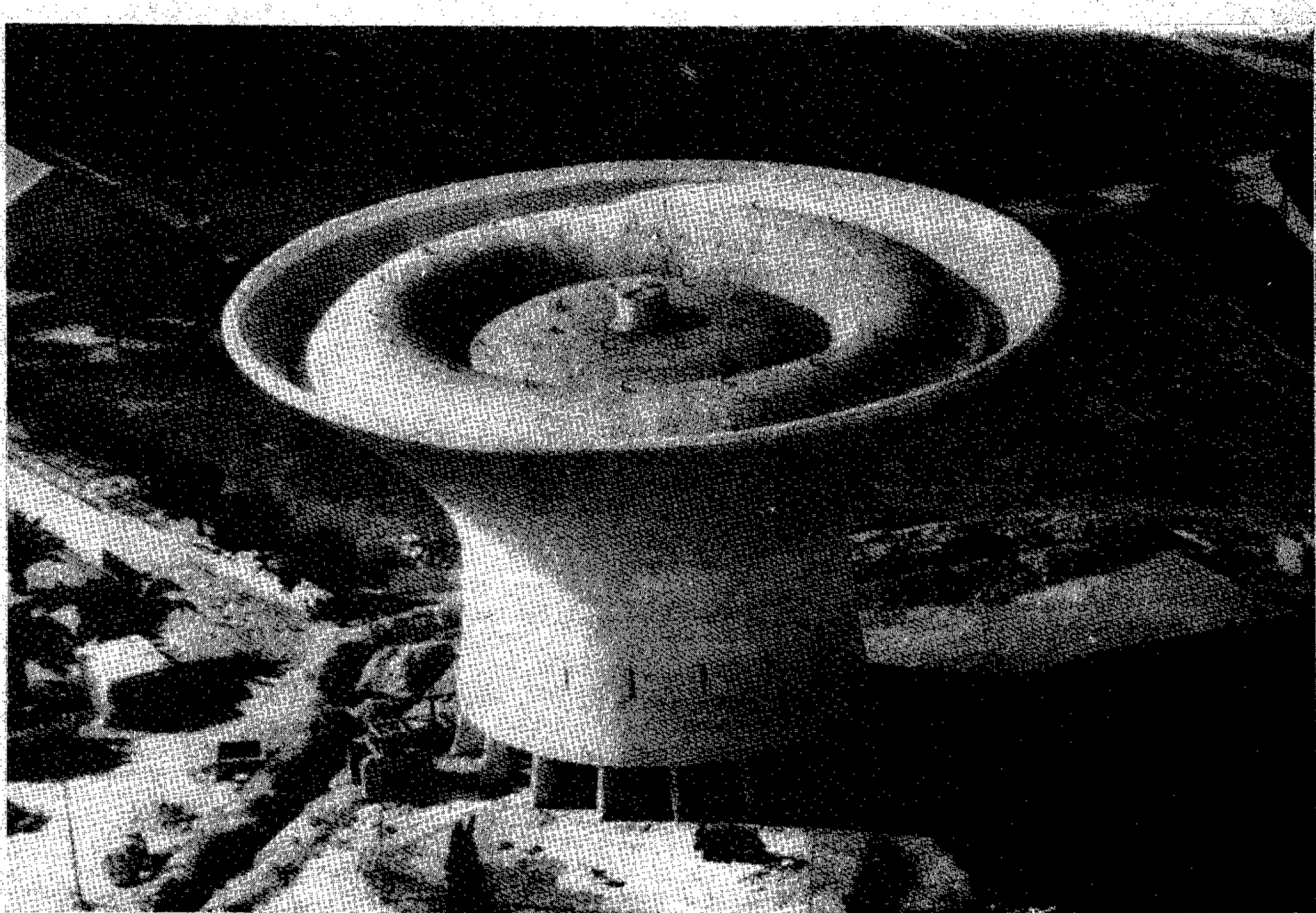
# bulletin

**of the international association for shell and spatial structures**

Prof. D. h-c. Eng. E. TORROJA, founder



# Contributions to the Bulletins of the IASS

The Bulletin of the IASS welcomes contributions pertaining to the design, analysis, construction and other aspects of the technology of all types of shells and spatial structures. Papers describing realizations of projects are particularly solicited. All material submitted for publication shall be evaluated as to editorial and technical content. The Editorial Committee reserves the right to accept or reject any manuscript.

Manuscripts shall be submitted in triplicate. The text shall be typed double spaced on one side of standard size sheets. Submit the originals and two copies of all figures, graphs and photographs. Originals of all graphs and figures shall be india ink drawings. Originals of photographs shall be glossy prints of a size suitable for direct reproduction.

Maximum length of manuscripts shall be the equivalent of 20 typewritten double-spaced pages. Brevity is most strenuously encouraged. The organization of all manuscripts shall be as follows.

**Title**
**Author** (name, academic degree, professional affiliation)
**Summary** (not more than 200 words)
**Notation** (where applicable)
**Introduction** (including scope of the paper, and statement of the problem)
**Text**
**Conclusions**
**Acknowledgements** (where appropriate)
**References**
**Appendices** (where applicable)

In addition, each manuscript shall be accompanied by a list of figures and tables. Except where mathematical derivations constitute the essence of the paper, they should be relegated to appendices, with only the final formulae or results presented in the body of the text. References shall be listed as per recognized international conventions. Authors are requested to cite specific page when referring to a book. A book shall be listed among the references only once. If it is cited a number of times in the text, cite as follows: (cf. Ref. N. p. nn).

Contributions must contain material previously not published, or not readily available to the members of the Association. Submittal of a manuscript shall be interpreted as constituting a grant of publication rights to the IASS. Authors of accepted papers will receive twenty-five reprints free of charge after the publication of the article in the bulletin.

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# bulletin

**of the International Association for Shell and Spatial Structures**

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Vol 31 (1990) n. 1-2
n. 102-103





**secretariat: Alfonso XII, 3 - 28014 Madrid (Spain)**
SODEGRAF ISSN: 0304-3622 Depósito legal: M. 1444-1960

The continuing development of design, analysis and construction techniques of spatial structures in general, and of shell structures in particular, has resulted in an increasing fund of information of practical interest to Architects, Engineers and Contractors. The **International Association for Shell and Spatial Structures**, founded by Eduardo Torroja in 1959, has as its goal the achievement of further progress through an interchange of ideas between all those interested in these structural systems. To this end, the IASS organizes symposia and congresses, sponsors the activities of working groups and the publication of their reports, and publishes this *Bulletin*.

All those interested in any aspect of design, analysis and construction of spatial or shell structures, as well as those interested in the research into their behavior, are welcome as members of the IASS.

For additional information about the Association and for membership application forms please write the Secretariat of the IASS, Alfonso XII, 3, 28014 Madrid (Spain).

## DISCUSSION OF PAPERS PUBLISHED IN THE BULLETIN OF THE IASS

A discussion of all technical papers published in the *Bulletin* of the IASS is encouraged. The deadline for submittal of the discussion is one year from the date of publication of the original paper.

Manuscripts of discussions should be submitted in triplicate. The length of a discussion should not exceed 6 typewritten double-spaced pages. References cited in the original paper should not be repeated, but should be cited by their original number. The rules of manuscript preparation which apply to the original submittal shall apply also to discussion manuscripts.

The text of all discussions submitted for publication in the *Bulletin* of the IASS will be reviewed by referees appointed by the Editorial Committee, which reserves the sole right to accept or reject any manuscript.

# Nonlinear material, geometric and time-dependent analysis of reinforced and prestressed concrete shells

ALEX C. SCORDELIS [1]

**Summary** Reinforced and prestressed concrete designs have been widely used for thin concrete shells in buildings, bridges, tanks, cooling towers, offshore oil platforms, nuclear containment vessels, and many other structures. During the past decade much research has been conducted in the development of nonlinear analytical models, methods of analysis and computer programs to trace the structural response of these structures under increasing loads through their elastic, cracking, inelastic and ultimate ranges.
In this paper, first, a brief historical review of this research in many countries is presented. This is followed by a more detailed description of a continuing research program, which has been conducted at the University of California at Berkeley since 1973 on the nonlinear analysis of thin concrete shells. Numerial results obtained with some recently developed computer programs are presented, which indicate that in some cases an increase and in other cases a large reduction in the calculated ultimate load occurs as each of the nonlinear factors is included in the computer analyses. Finally, recommendations for future research, development and applications in this field of study are presented.

## 1. INTRODUCTION

Reinforced and prestressed concrete designs have been widely used for thin concrete shells utilized in buildings, bridges, tanks, cooling towers, offshore oil platforms, nuclear containment vessels, and many other structures. The design of these structures must satisfy the requirements of safety and serviceability. While this can be accomplished in most cases by following approximate or empirical procedures prescribed in codes or recommended practices, it is desirable to have refined analytical models and methods available which can trace the structural response of these structures throughout their construction and service load history and under increasing loads through their elastic, cracking, inelastic, and ultimate ranges. Such refined analytical methods, after having been verified by selected experimental results, may be used to study the effects of important parameters in a systematic way to provide a firmer basis for the codes and specifications on which usual designs are based, or they may be used directly in the analysis and design of unusual and complex structures.

During the past decade much research has been conducted in the development of nonlinear methods of analysis and computer programs to achieve the above goal. In this paper, first a brief review of this research in many countries is presented. This is followed by a more detailed description of a continuing research program which has been conducted at the University of California at Berkeley since 1973 on analytical models, computer programs, and efficient numerical procedures for the material, geometric, and time-dependent nonlinear analysis of reinforced and prestressed concrete thin shells. In the nonlinear analysis of all of these thin concrete shell structural systems, a unified finite element tangent stiffness formulation, coupled with a time-step integration scheme, is used to trace their quasistatic response up to ultimate failure.

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In this paper the methods of analysis and computer programs, which are based upon a composite layered finite element displacement model, are briefly described. Nonlinear material stress-strain relationships are used for the concrete, reinforcing steel, and prestressing steel throughout the nonlinear finite element analysis to determine the displacements, crack patterns, and strains and stresses in the concrete and reinforcing and prestressing steel at any time up to failure, under any load or environmental history. Time-dependent effects due to load history; temperature history; creep, shrinkage, and aging of the concrete; and prestress losses due to these effects, as well as others such as anchor slip, friction, and relaxation, are all included in the analysis.

Numerical results for thin concrete shells, obtained with recently developed and available computer programs, are presented which indicate that in some cases an increase and in other cases a large reduction in the calculated ultimate load occurs as each of the nonlinear factors is included in the computer analysis.

Finally, recommendations for future research, development, and applications in this field of study are presented.

## 2. BRIEF HISTORICAL REVIEW

With the advent of computers and the finite element method (FEM) in the 1950's and 1960's many linear analysis programs were developed for thin shells in which they were assumed to be made of a linear, homogeneous and uncracked material. The first application of the FEM to cracked reinforced concrete beams was published in 1967 by Ngo and Scordelis (1). Subsequent applications to the nonlinear analysis of all types of reinforced concrete structures were made by researchers from various countries and their colleagues and/or students.

Major contributors to the development of computer programs for the nonlinear analysis of reinforced and prestressed concrete shells in the 1970's and 1980's have included Argyris (2, 3), Bergan (4, 5), Gupta (6, 7), Hinton (8), Mang (9, 10, 11), Melhorn (12, 13), Owen (14), Ramm (15, 16), Rashid (17, 18), Schnobrich (19, 20, 21, 22, 23), Scordelis (24, 25, 26, 27, 28), as weell as other researches. Only a few of the many publications for which the above have been authors or co-authors are given in the reference list of this paper as examples of their work.

Their research has generally dealt with one or more of the following topics: (1) material, geometric and time dependent nonlinearities; (2) element selection for the shell proper and edge beams or stifferners; (3) analytical modeling, constitutive relationships and failure theories, either independently or as part of a composite material, for the concrete, reinforcing steel and prestressing steel in the elements; (4) concrete cracking criteria and models; (5) tension stiffening; (6) creep, shrinkage, and temperature effects; (7) prestress losses during the post-tensioning operations due to friction and anchor slip and after transfer of prestress due to the creep and shrinkage of concrete, the relaxation of the prestressing steel and the effects of load and temperature history; and (8) numerical solution strategies for the nonlinear analyses in the computer. While much progress has been made on these topics, general agreement on some of the topics is still lacking and thus more research needs to be done.

Applications of the above research and computer programs have generally been for special problems and studies in connection with long span roofs, nuclear containment vessels, offshore oil platforms, HP cooling towers, and liquid nitrogen gas tanks. The computer programs have not been widely used, primarily because of their complexity, need for verification, and general lack of widespread availability.

## 3. BERKELEY RESEARCH PROGRAM

### 3.1. Brief Review

About 16 years ago (1973), research on the nonlinear analysis of reinforced concrete shell structures was begun at Berkeley, recognizing that the demand for longer spans, thinner shells and more complex structures, made it desirable to have refined analytical models and computer programs, which could trace the complete structural response of these structures throughout their service load history and under increasing loads up to ultimate failure.

The capabilities desired were the nonlinear material, geometric and time dependent analysis of reinforced concrete shells with edge beams through their elastic, cracking, inelastic and ultimate ranges. The time dependent effects of load and temperature history and creep and shrinkage of the concrete should be taken into account. Well documented computer programs were to be written and made available to other interested researchers for comparisons and to structural engineers for applications to complex problems encountered in actual structural designs.

Initially in 1973, a computer program (NARCS) by Lin (29) which incorporated only the nonlinear material properties of the concrete and reinforcing steel was developed, and then successively at about three year intervals time-dependent effects in (NOTACS) by Kabir (30), then edge beams and then nonlinear geometry in (NOPARC) by Van Greunen and in (NASHL) by Chan (32) were added. The present computer program for reinforced concrete shells with edge beams (NASHL) incorporates all of these effects and gives as final output for any load and time up to failure: (1) joint displacement; (2) internal strains and stresses in the concrete and reinforcing steel; and (3) crack patterns.

In 1988 as part of a joint U.S.-Spain research project between the University of California at Berkeley and the University of Barcelona, Roca (33) of Spain, under the supervision of Mari, added to (NASHL) the capability of including curved threee dimensional prestressing tendons in the curved shell surface and in the edge beams. This latest computer program has been designated (NASHL1).

A detailed description of the theoretical development, analytical models and elements, material properties, solution tecniques, and input-output for the computer programs (NASHL) and (NASHL1) can be found in Refs. (32) and (33). A brief description of some of these features is given below. Anyone interested in obtaining copies of any of the research reports (29, 30, 31, 32, 33) referred to above or the computer programs themselves should write to the author for further information.

### 3.2. Analytical Model

In the computer program, the analytical model consists of a series of joints interconnected by shell and beam elements (Fig. 1). The shell element is a nine

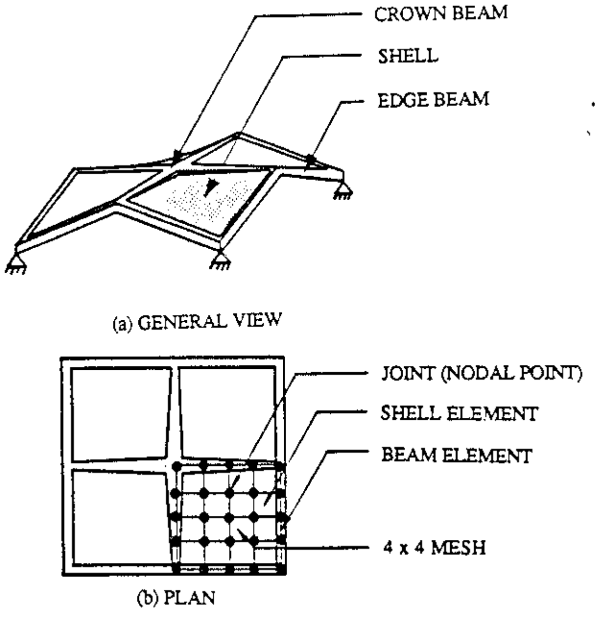


**Fig. 1. Analytical Model for NASHL Computer Program.**

node, two dimensional curved isoparametric element with 5 DOF at each node (Fig. 2). The element thickness is divided into concrete and reinforcing steel layers and each layer is assumed to be under a two dimensional stress state. The steel reinforcement can be placed in any layer and in several directions if desired. Cracking and nonlinear material response is traced layer by layer under increasing load and the time dependent effects of creep and shirnkage. An

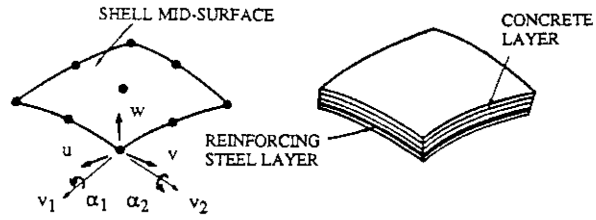


**Fig. 2. Layered Isoparametric RC Shell Element.**

updated Lagrangian formulation is used to take the effects of changing structural geometry into account.

By adding two straight one dimensional beam elements to the side of a curved shell element, thin shells with edge beams can be analyzed (Fig. 3). Each beam element with 12 DOF is prismatic (Fig. 4); but has an arbitrary cross-section made up of discrete numbers of concrete and reinforcing steel filaments for which the uniaxial strains and stresses are monitored.

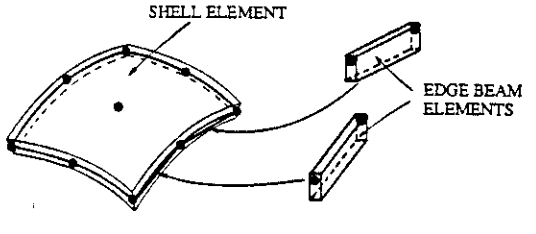


**Fig. 3. Addition of Two Beam Elements to Curved Shell Element.**

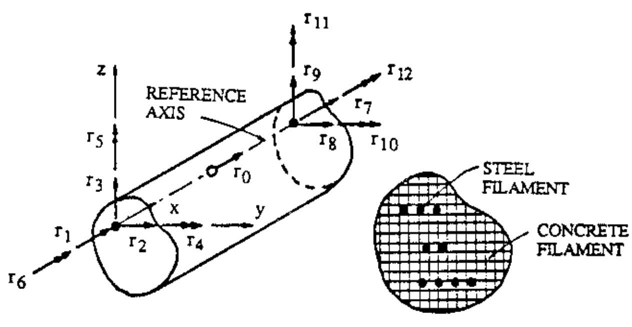


**Fig. 4. Filament RC Beam Element.**

In order to simulate elastic supports, obtain support reactions, or enforce irregular boundary conditions, linear elastic spring elements can be placed in any specified direction at discrete nodal points.

### 3.3. Prestressing Tendon Analytical Models

Shell prestressing tendons, added by Roca(33) in NASHL1, are individually as arbitrary spatial curves contained in the shell thickness. The geometric treatment of tendons is based upon a method first introduced by Hofstetter(11) where analytical parametric expressions are used for the definition of the mid surface of the shell as well as the tendon curves; then, the path of the tendons through the shell element mesh as well as the limiting points of each tendon segment included in a shell element are automatically determined.

Once generated, the tendon segments, together with the shell elements, are interpolated by means of shape functions to allow an automatic calculation of their geometric properties as well as further updating of the whole geometry in the nonlinear geometric analysis.

Distributed tangential and normal interactive forces between concrete and prestressing tendons are calculated taking into account frictional and anchor slip forces, and then, together with the end tendon segment forces, they are converted to consistent nodal loads according to the general procedure used for external loads. This method insures that each set of forces is self-balanced for each individual shell element.

Bonded or unbonded post-tensioned as well as pretensioned tendons may be considered.

For bonded tendons, the axial strain, at any stage of the analysis, is calculated from the strain field of the shell element, by a tensor transformation. For an unbonded tendon, the axial strain is calculated from the total increment of elongation caused in it. Taking into account the effect of friction, a distribution of strain throughout the length of the tendon can be obtained which when integrated equals the total elongation.

Each edge beam prestressing tendon is divided into a number of segments, each of which is straight, spans a single beam element, and is assumed to have a constant force. Using vector algebra to define the tendon geometry-nodal forces, strains and stresses can then be monitored throughout the analysis.

### 3.4. Concrete Material Properties

For the concrete layers of the shell elements, a biaxial orthotropic material law based upon nonlinear elasticity, together with a failure envelope, are used to represent the nonlinear behavior of the concrete using equivalent uniaxial strains. A typical equivalent uniaxial stress-strain curve, which is dependent on the existing biaxial stress ratio, consists in compression of a parabolic curve up to the point of maximum stress ($\sigma_{jc}$, $\varepsilon_{jc}$), followed by a linear unloading curve to the point of final concrete crushing ($\sigma_{cu}$, $\varepsilon_{cu}$), where $\sigma_{cu} = 0.2\ \sigma_{ic}$ and $\varepsilon_{cu} = 4\ \varepsilon_{ic}$. In tension, smeared cracking over each shell element layer at a limiting tensile stress ($\sigma_{it}$ and two directional cracks orthogonal to each other are assumed. Once cracking occurs, in one direction, the concrete stress normal to the crack drops to zero and the concrete behaves as a uniaxial material in the orthogonal direction until both directions crack. At this point, the concrete can only carry shear stress by shear interlock between cracks. This is done by modifying the shear stiffness by a shear retention factor $\beta$, where $0 \leq \beta \leq 1$.

It should be noted that the present version of NASHL uses a fixed rather than a rotating crack model. Recent research by Schnobrich (23) and others have indicated that the rotating crack model may be more realistic. Future work in (NASHL) will incorporate the rotating crack model as an option.

### 3.5. Reinforcing Steel Properties

A bilinear uniaxial stress-strain relationship is assumed for the steel. Steel yields with a reduced modulus $E_{sh}$ after reaching its yield strength $f_y$ and any subsequent unloading is assumed to occur with the initial modulus $E_s$. The steel fails at the limiting strain of $\varepsilon_{su}$.

Once concrete cracks, the stress normal to the crack is zero and the crack remains fixed in direction; however, the presence of the steel reinforcement will permit the concrete to carry some tension between cracks. This additional tension is conveniently lumped at the steel level and appears as an added tension stiffening to the steel. This procedure is used in NASHL because it takes into account the effects of the steel location in the thickness direction and the crack direction relative to the reinforcing direction.

### 3.6. Prestressing Steel Properties

Besides a large difference in the magnitude of the ultimate tensile strength, the stress-strain curve for the prestressing steel differs from that of the reinforcing steel in that it has no definite yield plateau. Thus, a multilinear stress-strain curve is adopted for the prestressing steel. In addition, the usual empirical formulas are used for stress relaxation and friction properties of the prestressing steel.

### 3.7. Torque-Twist Relationship for Beam Elements

A trilinear model is used for the effective torque-twist response for the reinforced concrete beam elements.

The curve is completely defined by three points: the torsional moment $T_{cr}$ at cracking and the corresponding twist $\alpha_{cr}$, the torsional moment $T_{yp}$ at full yielding of the longitudinal steel reinforcement and the corresponding twist $\alpha_{yp}$, and the ultimate twist at failure $\alpha_u$. These points can be found from available experimental test data. In the present NASHL program it is assumed that longitudinal bending and torsional effects in the beam elements are completely uncoupled. Improvements in this model to incorporate coupling are needed and are presently being studied by various researchers.

### 3.8. Creep and Shrinkage of the Concrete

A numerical formultion, developed by Kabir 30) is used for creep of the concrete in which the principle of superposition is assumed to be valid for evaluating the creep strain $\varepsilon^c(t)$ at any time t as expressed by the following superposition integral:

$$\varepsilon^c(t) = \int_0^t C(\tau, t-\tau, T) \frac{\partial\sigma(\tau)}{\partial\tau} d\tau \qquad [1]$$

in which $C(\tau, t-\tau, T)$ is the specific creep function dependent on age of loading $\tau$ and temperature variation T, and $\alpha(\tau)$ is the stress applied at time $\tau$.

The total creep strain at any time t can be found as the sum of the independent creep strains produced by the stress changes at different ages with different durations of time up to t.

For the effect of temperature variations on creep, the concrete is assumed to obey the time shift principle.

Stress and temperature changes are assumed to occur only at distinct time steps $t_n$; $n=1, 2, ..., N$. And, for the calculation of the creep strain increment during a given time interval, the stress and the temperature are assumed to remain constant.

It has been found that certain forms of mathematical approximations for the specific creep functions, while representing experimental or empirical creep curves accurately, overcome the necessity of storing all the stress increments of previous time steps. Such a form is adopted as follows:

$$C(\tau, t-\tau, T) = \sum_{i=1}^{m} a_i(\tau) \left[1 - e^{-\lambda_i\phi(T)(t-T)}\right] \qquad [2]$$

in which m, $a_i(\tau)$, $\lambda_i, \phi(T)$ are determined by a least squares fit to experimental or empirical creep curves, such as those recommended by ACI or CEB-FIP.

An efficient numerical procedure for evaluating creep strains can be developed using the following definitions for incremental quantities of time steps, stresses, and creep strains:

$$\Delta t_n = t_n - t_{n-1} \qquad [3]$$

$$\Delta\sigma_n = \sigma_n - \sigma_{n-1} \equiv \sigma(t_n) - \sigma(t_{n-1}) \qquad [4]$$

$$\Delta\varepsilon^c_n = \varepsilon^c_n - \varepsilon^c_{n-1} \equiv \varepsilon^c(t_n) - \varepsilon^c(t_{n-1}) \qquad [5]$$

Combining Eq. (1) to (5), after some extensive algebraic manipulations, the recursive relations necessary for calculating the increment of creep strain $\Delta\varepsilon^c_n$ at a time step $t_n$ are as follows:

$$\Delta\varepsilon^c_n = \sum_{i=1}^{m} A_{i,n} \left[1 - e^{-\lambda_i\phi(T_{n-1})\Delta t_n}\right] \qquad [6]$$

$$A_{i,n} = A_{i,n-1}\left[e^{-\lambda_i\phi(T_{n-2})\Delta t_{n-1}}\right] + \Delta\sigma_{n-1}\, a_i(t_{n-1}) \qquad [7]$$

$$A_{i,2} = \Delta\sigma_1 a_i(t_1) \qquad [8]$$

A very important advantage of the above formulation is that the computation for each new creep strain increment requires only the stress history of the last time step and not the total stress history. The above procedure is applied in a uniaxial direction in the filaments of the beam elements and in the biaxial principal stress directions in the layers of the shell elements.

The shrinkage strains can be found form experimental data or empirical formulas such as those recommended by ACI or CEB-FIP. These shrinkage strains are non-stress originated and are assumed to be composed of axial components only and are uniform in all directions.

### 3.9. Nonlineary Geometry Effects

The important steps in handling nonlinear geometry are the inclusion of the strains due to rotations and the updating of the structural geometry.

The procedure used is based upon small strains and small incremental rigid body rotations. An updated Lagrangian formulation is used where all the stress and strain states are referred to the last known configuration. The assumption of small strains implies linear superposition of strain increments and the assumption of small rotations implies that rotations can be treated as vectors.

### 3.10. Methods of Analysis

The external loads R are assumed to be applied only at the joints (Fig. 1). The external joint load history, temperature history, short-time stress-strain curves, time-dependent material properties, and boundary conditions are given. The unknown joint displacements r and the internal strains $\varepsilon$ and stresses $\sigma$ in the concrete and the reinforcing steel, at any point in the structure are to be found for any instant of time. The resulting load-displacement (R vs. r) relationship will be nonlinear because of possible nonlinear material, geometric, or timedependent effects.

To incorporate time-dependent nonlinearities, the time domain is divided into a discrete number of intervals, and a step forward integration is performed in which increments of displacements and strains are successively added to the previous totals as the solution marches forward in the time domain.

At each step, a direct stiffness analysis based on a displacement formulation is performed in the space domain, in which the resulting equilibrium equations will be nonlinear to be valid for the current state of material properties and geometry.

To account for geometric nonlinearity, an updated Lagrangian formulation is used, in which the direction of the local coordinate system is continuously updated as the structure deforms. Internal forces and stiffnesses are calculated in the local coordinate system for each element and transformed to the fixed global coordinate system where the equilibrium equations for the entire structure are assembled by the direct stiffness method and solved. Thus, the continuously changing displacement transformation matrix for each element takes into account the effect of geometric nonlinearity along with the nonlinear form of the strain-displacement relationship.

The total (Eq. 9a) or tangential (Eq. 9b), equilibrium equations in matrix form are:

$$Kr = R \qquad (9a)$$
$$K_t dr = dR \qquad (9b)$$

in which the stiffness matrices K or $K_t$ are functions of displacement r and material properties. Eqs. (9a) or (9b) are solved in the computer using an incremental load method with iterations within each load increment.

In NASHL an option is provided to use either the tangent stiffness or a constant stiffness during each single or series of iterations. In addition, a controlled displacement procedure can be specified when structures showing strain softenings or snap through phenomena are to be analyzed.

The NASHL computer programs make it possible to analyze reinforced and prestressed concrete shells of arbitrary shape with edge beams. At each time step external gravity, surface, pressure, prestressing and joint load increments as well as displacement increments and temperature changes can be specified as input. Several types of analysis can be performed: (1) linear elastic analysis (2) nonlinear analysis with any of following —nonlinear geometry, nonlinear material, time dependent effects— as specified in the input.

## 4. NUMERICAL EXAMPLES

Numerical results from four examples obtained using NASHL are presented below to illustrate the applications and capabilities of the program. Example 1 is a reinforced concrete (RC) cylindrical shell with reinforced (RC) or prestressed (PC) edge beams.

Example 2 is a RC and PC cylindrical shell with edge beams and prestressing in both the shell and the edge beams. Example 3 is a free form RC shell with and without initial imperfections. Example 4 is an RC gabled HP shell with crown and edge beams.

### 4.1. Example 1 - RC Cylindrical Shell with RC or PC Edge Beams (32.33)

A series of eleven cylindrical shells, simply supported at the two ends and constructed of reinforced concrete, have been tested experimentally by Bouma et al (34). The shells were 1/8 scaled models of actual full scale structures, and they had identical cross sections and edge beams, but different span lengths and amounts of reinforcement. In this example, the shell designated A2 in the test series is chosen (Fig. 5). The loadings on this model were the shell load of 41 psf of surface and a load of 33.6 lb/ft on each edge beam, and an additional 25% of these loads to account for the live load. The total load on the model was therefore 3360 lb. In the test, this load was proportionally increased until failure occurred.

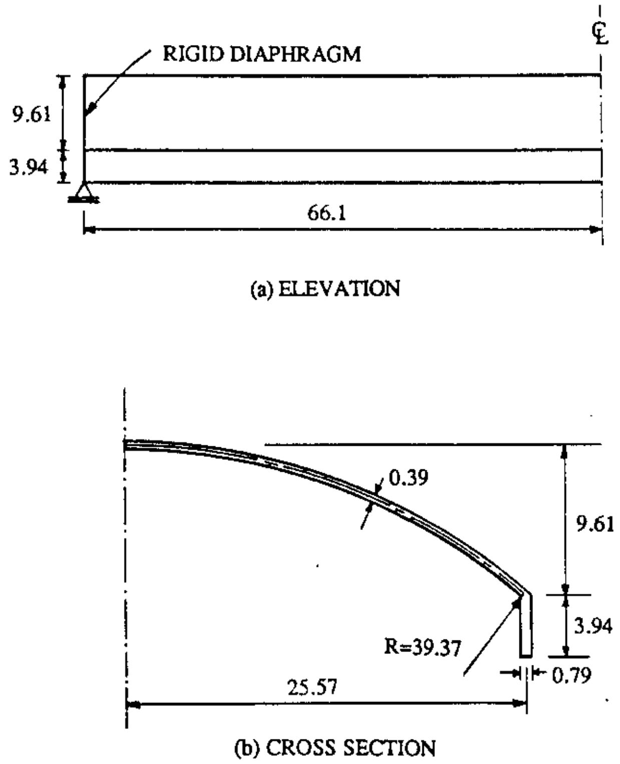


**Fig. 5. Ex. 1. Cylindrical Shell Geometry. (All dimensions in inches)**

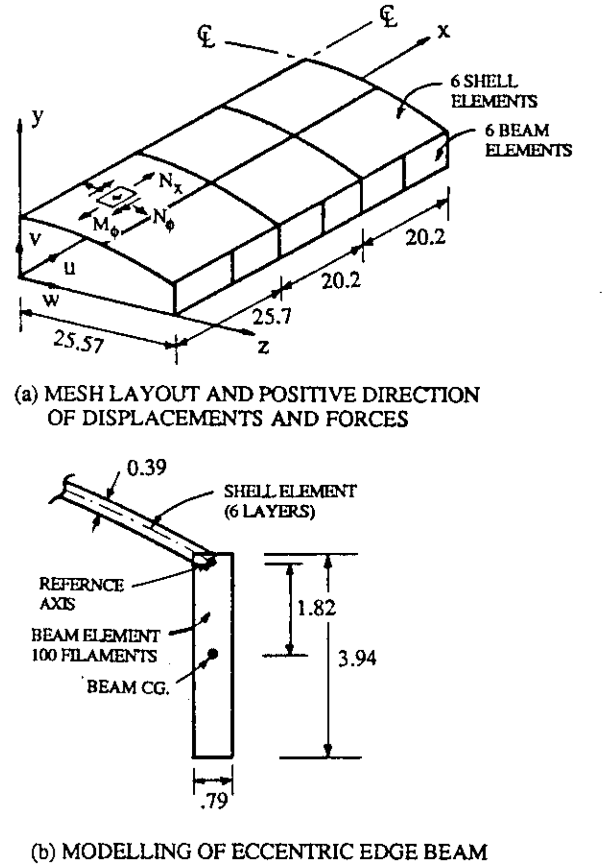


**Fig. 6. Ex. 1. Shell and Beam Mesh Layout. (All dimensions in inches)**

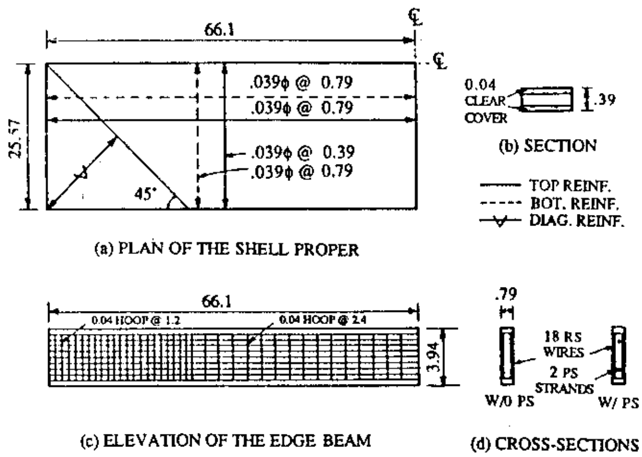


**Fig. 7. Ex. 1. Reinforcement for Shell and Edge Beams With and Without Edge Beam Prestressing. (All dimensions in inches)**

NASHL (32) was used to determine analytically the nonlinear response and ultimate load capacity of the shell. The geometry of the shell is shown in Fig. 5. Due to symmetry, onequarter of the shell is modelled by 6 shell elements and 6 beam elements (Fig. 6). The shell element is divided into 6 layers through the thickness while 10 layers are used for each of the two directions of the beam's cross section. The loading on the model is scaled proportionally to obtain a reference load of 1.0 psi on the shell's surface and 7.78 lb/in on each edge beam. The distributed load is replaced by the tributary nodal loads in the analysis.

The reinforcement details used in the analysis are shown in Fig. 7. The steel area in the edge beam is replaced by 10 equally spaced steel filaments. The material data for the concrete and steel were used in the analysis:

| *Concrete* | *Steel* |
|---|---|
| $E_o = 4.37 \times 10^3$ ksi | $E_s = 29.9 \times 10^3$ ksi |
| $f'_c = 4.12$ ksi | $E_{sh} = 0.0$ ksi |
| $f'_t = 0.711$ ksi | $f_y = 42.7$ ksi |
| $\nu = 0.3$ (assumed) | $\varepsilon_{su} = 0.1$ |
| $\varepsilon_c = 2\, f'_c/E_o$ | |

The resulting load-displacement response for the shell, obtained using NASHL is presented in Fig. 9 together with test results of Bouma et al. The agreement between the analytical and test results is satisfactory for the case where nonlinear geometry is included. For the analysis including nonlinear material only, an ultimate load of 8.4 k is obtained. This is close to the

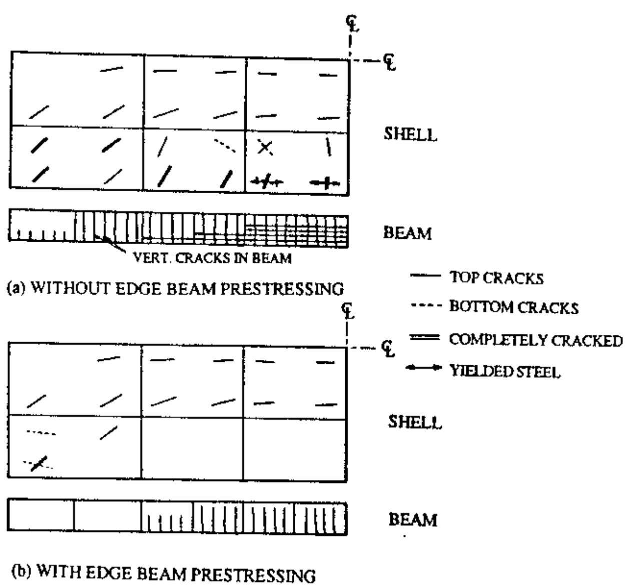


**Fig. 8. Ex. 1. Crack Patterns for Shell and Beams With and Without Edge Beam Prestressing.**

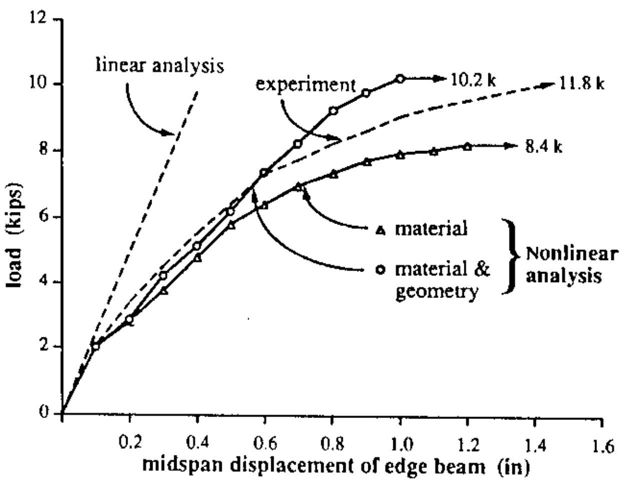


**Fig. 9. Ex. 1. Loads vs. Displacement Curves for RC Edge Beams.**

value obtained from beam theory. However, when nonlinear geometry is included in the analysis, an ultimate load of 10.2 k is obtained. This represents a 21% increase in strength over the case where nonlinear geometry is not considered, but it is still lower than the ultimate test load reported of 11.8 k.

Investigating the mode of failure for the case where nonlinear geometry is included indicates that the shell sustained initial yielding of the longitudinal steel causing larger displacements and more cracking and then finally failed in diagonal tension near the supporting points. The crack patterns are shown in Fig. 8a.

A statical check has also been carried out at mid-span, for the analysis where nonlinear geometry is included, at a load of 9.7 k just before the shell fails. The external midspan moment due to this load is 80.6 inch-kips. An integration of the internal stresses at midspan gives a tensile force of 7.86 kip and a compression force of 7.97 kips. Taking into account the deformed geometry, the intrenal moment at midspan can be found by integrating longitudinal stresses times level arm to be 80.4 inch-kips. Thus the statical check on the analytical results is excellent.

More recently the same shell was analyzed using NASHL1 (33), with and without prestressing in the edge beams, which consisted of two straight bonded tendons in each beam as shown in Fig. 7. The prestressing steel had $E_p = 29.0 \times 10^3$ ksi and $f_{py} = 247$ ksi. A comparison of the two analyses is shown in Fig. 10. It can be observed that the main effects of adding prestressing have been to significantly increase the range of linear behavior and to increase the ultimate load capacity to 12.0 k. A comparison of the analytical crack patterns for the two shells at the same load level (Fig. 8) indicates the addition of prestressing into the edge beam has also significantly reduced the cracking.

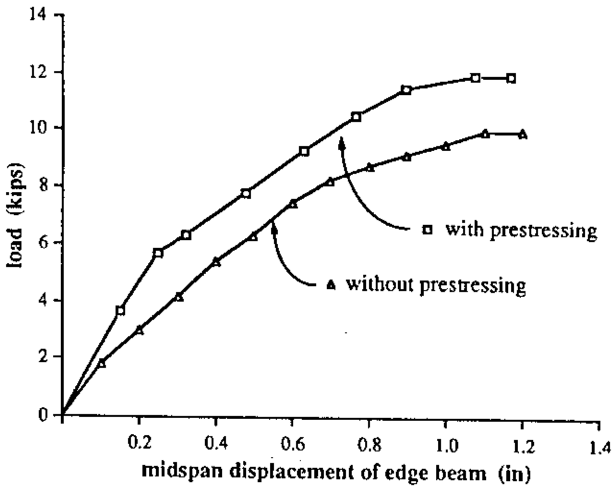


**Fig. 10. Ex. 1. Load vs. Displacement Curves for RC and PC Edge Beams.**

### 4.1. Example 2 - Cylindrical Shell with Prestressing in Both the Shell and the Edgebeams

Bouma (34) also tested a cylindrical shell with prestressing both in the shell and the edgebeams. The geometry of the shell, the reinforcement and the prestressing tendons are shown in Figs. 11, 12, and 13. All prestressing tendons are single strands of diameter 2 mm each. The prestressing tendons in the shell and the edge beams have a profile which is defined by the following equation:

$$\beta = a_i \alpha + b_i \qquad [10]$$

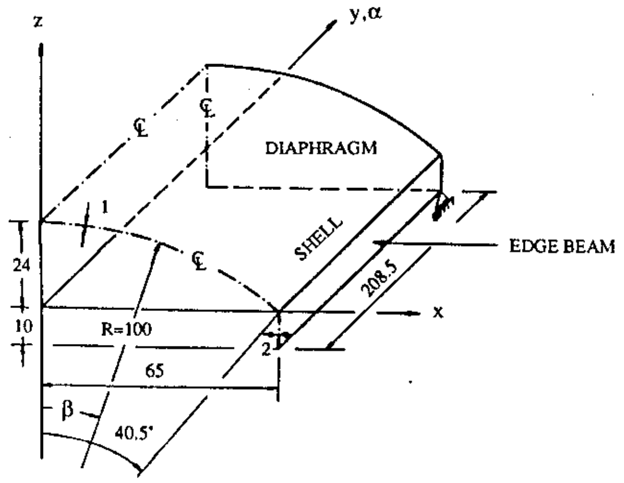


Fig. 11. Ex. 2. Prestressed Cylindrical Shell Geometry. (All dimensions in cm)

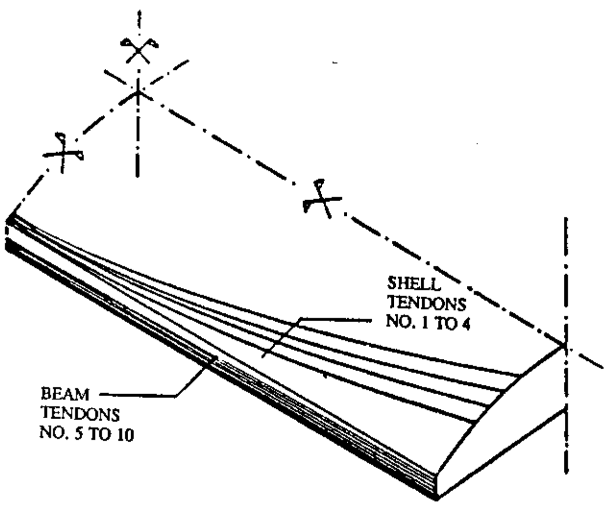


Fig. 12. Ex. 2. Prestressed Tendons in Shell and Edge Beam.

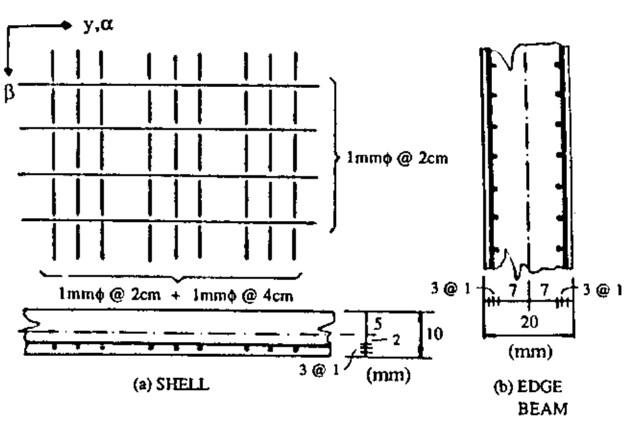


Fig. 13. Ex. 2. Reinforcement for Shell and Edge Beams.

where $a_i$ and $b_i$ are constants given for each tendon. $\beta$ and ($\alpha$ are as defined in Fig. 11. The following material data for the concrete and steel was used in the analysis.

| Concrete | Reinforcing Steel |
|---|---|
| $E_c = 33{,}675$ N/mm$^2$ | $E_s = 210{,}000$ N/mm$^2$ |
| $f'_c = 27.670$ N/mm$^2$ | $E_{sh} = 10{,}500$ N/mm$^2$ |
| $f'_t = 3.4$ N/mm$^2$ | $f_y = 300$ N/mm$^2$ |
| $\nu = 0.2$ | $\varepsilon_{su} = 0.1$ |
| $\tau_c = 25$ KN/m$^3$ | |

**Prestressing Steel**

$$E_p = 200{,}000 \text{ N/mm}^2$$
$$f_{py} = 1{,}700 \text{ N/mm}^2$$

The shell was first analyzed by Hofstetter(11). Recently, Roca(33) analyzed the same shell using NASHL1. Due to symmetry, only one quarter of the shell is used in the analytical model. Eight elements were used to model the shell, whereas the edge beam was modelled with four shell elements.

A comparison of the load-displacement plots obtained by the NASHL1 analysis and the experimental results is shown in Fig. 14. NASHL1 results indicate vertical cracking in the edge beams at a load level of 1.5 times the design load. Longitudinal cracks appear along the crown of the shell at a load level of about twice the design load which coincided with a distinct change in the slope of load-deflection curve (Fig. 14).

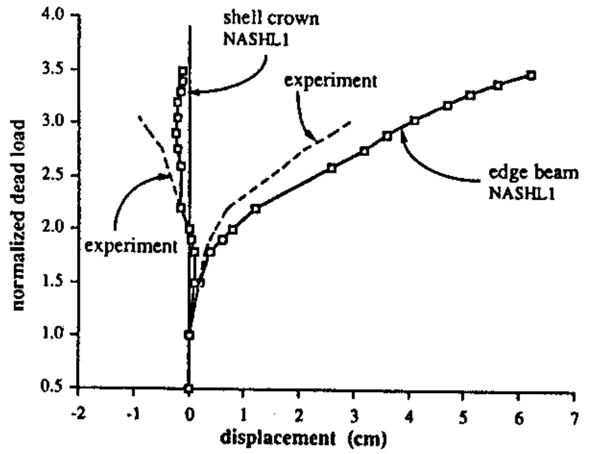


Fig. 14. Ex. 2. Loads vs. Midspan Displacement at Edge Beam and at Shell Crown.

These observations are in close agreement with Bouma's experimental results and also Hofstetter's analysis. NASHL1 also correctly predicted the experimental change in sign of vertical displacement at the crown (Fig. 14).

### 4.3. Example 3 - RC Free From Shell With and Without Initial Imperfections

Kolleger(13) recently studied a free form reinforced concrete shell (Fig. 15) both experimentally and anly-

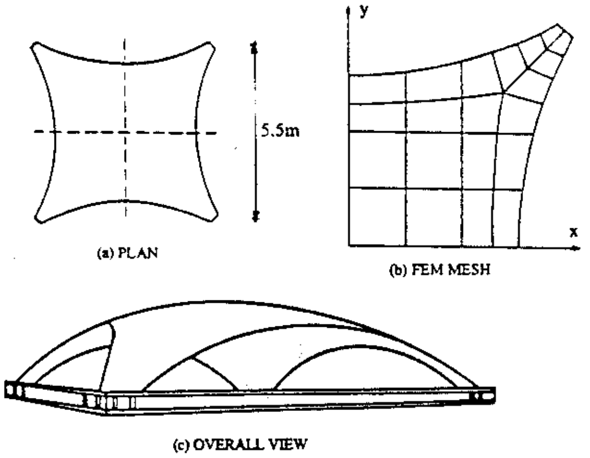


**Fig. 15. Ex. 3. Free Form RC Shell.**

tically. The shell has plan dimensions of 5.5 m x 5.5 m, the maximum height to the crown being 1 m. The shell has a thickness of 22 mm at the crown. At the supports the thickness is increased progressively up to 58 mm at the lowest point. The shell reinforcement is shown in Figs. 17 and 18. He studied the influence of a geometric imperfection in the shape of the shell on the ultimate load carried by the shell.

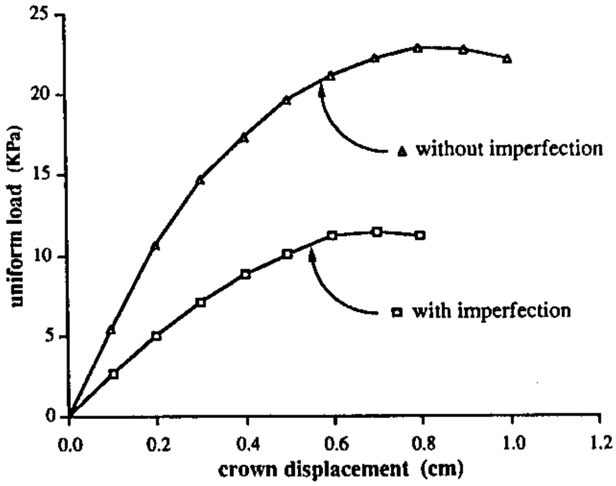


**Fig. 16. Ex. 3. Load vs. Crown Displacement With and Without Imperfections.**

Roca (33) analyzed the same shell with and without imperfections using NASHL1. Because of symmetry, only one quarter of the shell had to be used in the analytical model. The finite element mesh used in the analysis is shown in Fig. 15 (c). The geometry of the shell with imperfection was based on a deformed shape obtained in a previous analysis. This deformed shape was generated by subjecting the shell to a uniform load of 2 Kpa over the horizontal projection and factoring the displacements thus obtained such that

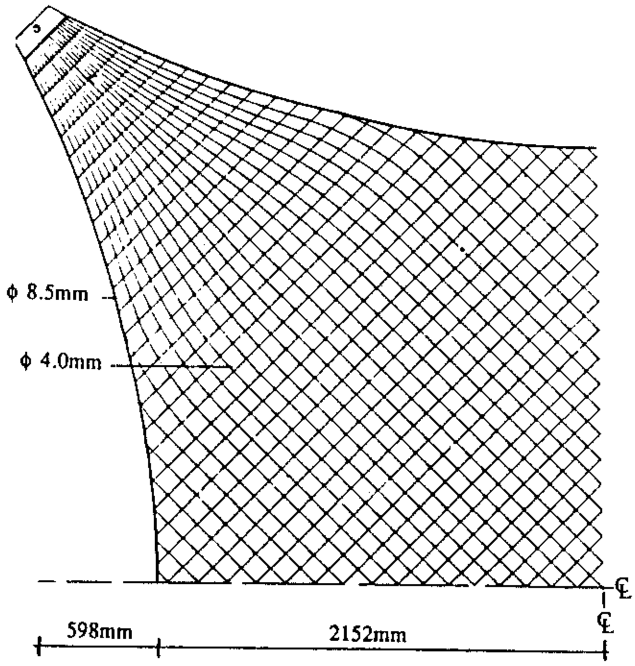


**Fig. 17. Ex. 3. Shell Reinforcement.**

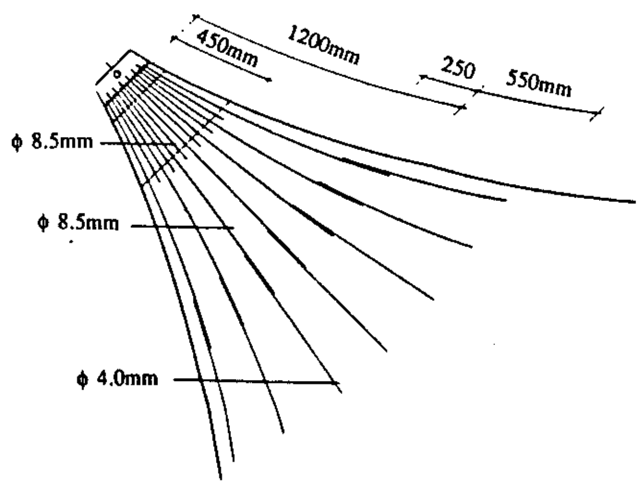


**Fig. 18. Ex. 3. Additional Reinforcement at Support.**

the deflection at the crown was 2 cm. This shape in an unstressed state was then used to specify initial imperfections before applying external load in the nonlinear analysis. The material properties used in the analysis for the concrete and reinforcing steel were:

| **Concrete** | **Steel** |
|---|---|
| $E_c = 32.000\ N/mm^2$ | $E_s = 210{,}000\ N/mm^2$ |
| $f'_c = 60\ N/mm^2$ | $E_{sh} = 0.0\ N/mm^2$ |
| $f'_t = 6\ N/mm^2$ | $f_y = 600\ N/mm^2$ |
| $\nu = 0.2$ | $\varepsilon_{su} = 0.1$ |
| $\tau_c = 25\ KN/m^3$ | |

The load versus crown displacement plot for the two cases, with and without imperfections, are shown in Fig. 16. It can be seen that the geometric imperfection has caused a dramatic reduction in the ultimate load carrying capacity of the structure: A geometric imperfection of the order of the thickness of shell has caused a reduction of about 50% in the ultimate load carried by the structure, emphasizing that nonlinear geometry must be considered in the analysis.

### 4.4. Gabled HP Shell with Crown and Edge Beams

In this example a gabled HP shell is analyzed to determine its response behavior under the influence of dead and live loads on the shell. The importance of the nonlinear geometry and of the creep and shrinkage effects are demonstrated.

The dimensions of the gabled HP shell used in the present example are shown in Fig. 19. The shell has plan dimensions of 80 by 80 ft—a rise of only 8 ft, and a constant thickness of 3 in. The size of the crown beams and the edge beams are, respectively, B x H and b x h. A number of nonlinear analyses have been carried out using NASHL, all with the same edge beam dimensions b x h = 12 x 16 in., but with two different sizes for the crown beam, B x H = 24 x 8 in. and 48 x 12 in. Only the results for the larger crown beam case (B x H = 48 x 12 in.) will be presented here to show the undesiderable effect of using an oversized beam. In this example selected results are presented to illustrate the dramatic decrease in the ultimate load which can occur when nonlinear geometric, material, and time dependent effects are included. Because of symmetry, only one quarter of the shell had to be used in the analytical model. For the NASHL analyses either a 4 x 4 or 2 x 2 mesh element layout was used.

The reinforcement layout (Fig. 20) for the shell and beams is designed based upon membrane theory for dead load (DL) plus a live load (LL) of 20 psf.

The material properties used in the analysis for the concrete and reinforcing steel are:

| Concrete | Steel |
|---|---|
| $E_o = 3.33 \times 10^3$ | $E_s = 29.0 \times 10^3$ ksi |
| $f'_c = 3.0$ksi | $E_{sh} = 0.0$ ksi |
| $f'_t = 0.471$ ksi | $f_y = 60.0$ ksi |
| $\nu = 0.15$ | $\varepsilon_{su} = 0.1$ |
| $\beta = 0.5$ | |
| $\varepsilon_c = 2\, f'_c/E_o$ | |

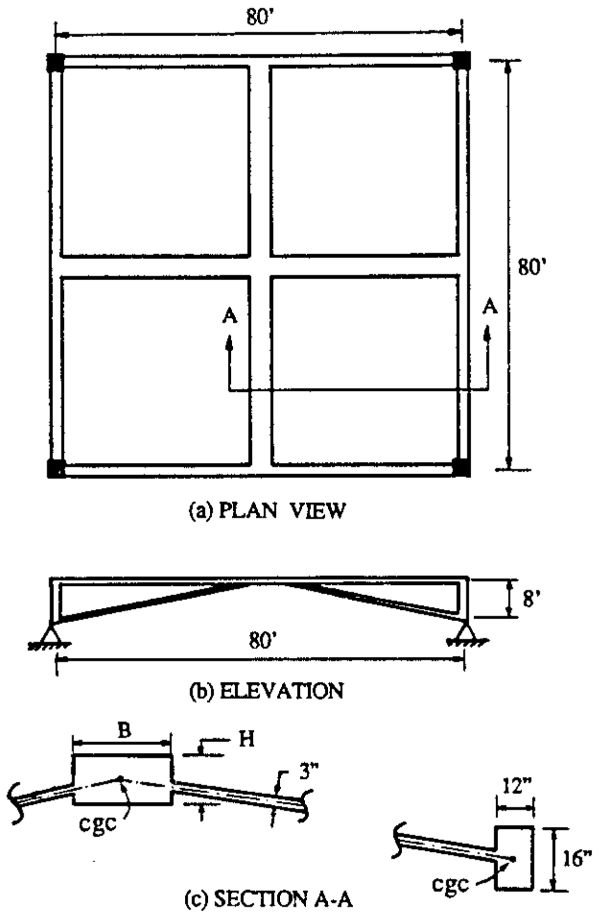


Fig. 19. Ex. 4. Gabled HP Shell Geometry.

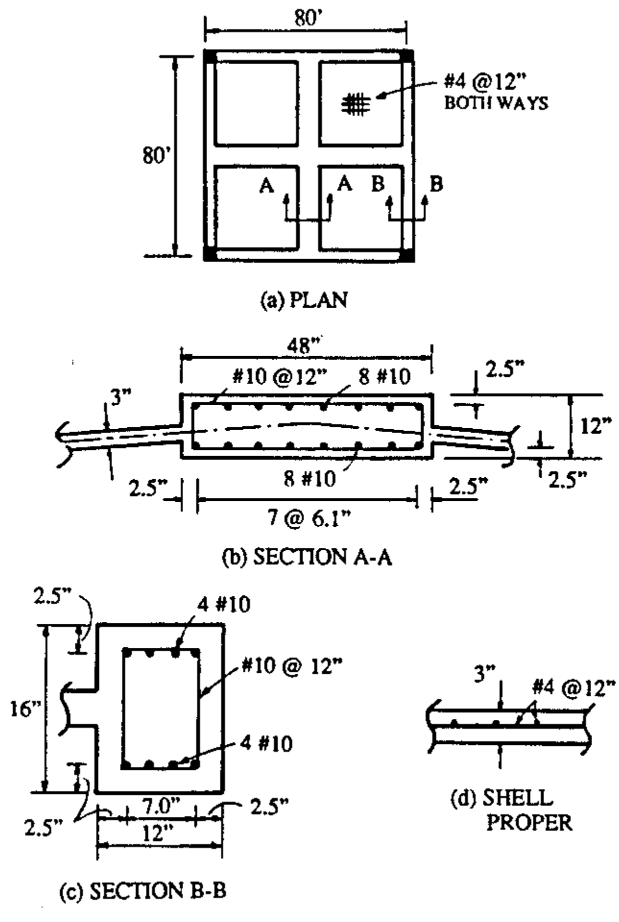


Fig. 20. Ex. 4. Reinforcement for Shell and Beams.

ACI recommendations for the creep and shrinkage effects are used for the time dependent analysis.

All the nonlinear analyses are perfomed by first applying the total dead load of the shell plus beams and then adding multiples of the 20 psf live load until failure occurs.

Crown displacement results from the analysis of a homogeneous uncracked concrete model without (linear) and with nonlinear geometry effects included (Fig. 21) indicates that the ultimate load in the latter case is DL + 11.8 LL. The response is almost linear up to a load of DL + 10.0 LL after which the continuous

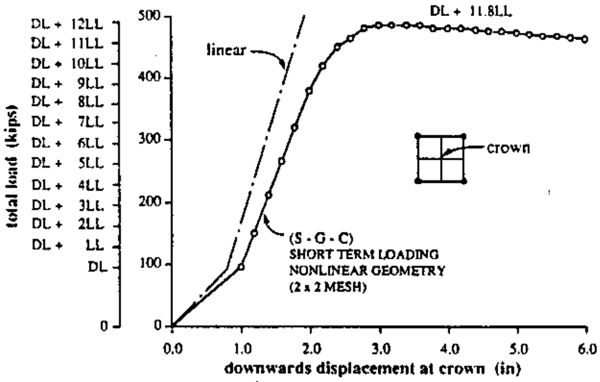


**Fig. 21. Ex. 4. Load vs. Crown Displacements for Uncracked Homogeneous Concrete Analytical Model.**

change in geometry causes local instability in the vicinity of the crown where the curvatures of the shell are smallest (Fig. 24) and the axial force is the largest.

Figure 22 shows the results of three analyses for a reinforced concrete analytical model of the HP gable shell which included nonlinear material effects usch as cracking, etc. First, for a short time, loading with only nonlinear materials, the ultimate load was DL + 6.5 LL. Second, for a short-time loading with nonlinear material and geometry, the ultimate load drops to DL + 3.3 LL. Finally, for a case in which the dead load is first applied and then left on for five months during which time creep and shrinkage take place resulting in increased deflections and redistribution of stresses and then increments of live load are applied on the structure with nonlinear material and geometry, the ultimate drops to DL + 1.5 LL. This illustrates the large decrease in the ultimate load that can occur in shells of this type as each nonlinearity is included in the analysis. Generally similar results have been found recently by Gallegos and Schnobrich (23).

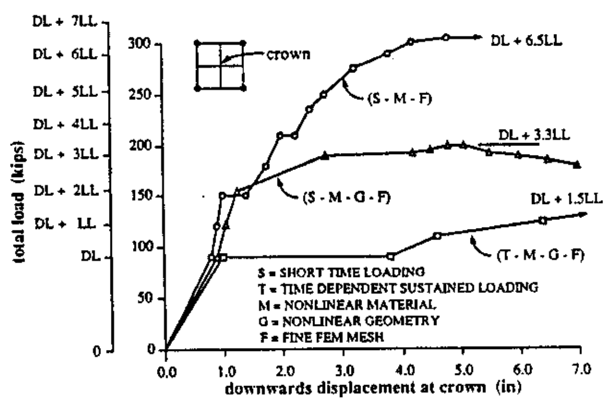


**Fig. 22. Ex. 4. Load vs. Crown Displacements for Cracked, etc. Reinforced Concrete Analytical Model.**

This reduction in load capacity is primarily due to the redistribution of axial force from the shell to the crown beam and thereby an amplification of the $P - \Delta$ effect in the crown region. This redistribution occurs due to cracking and the time dependent effects of creep and shrinkage.

The crack patterns at ultimate load for the three nonlinear material analyses (Fig. 22) are shown in Fig. 23. In all cases cracking is initiated near the support and propagates upwards along the diagonal. The failure

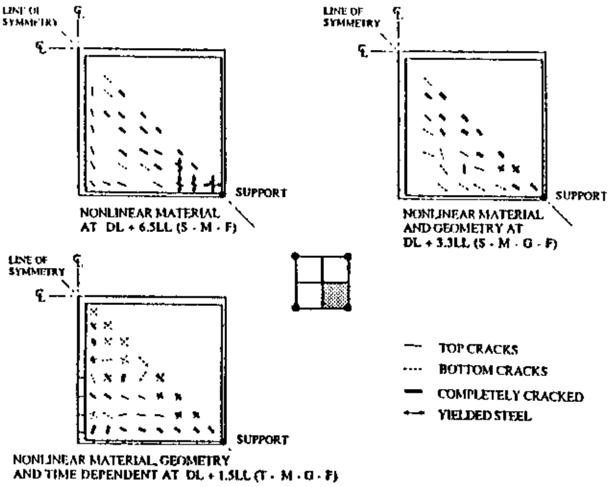


**Fig. 23. Ex. 4. Crack Patterns at Ultimate Load for the Three Nonlinear Load Cases in Fig. 22.**

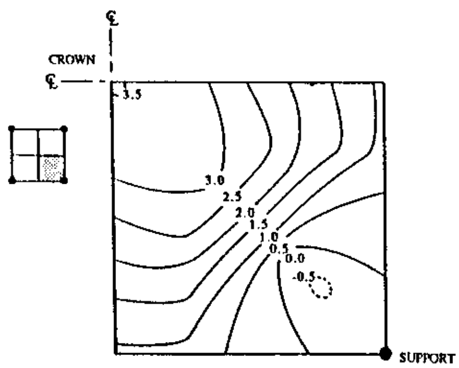


Fig. 24 Ex. 4 - Displacement Contours at DL+11.8(LL) for the Nonlinear Geometry Load Case in Fig. 21.

mode is ductile for the case of short time loading with only nonlinear material. However, when both nonlinear material and geometry are included, the modes of failure are brittle for both the short time and the sustained load on the shell. In the latter case the additional cracking in the supporting members further deteriorates the shell stiffness and results in the ultimate collapse of the structure.

## 3. FUTURE RESEARCH, DEVELOPMENT AND APPLICATIONS

A review of research, development and applications in the nonlinear analysis of reinforced and prestressed concrete shells has been presented. This work was initiated by a few investigators in the early 1970's. It has been pursued by an increasing number of investigators in the 1980's, so that substantial progress has been made during the past decade in nonlinear analytical models, methods of analysis and computer programs to predict the nonlinear material, geometric and time dependent response of reinforced and prestressed concrete shells through their elastic, cracking, inelastic and ultimate ranges.

Future research in this field needs to be done in the verification of these solutions by comparing analytical results with available experimental results or carefully proposed tests (see for example Ref. 18). Standard test cases need to be established for this purpose. Basic research needs to be continued in the analytical modeling of the composite material including cracking, tension stiffening, creep and shrinkage, the torsion-bending coupling in beam members, and for other phenomena. Only a limited number of investigations to date have incorporated prestressing into the analytical models, which introduces a number of additional complexities into the solution, which need to be further studied. A segmental analysis, capability which can trace the response of a structure through any sequence of construction, should be added to the solution.

Computer program development needs to be continued for these complex solutions, so that the input and output is clear and easily interpreted. Pre and post processing programs, including computer graphics, need to be implemented before the solutions will be used extensively. Most important of all the computer programs should be well documented and available to those who wish to use them.

Future applications to special complex problems in important and high cost projects will undoubtedly increase, because of a lack of any reasonable alternative. Studies of important design problems such as buckling or time dependent response of various types of RC and PC shells should be undertaken to establish realistic design rules.

## 6. CONCLUSIONS

As one who has had a keen interest in teaching, research and consulting in the field of reinforced and prestressed concrete shells during the past thirty years, I believe tremendous progress has been made in our analytical capability to predict their response. Of course, the ultimate goal of this improvement in capability is its implementation into design so as to produce better, safer, and more economical structures, regardless of their complexity. This should be the continuing, unending and exciting challenge for all structural engineers.

## 7. ACKNOWLEDGEMENTS

The contributions of the many graduate students cited and not cited in this paper to this field of study are gratefully acknowledged. Special thanks should go to Sajid Abbas and Pedro Roca for their assistance in preparing this paper.

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