}{3.5\sqrt{f_c'}}}\right)$$

The curve representing this equation is plotted on Fig. 20, as a solid line. The equation may be simplified to the form:

$$v_{cw} = 3.5\sqrt{f_c'} + 0.3 f_{pc}$$

which is shown by the dashed lines.

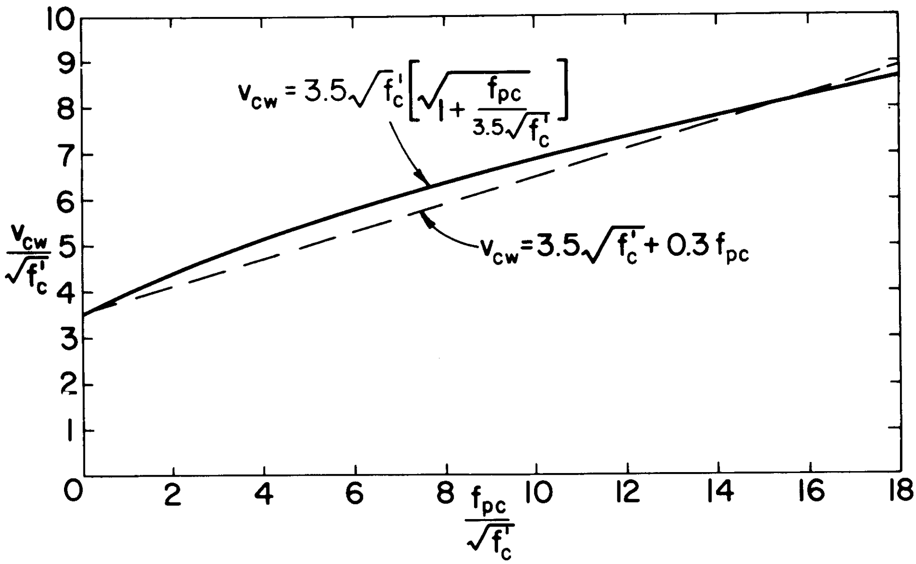

Fig. 20—Relationship between nominal shear stress at Type 2 cracking and compressive stress at centroid

The nominal shear stress, v_{cw}, equals the shear force due to loads, V_{cw}, minus the counteracting component of the prestressing force, V_p, divided by $b'd$

$$v_{cw} = \frac{V_{cw} - V_p}{b'd}$$

Substituting this expression for v_{cw}

$$\frac{V_{cw} - V_p}{b'd} = 3.5\sqrt{f_c'} + 0.3 f_{pc}$$

and

$$V_{cw} = b'd\left(3.5\sqrt{f_c'} + 0.3 f_{pc}\right) + V_p\,,$$

which is Eq. (26-13).

In Eq. (26-13), d is taken as 80 percent of the total beam depth or as the value used in Eq. (26-12) whichever is greater. This is valid because the section is uncracked. Thus the web area available to compute nominal shear stress is not directly a function of the location of the centroid of the prestressing steel. The value of 80 percent is substantiated by test data which showed that 80–85 percent of the total depth yielded reasonable results for this type of analysis.

Lightweight aggregate concrete—The equations for shear in lightweight aggregate prestressed concrete beams were derived from the equations for normal weight concrete, Eq. (26-12) and (26-13), by analogy, using the provisions of Section 1708 as a guide. Separate equations for lightweight aggregate concrete are necessary because such concrete can have a lower tensile strength than normal weight concrete of the same compressive strength.

Section 1708(a)1 requires that the limiting value for shear stress on the unreinforced web of a lightweight aggregate concrete member may not exceed $0.3\,\phi\,F_{sp}\sqrt{f_c'}$.

Section 1701(c) requires that for normal weight concrete this limiting value be $2\,\phi\sqrt{f_c'}$.

For these two limiting values to be equivalent, the value of F_{sp} for normal weight concrete must be 6.67, which is in reasonable agreement with the test value found for sand and gravel concrete. From this relationship, Eq. (26-12A) and (26-13A) were derived by analogy.

Guide for application—The web reinforcement in excess of the specified minimum can be governed by either V_{ci}, Eq. (26-12), or V_{cw}, Eq. (26-13). Both values must be computed unless the procedure can be abbreviated, for uniform loads, by realizing that V_{cw} usually governs near the support, and V_{ci} near the quarter point. The center portion of the span is usually governed by the minimum value of V_{ci}.

It may be advantageous to determine the web steel requirements by plotting the various allowable and prevailing shears along the member. Fig. 21 is such a typical and schematic diagram with the computed values of V_{cw}, V_{ci}, and $V_{ci\,min}$ at various points along the span. Plotting enough points to define the curves illustrates where each type of shear governs and its relationship to the actual

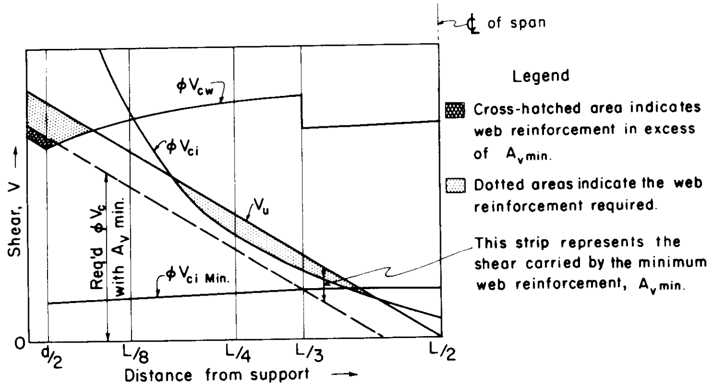

Fig. 21—Schematic diagram of shear computations for V_{cw}, V_{ci}, and $V_{ci\ min}$

ultimate shear force, V_u. This visual interpretation of Eq. (26-10) indicates where V_u exceeds V_{cw} or V_{ci} and where extra stirrups must be provided.

General comments—These provisions for design in shear were developed from tests on beams having either rectangular or I-shaped cross sections, with webs of constant width. If the web width of a member varies over its height, the use of the minimum width, as required by the code, will lead to conservative designs.

In lieu of the term *d*, which is specified differently for most of the equations of Section 2610, an effective depth d_1 could be defined as either the distance from the extreme compression fiber to the centroid of the prestressing tendons, or 80 percent of the over-all depth of the member at the cross section under consideration, whichever is greater.

This value d_1 could replace *d* in all equations except Eq. (26-12) and Eq. (26-12A).

Attention is called to the provision at the end of Section 2610(c), that not even the minimum web reinforcement need be provided if it is shown by tests that the required ultimate flexural and shear capacity can be developed without web reinforcement. The problem is the determination of a parameter that will accurately predict when no web reinforcement is required. In the absence of such a parameter, minimum shear steel is required unless load tests have shown otherwise.

When such tests are made, care should be taken to reproduce as closely as possible the conditions of loading, and to simulate the support conditions of the actual structure.

2611—Bond

The provisions of section 2611 apply to pretensioned, prestressed concrete members using three or seven-wire steel prestressing strands. They are intended to ensure that failure of a pretensioned prestressed member shall not occur by the strand pulling through the concrete as a result of failure in flexural bond.