Recommendations for Estimating Prestress Losses

Prepared by

PCI Committee on Prestress Losses

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This PCI Committee report summarizes data on creep and shrinkage of concrete and steel relaxation, and presents both a general and a simplified design procedure for using these data in estimating loss of prestress after any given time period. A Commentary explains the design provisions. Detailed design examples for pretensioned and post-tensioned concrete structures explain the procedures.

CONTENTS

COMMITTEE STATEMENT

This recommended practice is intended to give the design engineer a comprehensive summary of research data applicable to estimating loss of prestress. It presents a general method whereby losses are calculated as a function of time.

This report contains information and procedures for estimating prestress losses in building applications. The general method is applicable to bridges, although there are some differences between it and the AASHTO *Standard Specifications for Highway Bridges* with respect to individual loss components.

A precise determination of stress losses in prestressed concrete members is a complicated problem because the rate of loss due to one factor, such as relaxation of tendons, is continually being altered by changes in stress due to other factors, such as creep of concrete. Rate of creep in its turn is altered by change in tendon stress. It is extremely difficult to separate the net amount of loss due to each factor under different conditions of stress, environment, loading, and other uncertain factors.

In addition to the foregoing uncertainties due to interaction of shrinkage, creep, and relaxation, physical conditions, such as variations in actual properties of concrete made to the same specified strength, can vary the total loss. As a result, the computed values for prestress loss are not necessarily exact, but the procedures here presented will provide more accurate results than by previous methods which gave no consideration to the actual stress levels in concrete and tendons.

An error in computing losses can affect service conditions such as camber, deflection, and cracking. It has no effect on the ultimate strength of a flexural member unless the tendons are unbonded or the final stress after losses is less than $0.5f_{pu}$.

It is not suggested that the information and procedures in this report provide the only satisfactory solution to this complicated problem. They do represent an up-to-date compromise by the committee of diverse opinions, experience and research results into relatively easy to follow design formulas, parameters, and computations.

CHAPTER 1—GENERAL ASPECTS RELATED TO PRESTRESS LOSSES

1.1—Tensioning of Prestressing Steel

1.1.1—Pretensioned construction

For deflected prestressing steel, loss *DEF*, occurring at the deflecting devices, should be taken into account.

1.1.2—Friction in post-tensioned construction

Loss due to friction in post-tensioned construction should be based upon wobble and curvature coefficients given below, and verified during stressing operations. The losses due to friction between the prestressing steel and the duct enclosure may be estimated by the following equation:

$$FR = T_o[1 - e^{-(Kl_{tx}+\mu\alpha)}] \qquad (1)$$

When $(Kl_{tx} + \mu\alpha)$ is not greater than 0.3, the following equation may be used:

$$FR = T_o(Kl_{tx} + \mu\alpha) \qquad (2)$$

Table 1 gives a summary of friction coefficients for various post-tensioning tendons.

1.2—Anchorage

Loss *ANC*, due to movement of prestressing steel in the end anchorage, should be taken into account. Slip at the anchorage will depend upon the particular prestressing system utilized and will not be a function of time. Realistic allowance should be made for slip or take-up as recommended for a given system of anchorage.

1.3—Transfer of Prestress

Loss due to elastic shortening may be calculated according to the provisions in this recommended practice. The concrete shortening should also include that resulting from subsequent stressing of prestressing steel.

1.4—Effect of Members in Structures

Loss of prestress of a member may be affected by connection to other structural elements or composite action with cast-in-place concrete. Change in prestress force due to these factors should be taken into account based on a rational procedure that considers equilibrium of forces and strain compatibility.

Table 1. Friction coefficients for post-tensioning tendons.

Type of tendon	Wobble coefficient, *K*, per foot	Curvature coefficient, μ
Tendons in flexible metal sheathing		
Wire tendons	0.0010 - 0.0015	0.15 - 0.25
7-wire strand	0.0005 - 0.0020	0.15 - 0.25
High strength bars	0.0001 - 0.0006	0.08 - 0.30
Tendons in rigid metal duct		
7-wire strand	0.0002	0.15 - 0.25
Pre-greased tendons		
Wire tendons and 7-wire strand	0.0003 - 0.0020	0.05 - 0.15
Mastic-coated tendons		
Wire tendons and 7-wire strand	0.0010 - 0.0020	0.05 - 0.15

CHAPTER 2—GENERAL METHOD FOR COMPUTING PRESTRESS LOSSES

2.1—Scope

2.1.1—Materials

2.1.1.1—*Lightweight concrete*—Lightweight aggregate concrete with a unit weight between 90 and 125 lb per cu ft where the unit weight varies because of replacement of lightweight fines with normal weight sand.

2.1.1.2—*Normal weight concrete*—Concrete with an approximate unit weight of 145 lb per cu ft where all aggregates are normal weight concrete aggregates.

2.1.1.3—*Prestressing Steel*—High strength prestressing steel that has been subjected to the stress-relieving process, or to processes resulting in low relaxation characteristics.

2.1.2— Prestressed units

Linearly prestressed members only. Excluded are closed sections prestressed circumferentially.

2.1.3—Curing

2.1.3.1—*Moist cure*—Impermeable membrane curing or other methods to prevent the loss of moisture from the concrete.

2.1.3.2—*Accelerated cure*—Curing in which the temperature of the concrete is elevated to not more than 160F for a period of approximately 18 hours, and steps are taken to retain moisture.

2.1.4—Environment

Prestressed concrete subjected to seasonal fluctuations of temperature and humidity in the open air or to nominal room conditions is covered.

The values for *UCR* and *USH* are based on an average ambient relative humidity of 70 percent.

2.2—Total Loss

2.2.1—Pretensioned construction

$$TL = ANC + DEF + ES + \sum_{t_1}^{t} (CR + SH + RET)$$

2.2.2—Post-tensioned construction

$$TL = FR + ANC + ES + \sum_{t_1}^{t} (CR + SH + RET)$$

2.3—Loss Due to Elastic Shortening *(ES)*

Loss of prestress due to elastic shortening of the concrete should be calculated based on the modulus of elasticity of the concrete at the time the prestress force is applied.

$$ES = f_{cr}(E_s/E_{ci}) \quad (5)$$

2.3.1—Pretensioned construction

In calculating shortening, the loss of prestress shall be based upon the concrete stress at the centroid of the prestressing force at the section of the member under consideration. This stress, f_{cr}, is the compressive stress due to the prestressing force that is acting immediately after the prestress force is applied minus the stress due to all dead load acting at that time.

2.3.2—Post-tensioned construction

The average concrete stress between anchorages along each element shall be used in calculating shortening.

2.4—Time-Dependent Losses (General)

Prestress losses due to steel relaxation and creep and shrinkage of concrete are inter-dependent and are time-dependent. To account for changes of these effects with time, a step-by-step procedure can be used with the time interval increasing with age of the concrete. Shrinkage from the time when curing is stopped until the time when the concrete is prestressed should be deducted from the total calculated shrinkage for post-tensioned construction. It is recommended that a minimum of four time intervals be used as shown in Table 2.

When significant changes in loading are expected, time intervals other than those recommended should be used. Also, it is neither necessary, nor always desirable, to assume that the design live load is continually present. The four time intervals above are recommended for minimum non-computerized calculations.

Table 2. Minimum time intervals.

Step	Beginning time, t_1	End time, t
1	Pretensioned anchorage of prestressing steel Post-tensioned: end of curing of concrete	Age at prestressing of concrete
2	End of Step 1	Age = 30 days, or time when a member is subjected to load in addition to its own weight
3	End of Step 2	Age = 1 year
4	End of Step 3	End of service life

2.5—Loss Due to Creep of Concrete *(CR)*

2.5.1—Loss over each step

Loss over each time interval is given by

$$CR = (UCR)(SCF)(MCF) \times (PCR)(f_c) \quad (6)$$

where f_c is the net concrete compressive stress at the center of gravity of the prestressing force at time t_1, taking into account the loss of prestress force occurring over the preceding time interval.

The concrete stress f_c at the time t_1 shall also include change in applied load during the preceding time interval. Do not include the factor *MCF* for accelerated cured concrete.

2.5.2—Ultimate creep loss

2.5.2.1—*Normal weight concrete (UCR)*

Moist cure not exceeding 7 days:

$$UCR = 95 - 20E_c/10^6 \geqq 11 \quad (7)$$

Accelerated cure:

$$UCR = 63 - 20E_c/10^6 \geqq 11 \quad (8)$$

2.5.2.2—*Lightweight concrete (UCR)*

Moist cure not exceeding 7 days:

$$UCR = 76 - 20E_c/10^6 \geqq 11 \quad (9)$$

Accelerated cure:

$$UCR = 63 - 20E_c/10^6 \geqq 11 \quad (10)$$

Table 3. Creep factors for various volume to surface ratios.

Volume to surface ratio, in.	Creep factor *SCF*
1	1.05
2	0.96
3	0.87
4	0.77
5	0.68
>5	0.68

Table 4. Creep factors for various ages of prestress and periods of cure.

Age of prestress transfer, days	Period of cure, days	Creep factor, *MCF*
3	3	1.14
5	5	1.07
7	7	1.00
10	7	0.96
20	7	0.84
30	7	0.72
40	7	0.60

Table 5. Variation of creep with time after prestress transfer.

Time after prestress transfer, days	Portion of ultimate creep, *AUC*
1	0.08
2	0.15
5	0.18
7	0.23
10	0.24
20	0.30
30	0.35
60	0.45
90	0.51
180	0.61
365	0.74
End of service life	1.00

Table 6. Shrinkage factors for various volume to surface ratios.

Volume to surface ratio, in.	Shrinkage factor *SSF*
1	1.04
2	0.96
3	0.86
4	0.77
5	0.69
6	0.60

2.5.3—Effect of size and shape of member (SCF)

To account for the effect of size and shape of the prestressed members, the value of *SCF* in Eq. (6) is given in Table 3.

2.5.4—Effect of age at prestress and length of cure (MCF)

To account for effects due to the age at prestress of moist cured concrete and the length of the moist cure, the value of *MCF* in Eq. (6) is given in Table 4. The factors in this table do *not* apply to accelerated cured concretes nor are they applicable as shrinkage factors.

2.5.5—Variation of creep with time (AUC)

The variation of creep with time shall be estimated by the values given in Table 5. Linear interpolation shall be used between the values listed.

2.5.6—Amount of creep over each step (PCR)

The portion of ultimate creep over the time interval t_1 to t, *PCR* in Eq. (6), is given by the following equation:

$$PCR = (AUC)_t - (AUC)_{t_1} \quad (11)$$

2.6—Loss Due to Shrinkage of Concrete *(SH)*

2.6.1—Loss over each step

Loss over each time interval is given by

$$SH = (USH)(SSF)(PSH) \quad (12)$$

2.6.2—Ultimate loss due to shrinkage of concrete

The following equations apply to

both moist cured and accelerated cured concretes.

2.6.2.1—*Normal weight concrete (USH)*

$$USH = 27{,}000 - 3000E_c/10^6 \quad (13)$$

but not less than 12,000 psi.

2.6.2.2—*Lightweight concrete (USH)*

$$USH = 41{,}000 - 10{,}000E_c/10^6 \quad (14)$$

but not less than 12,000 psi.

2.6.3—Effect of size and shape of member (SSF)

To account for effects due to the size and shape of the prestressed member, the value of *SSF* in Eq. (12) is given in Table 6.

2.6.4—Variation of shrinkage with time (AUS)

The variation of shrinkage with time shall be estimated by the values given in Table 7. Linear interpolation shall be used between the values listed.

2.6.5—Amount of shrinkage over each step (PSH)

The portion of ultimate shrinkage over the time interval t_1 to t, *PSH* in Eq. (12), is given by the following equation:

$$PSH = (AUS)_t - (AUS)_{t_1} \quad (15)$$

2.7—Loss Due to Steel Relaxation *(RET)*

Loss of prestress due to steel relaxation over the time interval t_1 to t may be estimated using the following equations. (For mathematical correctness, the value for t_1 at the time of anchorage of the prestressing

Table 7. Shrinkage coefficients for various curing times.

Time after end of curing, days	Portion of ultimate shrinkage, *AUS*
1	0.08
3	0.15
5	0.20
7	0.22
10	0.27
20	0.36
30	0.42
60	0.55
90	0.62
180	0.68
365	0.86
End of service life	1.00

steel shall be taken as 1/24 of a day so that log t_1 at this time equals zero.)

2.7.1—Stress-relieved steel

$$RET = f_{st}\{[\log 24t - \log 24t_1]/10\} \times [f_{st}/f_{py} - 0.55] \quad (16)$$

where

$$f_{st}/f_{py} - 0.55 \geq 0.05$$
$$f_{py} = 0.85 f_{pu}$$

2.7.2—Low-relaxation steel

The following equation applies to prestressing steel given its low relaxation properties by simultaneous heating and stretching operations.

$$RET = f_{st}\{[\log 24t - \log 24t_1]/45\} \times [f_{st}/f_{py} - 0.55] \quad (17)$$

where

$$f_{st}/f_{py} - 0.55 \geq 0.05$$
$$f_{py} = 0.90 f_{pu}$$

2.7.3—Other prestressing steel

Relaxation of other types of prestressing steel shall be based upon manufacturer's recommendations supported by test data.

CHAPTER 3—SIMPLIFIED METHOD FOR COMPUTING PRESTRESS LOSSES

3.1—Scope

Computations of stress losses in accordance with the General Method can be laborious for a designer who does not have the procedure set up on a computer program. The Simplified Method is based on a large number of design examples in which the parameters were varied to show the effect of different levels of concrete stress, dead load stress, and other factors. These examples followed the General Method and the procedures given in the Design Examples.

3.2—Principles of the Simplified Method

3.2.1—Concrete stress at the critical location

Compute f_{cr} and f_{cds} at the critical location on the span. The critical location is the point along the span where the concrete stress under full live load is either in maximum tension or in minimum compression. If f_{cds} exceeds f_{cr} the simplified method is not applicable.

f_{cr} and f_{cds} are the stresses in the concrete at the level of the center of gravity of the tendons at the critical location. f_{cr} is the net stress due to the prestressing force plus the weight of the prestressed member and any other permanent loads on the member at the time the prestressing force is applied. The prestressing force used in computing f_{cr} is the force existing immediately after the prestress has been applied to the concrete. f_{cds} is the stress due to all permanent ((dead) loads not used in computing f_{cr}.

Table 8. Simplified method equations for computing total prestress loss (*TL*).

Equation number	Concrete weight		Type of tendon			Tensioning		Equations
	NW	LW	SR	LR	BAR	PRE	POST	
N-SR-PRE-70	X		X			X		$TL = 33.0 + 13.8f_{cr} - 4.5f_{cds}$
L-SR-PRE-70		X	X			X		$TL = 31.2 + 16.8f_{cr} - 3.8f_{cds}$
N-LR-PRE-75	X			X		X		$TL = 19.8 + 16.3f_{cr} - 5.4f_{cds}$
L-LR-PRE-75		X		X		X		$TL = 17.5 + 20.4f_{cr} - 4.8f_{cds}$
N-SR-POST-68.5	X		X				X	$TL = 29.3 + 5.1f_{cr} - 3.0f_{cds}$
L-SR-POST-68.5		X	X				X	$TL = 27.1 + 10.1f_{cr} - 4.9f_{cds}$
N-LR-POST-68.5	X			X			X	$TL = 12.5 + 7.0f_{cr} - 4.1f_{cds}$
L-LR-POST-68.5		X		X			X	$TL = 11.9 + 11.1f_{cr} - 6.2f_{cds}$
N-BAR-POST-70	X				X		X	$TL = 12.8 + 6.9f_{cr} - 4.0f_{cds}$
L-BAR-POST-70		X			X		X	$TL = 12.5 + 10.9f_{cr} - 6.0f_{cds}$

Note: Values of TL, f_{cr}, and f_{cds} are expressed in ksi.

3.2.2—Simplified loss equations

Select the applicable equation from Table 8 or 9, substitute the values for f_{cr} and f_{cds} and compute TL or f_{se}, whichever is desired.

3.2.3—Basic parameters

The equations are based on members having the following properties:

1. Volume-to-surface ratio = 2.0.
2. Tendon tension as indicated in each equation.
3. Concrete strength at time prestressing force is applied:
   3500 psi for pretensioned members
   5000 psi for post-tensioned members
4. 28-day concrete compressive strength = 5000 psi.
5. Age at time of prestressing:
   18 hours for pretensioned members
   30 days for post-tensioned members
6. Additional dead load applied 30 days after prestressing.

Compare the properties of the beam being checked with Items 1 and 2. If there is an appreciable difference, make adjustments as indicated under Section 3.4.

It was found that an increase in concrete strength at the time of prestressing or at 28 days made only a nominal difference in final loss and could be disregarded. For strength at prestressing less than 3500 psi or for 28-day strengths less than 4500 psi, an analysis should be made following Design Example 1.

Wide variations in Items 5 and 6 made only nominal changes in net loss so that further detailed analysis is needed only in extreme cases.

3.2.4—Computing f_{cr}

$$f_{cr} = A_s f_{si}/A_c + A_s f_{si} e^2/I_c - M'e/I_c \qquad (18)$$

Table 9. Simplified method equations for computing effective prestress (f_{se}).

Equation Number	Concrete weight		Type of tendon			Tensioning		Equations
	NW	LW	SR	LR	BAR	PRE	PQST	
N-SR-PRE-70	X		X			X		$f_{se} = f_t - (33.0 + 13.8f_{cr} - 11f_{cds})$
L-SR-PRE-70		X	X			X		$f_{se} = f_t - (31.2 + 16.8f_{cr} - 13.5f_{cds})$
N-LR-PRE-75	X			X		X		$f_{se} = f_t - (19.8 + 16.3f_{cr} - 11.9f_{cds})$
L-LR-PRE-75		X		X		X		$f_{se} = f_t - (17.5 + 20.4f_{cr} - 14.5f_{cds})$
N-SR-POST-68.5	X		X				X	$f_{se} = f_{st} - (29.3 + 5.1f_{cr} - 9.5f_{cds})$
L-SR-POST-68.5		X	X				X	$f_{se} = f_{st} - (27.1 + 10.1f_{cr} - 14.6f_{cds})$
N-LR-POST-68.5	X			X			X	$f_{se} = f_{st} - (12.5 + 7.0f_{cr} - 10.6f_{cds})$
L-LR-POST-68.5		X		X			X	$f_{se} = f_{st} - (11.9 + 11.1f_{cr} - 15.9f_{cds})$
N-BAR-POST-70	X				X		X	$f_{se} = f_{st} - (12.8 + 6.9f_{cr} - 10.5f_{cds})$
L-BAR-POST-70		X			X		X	$f_{se} = f_{st} - (12.5 + 10.9f_{cr} - 15.7f_{cds})$

Note: Values of f_t, f_{st}, f_{cr}, and f_{cds} are expressed in ksi.

3.2.5—Tendon stress for pretensioned members

Except for members that are very heavily or very lightly* prestressed, f_{si} can be taken as follows:

For stress-relieved steel

$$f_{si} = 0.90\, f_t \qquad (19)$$

For low-relaxation steel

$$f_{si} = 0.925\, f_t \qquad (20)$$

3.2.6 Tendon stress for post-tensioned members

Except for members that are very heavily or very lightly* prestressed, f_{si} can be taken as

$$f_{si} = 0.95\,(T_o - FR) \qquad (21)$$

3.3—Equations for Simplified Method

3.3.1—Total prestress loss

The equations in Table 8 give total prestress loss TL in ksi. This value corresponds to TL shown in the summaries of Design Examples 1 and 3.

3.3.2—Effective stress

The equations in Table 9 give effective stress in prestressing steel under dead load after losses. This value corresponds to f_{se} shown in the summary of Design Example 1.

As shown in the summary of Design Example 1, the stress existing in the tendons under dead load after all losses have taken place is the initial tension reduced by the amount of the total losses and increased by the stress created in the tendon by the addition of dead load after the member was prestressed. The increase in tendon stress due to the additional dead load is equal to $f_{cds}\,(E_s/E_c)$.

$$f_{se} = f_t - TL + f_{cds}(E_s/E_c) \qquad (22)$$

3.3.3—Explanation of equation number

The equation number in Tables 8 and 9 defines the conditions for which each equation applies:

1. The first term identifies the type of concrete.
   N = normal weight = approximately 145 lb per cu ft
   L = lightweight = approximately 115 lb per cu ft
2. The second term identifies the steel in the tendon:
   SR = stress-relieved
   LR = low-relaxation
   BAR = high strength bar
3. The third term identifies the type of tensioning:
   PRE = pretensioned and is based on accelerated curing
   POST = post-tensioned and is based on moist curing
4. The fourth term indicates the initial tension in percent of f_{pu}:
   For *pretensioned* tendons it is the tension at which the tendons are anchored in the casting bed before concrete is placed.

   For *post-tensioned* tendons it is the initial tension in the tendon at the critical location in the concrete member after losses due to friction and anchor set have been deducted.

*When f_{cr} computed by Eq. (18) using the approximations for f_{st} is greater than 1600 psi or less than 800 psi the value of f_{st} should be checked as illustrated in Design Example 2.

3.4—Adjustment for Variations from Basic Parameters

3.4.1—Volume-to-surface ratio

Equations are based on $V/S = 2.0$

V/S ratio	1.0	2.0	3.0	4.0
Adjustment, percent	+3.2	0	−3.8	−7.6

Example: For $V/S = 3.0$, decrease TL by 3.8 percent.

3.4.2—Tendon stress

3.4.2.1—*Pretensioned tendons*

Pretensioned tendons are so seldom used at stresses below those shown for the equations in members where the final stress is important, that examples covering this condition were not worked out. Design Example 1 can be followed if necessary.

3.4.2.2—*Post-tensioned tendons*

Equations are based on $f_{si} = 185{,}000$ psi. If f_{si} is less than 185,000 psi reduce the total stress loss:

For stress-relieved strands

$$\Delta TL = 0.41\,(185{,}000 - f_{si}) \qquad (23)$$

For low-relaxation strands

$$\Delta TL = 0.09\,(185{,}000 - f_{si}) \qquad (24)$$

For high strength bars which are based on $f_{si} = 0.70\, f_{pu}$

$$\Delta TL = 0.09\,(0.70\, f_{pu} - f_{si}) \qquad (25)$$

If f_{si} is greater than the value used in preparing the equations, ΔTL will be a negative number and will therefore increase the value of TL. Note that f_{si} is limited to a maximum of $0.70\, f_{pu}$ by ACI 318-71.

COMMENTARY

In this report a wide range of data has been assimilated to develop a general method for predicting loss of prestress, but including specific numerical values. In addition, creep and shrinkage of concrete and steel relaxation are presented as functions of time. By calculating losses over recommended time intervals, it is possible to take into account the interdependence of concrete and steel information.

It must be emphasized that losses, per se, are not the final aim of calculations. What is determined is the stress remaining in the prestressing steel. The stress remaining, however, must be evaluated using rational procedures.

The notation commonly used in the ACI Building Code is adopted wherever possible. For new terms, descriptive letters are used.

References are listed chronologically. Not all the references listed were used in developing the numerical recommendations. They are included because of their value in understanding time-dependent behavior.

Chapter 1—General Aspects Related to Prestress Losses

1.1—Friction losses during post-tensioning are estimated using familiar equations, but with up-dated coefficients (see Reference 30). No known systematic study has been made of losses that occur at deflecting devices in pretensioned construction. The provision is to warn that friction at these points may produce conditions where the desired steel stresses are not reached.

1.2—Seating losses are particularly important where the length of prestressing steel is short. For this condition, tolerances in seating deformation should not be overlooked.

1.3—The effect of post-tensioning of each individual tendon on previously anchored tendons should be considered. This applies, of course, when pretensioned and post-tensioned systems are combined.

1.4—Many problems have occurred because of unaccounted restraint and the effects of volume changes of concrete cast at different times. There are several references that give techniques for calculating these effects (References 6, 9, 10, 12, 20, 29, 30). This reminder is included here, even though losses are but one factor influenced by structural integration.

Chapter 2—General Method for Computing Prestress Losses

This section presents the range of data studied and, consequently, the range of applicability. Extrapolation has been avoided beyond documented data. In effect, this also shows where additional research is needed. Some practices and conditions common to certain areas of the country cannot be incorporated because no information is available. It is in this situation that experience, engineering judgment, and local sources of information are depended upon.

2.2—Total loss of prestress is the sum of losses due to individual factors. Eqs. (3) and (4) list the factors to be taken into account for each type of construction. The terms *ANC, DEF*, and *FR* are defined in Section 1. The remaining terms are defined in Section 2. Losses due to creep, shrinkage, and steel relaxation are the sum of losses during each time interval described in Section 2.4.

2.3—It is not desirable to be "conservative" and assume a low value for the modulus of concrete. The estimated modulus should reflect what is specified for minimum concrete strength and what is specified for permissible variation of concrete strength.

2.4—The step procedure is recommended to realistically approach the actual behavior of prestressed concrete. By this technique, it is possible to evaluate loss of prestress with change in time and change in stress. What is done here is to take into account the interdependence of one deformation on the other. Specific steps are outlined in succeeding sections.

2.5—Ultimate creep is the total amount of shortening measured on standard 6 x 12-in. cylinders. Fig. 1 illustrates that values differ according to the type of cure and the type of concrete (References 2, 14, 15, 21, 24). Ultimate creep is affected by relative humidity as shown in Fig. 2 (References 3, 7, 8, 11, 17). In standard tests, specimens are stored at 50 percent relative humidity. Average relative humidity over the majority of the United States is 70 percent (Reference 18). Eqs. (7), (8), (9), and (10), therefore, are based on data shown in Figs. 1 and 2.

Fig. 3 presents the relationship between creep and the size and shape of the test specimen (see References 7, 11, 17, 19). To apply standard creep data to actual members, a creep factor *SCF* is introduced. Values of *SCF* versus volume to surface ratios are listed in Section 2.5.3.

Similarly, age at loading and extent of moist-cure affect the amount of ultimate creep. Data in Fig. 4 (see Referenecs 3, 4, 11, 14) illustrate the trend. For moist-cured concretes, *UCR* is modified by the factor *MCF* in Section 2.5.4.

A generalization of creep-time data (References 2, 15, 24) was developed to take into account the rate of shortening due to creep. A typical creep curve is shown in Fig. 5. The portion of ultimate creep *AUC* at a given time is listed in Table 5. The amount of creep *PCR* in a given time interval is simply the difference of the amount of creep at the beginning and end of the time interval.

The terms, *SCF, MCF*, and *PCR* are non-dimensional. *UCR* was developed from creep data expressed as strain in millionths per psi concrete stress. By multiplying these data by the steel modulus of elasticity, unit steel stress per unit concrete stress was obtained.

Therefore, in Eq. (6), steel stress is obtained by multiplying concrete stress f_c by the product of these four non-dimensional terms. As stated in Section 2.5, f_c is the net concrete stress that results from at least full dead load and possibly some portion of the live load. The amount of live load, if any, present for extended periods is left to the engineer's judgment.

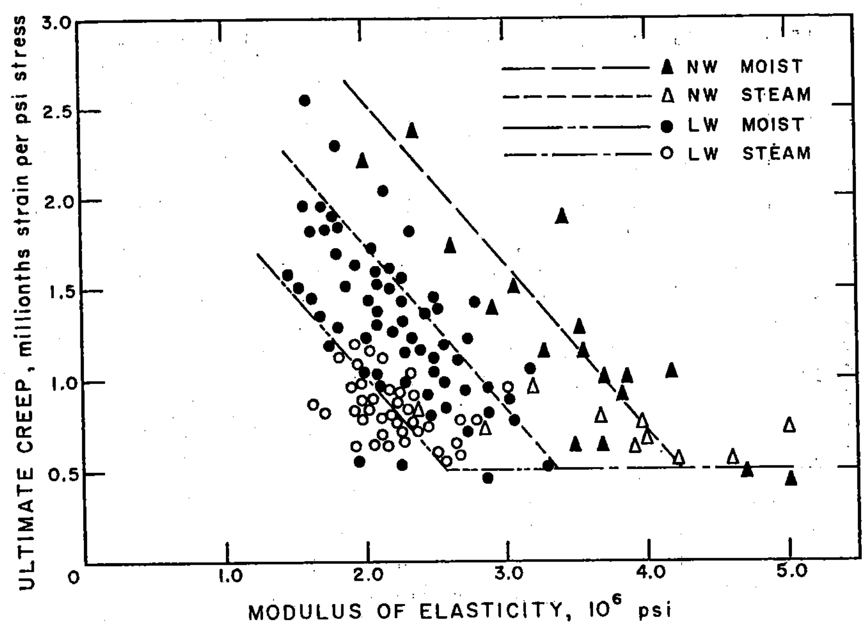

Fig. 1. Ultimate creep versus modulus of elasticity.

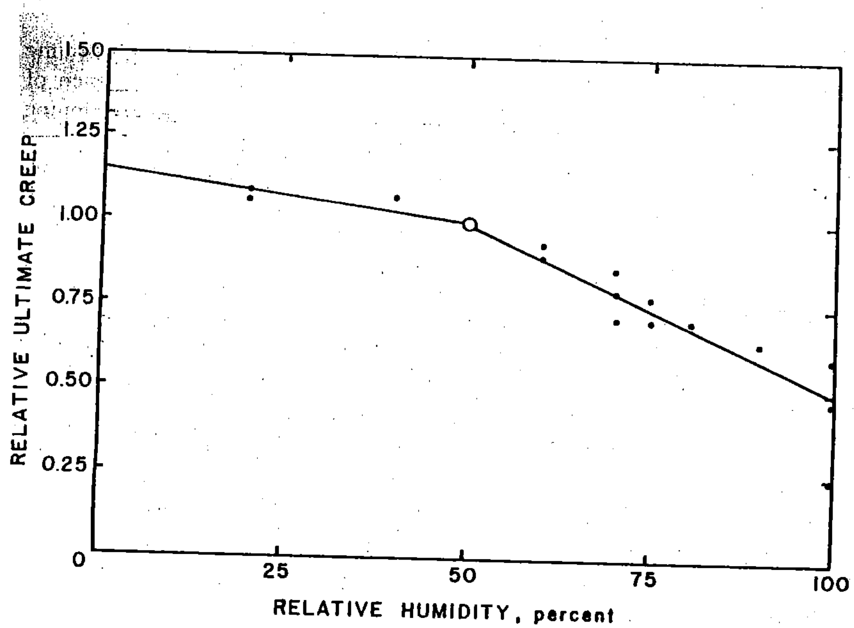

Fig. 2. Relative ultimate creep versus relative humidity.

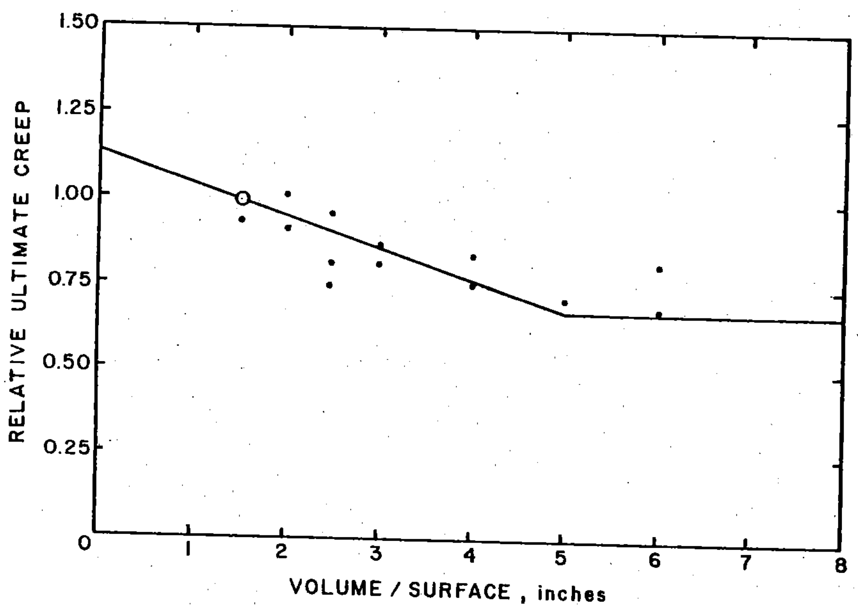

Fig. 3. Relative ultimate creep versus volume to surface ratio.

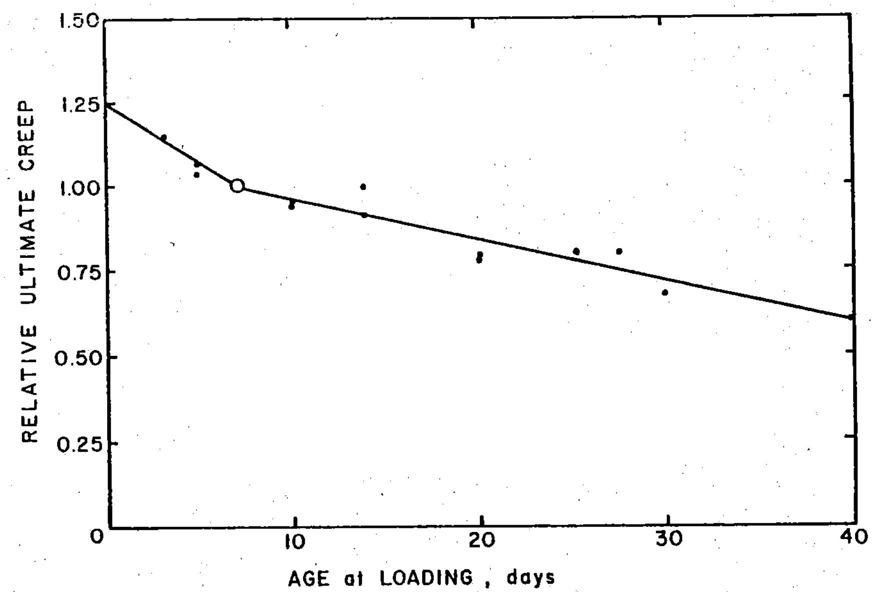

Fig. 4. Relative ultimate creep versus loading age.

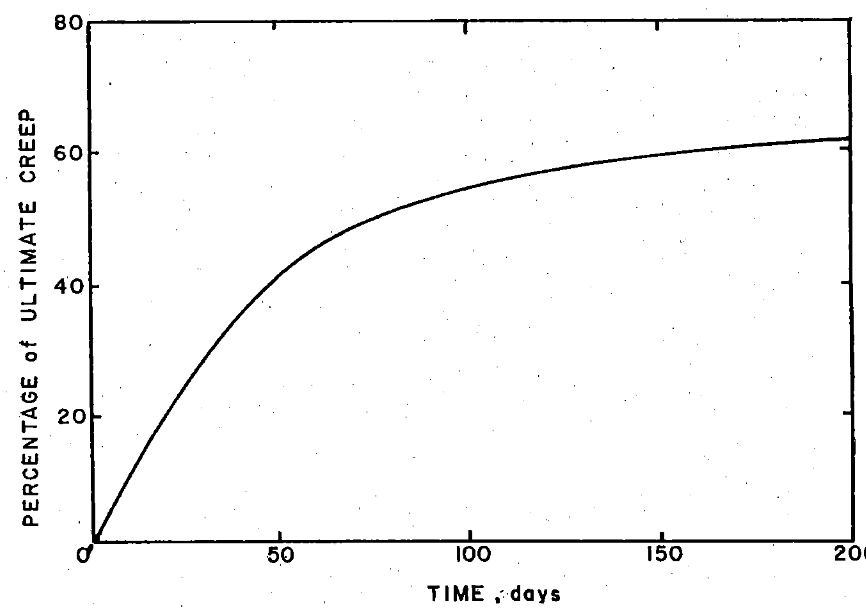

Fig. 5. Percentage of ultimate creep versus time.

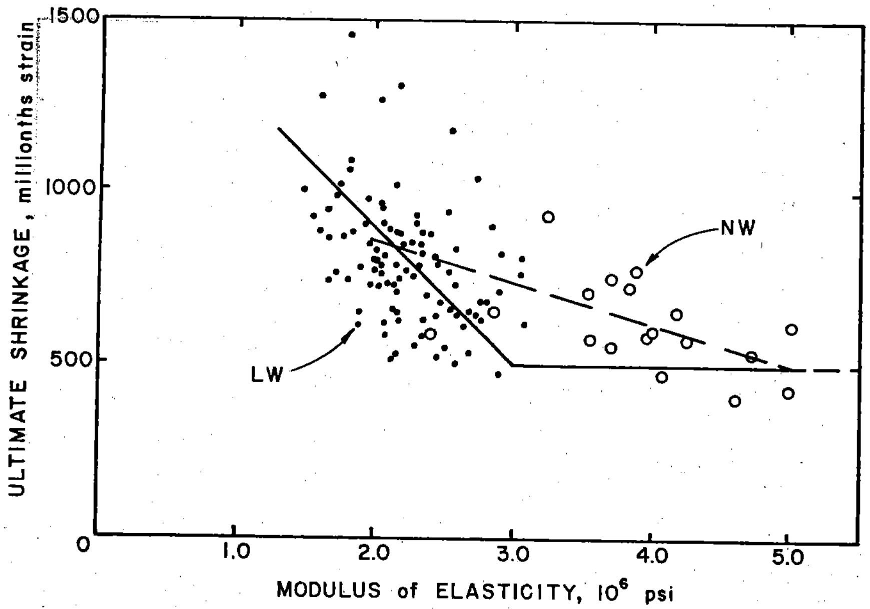

Fig. 6. Ultimate shrinkage versus modulus of elasticity.

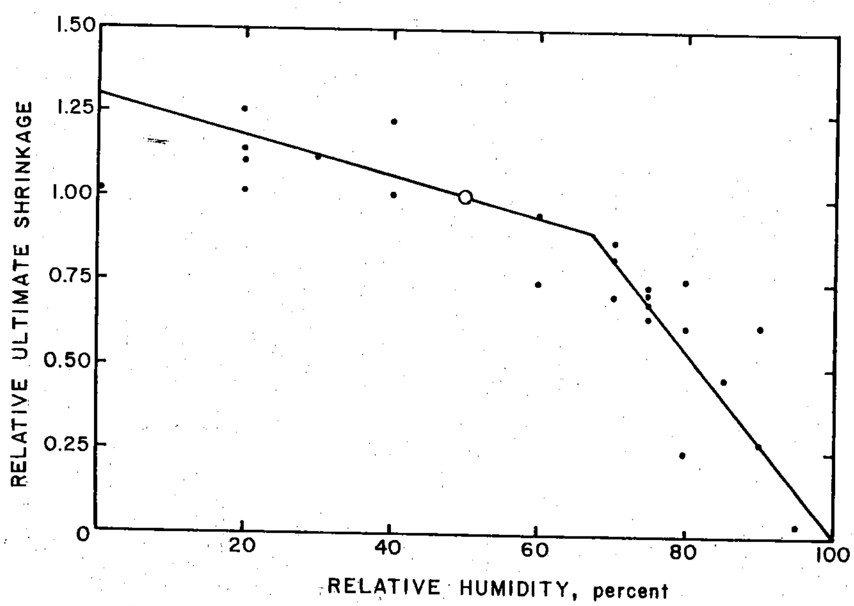

Fig. 7. Relative ultimate shrinkage versus relative humidity.

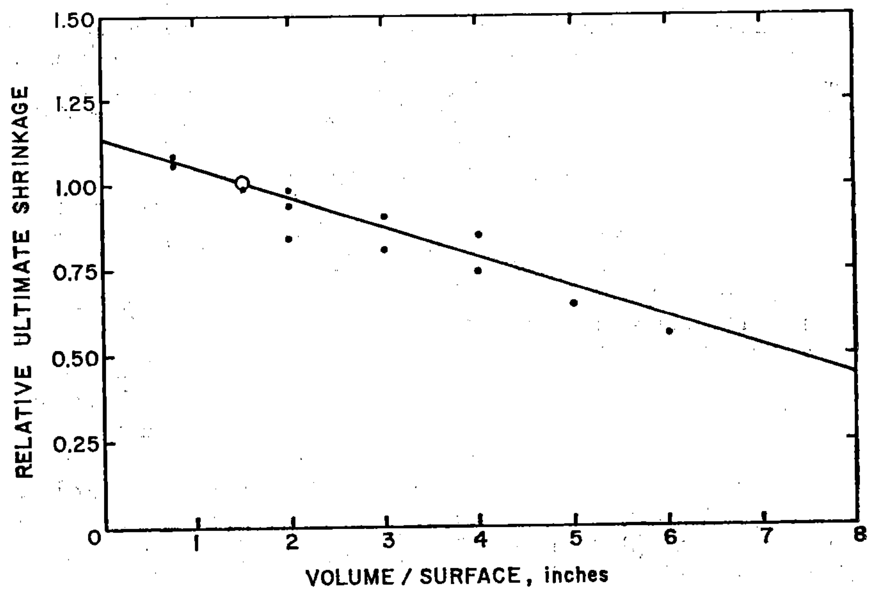

Fig. 8. Relative ultimate shrinkage versus volume to surface ratio.

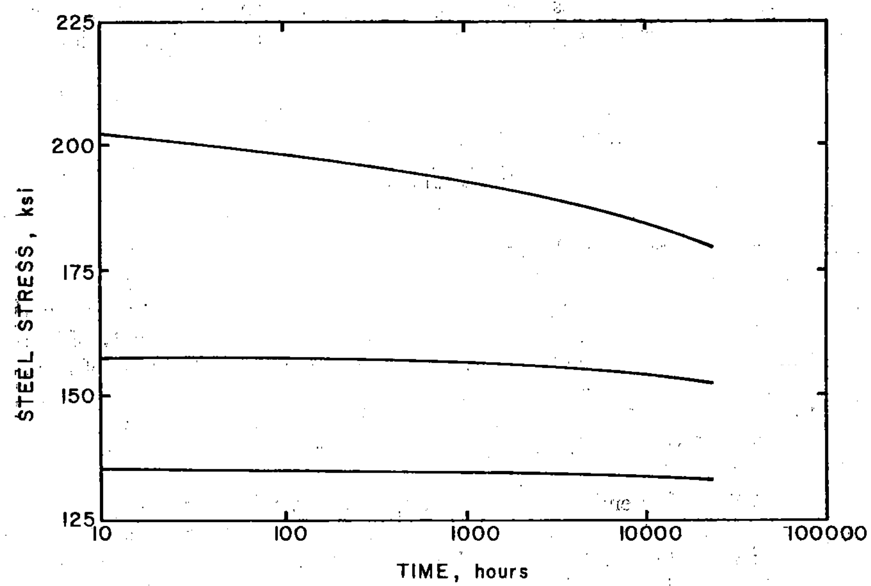

Fig. 9. Prestressing steel stress versus time for stress-relieved steel.

2.6—Comments on loss due to shrinkage are similar to those for creep, except that concrete stress is not a factor. Ultimate shrinkage strain from standard tests is shown in Fig. 6 (References 1, 2, 14, 15, 21, 24). These data were modified to 70 percent relative humidity using information illustrated in Fig. 7 (References 3, 7, 8, 11, 17), and multiplied by the steel modulus of elasticity to obtain Eqs. (13) and (14). *USH* is the loss in steel stress due to shrinkage shortening.

USH is influenced by the size and shape of the prestressed member. Shrinkage volume/surface data are presented in Fig. 8 (References 7, 11 17, 19). The size factor *SSF* is given in Table 6. By multiplying *USH* by *SSF*, standard test data can be applied to actual prestressed members. The variation of shrinkage with time was generalized using data in References 2, 15, and 24. This information is given in Table 7 as the portion of ultimate shrinkage *AUS* for a specific time after the end of curing.

The amount of shrinkage *PSH* occurring over a specific time interval is the difference between the amount of shrinkage at the beginning of the time interval and that at the end of the time interval.

The loss of prestress over one time interval due to shrinkage of concrete is stated in Eq. (12).

2.7—Eqs. (16) and (17) (References 13, 32) give the loss of prestress due steel relaxation. The time t_1 in Step 1, listed in Section 2.4, is at anchorage of the prestressing steel. In pretensioned construction, where elevated temperatures are used in curing, losses during curing can be studied more closely using References 5, 22, and 26.

Fig. 9 shows typical steel relaxation with time under constant strain. It is seen that losses are less for lower initial stresses. This illustrates that by taking into account concrete shortening and steel relaxation over a previous time period, subsequent losses are less than that under assumed constant strain.

NOTATION

A_c = gross cross-sectional area of concrete member, sq in.

A_s = cross-sectional area of prestressing tendons, sq in.

ANC = loss of prestress due to anchorage of prestressing steel, psi

AUC = portion of ultimate creep at time after prestress transfer

AUS = portion of ultimate shrinkage at time after end of curing

CR = loss of prestress due to creep of concrete over time interval t_1 to t, psi

DEF = loss of prestress due to deflecting device in pretensioned construction, psi

e = tendon eccentricity measured from center of gravity of concrete section to center of gravity of tendons, in.

E_c = modulus of elasticity of concrete at 28 days taken as $33w^{3/2}\sqrt{f_c'}$, psi

E_{ci} = modulus of elasticity of concrete at time of initial prestress, psi

E_s = modulus of elasticity of steel, psi

ES = loss of prestress due to elastic shortening, psi

f_{cds} = concrete compressive stress at center of gravity of prestressing force due to all permanent (dead) loads not used in computing f_{cr}, psi

f_c = concrete compressive stress at center of gravity of prestressing steel, psi

f_c' = compressive strength of concrete at 28 days, psi

f_{ci}' = initial concrete compressive strength at transfer, psi

f_{cr} = concrete stress at center of gravity of prestressing force immediately after transfer, psi

f_{pu} = guaranteed ultimate tensile strength of prestressing steel, psi

f_{py} = stress at 1 percent elongation of prestressing steel, psi

f_{se} = effective stress in prestressing steel under dead load after losses

f_{si} = stress in tendon at critical location immediately after prestressing force has been applied to concrete

f_{st} = stress in prestressing steel at time t_1, psi

f_t = stress at which tendons are anchored in pretensioning bed, psi

FR = friction loss at section under consideration, psi

I_c = moment of inertia of gross cross section of concrete member, in.[4]

K = friction wobble coefficient per foot of prestressing steel

l_{tx} = length of prestressing steel from jacking end to point x, ft

MCF = factor that accounts for the effect of age at prestress and length of moist cure on creep of concrete

M_{ds} = moment due to dead weight added after member is prestressed

M' = moment due to loads, including weight of member, at time prestress is applied to concrete

P = final prestress force in member after losses

P_o = initial prestress force in member

PCR = amount of creep over time interval t_1 to t

PSH = amount of shrinkage over time interval t_1 to t

RE = total loss of prestress due to relaxation of prestressing steel in pretensioned construction, psi

REP = total loss of prestress due to relaxation of prestressing steel in post-tensioned construction, psi

RET = loss of prestress due to steel relaxation over time interval t_1 to t, psi

SCF = factor that accounts for the effect of size and shape of a member on creep of concrete

SH = loss of prestress due to shrinkage of concrete over time interval t_1 to t, psi

SSF = factor that accounts for the effect of size and shape of a member on concrete shrinkage

t = time at end of time interval, days

t_1 = time at beginning of time interval, days

T_o = steel stress at jacking end of post-tensioning tendon, psi

T_x = steel stress at any point x, psi

TL = total prestress loss, psi

UCR = ultimate loss of prestress due to creep of concrete, psi per psi of compressive stress in the concrete

USH = ultimate loss of prestress due to shrinkage of concrete, psi

w = weight of concrete, lb per cu ft

α = total angular change of post-tensioning tendon profile from jacking end to point x, radians

μ = friction curvature coefficient

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DESIGN EXAMPLES

The following three design examples were prepared solely to illustrate the application of the preceding recommended methods. They do not necessarily represent the real condition of any real structure.

Design aids to assist in calculating prestress losses are included in the *PCI Design Handbook* (see Reference 35). The aids will reduce the calculations required. However, detailed study of losses and time-dependent behavior will follow the steps outlined in the design examples.

The first example applies the general method to a pretensioned double-tee and the second example uses the simplified method for the same member. The third example problem illustrates the general method for a post-tensioned structure.

In these examples it is assumed that the member geometry, load conditions, and other parameters have been defined. Consequently, the detailed moment and stress calculations are omitted.

DESIGN EXAMPLE 1

Pretensioned Double Tee

Reference: *PCI Design Handbook*, p. 3-33.

Data: Double-tee section 10LDT32 + 2. Strand pattern 128-D1.

Steam cured, ligthweight double-tee (115 lb per cu ft) with 2-in. topping of normal weight concrete (150 lb per cu ft).

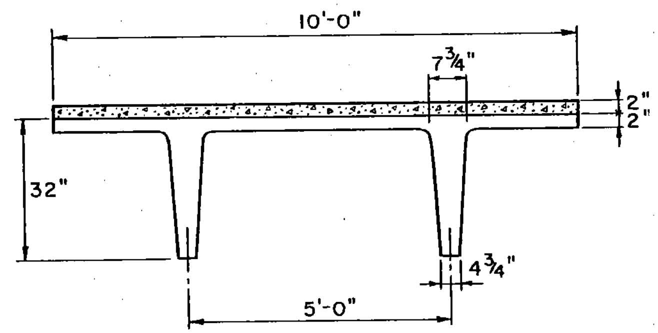
The beam is designed to carry a live load of 40 psf over a 70-ft span.

Required: Calculate the losses at the critical section, taken as 0.4 span in the *PCI Design Handbook*. $f'_{ci} = 3500$ psi, $f_c' = 5000$ psi

Section properties:

Non-composite

$A = 615$ in.^{2}
$I = 59{,}720$ in.^{4}
$y_b = 21.98$ in.
$y_t = 10.02$ in.
$Z_b = 2717$ in.^{3}
$Z_t = 5960$ in.^{3}
Weight: 491 lb per ft



Design Example 1. Cross section of double-tee beam.

Composite

$I = 83{,}001$ in.^{4}
$y_{bc} = 25.40$ in.
$y_{tc} = 8.60$ in.
$Z_b = 3268$ in.^{3}
$Z_t = 9651$ in.^{3}
Weight: 741 lb per ft

The beam is prestressed by twelve ½-in. diameter 270-grade strands, initially tensioned to 0.70 f_{pu}.

Eccentricity of strands:

At ends = 12.98 in.
At center = 18.73 in.
$f_{py} = 230$ ksi

Transfer at 18 hours after tensioning strand, topping cast at age 30 days.

Losses—Basic data

$f'_{ci} = 3500$ psi
$E_{ci} = 115^{1.5}(33\sqrt{3500})$
$= 2.41 \times 10^6$ psi
$f_c' = 5000$ psi
$E_c = 115^{1.5}(33\sqrt{5000})$
$= 2.88 \times 10^6$ psi

Volume to surface ratio = 615/364
= 1.69

$SSF = 0.985$
$SCF = 0.988$

$UCR = 63 - 20(E_c/10^6)$
(but not less than 11)
$= 63 - 20(2.88) = 11$

$USH = 41{,}000 - 10{,}000(E_c/10^6)$
(but not less than 12,000)
$= 41{,}000 - 10{,}000(2.88)$
$= 12{,}200$ psi

At critical section
$e = 12.98 + 0.8(18.73 - 12.92)$
$= 17.58$ in.

$(UCR)(SCF) = 10.87$

$(USH)(SSF) = 12{,}017$ psi

Stage 1: Tensioning of steel to transfer

$t_1 = 1/24$ day
$t = 18/24$ day
$f_{st} = 189{,}000$ psi

$RET = f_{st}[(\log 24t - \log 24t_1)/10] \times [f_{st}/f_{py} - 0.55]$
$= 189{,}000[(\log 18)/10][189/230 - 0.55]$
$= 6450$ psi

Dead load moment at 0.4 span
$M_{DL} = w(x/2)(L - x)$
$= (491/1000)(28/2)(70 - 28)$
$= 289$ ft-kips

Stress at center of gravity of steel due to M_{DL}

$f_c = [289{,}000(12)/59{,}720]\,17.58$
$= 1020$ psi (tension)

Assume $ES \approx 13$ ksi, then
$f_{si} = 189.0 - 6.45 - 13.0$
$= 169.55$ ksi
$P_o = 169.55(12)(0.153)$
$= 311.3$ kips

Stress at center of gravity of steel due to P_o:
$f_e = 311{,}300/615 +$
$311{,}300\,(17.58^2/59{,}720$
$= 2117$ psi (compression)
$f_{cr} = 2117 - 1020 = 1097$ psi
$ES = f_{cr}(E_s/E_c)$
$= 1097(28.0/2.41)$
$= 12{,}750$ psi ≈ 13 ksi (ok)
$SH = CR = 0$

Total losses in Stage 1 =
$6450 + 12{,}750 = 19{,}200$ psi

Stage 2: Transfer to placement of topping after 30 days

$t_1 = 18/24$ day
$t = 30$ days
$PCR = 0.35$
$PSH = 0.42$
$f_{st} = 189{,}000 - 19{,}200$
$= 169{,}800$ psi
$RET = 169{,}800\ [(\log 720 - \log 18)/10] \times [169.8/230 - 0.55]$
$= 5119$ psi

$CR = 10.87(0.35)(1097) = 4173$ psi

$SH = 12{,}017(0.42) = 5047$ psi

Total losses in Stage 2
$5119 + 4173 + 5047 = 14{,}339$ psi

Moment due to weight of topping
$250(28/2)(70 - 28) = 147{,}000$ ft-lb

Stress at center of gravity of steel due to weight of topping
$147{,}000(12)(17.58)/59{,}720 = 519$ psi

Increase in strand stress due to topping
$519(28.0/2.88) = 5048$ psi

Strand stress at end of Stage 2
$169{,}800 - 14{,}339 + 5048 = 160{,}509$ psi

Stage 3: Topping placement to end of one year

$t_1 = 30$ days
$t = 1$ year $= 365$ days
$PCR = 0.74 - 0.35 = 0.39$
$PSH = 0.86 - 0.42 = 0.44$
$f_{st} = 160{,}509$ psi

$RET = 160{,}509\,[(\log 8760 - \log 720/\log 720/10] \times [160.5/230 - 0.55]$
$= 2577$ psi

$f_c = 2117(160{,}509/169{,}550) - 1020 - 519$
$= 465$ psi
$CR = 10.87(0.39)(465) = 1971$ psi
$SH = 12{,}017(0.44) = 5287$ psi

Total losses in Stage 3
$2577 + 1971 + 5287 = 9835$ psi

Summary of steel stresses at various stages (Design Example 1)

Stress level at various stages	Steel stress, ksi	Percent
Strand stress after tensioning and deflection ($0.70f_{pu}$) . .	189.0	100.0
Losses:		
Elastic shortening = 12.75		6.7
Relaxation: 6.45+ 5.12 + 2.58 + 2.54 = 16.69		8.8
Creep: 4.17 + 1.97 + 0.97 = 7.11		3.8
Shrinkage: 5.05 + 5.29 + 1.68 = 12.02		6.4
Total losses, TL	48.57	25.7
Increase of stress due to topping	5.05	2.7
Final strand stress under total dead load (f_{se})	145.48	77.0

Stage 4: One year to end of service life

$t_1 = 1$ year
$t =$ end of service life (say 40 years)
$PCR = 1 - 0.74 = 0.26$
$PSH = 1 - 0.86 = 0.14$
$f_{st} = 160{,}509 - 9835 = 150{,}674$ psi
$RET = 150{,}674\,[(\log 350{,}400 - \log 8760)/10] \times [(150.7/230) - 0.55]$
$= 2537$ psi
$f_c = 2117(150{,}674/169{,}550) - 1020 - 519$
$= 343$ psi
$CR = 10.87(0.26)(343) = 969$ psi
$SH = 12{,}017(0.14) = 1682$ psi

Total losses in Stage 4 =
$2537 + 969 + 1682 = 5188$ psi

DESIGN EXAMPLE 2

Application of Simplified Procedure to Design Example 1

Compute f_{cds}

$f_{cds} = eM_{ds}/I$
$= 17.58(147)(12)/59{,}720$
$= 0.519$ ksi

Compute f_{cr}

$f_{cr} = A_s f_{si}/A_c + A_s f_{si} e^2/I_c + M'\,e/I_c$
$f_{si} = 0.90 f_t = 0.90(189) = 170.1$ ksi
$f_{cr} = 1.84(170.1)/615 + 1.84(170.1)(17.58)^2/59{,}720 - 289(12)(17.58)/59{,}720$
$= 0.509 + 1.620 - 1.021$
$= 1.108$ ksi

Equation L-SR-PRE-70 from Table 8 is
$TL = 31.2 + 16.8 f_{cr} - 3.8 f_{cds}$
$= 31.2 + 16.8(1.108) - 3.8(0.519)$
$= 31.2 + 18.61 - 1.97$
$= 47.84$ ksi

Adjustment for volume to surface ratio $= 1.69$

Use a straight-line interpolation between adjustment values for $V/S = 2.0$ and $V/S = 1.0$

Adjustment $= (0.31)(3.2) = +0.99\%$

Net $TL = 1.0099(47.84) = 48.31$ ksi
In Design Example 1, $TL = 48.57$ ksi
Difference $= 0.26$ ksi

Compute f_{se}

To find f_{se} in accordance with discussion under Section 3.32, and stress in tendons due to dead load applied after member was prestressed.

This stress is equal to
$f_{cds}(E_s/E_c) = 0.519(28/2.88) = 5.05$ ksi
$f_{se} = 189 - 48.31 + 5.05 = 145.74$ ksi

Note that f_{se} can also be computed from the equations shown in Table 9.

Equation L-SR-PRE-70 from Table 9 is
$f_{se} = f_t - (31.2 + 16.8 f_{cr} - 13.5 f_{cds})$
$= 189 - (31.2 + 16.8 \times 1.108 - 13.5 \times 0.519)$
$= 189 - (31.2 + 18.61 - 7.01)$
$= 189 - (42.8)$

An adjustment for variations in the basic parameters should be applied to the quantity in parentheses. In this case, adjust for a V/S of 1.69. The adjustment is +0.99 percent. The adjusted quantity becomes
$1.0099(42.8) = 43.22$
$f_{se} = 189 - 43.22 = 145.78$ ksi

Checking the assumed value of f_{si}:

In the application of the simplified method to Design Example 1, the value of f_{si} was assumed to be 170.1 ksi.

The following procedure can be used to check the accuracy of this assumed value.

For this example the exact value of f_{si} is the initial stress of 189 ksi reduced by strand relaxation from tensioning to release and by loss due to elastic shortening of the concrete as the prestressing force is applied.

From Section 2.7.1, the relaxation loss in a stress-relieved strand is

$$RET = f_{st}[(\log 24t - \log 24t_1)/10] \times [f_{st}/f_{py} - 0.55]$$

For stress-relieved strand
$f_{py} = 0.85(270) = 229.5$ ksi

By definition in Section 2.7.1, when time is measured from zero, $\log 24t_1 = 0$

$$RET = 189[(1.255 - 0)/10][189/229.5 - 0.55] = 6.49 \text{ ksi}$$

Stress loss due to elastic shortening of concrete

$$ES = (E_s/E_c)f_{cr} = (28/0.24)1.108 = 12.93 \text{ ksi}$$

Then, $f_{si} = 189 - 6.49 - 12.93 = 169.58$ ksi
and $0.90\, f_t = 170.10$ ksi

Therefore, there is a $170.10 - 169.58 = 0.52$ ksi stress error in f_{si}.

Consequently, in this particular case there is no need for a second trial.

As an example, assume a large error in the estimated f_{si}, say 10 ksi, and check its effect. The strand relaxation will not change.

Therefore, the change in ES will be
$\Delta ES = (10/170.1)12.93 = 0.76$ ksi

If desired, the original estimate of f_{si} can be adjusted by 10 ksi and f_{cr} can be recalculated. One such cycle should always give an adequate accuracy.

DESIGN EXAMPLE 3

Post-Tensioned Unbonded Slabs

The following is a procedure for calculating the prestress losses in the longitudinal tendons which extend from end to end of the slab (see sketch showing floor plan and tendon profiles).

Data

$w = 150$ lb per cu ft
f_c' (28 days) = 4000 psi

Prestressed at age 4 days
$f_c' = 3000$ psi

Moist cured 7 days.

Loads

7½-in. slab = 94 psf
Superimposed load = 60 psf
The tendon profile shown is designed to balance 85 psf.

Friction Loss (FR)

The slab is prestressed by 270-grade, ½-in. diameter strand, pregreased and paper wrapped.

Coefficient of friction, $\mu = 0.08$
Wobble coefficient, $K = 0.0015$
$f_{py} = 230$ ksi.

Angular changes along tendon will be:

$$\theta_{AB} = 2(2.5)/[12(12)] = 0.0347 \text{ radians}$$

$$\theta_{BC} = \theta_{EF} = \theta_{FG} = \theta_{KL} = 2(4.0)/[12(9.6)] = 0.0694 \text{ radians}$$

$$\theta_{CD} = \theta_{DE} = \theta_{GH} = \theta_{HK} = 2(1.0)/[12(2.4)] = 0.0694 \text{ radians}$$

Angular change between A and L

$$\alpha = 0.0347 + 4(0.0694) + 4(0.0694) = 0.59 \text{ radians}$$

FR at L (middle of length of slab)

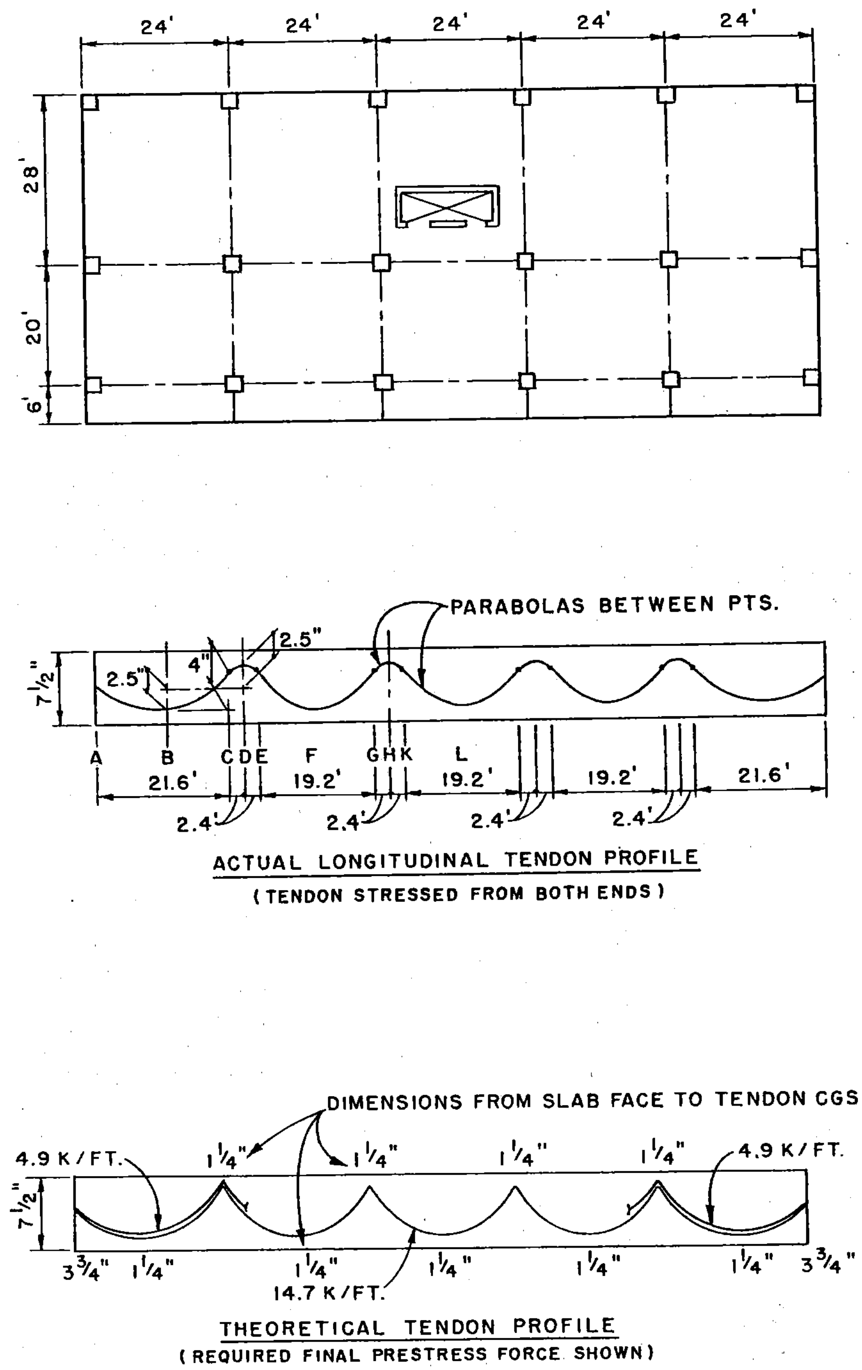
$$= T_o[1\text{-}e^{-(KL+\mu\alpha)}]$$
$$= T_o[1\text{-}e^{-\{(0.0015)(60)+(0.08)(0.59)\}}]$$
$$= T_o[1\text{-}e^{-(0.090+0.047)}]$$
$$= 0.128\, T_o$$

The distribution of frictional loss is not uniform, but nearly proportional to $(K + \mu\alpha/L)$. However, the variation of strand stress before anchoring is approximately as shown on p. 72.

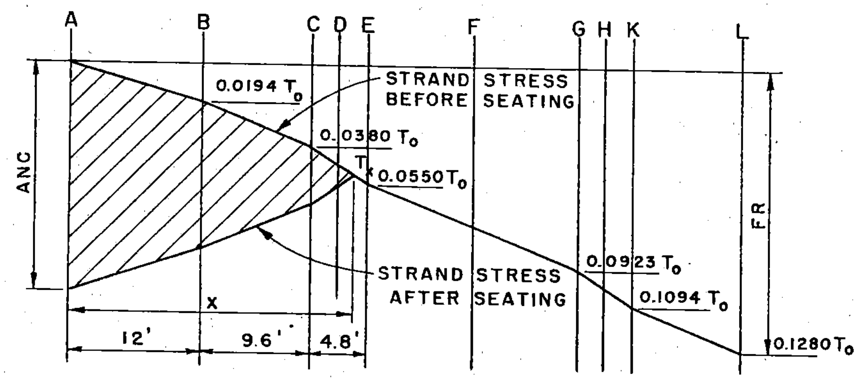
Anchorage loss (ANC)

Anchorage set in a single strand anchor
= 1/8 in.
= Shaded area in diagram × $(1/E_s)$
Area = (1/8)29,000 = 3625 ksi-ft
= 302 ksi-in.

The maximum strand stress after seating of anchorage occurs x ft from end,



Design Example 3. Plan and tendon profiles of post-tensioned unbonded slab.



Approximate variation of strand stress.

and this stress T_x must not exceed $0.70f_{pu} = 189$ ksi.

$$T_o - T_x = 0.0380\ T_o + (0.0550 - 0.0380)T_o(x - 21.6)/4.8$$

$$\text{Area} = (T_o - T_x)12 + (0.9806\ T_o - T_x)21.6 + (0.9620\ T_o - T_x)(x - 12)$$

Therefore

$$T_x = 0.9620T_o = 0.0170T_o(x - 21.6)/4.8 = 189 \text{ ksi}$$

$$(T_o - T_x)12 + (0.9806T_o - T_x)21.6 + (0.9620T_o - T_x)(x - 12) = 302 \text{ ksi-ft}$$

These equations can be solved by trial and error.

Approximate solution:

$x = 25.5$ ft
$T_o = 200$ ksi
$T_x = 192.4 - 2.8 = 189.6 \approx 189$ ksi

$$\text{Area} = 124.8 + 140.8 + 37.8 = 303.4 \approx 302 \text{ ksi-ft (ok)}$$

For initial end tension before anchorage
$T_o = 200$ ksi ($\approx 0.74\ f_{pu}$)

Maximum stress after anchorage
$T_x = 189.6$ ksi

$ANC = 2(T_o - T_x) = 20.8$ ksi
$T_A = 200 - 20.8 = 179.2$ ksi
$FR = 0.128\ T_o = 25.6$ ksi
$T_L = 200 - 25.6 = 174.4$ ksi

Average stress after anchorage:
$T_B = 183.2$ ksi, $T_c = 186.9$ ksi

$$T_{av} = (T_o/60)(0.5)[(0.896)(12) + (0.916)(21.6) + (0.935)(13.5) + (0.948)(4.8) + (0.945)(20.1) + (0.908)(24.0) + (0.891)(14.4) + (0.872)(9.6)] = 182.8 \text{ ksi}$$

Elastic shortening loss (ES)

In post-tensioned structural members, the loss caused by elastic shortening of concrete is only a fraction of the corresponding value in pretensioned members.

The fraction varies from zero if all tendons are tensioned simultaneously to 0.5 if infinitely many sequential steps are used.

In a slab, strands are spaced far apart and it is unlikely that the stretching of one strand will affect stresses in strands other than those immediately neighboring.

A factor of 0.25 will be used.

$$ES = 0.25(E_s - E_{ci})f_{cr}$$

In a design such as this in which the prestress approximately balances the dead load, and the level of prestress is low, a sufficiently close estimate of f_{cr} can be obtained by using the average prestress P/A.

The design final prestress force is 14.7 kips per ft for interior spans and 19.6 kips per ft for end spans. Assuming a long-term prestress loss of 15 percent, the initial prestress force will be 17.3 kips per ft and 23.1 kips per ft, respectively. The average strand stress after anchorage is 182.8 ksi.

Therefore, the required area of steel for the end spans is

$$A_s = 23.1/182.8 = 0.126 \text{ sq in. per ft}$$

This required area is supplied by ½-in. diameter strands spaced at 14 in.

$$A_s = 0.131 \text{ sq in. per ft}$$

Every fourth strand will be terminated in the first interior span, leaving an A_s of 0.098 sq in. per ft.

The actual initial prestressing forces are:

End span
$0.131(182.8) = 23.9$ kips per ft

Interior span
$0.098(182.8) = 17.9$ kips per ft

The average concrete stresses are 266 and 199 psi, respectively.

$$f_{cr} \approx (1/120)[(266)(48) + (199)(72)] = 226 \text{ psi}$$

$$E_{ci} = 33w^{1.5}\sqrt{f_{ci}'} = 33(150)^{1.5}\sqrt{3000} = 3.32 \times 10^6 \text{ psi}$$

$$ES = 0.25(226)(29/3.32) = 494 \text{ psi}$$

After all the strands have been tensioned and anchored:

$T_{av} = 182.8 - 0.494 = 182.3$ ksi

At midspan of the middle span
Strand stress (at L)
$174.4 - 0.494 \approx 173.9$ ksi

$$f_{cr} \approx [(173.9)(0.098)]/[(12)(7.5)] \approx 0.189 \text{ ksi}$$

Long-term losses

The calculation for long-term losses will be for the midspan of the middle span (at Section L).

Stage 1: To 30 days after prestressing

Relaxation:

$t_1 = 1/24$ day
$t = 30$ days
$f_{st} = 173.9$ ksi
$f_{st}/f_{py} = 0.756$

$$RET = f_{st}[(\log 24t - \log 24t_1)/10] \times [(f_{st}/f_{py}) - 0.55] = 173{,}900(0.2857)(0.206) = 10{,}230 \text{ psi}$$

Creep:

$CR = (UCR)(SCF)(MCF)(PCR)(f_c)$
$UCR = 95 - (20E_c/10^6)$
but not less than 11 psi
$E_c = 33(150)^{1.5}\sqrt{4000} = 3.83 \times 10^6$ psi
$UCR = 95 - 76.6 = 18.4$ psi

V/S ratio $= 0.5$(slab thickness) $= 0.5(7.5) = 3.75$ in.

$SCF = 0.80$
$MCF = 1.07$ (estimated)
$(UCR)(SCF)(MCF) = 15.75$
$f_c = f_{cr} = 189$ psi
$PCR = 0.35$
$CR = 15.75(0.35)(189) = 1042$ psi

Shrinkage:

$SH = (USH)(SSF)(PSH)$
$USH = 27{,}000 - (3000\ E_c/10^6)$
but not less than 12,000

$USH = 27{,}000 - 11{,}490 = 15{,}510$ psi
$V/S = 3.75$ in.
$SSF = 0.79$
$(USH)(SSF) = 12{,}270$ psi
Time after end of curing = 27 days
$PSH = 0.402$
$SH = 12{,}270(0.402) = 4933$ psi

Total losses in Stage 1:

$$RET + CR + SH = 10{,}230 + 1042 + 4933 = 16{,}205 \text{ psi}$$

Tendon stress at end of Stage 1

$$173{,}900 - 16{,}205 = 157{,}695 \text{ psi}$$

Concrete fiber stress

$$189(157{,}695/173{,}900) = 171.4 \text{ psi}$$

Stage 2: To 1 year after prestressing

Relaxation:

$t_1 = 30$ days
$t = 1$ year $= 365$ days
$f_{st} = 157{,}695$ psi
$f_{st}/f_{py} = 157.7/230 = 0.671$

$$RET = 157{,}695\,[(\log 8760\text{-}\log 30)/10] \times [(0.671 - 0.55)] = 2070 \text{ psi}$$

Creep:

$f_c = 171.4$ psi
$PCR = 0.74 - 0.35 = 0.39$
$CR = 15.75(0.39)(171.4) = 1053$ psi

Shrinkage:

$PSH = 0.86 - 0.402 = 0.458$
$SH = 12{,}270(0.458) = 5620$ psi

Total losses in Stage 2:

$$2070 + 1053 + 5620 = 8743 \text{ psi}$$

At end of Stage 2, tendon stress at Section L

$$157{,}695 - 8743 = 148{,}952 \text{ psi}$$

Concrete fiber stress

$$189(148{,}952/173{,}900) = 161.9 \text{ psi}$$

Stage 3: To end of service life (taken as 50 years)

Relaxation:

$t_1 = 1$ year $= 365$ days
$t = 50$ years $= 18{,}250$ days
$\log 24t - \log 24t_1 = 1.699$
$f_{st} = 148{,}952$ psi
$f_{st}/f_{py} = 0.634$

$$RET = 148{,}952(0.1699)(0.084) = 2126 \text{ psi}$$

Creep:

$f_c = 161.9$ psi
$PCR = 1 - 0.74 = 0.26$
$CR = 15.75(0.26)(161.9) = 663$ psi

Shrinkage:

$PSH = 1 - 0.86 = 0.14$
$SH = 12{,}270(0.14) = 1718$ psi

Summary of steel stresses at various stages (Design Example 3)

Stress level at various stages		Steel stress, ksi	Percent
Tensioning stress at end		200	
Average stress after seating		182.8	
Middle section stress after seating		174.4	100.0
Losses:			
Elastic shortening	0.5		0.3
Relaxation	14.4		8.2
Creep	2.7		1.6
Shrinkage	12.3		7.0
Total losses after seating		29.9	17.1
Final strand stress at middle section without superimposed load		144.5	82.9

Total long-term losses:

$RET = 10{,}230 + 2070 + 2126 = 14{,}426$ psi
$CR = 1042 + 1053 + 663 = 2{,}758$ psi
$SH = 4933 + 5620 + 1718 = 12{,}271$ psi
Total losses $= 29{,}355$ psi

This example shows the detailed steps in arriving at total losses. It is not implied that this effort or precision is required in all design situations.

The *PCI Post-Tensioning Manual* provides a table for approximate prestress loss values which is satisfactory for most design solutions. The value recommended for slabs with stress-relieved 270-kip strand is 30,000 psi which compares with the calculated value of 29,355 psi.

Final tendon stress at Section L

$$173.9 = 29.4 = 144.5 \text{ ksi}$$

Percentage loss after anchorage

$(174.4 - 144.5)/174.4 = 17.1$ percent but greater than 15 percent (assumed initially)

Assuming the same percentage loss prevails over the entire tendon length, the average prestressing forces after losses are 19.8 and 14.8 kips per ft, respectively, which are adequate when compared with the design requirements. Therefore, revision is not needed.

Discussion of this report is invited. Please forward your discussion to PCI Headquarters by December 1, 1975.