FLEXURAL REINFORCEMENT ESSENTIAL FOR PUNCHING SHEAR RESISTANCE OF SLABS

By Chandana Peiris and Amin Ghali

SYNOPSIS: The ductility and the strength of flat plate connections with their supporting columns are influenced by the concrete strength, the thickness of the slab and the shear and the flexural reinforcements. The present paper concentrates on the important effect of flexural reinforcement in the presence or the absence of shear reinforcement.

Keywords: ductility, flat-plates, punching shear, slab-column connections, strength, yield lines

Chandana Peiris: *is a PhD student in Department of Civil Engineering, University of Calgary, Alberta, Canada. He completed his BSc (2005) and MSc (2008) in Civil Engineering at University of Moratuwa, Sri Lanka.*

ACI Fellow Amin Ghali: *is professor emeritus, Civil Engineering, University of Calgary, Alberta, Canada. He is author of numerous papers and four books on analysis and design of concrete structures. He is member of ACI committees 421, Design of Reinforced Concrete Slabs; 343, Concrete Bridge Design and 435, Deflection of Concrete Building Structures*

INTRODUCTION

This paper is concerned with the design of non-pre-stressed concrete flat plates subject to shear. The critical zone for shear is at or near the connections of the flat plates with columns, where the flexure is also critical. Satisfying the code requirements for flexural reinforcement results in top reinforcement at the vicinity of the columns, whose ratio, ρ (=$A_s/(bd)$) is close to 0.008 or 0.01; lower or higher ratio can occur for small or large spans; where d = distance from extreme compression fibre to the centroid of flexural reinforcement; b = unit width.

The ACI 318[1] code requires that the reduced nominal shear strength $\phi_{shear}V_{shear}$ be equal or exceeds the factored shear force V_u at the shear critical section at $d/2$ from the column faces; where ϕ_{shear} = 0.75 = the shear strength reduction factor. The code specifies the shear stress v_n corresponding to the nominal strength by Equation 1 or Equation 2 in the absence or the presence of shear reinforcement, respectively:

$$\left.\begin{array}{l} v_n \equiv v_c = \text{the least of}: \quad \sqrt{f'_c}/3; \quad [2+(4/\beta)]\sqrt{f'_c}/12; \quad [(\alpha_s\, d/b_o)+2]\sqrt{f'_c}/12\ (\text{MPa}) \quad \text{or} \\ v_n \equiv v_c = \text{the least of}: \quad 4\sqrt{f'_c}; \quad [2+(4/\beta)]\sqrt{f'_c}; \quad [(\alpha_s\, d/b_o)+2]\sqrt{f'_c}\ (\text{psi}) \end{array}\right\} \qquad (1)$$

$$\left.\begin{array}{l} v_n = 0.17\sqrt{f'_c} + v_s \le 0.5\sqrt{f'_c}\ (\text{MPa}) \text{ or } v_n = 2\sqrt{f'_c} + v_s \le 6\sqrt{f'_c}\ (\text{psi}) \quad \text{Stirrups} \\ v_n = 0.25\sqrt{f'_c} + v_s \le 0.67\sqrt{f'_c}\ (\text{MPa}) \text{ or } v_n = 3\sqrt{f'_c} + v_s \le 8\sqrt{f'_c}\ (\text{psi}) \quad \text{Headed shear studs} \end{array}\right\} \qquad (2)$$

$$v_s = A_v\, f_{yt}/(b_o\, s) \qquad (3)$$

where f'_c = the specified compressive strength of concrete; β ($\ge$ 1.0) is the aspect ratio of column; α_s = 40, 30 or 20 for interior, edge or corner columns, respectively; A_v = the area of the vertical legs of the shear reinforcement on one peripheral line parallel to the column face; f_{yt} = the specified yield strength of the shear reinforcement; s = the spacing between peripheral lines of the shear reinforcement. At the critical section at $d/2$ from the outermost peripheral line of shear reinforcement, the nominal shear at punching failure is:

$$v_n = 0.17\sqrt{f'_c}\ (\text{MPa}) = 2\sqrt{f'_c}\ (\text{psi}) \qquad (4)$$

Equations 1 and 2 indicate that the nominal shear strength according to ACI 318 does not depend on the flexural reinforcement ratio. Eurocode 2[2] includes the flexural reinforcement ratio ρ; Figure 1 compares Eurocode 2 with ACI 318 for slabs having no shear reinforcement. The Eurocode 2 applies a reduction factor when d exceeds 8 in. (200mm).

In the absence of shear reinforcement, tests[3,4] show that the nominal shear strength given by Equation 1 cannot be reached in slabs having very low flexural reinforcement (e. g. when $\rho < 0.006$). This can occasionally occur in practice when the ratio of the thickness to the clear span is greater than the minimum specified in ACI 318. The purpose of the present research is to determine the minimum amount of flexural reinforcement that avoids shear failure at a shear stress lower than the strength, v_n predicted by ACI 318 (Equation 1 or Equation 2). The ACI 318 employs a modification factor reflecting the reduced mechanical properties of light-weight concrete relative to normal-weight concrete of the same f'_c; the modification factor (a multiplier of $\sqrt{f'_c}$) is not included here for simplicity of presentation.

RESEARCH SIGNIFICANCE

The ACI 318 code specifies the requirements for flat plates to possess the strength needed to resist a specified shear force. The requirements include the concrete strength, the geometry of the shear critical section and the shear reinforcement. However, recently published research shows that the strength predicted by the code cannot be experimentally reached when the flexural reinforcement in the column vicinity is not adequate. The present research explains why this occurs and develops the criterion for the design of the necessary flexural reinforcement that prevents premature punching. The published results of 49 physical tests and finite element analyses are used to calibrate the developed criterion.

STRENGTH DEPENDENCE ON FLEXURAL REINFORCEMENT RATIOS

Without Flexural reinforcement (ρ = 0), the nominal shear strength that can be considered in design is nil; this implies no reliance on arch action in flat plate design. Thus, a graph of V_n versus the flexural reinforcement ratio should start from zero, as shown in Figure 1 for Eurocode 2. However, in the practical zone of the graph, ρ close to 0.008 or 0.010, the strength predicted by the ACI 318 Code is not substantially different from Eurocode 2. With higher reinforcement ratio, a moderate increase in strength can be expected. Ignoring this increase as in ACI 318 simplifies the design. On the other hand, lower reinforcement ratio can result in a substantial drop in strength that cannot be ignored.

Compiled results of tests on slabs of thickness ≤ 12 in. (300 mm), having low flexural reinforcement ratio and no shear reinforcement, shows that punching occurs at a load V_{flex}, given by yield-line analysis. Muttoni[5] gives empirical equations for the angle of rotation and the width of critical crack at which failure occurs; the width is proportional to the effective depth d. Roughness of the crack surfaces increases the failure load. Muttoni's empirical equation predicts lower shear strength for thicker slabs and smaller aggregates. Punching of thick slabs can occur before the full creation of a yield line mechanism. The yield line analysis gives a reliable upper bound of the punching strength for slabs of any thickness with or without shear reinforcement. The punching strength of a slab 20 in. (500 mm) thick tested by Guandalini et al was 0.85 V_{flex}; the test value could be closely predicted by the empirical equations of Muttoni. Tests on slabs thicker than 12 in. (300 mm) are rare, making it difficult to calibrate with certainty the equations derived from tests. The upper bound of strength derived by yield line analysis is considered reliable, because it is based on equilibrium and compatibility.

The shear strengths predicted by codes are derived from tests such as the one shown in Figure 2-a; it represents a specimen of a square column connection with a simply supported square slab subjected to a shear force V. The nominal strength that the test gives is the smaller of:

$$V_{shear} = v_n\ b_o\, d \qquad \text{(Punching shear failure mode)} \qquad (5)$$

and

$$V_{flex} = 8\, m\, l_s / (l_1 - c) \qquad \text{(Flexural yield-line collapse mode; Figure 2b)} \qquad (6)$$

$$m = \rho\, f_y\, d^2 \left[1 - 0.59 \left(\rho\, f_y / f_c'\right)\right] \qquad (7)$$

where l_s, l_1 and c are dimensions shown in Figure 2b; m = the ultimate flexural strength of a strip of the slab of unit width in each of two orthogonal directions; ρ and f_y = the ratio of the top flexural reinforcement and its specified yield strength. The yield-line collapse assumes that yielding of the flexural reinforcement forms a mechanism of planar parts; the rotation of the parts relative to the column widens the crack surface adjacent to the column, reduces the shear resistance of aggregate interlock and eventually results in punching as a secondary cause of failure.

Based on this discussion, the nominal strength can hypothetically be taken equal to the smaller of $\phi_{flex}V_{flex}$ and $\phi_{shear}V_{shear}$; where ϕ_{flex}= 0.9 = the flexural strength reduction factor and ϕ_{shear} = 0.75 = the shear strength reduction factor, according to ACI 318. This hypothesis can be presented by a graph (Figure 3), whose abscissa represents the flexural reinforcement ratio, ρ ; the ordinate is the smaller of $[(\phi_{flex} V_{flex})/(b_o d\sqrt{f_c'})]$ and $[(\phi_{shear} V_{shear})/(b_o d\sqrt{f_c'})]$, respectively represented by lines OA and AB. In absence of shear reinforcement, with square column, having (c/d) smaller or equal to 4, the ordinate for line AB= 4.0 ϕ_{shear} when f_c' is in psi (or = 0.33 ϕ_{shear} when f_c' is in MPa); with stirrups or headed studs, the ordinate will be a constant value between 4.0 ϕ_{shear} and 6.0 ϕ_{shear} or between 4.0

ϕ_{shear} and 8.0 ϕ_{shear}, respectively when f_c' is in psi (between 0.33 ϕ_{shear} and 0.5 ϕ_{shear} or between 0.33 ϕ_{shear} and 0.67 ϕ_{shear}, when f_c' is in MPa). Line OA is approximately a straight line whose equation is:

$$V_{flex}/\left(b_0 d\sqrt{f_c'}\right)=\alpha_{flex}\,\rho \tag{8}$$

By approximating ($\rho f_y/f_c'$) by a constant, α_{flex} becomes a dimensionless parameter dependent upon the ratio ($\rho f_y/\sqrt{f_c'}$) and the geometry of the slab or the test setup:

$$\alpha_{flex}=\left[8l_s/(l_1-c)\right]\left[1-0.59\left(\rho f_y/f_c'\right)\right]\left(d/b_0\right)\left(f_y/\sqrt{f'_c}\right) \tag{9}$$

Equating reduced strengths ($\phi_{flex}\,V_{flex}$) and ($\phi_{shear}\,V_{shear}$), using Equation 5 and 8, gives:

$$\rho_{fs}=\left(\alpha_{flex}\right)^{-1}\left(v_n/\sqrt{f_c'}\right)\left(\phi_{shear}/\phi_{flex}\right) \tag{10}$$

This is the ratio of flexural reinforcement, in each of two orthogonal directions, that makes the shear force V_{flex} equal to the nominal shear strength permitted by ACI 318. When $\rho<\rho_{fs}$, the nominal shear strength that governs the design should be equal to a value, v_{govern}, smaller than v_n given by Equation 1 or Equation 2:

$$v_{govern}=v_n\left(\rho/\rho_{fs}\right) \tag{11}$$

Equations 6 and 8 are based on the test specimen and the yield-line pattern in Figure 2. Figure 4 represents the yield line pattern at the connection of an interior square column with a flat plate transferring shear force V_{flex}, whose value is given by:

$$V_{flex}=2\pi m/\left(1-2.8c/l\right) \tag{12}$$

In deriving Equation 12, the square column of side c is replaced by a circular column of the same area and zero bending moment is assumed at a circle of radius = 0.2 l; where l = the distance between the column centers in two orthogonal directions. The assumption of zero moment has the same effect as ignoring the bottom reinforcement in the vicinity of the column; elastic analysis of a plate subjected to uniform gravity load would give zero radial bending moment at approximately one-fifth the span[6].

In load-controlled tests (e. g. in the test represented in Figure 2), a monotonic increase of the applied force produces cracks and yielding of the flexural reinforcement adjacent to the column at an early stage. These cracks are commonly inclined because they are caused by a combination of flexure and shear. Well-anchored shear reinforcement, such as the vertical legs of stirrups or headed studs, control the widening of the cracks and delay the punching failure, causing an extension of the zone of in which the flexural reinforcement reaches yield; thus the test shows more ductility compared to a test without shear reinforcement. At punching, a sudden drop of the applied load is commonly recorded; but when the yield line pattern transforms the slab into a mechanism, no sudden increase in deflection or drop in the applied load occurs. For this reason, punching preceded by the development of a yield line mechanism can be reported as punching failure; while in fact punching is a secondary cause of failure and the primary failure is governed by the flexural strength; this is what can happen when the flexural reinforcement ratio is too small.

Muttoni's critical shear crack theory[5] attributes punching failure of slabs without shear reinforcement to the widening of inclined cracks, extending from the top of the slab and approaching the intersection of the bottom surface the column faces; the widening occurs when the slab rotates with respect to the column. The yield-line pattern creates a mechanism of slab parts that rotate freely and the cracks described by Muttoni widen causing the punching failure; this is considered here a secondary cause of failure, with the primary cause being the creation of the yield line pattern.

Appropriately designed shear reinforcement can delay failure until the development of a full yield pattern, exhibiting extensive ductility. For this to occur, the shear reinforcement should be designed to ensure that punching-shear strengths within and outside the shear-reinforced zone are greater than V_{flex}. The increase of the shear reinforcement beyond what is needed to satisfy these requirements does not change the strength or the ductility. On the other hand, a slab having no shear reinforcement can fail in punching without exhibiting much ductility when the flexural reinforcement ratio is large such that V_{flex} is much greater than V_{shear}; this is so because punching can occur before the extensive yielding of the flexural reinforcement associated with the yield-line mechanism.

Shear reinforcement such as the vertical legs of stirrups or headed studs intersect the inclined crack adjacent to the column faces and control its width. A yield line analysis gives an upper bound for the shear strength at the critical section adjacent to the column faces. However, slabs having well-anchored shear reinforcement, satisfying the requirements of ACI 318, will unlikely fail by punching at the shear critical section adjacent to the column. The yield line analysis is not relevant to the strength of the shear critical section outside the shear reinforcement zone.

Application of two equal and opposite horizontal forces at the tips of the column stubs, in Figure 2, combined with V, would transfer unbalanced moment, M between the column and the slab. If V is kept constant at a level below V_{shear} and M is gradually increased, a graph of $\phi_{flex} M_{flex} /(b_o d^2 \sqrt{f'_c})$ or $\phi_{shear} M_{shear} /(b_o d^2 \sqrt{f'_c})$ versus ρ would be similar to the graph of Figure 3; however, the yield line pattern at which flexural failure occurs will be different from that in Figure 2; M_{flex} = the unbalanced moment that produces a yield-line mechanism; M_{shear} = the unbalanced moment that causes punching shear; the present paper does not derive the graph. The remainder of the paper will calibrate the hypothetical graph of Figure 3 (or similar graphs) with published results of test specimens of slab-column connections transferring a shear force without unbalanced moment; some of the slabs have shear reinforcement. The calibration is also done using the results of finite element analyses.

YIELD LINE ANALYSIS OF PUBLISHED TESTS

The yield line patterns considered in this section are for tests whose results will be used to verify the proposed criterion. Figure 3 is the patterns of yield lines for square slabs, square columns and simply supported edges. Equation 6 will be used for analysis of the results of 26 slabs reported in references, 8, 9, 10 and 11.

Figure 5 is the yield line pattern for 11 square slabs, of side l_s tested by Guandalini et al[3]. The shear force corresponds to this yield line pattern is:

$$V_{flex} = \left\{ \frac{4\,l_s - c}{\left(l_s/\sqrt{2}\right)\cos(\pi/8) - c} + \frac{c}{\left(l_s/\sqrt{2}\right)\cos(\pi/8) - c/2} \right\} m \tag{13}$$

The yield line patters of 9 slabs tested by Birkle and Dilger[7] and one slab by Broms[12] is shown in Figure 6; the corresponding shear force is:

$$V_{flex} = \frac{16\,r_s \sin(\pi/8)}{r_1 - 2c/\pi}\, m \tag{14}$$

where, $2c/\pi$ equals radius of a circular column of perimeter same as that of a square column of side c; r_1 and r_s are dimensions shown in Figure 6.

Widianto et al[4] tested two square slabs supported at four points; the yield line pattern is shown in Figure 7; the corresponding shear force is:

$$V_{flex} = 4\frac{l_s - l_A - 2l_B}{l_1 - c}\, m + 4\frac{l_A}{l_1 - c}\, m_A + 8\frac{l_B}{l_1 - c}\, m_B \tag{15}$$

where l_1 is the distance between the centers of supports; l_s, l_A and l_B are defined in Figure 7; m_A, m_B and m are the flexural strength for unit width in the zones l_A, l_B and ($l_s - 2l_B - l_A$), respectively. Considering an average uniform flexural reinforcement throughout the slab also provide values of V_{flex} close to the values of V_{flex} obtain by Equation 15.

NOMINAL SHEAR STRENGTH OF TESTED SLABS

Table 1, Table 2 and Table 3 gives the main properties of 32 slabs without shear reinforcement and 17 slabs with shear reinforcement. All the slabs have been subjected to concentric shear force. The failure load, V_{test} is compared with V_{govern} , where V_{govern} is the smaller of V_{flex} and V_{shear}; where V_{flex} is the shear force that creates a yield line mechanism (Equation 8).

$V_{shear}= v_n b_0 d$, with v_n being the nominal shear stress at which punching occurs, according to ACI 318. The purpose of the comparisons of this section is to calibrate the hypothesis that the nominal strength, V_n that should be used in the design is equal to V_{govern} (reduced by ϕ_{flex} or ϕ_{shear}). Another way of stating the hypothesis is: The nominal shear strength can reach the value V_{shear} , given by the equations of ACI 318, only when the flexural reinforcement ratio $\rho \geq \rho_{fs}$, with ρ_{fs} being the reinforcement ratio that makes $V_{shear} = V_{flex}$. According to this hypothesis, a specimen having flexural reinforcement ratio $\rho \geq \rho_{fs}$ should be expected to resist a shear force $V_{shear} \geq V_{flex}$. Many tests on the literature are of this type; the conclusion commonly derived from such test is simply the code is conservative.

The values of the shear failure load, V_{test} are compared with V_{shear} and V_{flex} in Figure 8; where V_{shear} or V_{flex} are, respectively, values calculated in accordance with ACI 318 (Eq. (1) and (2)) or by yield line analysis. Figure 8 is a graph of (V_{flex}/V_{shear}) versus (V_{test}/V_{shear}) for 49 tests of which 17 slabs have shear reinforcement. The ratio (V_{flex}/V_{shear}) ≥ 1.0 indicates a slab for which the flexural reinforcement is sufficient to avoid creation of local yield line pattern that can induce premature punching before the load reaches V_{shear}; thus, for these slabs (V_{test}/V_{shear}) is expected to be ≥ 1.0 indicating that the code is safe or conservative; this is not the case for three relatively thick slabs ($h \geq 12$ in.); the reason can be the size effect[7], which is beyond the scope of the present paper. When (V_{flex}/V_{shear}) < 1.0, the hypothesis of the present paper predicts that V_{test} be less than the nominal strength V_{shear}. Figure 8 shows 9 slabs having the ratio (V_{test}/V_{shear}) < 1.0, because their ratio (V_{flex}/V_{shear}) < 1.0. Furthermore, the graph indicates that for the 9 slabs, the punching have occurred prematurely at a load that can be closely predicted by yield line analysis.

The slabs in Table 1, having no shear reinforcement are divided into 28 slabs of thickness ≤ 12 in. (300 mm) and 4 slabs of thickness 16 or 20 in. (400 or 500 mm). The ratio (V_{test}/V_{govern}) is greater than 1.0, verifying the hypothesis for 28 slabs of thickness not exceeding 12 in.; for one 12 in. slab tested by Birkle and Dilger (V_{test}/V_{govern}) = 0.9. For this slabs and for the two 20 in. slabs, V_{flex} given by yield line analysis can serve as upper bound of the nominal shear strength, but cannot predict the failure load.

For the slabs having shear reinforcement in Table 2, the yield line analysis gives V_{flex} applicable for the shear critical section adjacent to the column. The ACI 318 gives $V_{shear\ inner}$ and $V_{shear\ outer}$ for the shear critical section at $d/2$ from the column faces and from the outermost peripheral line of shear reinforcement, respectively. The smallest of the three shear strengths is listed as V_{govern} in column 14. The ratio (V_{test}/V_{govern}) is greater than 1.0 in all the listed slabs, indicating that the smallest V_{flex}, $V_{shear\ inner}$ and $V_{shear\ outer}$ reduced by ϕ_{flex} or ϕ_{shear} can be safely considered as nominal shear strength in design. It is noted that V_{flex} does not govern ($\rho > \rho_{fs}$) in all slabs of Table 2 except the first one. Similarly, when shear reinforcement is required in practice, the flexural reinforcement is likely greater than ρ_{fs}.

Table 3 highlights 9 tests having flexural reinforcement ratio $\rho \leq \rho_{fs}$. For this low ratio the yield line analysis gives a value V_{flex} very close to V_{test}. The mean and the standard deviation are 1.02 and 0.03, respectively.

Three tests having V_{test}/V_{govern} <0.9 were observed among the total analysed test results; PG-3[3], P500[8] and 10[7]. All these three specimens were having large slab thicknesses, 19.7, 21.7 and 11.8 in. respectively. Although the prediction of V_{shear} is independent from the thickness of the slab in ACI 318-11, several researchers and codes use a reduction factor to predict the strength of thick slabs. The discussion on the effect of thickness ("the size effect") is out of scope of this research.

EXPERIMENTAL VERIFICATION OF PREMATURE PUNCHING

The first slab listed in Table 1 was tested by the author and designed to verify that secondary failure by punching occurs when the yield line mechanism is created. The slab had no shear reinforcement; the flexural reinforcement ratio was 0.76 ρ_{fs}. Thus, the failure load in the test was expected to be equal to V_{flex} calculated by yield line analysis (Equation 6). The concrete dimensions of the specimens are given in Table 1. The measured values of f_c' and f_y and the results of the tests are given in Table 1. The results confirm that the secondary failure caused punching at V_{test} = 56.7 kip, while V_{shear} permitted by ACI 318 = 69.4 kip. The test also confirmed that V_{test} can be closely predicted by yield line analysis (V_{flex} = 54.4 kip).

Figure 9 compares the graphs of the load versus the central deflections measured in the test with the result of finite element analysis. Nonlinear 3D analysis software, ANACAP [13] was used. The finite element mesh for one-half specimen is shown in Figure 10. Several researches at University of Calgary have verified that ANACAP can predict the behaviour of slab-column connections with sufficient accuracy; this is here confirmed in Figure 9. The finite element software is used for the analysis of five slabs having variable flexural reinforcement ratio.

FINITE ELEMENT RESULTS

Table 4 gives a summary of the results of finite element analyses and compares the strengths obtained by the analyses with V_{flex} and V_{shear} . The values of f_c' and f_y used in the analysis are 4120 and 62900 psi, respectively (28.4 and 434 MPa). Slabs 1, 2 and 3 have flexural reinforcement ratio $\rho < \rho_{fs}$; equation 10 gives: ρ_{fs} = 0.56%. The failure load for the three tests is expected to be governed by V_{flex} determined by Equation 8. Column 7 compares the failure load determined by the finite element analysis, V_{FEA} with V_{flex}; the mean and the standard deviation for the ratio V_{FEA}/V_{flex} for the three tests are = 0.96 and 0.07, respectively.

EXAMPLE

Find the nominal shear strength for a flat plate of thickness 8 in. (203 mm), transferring a concentric shear force to a square column of side c = 12 in. (305 mm). The top flexural reinforcement ratio, in each of two orthogonal directions above the column, ρ = 0.006. The slab has no shear reinforcement.

Given data: centre-to-centre distance between adjacent columns in orthogonal directions $l = 18$ ft (5.5 m); f_c' = 4000 psi (28 MPa); f_y = 60 ksi (410 MPa); use ACI 318 equation for v_n (Equation 1 of present paper); ϕ_{flex} = 0.9; ϕ_{shear} = 0.75; d = 6.50 in. (165 mm) and b_0 = 74 in. (1900 mm).

The ultimate flexural strength per unit length with flexural reinforcement ratio ρ_{fs} (Equation 7);

$$m = \rho_{fs}\, f_y\, d^2 \left[1 - 0.59\left(\rho_{fs}\, f_y / f_c'\right)\right]$$

$$m = \rho_{fs}\,(60000)(6.5)^2\left[1 - 0.59\left(\rho_{fs}\, 60000/4000\right)\right] = 2410000\,\rho_{fs}\ \text{lb}$$

The term within the square bracket is ≃ 0.95. Equating the reduced V_{flex} to the reduced V_{shear} , using Equations 1, 2 and 5 gives;

$$0.9\ V_{flex} = 0.75\ V_{shear}$$

$$0.9(2m\pi)(1 - 5.4c/l) = 0.75 v_n b_0 d$$

$$0.9(2\pi)[1 - 5.4(12/216)]m = 0.75\left(4\sqrt{f_c'}\right)(74)(6.5)\ \text{(lb and in. units)}$$

$$\rho_{fs} = 0.0096$$

The nominal strength;

$$V_n = \left(\rho/\rho_{fs}\right)\phi_{shear} V_{shear}$$

$$V_n = (0.0060/0.0096)(0.75)\left[\left(4\sqrt{f_c'}\right)(74)(6.5)\right] = 57.1\ \text{kip (254 kN)}$$

CONCLUSION

Analysis of the results of 49 tests of several researches show that shear failure can occur at a load V_{test} smaller than the nominal shear strength V_{shear} permitted by ACI 318. In the majority of tests $V_{test} > V_{shear}$; in 9 tests V_{test} is close to V_{flex} ; where V_{flex} = the shearing force that creates a flexural yield line mechanism. Thus it is concluded that the design of the plates should consider that the nominal shear strength is equal to the smaller of V_{shear} and V_{flex}. The equations for V_{shear} are given in the code ACI 318. Equations for V_{flex} are given for interior columns. Similar equations are needed for the general cases of interior edge and corner columns transferring shearing forces and unbalanced moments.

NOTATION

A_s = area of longitudinal tension reinforcement

A_v = area of vertical legs of shear reinforcement on one peripheral line parallel to the column face

b_0 = perimeter of the shear critical section

c = side length of a square column

d = distance from extreme compression fibre to the centroid of flexural reinforcement

D = deflection of the slab at the column perpendicular to the plane of the slab

f'_c = the specified compressive strength of concrete

f_y = specified yield strength of the top flexural reinforcement

f_{yt} = specified yield strength of the shear reinforcement

l_A, l_B = zones with different reinforcement ratios in the slab of the test specimen

l_s = length of a side of a square test specimen

l_1 = distance between the supports of test specimen

m = ultimate flexural strength of a strip of the slab of unit width

m_A, m_B = ultimate flexural strength of a strip of the slab of unit width within l_A and l_B, respectively

M_{flex} = the unbalanced moment that produces a yield-line mechanism

M_{shear} = the unbalanced moment that causes punching shear

r_s, r_1 = refer Figure 6

s = spacing between peripheral lines of the shear reinforcement

v_c = nominal shear stress provided by the concrete

v_{govern} = smaller of v_n and $v_n(\rho/\rho_{fs})$

v_n = nominal shear stress

v_s = nominal shear stress provided by the shear reinforcement

V_c = nominal shear strength provided by the concrete

V_{FEA} = shear capacity recorded in finite element analysis

V_{flex} = shearing force that creates a flexural yield line mechanism

V_s = nominal shear strength provided by the shear reinforcement

V_{shear} = nominal shear strength according to the ACI 318-11

V_u = factored shear force

α_{flex} = dimensionless parameter to calculate the flexural yield line strength of the slab which depends on the dimensions of the specimen, support condition and $\rho f_y / \sqrt{f'_c}$ ratio

α_s = a constant use to calculate v_c of slabs

β = ratio of long to short dimensions of column

ρ = ratio of A_s to bd

ρ_{fs} = value of ρ when $V_{flex} = V_{shear}$

ϕ_{flex} = strength reduction factor for flexure (= 0.9)

ϕ_{shear} = strength reduction factor for shear (= 0.75)

REFERENCES

1. ACI 318-11, "Building Code Requirements for Structural Concrete and Commentary", American Concrete Institute, Farmington Hills, MI, 2011.

2. Eurocode 2, "Eurocode 2: Design of Concrete Structures – Part 1-1: General Rules and Rules for Buildings", European Committee for Standardization, Brussels, Belgium, 2003

3. Guandalini, S., Burdet, O.L. and Muttoni, A., "Punching of Slabs with Low Reinforcement Ratios", *ACI Structural Journal*, January- February 2009, V106.

4. Widianto, Bayrak, O. and Jirsa, J. O., "Two-Way Shear Strength of Slab-Column Connections: Reexamination of ACI 318 Provisions", ACI *Structural Journal*, March-April 2009, Vol 106, No2, pp 160-170.

5. Muttoni, A., "Punching shear strength of Reinforced Concrete Slabs without Transverse Reinforcement", *ACI Structural Journal*, July-August 2008, pp440-450.

6. Timoshenko, S. P. and Woinowsky-Krieger, S., *Theory of Plates and Shells*, 2nd edition, McGraw-Hill, NY, 1959, p 580

7. Birkle, G. and Dilger, W., "Influence of Slab Thickness on Punching Shear Strength", *ACI Structural Journal*, March-April 2008, Vol 105, No2, pp 180-188.

8. Li, K.K.L., Influence of Size on Punching Shear Strength of Concrete Slabs, M. Eng. Thesis, Department of Civil Engineering and Applied Mechanics, McGill University, Montreal, Quebec, 2000.

9. Mokhatar, A., "Design of Stud Shear Reinforcement for Concrete Flat Plates", PhD Thesis, Department of Civil Engineering, University of Calgary, 1982.

10. Pisanty, A., "Euro Codes and North American Codes Predictions of Punching Shear Capacity in View of Experimental Evidence", *SP-232*, American Concrete Institute, Farmington Hills, 2005, MI, PP 39-55.

11. Stein, T., Ghali, A. and Dilger, W., 2007, "Distinction Between Punching and Flexural Failure Modes of Flat Plates", *ACI Structural Journal*, May-June 2007, Vol 104, No3, pp 357-365.

12. Broms, C. E., "Ductility of Flat Plates: Comparison of Shear Reinforcement Systems", *ACI Structural Journal*, November-December 2007, Volume 104, No6.

13. ANATECH consulting Engineers (ANATECH), *Manual of concrete analysis program ANACAP, Version 3.0, User's Manual*, San Diego, August 2004.

Table 1− *Specimens without shear reinforcements subjected to concentric punching.*

1	2	3	4	5	6	7	8	9	10	11	12	13	14**	15
Reference	ID	h (in.)	d (in.)	c (in.)	l_1 (in.)	l_s (in.)	ρ (%)	f'_c (psi)	f_y (psi)	V_{flex} (kip)	V_{shear} (kip)	V_{test} (kip)	V_{govern} (kip)	V_{test}/V_{govern}
Slabs with thickness ≤ 12 in. (300 mm)														
–*	S-1	5.9	4.6	9.8	70.9	74.8	0.43	4120	62900	54.4	69.4	56.7	54.4	1.04
Li [8]	P100	5.3	3.9	7.9	28.5	36.4	0.97	5710	70800	139	56.4	74.2	56.4	1.31
	P150	7.5	5.9	7.9	39	46.9	0.90	5710	67400	239	98.8	131	98.8	1.33
	P200	9.4	7.9	7.9	49.2	57.1	0.83	5710	67400	361	151	203	151	1.35
Mokhatar [9]	AB1	5.9	4.5	9.8	70.9	74.8	1.40	5250	74800	186	75.7	91.7	75.7	1.21
Pisanty [10]	140/1	5.5	4.4	7.9	63	66.9	1.31	3830	58000	126	53.8	87.7	53.8	1.63
	140/2	5.5	4.4	7.9	63	66.9	1.31	3310	58000	124	50.0	79.8	50.0	1.60
	160/1	6.3	5.2	7.9	63	66.9	0.96	3630	58000	134	66.4	84.5	66.4	1.27
	160/2	6.3	5.2	7.9	63	66.9	0.96	2760	58000	130	57.9	100	57.9	1.73
	180/1	7.1	5.9	9.8	63	66.9	1.18	3380	58000	214	87.6	131	87.6	1.49
	180/2	7.1	5.9	9.8	63	66.9	1.18	3700	58000	216	91.6	136	91.7	1.49
	200/1	7.9	6.7	11.8	63	66.9	1.04	3500	58000	257	119	188	119	1.58
	200/2	7.9	6.7	11.8	63	66.9	1.04	3160	58000	254	113	185	113	1.64
Guandalini et al [3]	PG-1	9.8	8.3	10.2	59.1	59.1	1.50	4000	83100	504	155	232	155	1.49
	PG-2b	9.8	8.3	10.2	59.1	59.1	0.25	5870	80000	94.2	188	98.9	94.2	1.05
	PG4	9.8	8.3	10.2	59.1	59.1	0.25	4670	78400	91.9	168	91.9	91.9	1.00
	PG-5	9.8	8.3	10.2	59.1	59.1	0.33	4250	80500	122	160	127	122	1.03
	PG-10	9.8	8.3	10.2	59.1	59.1	0.33	4130	83700	126	158	121	126	0.95
	PG-11	9.8	8.3	10.2	59.1	59.1	0.75	4570	82700	276	166	174	166	1.05
	PG-6	4.9	3.8	5.1	29.5	29.5	1.50	5030	76300	99.1	38.2	53.5	38.2	1.40
	PG-7	4.9	3.9	5.1	29.5	29.5	0.75	5030	79800	61.1	40.7	54.4	40.7	1.34
	PG-8	4.9	4.6	5.1	29.5	29.5	0.28	5030	76100	30.8	51.0	31.4	30.8	1.01
	PG-9	4.9	4.6	5.1	29.5	29.5	0.22	5030	76100	24.5	51.0	26.0	24.5	1.06
Birkle and Dilger [7]	1	6.3	4.9	9.8	39.4	44.3	1.54	5250	70800	187	83.6	109	83.6	1.30
	7	9.1	7.5	11.8	59.1	62.4	1.30	5080	77000	367	165	186	165	1.12
	10	11.8	10.2	13.8	74.8	77	1.10	4550	76000	558	266	235	266	0.88
Widianto et al [4]	G0.5	6	5	16	68	168	0.37†	4550	60900	67.6	114	69.9	67.6	1.03
	G1.0	6	5	16	68	168	0.49†	4080	60900	92.1	108	90.2	92.1	0.98
Slabs with thickness ≥ 12 in. (300 mm) and ≤ 20 in. (500 mm)														
Li [8]	P300	13.6	11.8	7.9	69.9	77.8	0.76	5710	67900	683	282	310	282	1.1
	P400	17.7	15.7	11.8	69.9	77.8	0.76	5710	62800	1210	527	500	527	0.95
Slabs with thickness ≥ 20 in. (500 mm)														
Li [8]	P500	21.7	19.7	11.8	69.9	77.8	0.76	5710	62800	1880	753	603	753	0.8
Guandalini et al [3]	PG-3	19.7	18	20.5	112	118.1	0.33	4700	75400	579	759	486	579	0.84
Average														1.22

*Test by present authors done for this research.
** = the smaller of V_{flex} and V_{shear}.
† average ρ = (total area of the reinforcement in one direction)/($d\ l_s$)
1 psi = 0.00689 MPa; 1 kip = 1000 lb = 4.448 kN; 1 in. = 25.4 mm

Table 2− *Specimens with shear reinforcements subjected to concentric punching*.

1	2	3	4	5	6	7	8	9	10	11	12	13	14**	15
Reference	ID	h (in.)	d (in.)	c (in.)	l_1 (in.)	l_s (in.)	ρ (%)	f_c' (psi)	f_y (psi)	V_{flex} (kip)	V_{shear} (kip)	V_{test} (kip)	V_{govern} (kip)	V_{test} / V_{govern}
Stein et al [11]	V1	5.9	4.6	9.8	70.9	74.8	0.45	4310	63500	58.1	77.8	74	58.1	1.27
	V2	5.9	4.6	9.8	70.9	74.8	0.98	3800	63500	119	73.1	98.5	73.1	1.35
	V3	5.9	4.6	9.8	70.9	74.8	0.62	3730	63500	78.2	72.4	82.1	72.4	1.13
Mokhatar [9]	AB2	5.9	4.5	9.8	70.9	74.8	1.4	5470	74800	187	87.0	117	87.0	1.34
	AB3	5.9	4.5	9.8	70.9	74.8	1.4	3340	74800	172	90.6	123	90.6	1.35
	AB4	5.9	4.5	9.8	70.9	74.8	1.4	5970	74800	189	121	131	121	1.08
	AB5	5.9	4.5	9.8	70.9	74.8	1.4	5710	74800	188	118	131	118	1.11
	AB6	5.9	4.5	9.8	70.9	74.8	1.4	4150	74800	179	94.6	122	94.6	1.29
	AB7	5.9	4.5	9.8	70.9	74.8	1.4	5100	65000	164	112	129	112	1.15
	AB8	5.9	4.5	9.8	70.9	74.8	1.4	4330	65000	160	85.4	114	85.4	1.34
Birkle and Dilger [7]	2	6.3	4.9	9.8	39.4	44.3	1.54	4210	70800	180	106	129	106	1.22
	9	9.1	7.5	11.8	59.1	62.4	1.3	5100	77000	367	199	245	199	1.24
	12	11.8	10.2	13.8	74.8	77	1.1	4860	76000	562	331	342	331	1.03
	4	6.3	4.9	9.8	39.4	44.3	1.54	5510	70800	188	84.6	143	84.6	1.68
	8	9.1	7.5	11.8	59.1	62.4	1.3	5080	77000	367	186	236	186	1.27
	11	11.8	10.2	13.8	74.8	77	1.1	4350	76000	554	293	364	293	1.24
Broms [12]	18a	7.1	5.6	11.8	47.8	55.1	1.29	5570	81600	241	173	193	173	1.12

** = the smaller of V_{flex} and V_{shear}.
1 psi = 0.00689 MPa; 1 kip = 1000 lb = 4.448 kN; 1 in. = 25.4 mm

Table 3− *Results of the tests with* $V_{flex} < V_{shear}$. *Slab thickness* < 12 in. (300 mm).

1	2	3	4	5	6	7	8	9	10	11	12	13	14 **	15
Reference	ID	h (in.)	d (in.)	c (in.)	l_1 (in.)	l_s (in.)	ρ (%)	f_c' (psi)	f_y (psi)	V_{flex} (kip)	V_{shear} (kip)	V_{test} (kip)	V_{govern} (kip)	V_{test}/V_{flex}
– *	S-1	5.9	4.6	9.8	70.9	74.8	0.43	4120	62900	54.4	69.4	56.7	54.4	1.04
Guandalini et al [3]	PG-2b	9.8	8.3	10.2	59.1	59.1	0.25	5870	80000	94.2	188	98.9	94.2	1.05
	PG4	9.8	8.3	10.2	59.1	59.1	0.25	4670	78400	91.9	168	91.9	91.9	1.00
	PG-5	9.8	8.3	10.2	59.1	59.1	0.33	4250	80500	122	160	127	122	1.03
	PG-10	9.8	8.3	10.2	59.1	59.1	0.33	4130	83700	126	158	121	126	0.95
	PG-8	4.9	4.6	5.1	29.5	29.5	0.28	5030	76100	30.8	51.0	31.4	30.8	1.01
	PG-9	4.9	4.6	5.1	29.5	29.5	0.22	5030	76100	24.5	51.0	26.0	24.5	1.06
Widianto et al [4]	G0.5	6	5	16	68	168	0.37†	4550	60900	67.6	114	69.9	67.6	1.03
	G1.0	6	5	16	68	168	0.49†	4080	60900	92.1	108	90.2	92.1	0.98
Average														1.02
Standard deviation														0.03

*Test by present authors done for this research.
** = the smaller of V_{flex} and V_{shear}.
1 psi = 0.00689 MPa; 1 kip = 1000 lb = 4.448 kN; 1 in. = 25.4 mm

Table 4− *Finite element analysis results.* f_c' = 4120 psi (28.4 MPa)*;* f_y= 62900 psi (434 MPa) ; d = 4.6 in. (118 mm); h = 5.9 in. (150 mm); c = 9.8 in. (250 mm). *Dimensions are same as the test of the authors* (Table 1).

1	2	3	4	5	6	7	8
ID	ρ (%)	V_{FEA} (kip)	V_{flex} (kip)	V_{shear} (kip)	V_{govern} (kip)	V_{FEA}/V_{flex}	*Average of* V_{FEA}/V_{flex}
1	0.315	43.4	41.4	70.7	41.4	1.05	
2	0.42	52.1	54.7	70.7	54.7	0.95	
3	0.52	59.4	67.2	70.7	67.2	0.88	= 0.96
4	1.05	74.0	129.2	70.7	70.7	0.57	
5	1.26	78.3	151.9	70.7	70.7	0.52	

1 psi = 0.00689 MPa; 1 kip = 1000 lb = 4.448 kN; 1 in. = 25.4 mm

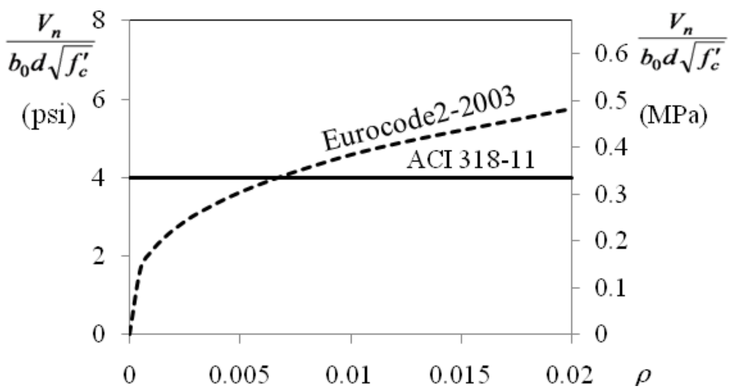

Figure 1− *Nominal shear strength, V_n versus flexural reinforcement ratio, ρ according to ACI 318 and Eurocode 2. Slab without shear reinforcement.* f_c' = 4000 psi (27.6 MPa); f_y = 60000 psi (414 MPa); d = 8 in. (203 mm); c =12 in. (304 mm); b_0 = 80 in. (2030 mm)

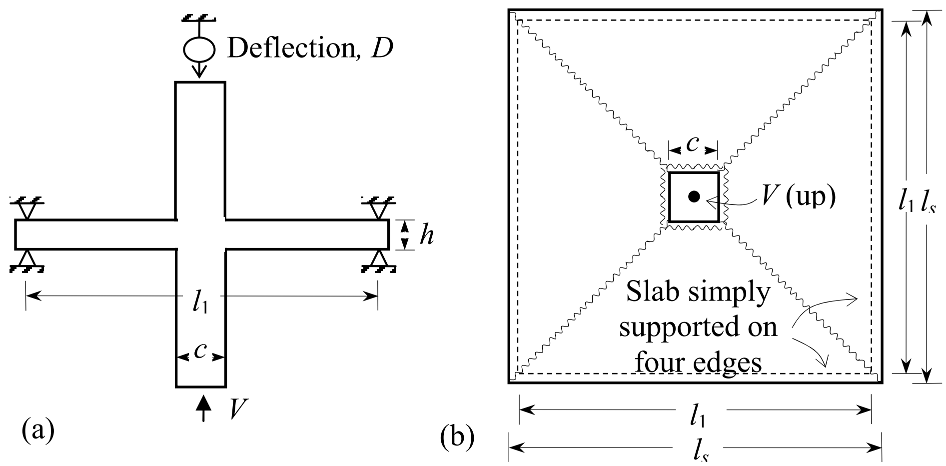

Figure 2− *Test specimen to develop empirical shear strength equations. (a) The test setup. (b) Yield line pattern.*

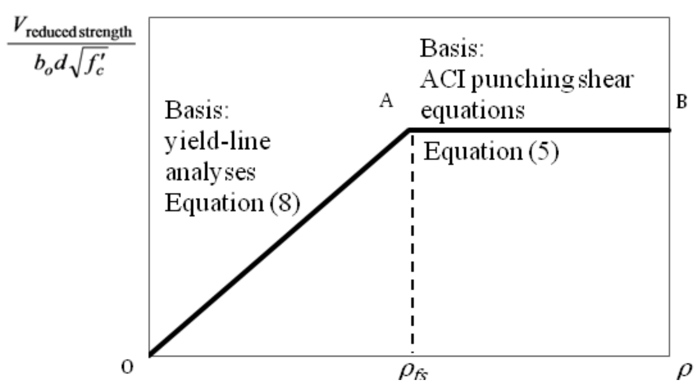

Figure 3− *Reduced shear strength (normalized) versus flexural reinforcement ratio, ρ. A proposed design criterion based on ACI 318 combined with yield-line analysis. The reduced shear strength (normalized) = the smaller of* $(\phi_{flex} V_{flex})/(b_o d\sqrt{f_c'})$ *and* $(\phi_{shear} V_{shear})/(b_o d\sqrt{f_c'})$.

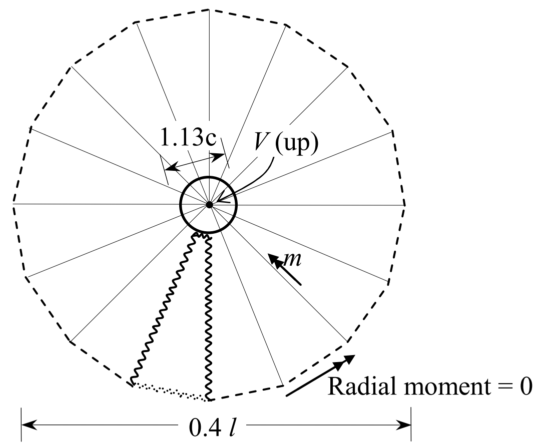

Figure 4– *Yield line pattern at the connection of an interior square column with a flat-plate transferring a shear force. The square column of side c is substituted by a circular column of diameter* = 1.13 *c*.

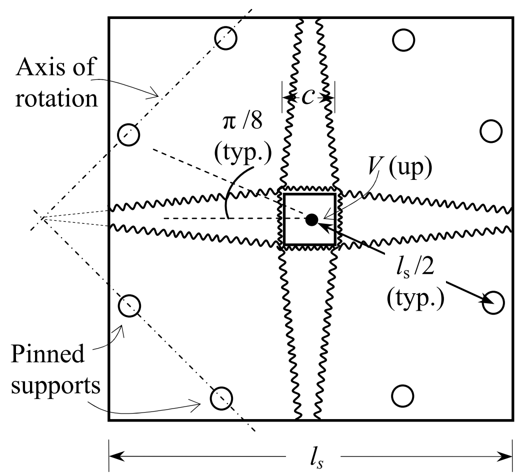

Figure 5– *Yield line patters for the test specimens of Guandalini et al* [3].

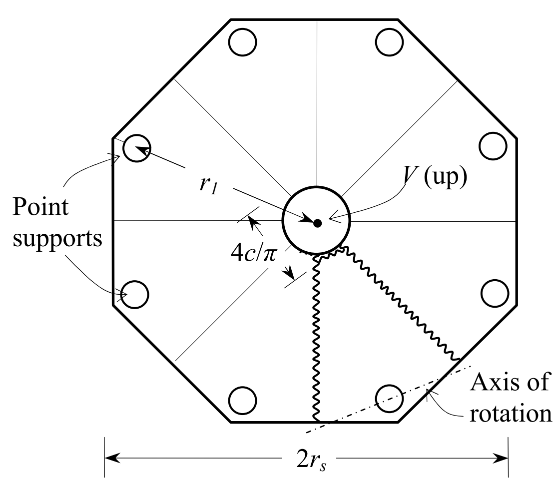

Figure 6– *Yield line pattern for an octagonal slab specimen having a circular column subjected to concentric force.*

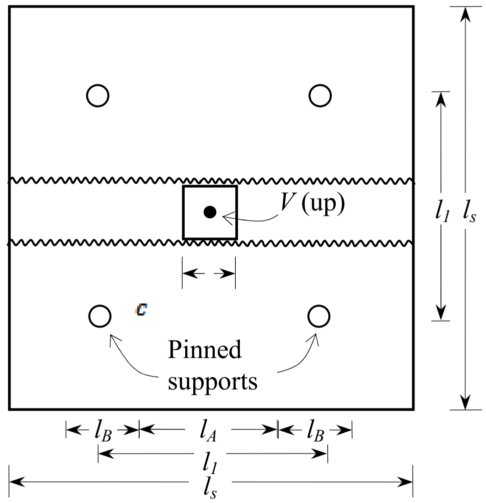

Figure 7− *Yield line patterns for the test specimens of Widianto et al*[4].

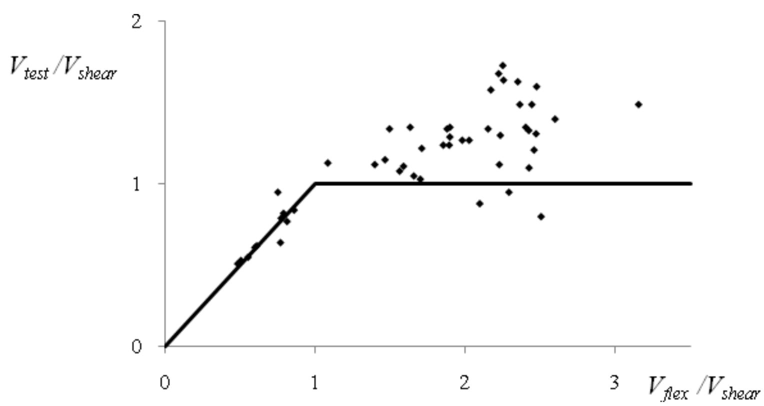

Figure 8− *Results of concentric punching shear tests*

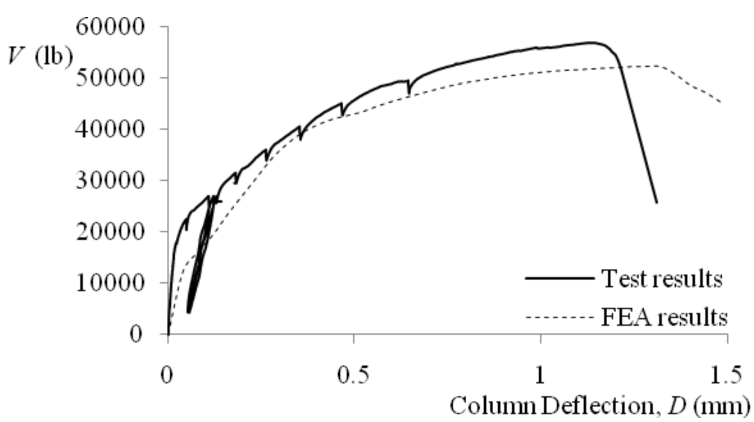

Figure 9− *Comparison between the results of the test by authors and FEA*

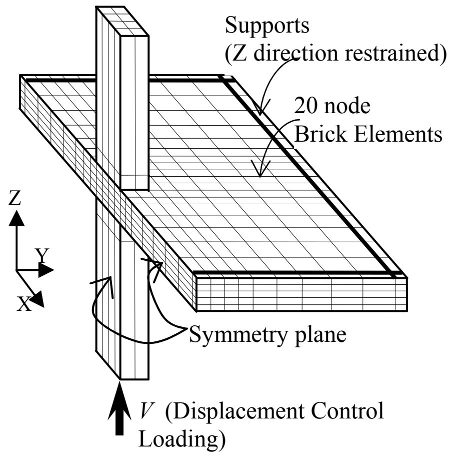

Figure 10− *Finite element mesh for half specimen*

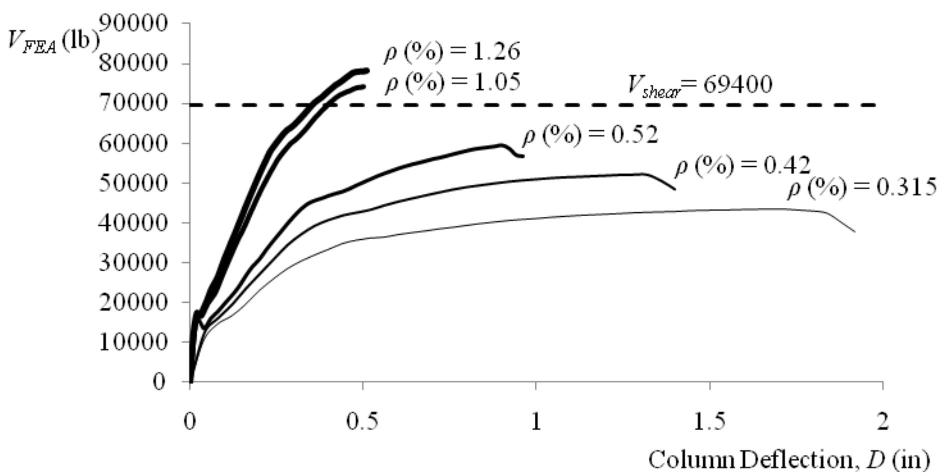

Figure 11− *Finite element analysis results.* f_c' = 4120 psi (28.4 MPa)*;* f_y = 62900 psi (434 MPa) *;* d = 4.6 in. (118 mm)*;* h = 5.9 in. (150 mm); c = 9.8 in. (250 mm).

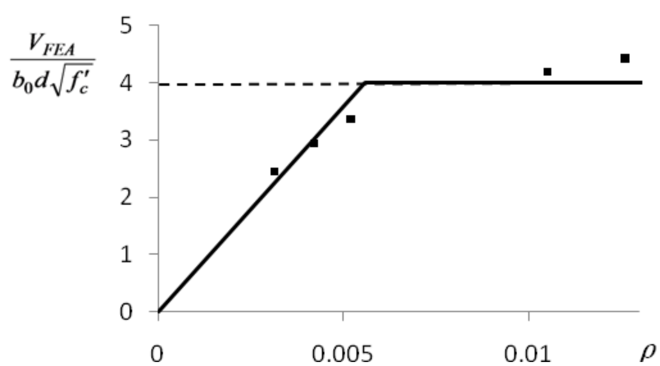

Figure 12− *Finite element analysis results*