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Shear Strength Prediction for Separate Categories of Simple Beam Tests

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The ultimate shear strength of simple rectangular beams under concentrated load is not only a function of the common design properties of f_c', p, slenderness a/d, and web reinforcement rf_{yw}, but also is strongly influenced by the beam support conditions. The accurate prediction of test beam strength must necessarily consider all of these factors. In this paper, available beam test data are separated into the distinct categories of different strength behavior, and simple empirical equations are derived for each catagory. Beginning with a previously developed equation for slender ($a/d <$ 2.5) beam shear strength, equations are developed for short ($a/d < 2.5$) indirect or side bracket loaded beams with or without vertical stirrups and short direct, top and bottom plate loaded beams with or without vertical stirrups. Good engineering prediction results with these equations, and a definite amount of dowel action is detectable in beams, with vertical stirrups, having two layers of longitudinal reinforcement.

Keywords: beams (supports); loads (forces); regression analysis; reinforced concrete; stirrups; structural design; supports; web reinforcement.

■ IN PREVIOUS PUBLICATIONS,[1] the following empirical relation was developed for the ultimate shear strength of simple, rectangular, slender ($a/d >$ 2.5) beams, without stirrups, under concentrated load: *

$$v_{u\,1} = \frac{V_u}{bd} = 60\left(f_c'\,p\,\frac{d}{a}\right)^{1/3} \qquad (1)$$

It was found that Eq. (1) is a good predicator of the limited amount of data for special short beams ($a/d < 2.5$) under indirect loading conditions as provided by side flanges or brackets. The object of this paper is to develop prediction equations for the remaining categories of simple beam shear tests, under concentrated load:

1. Short beams ($a/d < 2.5$) with the direct loading conditions of top load and bottom support plates or blocks

2. Slender beams, and indirect load short beams, with vertical stirrups

3. Short beams with direct load, with vertical stirrups

It will be seen that fortunate circumstances, perhaps more empirical than theoretical, allow the resulting prediction equations to be simple modified forms of the basic shear Eq. (1).

PURPOSE OF STUDY

The equations for beam shear strength in ACI 318-63 are not the most accurate representations of test beam behavior (see, for example, the scatter in the plot of Fig. 5.1 and 6.1 of the ACI-ASCE Committee 426 report[12]). However, these equations will most probably continue to exist in the Code because of a desire to preserve established

*Note that in Eq. (1), the constant value of 60 is used for simplicity rather than 61.2 as in Reference 1.

design procedures, and for generally conservative prediction over a wide range of beam types and loading conditions. Yet, structural engineers in all fields, have a real need for a simple, accurate method of evaluating shear strength for the aforementioned specific categories of test beams. For example: researchers need prediction for the planning of the range of variables in future test series, and for the analysis of the effects of web reinforcement, section shape, axial load, and torsion on shear strength; designers and teachers need to relate actual structural beam conditions to the strength information provided by the nearest appropriate category of beam tests; and code committee members require not only strength behavior, but also a knowledge of the variability or scatter of test results, for the formulation of new design equations and safety provisions for the shear strength of both special and regular structures such as containment vessels, shear wall systems, and precast elements.

While some excellent theories have been advanced recently to give us a good qualitative understanding of the entire shear failure mechanisms, the complex nature of the phenomenon and the very real lack of quantative knowledge concerning material properties such as combined load-deformation and fracture behavior, prohibits the practical prediction of strength by these theories. At least, for the present time, we must rely on simple empirical predictors in terms of the commonly available design properties, namely, f_c', p, a/d, and rf_{yw}.

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SHORT BEAMS UNDER DIRECT LOAD WITHOUT WEB REINFORCEMENT

Introduction and background

In studies by Kani[2] and Leonhardt[3] of beams with direct load conditions of top load and bottom support plates, it is shown that a positive change occurs in the rate of shear strength increase when the a/d ratio is decreased below a value of about 2.5. This positive rate change is so important, when compared to the rate of strength increase with decreasing a/d values above 2.5, as represented by Eq. (1), that direct load shear tests must be separated into two major categories: slender beams ($a/d > 2.5$), and short beams ($a/d < 2.5$). A wide scatter of prediction error results if a strength behavior equation is not adapted to recognize these two distinct categories.

The reason for the sharp change in the rate of strength increase for direct load short beams is due to a type of arch action that forms between the top load and bottom support plates when a/d falls below 2.5. This arch action definitely requires direct load conditions. In Reference 1 it is shown that the rate change does not occur in special short beams with an indirect load condition created by load and support force transmission to the beam by side flanges.

Due to the real testing problems concerning the difficulty of obtaining shear failure, rather than flexural failure, in some slender beams, a large portion of available test data falls in the category of direct load short beams. These data can provide useful information concerning short beams in actual structures such as: beams with offset columns, beams in bents for support of stringers in bridges and precast buildings, and certain types of shear walls. In research, many continuous beam tests, and shear-flexure-torsion experiments are in the short beam region. However, thus far, no theoretical procedure has been able to formulate a prediction equation that can accurately represent or integrate the general behavior of the available test data. It is desired here to develop a strength equation for these beams so as to serve the immediate need for engineering prediction. The development will rely on empirical methods and a simple, but rather successful, intuitive model of arch action.

Formulation of a lower bound strength predictor

Data from 108 short beam tests with direct load conditions were assembled from References[2, 4, 5, 6, 7, and 8], and a multiple regression analysis provided the preliminary equation for the central behavior of the data:

$$\bar{v}_u = 1670\ (f_c')^{0.26}\ (p)^{0.47}\ (d/a)^{1.30} \qquad (2)$$

with a coefficient of variation of error of 19 percent. This wide spread of error is due mainly to the following characteristic of the data. For twin beams, with nearly identical sets of f_c', p, and d/a values, selected from both within and between the different test sources, there can occur widely different values of strength v_u. This condition is shown in Fig. 1 where the enlarged points representing twin beams are joined by a vertical line. These differences in twin beam strengths may be due to variations in concrete tensile strength, load block size, loading rate, and beam section configuration. If we desire to predict in terms of the common available design parameters of f_c', p, and a/d, then central behavior prediction will necessarily possess the dispersion due to the neglect of these aforementioned factors.

Therefore, if we depart from the strictly empirical regression method of curve fitting to central behavior, we may arrive at a useful predictor of behavior by the following intuitive derivation; which was inspired by the values of the exponents in Eq. (2).

For values of a/d greater than 2.5, the slender beam strength is predicted by Eq. (1). As a/d decreases below 2.5, the short beam arch action of load and support blocks creates strengths greater than predicted by Eq. (1). The simplest mathematical model that could represent this strength increase is an equation formed by the multiplication of Eq. (1) by a linear arch action factor given by:

$$\frac{\text{Limit of } a/d \text{ for slender beam action of Eq. (1)}}{\text{Short beam } a/d \text{ or arch description}} = \frac{2.5}{a/d}$$

Thus, the trial prediction equation for short beam strength with direct load is:

$$v_u = \left[\text{Eq. (1)}\right] \cdot \left(\frac{2.5}{a/d}\right)$$

$$= 60\ [f_c'\ p\ d/a]^{1/3} \cdot \left(\frac{2.5}{a/d}\right)$$

$$v_u = 150\ (f_c'\ p)^{1/3}\ (d/a)^{4/3}$$

for $a/d < 2.5$ (3)

Note that this simple linear model has exponent values which with consideration of the possible range of sampling variation, are not essentially different from the exponents in the regression Eq. (2). It now remains to be seen whether the trial Eq. (3) represents some useful engineering property of the variable ensemble of the short beam test data, with its established high and low

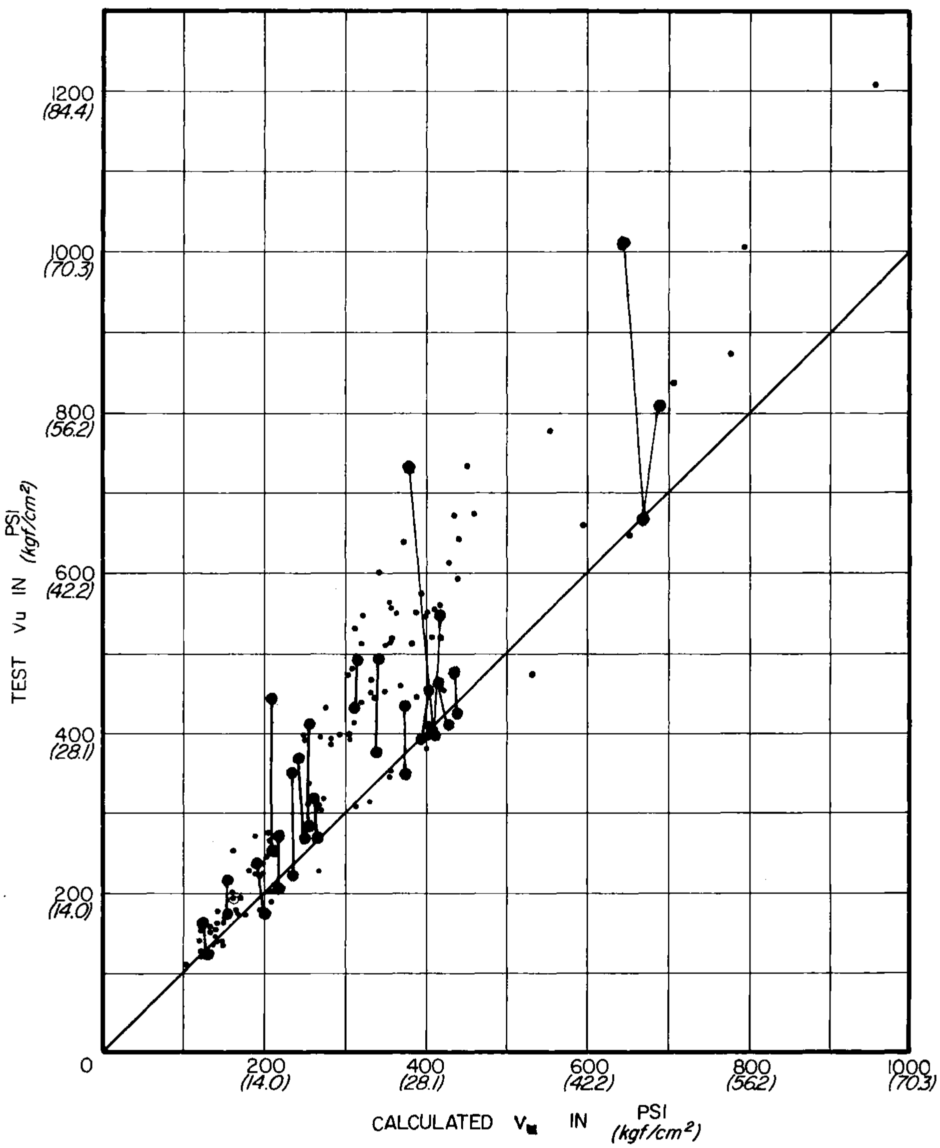

Fig. 1—Ultimate shear stress prediction by Eq. (3) for direct load short beams without web reinforcement

strengths of twin beam pairs. This is best illustrated by Fig. 1 which represents a plot of actual v_u test values (ordinates), and the corresponding Eq. (3) calculated values (abscissas). The most striking result is the excellent agreement of Eq. (3) with the enlarged points representing the low strength of the twin beam pairs. Further, there is very low scatter of those points below the prediction line for the entire strength range. If we wish to predict short beam strength in terms of f_c', p, and a/d, then it appears that Eq. (3) possesses good engineering qualities of lower bound strength prediction with generally non wasteful prediction of the possible, but uncertain high strength. Also, it will be seen that Eq. (3) has the important quality of providing an excellent representation of the concrete section contribution to the strength of short beams with vertical stirrups.

SHEAR STRENGTH OF BEAMS WITH VERTICAL STIRRUPS

Introduction

The current engineering method of representing the ultimate shear strength of beams with vertical stirrups is based on the superposition of the vertical concrete section force and stirrup yield force acting on the diagonal shear crack. In terms of stress (division of forces by bd) we have the standard form:

$$v_u = v_c + rf_{yw} \tag{4}$$

In the ACI-ASCE report[12], the wide scatter of test points in Fig. 6.1 indicates that the ACI 318-63 version of Eq. (4) is in need of improvement. A large source of scatter may be due to the fact that our code predictor of the concrete contribution v_c does not recognize the wide difference in strength behavior between slender beams and direct load short beams. Therefore, it is useful to investigate the capabilities of the general superposition form of Eq. (4) when v_c is predicted by Eq. (1) for slender beams and indirect load short beams, and by Eq. (3) for direct load short beams.

The general form of Eq. (4) certainly may deviate from recent theoretical models, such as the indeterminate arch truss of the shear failure mechanism. However, if it is able to furnish acceptable prediction of test results, then it will most probably continue to be used for practical applications. The use of the shear failure stress for beams without web reinforcement to represent the concrete contribution v_c in Eq. (4) has been considered conservative because of the following effects which may develop in beams with stirrups:

1. Extra concrete force contribution due to the prevention of diagonal crack propagation by the stirrups; to be termed as v_{xc}

2. Extra dowel action force from longitudinal tensile reinforcement due to the support, and prevention of longitudinal splitting by the stirrups; to be termed as v_{xD}.

The qualitative existence of the effects v_{xc} and v_{xD} may well be visualized, and in the following studies of each separate test beam category it will be possible to obtain information concerning their respective quantitative significance.

Slender beams with vertical stirrups

The proposed prediction equation is, with the use of the slender beam Eq. (1) for v_c:

$$v_u = v_{u1} + rf_{yw}, \text{ for } a/d > 2.5 \tag{5}$$

where

$$v_{u1} = 60\ (f_c'\, p\, d/a)^{1/3}$$

This equation was used to predict the ultimate shear stress of the slender beam data in References 4g, 4i, 9, 10, 11; with the data subject to the ACI 318-63 stirrups limitations of $rf_{yw} > 60$ psi and $s/d < 0.5$. Fig. 2 shows the test ordinates and Eq. (5) calculated abscissa plots for slender beams with one layer and two layers of longitudinal reinforcement. The general prediction accuracy of Eq. (5) is seen to be satisfactory. No significant value of either of the extra v_{xc} or v_{xD} effects can be detected for the beams with one layer of bars. It may be that v_{xc} and v_{xD} do occur during some intermediate stage of the failure mechanism, but these may diminish as the full yield state develops in the stirrups. For the beams with two layers of bars, the stable high tendency of the test points indicated the development of a significant dowel action v_{xD}. A supplementary investigation of data not conforming with the restrictions of $rf_{yw} > 60$ and $s/d < 0.5$ showed greater variability and some strengths significantly below the calculated values.

Short indirect load beams with stirrups

Since Reference 1 has provided the indication that Eq. (1) predicts the ultimate shear strength of indirect load short beams, it is logical to use this Eq. (1) for the v_c in this type of beams, with stirrups. Therefore, the proposed prediction equation for these beams is again Eq. (5). The data are indeed sparse; namely, one side flange loaded beam by Leonhardt[13] (Code L), one steel side bracket loaded beam by Muto[14,15] (Code M), and the side flange loaded beam with $rf_{yw} = 0$ by Ferguson from Reference 1 (Code F). However, the excellent agreement shown by these three beams on Fig. 2 gives no reason to reject the validity of Eq. (5) for indirect load short beams. It is of course necessary to use the actual a/d less than 2.5.

The ability to predict the strength of these beams is important because of their close resemblance to special structure load conditions, such as floor slab lateral inertia load on shear walls.

Also, the short shear span, without arch action, provides shear strength behavior in a region of low flexural stress, and therefore may approximate the shear behavior of beam sections close to the inflection points of continuous spans.

Direct load short beams with vertical stirrups

Having Eq. (3) for the v_u of direct load short beams without stirrups, this value was used as the concrete contribution v_c in short beams with vertical stirrups. The resulting simple equation is:

$$v_u = v_{u2} + rf_{yw} \qquad (6)$$

where

$$v_{u2} = 150\ (f_c' p)^{1/3}\ (d/a)^{3/4}$$

Eq. (6) was used to predict the short beam test data in Reference 4h, 4i, and 4j. Good engineering prediction results when the short beam data were within the following limitations:

$f_c' > 2500$ ···· A few beam tests with f_c' near 2000 psi were 10 to 15 percent low

$f_c' < 6000$ ···· A few beam tests with $f_c' > 6000$ were 10 to 15 percent high

$a/d > 1.5$ ···· Beams with a/d near unity do not appear to develop full stirrup force

$rf_{vw} < v_{u2}$ ···· Beams with rf_{yw} larger than v_{u2} do not appear to develop full stirrup force

The prediction results of Eq. (6) are shown in Fig. 2. The beams (see References 4h and 4j), with two layers of longitudinal reinforcement, do appear to develop extra dowel action strength v_{xD}. Also, it is interesting to note that apart from this rather stable extra strength increase due to two layers of reinforcement, there is none of the high positive error due to nonpredicted extra strength such as that shown in Fig. 1 for short beams without stirrups. It is possible that beams with stirrups may occasionally develop the upper level of concrete arch action strength during some early stage of failure, but this unstable high strength reverts back to the stable lower level of strength v_{uA} when the cracks develop and the stirrups have reached their yield capacity. It may be added that the practical value of the arch action Eq. (3) is greatly enhanced by the fact that it provides an accurate representation of the concrete contribution in Eq. (6).

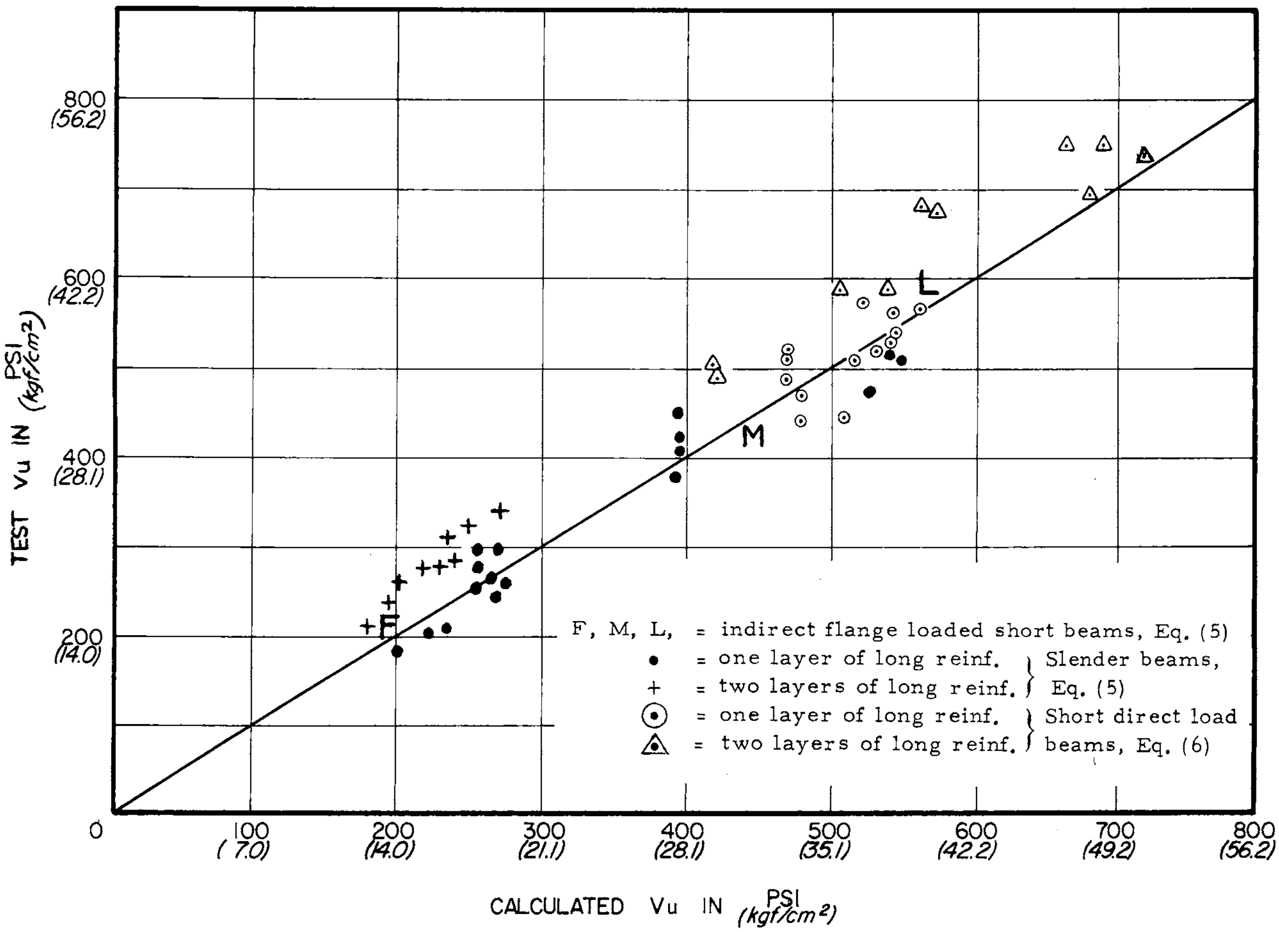

Fig. 2—Ultimate strength prediction by Eq. (5) and (6) for slender and short beams with vertical stirrups

CONCLUSIONS

Summary

Beginning with the available predictor of the ultimate shear strength of slender beams without vertical stirrups:

$$v_{u1} = 60 \left(f_c' \, p \, \frac{d}{a}\right)^{1/3}$$

it is possible to obtain good prediction of the ultimate shear strength of the following categories of beam tests with or without ($rf_{yw} = 0$) vertical stirrups:

Slender beams and indirect load short beams:

$$v_u = v_{u1} + rf_{yw}$$

Direct load short beams ($1.5 < a/d < 2.5$):

$$v_u = v_{u1}\left(\frac{2.5}{a/d}\right) + rf_{yw} = v_{u2} + rf_{yw}$$

The simplest forms of mathematical modeling were used to develop these equations. They are intended for the purpose of practical prediction, and their particular forms are not meant to convey any detailed information concerning the actual or theoretical mechanisms of shear failure.

Limitations

It is important to realize that the above equations represent the behavior of laboratory test beams with their specific load-support conditions, and with short time single direction, concentrated load. While they provide improved accuracy and flexibility, structural engineers must use careful judgment to adapt these expressions of test behavior to the actual support conditions of beams in structures and to their actual loading which may be repeated cycles of single direction, or two-directional stress reversals under concentrated and distributed load.

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APPENDIX—NOTATION

A_s = area of longitudinal tensile steel, sq in.
A_v = area of one vertical stirrup, sq in.
a = shear span, in.
b = beam width, in.
d = beam effective depth to steel, in.
f_c' = companion concrete cylinder strength, psi
f_{yw} = stirrup yield stress, psi
p = steel ratio A_s/bd
r = stirrup ratio A_v/bs
s = stirrup spacing, in.
V_u = ultimate shear strength, lb
v_u = V_u/bd = ultimate shear stress, psi
v_{u1} = prediction of v_c by Eq. (1), psi
v_{u2} = prediction of v_c by Eq. (3), psi
v_c = concrete contribution in Eq. (4), psi
v_{xc} = extra concrete contribution, psi
v_{xD} = extra dowel action, psi

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