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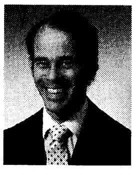
Tensile Capacity of Anchors with Partial or Overlapping Failure Surfaces: Evaluation of Existing Formulas on an LRFD Basis

by C. Ben Farrow and Richard E. Klingner

This study concerns the prediction of tensile capacity, as governed by concrete cone failure, of single anchors located close to a free edge and multiple anchor groups located far from a free edge and installed in uncracked, unreinforced concrete. A total of 160 data points is available for single anchors close to a free edge, while 185 data points are available for multiple anchors. A total of 31 data points consisting of data on high-strength anchors previously compiled by Klingner et al., Collins et al., and Cannon is accessible from tests on single anchors failing by fracture of the anchor steel.

Using common definitions and nomenclature for all variables and material properties, each data set is placed in a data base using SI units and concrete cube strengths. The concrete failure data are then compared with capacities predicted by the three existing methods: the 45 deg cone method of ACI 349-90, Appendix B; a variable angle cone (VAC) method; and the concrete capacity (CC) method. Observed data are compared against these existing methods in terms of average square error and load and resistance factor design (LRFD). Finally, using the principles of LRFD, and following the design procedure of ACI 349-90, the probability of steel failure or concrete cone failure under known loads and concrete cone failure under unlimited loads is calculated. Based on those comparisons, each approach is evaluated with respect to accuracy and suitability for use in design.

For single anchors located near a free edge and for multiple closely spaced anchors, the CC method fits most of the data better than either the ACI 349-90 method or the VAC method, and gives lower and more consistent probabilities of failure. This is especially evident at very shallow and very deep embedments.

Keywords: anchor bolts; anchors (fasteners); safety; structural design.

INTRODUCTION

The overall objective of this research is to evaluate the accuracy and suitability for use in design of specific methods for predicting tensile anchor capacity as governed by concrete cone failure. A previous study by Klingner et al. focused on evaluating each of the methods for single anchors far from a free edge and from other anchors.[1] In particular, this study focuses on single anchors located close to a free edge, and on multiple closely spaced anchors located far from a free edge. All data studied here refer to tensile anchors installed in uncracked, unreinforced concrete.

REVIEW OF AVAILABLE ANCHOR FAILURE DATA

Concrete cone failure data

All information on concrete failure was obtained from a data base compiled by Fuchs and Breen using results of tests on anchor bolts, headed studs, undercut anchors, and expansion anchors. Additional test data were obtained through correspondence with Cannon, Eligehausen, and Orr.

The original data base of Fuchs and Breen contains the results of over 1500 tests on anchors. These tests include anchors tested in shear and tension for a wide range of anchor configurations. Recent correspondence with Cannon, Eligehausen, and Orr has provided data from approximately 50 additional tests on anchors loaded in tension. Considering only those anchors loaded in tension and failing by formation of a concrete cone, the data bases yielded a total of 160 data points for single anchors located near a free edge, and 185 data points for multiple closely spaced anchors. The latter included two-anchor groups, four-anchor groups, and 16-anchor groups (connections with four groups of four anchors). Because most of the concrete cone failure results were obtained from European tests, the original concrete data base was prepared in SI units. Thus, concrete strength is expressed in terms of cube strength. Although the original data base contains tests conducted in both Europe and the U. S., Eligehausen, Fuchs, and Breen report no difference in tension test results between the two groups of data.[2]

No adhesive or bonded anchors are included in this study. All anchors included in this study were tested under concentric static loading. This factor is of special significance in comparing the concrete capacity (CC) method with either the ACI 349-90 method or the variable angle cone (VAC) meth-

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Steel failure data

Data on steel failures were obtained from a data base previously published by Klingner et al,[3] from tests performed by Collins,[4] and from recent correspondence with Cannon using results of tests on anchor bolts and headed studs. The data refer to single anchor tests only. This data base includes tests on both high-strength anchors and regular anchors. Since many segments of the anchor industry are moving toward the use of high-strength anchors, this study uses only the data available on high-strength anchors in its evaluation of the probability of failure of each of the three methods. A total of 31 tests on high-strength anchors is utilized. Because most of the steel failure data were obtained from U.S. tests, the original data base was prepared in U.S. units. Concrete strength is expressed in terms of cylinder strength.

Selection of data to be excluded

Some data on the original concrete data files were excluded from further consideration based on the following criteria:

Single anchors near a free edge

1. Test data from single anchors failing by side blowout were excluded. Anchors placed close to a free edge can fail prematurely by creation of a partial cone,[3,5] or by side blowout.[3,5-7] The edge distance within which a partial cone forms is far greater than that at which side blowout is a problem. This explains why this data base contains few points with very low ratios of edge distance to embedment.

2. Test data from anchors located sufficiently far from the edge so that a complete failure surface develops are not included in the data base. The limiting edge distance chosen is that distance at which all three methods (ACI 349-90, VAC, and CC) consider the anchor as a single anchor, uninfluenced by edge conditions. The shallowest cone angle predicted by the VAC method is 28 deg.[3,8] This value is less than that of either the ACI 349-90 method (45 deg cone) or the CC method (35 deg pyramid). The 28 deg angle implies that anchors are affected by a free edge if they are placed closer to the free edge than (h_e/tan 28 deg) + d/2 or (1.9 h_e + d/2). However, none of the anchors from the data base that were located close to a free edge have an embedment depth small enough to meet this limiting criterion. Hence, the governing formula is the CC method, with its assumption of approximately a 35 deg pyramid. This method is influenced by edge distances if anchors are less than 1.5h_e from the edge.

Multiple closely spaced anchors

1. Test data from multiple anchors located sufficiently far from other anchors or located close to a nearby edge are not included in the data base. The limiting center-to-center distance of the anchors chosen is the spacing at which all three methods (ACI 349-90, VAC, and CC) evaluate the group of anchors as an equivalent number of single anchors with no influence from other anchors. The shallowest cone angle predicted by the VAC method is 28 deg.[3,8] This value is less than that of either the ACI 349-90 method (45 deg cone) or the CC method (35 deg pyramid). The 28 deg angle implies that anchors are affected by other anchors if they are placed closer to anchors other than 2([h_e/tan 28 deg] + d/2) or 2(1.9 h_e + d/2). However, no anchors from the data base tested as part of an anchor group have an embedment depth small enough to meet this limiting criterion. Hence, the governing formula is the CC method, with its assumption of approximately a 35 deg pyramid. This method assumes that anchors will interact if spaced closer than 3.0h_e.

2. A series of nine tests is available involving unsymmetrically spaced group anchors. Specifically, the tests are groups of 16 anchors in which the anchors are cast in groups of four, symmetrically placed about a center point. Because the spacing between anchors within each group of four is not the same as the spacing between an anchor and the nearest anchor in an adjacent group, this pattern cannot be described by a single ratio of anchor spacing-to-embedment depth. These points are not included in the calculations to simplify the calculation process and avoid the influence of possible eccentric load.

ACI 349-90 has requirements for the minimum bearing area of the anchor head. The concrete data base has not been checked for compliance with those requirements. This is especially important to the single anchors located near a free edge, since the design of some of the anchors may be governed by this criteria. However, as previously stated, all "blowout" failures are excluded from the data base.

Several design guides place minimum limitations on the diameter of the anchor shaft and the minimum anchorage depth. For instance, the UEAtc[9] limits shaft diameter to 0.236 in. (6 mm) and embedment depth to 1.575 in. (40 mm). However, no check is made to insure that these criteria are met in this data base.

PRINCIPAL METHODS FOR PREDICTING CONCRETE CONE CAPACITY

General

In this section, the three principle existing methods for predicting concrete cone capacity are described mathematically for both single anchors near a free edge and multiple anchors. All equations are expressed in SI units (N, mm, and concrete cube strength). Note that in evaluating the ACI 349-90 and VAC methods for predicting concrete capacity, the head diameter d_h is assigned a value equivalent to the actual diameter of the anchor. This, it is believed, is consistent with the latest deliberations of ACI Committee 349. When capacity prediction methods are used for design, they are expressed in terms of specified concrete compressive strength (denoted by f_{cc}'). When capacity prediction methods are compared with test results, the actual concrete compressive strength is used (denoted by f_{cc}). For the equations to remain consistent with their original sources, they are written using the notation f_{cc}. Variables used in these three capacity prediction methods are defined as follows:

Table 1—Comparison of error using each method for single anchors near a free edge far from other anchors, SI units*

Ratio of edge distance-to-embedment depth	Error, ÷N× mm		
	ACI 349-90 method	VAC method	CC method
0.45-0.60	16,770	16,882	4955
0.601-0.75	12,016	12,096	5111
0.751-0.90	15,555	15,559	2696
0.901-1.05	22,485	22,495	5677
1.051-1.20	10,693	10,849	3554
1.201-1.35	32,345	32,286	1801
1.351-1.50	5960	5943	5319

*Square root of sum of square error.

A_p = actual projected area of anchor or anchor group

$A_{p,o}$ = projected area of all anchors not limited by edge or spacing influences

h_e = embedment length, measured from free surface to the bearing surface of the anchor head, or, for expansion anchors, to the point of bearing of the expansion mechanism

d_h = diameter of anchor head, taken as anchor diameter

θ = cone angle, measured from the failure surface to a plane perpendicular to the anchor axis

= 45 deg for $h_e \geq 127$ mm (5 in.)

= 28 deg + (0.13386 h_e) deg for $h_e < 127$ mm (5 in).

Method of ACI 349-90, Appendix B[10]

Nominal cone capacity of concrete is based on a maximum tensile stress of $4\sqrt{f_c}$ (psi units), idealized as uniformly distributed on the projected area of a 45 deg stress cone radiating toward the attachment from the bearing edge of the anchor (Reference 10). Locating an anchor near a free edge reduces the projected area of the 45 deg stress cone if the 45 deg cone intersects the edge of the concrete. Similarly, locating an anchor near other anchors reduces the projected area of the 45 deg stress cone due to overlapping failure surfaces. The ACI 349-90 equation for various cases of edge distance and spacing can be expressed in general form. When using SI units, the concrete cone capacity is determined by

Table 2—Comparison of error using each method for multiple closely spaced anchors far from a free edge, SI units*

Ratio of spacing-to-embedment depth	Error, ÷N × mm		
	ACI 349-90 method	VAC method	CC method
0.27-0.45	10,081	10,081	10,128
0.451-0.60	2676	2795	3513
0.601-0.75	2577	2577	3150
0.751-0.90	4559	4524	1467
0.901-1.05	2687	2623	1900
1.051-1.20	1748	1827	1514
1.201-1.35	3522	2381	3018
1.351-1.50	5877	5850	2355
1.501-2.00	3374	2880	2393
2.001-2.50	3658	3004	2728
2.501-3.00	3384	2014	2433

*Square root of sum of square error.

$$\frac{A_p}{A_{p,o}} 0.96 \sqrt{f_{cc}} h_e [h_e + d_h] \tag{1}$$

Variable angle cone method[3,8]

The VAC method is identical to that of Appendix B of ACI 349-90, except that the maximum tensile stress is assumed to be distributed uniformly on the projected area of a variable angle stress cone radiating toward the attachment

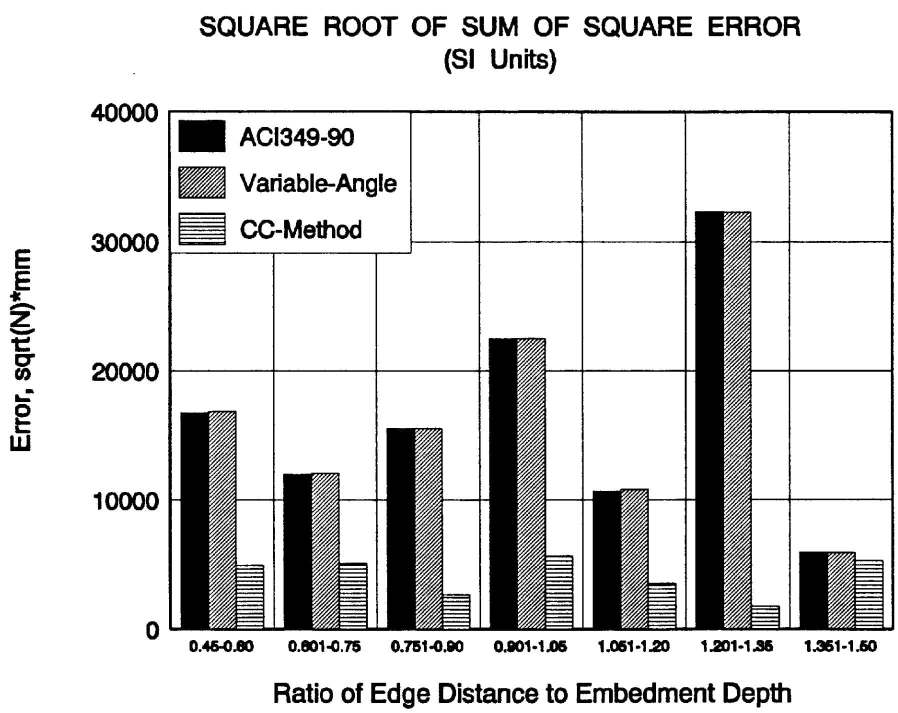

Fig. 1—Comparison of error using each method for single anchors near free edge (square root of sum of square error)

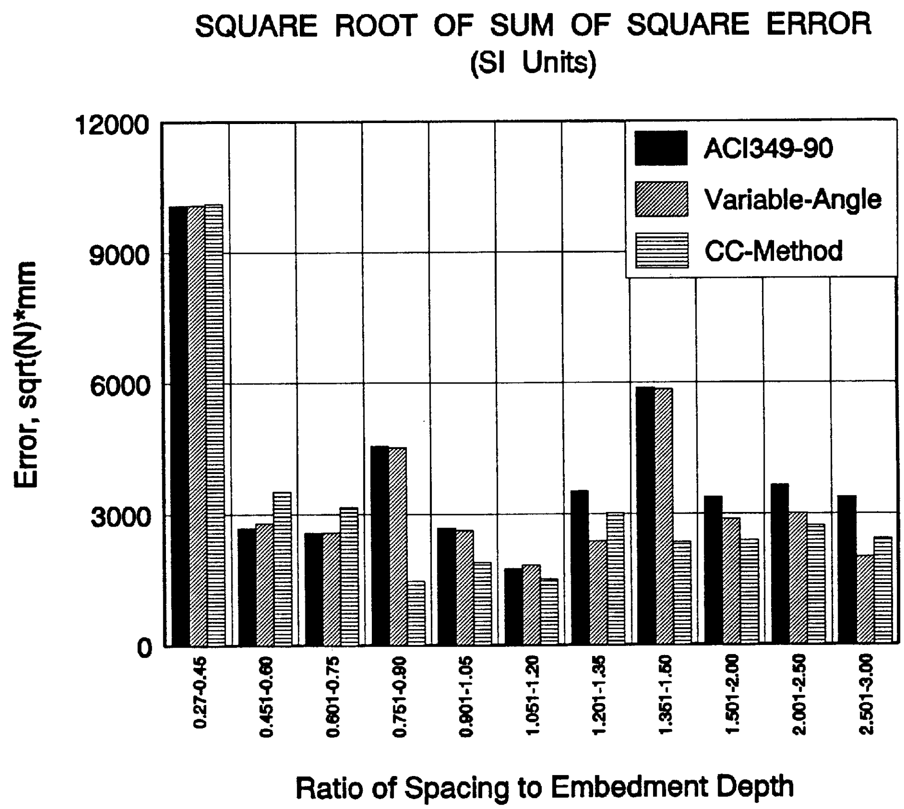

Fig. 2—Comparison of error using each method for multiple closely spaced anchors (square root of sum of square error)

from the bearing edge of the anchor. The VAC equation for various cases of edge distance and spacing can be expressed in general form.[3,8] When using SI units, the concrete cone capacity is determined by

$$\frac{A_p}{A_{p,\,o}} 0.96\sqrt{f_{cc}}\left(\frac{h_e}{\tan\theta}\right)\left[\frac{h_e}{\tan\theta}+d_h\right] \quad (2)$$

CC method[5]

Locating an anchor near a free edge reduces the projected area of the 35 deg stress pyramid if the 35 deg pyramid intersects the edge of the concrete. Similarly, locating an anchor near another anchor reduces the projected area of the 35 deg stress pyramid if the 35 deg pyramid intersects the stress pyramid of the other anchor. The CC method adopts the ACI 349-90 procedure by multiplying by the ratio of the actual projected area (net area) to the projected area of the anchor not limited by edge influences (gross area). When using SI units, the concrete cone capacity is found by

$$\frac{A_p}{A_{p,\,o}}\psi_{SN}\psi_{ec} \quad 15.5\sqrt{f_{cc}}h_e^{1.5} \quad \text{for Headed Studs}$$

$$\frac{A_p}{A_{p,\,o}}\psi_{SN}\psi_{ec} \quad 13.5\sqrt{f_{cc}h_e^{1.5}} \text{for Undercut or Expansion Anchors} \quad (3)$$

where

ψ_{SN} = 1 if c_1/h_e Š1.5
= 0.25 (2.5 + c_1/h_e) if $c_1/h_e < 1.5$
$\psi_{ec} = 1/(1 + 2e/(3h_e)) \leq 1$
e = eccentricity $\leq s/2$

In the previous equations, c_1 is the edge distance and s is the anchor spacing. The eccentricity factor is assumed equal to 1.0 (that is, concentric loading) for all failure data included in this study. The variables used in these equations are further defined in Reference 5.

COMPARISON OF EXISTING METHODS WITH AVAILABLE DATA

The equations previously given for each of the three methods (SI units) are normalized by dividing by $\sqrt{f_{cc}}$ and the number of anchors in the group. These normalized results for both single anchors near a free edge and multiple closely spaced anchors are then compared with available failure data by evaluating the square root of the sum of the squares error for both single anchors near a free edge and multiple closely spaced anchors. All comparisons are presented graphically in terms of SI units.

Tables 1 and 2 and Fig. 1 and 2 demonstrate that, for most embedment depths, the CC method has a square error lower than that of either the ACI 349-90 or the VAC methods. However, for multiple anchors with small ratios of spacing-to-embedment depth ($s/h_e \leq 0.75$), the ACI 349-90 and VAC methods have a lower square error than the CC method.

The CC method seems particularly advantageous when examining the square root of the sum of the square error for single anchors near a free edge and far from other anchors. The CC method has a lower error than either of the other two methods. This is true mainly because of the effect of including all embedment depths in each range of edge distance-to-embedment depth. The ACI 349-90 and VAC methods do not fit available failure data very well at large embedment depths. The effect of this divergence is to increase the sum of the square error for every range of edge distance-to-embedment depth.

No distinct conclusion can be drawn from the square error data for multiple anchors. There are only two cases in which the CC method is noticeably different than either the ACI 349-90 or VAC methods ($0.75 \leq s_1/h_e \leq 0.90$ and $1.35 \leq s_1/h_e \leq 1.50$). In both cases, a large number of tests with large embedment depths and the same configuration are present in the available data. This seems to imply that the CC method best fits the available data for cases in which the embedment depth is large.

For single anchors located near a free edge, the VAC method has a square error only slightly different from that of the ACI 349-90 method. However, some advantage can be obtained for some ratios of spacing-to-embedment depth for the case of multiple anchors.

Tables 1 and 2 are consistent with Fig. 1 and 2. However, this method of error analysis does not present a complete picture of the reliability of a given formula. It assigns more weight to data points located far from the values predicted by the equation under consideration. A few data points lying far from the curve can have as much effect as a larger number of points close to the curve. Because each data point does not contribute equally in the error analysis, some distortion is created. Also, this method does not distinguish systematic error from random error. An examination of plots of predicted capacity-versus-actual capacity illustrates that the ACI 349-90 cone method is consistently conservative for shallow embedments.[11,12] This fact is not disclosed in comparisons of square error.

LRFD IMPLICATIONS OF EXISTING METHODS

General

Each of the three capacity prediction methods is evaluated in terms of load and resistance factor design (LRFD) for both single anchors near a free edge and multiple closely spaced anchors. In particular, both the probability of steel fracture or concrete cone failure under known loads and the probability of concrete cone failure under unlimited loads are calculated. Subsequently, each approach is compared on the basis of accuracy and suitability for use in design.

Several assumptions have been made to facilitate probability of failure calculations. First, all three methods are compared using the concrete cone understrength factor of ACI 349-90 ($\phi = 0.65$), as well as the corresponding load factor (1.7). The effect of both the understrength factor and the load factor is to decrease the probability of failure.

Second, both concrete cone and steel fracture data are presumed to be normally distributed. This supposition is based on a previous study, prepared for the Tennessee Valley Authority,[1] on single anchors far from a free edge and remote from other anchors. For this study, the assumption of a normal distribution is judged reasonable if sufficient data is available for study.

Probability of failure under known loads

The objective of calculating the probability of steel failure or concrete cone failure under known loads is to determine the safety of a single anchor near a free edge or a group of multiple closely spaced anchors designed according to the load and understrength factors of ACI 349-90.

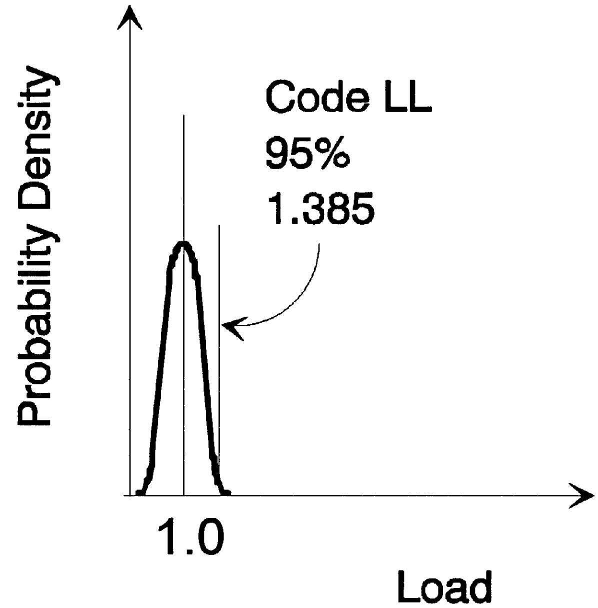

Fig. 3—Assumed statistical distribution of anchor loads

To determine this probability of failure, three distinct distributions must be defined: 1) the applied loading on the anchor; 2) the capacity of a tensile anchor, as governed by steel fracture; and 3) the capacity of a tensile anchor, as governed by concrete cone failure. As stated previously, both steel resistance and concrete resistance are assumed to follow a normal distribution. In addition, this study assumes that the loading applied to the anchor follows a normal distribution. No research was performed to determine distribution of anchor loads. These assumptions uniquely define the probabilities of failure. If loads or resistances had been assumed to be distributed in some other way (log-normal), the probabilities of failure would change. However, the same principles would have been followed in computing each probability of failure.

Statistical distribution of applied loading

For this analysis, the load is assumed to be known and distributed according to available statistical data. Moreover, anchor loads are presumed to be distributed normally, with an arbitrary mean of 1.0 and an arbitrary coefficient of variation of 0.2 (Fig. 3). No units are specified with this distribution because the distribution is independent of the units. Furthermore, the application of the loading curve is dependent only on the relationship of this curve to the curves of steel resistance and concrete resistance. Provided that units are consistent, they are otherwise unimportant.

Measurements of live load on typical office buildings have shown that building codes generally specify live loads at the 95 percentile value.[13] In other words, the prescribed value is greater than or equal to 95 percent of the observed load values. On that basis, the design load is fixed at the 95 percentile value of the assumed average load distribution. According to

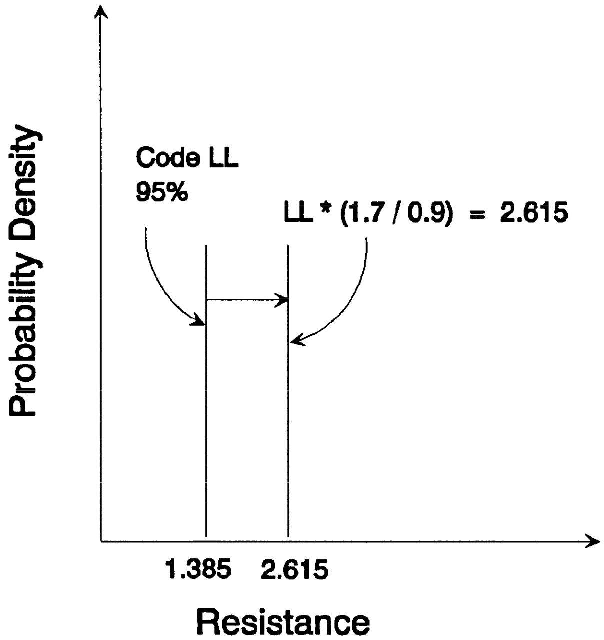

Fig. 4—Required steel resistance

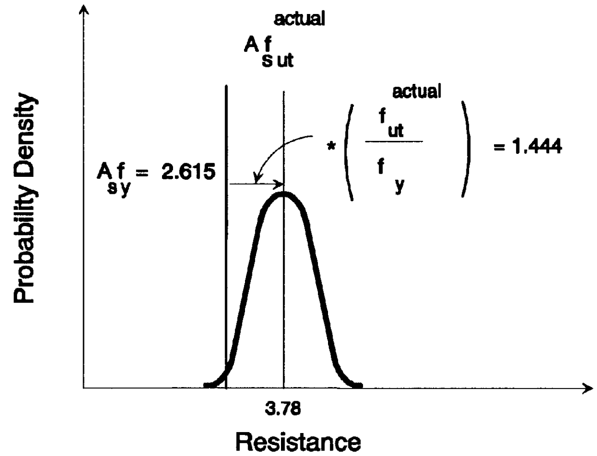

Fig. 5—Actual steel resistance

the assumed loading distribution, the 95 percentile load corresponds to a value of 1.385.

Statistical distribution of anchor resistances as governed by steel

Based on the 31 test values discussed previously, the statistical distribution of actual steel resistances divided by the ACI 349-90 predictions has a mean value of 1.444. Assuming the use of a normal distribution for steel resistance, the corresponding coefficient of variation is 0.156. Note that load factors and ϕ factors are not used in computing the mean and coefficient of variation. However, these factors are included in the probability of failure calculations.

According to ACI 349-90, the required steel resistance must be greater than or equal to the factored load. Design values for anchor steel resistances are defined as the smaller of either $\phi A_s f_y$ or $0.8 A_s f_{ut}$. Since the area of steel is constant for a given anchor, steel resistance is governed by the smaller of either f_y or 0.8 f_{ut}. For the high-strength anchors comprising the data base, the value $\phi A_s f_y$ always governs. For example, consider a typical A193-B7 anchor bolt. This anchor has a yield strength of 105 ksi and an ultimate strength of 125 ksi. Since $\phi f_y = 94.5 \text{ksi} < 0.8 f_{ut} = 100$ ksi, the yield strength governs.

Because ϕf_y governs the design of every anchor in the data base, the required steel resistance is calculated based on the yield criterion. Therefore, the minimum required steel resistance is equal to the design load (the 95 percentile value of the load distribution = 1.385), multiplied by the load factor for live load (1.7), and divided by ACI 349-90's ϕ factor for the yield strength of steel (0.90).

$$\text{Minimum required steel resistance} = \frac{1.385 \times 1.7}{0.9} = 2.615 \quad (4)$$

This required steel resistance is shown in Fig. 4.

Thus, the mean of the actual steel resistance can be defined as the product of the required steel resistance (2.615) and the calculated mean of the actual capacity-to-predicted capacity from the high-strength steel data (1.444).

$$\text{Actual mean steel resistance} = 2.615 \times 1.444 = 3.78 \quad (5)$$

As depicted in Fig. 5, the coefficient of variation of the actual steel resistance is the same as the coefficient of variation for available data on high-strength anchors.

In addition to the required steel resistance, one can define a theoretical steel resistance equal to the greater of either $\phi A_s f_y$ or $A_s f_{ut}$. If the required steel resistance is governed by f_y, then theoretical steel resistance is equal to the required steel resistance multiplied by the ratio of f_{ut} to f_y.

$$\text{Theoretical steel resistance} = 2.615 \times 1.2 = 3.138 \quad (6)$$

For purposes of this study, the ratio of f_{ut} to f_y is taken as 1.2. A comparison of the theoretical steel resistance and the steel resistance required by 0.8 $A_s f_{ut}$ is shown in Fig. 6. Observe that these values are relatively close, and that neither governs compared to $A_s f_y$.

Statistical distribution of anchor resistances as governed by concrete cone capacity

According to the provisions of ACI 349-90, the required nominal concrete strength of the anchor, reduced by an understrength factor of 0.65, must at least equal the specified ultimate tensile capacity of the anchor steel. As stated previously, the required nominal yield capacity of the anchor steel (yield strength of the anchor steel multiplied by the tensile stress area) is 2.615. Thus, the required nominal concrete capacity of the anchor is 2.615, multiplied by the ratio of specified ultimate strength to specified yield strength and reduced by the ϕ factor for concrete. As stated previously, the ratio of ultimate strength-to-yield strength is taken as 1.2. This theoretical concrete cone capacity is calculated as follows

$$\text{Theoretical concrete resistance} = 2.615 \times 1.2 \times \frac{1}{0.65} = 4.83 \quad (7)$$

These results are illustrated in Fig. 7.

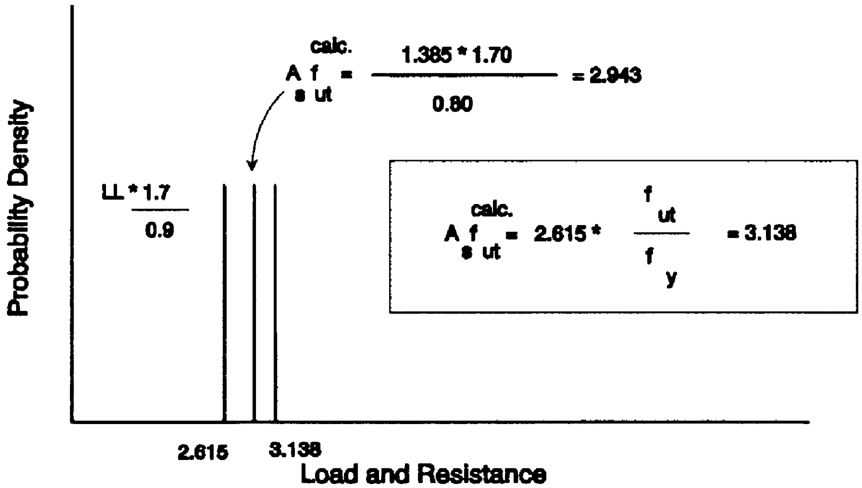

Fig. 6—Theoretical steel resistance

The actual concrete resistance mean is figured by multiplying the theoretical concrete resistance (4.83) by the ratio of actual concrete resistance-to-predicted concrete resistance. The ratio of actual concrete resistance-to-predicted concrete resistance is found from available concrete cone failure data from the data base (Fig. 8). Consider the CC method for single anchors near a free edge with edge distance-to-embedment depth ratios of 0.601-0.75

$$\textit{Actual mean concrete resistance} = 4.828 \times 1.10 = 5.31 \quad (8)$$

Combining load, steel resistance, and concrete resistance

Based on the calculated distributions for load, concrete resistance, and steel resistance, a distinct set of numbers can be selected that represents some combination of load, concrete resistance, and steel resistance. Each of these points is selected randomly based on the experimentally determined normal distributions. After comparing concrete resistance and steel resistance, the minimum of these values is selected. The quantity (resistance minus load) is then computed for distinct values of load, concrete resistance, and steel resistance. This process is repeated 10,000 times. The frequency for which the resistance is less than the load, divided by the total number of cases, represents the probability of failure. This can also be viewed graphically as the area under the (resistance minus load) curve (Fig. 9).

Probability of concrete cone failure under unlimited loads

The objective of calculating the probability of concrete cone failure under unlimited loads is to determine the probability of a failure when loads applied to a single anchor near a free edge or a group of multiple closely spaced anchors exceed those assumed during design. Such a probability of failure is of interest when the structure is subjected to a catastrophic event, such as a strong earthquake or extreme heat release. Such loads are highly unpredictable in magnitude. Because of this, design loads for extreme events are often based on the structural actions associated with the formation of a plastic mechanism, since the formation of such a mechanism sets an effective upper limit on the capacity. Thus, the phrase "unlimited loads" refers to loads that are limited only by the capacity of the weakest element in the anchoring system. The probability of concrete cone failure under unlimited loads is equivalent to the probability that steel capacity exceeds concrete capacity. To accomplish this task, distributions are designated for applied loading, steel resistance, and concrete resistance.

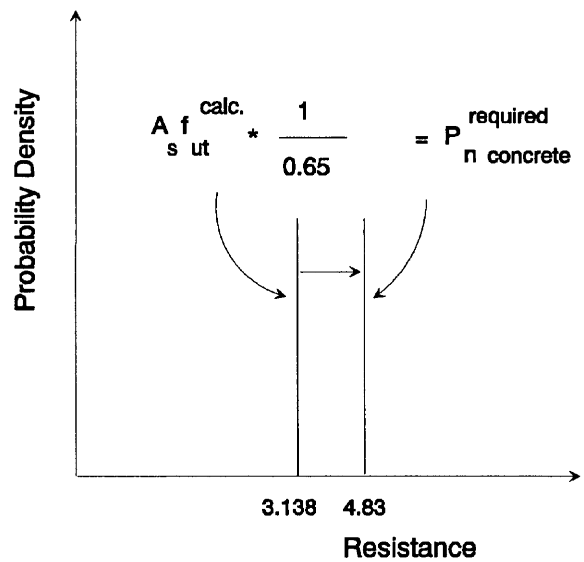

Fig. 7—Theoretical concrete resistance

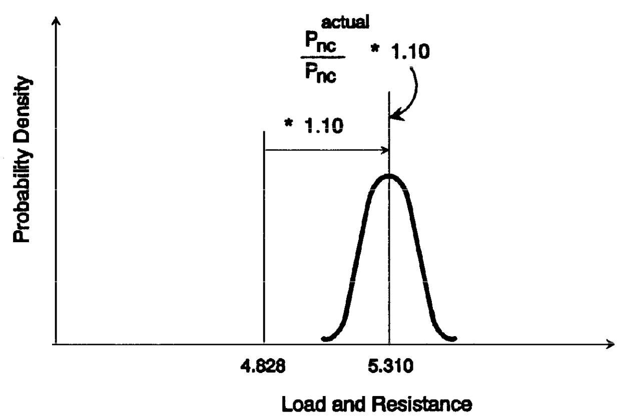

Fig. 8—Actual concrete resistance, single anchor with edge distance-to-embedment depth ratio = 0.601 - 0.75, CC method

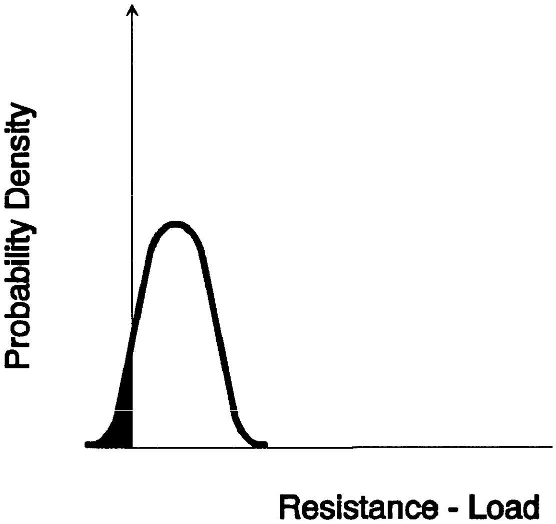

Fig. 9—Probability of failure under known loads

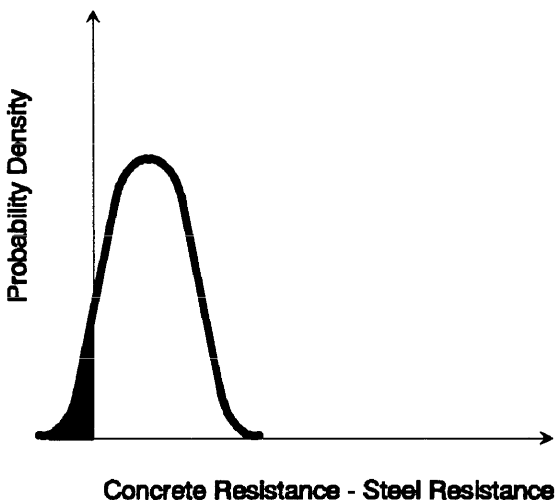

Fig. 10—Probability of concrete cone failure under unlimited loads

Table 3—Results of Monte Carlo analyses for probability of failure under known loads for single anchors near a free edge

Ratio of edge distance-to-embedment depth	Probability of failure, β = safety index					
	ACI 349-90 method		VAC method		CC method	
	β	Probability of failure	β	Probability of failure	β	Probability of failure
0.45-0.60	2.98	0.147 e2	2.70	0.351 e2	4.39	0.581 e5
0.601-0.75	3.49	0.214 e3	3.35	0.398 e3	4.25	0.107 e4
0.751-0.90	3.13	0.879 e3	3.83	0.629 e4	4.63	0.181 e5
0.901-1.05	3.27	0.546 e3	3.59	0.167 e3	4.43	0.468 e5
1.051-1.20	4.39	0.576 e5	3.11	0.943 e3	3.97	0.367 e4
1.201-1.35	4.57	0.242 e5	3.49	0.238 e3	4.74	0.107 e5
1.351-1.50	3.73	0.952 e4	3.78	0.781 e4	4.14	0.173 e4

Statistical distribution of applied loading

Although the loading to which the anchor is unlimited in this probability analysis, a distribution of loading must be assumed to obtain an initial design of the anchor. As previously stated, all applied loads are assumed to be distributed normally, with an arbitrary mean of 1.0 and an arbitrary coefficient of variation of 0.2. According to the assumed loading distribution, the 95 percentile load corresponds to a value of 1.385.

Table 4—Results of Monte Carlo analyses for probability of failure under known loads for multiple closely spaced anchors

Ratio of anchor spacing-to-embedment depth	Probability of failure, β = safety index					
	ACI 349-90 method		VAC method		CC method	
	β	Probbility of failure	β	Probbility of failure	β	Probbility of failure
0.27-0.45	4.16	0.159 e4	4.10	0.206 e4	4.45	0.437 e5
0.451-0.60	4.56	0.260 e5	4.35	0.676 e5	4.19	0.142 e4
0.601-0.75	4.40	0.548 e5	4.53	0.291 e5	4.32	0.772 e5
0.751-0.90	2.50	0.621 e2	4.21	0.129 e4	4.57	0.249 e5
0.901-1.05	4.59	0.222 e5	4.24	0.112 e4	4.59	0.225 e5
1.051-1.20	3.80	0.701 e4	3.97	0.359 e4	4.28	0.943 e5
1.201-1.35	4.50	0.347 e5	4.56	0.251 e5	4.50	0.333 e5
1.351-1.50	2.11	0.174 e1	3.28	0.519 e3	4.13	0.185 e4
1.501-2.00	4.22	0.121 e4	4.29	0.875 e5	4.46	0.405 e5
2.001-2.50	4.04	0.261 e4	4.64	0.176 e5	4.56	0.252 e5
2.501-3.00	4.55	0.266 e5	4.66	0.155 e5	4.67	0.149 e5

Statistical distribution of anchor resistances as governed by steel

The procedure used to find the position of the normal curve is equivalent to the procedure followed previously

$$Actual\ Mean\ Steel\ resistance = 2.615 \times 1.444 = 3.76 \quad (9)$$

$$Minimum\ required\ steel\ resistance = \frac{1.385 \times 1.7}{0.9} = 2.615 \quad (10)$$

As before, the coefficient of variation of the actual steel resistance is taken as the coefficient of variation of the available data on high-strength anchors.

Statistical distribution of anchor resistances as governed by concrete cone capacity

The procedure used to find the position of the normal curve is equivalent to the procedure followed previously

$$Theoretical\ concrete\ resistance = 2.615 \times \frac{1}{0.65} \times 1.2 = 4.83 \quad (11)$$

Combining steel resistance and concrete resistance

Given the distributions of steel resistance and concrete resistance, the probability of having a concrete cone capacity less than steel capacity can be calculated. Based on the calculated distributions for concrete resistance and steel resistance, a distinct pair of numbers can be selected that represents some combination of concrete strength and steel strength. Each of these points is selected randomly, based on the experimentally determined normal distributions. The quantity (concrete resistance minus steel resistance) is computed for distinct values of concrete resistance and steel resistance. The process is repeated 10,000 times. The

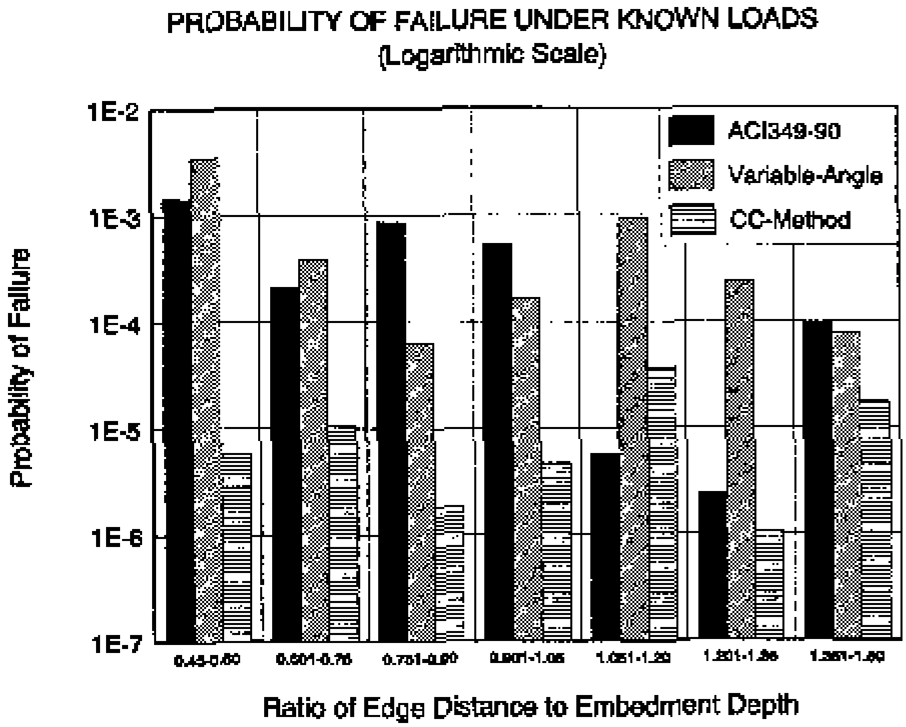

Fig. 11—Probability of failure under known loads for single anchors near a free edge

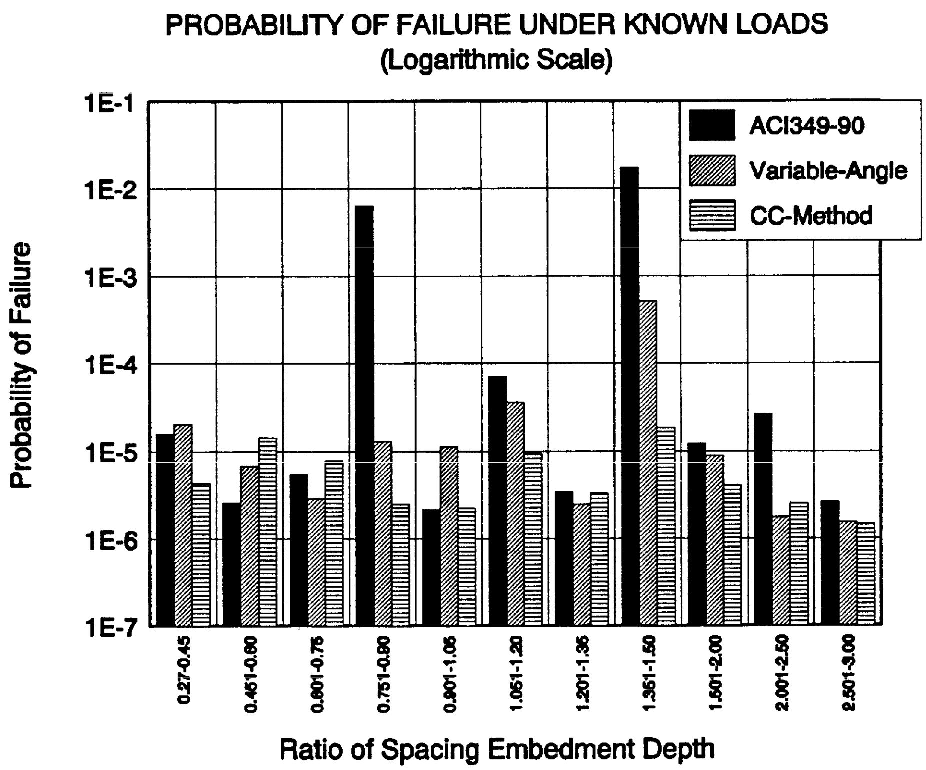

Fig. 12—Probability of failure under known loads for multiple closely spaced anchors

frequency for which concrete resistance is less than steel resistance, divided by the total number of cases, represents the probability of concrete cone failure. This can also be viewed graphically as the area under the (concrete minus steel) curve (Fig. 10).

DISCUSSION OF LRFD RESULTS

Probability of failure under known loads

The probabilities of steel or concrete failure under known loads are detailed in Tables 3 and 4, and are illustrated in Fig. 11 and Fig. 12. Current design practice for reinforced concrete structures with average consequences of failure accepts designs with probabilities of failure of about 0.0005, corresponding to β values of 3.0 to 3.5.[13] The β value, termed the "safety index," is defined as illustrated in Fig. 13.

For single anchors located near a free edge and far from other anchors, the CC method has the lowest probability of failure for all except one range of edge distance-to-embedment depth ratios. The lowest value of β for any range of edge distance-to-embedment depth for the CC method is

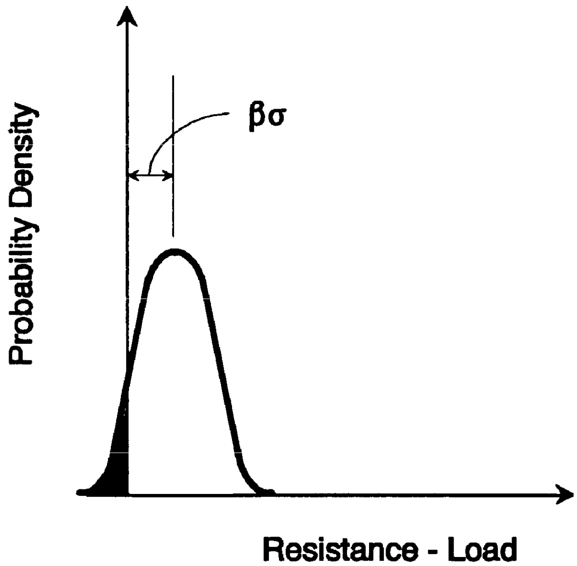

Fig. 13—Definition of β (safety index)

3.97. In contrast, the ACI 349-90 method's lowest value for β is 2.98. In addition, three ranges for the ACI 349-90 method have β values between 3.0 and 3.5, with all other β values greater than 3.5. For five of the seven ranges of edge distance-to-embedment depth, no difference is evident between the ACI 349-90 and VAC methods. In the remaining ranges, the ACI 349-90 method has a lower probability of failure than the VAC method.

As with single anchors near a free edge, the CC method generally has the lowest probability of failure for all ratio ranges of spacing-to-embedment depth for multiple closely spaced anchors. The lowest value of β for any ratio range of edge distance-to-embedment depth for the CC method is 4.13. In contrast, the ACI 349-90 method has two ranges with values of β less than 2.5. All other ranges have β values greater than 3.5. The VAC method predicts probabilities of failure similar to the ACI 349-90 method, except for two ranges of spacing-to-embedment depth where the β value is lowest. This is because the VAC method predicts the failure load more consistently for various embedment depths.

Table 5—Results of Monte Carlo analyses for probability of concrete cone failure under unlimited loads for single anchors near a free edge

Ratio of edge distance-to-embedment depth	Probability of concrete cone failure, β = safety index					
	ACI 349-90 method		VAC method		CC method	
	β	Probability of failure	β	Probability of failure	β	Probability of failure
0.45-0.60	0.43	0.333	-0.69	0.754	2.52	0.592 e2
0.601-0.75	1.01	0.156	-0.97	0.834	2.14	0.163 e1
0.751-0.90	0.59	0.279	-0.43	0.666	5.59	0.112 e7
0.901-1.05	1.19	0.117	-0.02	0.509	2.56	0.521 e2
1.051-1.20	2.69	0.356 e2	0.02	0.494	1.56	0.594 e1
1.201-1.35	2.62	0.436 e2	0.37	0.356	4.41	0.519 e5
1.351-1.50	1.68	0.467 e1	-0.37	0.645	1.37	0.851 e1

Probability of concrete cone failure under unlimited loads

The probabilities of concrete cone failure under unlimited loads are detailed in Tables 5 and 6, and illustrated in Fig. 14 and 15. For single anchors near a free edge, the VAC method is associated with a consistently larger probability of concrete cone failure than either the ACI 349-90 method or the CC method for all ranges of edge distance-to-embedment depth ratios. In addition, the ACI 349-90 method has a probability of failure much larger than the CC method for the lowest four ratio ranges of edge distance-to-embedment depth.

For multiple closely spaced anchors, the ACI 349-90 method has probabilities of concrete cone failure within two

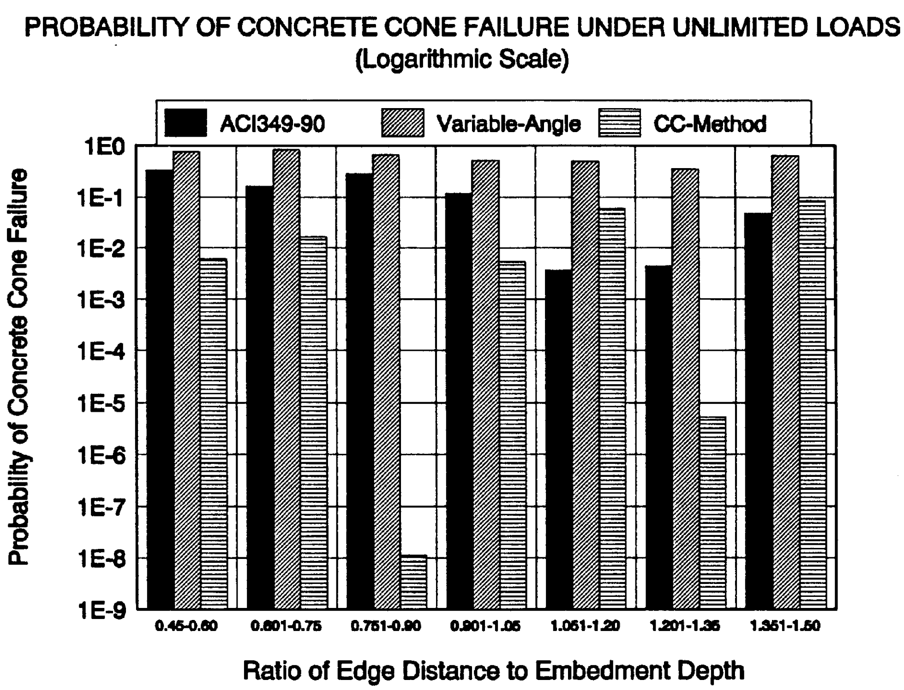

Fig. 14—Probability of concrete cone failure under unlimited loads for single anchors near a free edge

orders of magnitude of those from the CC method for all but two ranges of spacing-to-embedment depth ratios (s_1/h_e = 0.751 - 0.90 and s_1/h_e = 1.201 - 1.35). However, in those two ranges it is difficult to draw definitive conclusions regarding the relative merits of the ACI 349-90 and CC methods. This is because both ranges contain a large number of points in which the embedment depth and the configuration of the anchor group are constant. Thus, if all variables affecting the actual failure load remain constant for these tests, the failure loads would be the same. Consequently, these results lead to a constant ratio of actual-to-predicted failure load. This provides an excellent approximation to the current use of an assumption of the normal curve. However, there are approximately three different groups of tests in each range of spacing-to- embedment depth, so the assumption of the normal distribution is no longer a strong premise. Examination of Fig. 15 indicates that the CC method has more consistent probabilities of concrete cone failure under unknown loads than either the ACI 349-90 method or the VAC method.

General limitations of analyses

1. Loads and resistances are assumed to be normally distributed. Safety factors computed by the analysis could increase or decrease as a result of different presupposed distributions.

2. Actual concrete strength is presumed to equal the specified value. This premise is conservative because actual concrete strength usually exceeds specifications.

3. A single representative value is assumed for the ratio of specified ultimate steel strength-to-specified yield strength. This value could be made more accurate by computing the ratio separately for each anchor in the data base.

Table 6—Results of Monte Carlo analyses for probability of concrete cone failure under unlimited loads for multiple closely spaced anchors

Ratio of anchor spacing-to-embedment depth	Probability of concrete cone failure, β = safety index					
	ACI 349-90 method		VAC method		CC method	
	β	Probability of failure	β	Probability of failure	β	Probability of failure
0.27-0.45	1.88	0.301 e1	1.97	0.248 e1	2.71	0.339 e2
0.451-0.60	2.83	0.239 e2	1.88	0.299 e1	1.89	0.291 e1
0.601-0.75	2.77	0.284 e2	2.82	0.244 e2	2.55	0.537 e2
0.751-0.90	0.65	0.258	0.29	0.385	3.45	0.280 e3
0.901-1.05	6.30	0.150 e9	1.41	0.769 e1	5.57	0.128 e7
1.051-1.20	1.76	0.390 e1	1.23	0.110	2.21	0.135 e1
1.201-1.35	3.08	0.105 e2	6.37	0.938 e10	4.76	0.958 e6
1.351-1.50	0.81	0.210	0.56	0.287	1.93	0.265 e1
1.501-2.00	2.26	0.119 e1	2.12	0.172 e1	3.07	0.106 e2
2.001-2.50	2.04	0.204 e1	3.85	0.594 e4	2.92	0.175 e2
2.501-3.00	7.09	0.665 e12	5.35	0.445 e7	8.03	0.500 e15

SUMMARY AND CONCLUSIONS

Summary

The overall objective of this research has been to evaluate the accuracy and suitability for design of three different methods for predicting anchor capacity as governed by concrete cone failure. In particular, this study has focused on single anchors located close to a free edge and multiple anchors located far from a free edge. ThE objective has been accomplished by the following steps:

1. A total of 160 data points is available from tests on single anchors close to a free edge in which failure occurs by formation of a concrete cone. Using common definitions and nomenclature for all variables and material properties, those data have been placed on a data base in terms of SI units and concrete cube strengths.

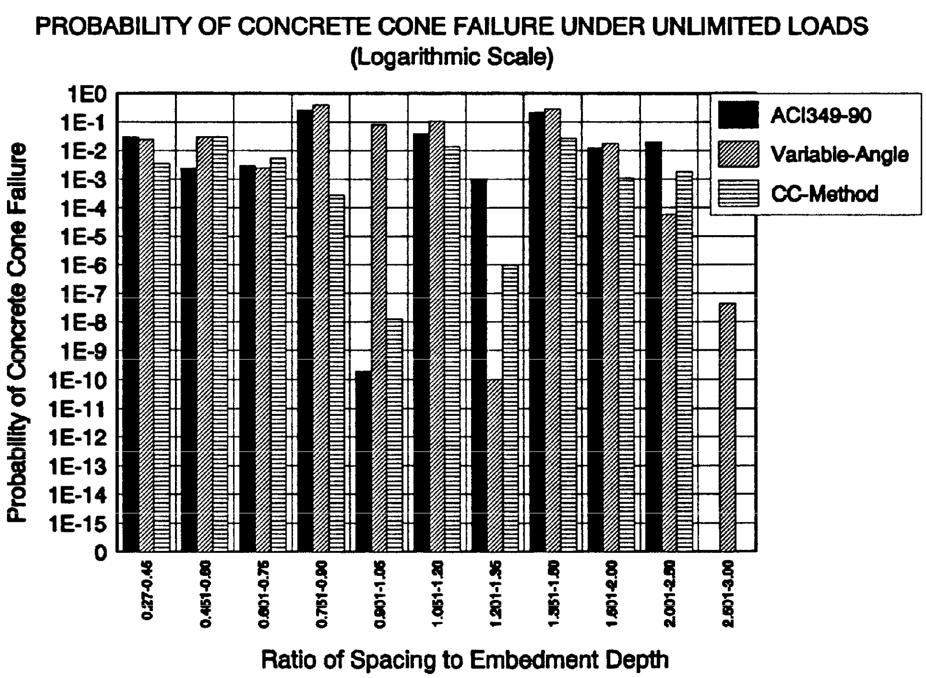

Fig. 15—Probability of concrete cone failure under unlimited loads for multiple closely spaced anchors

2. A total of 185 data points is available from tests on multiple anchors far from a free edge in which failure occurs by formation of a concrete cone. These data on multiple anchors include data points on groups of both two and four anchors. Using common definitions and nomenclature for all variables and material properties, those data are placed on a data base in SI units and in terms of concrete cube strengths.

3. Data for single anchors located close to a free edge have been compared with the predictions of three existing methods: a) the 45 deg cone method of ACI 349-90; b) the VAC method; and c) the CC method (exponent of 1.5). The comparisons are in terms of concrete cone capacity, normalized by $\sqrt{f_{cc}}$ as a function of embedment depth.

4. Data for multiple anchors have been compared with predictions of three existing methods: a) the 45 deg cone method of ACI 349-90; b) the VAC method; and b) the CC method (exponent of 1.5). The comparisons are, again, in terms of normalized concrete cone capacity as a function of embedment depth.

5. For single anchors near a free edge, square errors have been computed for each method for different ranges of the ratio of edge distance-to-embedment depth.

6. For multiple closely spaced anchors, square errors have been computed for each method for different ranges of the ratio of anchor spacing-to-embedment depth.

7. The probability of steel or concrete failure under known loads and concrete cone failure under unlimited loads has been determined for single anchors located near a free edge. Probabilities of failure have been evaluated for different ranges of the ratio of edge distance-to-embedment depth.

8. The probability of steel fracture or concrete cone failure under known loads and of concrete cone failure under unlimited loads has been determined for multiple closely spaced anchors. Probabilities of failure have been evaluated for different ranges of the ratio of anchor spacing-to-embedment depth.

9. Based on the previous comparisons of square error and probability of failure, each method has been evaluated with respect to accuracy and design suitability in terms of probability of failure for single anchors located near a free edge and for multiple closely spaced anchors.

CONCLUSIONS

1. For most embedment depths, the CC method has a square error lower than that of either ACI 349-90 or the VAC method. This is true both for single anchors located close to a free edge and for multiple closely spaced anchors.

2. For single anchors located close to a free edge, and also for multiple closely spaced anchors, all three methods have values of the safety index β that are comparable with accepted safety index values for similar structural elements. This implies that all three methods give sufficiently low probabilities of failure under known loads for all ratios of edge distance-to-embedment depth and of spacing-to-embedment depth.

3. Under unlimited loads, the probabilities of concrete cone failure associated with the CC method are generally lower than those of either the VAC method or the ACI 349-90 method. This is certainly true for single anchors near a free edge. For multiple closely spaced anchors, the CC method gives probabilities of failure that are less than or equal to the probabilities of the other two methods.

4. The primary advantage of the ACI 349-90 approach may be its conical idealization of the failure surface, which is easy to visualize. However, calculations of net projected area for multiple anchors are very complex in practice. The CC method uses the basic principle of idealizing the failure surface along with a relatively simple calculation for net projected area. For this reason, the CC method can be regarded as at least as designer-friendly.

5. It is believed that LRFD approaches such as that used here give a realistic view of the comparative safety of different anchor design methods. As additional data become available, they should be added to the data base. If new methods for predicting anchor capacity are proposed, they should be checked against the entire data base, using this same approach.

ACKNOWLEDGMENTS

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NOTATION

A_p	= actual projected area of anchor or anchor group
$A_{p,o}$	= projected area of all anchors not limited by edge or spacing influences
d	= diameter of anchor
d_h	= diameter of anchor head, taken as anchor diameter
f_c	= actual concrete cylinder compressive strength
f_c'	= specified concrete cylinder compressive strength
f_{cc}	= actual concrete cube compressive strength
f_{cc}'	= specified concrete cube compressive strength
h_e	= embedment length, measured from free surface to bearing surface of anchor head
ϕ	= cone angle, measured from plane perpendicular to anchor axis
Ψ_{SN}	= factor used in CC method to account for anchors close to free edge
ϕ_{ec}	= factor used in CC method to account for eccentricity of applied loading

REFERENCES

1. Farrow, C. B.; Frigui, I.; and Klingner, R. E., "Tensile Capacity of Single-Headed Anchors in Concrete: Evaluation of Existing Formulas on an LRFD Basis," *Report* to the Tennessee Valley Authority, University of Texas, Austin, Mar. 1992.

2. Breen, J. E.; Eligehausen, R.; and Fuchs, W., "Comparison of Procedures for Concrete Capacity," *Draft Study* No. 12/14-91/10, Sept. 1991.

3. Klingner, R. E., and Mendonca, J. A., "Tensile Capacity of Short Anchor Bolts and Welded Studs: A Literature Review," ACI JOURNAL, *Proceedings* V. 79, No. 4, July-Aug. 1982, pp. 270-279.

4. Collins, D. M.; Klingner, R. E.; and Polyzois, D., "Load-Deflection Behavior of Cast-in-Place and Retrofit Concrete Anchors Subjected to Static, Fatigue, and Impact Tensile Loads," *Research Report* CTR 1126-1, Center for Transportation Research, University of Texas, Austin, Feb. 1989.

5. CEB Task Group VI/5, "Fastenings to Reinforced Concrete and Masonry Structures: Design and Detailing, V. 1—Behavior under Monotonic, Sustained, Fatigue, Seismic, and Impact Loadings," Task Group/5, Embedments, Euro-International Concrete Committee (CEB), Aug. 1991.

6. Lee, D. W., and Breen, J. E., "Factors Affecting Anchor Bolt Development," *Research Report* No. 88-1F, Center for Highway Research, University of Texas, Austin, Aug. 1966.

7. Hasselwander, G. B.; Jirsa, J. O.; Breen, J. E.; and Lo, K., "Strength and Behavior of Anchor Bolts Embedded Near Edges of Concrete Piers," *Research Report* No. 29-2F, Center for Highway Research, University of Texas, Austin, May 1977.

8. "General Anchorage to Concrete," *TVA Civil Design Standard* No. DS-C.1.7.1, Tennessee Valley Authority, Knoxville, Tennessee, 1984.

9. "UEAtc--European Union of Agrément," UEAtc Directives for Assessment of Anchor Bolts, M.O.A.T. No. 42, London, 1986.

10. ACI Committee 349, "Code Requirements for Nuclear Safety-Related Structures (ACI 349-90)," American Concrete Institute, Detroit, 1985.

11. Frigui, Imed, "Evaluation of Single Tensile Anchors on an LRFD Basis," MS thesis, University of Texas, Austin, Aug. 1992.

12. Farrow, C. Ben, "Evaluation of Tensile Anchors with Partial or Overlapping Failure Cones on an LRFD Basis," MS thesis, University of Texas, Austin, Aug. 1992.

13. MacGregor, J. G., *Reinforced Concrete Mechanics and Design*, Prentice-Hall, Englewood Cliffs, New Jersey, 1988.