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SHEAR STRENGTH OF CONTINUOUS PLATES

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INTRODUCTION

A perennial problem in the design of flat plate reinforced concrete structures is the plate-column connection. A region of high shear and moment, this connection has proven to be the Achilles' heel of flat plate construction and, consequently, it has been the subject of numerous studies. While attempts have been made to model mathematically the shear strength behavior of flat plates (6,7), no satisfactory model yet exists and current design procedures (1) are necessarily empirical.

Test specimens used in studying parameters affecting shear strength have for the most part consisted of square, rectangular, or circular specimens with edges free to lift, and centrally loaded through a column stub. Footing type specimens such as these do not fully model the plate-column connection in the following respects:

1. Assuming elastic conditions, the distribution of moments and shears around the column in the footing type model is different than for a real structure.
2. In-plane forces which may be present in the real structure are absent in the model.
3. Redistribution of forces which can take place in the real structure with progressive increase in load is largely absent in the model.

A test specimen which more closely simulates the plate-column connection in

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TABLE 1.—DESCRIPTION OF TEST SERIES

Mark (1)	b, in inches[a] (2)	r, in inches (3)	d, in inches (4)	r/d (5)	$\rho-$ (6)	$\rho+$ (7)	f'_c, in pounds per square inch[b] (8)	f_{sp}, in pounds per square inch (9)	f_y, in kips per square inch[c] (10)	V_u, in kips[d] (11)	v_u, in pounds per square inch (12)	$v_u/\sqrt{f'_c}$ (13)
2S1–1	12	3	1.5	2	0.01	0.005	4,000	388	43.9	9.65	536	8.49
3S1–2	18	4.5	1.5	3	0.01	0.005	3,330	352	43.9	10.48	388	6.72
4S1–3	24	6	1.5	4	0.01	0.005	3,010	334	43.0	11.54	321	5.85
3C1–4	18	4.5	1.5	3	0.01	0.005	3,200	335	43.0	13.13	487	8.61
6S1–5	36	9	1.5	6	0.01	0.005	3,070	356	43.0	17.60	326	5.89
8S1–6	48	12	1.5	8	0.01	0.005	2,970	323	42.8	20.28	282	5.16
2S2–7	12	3	1.5	2	0.02	0.01	3,370	338	46.1	11.13	619	10.68
4S2–8	24	6	1.5	4	0.02	0.01	3,140	349	59.6	15.54	432	7.67
6C1–9	35.3	8.82	1.5	5.88	0.01	0.005	3,730	434	56.8	21.63	409	6.68
8S2–10	48	12	1.5	8	0.02	0.01	3,810	418	56.1	25.65	356	5.77
2C1–11	12	3	1.5	2	0.01	0.005	2,890	295	56.0	8.77	487	9.07
4C1–12	24.25	6.06	1.5	4.04	0.01	0.005	3,215	422	56.1	16.30	448	7.90
8C1–13	47.5	11.89	1.5	7.93	0.01	0.005	3,480	358	56.0	22.68	318	5.40
6S2–14	36	9	1.5	6	0.02	0.01	2,990	300	57.5	18.01	334	6.10
4C2–15	24.3	6.07	1.5	4.05	0.02	0.01	3,120	323	55.0	21.70	595	10.65

[a] 1 in. = 2.54 cm.
[b] 1 psi = 6895 N/m^2.
[c] 1 ksi = 6.895 N/m^2.
[d] 1 kip = 4448 N.

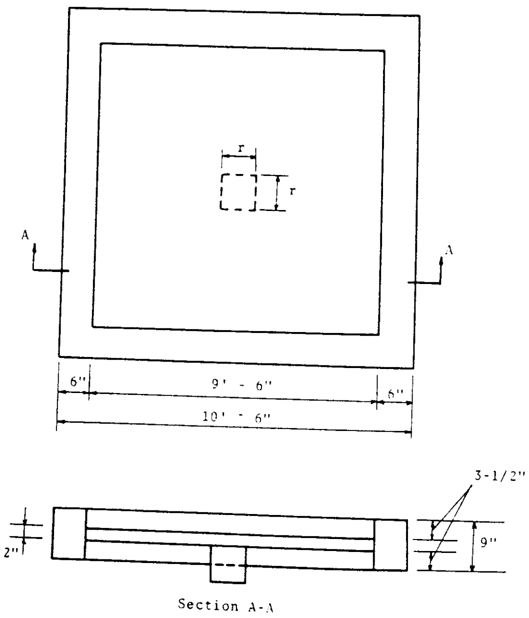

FIG. 1.—GEOMETRY OF TEST SPECIMEN

continuous two-way construction has been developed (9). The results of the tests made using this new type specimen are described herein.

TESTING PROGRAM

Description of Test Specimens.—Each specimen consisted of a square concrete plate cast integrally with a surrounding ring beam and a central column, as shown in Fig. 1. The only geometric variables were the column size and shape. Both square and circular columns were tested. All pertinent variables are summarized in Table 1. The Mark in Table 1 is translated as follows. The first numeral is the r/d ratio, the letter indicates a square (S) or circular (C) column, the second numeral is the reinforcing ratio as a percentage for the negative steel over the column, and the final number refers to the sequence of testing.

The ring beams and the column were heavily over-reinforced to prevent their premature failure. The slab reinforcement consisted of No. 2 deformed bars. Stress-strain curves for the No. 2 bars showed a sharply defined yield plateau with strain-hardening beginning at about 1-1/4 % strain. The reinforcing layout for a slab having 1 % negative steel is shown in Fig. 2. The concrete was made using sand, 0.5-in. stone aggregate, and type 3 cement.

Test Setup and Procedure.—Each specimen was cast in a form on the testing lab floor, moist cured in place 7 days or more and then placed on the testing frame as shown in Fig. 3. A strain-gage was attached to the concrete adjacent to the column before removing the specimen from its form. After placing in the test frame the supports were adjusted until the gage indicated zero strain. This was done so that the specimen before loading was not subjected to any differential support settlement stresses. The effects of dead load were ignored as being small. Uniform loading was provided through an air bag and was monitored using a precision dial manometer. Reactions were measured using tube dynamometers. The details of the loading and reaction measuring systems were reported in an earlier paper (11).

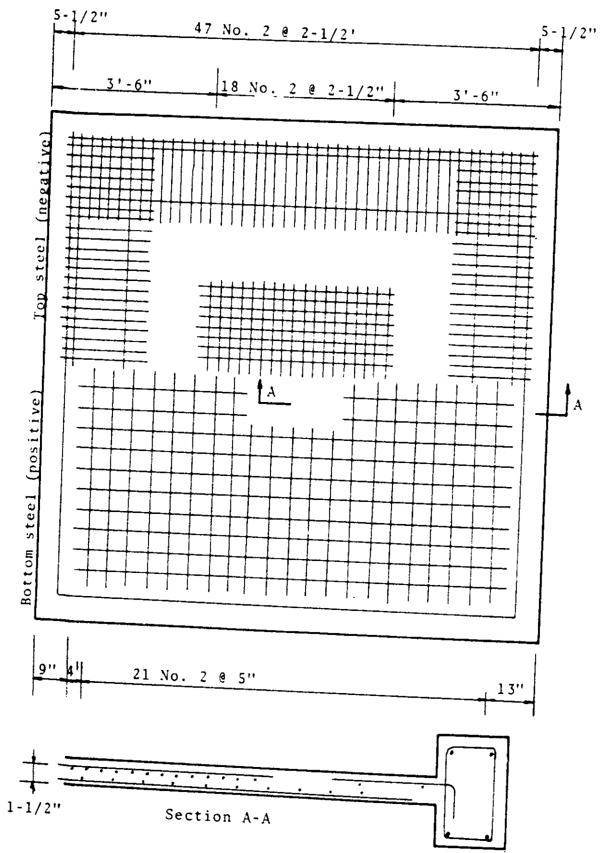

FIG. 2.—LAYOUT OF 1 % REINFORCING

The procedure followed in testing one slab consisted of preparing the slab for loading, applying load in increments, measuring reaction and deflection data, and marking cracks for each increment. Complete details of all aspects of the test program are given elsewhere (3, 10).

RESULTS

Elastic Analyses.—Analyses to determine the elastic behavior of the test specimen were made using the finite difference method (5). Fig. 4 shows

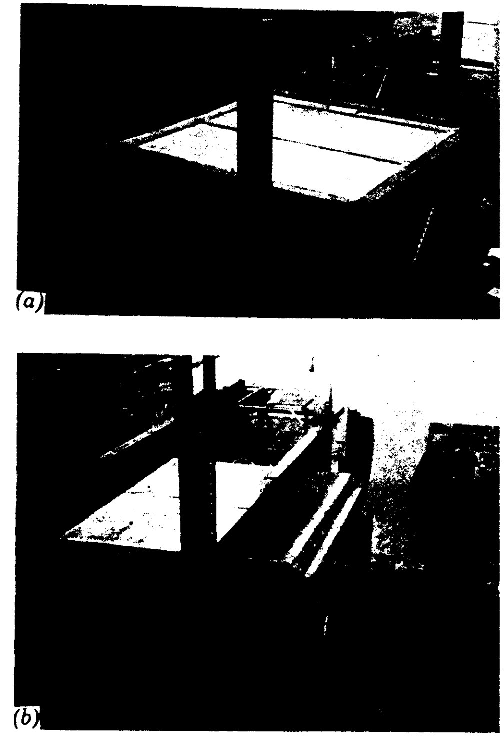

FIG. 3.—TEST SPECIMEN: (*a*) WITH AIR BAG IN PLACE AND (*b*) WITH REACTION FRAME IN PLACE

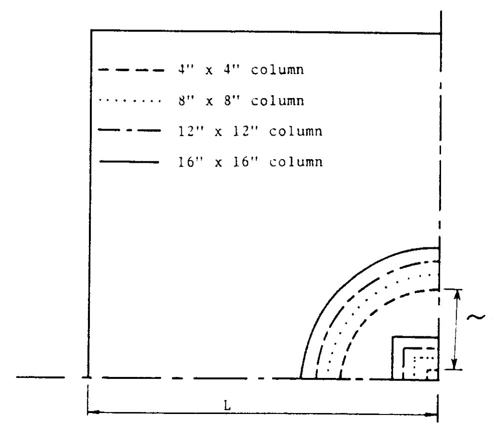

FIG. 4.—LINES OF CONTRAFLEXURE FOR PRINCIPAL MOMENTS FOR ONE-FOURTH OF SPECIMEN

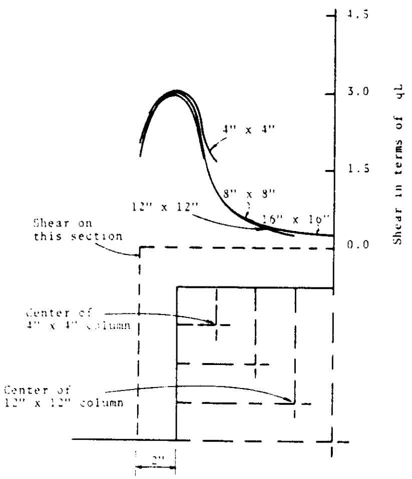

FIG. 5.—SHEAR AT 2 IN. FROM COLUMN FACE

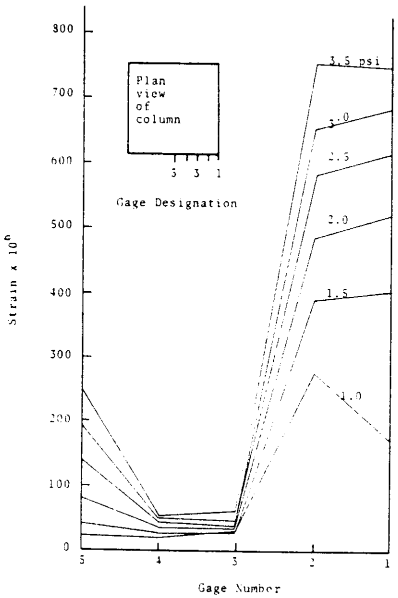

FIG. 6.—STRAINS IN SQUARE COLUMN

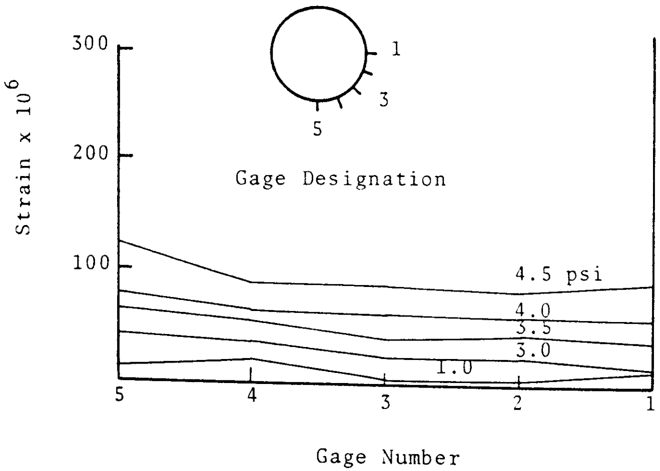

FIG. 7.—STRAINS IN ROUND COLUMN

the locations of lines of contraflexure around the column in the test specimen. These have the same general shape as the lines of contraflexure around column supporting an interior panel in a large array of continuous identical panels. However, the lines for the test specimen are located at about one-fourth the span from the column face rather than the one-sixth

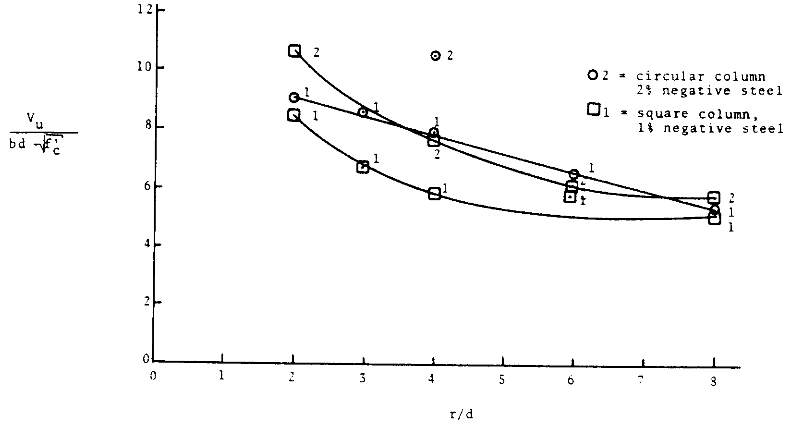

FIG. 8.—VARIATION OF STRENGTH WITH SIZE

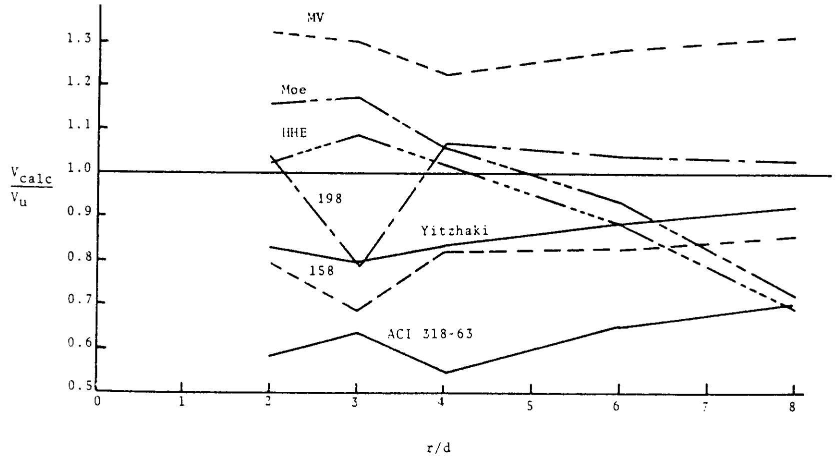

FIG. 9.—VARIATION OF AVERAGE V_{calc}/V_u WITH SIZE

span found in continuous plates. Thus while the test specimen models continuous two-way construction, it does not provide an exact model for interior plates.

The distribution of shearing forces close to the column face are shown in Fig. 5. The expected stress concentrations near the column corners are shown.

TABLE 2.—COMPARISONS OF CALCULATED AND TEST STRENGTHS

Mark (1)	V_u, in kips[a] (2)	V_{flex}, in kips (3)	V_{MOE}, in kips (4)	V_{MOE}/V_u (5)	V_{HHE}, in kips (6)	V_{HHE}/V_u (7)	V_{MV}, in kips (8)	V_{MV}/V_u (9)	V_{YITZ}, in kips (10)	V_{YITZ}/V_u (11)
2S1–1	9.65	24.4	11.66	1.208	10.86	1.125	13.30	1.378	7.53	0.780
3S1–2	10.48	26.7	13.87	1.323[b]	12.90	1.231[b]	15.43	1.472[b]	9.30	0.888
4S1–3	11.54	25.0	14.66	1.270	13.69	1.186	16.93	1.467	11.02	0.955
3C1–4	13.13	25.3	13.48	1.027	12.27	0.934	15.00	1.142	9.23	0.703
6S1–5	17.60	26.6	15.52	0.882	15.12	0.859	21.29	1.210	14.72	0.836
8S1–6	20.28	31.4	14.22	0.701[b]	13.18	0.650[b]	25.86	1.275	18.33	0.904
2S2–7	11.13	43.7	11.82	1.062	10.43	0.937	13.49	1.212	9.34	0.839
4S2–8	15.54	57.7	17.90	1.152	16.83	1.083	20.67	1.330	15.09	0.971
6C1–9	21.63	38.5	18.82	0.870	19.60	0.906	25.47	1.178	15.89	0.734
8S2–10	25.65	65.0	19.62	0.765	19.78	0.771	35.69	1.391	25.58	0.997
2C1–11	8.77	29.4	10.53	1.200	8.86	1.010	12.01	1.370	7.84	0.894
4C1–12	16.30	34.3	16.39	1.005	17.74	1.088	18.97	1.164	11.98	0.735
8C1–13	22.68	39.3	16.15	0.712	15.14	0.668	29.00	1.279	19.81	0.873
6S2–14	18.01	56.2	19.10	1.061	16.28	0.904	26.20	1.455	19.66	1.091[b]
4C2–15	21.70	52.5	17.67	0.814	15.62	0.720	20.46	0.943[b]	14.78	0.681[b]
Mean				1.003		0.938		1.284		0.859
Median				1.026		0.934		1.279		0.873
Standard deviation				0.204		0.183		0.147		0.118

[a] 1 kip = 4,448 *N*.
[b] High and low values in column.

Also it is seen that the peak shearing force is essentially independent of column size.

Behavior.—The effect of column shape on observed behavior is shown in Figs. 6 and 7 which give plots of strains measured in the columns about 1/2 in. beneath the bottom surface of the plate. These plots give graphic evidence of the strain concentrations present at the corners of square columns.

Each slab exhibited extensive cracking prior to failure. For all slabs failure was marked by a complete separation of column and slab concrete at the periphery of the column, i.e., the typical punching failure. The axial force in the column at failure is given in Table 1 and is designated V_u.

Strength.—A nondimensional plot showing the variation of punching strength with column size is given in Fig. 8. It is seen that for all other factors equal, a circular support permitted the development of greater shear strength than did a square support of equal periphery. Also doubling the steel increased shear strength by only 15 %-25 %.

ANALYSES

Empirical equations or procedures for the predictions of shear strength have been reported by Moe (8), Hognestad, Elstner, and Hanson (4), Yitzhaki (12), Kinnunen and Nylander (6), Kinnunen (7), and Mowrer and Vanderbilt (9), and are considered in Appendix I. These equations (4, 8, 9, 12) and procedures (6, 7) were used to predict the strength of each test specimen and the results

are shown in Fig. 9 in the form of ratios of calculated to test strengths plotted versus r/d. It is seen that no equation or procedure consistently predicted strength within an acceptable margin of error. Also shown in Fig. 9 is the strength computed using the provisions of ACI 318-63 (1). Observe that while generally the code equation provides a reasonable factor of safety, there is a definite decrease in factor of safety with increasing r/d, a trend which should be considered in the design of slabs with large shear heads.

Note that the equations by Moe and Hognestad, Hanson, and Elstner were not intended for use with r/d greater than about 3. Also the procedures developed by Kinnunen and Nylander involved a complex iterative technique and are most easily implemented through the use of a computer program.

SUMMARY AND CONCLUSIONS

The results of tests of 15 specimens of a new type of test specimen have been reported. The new type test specimen was developed to more closely simulate the state of stress around a column supporting an interior panel. Test variables were the size and shape of column and amount of reinforcing. The results are summarized in Tables 1-3 and Figs. 4-9.

The following conclusions applicable to this test series may be drawn:

1. The shear strength was a function of column shape, as well as size, with circular columns showing higher strength than square columns of equal pe-

riphery. This difference is attributed to stress concentrations present at the corners of square columns.

2. Doubling the reinforcing resulted in only a modest increase in shear strength.

3. None of the available equations or procedures for predicting shear strength proved to be a consistently reliable estimator.

4. The results obtained using the new type of test specimen are not greatly different from those obtained using footing type specimens. Therefore, it appears that the footing type specimens are satisfactory for use in making parameter studies.

TABLE 3.—COMPARISONS OF CALCULATED AND TEST STRENGTHS

Mark (1)	V_{158}, in kips[a] (2)	V_{158}/V_u (3)	V_{198}, in kips (4)	V_{198}/V_u (5)	V_{CODE}, in kips (6)	V_{CODE}/V_u (7)
2S1—1	7.00	0.725	8.33	0.863	5.80	0.601
3S1—2	8.13	0.775	9.49	0.905	7.06	0.674
4S1—3	8.83	0.765	10.70	0.928	8.39	0.727
3C1—4	7.82	0.595	8.87	0.675[b]	7.91	0.603
6S1—5	11.04	0.627	13.25	0.753	11.87	0.674
8S1—6	13.06	0.644	15.70	0.774	15.01	0.740[b]
2S2—7	9.42	0.846	13.50	1.213	5.33	0.479
4S2—8	15.84	1.019	22.65	1.457	8.57	0.552
6C1—9	13.48	0.623[b]	15.58	0.720	13.93	0.644
8S2—10	30.96	1.207	37.48	1.461	17.00	0.663
2C1—11	7.11	0.810	8.89	1.014	5.88	0.670
4C1—12	11.73	0.720	13.61	0.835	8.38	0.514
8C1—13	16.52	0.728	19.54	0.861	15.70	0.692
6S2—14	22.27	1.237[b]	29.68	1.648[b]	11.72	0.651
4C2—15	17.15	0.791	22.58	1.040	8.27	0.381[b]
Mean		0.808		1.010		0.618
Median		0.765		0.905		0.651
Standard deviation		0.199		0.300		0.099

[a] 1 kip = 4448 N.
[b] High and low values in column.

APPENDIX I.—EMPIRICAL STRENGTH EQUATIONS

In 1961, Moe (8) published the results of an extensive study of shear in footings of slabs. All of the specimens considered failed in shear and were either simply supported or rested on a bed of springs. Based on his test results of 43 slabs and a statistical analysis of 37 other slabs and 106 footings, he developed the following equation:

$$\frac{V_u}{bd\sqrt{f'_c}} = \frac{15\left(1 - 0.075\dfrac{r}{d}\right)}{1 + \dfrac{5.25\,bd\sqrt{f'_c}}{V_{flex}}} \quad \text{(1)}$$

Other investigators have revised Eq. 1 to fit other data. In 1964 Hanson, Hognestad and Elstner (4) tested six lightweight concrete slabs, identical to three slabs tested by Moe, to aid in the development of the 1963 ACI Building Code. To account for the strength properties of lightweight concrete they introduced the splitting strength, f_{sp}. Using f_{sp} equal to $6.7\sqrt{f'_c}$ for normal weight concrete, Eq. 1 was recast as

$$\frac{V_u}{bd} = \frac{2.24\left(1 - .075\dfrac{r}{d}\right)f_{sp}}{1 + \dfrac{.784\,bd\,f_{sp}}{V_{flex}}} \quad \text{(2)}$$

In 1966 Mowrer and Vanderbilt (9), utilizing the results of tests of 51 light and normal weight slabs, developed the following variation of Eq. 1:

$$\frac{V_u}{bd\sqrt{f'_c}} = \frac{9.7\left(1 + \dfrac{d}{r}\right)}{1 + \dfrac{5.25\,bd\sqrt{f'_c}}{V_{flex}}} \quad \text{(3)}$$

Based on considerations of flexural strength, Yitzhaki (12) developed in 1966 the following strength equation:

$$V_u = 8\left(1 - \frac{\omega}{2}\right)d^2\,(149.3 + 0.164\,\rho f_y)\left(1.0 + 0.5\frac{r}{d}\right) \quad \text{(4)}$$

Based on Moe's equation the following design equation was developed for inclusion in the 1963 ACI Building Code (1):

$$V_u = b_o d\;4\phi\sqrt{f'_c} \quad \text{(5)}$$

in which the capacity reduction factor, ϕ, = 0.85.

In 1960 Kinnunen and Nylander (6), utilizing the results of tests of circular test specimens, developed an idealization of the failure mechanism around circular columns. On the basis of this idealization they developed a mathematical model for predicting punching strength. As numerous parameters used in the model are based on test results, the final procedure is here dubbed semiempirical. This model was modified by Kinnunen in 1963 (7) to account for two-way reinforcement and dowel action.

The procedure followed in either method consists of computing the strength using each of two equations. When both answers obtained differ by only 2 % or 3 %, the average is taken as the computed strength. Many iterations are usually required before convergence. For details see Refs. 2 and 9.

APPENDIX II.—REFERENCES

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APPENDIX III.—NOTATION

The following symbols were used in this paper:

b = perimeter of column;
b_o = design perimeter at $d/2$ from column face;
d = effective depth from compressive face of concrete to centroid of tensile steel;
f'_c = compressive cylinder strength of concrete;
f_{sp} = splitting strength of concrete;
f_y = yield stress of steel;
L = span defined in Fig. 4;
q = uniformly distributed load, force per unit area, Fig. 5;
r = length of side of square column or $b/4$ for round columns;
V_{CODE} = capacity computed using equation from ACI 318-63 (1);

V_{HHE} = capacity computed using Hognestad, Hanson, and Elstner's equation (4);
V_{MOE} = capacity computed using Moe's equation (8);
V_{MV} = capacity computed using equation developed by Mowrer and Vanderbilt (9);
V_{YITZ} = capacity computed using Yitzhaki's equation (12);
V_{calc} = calculated shear strength;
V_{flex} = axial force in column corresponding to flexural capacity determined using yield line theory;
V_u = ultimate shear load;
V_{158} = capacity computed using procedure developed by Kinnunen and Nylander (6);
V_{198} = capacity computed using refinement of aforementioned procedure developed by Kinnunen (7);
v_u = V_u/bd = shear stress at critical section;
ρ = reinforcing ratio; and
ω = $\rho f_y/f'_c$.

Conversion Factors.

1 in. = 2.54 cm;
1 ft = 30.48 cm;
1 kip = 4448 N;
1 psi = 6895 N/m^2; and
1 ksi = 6.895 N/m^2.