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Shear Reinforcement for Deflection Ductility of Flat Plates

by Carl Erik Broms

The deflection ductility of flat plates with different types of shear reinforcement is tested. Excellent performance is achieved with a combination of bent bars and stirrups, even at reinforcement ratios exceeding $0.75\rho_b$. Code requirements for flat plates are critically reviewed and design recommendations are given for the bent bar/stirrup reinforcement arrangement.

Keywords: bent-up bars; building codes; **deflection; ductility; flat concrete plates;** flexural strength; punching shear; **reinforcing steels; shear strength;** stirrups; structural design; tests.

Primary building structures should be designed in such a way that they exhibit a ductile failure mode when subjected to catastrophic loading. Large deformations (and for concrete structures, excessive cracking) will then precede and forewarn of the final collapse.

Reinforced concrete structures subjected to flexure and shear will behave in the desired manner provided that reinforcement strength, rather than concrete strength, determines their load-carrying capacity. The design rules of modern building codes[1,2] reflect this approach, except for the one very important exception of design rules for flat plates.

The flat plate is a reinforced concrete slab that has no drop panels, and which is supported by columns without capitals. A disadvantage of this type of structure is the great risk of brittle punching failure at the slab-column connection. Flat plates are therefore sometimes provided with some type of shear reinforcement, the primary task of which is to increase the load-carrying capacity of the slab. But, since the codes normally have no requirements for the deflection ductility, the code design rules may lead to shear-reinforced flat plates that exhibit less safety against punching failure than do flat plates without shear reinforcement.[3]

It must be emphasized again that ductility of a flat plate is not automatically achieved merely by insuring that its nominal punching capacity according to Reference 1 is greater than its nominal flexural capacity; see Reference 4.

This author's opinion is that the shear reinforcement for flat plates should be so designed and arranged that a ductile failure mode is guaranteed. As for beams, a minimum area of shear reinforcement must therefore be provided, even at a shear force that is less than the punching shear capacity provided by the concrete (see ACI 318-83,[1] Section 11.5.5.1).

The behavior of a continuous flat plate in the vicinity of an interior column is often simulated in tests by specimens simply supported on a circle, where the radius is considered to correspond to the distance from the center of the column to the line of contraflexure in the continuous flat plate.

The relationship between the failure load and the deflection of such a specimen is shown in Fig. 1. If the amount of flexural reinforcement is large enough to produce a punching failure, and if no shear reinforcement is provided, then punching will take place at a very small deflection (Point B). If, on the other hand, effective shear reinforcement is installed, then the ultimate deflection will increase, and the flexural capacity of the slab will be reached (Point C). Only then can the reinforcement within the column strip of a continuous flat plate be considered completely effective. The structure still cannot be considered ductile, however, until a marked yield-plateau is achieved (curve Branch C-D), and not until then will a continuous flat plate have the capacity to redistribute bending moments from the supports to the midspan.

RESEARCH SIGNIFICANCE

This paper reports on tests conducted by the author at the Royal Institute of Technology, Stockholm. The tests were aimed at achieving ductile behavior of reinforced concrete flat plates by means of easy-to-install conventional reinforcing bars. Different types and arrangements of shear reinforcement at interior columns with symmetrical loading were investigated. Design

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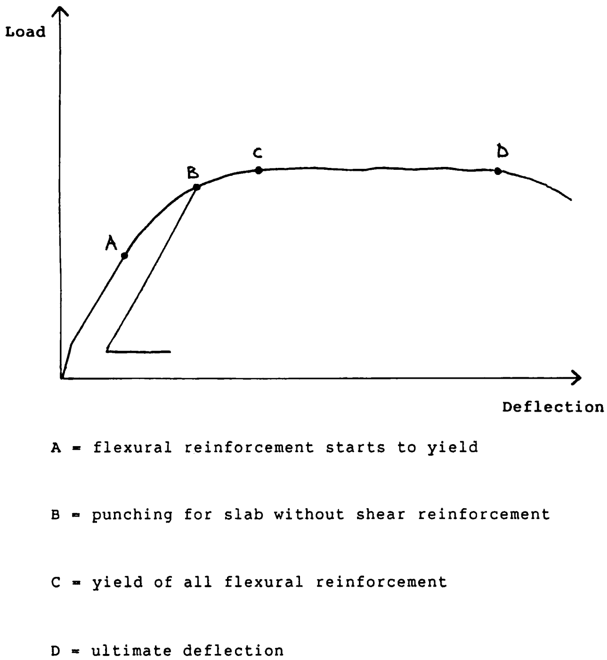

Fig. 1—Typical load-deflection curves for flat plate test specimens with and without effective shear reinforcement

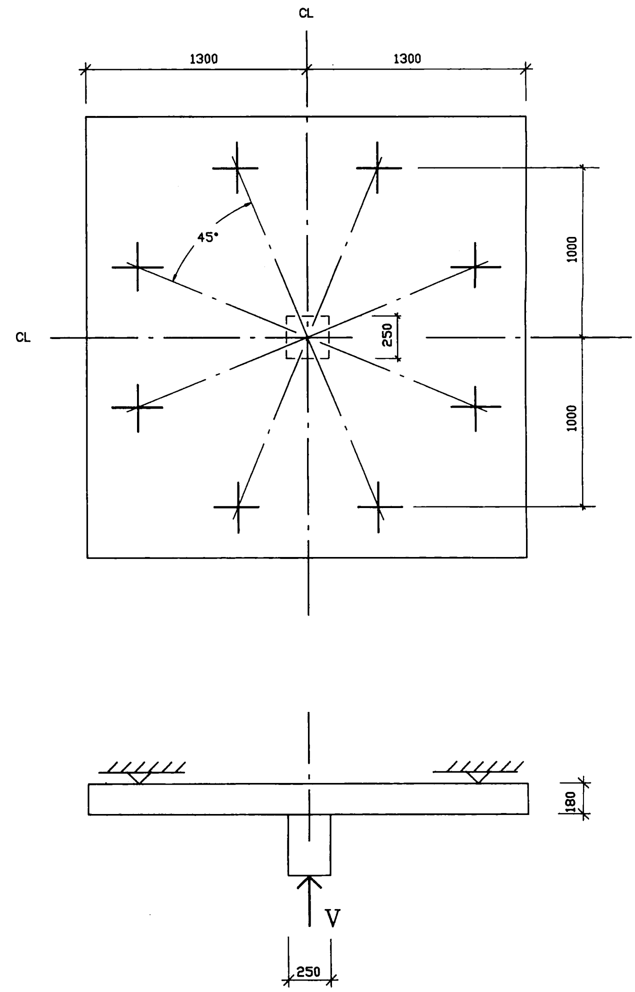

Fig. 2—Test setup

recommendations are given for the arrangement with the best performance, and a numerical example is presented.

TEST DATA

The test series (Fig. 2 illustrates the test setup) consisted of eight specimens, all of which had the same dimensions, but with different reinforcement arrangements according to Table 1 and Fig. 3 through 6.

The test setup was intended to simulate the conditions in the vicinity of an interior column in a continuous flat plate. The 2600 mm (102 in.) square, 180 mm (7.1 in.) thick slabs were simply supported at eight points. The supports were symmetrically distributed on the sides of a 2000 mm (78 in.) square.

The deflection of the bottom surfaces of the slabs were recorded at the column faces and at the supports. The difference between the average values for these two position types is defined as the deflection of the slab as shown in Fig. 3 through 6. Concrete and reinforcement strains were recorded for Specimens 6, 7, and 8 at representative locations.

All specimens were cast with normal density concrete. The concrete strength was recorded on 150 mm (5.85 in.) test cubes that were cured and stored under the same conditions as the test specimens. The corresponding moist-cured cylinder strength f_c' has been calculated by means of Eq. (1)

$$f_c' = 0.7 f_{cube}' + 0.5 \text{ (MPa)} \tag{1}$$
$$f_c' = 0.7 f_{cube}' + 72.5 \text{ (psi)}$$

The concrete strength was deliberately kept relatively low to simulate its normal state, where the concrete strength in place is assumed to be less than the specified strength.

The load was increased in 25 kN steps. The hydraulic jack was mechanically locked after each step. After approximately 10 min, the load and the deflections were recorded. At ductile behavior of the slab the load steps were changed to 5-mm (0.2 in.) deflection steps.

Specimen 1 had no shear reinforcement. It was intended to be a reference specimen and a test specimen for the theory previously presented by the author in Reference 4.

Specimens 2 through 5 were provided with multiple U-stirrups as shear reinforcement. The stirrups were arranged in the form of a cross in Specimens 2 and 3. In Specimens 4 and 5, the stirrups were evenly distributed within a square, symmetrically around the column. In both cases the stirrups only enclosed the top tension reinforcement as recommended in many building codes (the stirrups are easily installed in this manner). The stirrups were designed to at least resist the shear force that corresponded to the theoretical flexural capacity of the specimens.

Table 1 — Reinforcement arrangement and material strength

Specimen	Flexural reinforcement: Top / Bottom	f_y/f_u, MPa	Shear reinforcement: Bent bars / Stirrups	f_y/f_u, MPa	Concrete strength f'_c, MPa
1	24 + 24 Φ 12 s 80	681/830	—	—	21
	—	—	—	—	
2	16 + 16 Φ 12 s 120	681/830	—		24
	—	—	4 × 2 Φ 8 s 120	471/575	
3	23 + 23 Φ 12 s 80	681/830	—	—	23
	—	—	3 × 2 Φ 8 s 80	471/575	
4	24 + 24 Φ 10 s 90	656/783	—	442/655	15
	—	—	2 Φ 6 s 90/90		
5	12 + 12 Φ 12 s 180 12 + 12 Φ 10 s 180	684/827 656/783	—		17
	—	—	2 Φ 6 s 90/90	442/655	
6	10 + 10 Φ 10 s 180 10 + 10 Φ 8 s 180	665/793 678/817	4 + 4 Φ 12	671/807	20
	28 + 28 Φ 6 s 90	454/658	Φ 6 s 90/90	454/658	
7	20 + 20 Φ 12 s 90	691/969	4 + 4 Φ 16	656/950	17
	28 + 28 Φ 8 s 90	467/571	Φ 6 s 90/90	496/658	
8	20 + 20 Φ 12 s 90 28 + 28 Φ 8 s 90	691/969 467/571	4 + 4 Φ 16 Φ 6 s 90/90	656/950 496/658	18

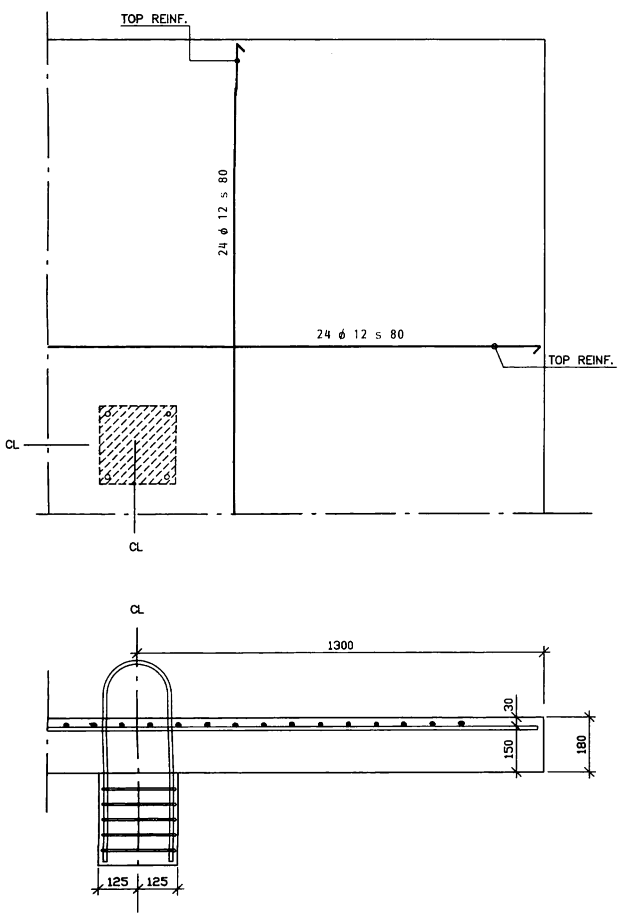

Fig. 3—Reinforcement arrangement for Specimen 1

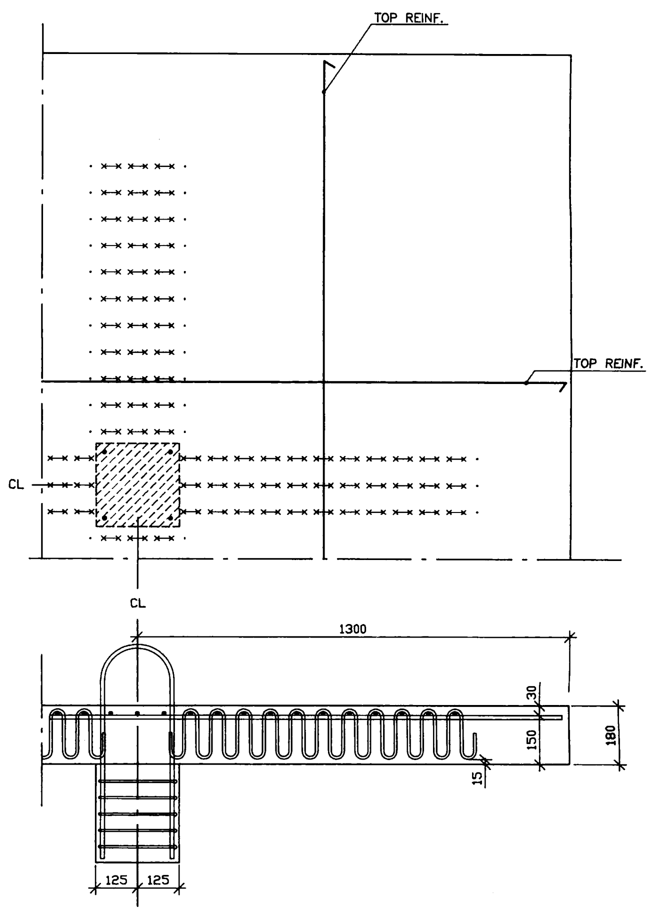

Fig. 4—Reinforcement arrangement for Specimens 2 and 3

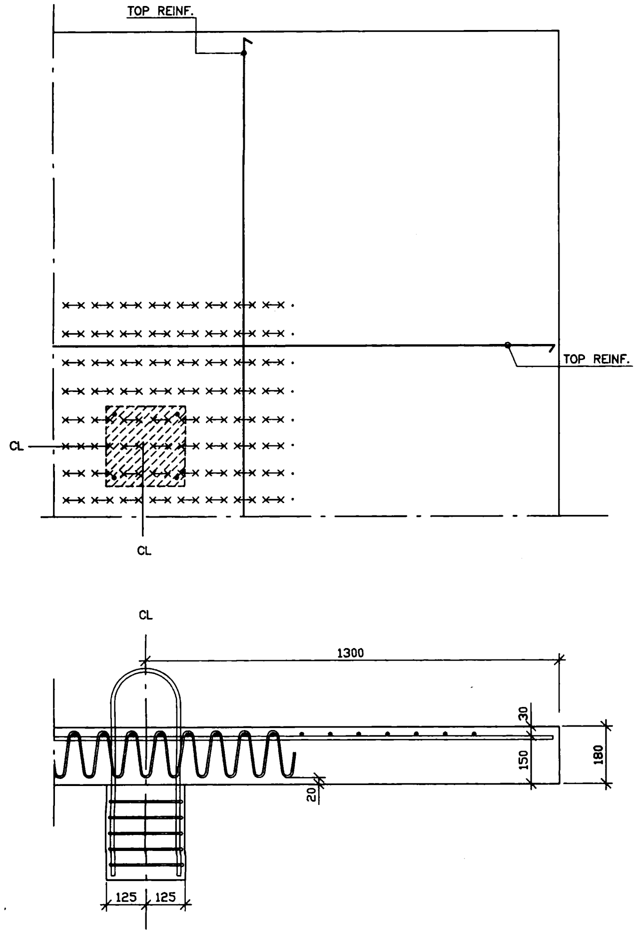

Fig. 5—Reinforcement arrangement for Specimens 4 and 5

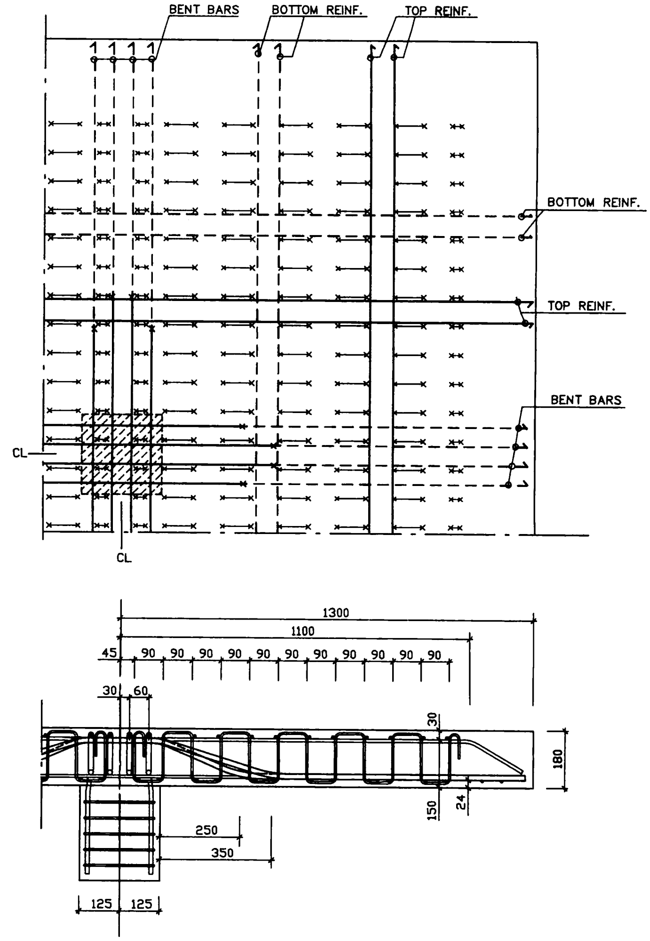

Fig. 6—Reinforcement arrangement for Specimens 6, 7, and 8

The bottom reinforcement was ommitted in Specimens 2 through 5 to obtain conservative test results. The contribution of such reinforcement to the punching shear capacity is, however, small (see Reference 5, p. 211).

Specimens 6, 7, and 8 were all provided with a combination of bent bars and multiple U-stirrups as shear reinforcement. The bent bars were designed so that the vertical component of their yield force would be approximately equal to the shear force corresponding to the flexural capacity of the slabs. The bent bars were supplemented by U-stirrups to avoid secondary "shear failures" outside the bent bars. The stirrups only enclosed the bottom reinforcement to ease the installation of all other bars (no "tacking" of bars was involved).

The multiple U-stirrup arrangement was identical in Specimens 6 through 8, but the amount of flexural reinforcement and bent bars in Specimens 7 and 8 was larger than in Specimen 6. Specimen 8 differed from Specimen 7 by the openings around the column, as shown in Fig. 7.

TEST RESULTS

The deflection-load relationships of the tested specimens are shown in Fig. 8, and a comparison of measured loads with calculated capacities in accordance with Reference 1 is made in Table 2.

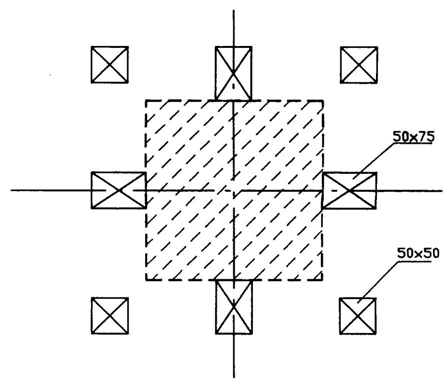

Fig. 7—Openings in Specimen 8

The reinforcement ratio at balanced strain conditions has been calculated according to Eq. (2), (3), and (4) (see Reference 2)

$$\rho_b = \beta_1 \frac{f'_c}{f_y} \cdot \frac{600}{600 + f_y} + \rho' \left(\frac{f'_{sb}}{f_y}\right) \tag{2}$$

$$\beta_1 = 0.85 \text{ (if } f'_c \leqslant 30 \text{ MPa)} \tag{3}$$

$$f'_{sb} = 600 - \frac{d'}{d}(600 + f_y) \leqslant f_y \qquad (4)$$

Note that the factor 0.85, according to Reference 2, is omitted in Eq. (2) to obtain a conservative estimate of the actual value ρ/ρ_b.

Specimen 1, which had no shear reinforcement, suddenly collapsed by punching at 475 kN (107 kip) load level and 10-mm (0.39 in.) deflection, when the load was increased from the stable load level 435 kN. The straight load-deflection curve indicates that no reinforcement yielding took place. The ultimate load and deflection correspond very well with the predicted values according to Reference 4, 446 kN (98 kips) and 10.2 mm (0.4 in.), respectively.

Specimens 2 through 5 were provided with various multiple U-stirrup arrangements enclosing the top tension reinforcement; all of them failed by punching. In Specimens 2, 4, and 5, the failure was caused by a steep shear crack near the column. The cracks passed between the stirrup legs thus leaving the stirrups ineffective. In Specimen 3, a shear crack outside the shear-reinforced zone caused the failure. Since Specimens 2 through 5 failed by punching, none of them exhibited the desirable ductile behavior. In fact—despite the heavy shear reinforcement—they did not even reach the upper limit for punching shear stress 0.5 $\sqrt{f'_c}$ MPa (6 $\sqrt{f'_c}$ psi), according to Reference 1.

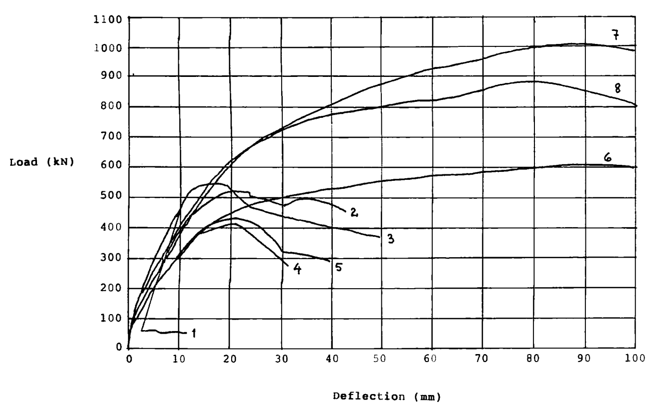

Fig. 8—Load-deflection curves for all tested specimens

Specimens 6 through 8 were provided with a combination of bent bars and stirrups as shear reinforcement. The stirrups only enclosed the bottom compression reinforcement. It is evident from Fig. 8 that these slabs behaved in a very ductile manner, with an ultimate deflection of approximately 10 times that of the slab without shear reinforcement. The ultimate load was not reached until the concrete at the bottom surface—due to its great curvature—was crushed across the entire width of the specimens along the two principal axes (see Fig. 9).

The ultimate deflection was so large that membrane action of the slabs developed; therefore, the ultimate loads of Specimens 6 through 8 exceeded the predicted flexural capacities. The flexural reinforcement and the bent bars were Swedish Type Ks60 deformed bars with a yield strength exceeding 650 MPa (94 ksi).

Strain gages were used to record the relationships between the total load V and the load that was transferred to the column via bent bars V_i. A typical result—that of Specimen 7—is shown in Fig. 10. It is evident that the concrete contribution V_c to the shear capacity was small. It is also evident that bent bars with high yield strengths can be used.

In contrast to Specimens 2 through 5, the capacity of Specimens 7 and 8 exceeded the upper limit 0.5 $\sqrt{f'_c}$ MPa (6 $\sqrt{f'_c}$ psi) for the nominal punching shear stress according to Reference 1 by more than 100 and 200 percent, respectively.

The crack patterns of Specimens 7 and 8 at ultimate load is shown in Fig. 11 and 12. The photographs are taken after removal of the test equipment. No penetration of the columns into the slab could be observed in spite of the considerable widths of the flexural cracks near the column, as shown in Fig. 13.

The great ductility of Specimens 6 through 8 is perhaps most visible in the photograph in Fig. 14. Note that all photographs show specimens that still have their full load capacities. Testing was stopped when the deflections of the slabs reached approximately 100 mm (3.9 in.), since the test equipment did not permit any larger deflections.

Table 2 — Test results and comparison with ACI 318-83

				Test results		Calculated properties (ACI 318-83)						
Specimen	ρ, percent	ρ', percent	$\frac{\rho}{\rho_b}$	V_y, kN	V_{test}, kN	V_y, kN	V_s, kN	V_c, kN	V_n, kN	$V_{n_{max}}$, kN	V_{calc}, kN	$\frac{V_{test}}{V_{calc}}$
1	0.94	—	0.77	—	≃435	769	—	367	—	—	367	1.18
2	0.63	—	0.45	523	523	547	800	392	996	588	547	0.96
3	0.94	—	0.70	—	558	744	900	384	1092	576	576	0.97
4	0.58	—	0.63	—	415	529	528	310	683	465	465	0.89
5	0.71	—	0.70	—	434	647	528	330	693	495	495	0.88
6	0.65*	0.21	0.48	≃495	608	457	456	358	635	537	457	1.33
7	1.11*	0.37	0.92	≃775	1006	765	764	330	929	495	495	2.03
8	1.31*	0.47	0.99	≃715	878	706	764	187	857	280	280	3.14

*Measured on 1000-mm width, including bent bars.

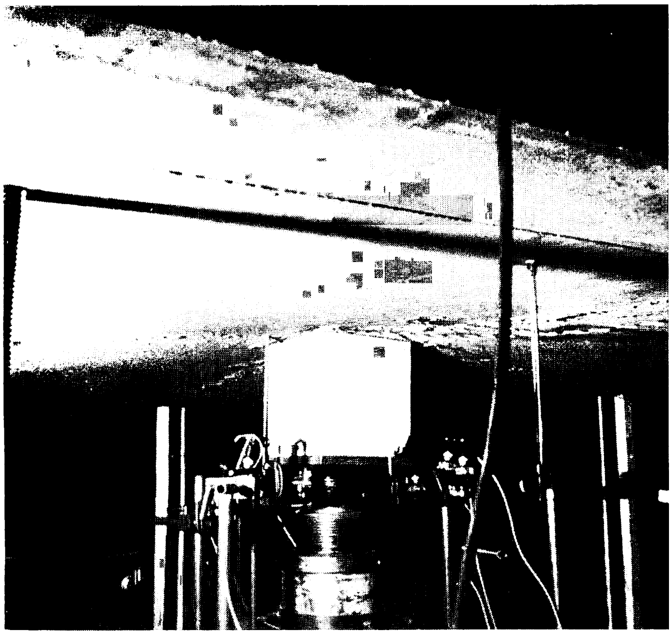

Fig. 9—Specimen 6. Spalling of bottom surface across the entire specimen

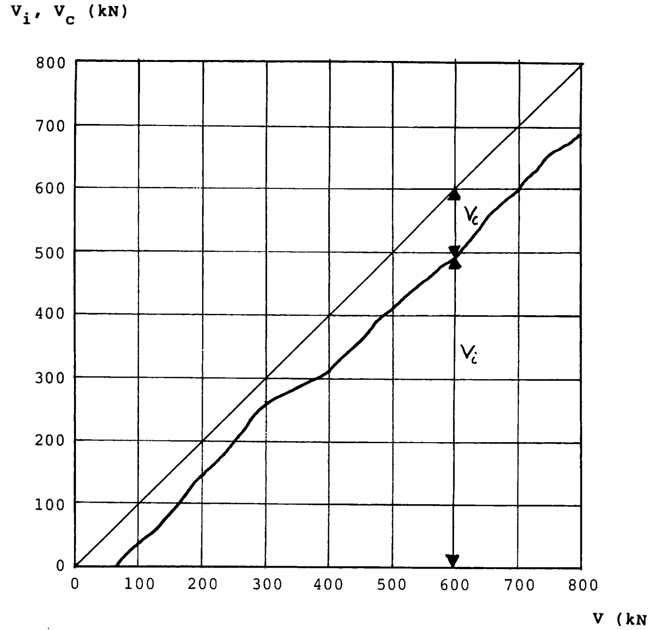

Fig. 10—Specimen 7. Shear force V_i *carried by bent bars versus total shear force* V

DESIGN RECOMMENDATION

The tests have demonstrated that flat plates with a shear reinforcement arrangement in accordance with Fig. 6 will exhibit a ductile behavior similar to that of ordinary reinforced slabs supported by beams or walls, provided that the following design recommendations are followed:

The bent bars shall be distributed within the column width. The inner bends shall be placed at the column face and the bars shall be given a moderate inclination so that the outer bends are placed at an average distance of $2d$ from the column face.

The bent bars shall be given full development length, but, in order for the bars to be effective for membrane action of the slab, they shall extend at least the distance $0.25\ l_n$ from the face of the column.

The vertical component V_i of the yield force in the bent bars shall be at least equal to the shear force that corresponds to the flexural capacity of the slab.

The flexural reinforcement shall be designed for the total moment M_o according to Eq. (5)

$$M_o = \frac{w_u \cdot l_2 \cdot l_1^2}{8} - \Delta M \tag{5}$$

$$\Delta M = 0.25 w_u\, l_1\, l_2\, (0.5c_1 + 0.25c_2 + 0.8 \cdot 2d) \tag{6}$$

When calculating the size of the concrete stress block at flexure, the reinforcement area A_s shall include the area of bent bars in the direction considered

$$a = \frac{A_s f_y}{0.85 f_c'\ b} \tag{7}$$

where b = half the width of the column strip.

The stirrups shall enclose the bottom flexural reinforcement and they shall be designed to transfer the shear force to the bent bars along the critical section defined in Fig. 15. The concrete contribution V_c may then not be taken as being greater than

$$V_c \leqslant \frac{1}{6} b_{eff} d \sqrt{f_c'},\ \text{MPa} \qquad V_c \leqslant 2\, b_{eff} d \sqrt{f_c'},\ \text{psi} \tag{8}$$

where b_{eff} = total effective length of critical section (Fig. 15).

The stirrup contribution V_s to the shear strength along the critical section may not be taken as greater than $2V_c$, according to Eq. (8). A minimum area of stirrups in accordance with Eq. (11-14) of ACI 318M-83[1] shall always be provided.

The stirrups shall be evenly distributed around the column and cover a circle outside of which the shear stress in the slab is less than $1/6 \sqrt{f_c'}$ MPa ($2 \sqrt{f_c'}$ psi). The stirrup legs shall be given a spacing which is not greater than the lesser of 0.6d and $16d_b$, where d_b is the nominal diameter of the bottom flexural bars.

The stirrups shall be given a diameter not exceeding 7 mm (*D*6) and the inside diameter of the bends shall be $4d_b$. The design yield strength shall not exceed 340 MPa (50 ksi) in order to avoid splitting of the concrete inside the upper bends.

Openings in the slab inside the critical perimeter indicated in Fig. 15 affect neither the strength nor the stiffness of the slab, provided that there is space enough for the bent bars to pass within the column width. (Compare the load-deflection curves for Specimens 7 and 8 in Fig. 8.)

The previous design recommendations have been verified for such high reinforcement ratios as ρ =

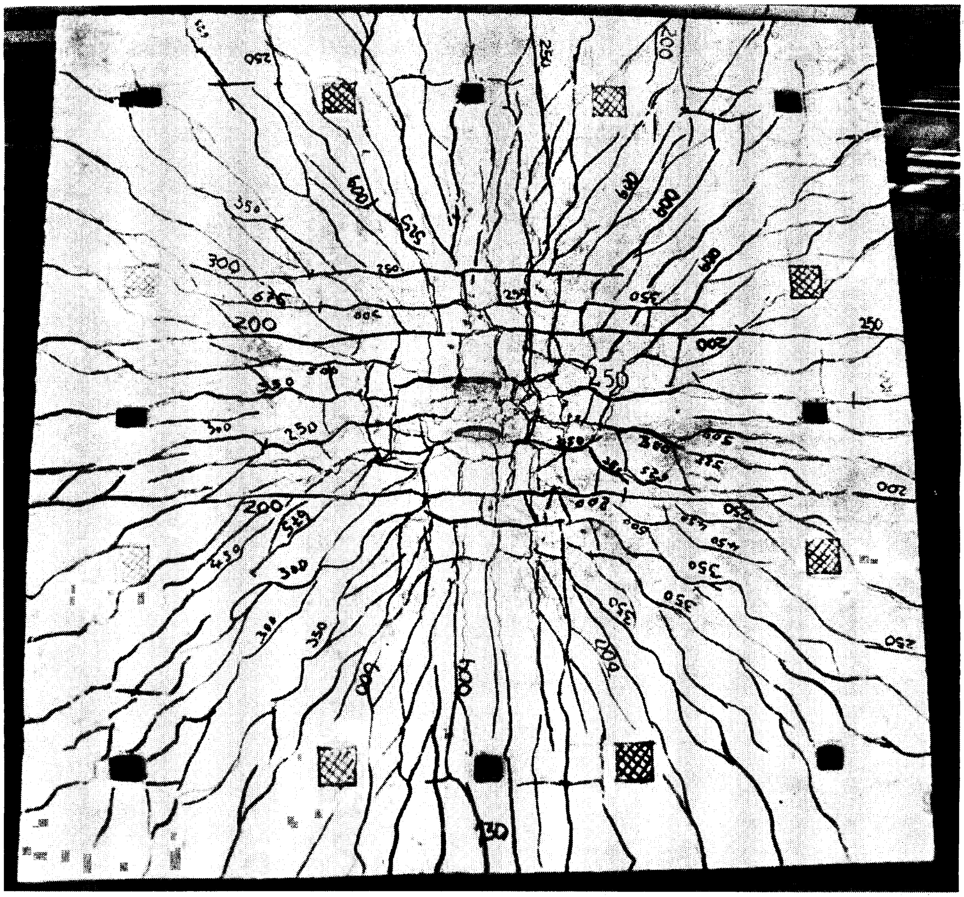

Fig. 11—Specimen 7. Crack pattern at ultimate load

$0.9\rho_b$. (When calculating ρ, the area of the bent bars in the direction considered shall be added to the support flexural reinforcement within half the column strip.)

Fabrication and installation

The stirrups shall preferably be prefabricated from welded deformed wire fabric, as shown in Fig. 16. In that way, stable and labor-effective installation units can be achieved, as demonstrated in Fig. 17.

The arrangement of the stirrups permits the bent bars as well as the bottom and top flexural reinforcement to be installed after the stirrups. It is, however, necessary that the designer coordinates the spacing of the flexural reinforcement with the spacing of the stirrups.

Numerical example

Calculate the theoretical ultimate capacity of Specimen 7.

From Table 1	MPa
Top reinforcement 20 ϕ 12 s 90	$f_y = 691$
Bottom reinforcement 28 ϕ 8 s 90	$f_y' = 467$
Bent bars 4 + 4 ϕ 16	$f_y = 656$
Stirrups ϕ 6 s 90/90	$f_y = 496$
Concrete	$f_c' = 17$
$d = 150$ mm	
$d' = 24$ mm	

Flexural capacity
Half column strip $\simeq$ 1000 mm
Guess neutral axis position $c = 74$ mm

$$f_s' = \frac{c - d'}{c} \cdot 0.003\, E_s = \frac{50}{74} \cdot 600 = 405 \text{ MPa} < f_y'$$

$$a = \frac{1}{0.85 \cdot 17}\left(\frac{8 \cdot 113 \cdot 691}{1000} + \frac{4 \cdot 201 \cdot 656}{1000} - \frac{12 \cdot 50 \cdot 405}{1000}\right) = 62.9 \text{ mm}$$

$$c = \text{a} \cdot 0.85^{-1} = 74.01 > 74 \text{ mm} \qquad \text{OK}$$

$$M_n = \left(A_s f_y - A_s' \cdot f_s'\right)\left(\text{d} - \frac{a}{2}\right) + A_s' \cdot f_s'\,(d - d')$$

$M_n = (20 \cdot 113 \cdot 691 - 28 \cdot 50 \cdot 405)\,(150 - 0.5 \cdot 62.9) \cdot 10^{-6} + 28 \cdot 50 \cdot 405\,(150 - 24) \cdot 10^{-6} = 189.4$ kNm

$$M_{max} = \frac{V_y}{8}\,(2 \cdot 1.0 + 2 \cdot 1.0 \cdot \tan 22.5) - \frac{V_y}{4}\,(0.5 \cdot 0.250 + 0.25 + 0.8 \cdot 0.300) = 0.2467\, V_y \text{ kNm}$$

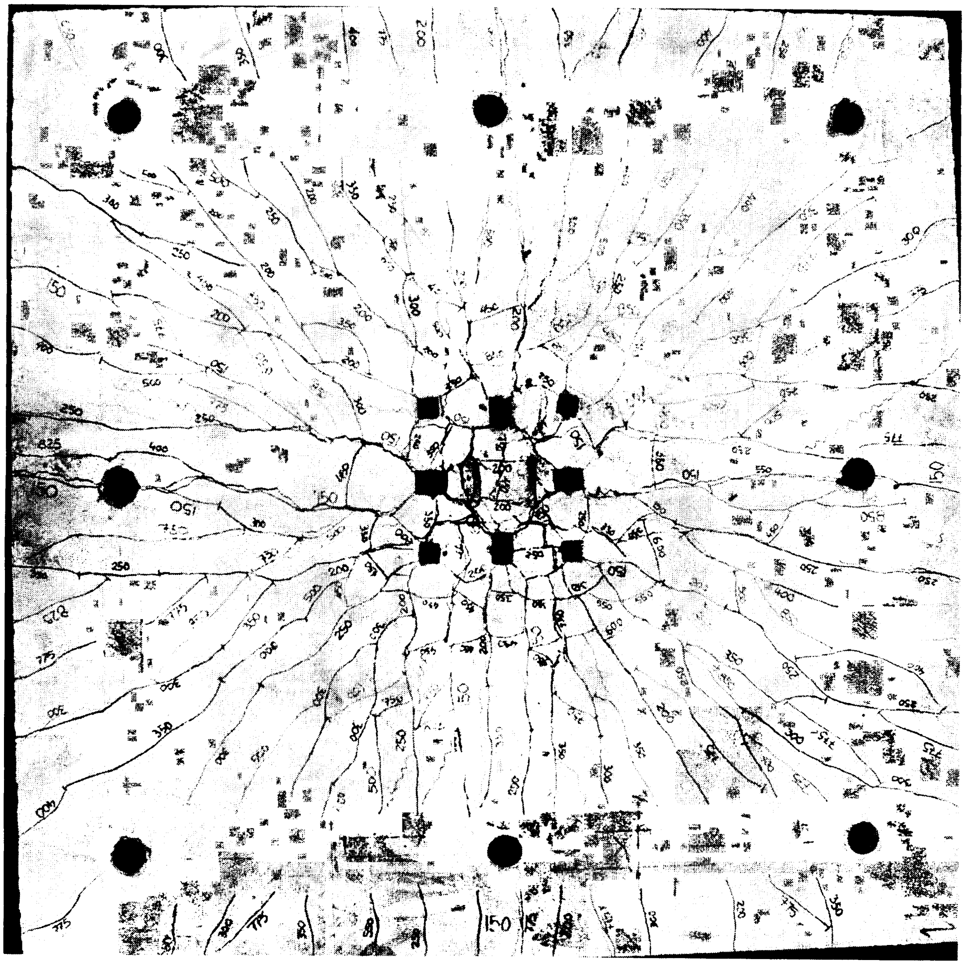

Fig. 12—Specimen 8. Crack pattern at ultimate load

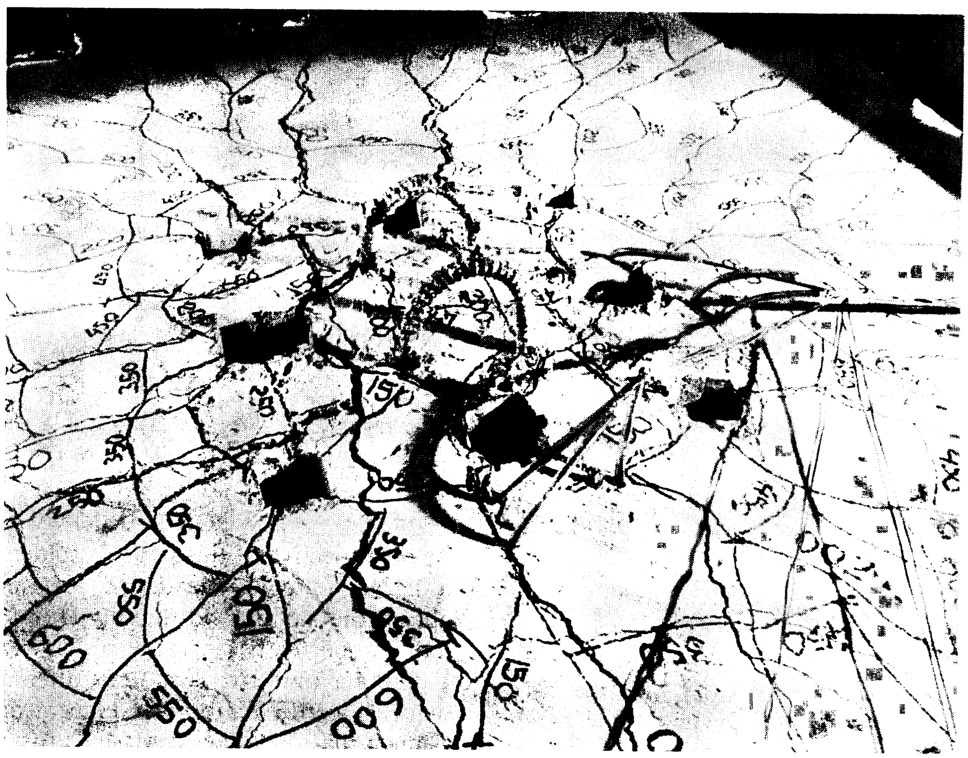

Fig. 13—Specimen 8. Close-up of cracks in the column region

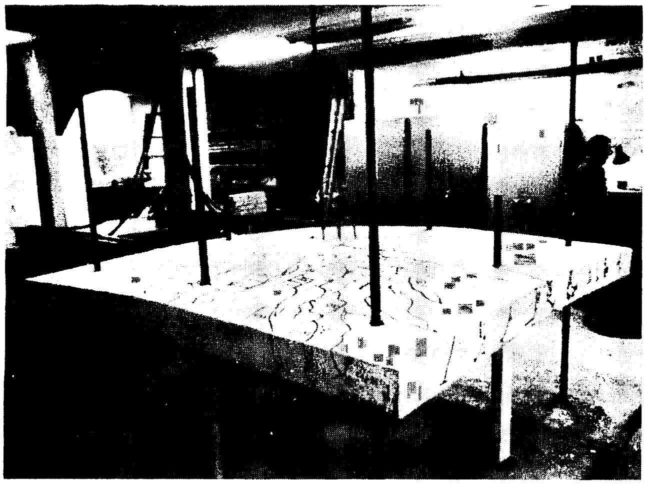

Fig. 14—Specimen 6 after removal of test equipment

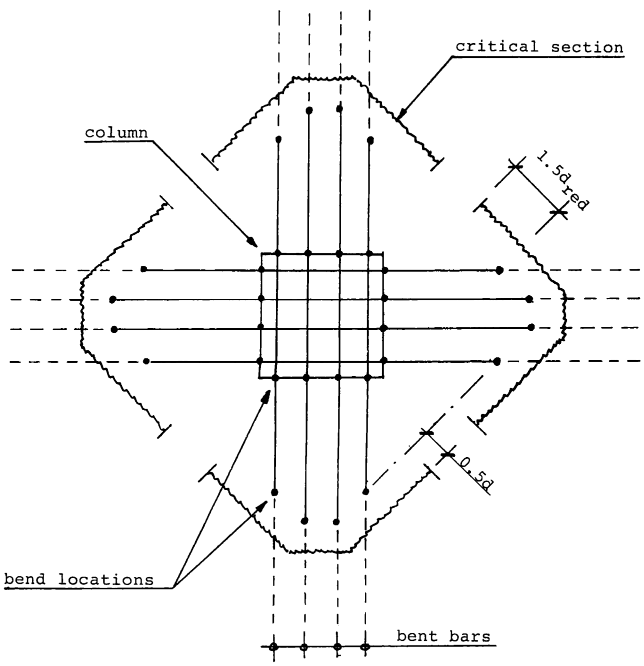

Fig. 15—Critical section for the design of stirrups

$M_{max} = M_n \rightarrow 0.2467\ V_y = 189.4$
$V_y = \underline{768}$ kN

Shear capacity at column
$4 + 4\ \phi\ 16$
$f_y = 656$ MPa

$$V_i = 8 \cdot 201 \left[\frac{114}{\sqrt{114^2 + 250^2}} + \frac{114}{\sqrt{114^2 + 350^2}}\right) \cdot 656 \cdot 10^{-3} = \underline{764}\ \text{kN} \simeq V_y \qquad \text{OK}$$

Shear capacity at critical section
$d_{red} = 114$ mm
$d = 150$ mm
Graphical solution gives $b_{eff} \simeq 4(350 + 350) = 2800$ mm

$$V_c = \tfrac{1}{6}\sqrt{17} \cdot 0.150 \cdot 2.8 \cdot 10^3 = 289\ \text{kN}$$

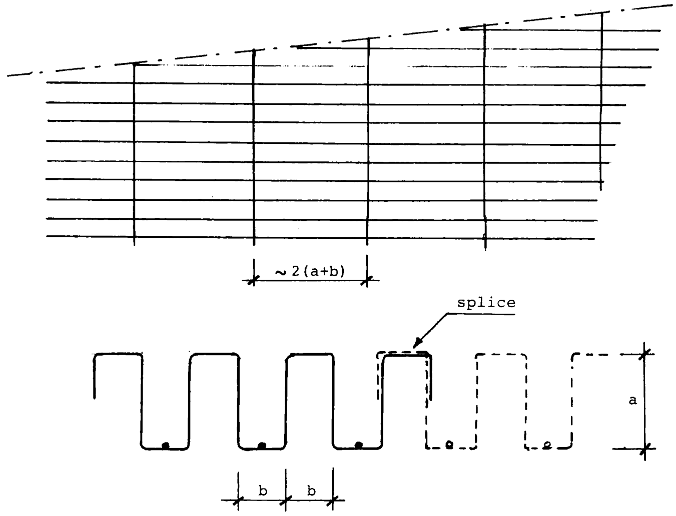

Fig. 16—Prefabricated stirrups made from welded wire fabric

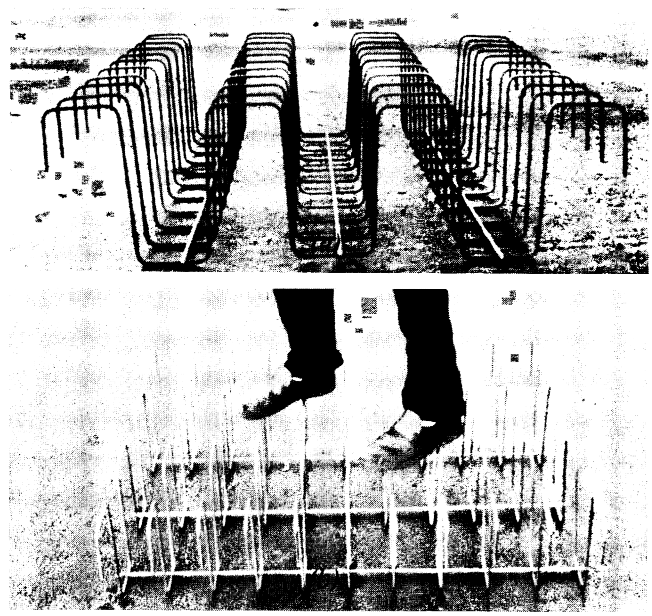

Fig. 17(a) and (b)—Stable and easy-to-handle prefabricated stirrup unit

Stirrups ϕ 6 s 90/90 $\qquad f_y = 340$ MPa

$$V_s = \frac{\pi \cdot 6^2}{4} \cdot 340 \cdot \frac{150}{90} \cdot \frac{2800}{90} \cdot 10^{-3} = 498\ \text{kN} < 2V_c \qquad \text{OK}$$

$$V_c + V_s = \underline{787}\ \text{kN} > V_y \qquad \text{OK}$$

CONCLUSIONS

One objective of this paper is to demonstrate that by using very simple precautions, it is indeed possible to obtain the same good ductility for flat plates as for slabs supported by beams or walls.

Another objective is to draw attention to the inconsequence of the majority of current building codes inasmuch that they have provisions for beams to achieve a ductile shear failure mode, but at the same time accept a brittle punching failure mode for flat plates.

The tests performed by the author have shown that ordinary shear reinforcement in the form of stirrups enclosing only the tension flexural reinforcement are not effective enough to give flat plates the desired ductility. On the other hand, a combination of bent bars and multiple U-stirrups enclosing the bottom compression reinforcement gave the specimens excellent performance.

The tested bent bar and stirrup combination is easy to install, and particularly so if the stirrups are prefabricated from welded wire fabric.

The proposed system is only meant for interior columns. Suitable arrangements for edge columns and corner columns have not yet been developed.

NOTATION

a = depth of equivalent rectangular stress block
b_o = perimeter of critical section at the distance $d/2$ from column face
b_{eff} = effective length of critical section as defined in Fig. 15
c = distance from extreme compression fiber to neutral axis
c_1 = size of rectangular column in the direction that moments are being determined
c_2 = size of rectangular column transverse to the direction that moments are being determined
d = distance from extreme compression fiber to centroid of tension reinforcement
d' = distance from extreme compression fiber to centroid of compression reinforcement
d_b = nominal diameter of bar
d_{red} = distance from centroid of horizontal legs of bent bars to centroid of tension reinforcement
f'_c = compressive cylinder strength of concrete
f'_{cube} = compressive cube strength of concrete
f_y = yield strength of reinforcement
f_u = tensile strength of reinforcement
l_1 = length of span in direction that moments are being determined, measured center-to-center of supports
l_2 = length of span transverse to l_1, measured center-to-center of supports
l_n = clear span measured face-to-face of supports
s = spacing of reinforcement, mm (Table 1) (16 ϕ 12 s 120 means 16 bars with the diameter 12 mm and with spacing 120 mm)
w_u = factored load per unit area
V = shear force transferred from slab to column
V_c = nominal shear strength provided by concrete = $1/3\ b_o d \sqrt{f'_c}$ (with f'_c in MPa) in Table 2
V_{calc} = calculated capacity of test specimen (the lesser of V_y, V_n, and $V_{n_{max}}$)
V_i = vertical component of the yield force in bent bars
V_n = nominal shear strength = $V_c/2 + V_s$ in Table 2
$V_{n_{max}}$ = $0.5\ b_o d \sqrt{f'_c}$ (with f'_c in MPa) in Table 2
V_y = shear force at yield of all flexural reinforcement
V_s = shear strength provided by shear reinforcement
V_{test} = ultimate capacity for test specimen
ρ = ratio of tension reinforcement
ρ_b = reinforcement ratio producing balanced strain conditions

CONVERSION FACTORS

1 mm	=	0.0394 in.
1 m	=	3.281 ft
1 kN	=	0.2248 kip
1 MPa	=	145 psi
$\sqrt{f'_c}$ (MPa)	=	12 $\sqrt{f'_c}$ (psi)

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