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Punching Shear Design of Earthquake-Resistant Slab-Column Connections

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During an earthquake, unbalanced moments can produce significant shear stresses that increase the vulnerability of slab-column connections to brittle punching failure. This paper proposes design requirements for earthquake-resistant slab-column connections. The proposed requirements include the value of the unbalanced moment to be used in punching shear design. An upper limit for the design value of the unbalanced moment is suggested based on the flexural capacity of the slab. Use of the upper limit of the design moment ensures that punching shear failure is prevented. The paper also shows that the use of shear reinforcement, particularly shear studs with mechanical anchors, significantly enhances the ductility of slab-column connections under reversed cyclic loading.

Keywords: column capital; drop panel; ductility; flat slab; punching shear; shear reinforcement; stud.

INTRODUCTION

Brittle punching failure can occur due to the transfer of shearing forces and unbalanced moments between slabs and columns. During an earthquake, the unbalanced moments can produce significant shear stresses in the slab. As a result, many flat slab structures have collapsed by punching shear in past earthquakes.[1-3] During the 1985 Mexico City earthquake, 91 waffle- and flat-slab buildings collapsed.[3]

A solution sometimes used in practice to augment punching resistance is to increase the slab thickness by drop panels or shear capitals. This paper shows that even though drop panels and shear capitals enhance the punching strength of slabs, they do not improve the ductility, which is an essential requirement of earthquake-resistant structures. It is also shown that shear capitals do not enhance the punching strength when columns transfer large moment reversals to the slab.

Shear reinforcement in the slab provides the ductility necessary for earthquake-resistant slab-column connections. Design requirements for this shear reinforcement are proposed in this paper. The suggested design procedure supplements the ACI 318-99[4] provisions and recommendations of ACI 421.1R-99[5] for punching shear design. Additional design requirements specified in Chapter 21 of the ACI 318-99 building code should also be satisfied.

The ACI 318-99 Code allows analysis of flat slab structures as plane frames. When the frame is subjected to large horizontal forces, however, there is no consensus on the width of slab strip to be included in the frame model and on how to allow for cracking. These modeling parameters significantly affect the resulting values of the moments transferred between slabs and columns. A design procedure is suggested herein to determine an upper limit for the moments that can be transferred between the slab and the column in an earthquake.

This paper focuses on design of earthquake-resistant slab-column connections both without shear reinforcement and with stud shear reinforcement (SSR). The proposed design procedure is based on numerical analyses as well as experimental research on reinforced concrete flat slabs subjected to cyclic moment reversals simulating earthquakes. Although reference is made here to the Uniform Building Code (UBC 97),[6] the proposed design procedure is general and thus should apply to any earthquake zone.

RESEARCH SIGNIFICANCE

Brittle punching failure is a problem that unnecessarily limits the widespread use of flat plates in active earthquake zones. This may be partly due to the fact that the ACI 318-99 Code does not have provisions for analysis and design of earthquake-resistant flat plates.[4] This paper presents a complete procedure for the analysis and design of earthquake-resistant slab-column connections. The suggested procedure is justified by results of experiments and finite element analyses.

LATERAL INTERSTORY DRIFT

Flat slab-column frames are very flexible unless they are provided with primary structural systems that control sidesway. The slab-column connections must have sufficient ductility to undergo the lateral deformations of the primary lateral-force-resisting system without loss of their ability (due to punching) to carry the gravity loads applied during or after the earthquake. The connections, however, should not be considered as part of the primary system resisting lateral forces.

It has been suggested that a primary system should be provided to limit the interstory drift ratio of concrete structures to 1.5%.[7] The interstory drift ratio DR_u is defined as the lateral displacement of one level (or floor) relative to the level above or below, divided by the floor height, and it includes inelastic deformations. The structural elements must have the capability to undergo this 1.5% drift ratio without failure. The 1.5% drift ratio has been frequently adopted as a minimum drift ratio that slab-column connections must be able to undergo without failure.[8,9]

The UBC 97 code[6] requires that DR_u be less than 0.025 (2.5%) for structures having a fundamental period less than 0.7 s. For structures having a fundamental period of 0.7 s or greater, DR_u must not exceed 0.020 (2.0%). Thus, the interstory drift ratio of flat slabs provided with a primary lateral-force resisting system must not exceed 2.0 or 2.5%, depending on

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the fundamental period of the structure. The fundamental period is to be calculated according to the UBC 97 provisions.

It is recommended here that flat slab structures located in seismic zones should be provided with a primary lateral force-resisting system, such as shear walls. The primary system should have sufficient stiffness to control the sidesway of the structure. The frequently recommended limit of 1.5% drift ratio, or the UBC 97 specified 2.0 or 2.5% drift ratio, should be satisfied. When no shear reinforcement is provided in the slab, the limiting 1.5% drift ratio should apply; furthermore, the shearing force V_u transferred between the slab and column should not exceed a limit specified below.

EFFECT OF GRAVITY LOAD ON INTERSTORY DRIFT CAPACITY

Figure 1 shows the variation of ultimate interstory drift ratio DR_u with the ratio $(V_u/\phi V_c)$ for interior slab-column connections transferring constant shearing forces V_u, and with reversals of cyclic moments of increasing magnitude. The Curves 1, 2, and 3 shown in Fig. 1 represent, respectively, the best fit of results of experiments reported in the literature[9-16] on interior slab-column connections with no shear reinforcement, with conventional stirrups, and with SSR. The SSR is composed of vertical rods anchored mechanically near the bottom and top surfaces of the slab. The variable V_u represents the ultimate shearing force transferred between the slab and the column at the moment of earthquake occurrence; V_c is the punching shear capacity of the slab-column connection without shear reinforcement and with no moment transfer; and ϕ is the strength-reduction factor taken as 1.0 (from experimental data). According to ACI 318-99,[4] V_c is usually governed by Eq. (1) (in inch and pound units):

$$V_c = 4 b_o d \sqrt{f_c'} \tag{1}$$

where b_o is the perimeter length of the punching shear-critical section at $d/2$ from column face (Fig. 2), d is the effective slab depth, and f_c' is the specified concrete compressive strength. In special cases, the code specifies smaller values for V_c.

Figure 1 indicates that the capability of the slab-column connection to withstand interstory drift without failure decreases with the increasing magnitude of the applied gravity loads. The horizontal line at an ultimate drift ratio intersects Curve 1 at $(V_u/\phi V_c) \approx 0.4$, indicating that slabs with no shear reinforcement will satisfy the required 1.5% drift ratio only if V_u does not exceed $0.4\phi V_c$. The horizontal dashed lines plotted in Fig. 1, corresponding to the 2.0 and 2.5% drift ratios specified by the UBC 97, intersect Curve 1 at 0.32 and 0.25, respectively (Fig. 1). Thus, it is recommended that for DR_u = 2.0 or 2.5%, the slab be provided with shear reinforcement when V_u is greater than 0.25 ϕV_c.

Figure 3 is similar to Fig. 1, but the two curves shown in Fig. 3 represent experiments of edge slab-column connections, with and without SSR, for transferring shearing forces and cyclic moment reversals.[17,18] The graphs indicate that for edge slab-column connections with no shear reinforcement, DR_u = 1.5 or 2.5% only when V_u is limited to 0.5 or 0.4 of ϕV_c, respectively. Comparison of Fig. 1 and 3 indicates that somewhat higher limits on $(V_u/\phi V_c)$ can be allowed for edge connections than for interior connections. For simplicity, a single pair of limits on V_u is suggested in the following section. In addition to the limitation of V_u, absence of shear reinforcement should be allowed only when the maximum shear stress is checked as discussed as follows.

Figures 1 and 3 show that the curves that represent experiments of slab-column connections with shear reinforcement fall well above the horizontal lines. Although Curves 2 and 3 of Fig. 1 are not accurate because of the scatter of the experimental data, the accuracy of the curves does not change the conclusion that no limit on $V_u/\phi V_c$ is needed for slabs with a minimum amount of shear reinforcement in the form

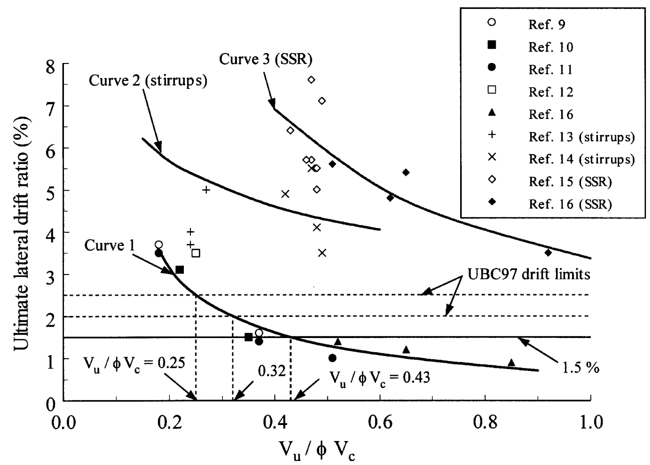


Fig. 1—Effect of gravity loads on lateral drift capacity of interior slab-column connections.

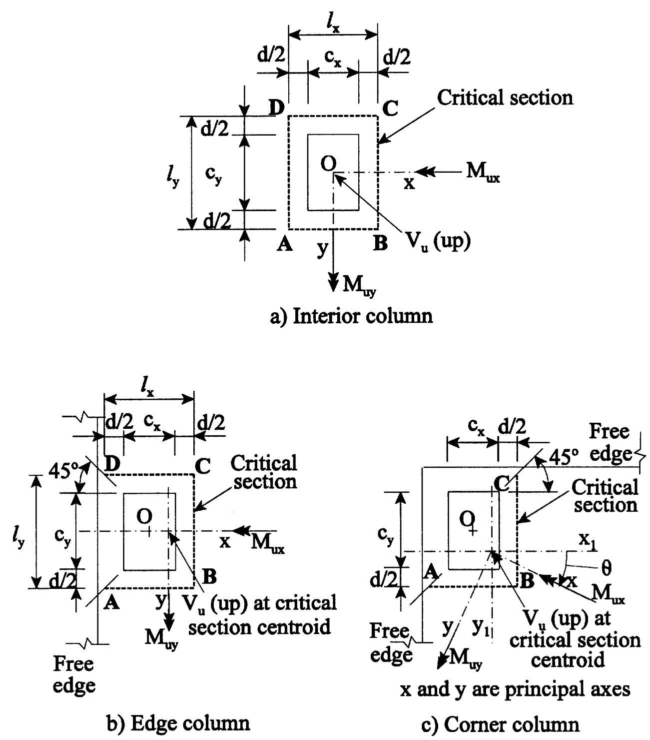


Fig. 2—Critical sections for punching shear at d/2 *from column face (ACI 318-99 code[4]).*

of stirrups or SSR to achieve the 2.0 or 2.5% interstory drift ratios specified by UBC 97. Figure 1 also shows that the drift capability without punching failure of slabs with SSR is higher than that of slabs with conventional stirrups.

PROPOSED REQUIREMENTS FOR EARTHQUAKE-RESISTANT SLAB-COLUMN CONNECTIONS

As mentioned previously, the primary structural system should be designed for full-earthquake lateral forces. In addition to this requirement, slab-column connections should be provided with shear reinforcement. This reinforcement should not be not less than the minimum amount specified in the following section, except when the value of the factored shearing force V_u transferred between the slab and the column is less than $0.25\phi V_c$ (where ϕ is the strength-reduction factor for shear equal to 0.85). This requirement is to ensure that the connections can withstand a drift ratio DR_u of 2.5%. If it can be shown by analysis that DR_u does not exceed 1.5%, shear reinforcement should be provided if V_u exceeds $0.4\phi V_c$ (instead of $0.25\phi V_c$).

MINIMUM AMOUNT OF SHEAR REINFORCEMENT

Earthquake-resistant slab-column connections can be built without shear reinforcement only when $V_u/\phi V_c$ is limited as specified in the preceding section. In addition, the maximum shear stress due to V_u, combined with unbalanced moment M_u should be smaller than ϕ multiplied by the nominal shear stress at the critical section at $d/2$ from the column face. Connections that do not satisfy these two conditions should be provided with a minimum amount of shear reinforcement such that the nominal shear stress satisfies the inequality

$$v_s = \frac{A_v f_{yv}}{b_o s} \geq 3\sqrt{f_c'} \tag{2}$$

with A_v being the area of shear reinforcement in one peripheral line parallel to column faces; s is the spacing between peripheral lines of shear reinforcement ($s \leq 0.75d$ for stud shear reinforcement); b_o is length of perimeter of the critical section at $d/2$ from column face (refer to Fig. 2); and f_{yv} is the specified yield strength of shear reinforcement. The distance between the column face and the outermost peripheral line of shear reinforcement should not be less than $3.5d$.

The minimum shear reinforcement recommended previously is based on past experiments.[15-18] In these experiments (Table 1), slab-column connections with different amounts of SSR could undergo substantial drifts before failure. The table indicates that even when the studs are spaced at the upper limit $s = 0.75d$, slab-column connections can undergo drift ratios much higher than 2.5% without punching failure, even when $(V_u/\phi V_c)$ is relatively high. For this reason, the minimum amount of SSR specified by Eq. (2) is smaller than what has been used in the experiments. No equation is given herein to replace Eq. (2) when stirrups are used because there have been no sufficient experiments with unbalanced moment reversals.

DUCTILITY OF SLAB-COLUMN CONNECTIONS

The methods for punching-shear strengthening of slab-column connections include provisions for drop panels, shear capitals, conventional stirrups, or SSR. Although all these methods are successful in increasing the punching strength, their effects on ductility are substantially different.

The term shear capital, used in this paper, refers to a uniform increase in slab thickness over a small area in the column vicinity. The shear capital is intended to increase the perimeter of the first shear-critical section adjacent to the column. Because the plan dimensions of shear capitals are small, they commonly do not contain reinforcement other than that of the column.

The most common types of shear reinforcement in slabs are the vertical legs of stirrups and SSR. Stirrups may not be effective as shear reinforcement in thin slabs because of inefficient anchorage of the stirrups' vertical legs. With SSR, the anchorage is provided mechanically by forged heads, or by a forged head at one end and a welded steel strip (referred to as a rail) at the other end.

Figure 4 represents load-deflection graphs for five slabs of 6 in. thickness.[18] The specimens represent connections of slabs with interior columns. Vertical load is applied on the column and transferred to the slab, which is simply supported on all four edges. The slabs have the same flexural reinforcement layout and almost the same material properties for concrete and flexural reinforcement. The slabs differ only in the punching strengthening method. One control slab[19] has no punching strengthening means. The others are strengthened by a drop panel,[18] a shear capital,[18] stirrups,[20] and SSR.[19] Within the drop panel and the shear capital, the slab thickness is increased from 6 in. to 9 in. The nominal shear stresses provided by the shear reinforcement (Eq. (2)) in Specimens B (stirrups) and AB5 (SSR) are 428 and 454 psi, respectively. Punching failure has occurred at a section within the shear-reinforced zone in Specimen B and at a section outside the

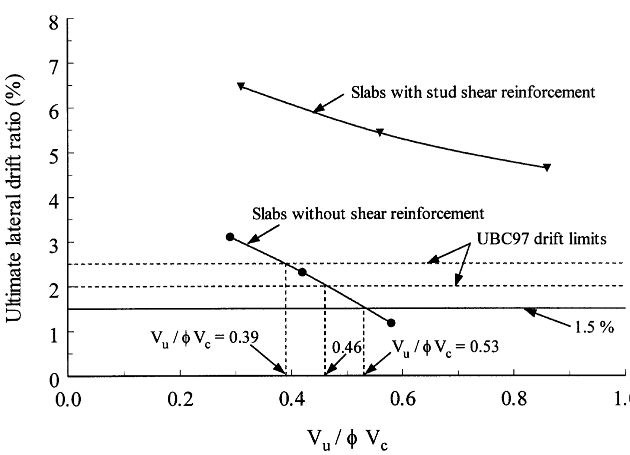


Fig. 3—Effect of gravity loads on lateral drift capacity of edge slab-column connections.

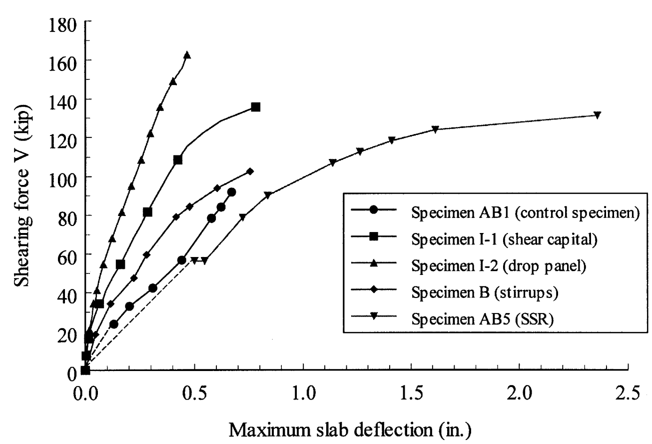


Fig. 4—Load-deflection curves of slabs with different punching strengthening methods.

shear-reinforced zone in Specimen AB5. (The first parts of the experimental curves shown in Fig. 4 for Specimens AB1 (control specimen) and AB5 are not available.)

Figure 4 shows that the strength is slightly increased by the provision of stirrups. The drop panel, the shear capital, and the SSR have provided significant strength increases. With the drop panel and the shear capital, however, the failure is brittle. The slab deflection at failure is much smaller compared to the slab with SSR, for which the failure may be described as ductile. Conventional stirrups have provided insignificant increases in ductility. Figure 4 indicates that provision of SSR is the most efficient means of punching strengthening of slab-column connections.

It has been verified by experimental research that SSR is also efficient when slab-column connections are subjected to cyclic moment reversals.[15-18] Samples of such experimental results are shown in Fig. 5. The figure shows hysteresis loops of unbalanced reversed moment versus lateral drift ratio of two edge slab-column connections.[17] The two connections are identical with the only difference being provision of SSR. Figure 5(a) shows hysteresis loops for the slab without shear reinforcement, whereas Fig. 5(b) shows the hysteresis loops of the slab with SSR spaced at $s = 0.75d$. Figure 5(a) indicates that without shear reinforcement, the strength drops with moment reversals and brittle failure occurs after a small number of cycles. With SSR, even when spaced at the upper limit ($s = 0.75d$), the ultimate drift ratio, ductility, and energy-dissipation capability are significantly enhanced.

SHEAR CAPITALS

Figure 6(a) and (b) show the inclination of punching shear cracks of a slab column connection transferring V_u combined with M_u. When M_u is relatively high, the inclination of the shear crack can be reversed (Fig. 6(b)), in which case the shear capital is not fully effective. This phenomenon has been predicted by finite element analyses[18] and confirmed by experiments.[12] Furthermore, with shear capitals, the punching shear failure is brittle; failure is accompanied by sudden separation and drop of parts of the shear capital.[18] For these reasons, it is recommended that shear capitals not be used as means of providing earthquake-resistant slab-column connections.

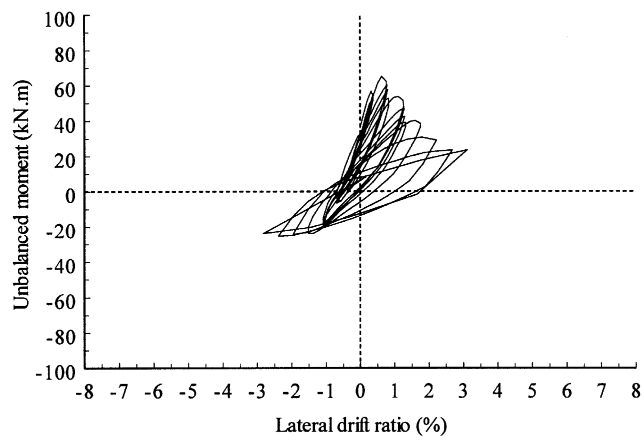
DESIGN VALUE OF UNBALANCED MOMENT TRANSFER

The ACI 318-99 Code gives different methods of analysis for flat slabs subjected to gravity loads with no lateral forces.[4] These include the direct design method and the equivalent frame method of analysis. Although it is recommended previously that flat slabs should not be employed in resistance of lateral forces, unavoidable moments are transferred between flat slabs and their supporting columns as the flat slab-column connections undergo the sidesway of the primary lateral-force-resisting system. Thus, slab-column connections should be designed for punching shear using the shearing force and unbalanced moments resulting from the applied gravity loads, combined with the unbalanced moments resulting from horizontal movements of the slab during earthquakes.

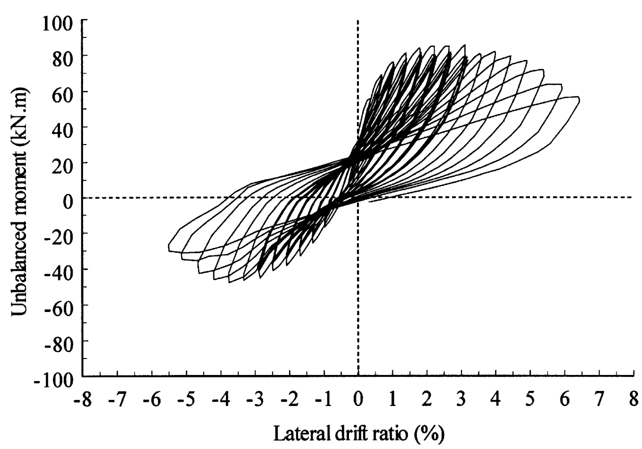
Table 1—Ultimate drift ratio of slab-column connections transferring cyclic moment reversals

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Slab	Reference no.	f_c', psi	$V_u/\phi V_c$	s/d	f_{yv}, ksi	$v_s/\sqrt{f_c'}$	DR_u, %
SJB-1	15*	4669	0.48	0.57	66.7	5.88	5.5
SJB-2		4974	0.47	0.57	66.7	5.70	5.7
SJB-3		4698	0.48	0.57	66.7	5.87	5.0
SJB-4		5757	0.43	0.57	66.7	5.30	6.4
SJB-5		4843	0.47	0.57	66.7	5.78	7.6
SJB-8†		5075	0.46	0.57	66.7	5.64	5.7
SJB-9†		4524	0.49	0.57	66.7	5.98	7.1
CD3	16	5162	0.92	0.79	53.8	4.43	3.5
CD4		4974	0.62	0.79	53.8	4.51	4.8
CD6		4553	0.65	0.39	53.8	12.3	5.4
CD7		4147	0.51	0.79	53.8	4.94	5.6
MG-3	17	4872	0.56	0.75	54.7	4.03	5.4
MG-4		4684	0.86	0.75	54.7	4.11	4.6
MG-5		4104	0.31	0.75	54.7	4.39	6.5
MG-6		4365	0.59	0.44	54.7	7.23	6.0
MG-10	18	4307	0.60	0.75	54.7	4.28	5.2

*Test variable in this series was concentration of slab flexural reinforcement in column vicinity.
†Slab-column connections transferring biaxial moments $M_{ux} = M_{uy}$ (Fig. 2(a)).



(a) Slab with no shear reinforcement



(b) Slab with stud shear reinforcement

Fig. 5—Hysteresis loops of unbalanced moment-versus-drift ratio of edge slab-column connections.[17]

There is no consensus on the value of M_u used in punching design of earthquake-resistant slab-column connections. The method suggested as follows may be used to determine the value of M_u for slab-column connections subjected to gravity loads and earthquake forces. An upper limit for M_u that need not be exceeded is also suggested in the following section.

The flat slab structure can be modeled as a plane frame for which the moments of inertia of the slab and its supporting columns are determined as per the equivalent frame method.[4] The flat slab-column frame should be combined with the primary structural system (such as shear walls) for analysis under earthquake lateral forces specified by UBC 97. Slab-column connections should be designed for the moments obtained from this analysis. Alternatively, the slab-column connection can be idealized by the equivalent frames shown in Fig. 7. For the frame model shown in Fig. 7(a), it is assumed that the length of spans l adjacent to the column is equal and the floor heights h_f above and below the considered level are equal. The moments of inertia of the column I_c and of the slab I_s are calculated as per the equivalent frame method of analysis.[4] Both moments of inertia in the slab-column joint area have infinite values; these areas are represented by the thick members shown in Fig. 7. Horizontal displacement Δ_s is introduced at the upper end of the columns as shown in Fig. 7. According to the UBC,[6] Δ_s is related to the maximum interstory drift, Δ_m, including inelastic deformations

$$\Delta_s = \frac{\Delta_m}{0.7R} \tag{3}$$

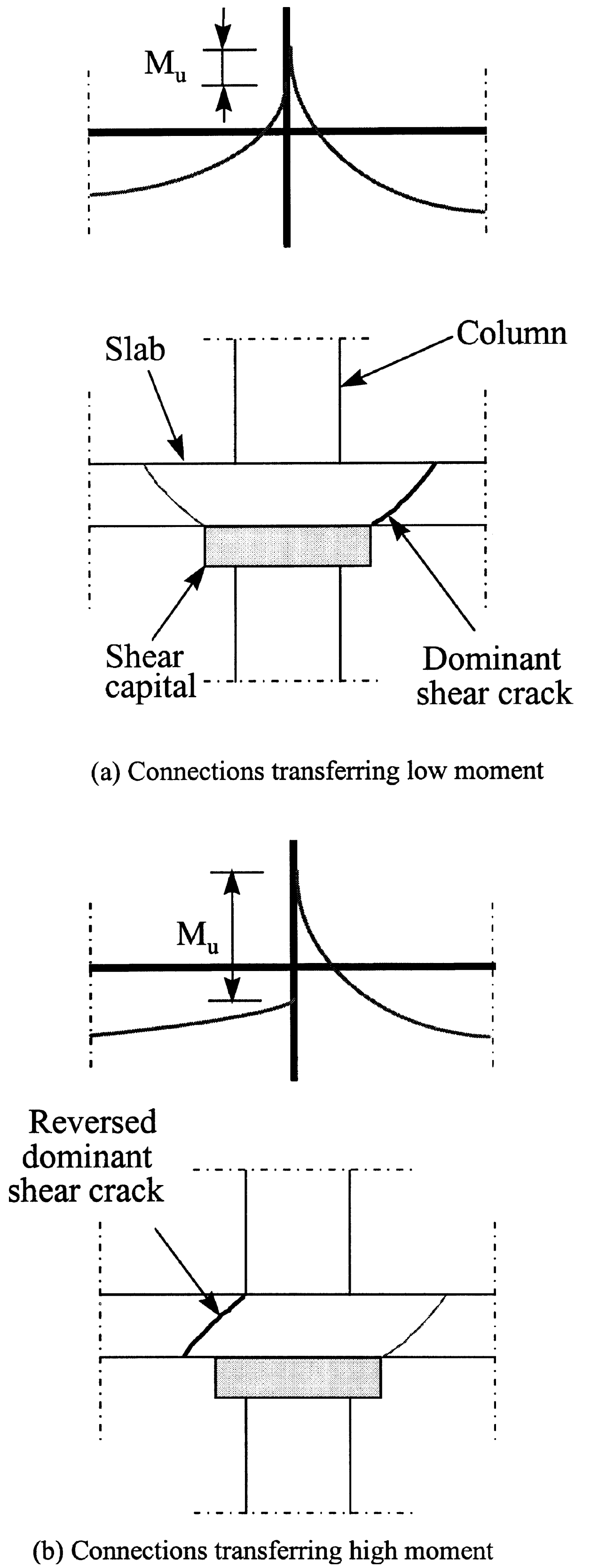


Fig. 6—Punching shear cracks in slab-column connections with shear capitals.

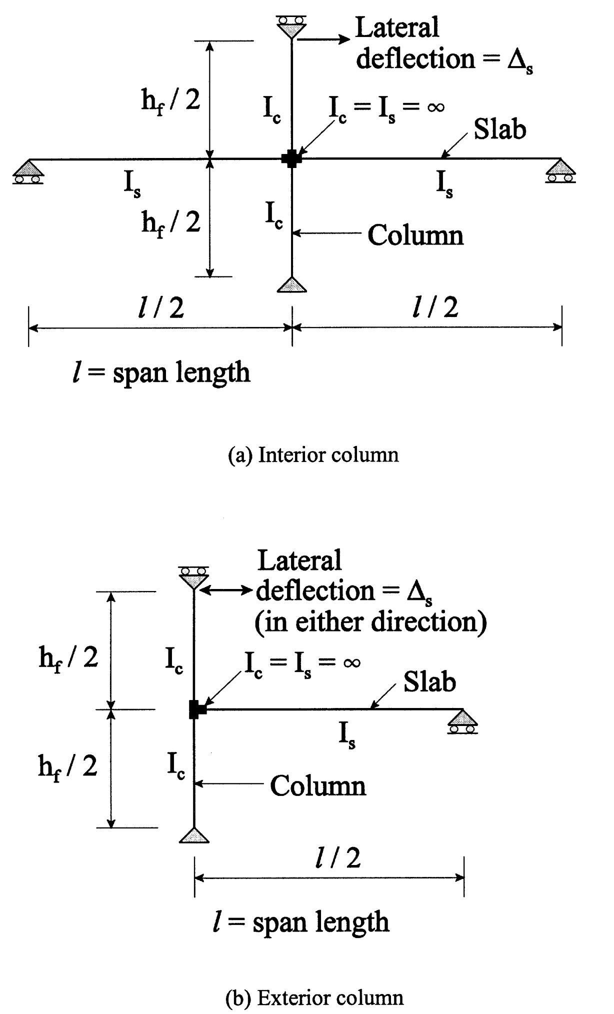


Fig. 7—Plane frame idealization of slab-column connections.

where R is a dimensionless coefficient specified by the UBC[6] representing the inherent overstrength and the global capacity of the primary system. The value of the factor R in Eq. (3) depends on the primary system employed to resist the lateral forces. The value of Δ_s, representing the elastic interstory drift can be estimated by any rational analysis; for example, it can be calculated by an elastic analysis of the primary system subjected to the lateral forces specified by the UBC.[6] The unbalanced moment due to the horizontal earthquake forces transferred between the column and the connected slab is equal to the sum of the two moments at the two column ends meeting at the connection. To account for the effect of cracking, Vanderbilt and Corely[21] recommend considering the moment of inertia of the slab as equal to 1/3 the value of the non-cracked slab, to obtain a conservative estimate of the interstory drift. To avoid underestimating the unbalanced moments transferred between the slab and the columns, it is recommended here that the unbalanced moments be determined by an elastic analysis as described previously, but with the moment of inertia of the slab equal to one half the value of the noncracked slab. The unbalanced moment caused by the vertical forces that can exist during earthquake should be added to the values obtained by the analysis discussed in this section; each should be multiplied by the appropriate load factor (according to ACI 318-99). The value of M_u to be used in punching shear design is the smaller of the upper limit given in the following section and the total factored unbalanced moment as determined previously.

UPPER LIMIT TO UNBALANCED MOMENT

The unbalanced moment M_u, determined by the provisions given as follows, is considered to be transferred between slabs and columns at the centroid of the punching shear-critical section at $d/2$ from column face (Fig. 2). Based on finite element analyses[18] as well as experiments[17] on slab-column connections transferring shearing force combined with moment reversals, an upper limit can be set for the value of M_u that need not be exceeded in the design of shear reinforcement in the slab

$$M_u \le \frac{M_{pr}}{\alpha_m} \tag{4}$$

where M_{pr} is the sum of the probable flexural strengths of opposite critical section sides of width ($c_x + d$ or $c_y + d$) when the transferred moment is about the x or y axes, respectively; and where c_x and c_y are column dimensions in the x and y directions, respectively (Fig. 2). The probable flexural strength should be based on the probable yield strength of the flexural reinforcement (Chapter 21 of ACI 318-99). This is considered here to be equal to $1.25A_s f_y$, where A_s is area of the flexural reinforcement and f_y is the specified yield strength. A_s represents the cross-sectional areas of the bars normal to two opposite sides of the shear-critical section (Fig. 2); only the top bars on one side and the bottom bars on the opposite side should be considered. The right-hand side of Eq. (4) represents the magnitude of the unbalanced moment that will develop the probable yield strength of the flexural reinforcement. When this occurs without punching shear failure, the slab-column connection will undergo substantial drift without losing the ability to transfer gravity loads, thus avoiding collapse. The empirical coefficient α_m (Eq. (4)) is expressed as

$$\alpha_m = 0.85 - \gamma_v - \left(\frac{\beta_r}{20}\right) \quad \text{for interior connections} \tag{5}$$

$$\alpha_m = 0.55 - \gamma_v - \left(\frac{\beta_r}{40}\right) + 10\rho \quad \text{for exterior connections} \tag{6}$$

where γ_v is the fraction of moment transferred by vertical shear stresses in the slab. Equations for γ_v, depending on the shape of the critical section, are given in ACI 421.1R-99.[5] Equation (5) applies for interior connections transferring M_{ux} or M_{uy} (Fig. 2(a)), for edge connections transferring M_{ux} (with M_{uy} = 0; Fig. 2(b)), and for corner connections transferring M_{ux} (with M_{uy} = 0; Fig. 2(c)); while Eq. (6) applies for the remaining cases. In Eq. (5) and (6), β_r is equal to l_y/l_x or l_x/l_y when the transferred moment is about the x or y axes, respectively, where l_x and l_y are projections of the critical section at $d/2$ from column face on its principal axes x and y, respectively (Fig. 2). In Eq. (6), ρ is the ratio of tensile flexural reinforcing bars passing through the projection of the shear-critical section (Fig. 2) in the direction in which moments are transferred. For example, ρ is the ratio of the tensile flexural reinforcing bars passing through Side BC of the critical section for the edge slab-column connection shown in Fig. 2(b) (when M_{ux} = 0). For the corner slab-column connection shown in Fig. 2(c) (with M_{ux} = 0), α_m is calculated by Eq. (6) with ρ given by

$$\rho = \frac{(A_{sx}\cos\theta + A_{sy}\sin\theta)}{l_y d} \tag{7}$$

where A_{sx} and A_{sy} are the cross-sectional areas of tensile reinforcing bars—within Sides BC and AB—running in the x and y directions, respectively; and θ is the angle between the principal and nonprincipal axes (refer to Fig. 2(c)); and l_y is the projection of AB and BC on the principal y axis of the shear-critical section.

When Eq. (4) is used to determine the upper limit for M_{uy} for an interior connection (with M_{ux} = 0; Fig. 2(a)), M_{pr} is the sum of the absolute values of the positive and negative flexural resistance of the two opposite Sections AD and BC, respectively (Fig. 2(a)). For an edge column connection (with M_{ux} = 0; Fig. 2(b)), M_{pr} should be calculated for the negative and positive flexural resistance of the critical section side parallel to the free edge (Side BC in Fig. 2(b)); the connection should be designed to resist V_u combined with each of the two moments. For an edge slab-column connection transferring M_{ux} (with M_{uy} = 0, Fig. 2(b)), M_{pr} is the sum of the absolute values of the positive and negative flexural resistance of the two opposite Sections CD and AB, respectively.

For a corner slab-column connection, M_{pr} is calculated for the negative and positive flexural resistance of slab strips of widths equal to projections of the critical section on its principal axes. The connection should be designed for punching shear when M_{pr} is transferred about the principal axis y (Fig. 2(c)). The connection should also be checked when M_{pr} is transferred about the principal axis x (Fig. 2(c)).

In a slab-column connection (Fig. 2(a) or (b)) with transferring constant V_u combined with M_{uy} of increasing magnitude, yielding is reached in the flexural reinforcing bars passing through critical section Sides AD and BC in Fig. 2(a) or Side BC in Fig. 2(b). The sum of the absolute values of the positive moment strength of Side AD and negative moment strength of Side BC (Fig. 2(a)) represents the product $\alpha_m M_{uy}$.

Similarly, $\alpha_m M_{uy}$ represents the negative moment strength of Side BC in Fig. 2(b). The quantity $\alpha_m M_{uy}$ can be determined from the results of nonlinear finite element analyses. Results of such analyses, which have helped in developing the empirical Eq. (5) and (6), are presented in Table 2 and 3 for interior and edge slab-column connections, respectively. The slab is simply supported on four or three edges in case of interior or edge slab-column connections, respectively (Fig. 8(a) and 8(b)). The magnitude of the constant force V_u, applied through the axis of the column, is one of the variables in the tables. The bottom reinforcement ratio in all slabs is 0.65%; while the top reinforcement ratio ρ is variable. The yield strength of flexural reinforcing bars is 58 ksi (400 MPa), except as otherwise mentioned. The validity of the finite element software[22] and the finite element idealization have been verified by physical experiments.[23] The dimensions of slab sides in Fig. 8(a) and 8(b) are 74.8 x 74.8 in.^{2} and 74.8 x 53.1 in.^{2}, respectively; the slab thickness is 6 in.; and the column dimensions are 9.8 x 9.8 in.^{2}

Table 2—Nonlinear finite element results of fraction of unbalanced moment transferred by flexure in interior slab-column connections

1	2	3	4	5	6	7
Slab	l_x/l_y	ρ, %	V_u, kips	α_{mFE}	α_m	α_{mFE}/α_m
I1	1.00	0.61	0	0.40	0.40	1.00
I2	1.00	0.61	16.9	0.39	0.40	0.98
I3	1.00	0.61	33.7	0.37	0.40	0.93
I4	1.00	0.61	67.4	0.43	0.40	1.08
I5	1.00	1.22	0	0.39	0.40	0.98
I6	1.00	1.22	16.9	0.39	0.40	0.98
I7	1.00	1.22	33.7	0.37	0.40	0.93
I8	1.00	1.22	67.4	0.44	0.40	1.10
I9*	1.00	1.22	33.7	0.37	0.40	0.93
I10†	1.00	1.22	33.7	0.39	0.40	0.98
I11	1.00	2.44	0	0.45	0.40	1.13
I12	1.00	2.44	16.9	0.42	0.40	1.05
I13	1.00	2.44	33.7	0.44	0.40	1.10
I14	1.00	2.44	67.4	0.52	0.40	1.30
I15	1.00	2.44	101.2	0.57	0.40	1.43
I16‡	1.00	2.44	0	0.44	0.40	1.10
I17§	1.00	2.44	33.7	0.47	0.40	1.18
I18‖	1.00	2.44	33.7	0.44	0.40	1.10
I19‡	1.00	2.44	33.7	0.43	0.40	1.08
I20#	1.00	2.44	33.7	0.45	0.40	1.13
I21‡	1.00	2.44	101.2	0.55	0.40	1.38
I22	0.39	1.22	0	0.64	0.54	1.19
I23	0.39	1.22	22.5	0.64	0.54	1.19
I24	0.39	1.22	33.7	0.62	0.54	1.15
I25	0.39	1.22	45.0	0.60	0.54	1.11
I26	0.39	1.22	67.4	0.64	0.54	1.19
I27	0.66	1.22	0	0.53	0.47	1.15
I28	0.66	1.22	33.7	0.47	0.47	1.00
I29	0.66	1.22	67.4	0.55	0.47	1.17
I30	1.52	1.22	0	0.34	0.32	1.06
I31	1.52	1.22	33.7	0.30	0.32	0.94
I32	1.52	1.22	67.4	0.37	0.32	1.16
I33	2.57	2.44	0	0.19	0.20	0.95
I34	2.57	2.44	16.9	0.19	0.20	0.95
I35	2.57	2.44	33.7	0.20	0.20	1.00
Mean						1.09
Standard deviation						0.12
Coefficient of variation						0.11

*f_y = 300 MPa.
†f_y = 500 MPa.
‡Slab with stud shear reinforcement; stud diameter is 3/8 in. (12 studs in each peripheral line).
§Slab with stud shear reinforcement; stud diameter is 1/8 in. (12 studs in each peripheral line).
‖Slab with stud shear reinforcement; stud diameter is 1/4 in. (12 studs in each peripheral line).
#Slab with stud shear reinforcement; stud diameter is 1/2 in. (12 studs in each peripheral line).

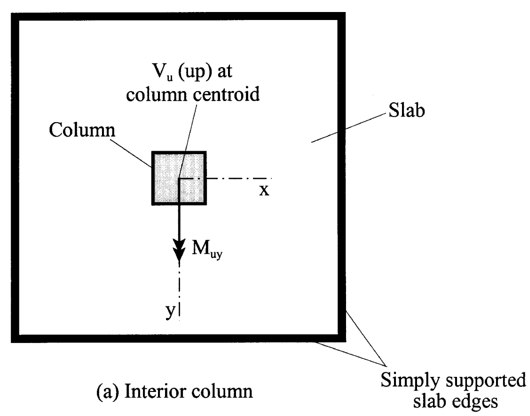
The ratios of $\alpha_{m\ finite\ element}$ to $\alpha_{m\ Eq.\ 5\ or\ 6}$ given in Column 7 of Table 2 and 3 have average values of 1.09 or 1.08, respectively, and coefficients of variation of 0.11 or 0.13, respectively. These results justify the proposed Eq. (4) through (6).

PUNCHING SHEAR STRENGTH DESIGN

Punching shear strength design should be according to the provisions of ACI 421.1R-99.[5] The factored shear stress at the critical section v_u is given by

$$v_u = \frac{V_u}{b_o d} + \frac{\gamma_{vx} M_{ux}}{I_x} y + \frac{\gamma_{vy} M_{vy}}{I_y} x \qquad (8)$$

where V_u and M_u are the ultimate values of shearing force and moment transferred between column and slab; b_o is length of perimeter of shear-critical section; subscripts x and y refer to centroidal principal axes of the critical section considered; (x,y) are coordinates of the point at which v_u is maximum; γ_{vx} and γ_{vy} are fractions of moments M_{ux} and M_{uy}, respectively, transferred by shear stresses; and I_x and I_y are



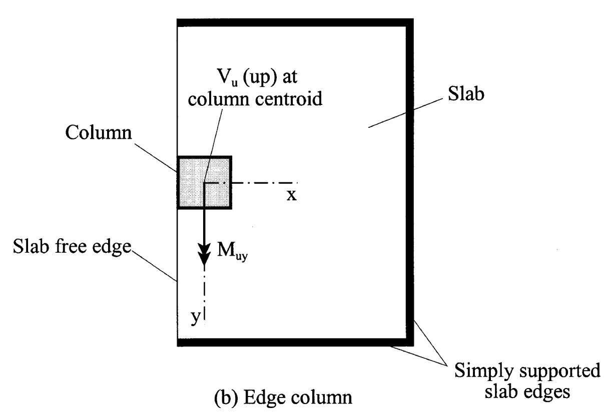


Fig. 8—Plan views of slab-column connections analyzed by finite element method.

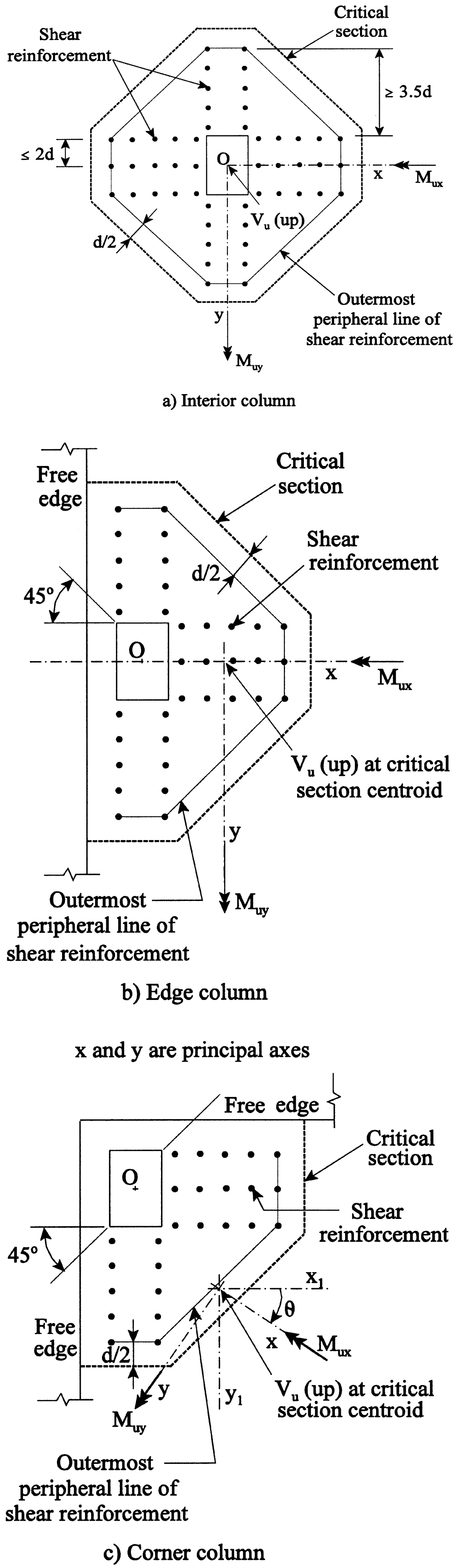


Fig. 9—Critical sections for punching shear outside shear-reinforced zone.

second moments of area of the critical section about the principal axes x and y, respectively. Shear failure is assumed to occur once the maximum shear stress v_u, calculated by Eq. (8), reaches $\phi\, v_n$ where v_n is the nominal shear stress.

The ACI 318 Code uses a parameter J in lieu of I in Eq. (8) and gives an equation defining J for the critical section in the shape of sides of a closed rectangle (refer to Fig. 2(a)). The value of J is slightly larger than I (by less than 3%). Because of lack of definition for J for other shapes of critical section and the small difference between I and J, the former is used in Eq. (8). With the critical section perimeter composed of straight segments, I_x is the sum of the quantity $[(dl/3)\,(y_1^2 + y_1 y_2 + y_2^2)]$ for each segment; where l is the length of the segment and $\{y_1, y_2\}$ are the coordinates of its ends. I_y can be calculated in a similar way.

According to ACI 421.1R-99,[5] when stud shear reinforcement is used, the nominal shear stress, v_n is given by (using inch and pound units)

$$v_n = v_c + v_s \le 8\sqrt{f_c'} \tag{9}$$

where v_c and v_s are the nominal shear stresses provided by concrete and shear reinforcement, respectively. The shear stress v_s is given by Eq. (2), while the nominal shear stress v_c is given by

$$v_c = 1.5\sqrt{f_c'} \tag{10}$$

The shear reinforcement should be extended away from the column so that the factored shear stress calculated by Eq. (8) at a critical section at $d/2$ outside the outermost peripheral line of studs (Fig. 9) does not exceed the nominal shear stress given by

$$v_n = 2.0\sqrt{f_c'} \tag{11}$$

For an earthquake-resistant slab-column connection, the distance between the column face and the outermost peripheral line of shear reinforcement should not be less than $3.5d$, as mentioned previously. It should be verified that the minimum amount of shear reinforcement is provided in Eq. (2). One of the purposes of the minimum amount of shear reinforcement recommended by Eq. (2) is to ensure that the slab-column connections can support factored gravity loads after the occurrence of inelastic deformations due to cyclic unbalanced moment transfer in an earthquake. Experiments have shown that this can be achieved by the provision of shear reinforcement.[18]

It is recommended here that the upper limit of $8\sqrt{f_c'}$ on v_n (Eq. (9)) should be waived when design value of the unbalanced moment M_u is determined by Eq. (4) through (7). This is because the maximum shear stress in this case (v_u) is caused mainly by M_u rather than by V_u. The absolute value of v_u due to M_u varies between zero and a maximum value; the maximum value is reached only at one point or one side of the critical section.

SUMMARY OF DESIGN STEPS

It is assumed here that the interstory drift ratio DR_u, including inelastic deformations, has been controlled by provision of the primary structural system. The steps explained previously for punching shear design of the slab-column connections are illustrated by the flow chart shown in Fig. 10. In the penultimate step in this figure, M_u should be calculated by taking into account the upper limit given by Eq. (4) through (6). The equations given by ACI 421.1R-99[5] should be applied to calculate the maximum shear stress and to design the SSR when required.

DESIGN EXAMPLE

The design of earthquake-resistant slab-column connections for punching shear is illustrated by the following numerical example of an interior column connection with a reinforced concrete slab (refer to Fig. 2(a)):

1. Assume a flat slab structure, with span lengths $l_1 = l_2 =$ 20 ft (where l_1 and l_2 are the span lengths in two orthogonal directions), floor height h_f = 12 ft, column size $c_x \times c_y$ = 16 x 16 in., slab thickness h = 8 in., I_s = 5120 in.^{4}, and I_c = 1980 in.^{4} The value of I_c is obtained by the equivalent frame method,[4] assuming that the flexibility of the equivalent column equals the sum of the flexibility of the actual column and the torsional strip of the slab. The drift ratio DR_u, including inelastic deformations, is 2.0%;

2. Design the interior slab-column connection for V_u combined with the earthquake-factored moment M_{uy}. Other data are: V_u = 80 kip; concrete cover = 0.75 in.; f_c' = 4000 psi; E_c = 3600 ksi; f_y = 60 ksi; f_{yv} = 50 ksi; ratios of top and bottom slab flexural reinforcements in the column vicinity are 0.9 and 0.5%, respectively; the nominal diameter of flexural reinforcement is 0.625 in.; and the diameter of the shear studs is 1/2 in. The effective slab depth d is 6.625 in. (that is, 8 in. – 0.75 in. – 0.625 in.);

3. Using b_o = 90.5 in. for the critical section shown in Fig. 2(a) and Eq. (1), $V_c = 4\ (90.5 \times 6.625)\sqrt{4000}$ = 151.7 kip, and $V_u/\phi\ V_c$ = 80 / (0.85 × 151.7) = 0.62. Because DR_u > 1.5% and $(V_u/\phi\ V_c)$ > 0.25, shear reinforcement is required;

4. The elastic interstory drift Δ_s is equal to $h_f\ (DR_u)$ divided by a factor, which, according to UBC 97, is 0.7R with R = 5.5 (assuming that the primary system consists of shear walls). This gives Δ_s = 0.02 (12 × 12) / (0.7 × 5.5) = 0.75 in., and the corresponding unbalanced moment determined from elastic analysis of the frame shown in Fig. 7(a) is M_{uy} = 3530 kip-in;

5. The probable flexural strengths provided by the top and bottom flexural reinforcements of critical section Sides BC and AD are 650 and 370 kip-in., respectively; thus, M_{pr} = 1020 kip-in. For this interior connection, γ_{vy} = 0.4; β_r = 1.0 and α_m = 0.4 (Eq. 5). It follows that the upper limit of M_{uy} (from Eq. (4)) is

$$M_{uy\ upper\ limit} = \frac{1020}{0.4} = 2550 \text{ kip-in.}$$

6. Because the upper limit is smaller than M_{uy} = 3530 kip-in., the slab-column connection should be designed to resist V_u = 80 kip and M_{uy} = 2550 kip-in. Properties of the critical section at d/2 from column face (Fig. 2(a)) are b_o = 90.5 in., I_y = 51,150 in.^{4}, and γ_{vy} = 0.40. The maximum shear stress at x = 11.31 in. (from Eq. (8)) is

$$v_u = \frac{80 \times 10}{90.5(6.625)} + \frac{0.40(2550 \times 10^3)(11.31)}{51{,}150} = 359 \text{ psi}$$

7. The nominal shear stress allowed without shear reinforcement is $\phi v_n = \phi(4\sqrt{f_c'}) = 0.85\ (4\sqrt{4000})$ = 215 psi, which is less than v_u. Thus, shear reinforcement is required;

8. The SSR shown in Fig. 11 is designed according to ACI 421.1R-99 recommendations.[5] For the chosen shear reinforcement, A_v = 2.36 in.^{2} (for 12 studs), s = 3.25 in., v_s = 401 psi (from Eq. (2)), v_c = 95 psi (from Eq. (10)), and v_n = 95 + 401 = 496 psi (which is greater than v_u / ϕ). Moreover, the value of v_s is greater than the minimum $3\sqrt{f_c'}$ (= 190 psi); and

9. Properties of the critical section at d/2 from the outermost peripheral line of studs (Fig. 11) are: b_o = 233 in., I_y = 991,630 in.^{4}, and γ_{vy} = 0.40. The maximum shear stress, which is at x = 37.31 in. and y = 9.37 in., calculates to be (using Eq. (8)):

$$V_u = \frac{80 \times 10^3}{233(6.625)} + \frac{0.40(2550 \times 10^3)(37.31)}{991{,}630} = 90 \text{ psi;}$$

Table 3—Nonlinear finite element results of fraction of unbalanced moment transferred by flexure in edge slab-column connections

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Slab	l_x/l_y	ρ, %	V_u, kips	α_{mFE}	α_m	α_{mFE}/α_m
E1	0.84	0.61	0	0.24	0.24	1.00
E2	0.84	0.61	16.9	0.22	0.24	0.92
E3	0.84	0.61	22.5	0.21	0.24	0.88
E4	0.84	0.61	34.1	0.25	0.24	1.04
E5	0.84	1.22	0	0.34	0.30	1.13
E6	0.84	1.22	16.9	0.29	0.30	0.97
E7	0.84	1.22	22.5	0.31	0.30	1.03
E8	0.84	1.22	28.1	0.32	0.30	1.07
E9	0.84	1.22	33.7	0.32	0.30	1.07
E10	0.84	1.22	38.1	0.37	0.30	1.23
E11	0.84	1.22	16.9	0.36	0.30	1.20
E12	0.30	1.22	16.9	0.70	0.49	1.43
E13	0.50	1.22	16.9	0.49	0.39	1.26
E14	1.28	1.22	16.9	0.25	0.23	1.09
E15	1.91	1.22	16.9	0.14	0.16	0.88
E16	2.33	1.22	16.9	0.13	0.12	1.08
E17	0.92	1.22	16.9	0.30	0.29	1.03
E18*	0.84	1.22	16.9	0.26	0.30	0.87
E19†	0.84	1.22	16.9	0.36	0.30	1.20
E20‡	0.84	1.22	46.1	0.38	0.30	1.27
E21	0.84	1.83	16.9	0.34	0.36	0.94
E22	0.84	2.44	0	0.51	0.43	1.19
E23	0.84	2.44	16.9	0.45	0.43	1.05
E24	0.84	2.44	22.5	0.45	0.43	1.05
E25	0.84	2.44	33.7	0.46	0.43	1.07
E26	0.84	2.44	45.0	0.48	0.43	1.12
E27	0.84	2.44	56.9	0.52	0.43	1.21
E28‡	0.84	2.44	0	0.40	0.43	0.93
E29‡	0.84	2.44	16.9	0.40	0.43	0.93
E30‡	0.84	2.44	33.7	0.43	0.43	1.00
E31‡	0.84	2.44	67.7	0.54	0.43	1.26
Mean						1.08
Standard deviation						0.14
Coefficient of variation						0.13

*f_y = 300 MPa.

†Slab with twice the number of reinforcing bars but with ρ = 1.22%.

‡Slab with stud shear reinforcement; stud diameter is 3/8 in. (7 studs in each peripheral line).

$$(v_u/\phi v_n) = 106 \text{ psi } < 2.0\sqrt{f_c'} = 126 \text{ psi}.$$

The distance between the column face and the outermost peripheral line of SSR = 25.375 in., which is greater than 3.5d (= 23.2 in.). Thus, the shear studs shown in Fig. 11 are adequate.

In this example, it is assumed that the primary system controlling the drift consists of shear walls, and the value of R = 5.5 specified by UBC 97 is employed in calculating Δ_s. When a smaller R-value is used for a different primary system, the shear reinforcement determined previously will not be changed. This is because the upper limit on M_u is used in the shear reinforcement design. In other words, the shear reinforcement shown in Fig. 11 represents an upper limit on the amount of shear reinforcement required for earthquake design.

CONCLUSIONS

Flat-plate structures should be provided with the primary structural system, such as shear walls, to limit the interstory drift ratio to the specified limits. Slab-column connections must be designed to undergo sidesway of the primary structural system. This paper presents a complete procedure for punching shear design of earthquake-resistant slab-column connections. The suggested design procedure is demonstrated by a design example of an interior slab-column connection. The following are the major conclusions:

To ensure ductility with connections that have no shear reinforcement for transferring shearing force V_u combined with unbalanced moment M_u, the value of V_u must be lower than a limit that depends upon the drift ratio. Furthermore, the maximum shear stress due to V_u combined with M_u must not exceed the limit given by ACI 318-99. If the two conditions are not satisfied, shear reinforcement is required.

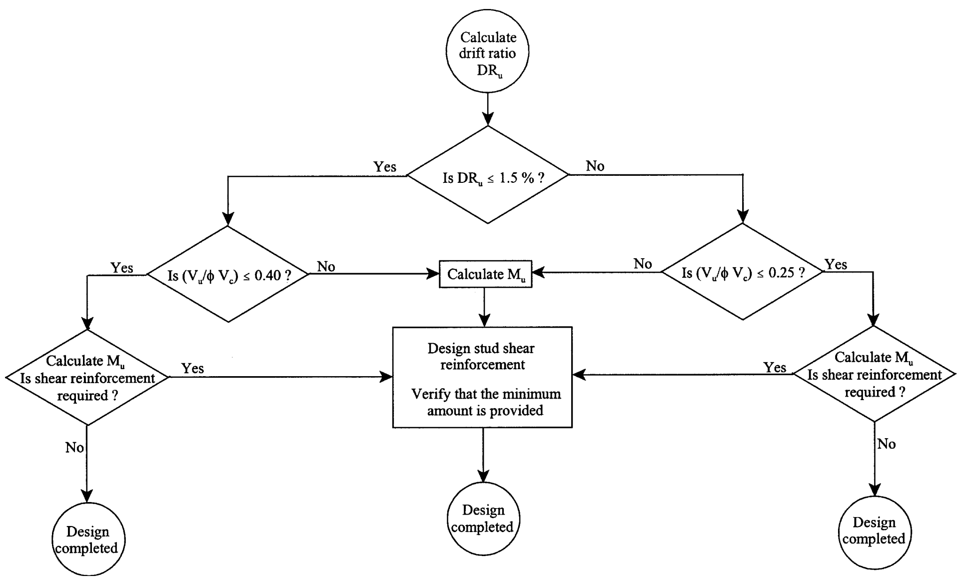


Fig. 10—Steps of punching shear design of earthquake-resistant slab-column connections.

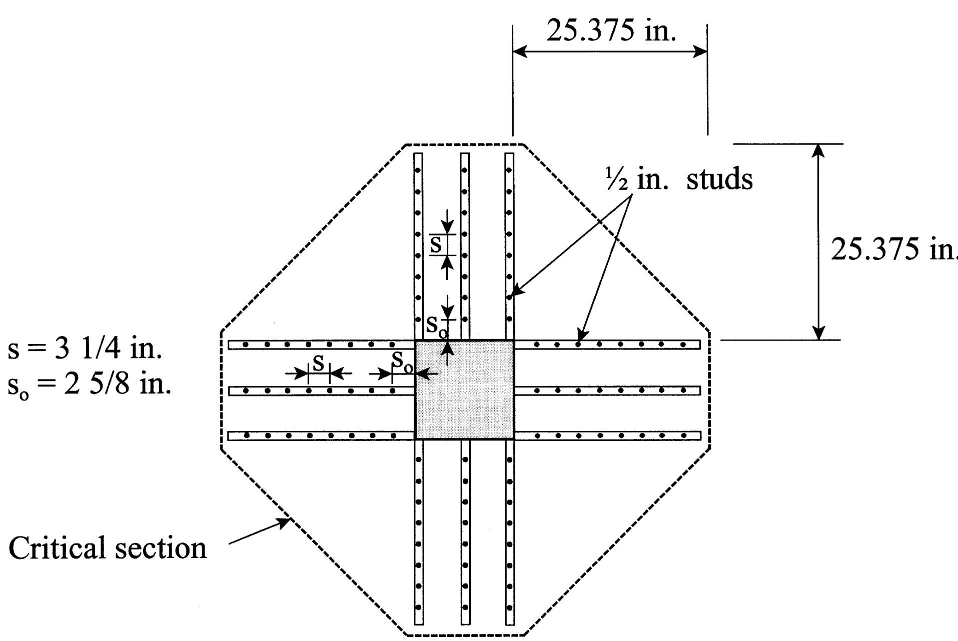


Fig. 11—Arrangement of stud shear reinforcement in design sample.

The value of M_u need not be greater than a specified limit, which is a function of the probable flexural strength of the slab. The design of shear reinforcement should be according to ACI Committee 421 report ACI 421.1R-99.[5] A minimum amount of stud shear reinforcement is suggested to ensure adequate ductility of earthquake-resistant slab-column connections.

Shear capitals should not be used as means of providing earthquake-resistant slab-column connections.

ACKNOWLEDGMENTS

This study has been supported financially by the Natural Sciences and Engineering Research Council of Canada, which is gratefully acknowledged.

NOTATIONS

A_s = area of flexural reinforcing bars
A_v = cross-sectional area of shear reinforcement on one peripheral line
b_o = length of perimeter of shear-critical section
c_x, c_y = column dimensions in the *x* and *y* directions, respectively
d = effective depth of slab
DR_u = ultimate lateral drift ratio of a slab-column connection
E_c = elastic modulus of concrete
f_c' = concrete uniaxial compressive strength
f_y, f_{yv} = specified yield strength of flexural and shear reinforcements, respectively
h = slab thickness
h_f = floor height
l = span length
l_x, l_y = projections of shear-critical section on its principal axes *x* and *y*, respectively
M = unbalanced moment transferred between slab and column
M_c = pure moment transfer capacity of a slab-column connection with no shear reinforcement
M_{pr} = probable flexural moment of resistance
M_u = factored unbalanced moment transferred between slab and column at shear-critical section centroid
s = spacing between peripheral lines of shear reinforcement
s_o = spacing between first peripheral line of shear reinforcement and column face
v_c = nominal shear stress provided by concrete in presence of shear reinforcement
v_n = nominal shear stress
v_s = nominal shear stress provided by shear reinforcement
v_u = maximum shear stress at critical section due to applied forces
V = shearing force transferred between column and slab
V_c = pure shear capacity of a slab-column connection with no shear reinforcement
V_u = applied shearing force at failure
α_m = factor used in calculation of upper limit for design moment of earthquake-resistant slab-column connections
α_s = factor that adjusts v_c for support type
β_c = ratio of long side to short side of concentrated load or reaction area
β_r = aspect ratio of the shear-critical section at $d/2$ from column face
γ_v = fraction of unbalanced moment transferred by vertical shear stresses at slab-column connections
Δ_S = interstory drift used in elastic frame analysis
ρ = slab flexural reinforcement ratio
ϕ = strength-reduction factor for shear according to ACI 318-99 Code (= 0.85)

CONVERSION FACTORS

1 in. = 25.4 mm
1 ft = 0.3048 m
1 kip = 4.448 kN
1 ft-kip = 1.356 kN-m
1 psi = 6.89×10^{-3} MPa;
$\sqrt{f_c'}$ in psi = $0.083\sqrt{f_c'}$ in MPa

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