Strength of Prestressed Concrete Members at Sections Where Strands Are Not Fully Developed

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In some precast, prestressed concrete flexural members, prestressing strands may not be completely developed at sections of high moment. This situation can be particularly critical when some of the strands are debonded near the ends of the member to reduce release stresses. Unfortunately, current design procedures overestimate the strength of the member at some sections and there have been instances in which failures under test or overload conditions have been observed. In this paper, a rational method for designing such members using strain compatibility is proposed and illustrated with examples. Code changes are also suggested.

In short span prestressed concrete flexural members, prestressing strands may not be developed at sections of high moment. In such cases, it is possible a premature failure may occur in the concrete due to strand slip. Further, it has become common practice in the precast concrete industry to debond prestressing strands at the ends of members in order to reduce stresses at release of prestress. When only a portion of the strands are debonded, zones are created where sections through the member will contain strands with unequal strains.

The effect of this strain differential on the ultimate moment capacity of a prestressed concrete member can be significant in terms of magnitude and mode of failure. Design engineers need to be aware of the behavior changes that occur when strands are debonded and account for differential strains in member design.

Section 12.9 of ACI 318-89[1] specifies the minimum development length for prestressing strand in pretensioned members.* This equation is:

$$l_d = (f_{ps} - 2f_{se}/3)d_b$$

where

l_d = development length, in.

f_{ps} = stress in the strand at nominal strength, ksi

f_{se} = effective stress in the strand after losses, ksi

d_b = nominal diameter of the strand, in.

* A considerable amount of evidence suggests that the present code provisions are unconservative. However, they are not scheduled for revision in ACI 318-95. The principles expressed in this paper are not affected by use of different equations for development length.

The Commentary to the ACI Code contains the diagram shown in Fig. 1.

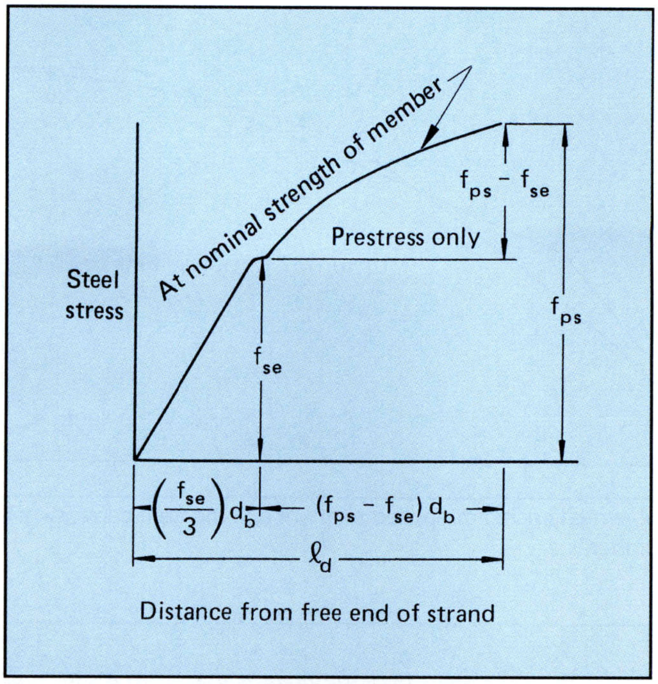


Fig. 1. Variation of steel stress with distance from free end of strand (Ref. 1).

PARTIALLY DEVELOPED STRAND

The ACI Code does not give guidelines for determining the flexural capacity at sections where strand is not fully developed. In fact, Section 12.9.2 states:

> "Investigation may be limited to cross sections nearest each end of the member that are required to develop full design strength under specified factored loads."

This statement seems to indicate that it is not necessary to check member capacity at regions nearer the end where moments are less than maximum, although the meaning of this paragraph is not entirely clear.

The diagram in Fig. 1 shows that strands will develop partial strength with partial development. It has been common practice to use these partial developments in design. The *PCI Design Handbook*[2] illustrates a method where the curve of Fig. 1 has been simplified to the bi-linear curve of Fig. 2. Example 4.2.11 in the *PCI Design Handbook* illustrates the procedure.

It is clearly a mistake not to check the capacity of a section at frequent intervals along the development length

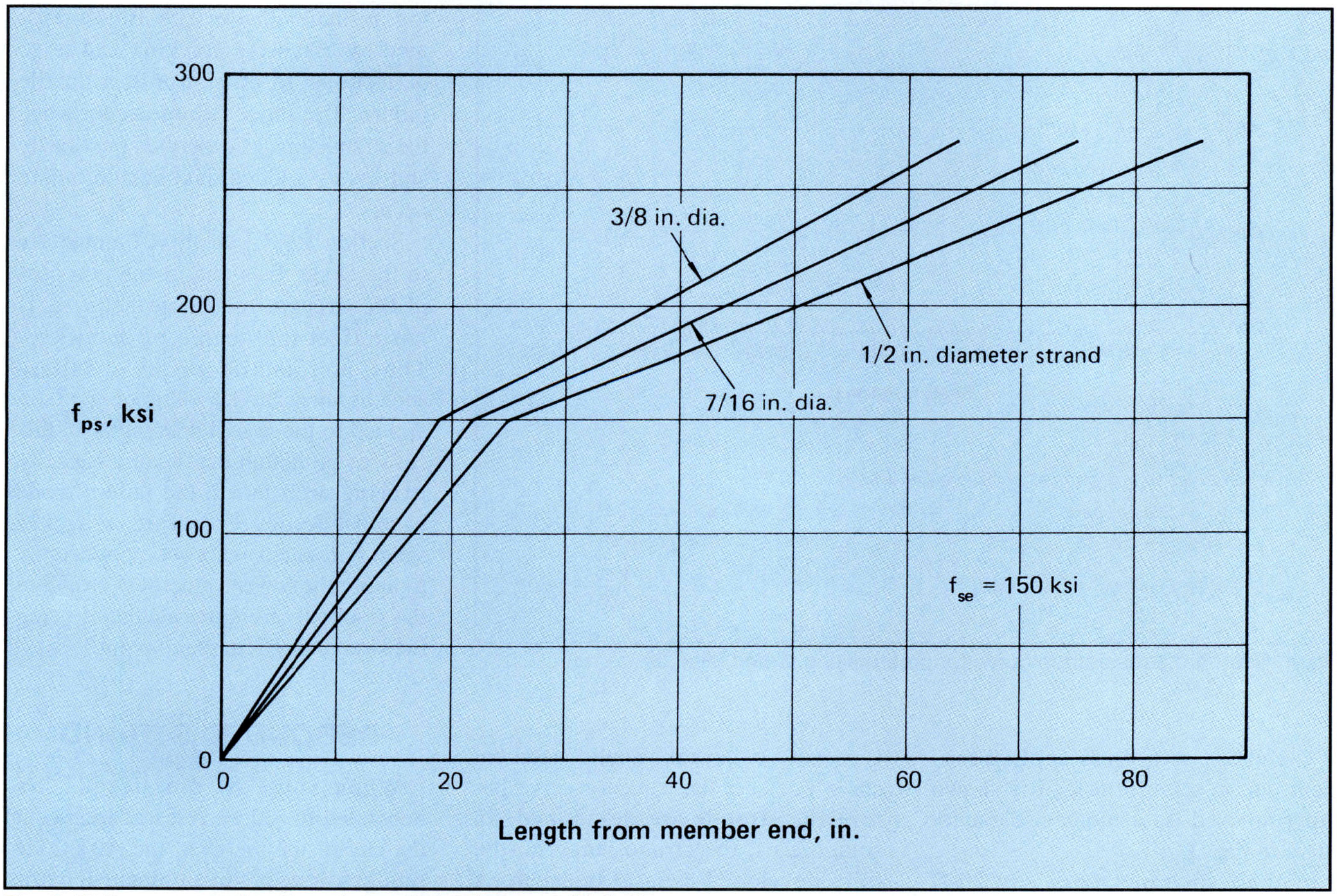


Fig. 2. Bi-linear approximation of strand development (Ref. 2).

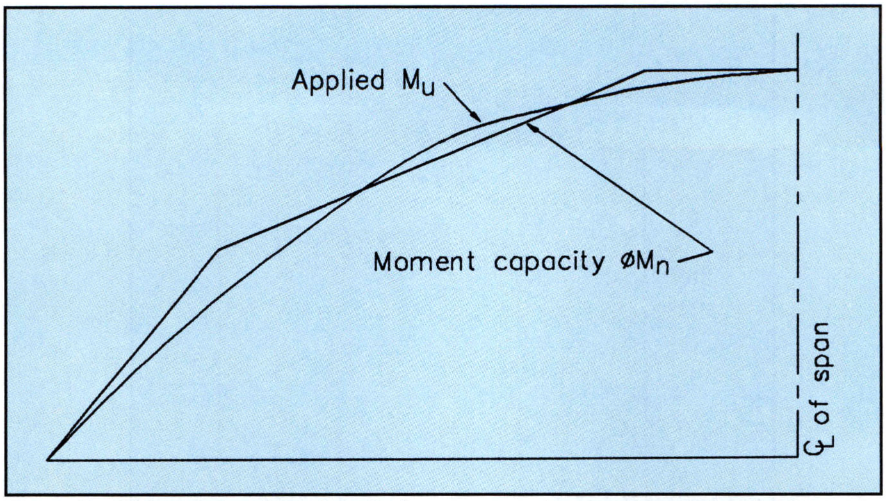
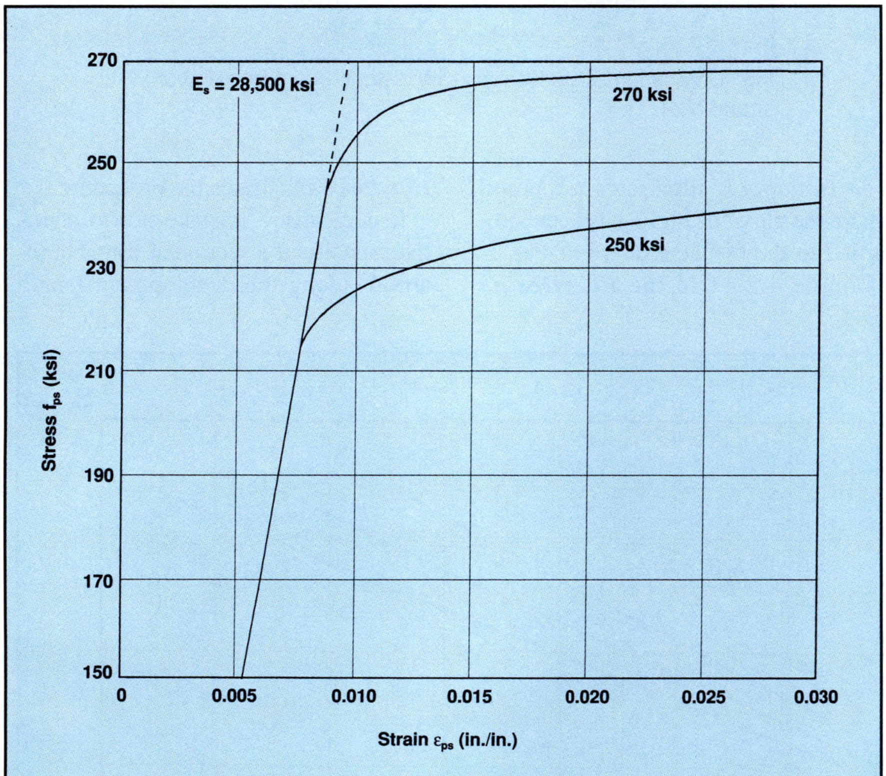


Fig. 3. Possible relationship of applied moment and moment capacity of a flexural concrete member.



These curves can be approximated by the following equations:

250 ksi strand

$\varepsilon_{ps} \le 0.0076$: $f_{ps} = 28{,}500\ \varepsilon_{ps}$ (ksi)

$\varepsilon_{ps} > 0.0076$: $f_{ps} = 250 - \dfrac{0.25}{\varepsilon_{ps}}$ (ksi)

270 ksi strand

$\varepsilon_{ps} \le 0.0086$: $f_{ps} = 28{,}500\ \varepsilon_{ps}$ (ksi)

$\varepsilon_{ps} > 0.0086$: $f_{ps} = 270 - \dfrac{0.04}{\varepsilon_{ps} - 0.007}$ (ksi)

Fig. 4. Standard stress-strain curve for prestressing strand (Ref. 2).

of members. A typical applied moment due to a uniform load is shown superimposed on a moment capacity curve in Fig. 3.

In short span members, say 20 ft (6 m) or less, with straight strands, it is possible to have the applied moment curve intersect the capacity curve before the strands are developed. In some cases, the strands may not be fully developed even at midspan. A number of failures have been observed in such members under overload or test conditions at values of 85 to 90 percent of calculated nominal strength.

BOND FAILURE MECHANISM

A bond failure starts as a flexural failure. If the strands cannot develop the required stress, then they slip. If all of the strands slip, and there is no supplemental reinforcement, the section is essentially unreinforced along the slipped length. Thus, at points where the modulus of rupture of the concrete is exceeded, a vertical flexural crack appears. This crack extends upward, reducing the effective depth and, hence, the area, $b_w d$, that can resist shear. When the effective shear strength is reduced until it is less than the shear stress, a classic diagonal tension crack develops and the member fails in a very sudden and brittle mode.

Thus, it is apparent that there are two possible flexural failure end points. The first occurs when the strand yields enough so that excessive compression occurs in the top of the member, resulting in final collapse. This is accompanied by extensive cracking and large deflections; in other words, a ductile failure. The other failure occurs when the strand slips as described previously and a very sudden, non-ductile failure occurs.

Section R9.3.1 of the Commentary to the Code[1] lists one of the purposes of the strength reduction factor, ϕ, is "to reflect the degree of ductility." Thus, non-ductile modes of failure such as shear have a value of $\phi = 0.85$ applied to the nominal strength. In this case, even though the flexural capacity is being determined, the failure mode is non-ductile. Therefore, it would seem consistent with code philosophy to apply the lower value of $\phi = 0.85$ to the nominal strength calculated using the strand slip failure end point.

DEBONDED STRAND

When some of the strands are debonded to reduce release stresses at the end of the member, the ACI Code requires longer development lengths in Section 12.9.3:

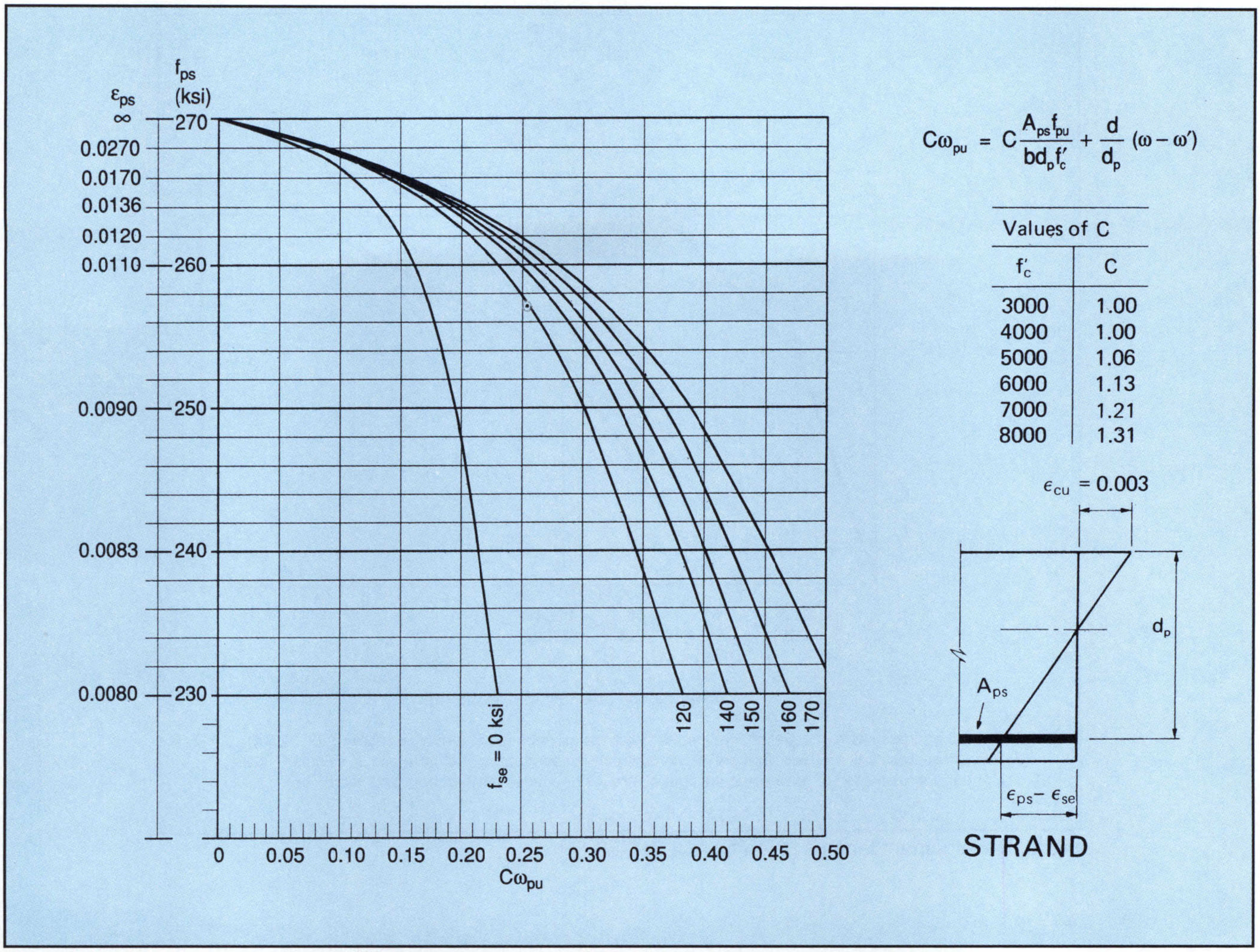


Fig. 5. Design aid for strain compatibility analysis of flexural member (Ref. 2).

"Where bonding of a strand does not extend to the end of member and design includes tension at service load in precompressed tensile zone as permitted by 18.4.2, development length specified in 12.9.1 shall be doubled."

Thus, the possibility of a critical section occurring within the strand development area increases and extends to longer span members.

It has been common practice to calculate the partial development stress in the debonded strands in a similar manner to that used for the strands that are bonded to the end except using the longer development length, and then numerically adding the capacities together. This is illustrated in Example 2A in the Appendix.

Unfortunately, this method does not account for compatibility of strains and can significantly overestimate the nominal strength of the member. Examples 2B, 2C, and 3 show how ca-

Table 1. Example 1 — All strands are bonded to the end of the member.

Example number	Distance from end	Assumptions	Nominal strength
A	Point where all strands are fully developed	N/A	672 ft-kips (911 kN-m) (maximum strength)
B1	3 ft (914 mm)	Neglect strains	466 ft-kips (632 kN-m)
B2	3 ft (914 mm)	Strain compatibility	423 ft-kips (574 kN-m)

Table 2. Examples 2 and 3 — Center strand is debonded for 5 ft (1524 mm) from end of member.

Example number	Distance from end	Assumptions	Nominal strength
2A	12 ft (3658 mm)	Neglect strains	637 ft-kips (864 kN-m)
2B	12 ft (3658 mm)	Consider strains Strand does not slip	467 ft-kips (633 kN-m)
2C	12 ft (3658 mm)	Consider strains Strand allowed to slip	540 ft-kips (732 kN-m)
3	14 ft (4267 mm)	Strand does not slip	541 ft-kips (734 kN-m)

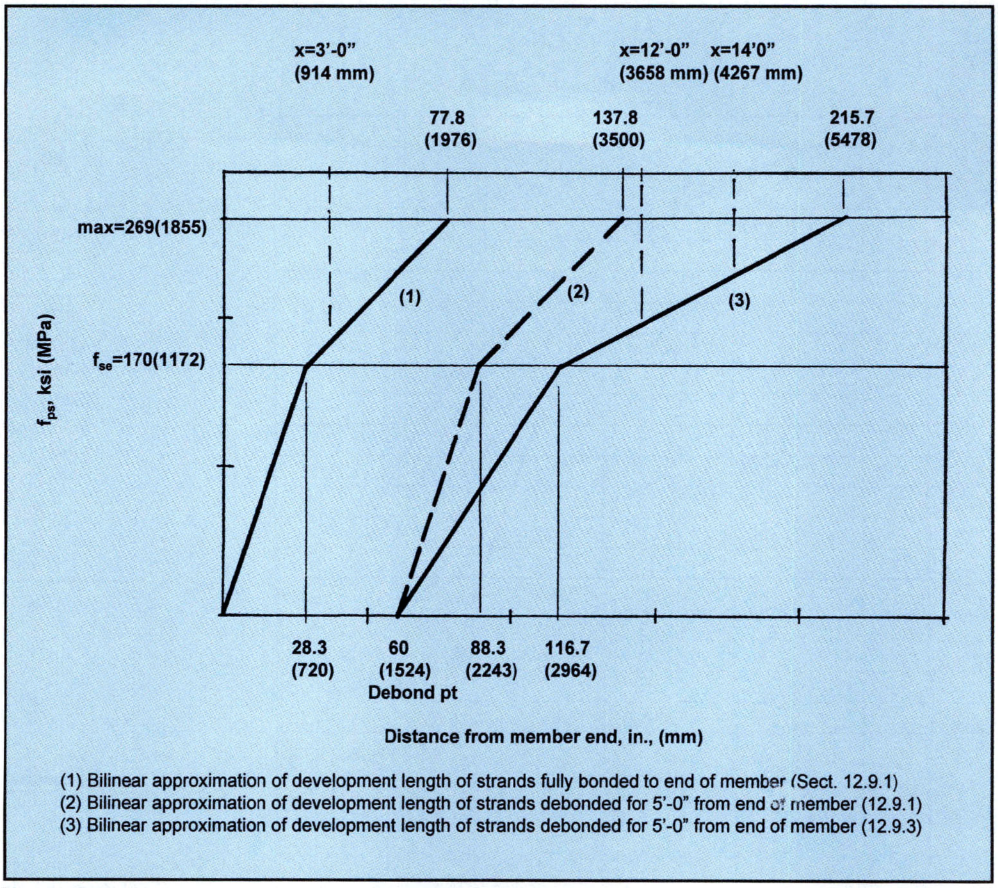


Fig. 6. Development lengths given by ACI 318-89.

pacities can be calculated taking strain compatibility into account.

STRAIN COMPATIBILITY ANALYSIS

Fig. 4 shows standard stress-strain curves for prestressing strand taken from the *PCI Design Handbook*.[2] While there is some variation in the stress-strain characteristics of strands, the PCI curves can be considered reasonable averages that are adequate for most design purposes. The equations at the bottom of the figure are approximations of the curves.

Note that the curves and equations are asymptotic to the specified breaking strength of the strand. This is probably somewhat conservative because in actual tests, nearly all strands break at a level higher than the specified strength. The equations indicate a precision that is not warranted, but are useful for computer design programs.

Fundamental strain compatibility analysis of a prestressed concrete member is an iterative procedure in which the depth of the neutral axis is adjusted until the compression and tension forces are equal. The strain limit for concrete is specified by ACI 318[1] as 0.003 in. per in. When determining the maximum capacity of a member, this value is assumed and the strain in the strand varies with the depth of the neutral axis.

The procedure is illustrated in Example 4.2.6 of the *PCI Design Handbook*.[2] Ref. 3 also provides a convenient design aid in which the value of f_{ps} can be determined graphically, eliminating the tedium of the iterations. This design aid is reproduced as Fig. 5 and is used in Example 1A in the Appendix.

When the strand is not fully developed, the limitation may be the stress of the strand because of slip. For analysis, then, the maximum stress in the strand is determined from Fig. 2 (or a similar assumption based on twice the development length for debonded strands), and the corresponding strain is determined (see Fig. 4). Adjustments in the neutral axis will then cause the strain in the concrete to vary. In order to evaluate the compressive force, the stress-strain characteristics of the concrete must be known or assumed.

Several expressions have been developed that reasonably approximate experimental data[3,4] and such expressions can be used. The ACI Code[1] suggests an equation in Section 8.5.1, and the Commentary implies that this value is valid for concrete stresses up to $0.45f_c'$. The calculations are not overly sensitive to the values of E_c, so it is adequate to use the Code equation for all values. The examples in the Appendix used the Code equation, rounded to 4300 ksi (29.650 MPa) for 5000 psi (35 MPa) concrete.

In nearly all prestressed concrete flexural design, it is assumed that all strands are stressed equally regardless of the differences in strain due to their relative vertical positions (see the strain diagrams in the Appendix). It is apparent that this is valid when the strains are on the upper or flat part of the steel stress-strain curve; but when they are on the more vertical part of the curve, the differences are greater.

This assumption is of little or no consequence when the strands are adequately anchored so that the failure mode is ductile yielding. However, when the failure mode is strand slip, it is possible that the lower placed strands can slip at lower load, perhaps causing a "zippering" effect. This further emphasizes the need for conservative development assumptions.

When some strands are debonded for part of the length, they may not be able to accommodate the strain associated with the straight line strain distribution. Thus, the nominal moment strength may be either that available with only the fully bonded strands or that available when the straight line strain distribution allows the stress in the debonded strand to be developed, and the stress in the fully bonded strands are reduced because of the reduced strains. Example 2C illustrates the former case and Examples 2B and 3 illustrate the latter case.

It is not always obvious which analysis will result in a higher allowable nominal strength. Computer programs could be set up to check both cases, whereas the conservative approach of neglecting the debonded strand until it is fully bonded may be more appropriate for hand calculations.

DESIGN EXAMPLES

Appendix A shows examples of strain compatibility analysis for different assumptions and conditions of debonding. The section chosen is a typical 10 ft (3048 mm) wide, 26 in. (660 mm) deep, pretopped double tee prestressed with ten 1/2 in. (13 mm) diameter low relaxation strands with a specified minimum ultimate strength of 270 ksi (1860 MPa). The results of the calculations are summarized in Tables 1 and 2.

In Example 1, the difference between using strain compatibility, i.e., considering the reduced concrete strains, and the common practice of neglecting the reductions is shown by the differences between the results of the calculations in Example B1 vs. Example B2; in this case, approximately 10 percent. In Example 2, the differences are even greater, with a maximum calculated nominal capacity of 540 ft-kips (732 kN-m) when concrete strains are considered; only 85 percent of the value calculated when strains are neglected.

Example 2 also illustrates the difference between the assumption of strand slip being the failure end point (Example 2B) and assuming that the debonded strand will slip and be ineffective up to the point that it is fully bonded (Example 2C); in which case, ductile yielding would be the failure end point, but with fewer strands contributing to the strength. For design purposes, the maximum of these two values should be used.

Thus, a conservative design for this member would assume that the nominal moment capacity is 540 ft-kips (732 kN-m) from the point that the eight bonded strands are developed [77.8 in. (1976 mm) from the end] to the point that the two unbonded strands are developed, 155.7 in. + 5 ft 0 in. debonded length or 215.7 in. (5479 mm) from the end, at which point it becomes the maximum value, 672 ft-kips (911 kN-m) (see Fig. 6).

Example 2 also shows that at 14 ft (4267 mm) from the end, the assumption of strand slip will yield a slightly higher nominal moment strength. However, if a value of $\phi = 0.85$ is applied to this value, as opposed to the $\phi = 0.9$, which can be used for the failure end point of ductile yielding, the usable moment strength is somewhat less.

RECOMMENDATIONS

The ACI Building Code should be revised to give guidelines on the calculation of nominal strength in the strand development area. These guidelines should include:

1. A requirement that the strength reduction factor of $\phi = 0.85$ be applied to the calculated nominal moment strength, M_n, when the failure end point is strand slip.

2. A requirement that for members with debonded strands, calculation of nominal strength in the development region be based on strain compatibility, or conservatively, the contribution of the debonded strand be neglected until it is fully bonded.

ACKNOWLEDGMENT

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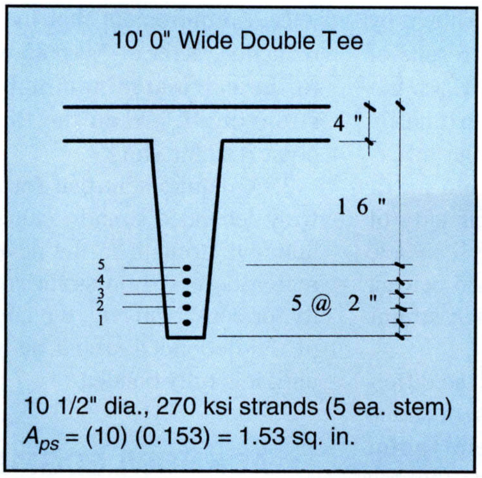

Assume:

Initial stress in strand = $0.75 f_{pu}$

Total losses = 16 percent

$f_{se} = 0.75(270)(0.84) = 170$ ksi

$E_{ps} = 28{,}500$ ksi

$$\varepsilon_{se} = \frac{170}{28{,}500} = 0.0060 \text{ in./in.}$$

$f'_c = 5000$ psi

Fully developed f_{ps} — Use Fig. 4.10.3 from the *PCI Design Handbook,* Fourth Edition.

$$C\omega_{pu} = C\frac{A_{ps}f_{ps}}{bd_p f'_c} = 1.06\frac{(1.53)(270)}{120(20)(5)} = 0.036$$

$f_{ps} = 269$ ksi

$$l_d = \left(f_{ps} - \frac{2f_{se}}{3}\right)d_b = \left[269 - \frac{2(170)}{3}\right]0.5 = 77.8 \text{ in.}$$

$2l_d = 155.7$ in.

$$\left(\frac{f_{se}}{3}\right)d_b = \frac{170}{3}(0.5) = 28.3$$

28.3(2) = 56.7 in.

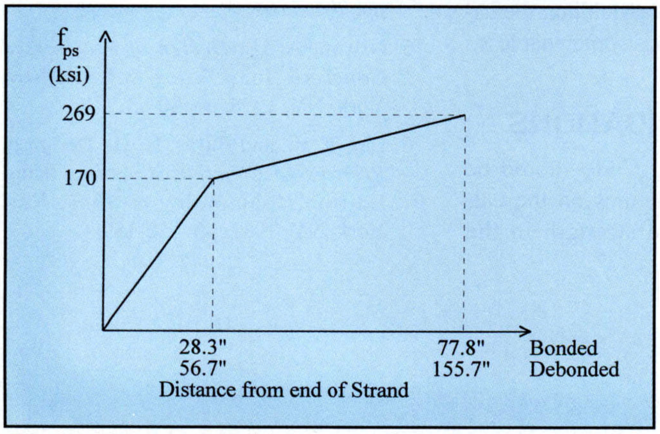

Fig. A1. Bi-linear strand development.

EXAMPLE 1 — All strands bonded to end of member.

Case A. Beyond 77.8 in. — Strands fully bonded.

$f_{ps} = 269$ ksi

$A_{ps} = 1.53$ in.^{2}

$d = 26 - 6 = 20$ in.

$T = C = A_{ps}f_{ps} = 1.53(269) = 411.6$ kips

$M_n = T(d - a/2)$

$$a = \frac{T}{0.85f'_c b} = \frac{411.6}{0.85(5)(120)} = 0.81 \text{ in.}$$

$$M_n = 411.6\left(20 - \frac{0.81}{2}\right) = 8066 \text{ in.-kips} = 672.2 \text{ ft-kips}$$

Case B1. At 3 ft 6 in. (42 in.) from end (neglecting strains). From Fig. A1:

$$f_{ps} = 170 + \frac{42 - 28.3}{77.8 - 28.3}(269 - 170) = 197.4 \text{ ksi}$$

$T = A_{ps}f_{ps} = (1.53)(197.9) = 302.0$ kips

$$a = \frac{T}{0.85f'_c b} = \frac{302.0}{(0.85)(5)(120)} = 0.59 \text{ in.}$$

$$M_n = T\left(d - \frac{a}{2}\right) = (302.9)\left(20 - \frac{0.59}{2}\right)$$

$= 5958$ in.-kips $= 496.5$ ft-kips

Case B2. At 3 ft 6 in. (42 in.) from end — Using strain compatibility:

$$f_{ps} = 170 + \frac{42 - 28.3}{77.8 - 28.3}(269 - 170) = 197.4 \text{ ksi}$$

$$\text{Maximum strain} = \varepsilon_{ps} = \frac{197.9}{28{,}500} = 0.00693$$

$\varepsilon_{sa} = \varepsilon_{ps} - \varepsilon_{se} = 0.00693 - 0.0060 = 0.00093$

Use trial and error to determine ε_c such that compression equals tension (see Fig. A2).

Try $\varepsilon_c = 0.00024$ in./in.

$$c = \frac{0.00024}{0.00093 + 0.00024}(24 \text{ in.}) = 4.92 \text{ in.}$$

Strain at centroid of strand:

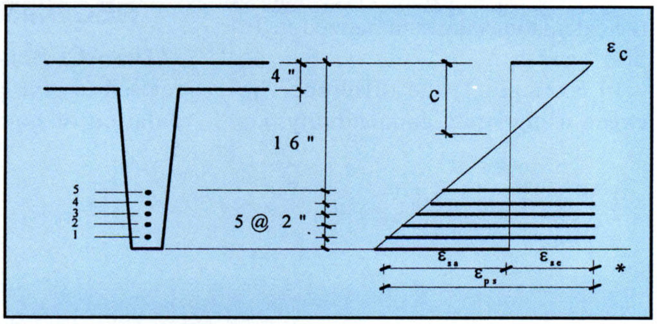

Fig. A2. Strain diagram.

$$\varepsilon_{sa} = \frac{15.08}{19.08}(0.00093) = 0.00074 \text{ in./in.}$$

$T = (0.00074 + 0.006)(28500)(1.53) = 293.9$ kips

Note: For unsymmetrical strand placement, individual groups of strands should be considered separately.

Compression — Use $E_c = 4300$ ksi

Stress at top of flange = (0.00024)(4300) = 1.03 ksi

$$\text{Stress at bottom of flange} = \frac{0.92}{4.92}(1.031 \text{ ksi}) = 0.19 \text{ ksi}$$

$$\text{Compression in flange} = (4 \text{ in.})(120 \text{ in.})\left(\frac{1.03+0.19}{2}\right)$$

= 292.8 kips

$$\text{Compression in web} = (6 \text{ in.})(0.92 \text{ in.})\left(\frac{0.19}{2}\right)(2 \text{ webs})$$

= 1.0 kips

Total compression = 293.8 kips ≈ T = 293.9 kips

Note: Some references[5,6] include a "decompression strain" in this analysis. In the examples shown, this has a negligible effect on the magnitude of the calculated moments.

From Fig. A3:

$M_n = (293.9)(15.71 + 2.76) - (1.0)(2.76)$

$M_n = 5425$ in.-kips = 452.1 ft-kips
(91 percent of value obtained by neglecting strains)

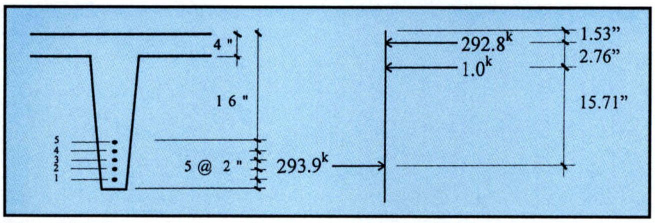

Fig. A3. Example 1 — Force couple.

EXAMPLE 2: Strand #3 is debonded for 5 ft from end. Find M_n at 12 ft from end.

Case A. If strains are ignored.
Eight fully bonded strands: $f_{ps} = 269$ ksi
Two debonded strands — 7 ft (84 in.) from start of strand bonding

$$\max f_{ps} = 170 + \frac{84-56.7}{155.7-56.7}(269-170) = 197.3 \text{ ksi}$$

$T = 8(0.153)(269) + 2(0.153)(197.3) = 389.6$ kips

$$d = 20 \text{ in.} \quad a = \frac{T}{0.85(5)(120)} = \frac{389.6}{510} = 0.76 \text{ in.}$$

$M_n = T(d - a/2) = 389.6(20 - 0.76/2)$
= 7643 in.-kips = 636.9 ft-kips

Case B. If strains are considered and debonded strand does not slip.

Maximum strain in #3 (debonded) =

$$\frac{f_{ps}}{E_{ps}} = \frac{197.3}{28{,}500} = 0.00692 \text{ in./in}$$

$\varepsilon_{sa} = \varepsilon_{ps} - \varepsilon_{se} = 0.00692 - 0.0060 = 0.00092$ in./in.
(see Fig. A2)

Try $\varepsilon_c = 0.00024$ in./in.

$$c = \frac{0.00024}{0.00024+0.00092}(24 \text{ in.}) = 4.96 \text{ in.}$$

Strain at centroid of strand:

$\varepsilon_s = 0.00092 + 0.006 = 0.00692$ in./in.

$f_{ps} = (0.00692)(28{,}500) = 197.3$ ksi

$T = (197.3)(1.53) = 301.8$ kips

Compressive stress at top of flange =
(0.00024)(4300 ksi) = 1.032 ksi

Compressive stress at bottom of flange =

$$\left(\frac{0.96}{4.96}\right)(1.032 \text{ ksi}) = 0.20 \text{ ksi}$$

Compression in flange =

$$\left(\frac{1.032+0.20}{2}\right)(120 \text{ in.})(4 \text{ in.}) = 295.6 \text{ kips}$$

Compression in webs =

$$(6 \text{ in.})(0.96)\left(\frac{0.20 \text{ ksi}}{2}\right)(2 \text{ webs}) = 0.2 \text{ kips}$$

Total compression = 295.6 + 0.2 = 295.8 kips ≈ 301.8 kips

Use Fig. A4 to calculate moments:

$M_n = (301.8)(15.6 + 2.96) - (0.2)(2.96)$

$M_n = 5601$ in.-kips = 467 ft-kips

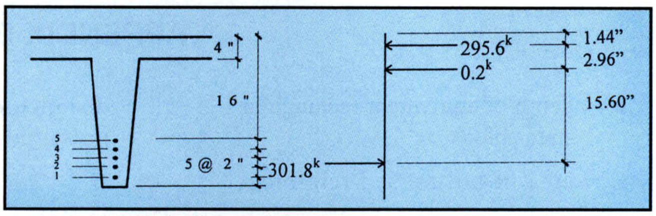

Fig. A4. Example 2 — Force couple.

Case C. If strains are considered and debonded strand slips, neglect debonded strands and determine moment capacity with eight fully bonded strands.

$$T = 8(0.153)(269) = 329.3 \text{ kips} \quad a = \frac{329.3}{510} = 0.65 \text{ in.}$$

$$M_n = T\left(d - \frac{a}{2}\right) = 329.3\left(20 - \frac{0.65}{2}\right) = 6480 \text{ in.-kips}$$

= 540 ft-kips > 467 ft-kips — capacity from Case C governs

EXAMPLE 3: Strand #3 is debonded for 5 ft from end — Find M_n at 14 ft from end.

Maximum stress in #3 at 9 ft (108 in.) from start of strand bonding.

$$\varepsilon_{ps} = \frac{221.3}{28.500} = 0.00776 \text{ in./in.} \quad \varepsilon_{sa} = 0.00176 \text{ in./in.}$$

Note: Because the debonded strand is in the center of the strand group, it is sufficiently accurate to use the average f_{ps} of 221.3 ksi.

$T = (1.53)(221.3 \text{ ksi}) = 338.6$ kips

From strain compatibility analysis: $a = 1.65$ in.

$M_n = 338.6(20 - 1.65/2) = 6493$ in.-kips $= 541.1$ ft-kips

This value is greater than $M_n = 540$ ft-kips if the debonded strand was assumed to slip. However, in that case, a value of $\phi = 0.9$ could be used, whereas in this example, a value of $\phi = 0.85$ should be used.

$\phi M_n = 0.9(540) = 486.0$ ft-kips (use for design)

$\phi M_n = 0.85(541.1) = 460.0$ ft-kips

Metric (SI) conversion factors: 1 in. = 25.4 mm; 1 in.^{2} = 645.2 mm^2; 1 psi = 0.006895 MPa; 1 ksi = 6.895 MPa; 1 kip = 4.448 kN; 1 ft-kip = 1.356 kN-m.

APPENDIX B — NOTATION

a = depth of equivalent rectangular stress block

A_{ps} = area of prestressed reinforcement

b = width of compression face of member

b_w = web width

c = distance from extreme compression fiber to neutral axis

C = compression force

d = distance from extreme compression fiber to centroid of tension reinforcement

d_b = nominal diameter of prestressing strand

d_p = distance from extreme compression fiber to centroid of prestressed reinforcement

E_c = modulus of elasticity of concrete

E_{ps} = modulus of elasticity of prestressing steel

f'_c = specified compressive strength of concrete

f_{ps} = stress in prestressed reinforcement at nominal strength

f_{pu} = specified tensile strength of prestressing tendons

f_{se} = effective stress in prestressed reinforcement (after allowance for all prestress losses)

l_d = development length of prestressing tendons

T = tensile force

ε_c = concrete strain

ε_{ps} = strain in tendons corresponding to f_{ps}

ε_{sa} = strain in tendons due to applied loads

ε_{se} = strain in tendons corresponding to f_{se}

ϕ = strength reduction factor