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Shrinkage Restraint and Loading History Effects on Deflections of Flexural Members

by Andrew Scanlon and Peter H. Bischoff

The effects of shrinkage restraint cracking and loading history on deflection of reinforced concrete flexural members are discussed. It is shown that deflections of lightly reinforced members (less than 0.8% reinforcement) are highly sensitive to both shrinkage restraint cracking and loading history, whereas deflections of more heavily reinforced members are insensitive to these effects at full service load. Results of a deflection example are presented along with recommendations for changes to the ACI 318 Building Code. Proposed changes include a) adoption of a cracking moment equal to two-thirds the value currently specified in the Code to account for shrinkage restraint stresses; and b) evaluation of deflection using an effective moment of inertia based on the full dead plus live service load to account for preloading from construction loads prior to installation of nonstructural elements. Both effects are evaluated using an alternative formulation for the effective moment of inertia that works well over a wide range of reinforcing ratios and for fiber-reinforced polymer (FRP)-reinforced concrete.

Keywords: beam; deflection; loading history; shrinkage restraint; slab.

INTRODUCTION

Time-dependent deflection of reinforced concrete members involves a complicated interaction of many factors including cracking, creep, shrinkage, and loading history. Uncertainties in material properties and loading exacerbate the problem further and make prediction of deflection a difficult task at the design stage. Nevertheless, engineers need to design structures that perform under service loads in a manner that satisfies serviceability requirements of the structure by providing an acceptable level of deflection control.

One method of deflection control is to compute deflection under specified conditions and compare the calculated value with a limit prescribed by the ACI 318 Building Code. To aid the engineer, a methodology or basis for the calculation is usually provided. It is not necessary for the calculated deflection to precisely match the deflection that actually occurs in the field because it is recognized that this is not possible due to the uncertainties involved. Nevertheless, the calculation procedure should take into account the most important factors affecting deflection to give reasonable results and make the deflection calculation meaningful.

This paper addresses two of the most important issues affecting deflection of concrete members: 1) the contribution of shrinkage restraint to cracking and its subsequent effect on flexural stiffness; and 2) the importance of loading history, particularly during the construction phase. An earlier paper (Bischoff and Scanlon 2007) addresses the question of the effect that cracking and tension stiffening have on flexural stiffness and recommends a new expression for the effective moment of inertia.

RESEARCH SIGNIFICANCE

This paper deals with building code requirements for deflection control and contains recommendations for modifications to the ACI 318 Building Code. Practical recommendations for a lower cracking moment to account for shrinkage restraint and preloading from construction loads are found to have a significant influence on deflection of lightly reinforced concrete members.

EFFECTIVE MOMENT OF INERTIA AND LONG-TERM MULTIPLIERS

In North American engineering practice, immediate deflections are most often calculated using engineering beam theory where the flexural rigidity of a beam is characterized by an elastic modulus and a constant moment of inertia (second moment of area) of the member cross section averaged over the member length. It is well known that cracking reduces flexural stiffness in concrete members and this effect is accounted for by using an effective moment of inertia I_e to model the gradual reduction in stiffness as load increases and cracking progresses along the member. An equation to model the gradual transition from uncracked to cracked stiffness was introduced by Branson (1963) and adopted by the ACI Code (ACI Committee 318 1971).

Subsequent research has shown that Branson's (1963) effective moment of inertia expression works well at moderate to high reinforcement ratios (over 1%) but overestimates member stiffness at lower reinforcement ratios typical of slab systems. Bischoff (2005) introduced an alternative formulation for the effective moment of inertia expression that has been shown to be applicable for all practical reinforcement ratios as well as FRP-reinforced members (Bischoff and Scanlon 2007). The new expression, recommended for adoption by the ACI 318 Code is given by either

$$\frac{1}{I_e} = \left(\frac{M_{cr}}{M_a}\right)^2 \frac{1}{I_g} + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^2\right] \frac{1}{I_{cr}} \geq \frac{1}{I_g} \tag{1a}$$

or

$$I_e = \frac{I_{cr}}{1 - \left(\frac{M_{cr}}{M_a}\right)^2 \left[1 - \frac{I_{cr}}{I_g}\right]} \leq I_g \tag{1b}$$

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where I_e is determined for the service load moment M_a at the critical section and is assumed constant along the length of member to provide a conservative estimate of deflection, M_{cr} defines the cracking moment and controls the amount of tension stiffening in the member response, I_g represents the gross (uncracked) moment of inertia, and I_{cr} is the cracked transformed moment of inertia.

To compute additional long-term deflections under sustained load accounting for creep and shrinkage, a time-dependent multiplier is typically used as a basis for calculation (ACI Committee 318 2005). In this approach, the immediate (short-term) deflection caused by sustained load $\Delta_{i,sus}$ is multiplied by a time-dependent multiplier to give

$$\Delta_{lt} = \lambda\Delta_{i,sus} \tag{2}$$

The time-dependent multiplier λ typically ranges between a value of 1 and 2 depending on the duration of loading and is reduced by the presence of compression reinforcement in the cross section.

SHRINKAGE RESTRAINT AND CRACKING MOMENT

It is well known that tensile stresses can be induced in a concrete member due to shrinkage under drying conditions when the member is restrained against volume change (ACI Committee 224 2001). Several sources of shrinkage restraint can be identified in concrete beams and slabs. These include embedded reinforcing bars, stiff supporting elements, adjacent portions of slabs placed at different times, and nonlinear distribution of shrinkage over the thickness of a member.

Development of these tensile stresses is time-dependent, as shown in Fig. 1(a) and (b), but the net effect is a reduction in flexural stiffness resulting from the formation of cracks due to the combined effects of stresses caused by shrinkage restraint and applied loading. Figure 1 also shows the relationship between the development of tensile strength (Fig. 1(a) and (b)), loading history (Fig. 1(c) and (d)), and subsequent deflection (Fig. 1(e)).

Restraint stresses decrease the cracking moment M_{cr} of the member under applied loads by reducing the effective tensile strength or modulus of rupture of the concrete. In other words

$$f_{re} = f_r - f_{res} \tag{3}$$

$$M'_{cr} = f_{re}I_g/y_t = \frac{f_{re}}{f_r}M_{cr} \tag{4}$$

where the rupture modulus of concrete $f_r = 7.5\sqrt{f'_c}$ (psi) is reduced by the restraint stress f_{res}. The value M_{cr} is the unrestrained cracking moment based on f_r and y_t is the distance from the centroidal axis of the uncracked section to the tension face of the section.

Member stiffness is reduced when a lower cracking moment is used in the expression for the effective moment of inertia. This concept was originally introduced by Scanlon and Murray (1982). They estimated a restraint stress in the order of one-half the stress given by the ACI 318-05 expression for the modulus of rupture ($7.5\sqrt{f_c'}$) and recommended using a reduced effective modulus of rupture f_{re} rounded off to $4\sqrt{f_c'}$ for calculating M_{cr} in deflection calculations based on Branson's (1963) expression for the effective moment of inertia. Scanlon and Murray (1982) also pointed out that shrinkage restraint stress has a more significant effect on flexural stiffness as the reinforcement ratio decreases. It is therefore a more important consideration for slabs than for more heavily reinforced beams.

Time-dependent stresses that develop in a member restrained with embedded reinforcing bars can be calculated based on consideration of equilibrium and strain compatibility for an assumed value of free shrinkage strain (Tam and Scanlon 1986, Gilbert 1999). For embedded bars in a flexural member, Gilbert (1999) proposed the following expression to calculate the restraint stress f_{res}

$$f_{res} = \frac{2.5\rho}{1 + 50\rho}E_s\varepsilon_{sh} \tag{5}$$

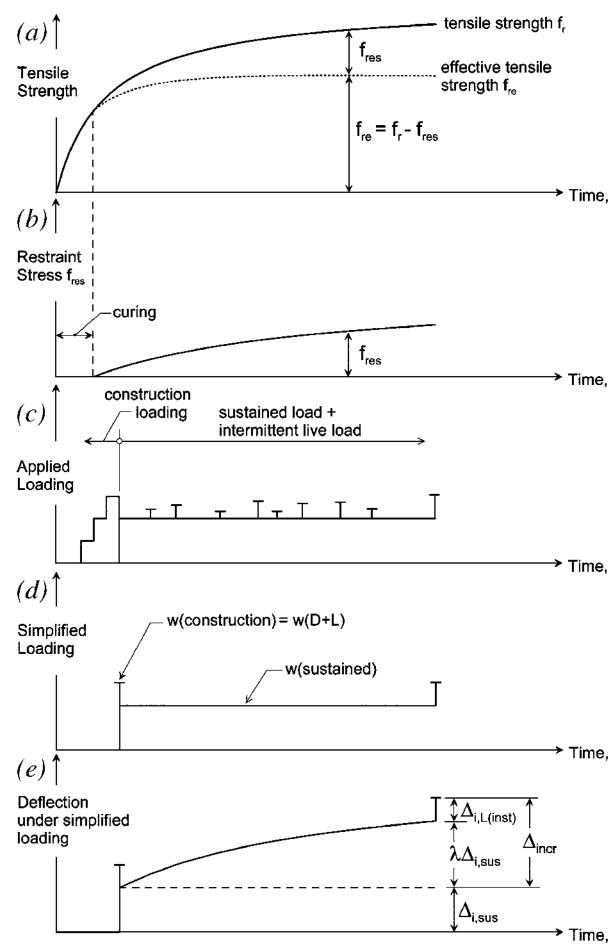

Fig. 1—Development of restraint stresses and typical loading history.

where the reinforcement ratio ρ equals A_s/bd and ε_{sh} is the design free shrinkage strain.

Gilbert's (1999) Eq. (5) is dependent on assumptions made regarding the long-term modular ratio $\bar{n}$ of concrete and depth of reinforcement, d, relative to the member height h. Full derivation of the expression for f_{res} is provided in the Appendix, with the resulting expression given by Eq. (6) assuming 80% of the ultimate shrinkage strain ε_{sh} has occurred for a rectangular beam cross section with d/h = 0.85.

$$f_{res} = \frac{2.635E_s\rho(0.8\times\varepsilon_{sh})}{1+2.1\bar{n}\rho} \approx \frac{2.1\rho E_s\varepsilon_{sh}}{1+2.1\bar{n}\rho} \quad (6)$$

For a typical concrete with $\bar{n}$ = 20, Eq. (6) gives a residual stress $f_{res} = 2.1\rho E_s\varepsilon_{sh}/(1 + 42\rho)$ that is slightly different from Gilbert's (1999) equation. Murashev et al. (1971) also proposed a similar expression.

Figure 2 plots the effective stress ratio f_{re}/f_r using Gilbert's (1999) Eq. (5), Scanlon and Murray's (1982) estimated value of one-half, and a range of values given by Eq. (6) for different concrete strengths. Also shown is Eq. (5) with the 2.5 factor changed to 1.5 as this was adopted by the Australian Standard (AS3600 2001) when shrinkage effects were first introduced. Results shown for a typical shrinkage strain of 0.075% are conservatively based on an assumed rupture modulus of $7.5\sqrt{f_c'}$ and have less an effect on concrete with a rupture modulus greater than $7.5\sqrt{f_c'}$.

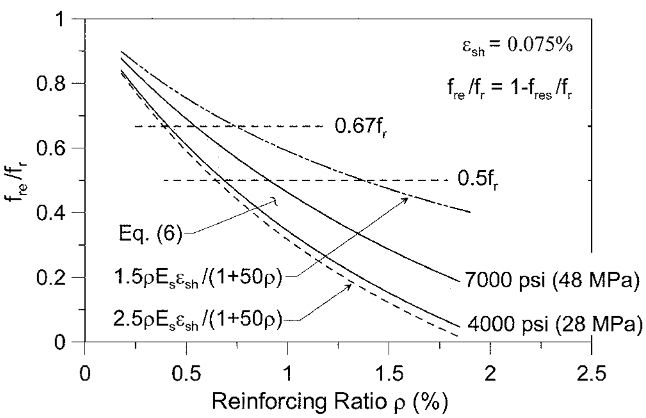

Fig. 2—Development of shrinkage restraint stresses.

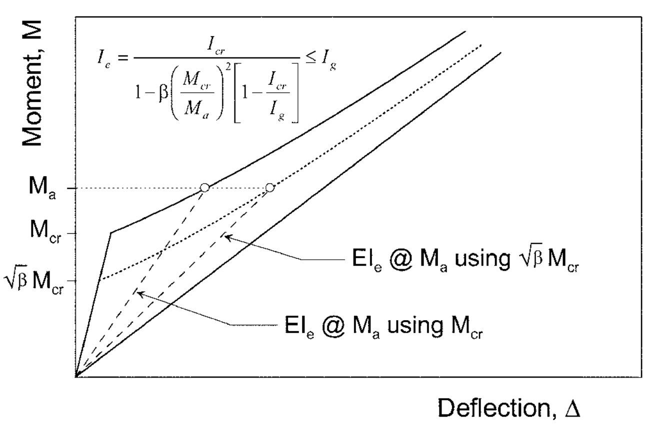

Fig. 3—Computed member response with reduced cracking moment.

The plot shows that the one-half factor provides a conservative estimate of the effective stress ratio compared with the other approaches at low reinforcement ratios when restraint due to embedded bars only is considered. The one-half factor corresponds to shrinkage restraint in a flexure member with a reinforcing ratio of approximately 0.8%, whereas a two-thirds factor corresponds to a lower reinforcing ratio of 0.5% and is more appropriate when used with Bischoff's (2005) expression for I_e as demonstrated in the following. Values depend, of course, on the amount of shrinkage assumed and actual value of rupture modulus.

Shrinkage restraint effects on cracking are beginning to be recognized in building code requirements. As noted previously, the Australian Standard (AS3600 2001) initially adopted Eq. (5) with a 1.5 factor in the numerator instead of the 2.5 value proposed by Gilbert (2001). The recent proposed draft of the Australian Standard (AS3600 2005) now uses the 2.5 factor. A 50% reduction in cracking moment was adopted by the Canadian Standard A23.3 in 1994 for two-way slab systems (CSA A23.3 1994), whereas recent changes to ACI 318 (ACI Committee 318 2008) evaluate service load deflections of slender (tilt-up) walls with a lower cracking moment equal to two-thirds of the code specified value.

Eurocode (CEN 2004) provisions account for the time-dependent loss of flexural stiffness by applying a one-half factor to the tension stiffening term used for long-term deflection calculations. This corresponds to a 30% reduction in the cracking moment and is equivalent to using a reduced cracking moment $M'_{cr} = \sqrt{\beta}\,M_{cr}$ as given by Eq. (7)

$$I_e = \frac{I_{cr}}{1-\eta\left(\frac{M'_{cr}}{M_a}\right)^2} = \frac{I_{cr}}{1-\eta\left(\frac{\sqrt{\beta}M_{cr}}{M_a}\right)^2} = \frac{I_{cr}}{1-\eta\beta\left(\frac{M_{cr}}{M_a}\right)^2} \quad (7)$$

where $\eta = 1 - I_{cr}/I_g$ as before, and β is a sustained load factor used by the Eurocode to account for a lower cracking moment (Fig. 3). Comparison with Eq. (1b) and (4) gives $\beta = (f_{re}/f_r)^2$, and setting β equal to 0.5 is almost the same as using a two-thirds factor applied to the cracking moment. The β factor can also account for the loss of stiffness that occurs from cracking under repeated loading (Bischoff 2005).

Figure 4 compares the computed member response using both Branson's (1963) and Bischoff's (2005) equations for a range of reinforcement ratios. Results are presented for a rectangular section using 4000 psi (27.6 MPa) concrete, Grade 60 (415 MPa) reinforcement, and assuming an effective depth to beam thickness (d/h) ratio equal to 0.85. Results are shown for the full cracking moment neglecting restraint stress and for a reduced cracking moment of $0.67M_{cr}$ for Bischoff's (2005) equation and $0.5M_{cr}$ for Branson's (1963) equation to account for restraint stress.

Whereas restraint stresses decrease the cracking moment considerably at higher reinforcing ratios (Fig. 2), this has little effect on member stiffness for reinforcing ratios greater than approximately 1% when the effective moment of inertia is computed under full service loads ($0.675M_n$ assuming a dead-to-live load ratio of 2:1). Similar conclusions were reached by Rangan and Sarkar (2001). The full service load is at least three times greater than the cracking moment when steel reinforcing ratios are greater than 1%, and the magnitude of cracking moment has little effect on member stiffness at this load level since $I_e \approx I_{cr}$ when $M_a/M_{cr} > 3$.

At lower reinforcement ratios (0.3 and 0.5%), the plots in Fig. 4 demonstrate the sensitivity of deflection to restraint stress as represented by the reduced cracking moment.

Shrinkage restraint can have a significant effect on member deflection for lightly reinforced members when the full service load moment is less than or slightly above the computed cracking moment (based on $f_r = 7.5\sqrt{f_c'}$). In this instance, even if the member is initially uncracked, cracking is likely to occur as restraint stresses develop over time. Computation of long-term deflection under sustained loads will then be significantly underestimated when based on an uncracked section as pointed out by Gilbert (1999). To account for this effect, calculation of deflection based on an effective moment of inertia is recommended using Bischoff's (2005) equation for I_e given by Eq. (1) with either a two-thirds factor applied to the cracking moment or assuming a reduced modulus of rupture $f_r = 5\sqrt{f_c'}$ psi ($0.4\sqrt{f_c'}$ in MPa).

LOADING HISTORY AND INSTALLATION OF NONSTRUCTURAL ELEMENTS

Building structures are often subjected to loads during construction before the concrete has attained its specified 28-day strength. Significant loads can arise from shoring and reshoring procedures in multi-story construction in addition to loads from personnel, equipment, and temporary storage of construction materials such as drywall or reinforcing bars. These loads can reach a level approaching and sometimes exceeding the design dead plus live load. Quite often the magnitude of load depends on the shoring/reshoring sequence used for construction (Grundy and Kabaila 1963). Others have also emphasized the importance of loads during construction on long-term serviceability of floor systems.

Requirements for design loads during construction are provided by SEI/ASCE 37-02 (2002). This standard specifies a uniform load of 50 lb/ft^2 (2.4 kPa) for average construction and 75 lb/ft^2 (3.6 kPa) for heavy construction. Actual loads are required to be used when construction does not meet the definitions provided in the standard. ACI 318 (2005) requires that construction loads not exceed the combination of superimposed dead load plus specified live load unless analysis indicates adequate strength to support such additional loads.

Figure 1(c) gives a typical representation of loading during and after the construction period. Loading during the construction phase is shown as a step function to represent a typical shoring/reshoring sequence, but can also represent construction loads due to personnel, equipment, and materials. Sustained loading after the construction phase consists of the dead load plus the sustained portion of live load. Variable live load is applied intermittently during the life of the structure.

The ACI 318-05 Building Code prescribes limits on live load deflection (when not supporting nonstructural elements likely to be damaged by large deflections) and incremental deflection occurring after installation of nonstructural elements. Whereas the ACI 318-05 Building Code does not prescribe a limit on total time-dependent deflection, the limit on incremental deflection indirectly limits the total deflection. In most cases, the deflection occurring after installation of nonstructural elements is the most critical condition. This deflection consists of time-dependent deflection from the sustained portion of load and immediate deflection from the remaining live load. Because the time-dependent deflection is typically based on a multiplier of immediate deflection from the sustained load (Eq. (2) and Fig. 1(e)), construction loads prior to installation of the nonstructural elements can

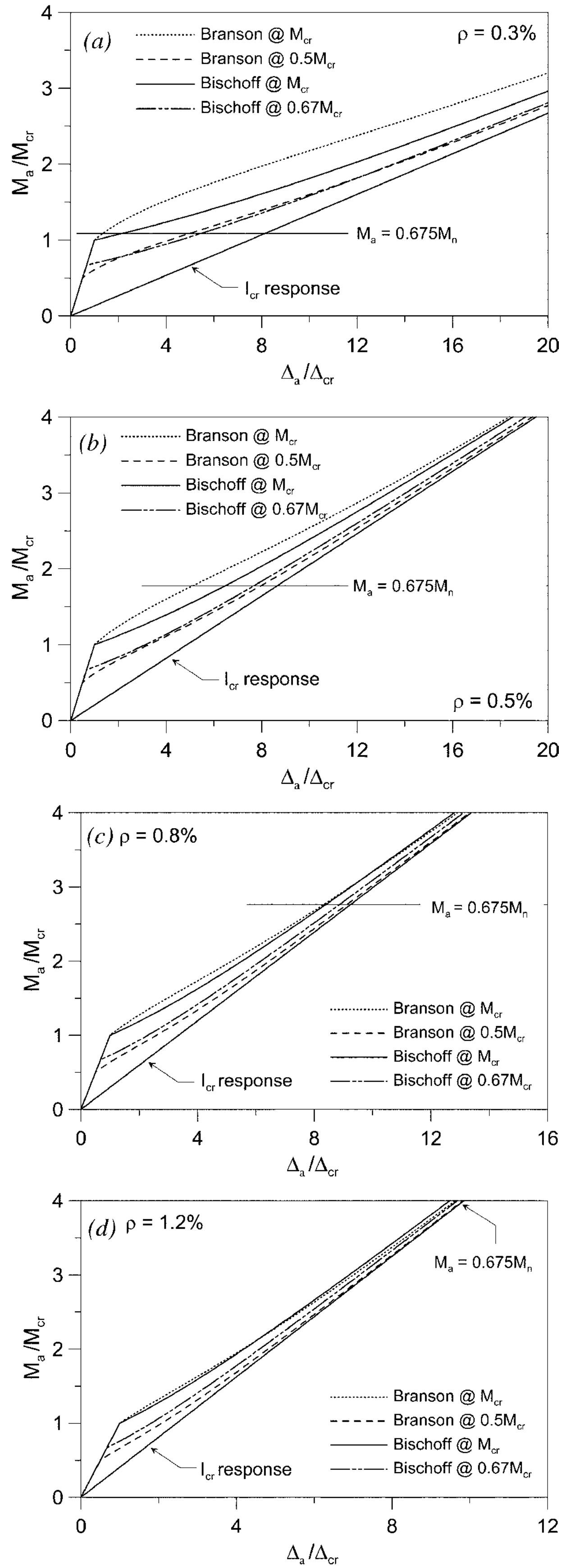

Fig. 4—Effect of cracking moment on member response for: (a) 0.3%; (b) 0.5%; (c) 0.8%; and (d) 1.2% reinforcing ratio.

affect the stiffness of the floor system and should be taken into account in the deflection calculations.

The simplified loading history shown in Fig. 1(d) is used as the basis for deflection checks. Maximum estimated construction loads are assumed to be applied just prior to installation of the nonstructural elements and are taken as the specified dead plus live load unless more detailed information is available. Immediate deflection under the sustained load level is then calculated using an effective moment of inertia corresponding to the full dead plus live load. Time-dependent deflection is taken as a multiple of this computed value. The same moment of inertia is also used to calculate the remaining live load deflection and is much simpler than considering different moments of inertia for dead load and dead plus live load as commonly assumed in the past.

Although methods are available for calculating construction loads due to shoring and reshoring, the method of construction and shoring sequence are usually unknown at the design stage when deflection checks are being made. Nor will the engineer know the stage at which nonstructural elements will be installed as this is usually the contractor's responsibility. The proposed simplified loading history recognizes that these loads can reach and sometimes exceed the specified design loads and, consequently, have a significant effect on the extent of cracking in the member prior to service loading.

Once the construction sequence is known, the corresponding loading history can be used to determine the maximum load expected during construction. ACI 318-05 requires that loads during construction should not exceed the specified design loads unless an analysis is performed to show that adequate resistance is available. This analysis would typically be carried out after the design phase.

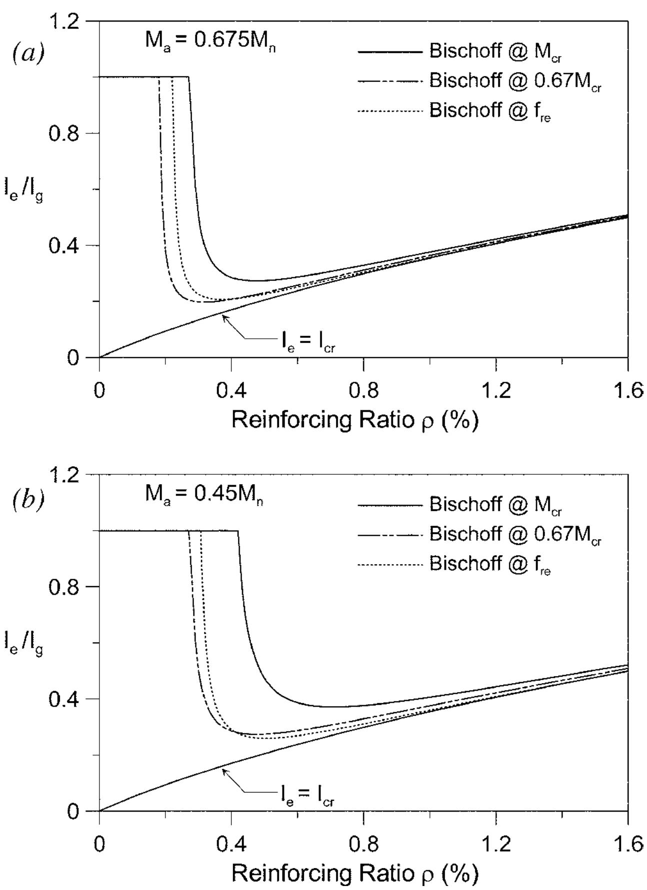

*Fig. 5—Effect of cracking moment and shrinkage restraint on effective moment of inertia for: (a) full service (*D + L*) load; and (b) dead (*D*) load only.*

COMBINED EFFECTS OF SHRINKAGE RESTRAINT AND LOADING HISTORY

The previous sections outline the need to consider both shrinkage restraint and loading history effects when calculating immediate and time-dependent deflections of concrete members. The sensitivity of calculated values to assumptions made relative to these two factors is discussed in this section.

Figure 5 shows the variation of I_e/I_g versus reinforcing ratio at the full service ($D + L$) load corresponding to $M_a = 0.675M_n$ and for dead (D) load only corresponding to $0.45M_n$. In each case, plots are made based on I_e computed using 1) the full value of M_{cr} for $f_r = 7.5\sqrt{f_c'}$; 2) two-thirds of the full value of M_{cr}; and 3) M_{cr} based on consideration of a restraint stress f_{res} calculated from Eq. (6).

Under full service load, all plots converge asymptotically to the line representing I_{cr} as the reinforcement ratio increases beyond approximately 0.8%. This occurs because the effective moment of inertia (Eq. (1)) begins to approach I_{cr}, as the ratio of M_a to M_{cr} increases with the increase in reinforcement ratio. The value I_e becomes insensitive to either the load level or cracking moment for reinforcing ratios greater than approximately 1.2% as indicated by Fig. 5(b), and the use of I_{cr} in deflection calculations is a reasonable approximation for the moment of inertia in this range. It is interesting to note that a 0.8% limit of reinforcing ratio at full service load is consistent with the ACI 318 requirements for deflection calculations prior to the 1971 edition (Bischoff and Scanlon 2007).

The situation is quite different as the reinforcement ratio drops below 0.8% toward minimum steel requirements for slab reinforcement. At these low reinforcing ratios, the I_e/I_g ratio begins to diverge rapidly from the line representing I_{cr}, eventually reaching unity when $M_a = M_{cr}$. Thus, the calculation of I_e becomes highly sensitive to the cracking moment and level of load considered. For example, at $\rho = 0.4\%$ with the full value of M_{cr} and dead load only, $I_e/I_g = 1.0$, whereas at full M_{cr} and full ($D + L$) service load the ratio is closer to 0.3. This corresponds to more than a three-fold difference in member stiffness depending on load level. On the other hand, differences between the I_e/I_g ratio at the two load levels are not as great when using a reduced cracking moment. For this example, the difference is approximately 50% (with the I_e/I_g ratio dropping from 0.3 for dead load only to 0.2 at the full service load).

The I_e/I_g ratio is obviously highly sensitive to both the cracking moment and load level for reinforcing ratios between 0.2 and 0.4%. This reinforcement range is typically used in floor slab systems. In this range, the applied moment is close to or below the cracking moment. These observations are consistent with results from Monte Carlo simulations indicating high variability of deflections when the applied moment is close to the cracking moment (Ramsay et al. 1979, Choi et al. 2004).

It is clear from these comparisons that the stiffness of lightly reinforced members is highly sensitive to the presence of shrinkage restraint stresses and the load level at or prior to the time deflection is considered.

DEFLECTION EXAMPLE

Results of deflection calculations are presented for a simply supported one-way slab with an 18 ft (5500 mm) span. This example demonstrates the sensitivity of computed values to assumptions made about the cracking moment and level of load prior to installation of the nonstructural elements. The slab, as shown in Fig. 6, has a thickness of 8 in. (200 mm) that does not satisfy minimum thickness requirements found in Table 9.5(a) of ACI 318-05 (h_{min} = L/20 = 10.8 in. [275 mm]). Hence, member deflection needs to be computed.

Loading consists of dead load w_D = 100 lb/ft^2 (4.8 kPa) and live load w_L = 70 lb/ft^2 (3.4 kPa). Part of the live load $w_{L(sus)}$ = 20 lb/ft^2 (1 kPa) is sustained. The concrete has a specified compressive strength of 4000 psi (27.6 MPa) and is reinforced with Grade 60 (415 MPa) steel bars. Strength requirements are satisfied using No. 4 (12.7 mm diameter) bars spaced at 6 in. (150 mm) on center for a reinforcing ratio ρ of 0.48%.

Four cases are summarized in Table 1 involving two loading histories and two values of cracking moment to demonstrate the effects of preloading and shrinkage restraint on computed values of deflection. Loading History LH1 corresponds to a sustained load of $w_{D+L(sus)}$ = 120 lb/ft^2 (5.8 kPa) applied at 28 days and is followed by the remaining live load $w_{L(inst)}$ = 50 lb/ft^2 (2.4 kPa) applied after a period of 5 years or more. Hence, the effective moment of inertia used to compute long-term deflection is based on a bending moment corresponding to a sustained load of 120 lb/ft^2 (5.8 kPa), whereas immediate deflection from the remaining live load is calculated using an effective moment of inertia based on a moment corresponding to the full service load (170 lb/ft^2 [8.2 kPa]).

Loading History LH2 is similar to Loading History LH1 except that the full dead plus live load w_{D+L} = 170 lb/ft^2 (8.2 kPa) is applied at 28 days to simulate the effect of precracking from construction loads prior to any long-term deflection occurring from the sustained loads. Deflection calculations in this instance are all made using an effective moment of inertia based on an applied moment corresponding to the full dead plus live load (170 lb/ft^2 [8.2 kPa]). Both the full cracking moment M_{cr} and two-thirds of M_{cr} are considered for Loading History LH2. Comparison is also made to deflection calculations prescribed in ACI 318-05 (ACI Committee 318 2005) corresponding to Loading History LH1 with the full cracking moment M_{cr} and using Branson's (1963) expression for the effective moment of inertia.

Incremental deflection of the slab increased 35% from a value of 0.74 in. (18.8 mm) in Case 1 to 1.0 in. (25.4 mm) in Case 2 when account was taken of preloading from construction loads. Progressive cracking under the action of construction loads decreased the member stiffness and this increased deflection under sustained loads while decreasing the immediate live load deflection that occurs subsequent to the long-term deflection. A large part of this difference occurs because the member in this example is uncracked under the action of sustained loads only. Residual stresses that developed from restraint to shrinkage decreased the member stiffness further and this increased deflection by another 40% to give a final deflection value of 1.4 in. (35.6 mm) for Case 3. Incremental deflection criteria were not satisfied with the final computed value of 1.4 in. (35.6 mm).

In contrast, the computed value of incremental deflection using the present approach prescribed in ACI 318-05 (ACI Committee 318 2005) gave a much lower computed deflection of 0.57 in. (14.5 mm) that easily satisfied the L/240 (0.90 in. [22.9 mm]) criterion and was not far from satisfying the

Table 1—Deflection calculation example

Case	Loading history	Effective cracking stress	Live load deflection: Computed, in. (mm)	Limit, in. (mm): L/360	Limit, in. (mm): L/180	Incremental deflection: Computed, in. (mm)	Limit, in. (mm): L/480	Limit, in. (mm): L/240
—	ACI 318*	$7.5\sqrt{f_c'}$	0.29 (7.4)	0.60 (15.2)	1.20 (30.5)	0.57 (14.5)	0.45 (11.4)	0.90 (22.9)
1	LH1	$7.5\sqrt{f_c'}$	0.46 (11.7)	0.60 (15.2)	1.20 (30.5)	0.74 (18.8)	0.45 (11.4)	0.90 (22.9)
2	LH2	$7.5\sqrt{f_c'}$	0.24 (6.1)	0.60 (15.2)	1.20 (30.5)	1.00 (25.4)	0.45 (11.4)	0.90 (22.9)
3	LH2	$5\sqrt{f_c'}$	0.34 (8.6)	0.60 (15.2)	1.20 (30.5)	1.40 (35.6)	0.45 (11.4)	0.90 (22.9)

*ACI 318 calculations are based on Branson's equation for I_e and loading corresponding to LH1. Calculations for Cases 1 to 3 are based on Bischoff's Eq. (1) for I_e.

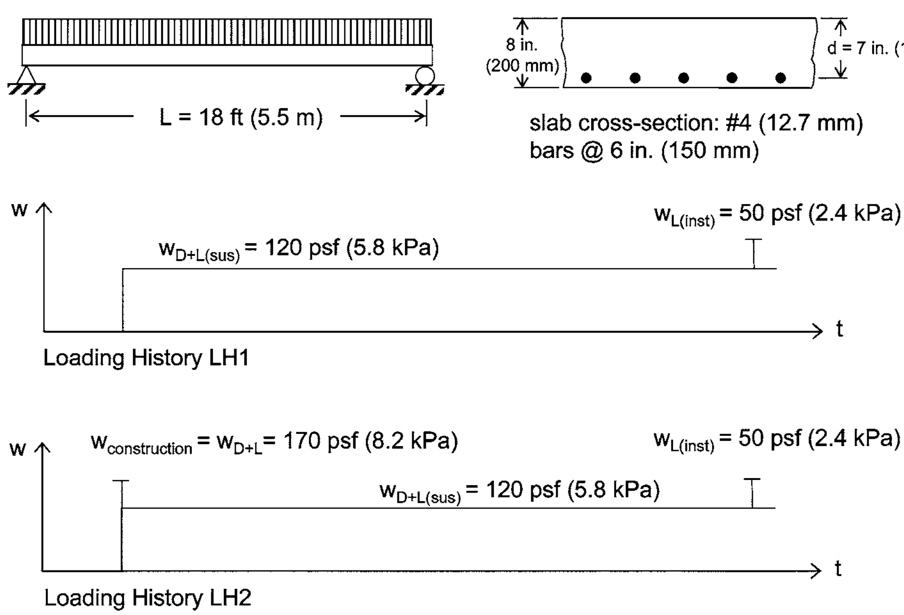

Fig. 6—Deflection example.

L/480 (0.45 in. [11.4 mm]) criterion. The computed value of 0.57 in. (14.5 mm) is almost one-third the value obtained when account is taken of preloading from construction loads, shrinkage restraint effects, and without the artificial stiffening effect observed with Branson's (1963) equation for lightly reinforced members such as slabs.

Recommendations made for shrinkage restraint and loading history are only expected to affect deflection calculations for lightly reinforced members. For example, computed values of deflection for a flexural member with a higher 1% reinforcing ratio only increase by approximately 6% when account is taken of the reduced stiffness from construction loading and lower cracking moment.

CONCLUSIONS AND RECOMMENDATIONS

It has been shown that deflection of flexural members with low reinforcement ratios (less than approximately 0.8%) is highly sensitive to shrinkage-restraint stress and early-age loading during construction. For reinforcement ratios greater than 1%, the cracked moment of inertia I_{cr} can be used as a reasonable and conservative approximation for the effective moment of inertia for most deflection calculations. For lower reinforcement ratios, the effective moment of inertia proposed by Bischoff (2005) (Eq. (1)) is recommended for deflection calculations taking into account potential shrinkage restraint cracking and construction loading as outlined in the following.

Restraint stress due to embedded bars and other sources of restraint can be accounted for by using a reduced effective cracking moment or modulus of rupture. A value equal to 2/3 of M_{cr} specified in the ACI 318-05 Building Code is recommended for use with the effective moment of inertia given by Eq. (1).

Construction load effects can be taken into account by using the effective moment of inertia corresponding to full dead load plus live load when calculating both the immediate deflection due to sustained load and immediate deflection due to live load. This simplifies calculations considerably because the effective moment of inertia needs to be computed for one loading case only.

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NOTATION

A_g = gross area of concrete section
A_s = area of reinforcing steel
b = beam or slab width
c = neutral axis depth of section
d = effective depth of tension reinforcement
E_c = elastic modulus of concrete
$\overline{E}_c$ = age-adjusted long-term modulus of concrete ($\overline{E}_c = E_c/(1 + \chi\phi)$)
E_{CFRP} = elastic modulus of carbon fiber-reinforced polymer (CFRP) bar
E_{GFRP} = elastic modulus of glass fiber-reinforced polymer (GFRP) bar
E_s = elastic modulus of reinforcing steel
e = eccentricity of reinforcement from centroidal location of gross cross section
F_c = tension force in concrete (from restraint to shrinkage)
F_s = compression force in steel reinforcement (from restraint to shrinkage)
f_c' = specified compressive strength of concrete
$f_{c,s}$ = concrete stress at level of reinforcement (from restraint to shrinkage)
f_r = rupture modulus of concrete
f_{re} = effective rupture modulus based on shrinkage restraint
f_{res} = stress from restraint to shrinkage
h = slab thickness or beam height
h_{min} = minimum thickness for deflection control
I_{cr} = cracked transformed moment of inertia
I_e = effective moment of inertia
I_g = gross (uncracked) moment of inertia
L = span length
M_a = service load moment at critical section
M_{cr} = cracking moment (unrestrained)
M'_{cr} = restrained cracking moment
M_n = nominal moment capacity
n = modular ratio (E_s/E_c)
$\overline{n}$ = age-adjusted long-term modular ratio ($E_s/\overline{E}_c$)
w_D = dead load (uniformly distributed)
w_{D+L} = dead plus live load (uniformly distributed)
$w_{D+L(sus)}$ = sustained (dead plus sustained live) load
w_L = live load (uniformly distributed)
$w_{L(inst)}$ = portion of live load that is not sustained
$w_{L(sus)}$ = sustained live load (uniformly distributed)
y_t = distance from centroidal axis of uncracked section to tension face of cross section
β = sustained load factor
χ = aging coefficient used in creep analysis
Δ_a = member deflection under service load
Δ_{cr} = member deflection just before cracking (based on uncracked section)
$\Delta_{i,L(inst)}$ = immediate (instantaneous) deflection from part of live load that is not sustained
Δ_{incr} = incremental deflection occurring after attachment of partitions ($\Delta_{lt} + \Delta_{i,L(inst)}$)
$\Delta_{i,sus}$ = immediate (short-term) deflection caused by sustained load
Δ_{lt} = additional long-term deflection ($\lambda\Delta_{i,sus}$)
ε_c = strain in concrete
$\varepsilon_{c,s}$ = strain in concrete at level of reinforcement
ε_s = strain in reinforcing steel
ε_{sh} = design or ultimate shrinkage strain
ϕ = creep coefficient
γ = neutral axis depth factor (c/h) for uncracked section
η = stiffness reduction factor ($1 - I_{cr}/I_g$)
λ = multiplier for long-term deflection
ρ = reinforcing ratio
ξ = eccentricity factor (e/h)

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APPENDIX—SHRINKAGE RESTRAINT STRESSES

This Appendix gives an adaptation of the rationale used by Gilbert (1999, 2003) to compute residual stress that develops in a reinforced concrete section when the reinforcement restrains shrinkage of the concrete.

Consider an uncracked rectangular section with a single layer of reinforcement as shown in Fig. A. Compressive stress develops gradually in the reinforcing steel as the concrete shrinks and the compressive force in the reinforcement F_s is resisted by a net tensile force F_c acting on the concrete section. The resulting tensile stress f_{res} that develops in the concrete reduces the applied moment needed to crack the member, such that

$$M'_{cr} = (f_r - f_{res})I_g/y_t \tag{A-1}$$

where the rupture modulus $f_r = 7.5\sqrt{f_c'}$, I_g is the gross moment of inertia and y_t is the distance from the centroidal axis of the cross section to the extreme fiber in tension.

The neutral axis of the concrete section is assumed to be located at midheight of the member (h/2) and the reinforcement has an eccentricity of $e = d - 0.5h = \xi h$ with $\xi = d/h - 0.5$ for a rectangular section. An exact analysis would include the effect of the concrete displaced by the reinforcing bars on the neutral axis location and net area of concrete.

Given that the compressive force F_s in the reinforcing steel is equilibrated by an equal and opposite tensile force $F_c = -F_s = -E_s A_s \varepsilon_s$ acting on the concrete section, the residual stress in the extreme tensile fiber of the concrete is given by

$$f_{res} = \frac{F_c}{A_g} + \frac{F_c e y_t}{I_g} = \frac{F_c}{bh} + \frac{F_c \xi h(h/2)}{(bh^3/12)} = \frac{F_c(1+6\xi)}{bh} \tag{A-2}$$

Hence, $F_c = f_{res}bh/(1 + 6\xi)$ and the tensile strain in the steel $\varepsilon_s = -F_c/E_sA_s$.

The stress in the concrete at the level of reinforcement is equal to

$$f_{c,s} = \frac{F_c}{bh} + \frac{F_c e(d-h/2)}{(bh^3/12)} = \frac{F_c(1+12\xi^2)}{bh} \tag{A-3}$$

Given that the strain in the concrete at the steel level equals the sum of the strain caused by stress $f_{c,s}/\bar{E}_c$ plus shrinkage strain ε_{sh}, then

$$\varepsilon_{c,s} = \frac{F_c(1+12\xi^2)}{\bar{E}_c bh} + \varepsilon_{sh} \tag{A-4}$$

Creep increases the elastic strain in the concrete and the age-adjusted long-term modulus $\bar{E}_c$ is used for concrete because the restraint stress develops gradually over time as the shrinkage increases. In this case, $\bar{E}_c = E_c/(1 + \chi\phi)$, where ϕ is the creep coefficient and χ is the aging coefficient.

Compatibility of strains at the level of reinforcement, $\varepsilon_{c,s} = \varepsilon_s$, gives a solution for F_c such that

$$\begin{aligned} &\frac{F_c(1+12\xi^2)}{\bar{E}_c bh} + \varepsilon_{sh} = -\frac{F_c}{E_sA_s} \Rightarrow \\ &F_c = \frac{-E_sA_s\varepsilon_{sh}}{1+\bar{n}\rho(d/h)(1+12\xi^2)} \end{aligned} \tag{A-5}$$

where $\varepsilon_s = -F_c/E_sA_s$, $\rho = A_s/bd$, and $\bar{n} = E_s/\bar{E}_c = n(1 + \chi\phi)$. Substitution of the force F_c into Eq. (A-2) gives a solution for the residual stress that develops in the concrete. Note that shrinkage is negative in this case.

$$f_{res} = -\frac{E_s\rho(d/h)(1+6\xi)\varepsilon_{sh}}{1+\bar{n}\rho(d/h)(1+12\xi^2)} \tag{A-6}$$

The d/h ratio for beams and slabs generally varies between 0.8 and 0.9. Assuming $d/h = 0.85$ gives $\xi = 0.35$, the residual stress f_{res} is then equal to

$$f_{res} = -\frac{2.635E_s\rho\varepsilon_{sh}}{1+2.1\bar{n}\rho} \tag{A-7}$$

Using an absolute value for shrinkage ε_{sh} and assuming 80% of the final shrinkage occurs before the beam cracks, gives

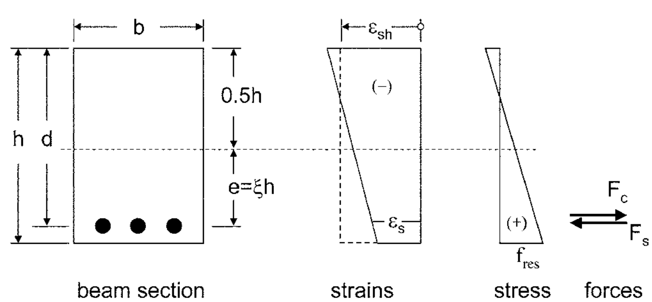

Fig. A—Section analysis for shrinkage.

$$f_{res} = \frac{2.635E_s\rho(0.8 \times \varepsilon_{sh})}{1 + 2.1\bar{n}\rho} \approx \frac{2.1\rho E_s\varepsilon_{sh}}{1 + 2.1\bar{n}\rho} \quad \text{(A-8)}$$

For a typical modular ratio of $n = 7.2$, creep coefficient $\phi = 2.25$, and $\chi = 0.8$, then $\bar{n} = n(1 + \chi\phi) = 7.2(1 + 0.8 \times 2.25) \approx 20$. This finally gives

$$f_{res} \approx \left(\frac{2.1\rho}{1 + 42\rho}\right)E_s\varepsilon_{sh} = \frac{45{,}700\rho}{1 + 42\rho} \text{ (psi)} \quad \text{(A-9)}$$

$$f_{res} \approx \frac{315\rho}{1 + 42\rho} \text{ (MPa)}$$

based on $E_s = 29 \times 10^6$ psi (200 GPa) and $\varepsilon_{sh} = 0.075\%$.

This is similar to the stress $f_{res} \approx 2.5\rho E_s\varepsilon_{sh}/(1 + 50\rho)$ given by Gilbert (2003) for a rectangular beam with $d/h = 0.9$. Note also that Murashev (Murashev et al. 1971) gives a very similar expression with $f_{res} = 2.25\rho E_s\varepsilon_{sh}/(1 + 2.25\bar{n}\rho)$.

The elastic modulus of fiber-reinforced polymer (FRP) reinforcing bars can be up to five times less than the stiffness of steel reinforcement and this reduces the residual stress that develops from restraint to shrinkage. For a glass FRP bar with $E = 5800$ ksi (40 GPa), $n = 1.45$, and $\bar{n} = 4$ using the assumptions made previously for long-term behavior. For $\varepsilon_{sh} = 0.075\%$ this then gives

$$f_{res} \approx \left(\frac{2.1\rho}{1 + 8.5\rho}\right)E_{GFRP}\varepsilon_{sh} = \frac{9135\rho}{1 + 8.5\rho} \text{ (psi)} \quad \text{(A-10)}$$

$$f_{res} \approx \frac{63\rho}{1 + 8.5\rho} \text{ (MPa)}$$

For carbon FRP (CFRP) bars with $E = 18{,}000$ ksi (124 GPa), $n = 4.5$, and $\bar{n} = 12.5$ using the assumptions made previously for long-term behavior. This gives

$$f_{res} \approx \left(\frac{2.1\rho}{1 + 26\rho}\right)E_{CFRP}\varepsilon_{sh} = \frac{28{,}350\rho}{1 + 26\rho} \text{ (psi)} \quad \text{(A-11)}$$

$$f_{res} \approx \frac{195\rho}{1 + 26\rho} \text{(MPa)}$$

assuming $\varepsilon_{sh} = 0.075\%$.

It should be noted that the aforementioned derivation applies to rectangular sections and is based on an equivalent restraint force acting on the concrete section alone. For nonrectangular sections with a neutral axis depth $c = \gamma h$ for the uncracked section and $e = \xi h$ with $\xi = d/h - \gamma$ ($\gamma = 0.5$ for a rectangular section)

$$f_{res} = \frac{F_c}{A_g}\left(1 + \frac{\xi h^2(1 - \gamma)}{I_g/A_g}\right) \text{ given that } y_t = h(1 - \gamma) \quad \text{(A-12)}$$

$$f_{c,s} = \frac{F_c}{A_g}\left(1 + \frac{\xi^2 h^2}{I_g/A_g}\right) \quad \text{(A-13)}$$

$$\varepsilon_{c,s} = \frac{F_c}{\bar{E}_c A_g}\left(1 + \frac{\xi^2 h^2}{I_g/A_g}\right) + \varepsilon_{sh} \quad \text{(A-14)}$$

$$F_c = \frac{-E_s A_s \varepsilon_{sh}}{1 + \bar{n}(A_s/A_g)(1 + \xi^2 h^2 A_g/I_g)} \quad \text{(A-15)}$$

$$f_{res} = -\frac{E_s(A_s/A_g)[1 + \xi h^2(1 - \gamma)(A_g/I_g)]\varepsilon_{sh}}{1 + \bar{n}(A_s/A_g)(1 + \xi^2 h^2 A_g/I_g)} \quad \text{(A-16)}$$