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A Study of Static and Dynamic Modulus of Elasticity of Concrete

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Project Objectives

The objectives of this work are:

(i) to obtain a better understanding of dynamic tests (based on vibration and wave propagation) used to compute the dynamic Young's Modulus (E_d) and to improve application by establishing consistent and accurate formulas to compute E_d and
(ii) to better understand the relationship between the static Young's modulus (E) in concrete and E_d. This project started formally on October 1, 2006 and was completed on September 30, 2007.

Project Deliverables

The following deliverables were provided by this project:

- Three progress reports, delivered to CRC in Fall 2006, Fall 2007 and Spring 2008;
- an oral technical presentation of progress to date to the CRC committee meeting during the Spring 2007 ACI conference in Atlanta;
- an oral technical presentation of overall progress to ACI Committee 363 (High Strength) during the Spring 2008 ACI conference in Los Angeles;
- this final report, which lists accomplishments achieved during the project. All three investigators contributed to the preparation and review of the oral presentations and final report.

Background and significance of work to concrete industry

The static Young's modulus (E) is defined as the ratio of the axial stress to axial strain for a material subjected to uni-axial load [Neville 1997]. It is important that E of concrete be known because engineers increasingly use this value in the structural design process. For example, E is needed to analyze the cross-sectional response of a reinforced concrete beam [Leet and Bernal 1997]. In recent years building specifications have even required a specific E of concrete to be met, mostly to limit excessive deformation and sway in tall buildings. For example, the designer of the Two Union Square Building in Seattle required concrete with E of at least 50 GPa [Godfrey 1987].

However, once a structure is erected the in situ elastic properties cannot be measured directly without damaging the structure itself. Most often E is inferred from the compressive strength (f_c) of companion cylinders, rather than being measured directly, through the application of established empirical relations. This approach often leads to overly conservative results because, in order to meet the minimum E requirement, concrete with much higher f_c is used than the specification requires, which leads to unnecessarily high material costs. Enhanced understanding of the relation between E and compressive strength, with respect to different types of concrete, would improve the efficacy of the estimation of E from strength.

Non-destructive dynamic methods can be used to estimate in-place E, but the meaning of the obtained dynamic modulus is uncertain because E_d is known to be different (higher) from that obtained by direct static testing of a cylinder drawn from the structure. Concrete is expected to show a nonlinear dependence between stress and strain, even at low values of deformation caused by quasi-static tests and dynamic tests based on stress-wave propagation [Powers 1938; Bell 1984]. Quasi-static experiments show a nonlinear dependence between stress and strain even at infinitesimal values of deformation for a wide range of materials (metal, stone, concrete, wood, glass, polymers, etc.). The nonlinear stress-strain relationship for concrete is well described by a quadratic parabola [Shkolnik 1996]. The non-linear behavior provides the bases for the conventionally accepted view why E_d is higher than E, since concrete is subjected to very small strains in dynamic testing. [Neville 1997]. Although this argument is conceptually satisfying, experimental test results show that this may not in fact be the the case: low-strain static test data agree with higher strain (stressed to 40% of ultimate) static test data, as shown in Figure 1. Therefore it is important to understand precisely how and why the E and E_d are related to each other.

Furthermore, it is known that E_d values for a given concrete obtained by different dynamic tests do not agree with each other [Philleo 1955]. In general, E_d obtained from pulsed wave propagation measurements are significantly higher than those obtained from vibration resonance measurements carried out on the same specimen. E_d varies significantly even within one type of measurement: E_d computed from vibration resonance of prismatic beams is known to be, on average, significantly higher than that computed from cylinders for the same concrete mixture.
The work will have significant impact on the technical community since more accurate monitoring and in-place control of concrete E would be enabled. As such the work would likely influence existing testing standards, concrete design specifications, concrete quality control, inspection of structures, and development of commercial NDE equipment. The work includes significant in-kind contributions from a certified construction materials testing firm and a concrete ready-mix supplier.

Scope of experimental tests

Test samples

The investigating team members received from CTLGroup a test data matrix from over 200 high strength concrete samples (150mmx300mm and 100mmx200mm cylinders). Measured compressive strength of these samples ranged from 24MPa to 161MPa, E from 25.3GPa to 61.7GPa, and testing age from 4 days to 730 days.

A range of concrete cylinder (100mmx200mm) and beam (150mmx150mmx530mm) samples were additionally cast at the laboratories of the University of Illinois. The concrete mixture proportions varied to include two different w/c, two different coarse aggregate types, with both air-entrained and non-air entrained conditions. Multiple samples were cast from each mixture. In addition, identically-shaped cylinder and beams samples comprised of nominally linearly-elastic and homogeneous material (aluminum) were obtained and tested.

Test methods

These samples were tested using the standard static loading method to determine Young's modulus of elasticity specified in ASTM C469 - 02e1, Standard Test Method for Static Modulus of Elasticity and Poisson's Ratio of Concrete in Compression (C469). The standard compressive strength of most of the cylindrical specimens was also measured, following standard procedure ASTM C39 / C39M - 05e2, Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens (C39). Resonance frequencies can be used to estimate E knowing the dimensions and mass of the specimen. The dynamic modulus values were obtained using the standard longitudinal and flexural vibration method specified in ASTM C215 – 02, Standard Test Method for Fundamental Transverse, Longitudinal, and Torsional Frequencies of Concrete Specimens (C215) (see Figure 2 for testing configuration), and through thickness ultrasonic pulse velocity (UPV) measurements specified by ASTM C597 – 02, Standard Test Method for Pulse Velocity Through Concrete (C597). An estimate of Poisson's ratio is needed to obtained E_d from the transverse resonance method in C215 and the UPV value from C597; an estimate of Poisson's ratio is not needed for the longitudinal resonance method in C215. The formula that relates longitudinal vibrational resonance frequency to E_d in ASTM C215 is based on basic one-dimensional, plane-section motion assumptions, whereas that for the flexural vibrations do account for rotary inertia effects.

E_d can be computed from P-wave velocity (V_P) using

$$E = \frac{V_P{}^2\, r(1+n)(1-2n)}{(1-n)}$$

where ρ is mass density and ν is Poisson's ratio.

Two modified vibration methods were also used: the two frequency method for cylinders [Subramaniam et al. 2000] and the Love's correction method for cylinders [Love 1944]. The two frequency method obtains an accurate estimate of both the dynamic Poisson's ratio and the dynamic modulus of elasticity from one vibration measurement where the frequencies of the first two longitudinal modes of a cylindrical specimen are measured:

$$u_d = A_1\left(\frac{f_2}{f_1}\right)^2 + B_1\left(\frac{f_2}{f_1}\right) + C_1$$

where $?_d$ = dynamic Poisson's ratio, f_1= first longitudinal resonance frequency, Hz and f_2= second longitudinal resonance frequency, Hz and A_1, B_1, and C_1 are constants based on the dimensions of the cylinder [Subramaniam et al., 2000]. The elastic modulus is then determined from the Poisson's ratio and the measured first longitudinal resonance frequency as follows:

$$E_d = 2(1+u_d)\, r \left(\frac{2p\, f_1 R_o}{f_n^1} \right)^2$$

where E_d = dynamic modulus of elasticity, Pa, $?_d$ = dynamic Poisson's Ratio, ? = density, kg/m^3, R_0= radius of the cylinder, m, $f_n^1 = A_2(u_d)^2 + B_2(u_d) + C_2$ and A_2, B_2, and C_2 are constants based on the dimensions of the cylinder [Subramaniam et al., 2000].

For the Love's correction method, the fundamental longitudinal resonance frequency (f) of a cylinder with length L and diameter d is measured, and a value of Poisson's ratio is assumed. Then E_d is computed as

$$E = \frac{4 f^2 L^2 r \left(1 + \frac{n^2 k^2 (2m-1)^2 p^2}{L^2}\right)}{(2m-1)^2}$$

where m=1 for the fundamental mode and $\kappa = d/2$v2 for cylinders. In this work, only the fundamental mode in cylinders was measured.

Project Findings

The project findings are based on the analysis of the experimental test data.

Dynamic test methods on control sample

The dynamic vibration methods were first applied to a homogenous, uniform material with known properties. The tests were carried out on beam and cylinder samples made from aluminum. The geometries of the aluminum samples replicate those of the concrete samples. The material properties of the aluminum were verified using ultrasonic P-wave and S-wave velocity measurements: E = 10.4x10^6 psi GPa and Poisson's ratio = 0.33. All four vibration tests were applied to the specimens: ASTM C215 using transverse vibrations (both beam and cylinder), ASTM C215 using longitudinal vibrations (both beam and cylinder), Love's correction for longitudinal vibrations (cylinders only) and the two-frequency method (cylinders only). The results from the tests are shown in Figure 3. The ASTM C215 measurements with longitudinal vibration consistently underestimate the actual value, while the ASTM C215 transverse measurements overestimate the actual value. The amount of under- and over-estimation increases when cylinders are used. The largest discrepancy with the actual value is provided by ASTM C215 using longitudinal resonances with cylindrical specimens. The Love's correction and two frequency methods give accurate estimates of E for Aluminum cylinders, when correct values of Poisson's ratio are assumed in the computations.

Dynamic test methods on concrete samples

The test results from the aluminum samples indicate that the various vibration tests provide different values of E for the same sample. This was confirmed by tests carried out on high strength concrete cylinder samples provided by CTLGroup. Figure 4 shows a comparison of E_d values obtained by ASTM C215 using the transverse and longitudinal modes measured on the same sample. The results confirm that the transverse vibration measurements are consistently higher than the longitudinal measurements. This discrepancy appears to widen as the E_d value increases. The agreement between longitudinal and transverse resonance values improves for all values of E_d when the Love's correction in applied to the longitudinal resonances, where a Poisson's ratio of 0.25 was assumed. This is shown in Figure 5.

One issue related to the use of Love's correction for cylinders, ultrasonic pulse velocity and ASTM C215 using the transverse mode methods is that a value of Poisson's ratio must be assumed or measured for the computation. Furthermore, it is known that both static and dynamic Poisson's ratio vary across a broad range of values for concrete, usually between 0.15 and 0.25, and that the dynamic and static values of Poisson's ratio are not strongly related to each other. Considering this, incorrect estimates of Poisson's ratio for concrete are likely when a single value is assumed for a given concrete. The effects of this possible mis-estimation on each method should be considered. Examination of the formulas for each method reveals that mis-estimates of Poisson's ratio affect differently the accuracy of E estimates from each method. ASTM C215 using longitudinal resonances and the two-frequency method for cylinders do not rely on an estimate of Poisson's ratio in the computation. Love's correction for cylinders shows only a small disruption of E estimates owing to mis-estimation of Poisson's ratio. ASTM C215 using transverse modes shows moderate disruption, while ultrasonic P-wave velocity shows large disruption. This last point suggests that E estimates from P-wave velocity are not reliable for concrete; this is confirmed by test results, shown in Figure 6, that show that E_d measurements from P-wave velocity significantly over-predict those obtained from ASTM C215 longitudinal vibration, when median values of Poisson's ratio are assumed for the concrete. Interestingly, the agreement between E measured by the two methods improves when an artificially high value of Poisson's ratio (0.28) is assumed for all concretes, as shown in Figure 7.

The behavior shown in Figures 6 and 7 is unexpected, and cannot be readily explained by conventional theory for elastic and homogeneous materials. It is possible, however, that composite nature of concrete has a role in this behavior. To investigate this, a set of special two-phase (cement paste and coarse aggregate) concrete samples with varying proportions of the phases were produced. Photos of the range two-phase concrete samples are shown in Figure 8. These samples represent the extreme cases of the two-phase mixture arrangement. Any global property of the two-phase composite should be bounded by the Voigt and Ruess (parallel and series configurations, respectively) combinations of the properties of the individual phases themselves. Several composite material properties were investigated across the different phase compositions. As expected, the material mass density of the composite follows the upper bound of the Voigt model combination of the density of the individual phases. Interestingly, the global static and dynamic (ASTM C215 using longitudinal mode) moduli are different from each other across the different phase compositions, as shown in Figure 9. Dynamic modulus is consistently higher than static modulus, and it tends to follow the upper-bound Voigt mixture rule. On the other hand, the static modulus lies in-between the upper and lower bounds. In other words, static and dynamic moduli follow different mixture behaviors in composite materials such as concrete. It is this difference that may cause dynamic moduli to be higher than static moduli in concrete. It should be noted that this difference between static and dynamic moduli is not observed in cement paste samples, which are more homogeneous.

Comparison of E and E_d for high strength concrete

Several attempts have been made to correlate static (E) and dynamic (E_d) moduli for concrete. The simplest of these empirical relations is proposed by Lydon and Balendran [Neville, 1997]

$$E = 0.83\,E_d.$$

Another empirical relationship for concrete's elastic moduli was proposed by Swamy and Bandyopadhyay and is now accepted as part of British testing standard BS8110 Part2:

$$E = 1.25E_d - 19$$

where both units of E and E_d are in GPa. This expression does not apply for lightweight concretes or concrete that contains more than 500 kg of cement per cubic meter of concrete [Neville, 1997].

Shkolnik proposed relations between dynamic and static moduli for concrete, based on thermo-fluctation theory, which depends on strain rate of loading and the temperature [Shkolnik 1996; Shkolnik 2005]. The vibration of atoms with average kinetic energy causes stresses on the atomic bonds of the same magnitude as the strength of materials. Thus at atomic-molecular level the fracture of a material is controlled by breaking of atomic bonds at thermal fluctuation. The thermal fluctuations of energy, exceeding the average thermal energy of atoms, naturally, depend on the time or rate of loading and predetermine the mechanical properties (modulus of elasticity, strength) of real materials. By coupling the nonlinear stress-strain material model with the kinetic (thermofluctuation) theory of solid strength, a physically proved expression was applied for modulus of elasticity of concrete of a material. Considering the samples provided by the CTLGroup, which were tested at a nominal value of stress/strength ratio of 0.24, rate of loading 0.24MPa/s, and temperature of 20 ^{o}C 9, the thermoflucutaion theory gives:

$$E = E_d - 5864$$

for units of MPa. The constant at the right side of equation is determined in accordance with ASTM C469.

For both lightweight and normal concretes, Popovics suggested a more general relationship between the static and dynamic moduli as a function of the density of the concrete

$$E_s = kE_d^{1.4}\rho^{-1}$$

where $k = 0.23$ for units of psi and ? is density, lbs/ft^3 [Popovics, 1975]. Whatever the relation between the moduli, it is thought to be unaffected by air entrainment, method of curing, condition at test, or the type of cement used [Neville, 1997].

In Figures 10-12, the fit of the four shown equations that relate E_d to E are shown for the high-strength concrete cylinder data obtained from CTLGroup. In addition, a best fit fine to the data is also shown. Only the fundamental longitudinal and fundamental transverse modes were provided by CTLGroup, so only three analyses methods are possible: ASTM C215 using longitudinal resonance for cylinders, ASTM C215 using transverse resonance for cylinders, and Love's correction for longitudinal vibration. For ASTM C215 using transverse vibration and the Love's correction method, a Poisson's ratio of 0.25 was assumed. The predictions for E_d are shown in Figures 10, 11 and 12, respectively. The mean absolute errors (in GPa) for each fit are also in the figures. In general the data show a linear, or nearly linear, dependence between E and E_d across the full range of modulus values. In all three data sets, the fitted line shows the best fit (lowest mean absolute error) and the equation proposed by Lyndon and Balendran show the poorest fit (highest mean absolute errors); it appears that the latter equation is inappropriate for high strength concrete. The remaining three equations show reasonable fit, where the mean absolute error is always below 2GPa. The best fit line exhibits absolute mean error below 1.3 GPa. It is interesting to note that the predictions using ASTM C215 with transverse resonance show approximately 10% higher error for the fit line, suggesting that the transverse resonance data may exhibit a higher degree of scatter.

More complete statistical analysis of the predicted E data is shown in Tables 1 and 2. In Table 1, column 1 has the lowest Mean E, STDEV, CV, and P-values at longitudinal and transverse vibrations. The relative mean deviations are all negative for longitudinal vibration, but for transverse vibration two of them are negative (columns 1 and 3), and two- positive (columns 2 and 4). The MAPE does not exceed 10% at longitudinal and transverse vibration. The column 2 illustrates the highest P-value at longitudinal vibration (P-value = 0.282), and column 4 at transverse vibration (P-value=0.827). The relative difference between the Mean E for column 4 at transverse vibration (E=49.157 GPa), and the average value of the Mean E for columns 2 and 3, i.e. (49.777+48.774)/2= 49.276 GPa, equals 0.2%. In Table 2, the relative mean deviation from E (C 469) decreased, and simultaneously column 2 and 4 changed the sign from negative to positive. The values of MAPE decreased (except column 2), and The P-values increased (columns 2, 3, 4) in comparison with corresponding data for longitudinal vibrations in Table 1. As a consequence of accepted assumptions (ν=0.25), the mean values of predicted dynamic modulus of elasticity obtained for longitudinal vibrations (Table 2) and transverse vibrations (Table 1) are equal within the limits of errors of measurements and calculations.

Conclusions

The following conclusions are drawn, based on the results presented and analyzed in this report:

- The dynamic and static modulus tests carried out at the University of Illinois laboratories confirm that E_d values vary, depending on method of measurement: ultrasonic pulse velocity (UPV – ASTM C597) gives the highest predictions of E_d, and longitudinal vibration the lowest. Of the vibration methods, longitudinal vibration of cylinders (L/D=2) give the least accurate prediction of E_d. A modified method (Love's correction) was introduced for longitudinal resonances for cylinders, which gives more accurate E_d results that agree with those from the transverse method. The Love's correction was applied to the CTLGroup data, providing a more accurate estimation of E from E_d, with an absolute mean error of estimate of 1.10 GPa. UPV may be able to provide accurate estimates of E, but only if excessively high values of Poisson's ratio are assumed in the calculations. More study in this area is warranted. It was found that the longitudinal and transverse methods of ASTM C215 give different results from the same sample: transverse provides higher dynamic modulus (E_d) estimates.

- As expected, E_d is always greater than E for concrete; however, this behavior may be caused by the *composite* nature of concrete, rather than *non-linear behavior* of concrete exposed by varying strain levels. Stress strain curvature analyses show that the expected non-linear behavior at higher strain levels associated with the static test is not clearly seen. Tests on specially prepared two-phase composite samples, however, do indicate the influence of composite nature on the E vs. E_d behavior.

- Several existing relations between dynamic and static modulus (E) were evaluated for the data. The results show that the relation between E_d and E is affected by nature of E_d data: the best relation to apply depends on how E_d was obtained. In general, though, the relation between E_d and E appears to be linear, or nearly linear. For longitudinal vibration, the British Standard equation (BS8110, Part 2) gives the best prediction of E, with an average squared error of 1.36 GPa. For transverse vibration and Love's correction of longitudinal vibration, the equation proposed by S. Popovics gives best predictions for E: 1.41 GPa and 1.10 GPa, respectively.

- A thorough understanding of the various factors affecting E_d and E in concrete should enable improved prediction of $E_{d.}$ Once this is established, vibration measurements should offer an effective, non-destructive, inexpensive and rugged method to estimate E for concrete.

Acknowledgements

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Figures

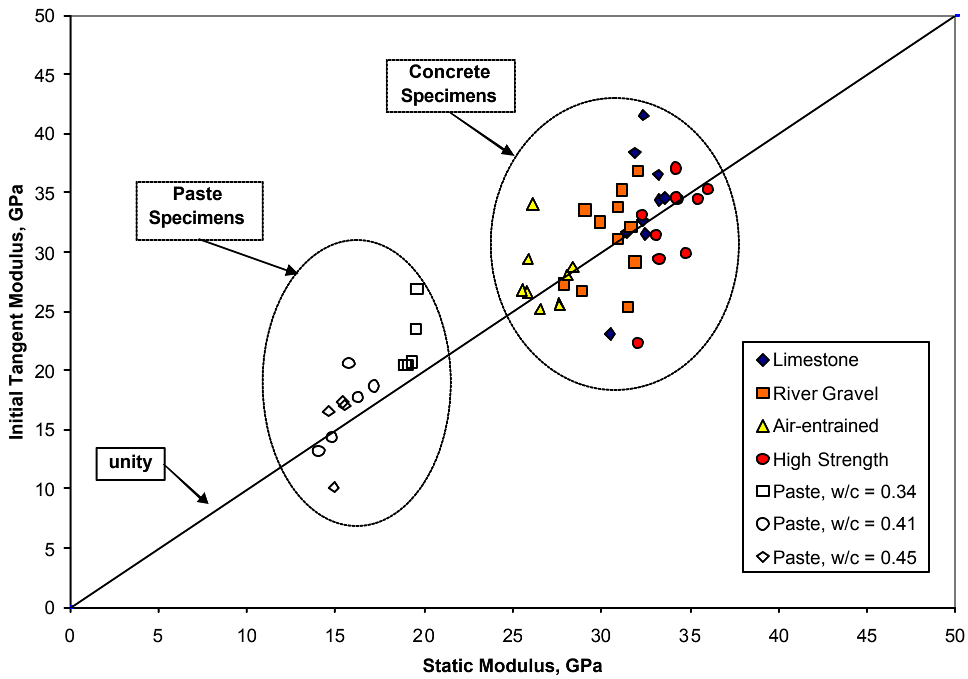


Figure 1. Comparison of low-strain (initial tangent modulus) and high strain (conventional chord modulus) for static compressive tests carried out on concrete and paste samples.

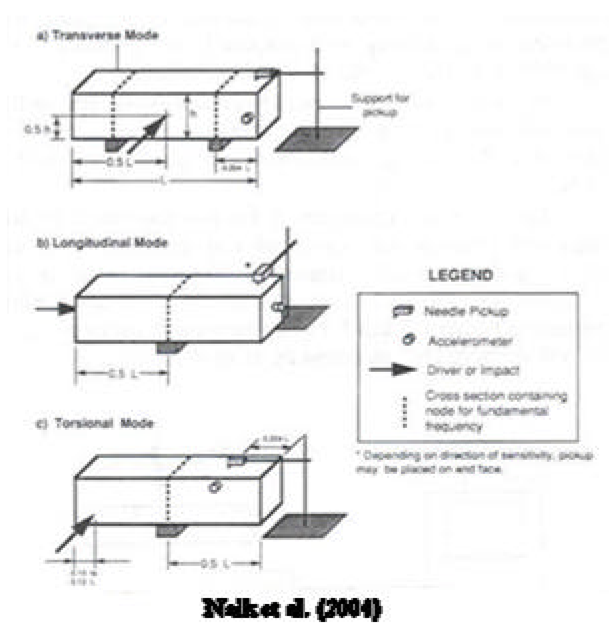


Figure 2. Testing configuration for standard vibrational resonance tests (Naik et a. 2004)

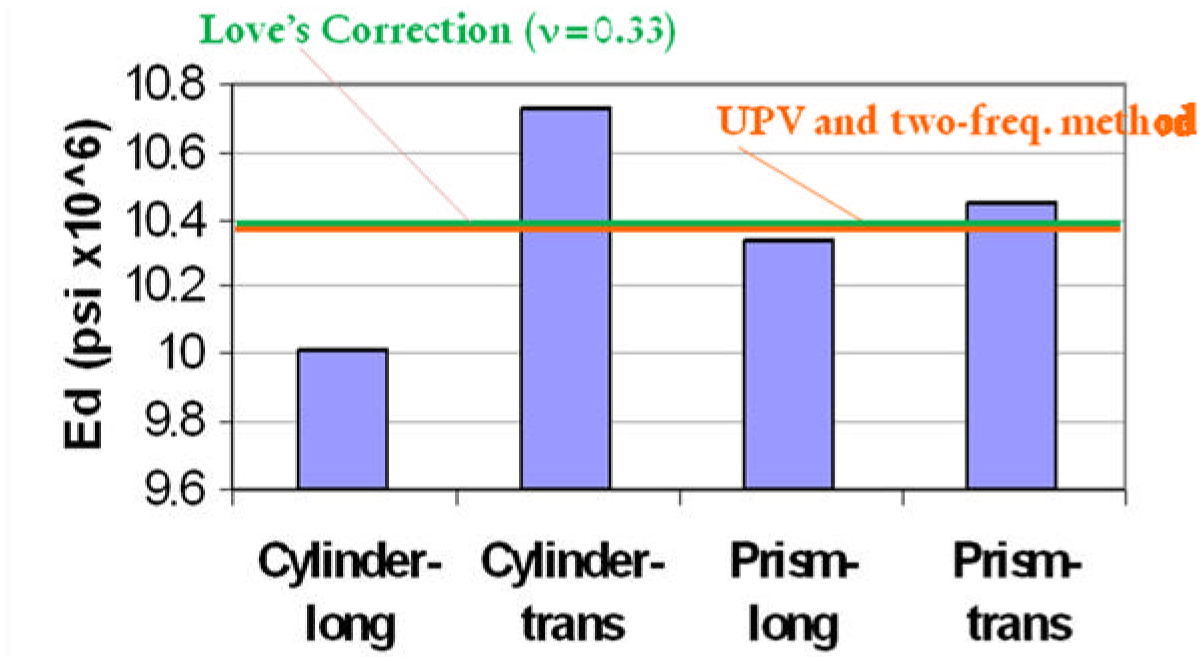


Figure 3. Comparison of longitudinal and transverse E_d values obtained using ASTM C215 method on aluminum samples. The expected E value is 10.4×10^6 psi and ν is 0.33.

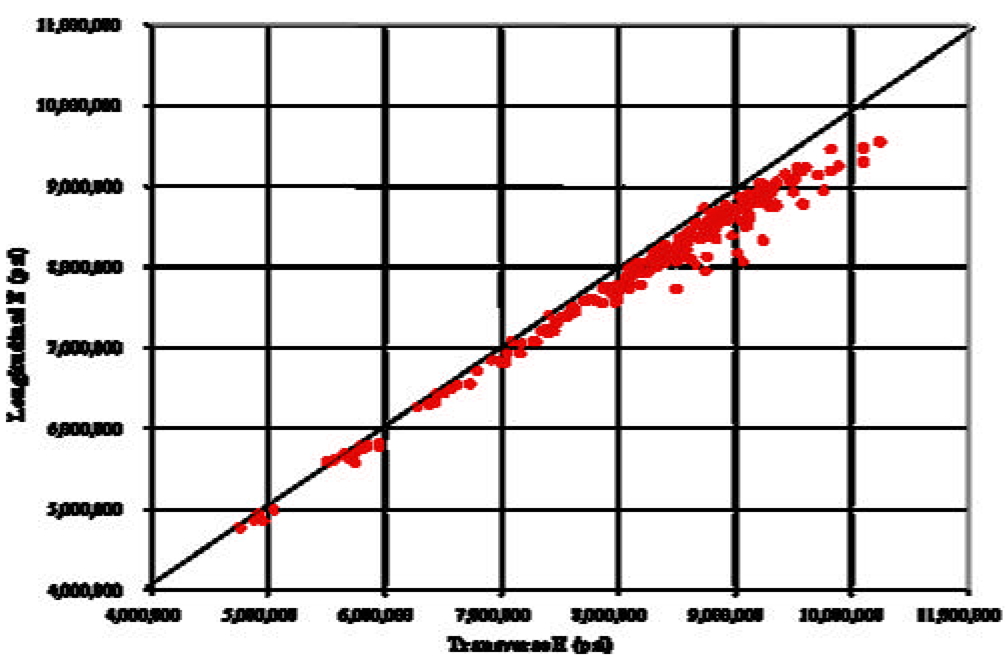


Figure 4. Comparison of longitudinal and transverse E_d values obtained using ASTM C215 method on concrete cylinders. Test data provided by CTLGroup.

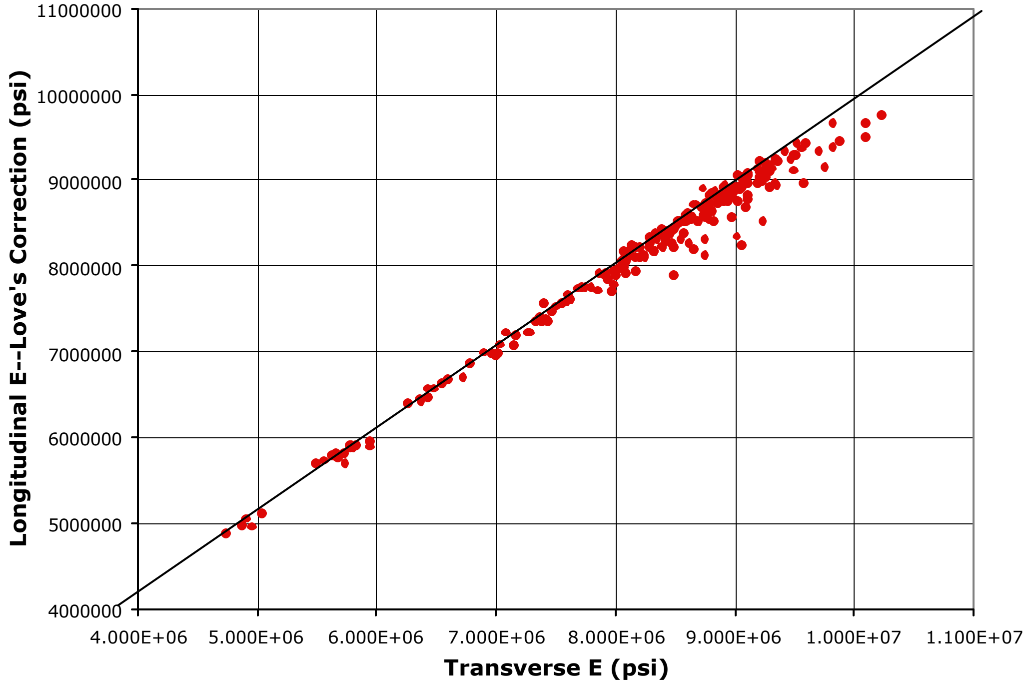


Figure 5. Comparison of longitudinal and transverse E_d values. Longitudinal values obtained using Love's correction (assumed v = 0.25) and transverse values obtained using ASTM C215 method on concrete cylinders. Test data provided by CTLGroup.

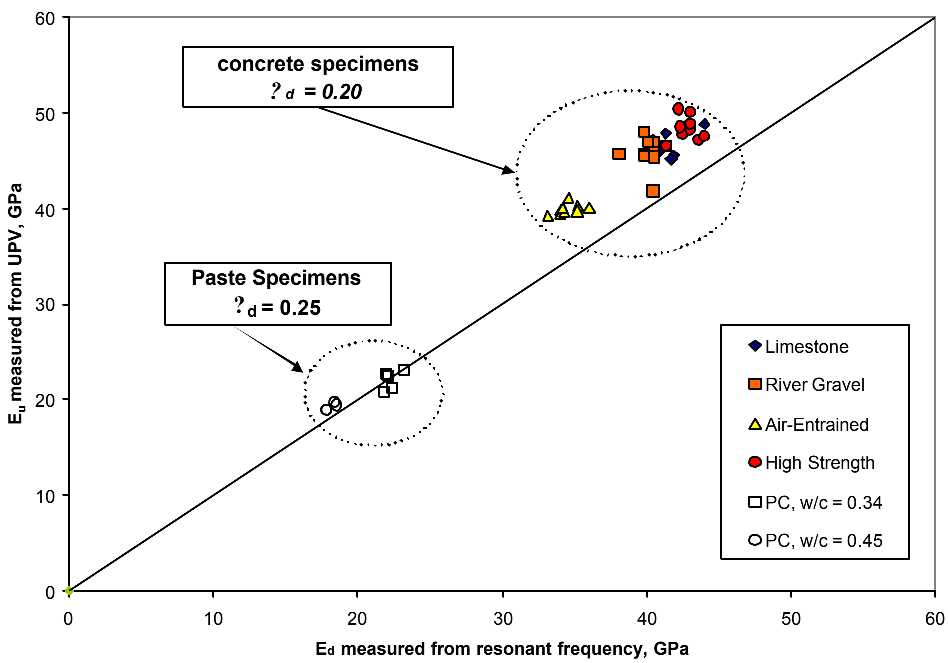


Figure 6. Comparison of E_d values obtained by longitudinal resonance frequency (ASTM C215) and ultrasonic pulse velocity (ASTM C597) tests carried out on concrete and paste cylinders. Median values of Possion's ratio were assumed for the predictions based on pulse velocity.

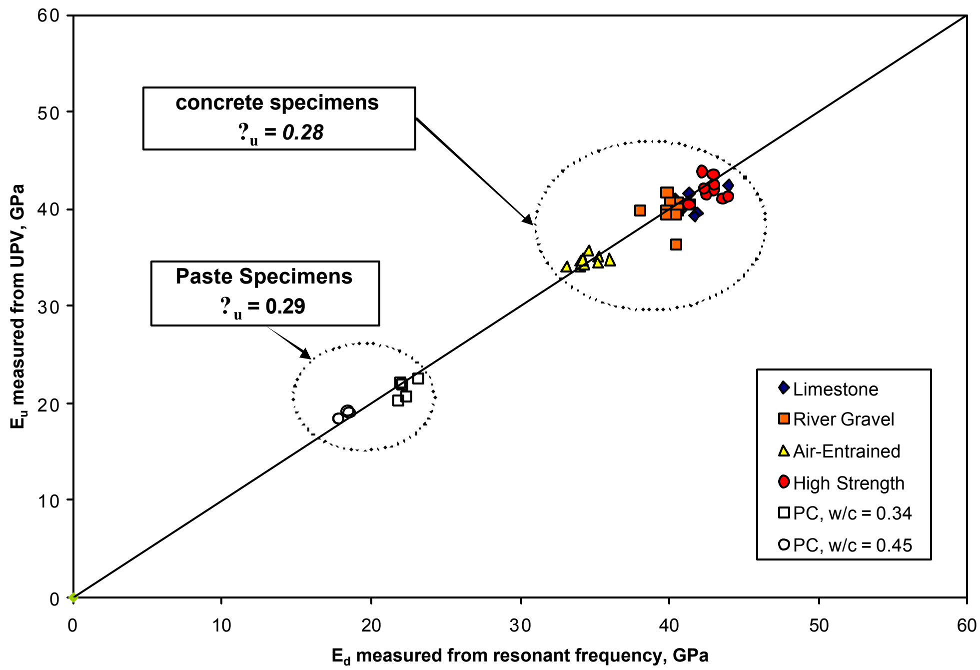


Figure 7. Comparison of E_d values obtained by longitudinal resonance frequency (ASTM C215) and ultrasonic pulse velocity (ASTM C597) tests carried out on concrete and paste cylinders. Unusually high values of Possion's ratio were assumed for the predictions based on pulse velocity.

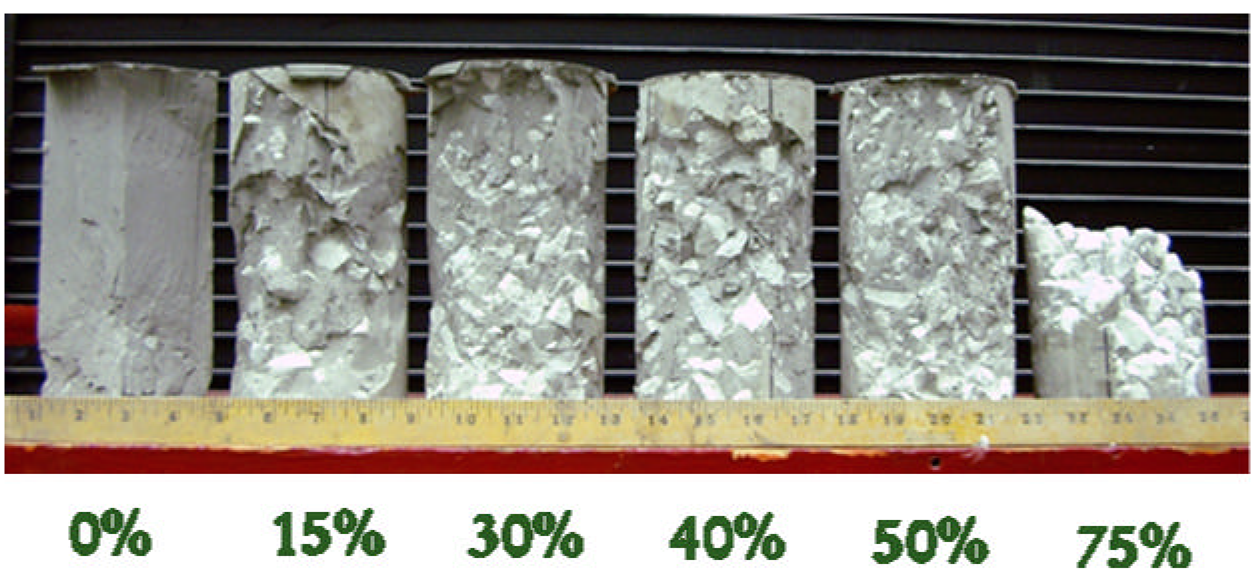


Figure 8. Two-phase limestone and cement paste samples, with limestone content ranging from 0 to 75% by mass.

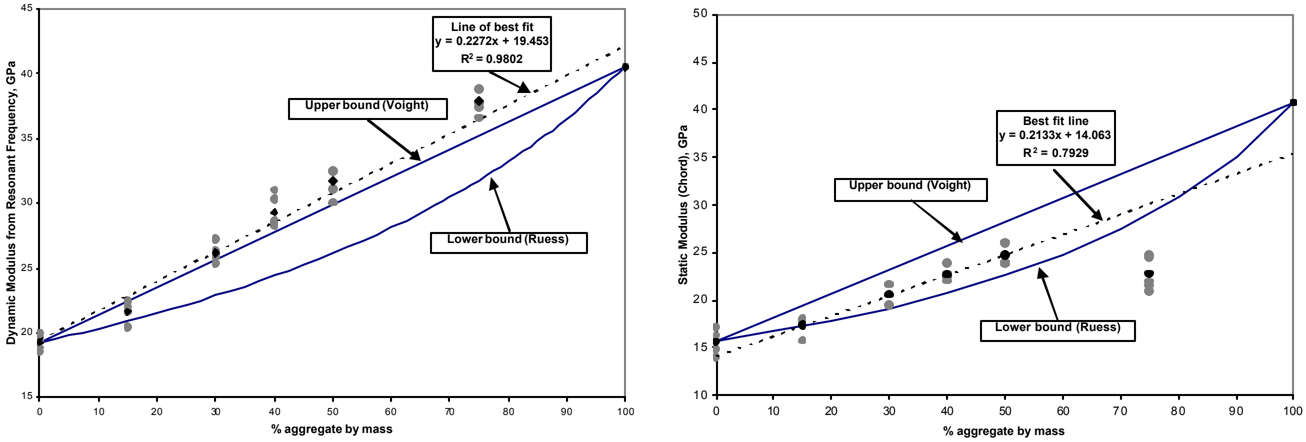
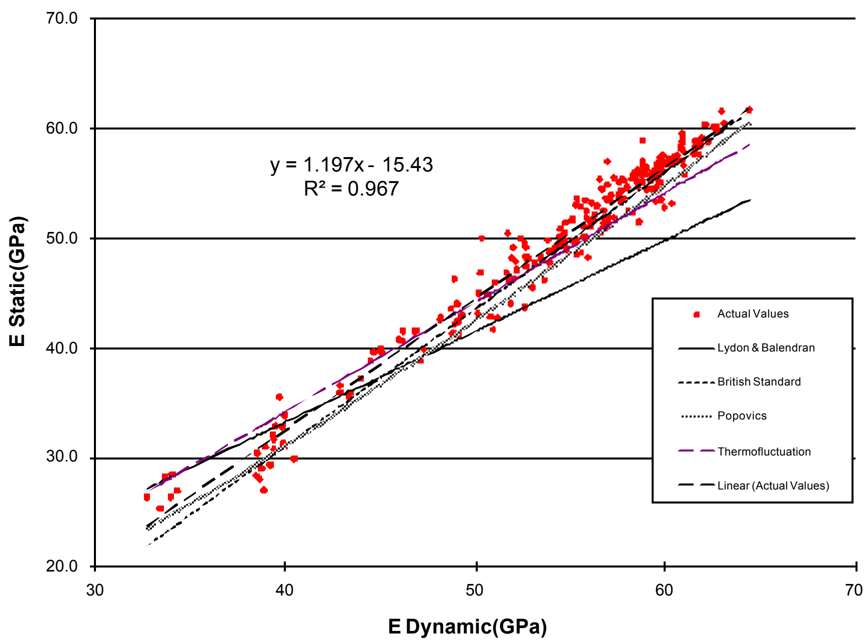
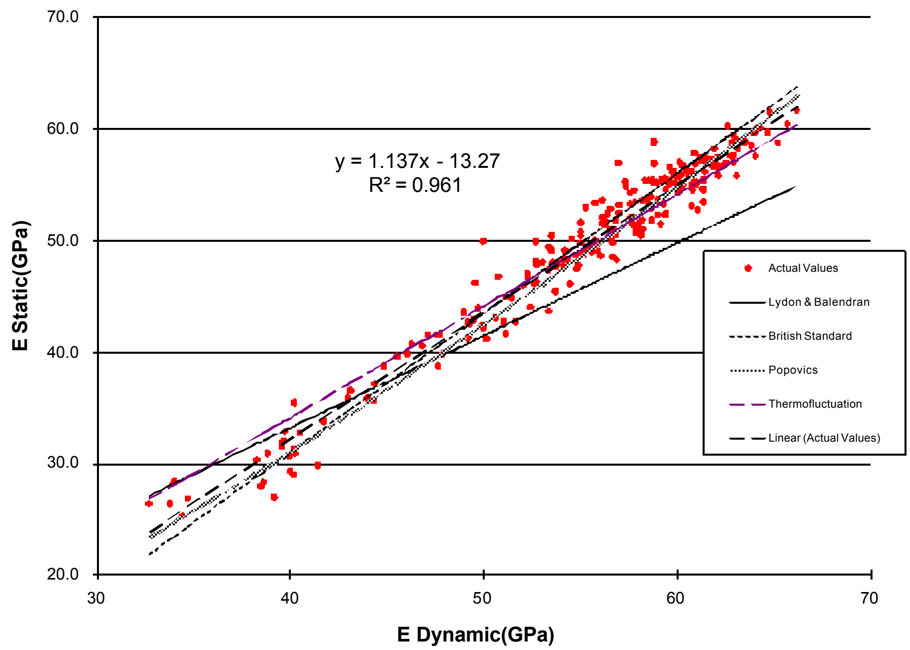


Figure 9. Variation of dynamic (left) and static (right) Young's modulus as a function of aggregate content in the two-phase composite concrete specimens. Ed determined using longitudinal resonance following the ASTM C215 procedure.



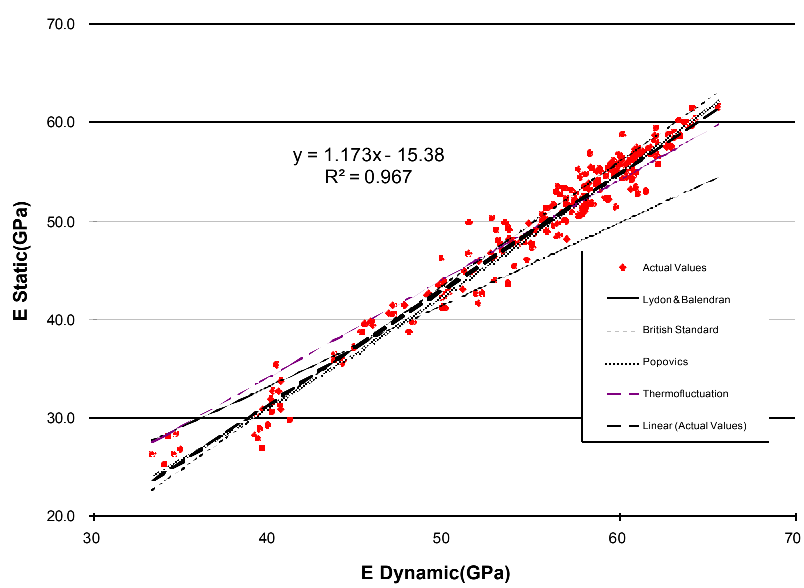
Lydon & Balendran	British Standard	Popovics	Thermofluctuation	Fitted line
4.758	1.363	1.885	1.943	1.176

Figure 10. Comparison of E (ASTM C469) vs. E_d (ASTM C215) data (points) using longitudinal resonance for concrete cylinders. Various relations between E and E_d are also shown as lines. The mean absolute error (in GPA) for each type of fit curve is shown in the table underneath. Data obtained from CTLGroup.



Lydon & Balendran	British Standard	Popovics	Thermofluctuation	Fitted line
4.044	1.524	1.410	1.500	1.280

Figure 11. Comparison of E (ASTM C469) vs. E_d (ASTM C215 using transverse resonance) data (points) for concrete cylinders. Various relations between E and E_d are also shown as lines. The mean absolute error (in GPA) for each type of fit curve is shown in the table underneath. Data obtained from CTLGroup.



Lydon & Balendran	British Standard	Popovics	Thermofluctuation	Fitted line
4.020	1.362	1.192	1.575	1.175

Figure 12. Comparison of E (ASTM C469) vs. E_d (Love's correction using longitudinal resonance) data (points) for concrete cylinders. Various relations between E and E_d are also shown as lines. The mean absolute error (in GPA) for each type of fit curve is shown in the table underneath. Data obtained from CTLGroup.

Tables

Table 1. Statistical comparison of E computed by the various relations and E_d (ASTM C215). The mean value of E obtained by ASTM C469 is 49.313 GPa, STDEV=8.39GPa and CV=0.17.

Longitudinal vibrations				
Data	E= 0.83Ed, (MPa). E, GPa	E=1.25 Ed - 19, E, GPa	E=0.00153 $Ed^{1.4}$, (psi) E, GPa	E= Ed-5864, (MPa). E, GPa
	1	**2**	**3**	**4**
Mean E, GPa	44.815	48.492	47.484	48.129
STDEV, GPa	5.653	8.513	8.070	6.811
CV.	0.130	0.180	0.170	0.140
Relative mean deviation from E (C469)	-0.082	-0.017	-0.036	-0.017
MAPE	0.094	0.031	0.042	0.041
P-value	0.000	0.282	0.014	0.086
Transverse vibrations				
	1	**2**	**3**	**4**
Mean E, GPa	45.688	49.777	48.774	49.157
STDEV, GPa	6.006	9.045	8.648	7.236
CV.	0.130	0.180	0.180	0.150
Relative mean deviation from E (C469)	-0.065	0.008	-0.011	0.003
MAPE	0.079	0.033	0.030	0.033
P-value	0.000	0.557	0.485	0.827

Table 2. Statistical comparison of E computed by the various relations and E_d using Love's correction (v=0.25). The mean value of E obtained by ASTM C469 is 49.313 GPa, STDEV=8.39GPa and CV=0.17.

Longitudinal vibrations				
Data	E= 0.83Ed, (MPa). E, GPa	E=1.25 Ed - 19, E, GPa	E=0.00153 $Ed^{1.4}$, (psi) E, GPa	E= Ed-5864, (MPa). E, GPa
	1	**2**	**3**	**4**
Mean E, GPa	45.711	49.842	48.819	49.209
STDEV, GPa	5.766	8.683	8.297	6.947
CV.	0.126	0.174	0.170	0.141
Relative mean deviation from E (C469)	-0.063	0.010	-0.009	0.005
MAPE	0.080	0.031	0.028	0.036
*P value	0.000	0.492	0.512	0.882