Using the strut and tie model

The use of an equilibrium truss model for the design of reinforced concrete deep beams is explained. The model is compared to test results and a design example is presented.

Design of Reinforced Concrete Deep Beams

by D. M. Rogowsky and J. G. MacGregor

Three procedures are currently used for the design of load transfer members such as deep beams:

—Empirical design methods,

—Two or three dimensional analysis, either linear or nonlinear, and

—By means of trusses composed of concrete struts and steel tension ties.

Most of this article will deal with the strut and tie model for deep beam design. Before doing so, the other two options will be examined briefly.

Empirical design methods

In Reference 1, the results of tests of one and two span deep beams are compared to the shear capacity calculated by the empirical design procedure in Section 11.8 of ACI 318-83, Standard Building Code.[2] The ratios of the test to calculated strengths range from 1.14 to 2.26 for simple beams suggesting that the ACI Code is conservative for such beams. When the ACI procedure is compared to tests of continuous deep beams, however, the ratios of test to calculated strengths range from 1.39 down to 0.48. Over half of the tests of continuous beams had measured strengths less than the strength predicted by the ACI Code. This discrepancy arises because the empirical design method given in Section 11.8 of the ACI Code is not based on a clear mechanical model of the behavior.

Design based on stress analysis

Fig. 1(b) shows the crack pattern in a two span continuous deep beam after testing. The solid lines in parts (a) and (c) of this figure show the total measured force in the longitudinal reinforcement immediately before failure. Measurements were made using Demec Gauges between targets on the steel, shown by dots in Fig. 1(b). The tensile force calculated using a standard two dimensional, elastic continuum, finite element program (SAP IV) is plotted in Fig.

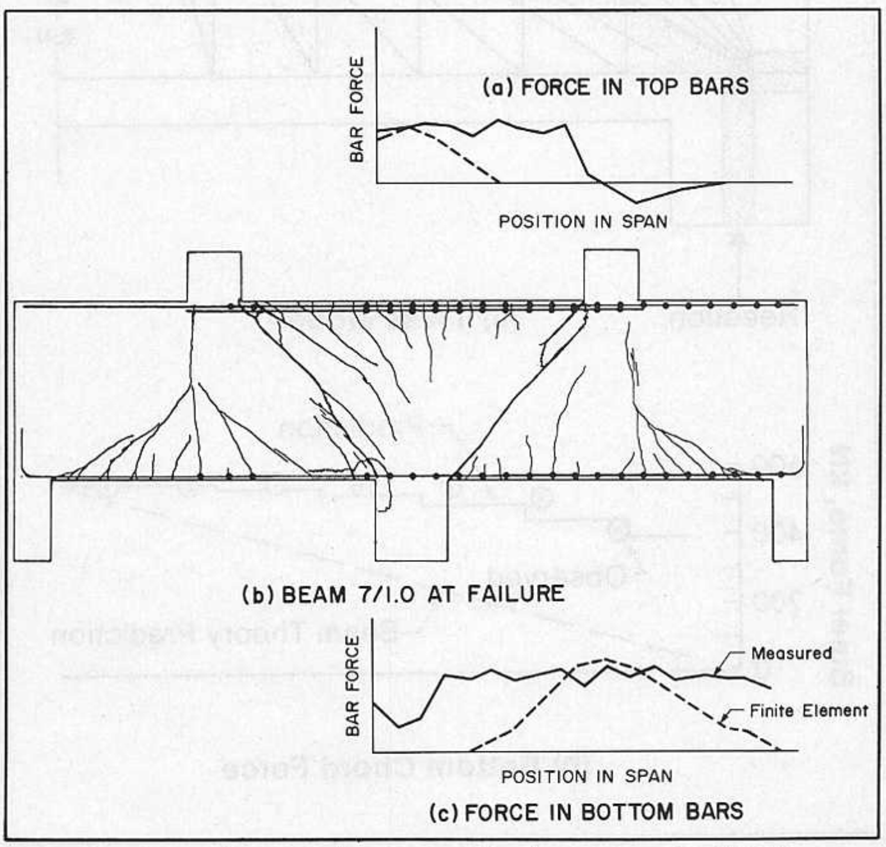

Fig. 1—Comparison of measured bar force and bar force computed by SAP IV analysis.

Keywords: beams (supports); **deep beams;** plasticity; **reinforced concrete;** shear strength; **structural design.**

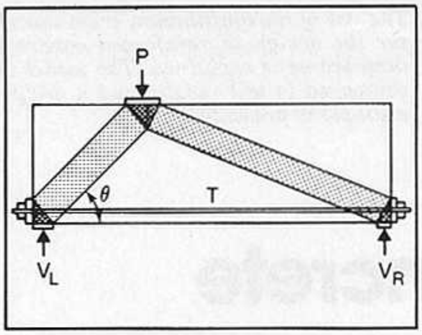

Fig. 2—An equilibrium truss model.

1(a) and (c) with dashed lines.

This tensile force was obtained as the total tension across vertical lines in the bottom half of the beam. It can be seen that there is little comparison between the bar forces from the finite element model and those measured in the test except at midspan and over the center support where the two agreed almost exactly. This is because the finite element model was no longer valid once the concrete cracked. If the longitudinal reinforcement had been curtailed according to the SAP IV tension force diagrams the authors believe the beam would have failed prematurely.

It is recognized that more exotic finite element analyses are available which recognize the low tensile strength of concrete and which would predict the behavior better. These programs are not widely available, however, and are too expensive for use in routine design. The poor agreement between the measured and computed bar forces suggests that the use of two dimensional stress analyses in the design of reinforced concrete should be approached with great caution.

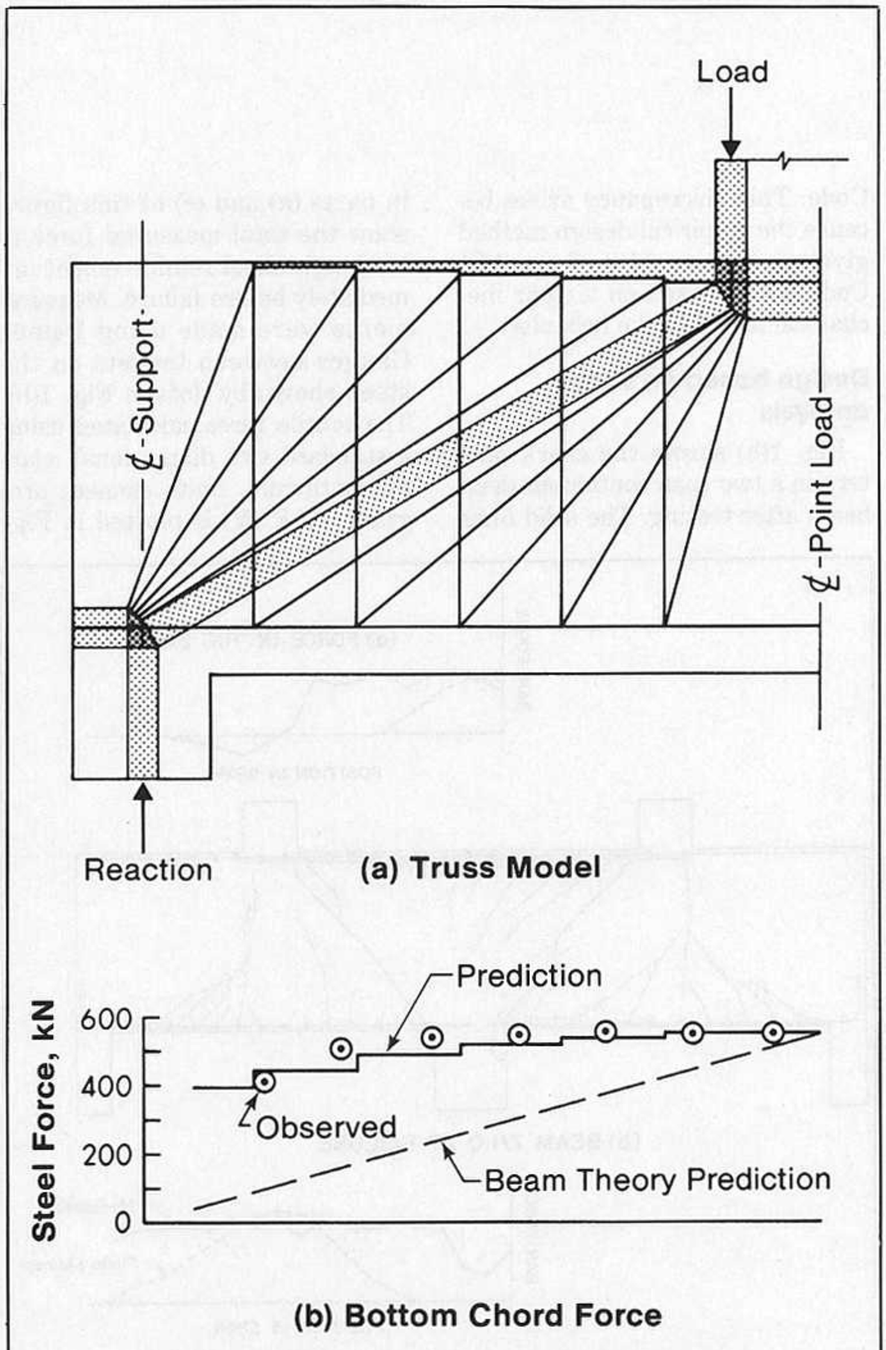

Fig. 3—Truss model for a simple beam with web reinforcement.

Equilibrium truss models for deep beams

The balance of this article deals with the use of equilibrium truss models for design of deep beams. In such a model the member is idealized as a series of tension ties, concrete struts, loads and supports interconnected at nodes to form a truss (Fig. 2). Such models have been proposed by a number of authors including Grob and Thürlimann,[3] Nielsen et al.,[4] and Marti.[5]

Assumptions

Equilibrium truss models assume or require that:

(1) Equilibrium must be satisfied,

(2) The concrete only resists compression and has an effective compressive strength f_{ce} equal to $\nu f'_c$ where the efficiency factor, ν, is usually less than 1.0,

(3) Steel is required to resist all tensile forces,

(4) The centroids of each truss member and the lines of action of all externally applied loads at a joint must coincide.

(5) Failure of the equilibrium truss model occurs when a concrete compression member crushes, or when a sufficient number of steel tension members reach yield to produce a mechanism.

These assumptions satisfy the lower-bound theorem of plasticity which states that:

> If an equilibrium distribution of stresses can be found which balances the applied load and is everywhere below yield or at yield, the structure will not collapse or will just be at the point of collapse. Since the structure can carry at least this applied load, it is a lower bound to the load carrying capacity of the structure.

While this theorem has a rigorous mathematical basis, it is obvious that if one can find a safe load path through the structure this will be a conservative lower bound to the true capacity of a structure since there may be, and often are, other load paths which could carry a greater load. Design procedures based on the lower bound theorem require the designer to identify at least one plausible load path and insure that no portion of the load path is overstressed.

The choice of load paths is limited by the deformation capacity of the beam. If the assumed load path differs greatly from the original load path in the cracked beam, the structure may be unable to undergo the force redistribution necessary to reach the assumed load path.

Components of equilibrium truss models

Equilibrium truss models are made up of five building blocks:

(1) Struts or concrete compression members, shown by light shading in Fig. 2 to 4, are in uniaxial compression with a uniform stress of f_{ce} at the ultimate load. The struts have a finite width and thickness which depend on the force in the member and the permissible stress level. The end faces of a strut are principal stress faces and must be perpendicular to the longitudinal axis of the strut.

(2) Ties or steel tension members are permitted to reach and sustain the yield stress f_y. Collapse of the beam does not occur when a single tension member reaches yield unless this is sufficient to convert the truss into a mechanism.

(3) Joints. The centroid of each truss member and the lines of action of all external loads at a joint must coincide. With such a concurrent force system there is no moment in the joint, making the assumption of a pinned joint reasonable.

Joints are accommodated by "hydrostatic stress elements" which are shown by dark shading in Fig. 2 to 4. The concrete within these elements has both in-plane principal stresses equal to f_{ce}. These elements may have any polygonal shape, but must have a uniaxial compression stress of f_{ce} acting on each face or facet of the element within the plane of the beam. (These elements do not have true hydrostatic stress since the stresses on the "free faces" will not be equal to f_{ce}. The Mohr's circle for the in-plane stresses does, however, plot as a point, as would be the case for true hydrostatic stress.)

The tension ties are shown anchored by bearing plates in Fig. 2 to 4. This is done as a convenient short-hand notation to emphasize that the tension tie anchorage must be able to positively anchor the bar and also that it must spread the anchored force over the depth of the nodal zone. In an actual beam, the anchorage would be accomplished with development length, hooks or in rare cases with end plates.

In a similar manner the loads are shown acting through bearing plates in Fig. 2. In a normal concrete structure this force transfer would occur where the beam supported a concrete column or was supported on such a column.

The requirement that the axis of loads and members meet at a point affects the size of the members meeting at that joint as shown in Fig. 5. When the tension tie is moved closer to the tension side of the beam, the size of the nodal element must decrease, causing a reduction in the size of the compression strut. This weakens the beam.

(4) Compression fans occur under point loads and over supports where a number of minor compression struts fan out, distributing the load or reaction to a number of stirrups. Compression fans can be seen in Fig. 3 and 6.

(5) Compression fields occur where parallel minor compression struts transmit force from one stirrup to another. Fig. 6 shows a short compression field between two compression fans.

Additional elements or building blocks are described by Marti.[5] To simplify the presentation only the five simple elements given here will be used in this article.

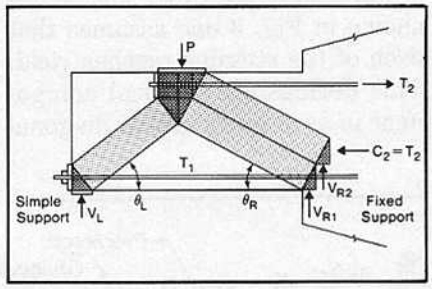

Fig. 4—Truss model for a beam with fixed-simple support conditions.

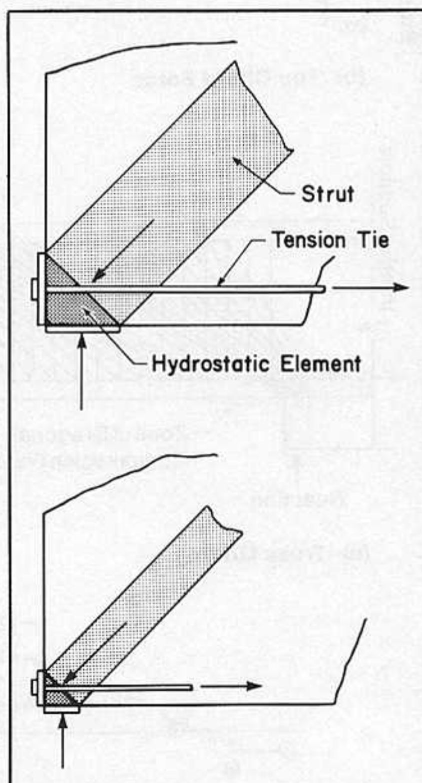

Fig. 5—Effect of tension tie location on size of compression strut.

Analysis of equilibrium truss models

The truss model in Fig. 2 is statically determinate. It does not require any assumed plasticity to evaluate its strength. From simple statics, $V = T \tan \theta$, where T is the force in the tension tie. The cross-sectional area of the compression strut must be sufficient to insure that the stress in the strut does not exceed f_{ce}.

The other truss models shown in the figures would normally be statically indeterminate. The plasticity available in the steel tension members is used to define or limit the force in enough truss members to make the structure statically determinate thus allowing a simple solution. For the truss shown in Fig. 3 one assumes that each of the stirrups reaches yield. This defines the vertical component in each of the minor diagonal struts shown by inclined lines. This in turn defines the horizontal and axial components in each minor strut.

The main reinforcement is also assumed to yield at midspan. Working from midspan outward, the force in the remaining truss members can be determined. The stepped line in Fig. 3 (b) shows the computed force in the lower chord members from this analysis. There will still be a significant force in the bottom chord at the face of the left support. Part of this force equilibrates the horizontal component of the major direct compression strut, while the remainder equilibrates the horizontal force components of the minor struts radiating upward from the support. Again, the compression struts should be checked to ensure that they are not overstressed.

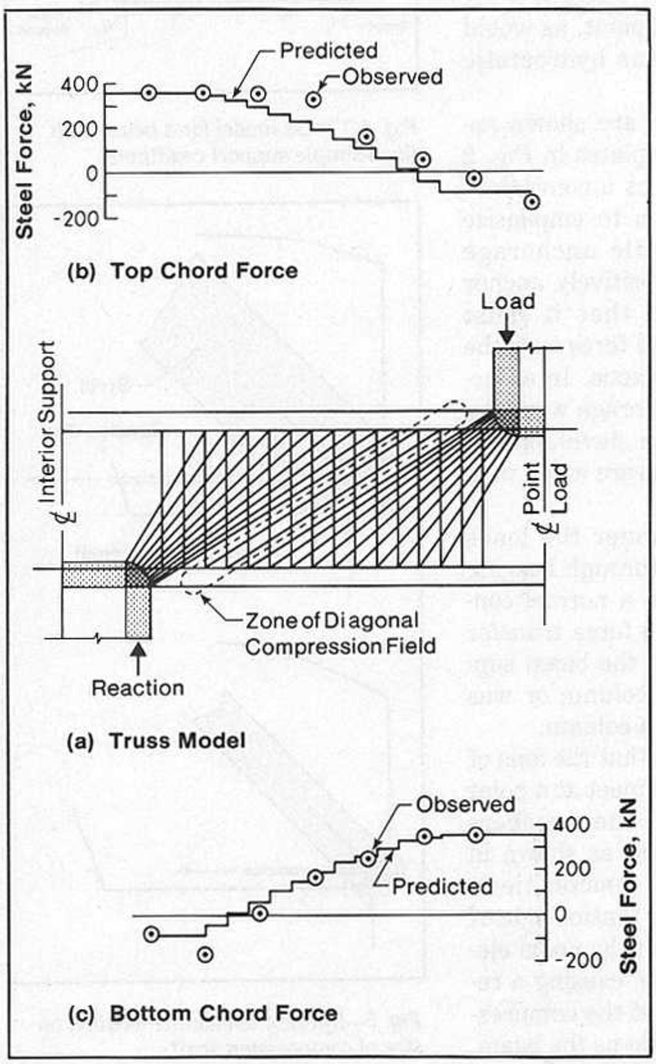

Fig. 6—Truss model for interior shear span of a two-span continuous deep beam with heavy stirrup reinforcement.

Behavior of concrete in web of beam, nodal zones

In design applications, it is desirable that the concrete compressive struts and nodal zones not fail before the longitudinal steel and stirrups yield to form a mechanism. This assures somewhat ductile behavior and, if the full mechanism can develop, the failure load will equal or exceed the design load. It is necessary, therefore to limit the stresses and deformations sustained by the concrete.

Effective concrete strengths

In deriving the equilibrium truss model the concrete struts and nodal zones were assumed to be stressed to $f_{ce} = \nu f'_c$ where ν is referred to as the the efficiency factor. Various authors have proposed different values for ν expressed here in terms of cylinder strength: the accurate method in the 1978 CEB Model Code[6] sets an upper limit on shear to avoid web crushing. This is equivalent to $\nu = 0.6$ based on the CEB safety factors. Allowing for differences in the CEB and ACI load and resistance factors, this would be equivalent to $\nu = 0.51$. M.P. Nielsen[4] and his co-workers have proposed:

$$f_{ce} = \left(0.7 - \frac{f'_c}{29{,}000}\right) f'_c \text{ (psi)}$$

$$f_{ce} = \left(0.7 - \frac{f'_c}{200}\right) f'_c \text{ (MPa)} \qquad (1)$$

Ramirez[7] has proposed:

$$f_{ce} = 30\sqrt{f'_c} \text{ (psi)}$$

$$f_{ce} = 2.5\sqrt{f'_c} \text{ (MPa)} \qquad (2)$$

Rogowsky[8] observed that the selection of the concrete truss was more important than the selection of a value of ν. He observed that if the truss selected differed excessively from the elastic distribution, full redistribution may not occur and the truss would fail prematurely, giving the appearance of a low value of ν. In his analysis, Rogowsky used $\nu = 0.85$. This value was used in drawing Fig. 3 and 6.

Schlaich and Weischede[9] observed that the compression struts in Fig. 5 have their least width at the nodal points and, in the real case, can increase in width between the nodal points. This induces tensile stresses transverse to the strut which limit the compressive strength of the strut. Schlaich and Weischede give a mini-truss model for the strut itself and a graph of the compressive strengths, $f_{ce} = \nu f_c'$ of the struts. If, for example, the effective width of the strut halfway between its ends is twice its width at the nodes, they find that ν is 0.67. Schlaich and Weischede recommend that the slope of the compression struts should be within ± 15 deg of the slope of the elastic compressive stress trajectories.

Based on extensive tests of uniformly strained shear panels, Collins and Mitchell[10] and Vecchio and Collins[11] have related the effective concrete strength to the principal tensile strain, ϵ_1, at right angles to the direction of the principal compressive stress (Fig. 7):

$$f_{ce} = \frac{f_c'}{0.8 + 170\epsilon_1} \qquad (3)$$

The principal tensile strain, ϵ_1, is obtained from a Mohr's circle for strain relating the strain ϵ_x parallel to the axis of the beam, the angle θ and the maximum strain ϵ_2 corresponding to the highest point in a compressive stress-strain curve for concrete, taken as 0.002. In lieu of an analysis for ϵ_x, Collins and Mitchell suggest it can be taken as 0.002. If ϵ_2 and ϵ_x are taken equal to 0.002, ν varies almost linearly from 0 when θ = 0 deg, to about 0.55 when θ = 45 deg, and back down to 0 when θ = 90 deg.[10]

The strength of the concrete in the nodal regions is generally taken as f_{ce}. In the 1984 Canadian Code,[12] however, it is taken as: (a) 0.85 f_c' in nodal zones bounded by compressive struts and bearing areas, (b) 0.75 f_c' in nodal zones anchoring only one tension tie, and (c) 0.60 f_c' in nodal zones anchoring tension ties in more than one direction. This is intended to reflect the incompatibility of strains between the tension tie and compressed nodal zone and the nonuniformity of stress in regions anchoring tension ties, particularly if the anchorage is accomplished by hooks or bond. These values should be multiplied by about 0.80 to account for the differences in load and resistance factors between the Canadian and ACI codes.

In the design example that follows, ν has been taken equal to 0.60 and this value is recommended for general use.

Allowable strut angles

When a slender concrete beam is loaded, the initial cracks form at about 45 deg. If there are well anchored stirrups present, subsequent cracks will form at a flatter angle θ, crossing the original cracks. In the compression fan regions the cracks range from θ to almost vertical. Many investigators have recognized the need for limiting the strut angles in beams: Thurlimann[3] proposed that tan θ be between 0.5 and 2 (θ between 26.6 and 63.4 deg) based on a study of the geometry of the crack openings. The 1978 CEB Model Code[6] reduced this range, allowing tan θ to fall between 3/5 and 5/3 (θ between 31 and 59 deg). Collins and Mitchell[10] do not limit θ but limit f_{ce} as a function of θ. Ramirez[7] and Rogowsky[8] suggest 25 to 65 deg. In the design example that follows the limits on θ have been taken as 25 and 65 deg.

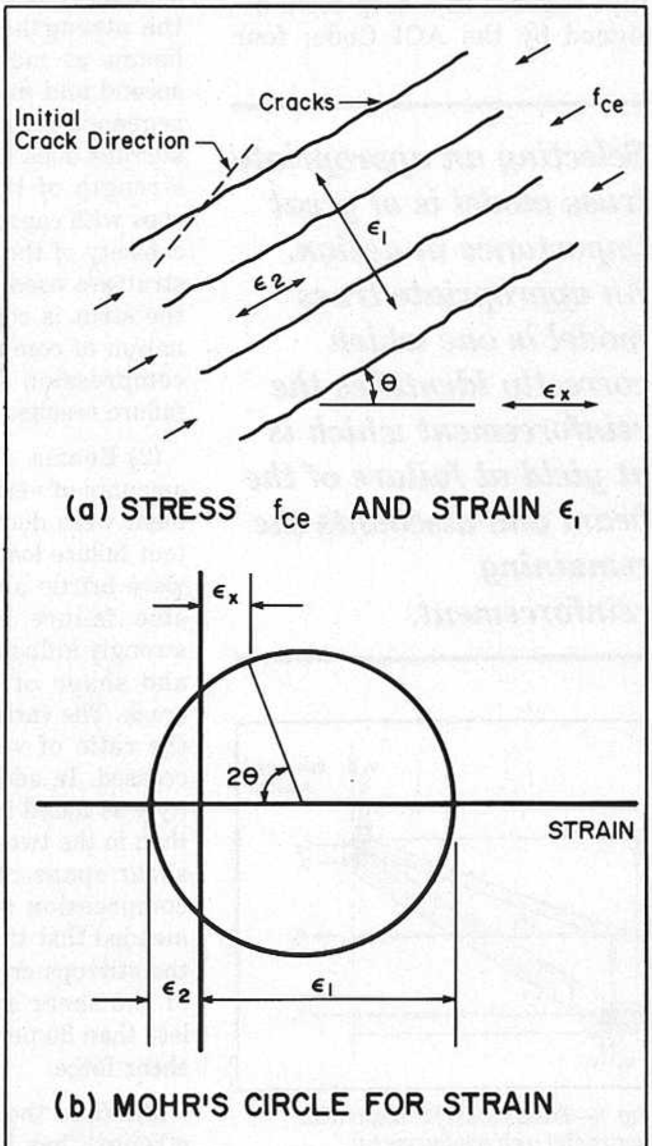

Fig. 7—Strains in a cracked web.

It should be noted that the flatter the angle of the compression diagonals, the longer the longitudinal bars must extend before they can be cut off. Thus, by reducing θ, the area of stirrups may be reduced, but the length of the main steel is increased.

Comparison of equilibrium truss model to beam tests

The equilibrium truss model was compared to a series of 24 large scale deep beams which included:[1,8]

(a) Seven simple span beams and 17 two span continuous beams, all loaded with one concentrated load per span,

(b) Shear span to depth ratios ranging from about 1 to about 2.5, and

(c) Various web reinforcement: none; the minimum vertical stirrups required in a deep beam designed by the ACI Code; four times the minimum stirrups; roughly half the minimum horizontal shear reinforcement required by the ACI Code; and about 1.5 times the minimum horizontal reinforcement.

Selecting an appropriate truss model is of great importance in design. An appropriate truss model is one which correctly identifies the reinforcement which is at yield at failure of the beam and discounts the remaining reinforcement.

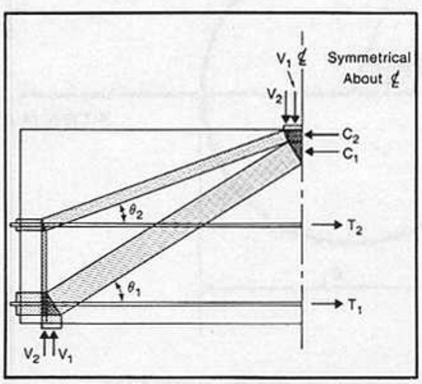

Fig. 8—Truss model for beam with horizontal web reinforcement.

The significant test observations are:

(1) In a deep beam the increase in shear strength with decrease in a/d is due to a direct compression strut from the load to the support.

This has two important consequences. The slope of the compression strut is the important parameter and is better defined by a/d than by M/Vd. Both are essentially the same in a simple beam, but in continuous beams M/Vd varies greatly depending on how close the critical section is to the point of inflection. This is one reason the ACI Code overestimated the strength of the continuous beams as mentioned earlier. A second and more significant consequence is that the addition of stirrups does little to enhance the strength of the beam until stirrups with capacity in excess of the capacity of the direct compression strut are used. When this occurs, the strut is replaced by a combination of compression fans and a compression field and a ductile failure results.

(2) Beams with significant amounts of vertical web reinforcement were ductile and had consistent failure loads. All other beams were brittle and had rather variable failure loads which were strongly influenced by the location and shape of the first inclined crack. The variability decreased as the ratio of vertical stirrups increased. In addition, less variability was found in the simple beams than in the two-span specimens. In shear spans containing a major compression strut, it is recommended that the shear capacity of the stirrups crossing the diagonal of the shear span should not be less than 30 percent of the applied shear force.

(3) While the effective concrete strength has been dealt with at length by some investigators, this variable was not found to be very significant. The specimens were under-reinforced so that the stirrups and much of the flexural steel yielded before the concrete crushed. This should be the typical case for beams designed in practice. In such a beam, the actual failure strength of the concrete is not nearly as significant as is the correct determination of which steel yields. In other words, selecting an appropriate truss model is of great importance in design. An appropriate truss model is one which correctly identifies the reinforcement which is at yield at failure of the beam and discounts the remaining reinforcement. If the model requires a redistribution of internal forces, such a redistribution must not exceed the deformation capacities of the various concrete elements involved. This is illustrated for three cases below.

Simple beams

Fig. 3(a) illustrates a truss model for one of the test beams. This beam was a simple beam with minimum vertical web reinforcement. The truss model consists of two compression fans indicated by the radiating lines plus a major compression diagonal shown by the inclined shaded region. The capacity of the equilibrium truss model was reached when the stirrups yielded and the longitudinal steel yielded at midspan. Because the beam was simply supported, very little redistribution was required to reach that state.

Fig. 3(b) compares the lower chord bar forces from the truss model, shown by the stepped line, with the measured bar forces near ultimate. The strain measurements agreed very well with the predicted bar forces. It is important to note that the longitudinal steel carried significant force at the faces of the supports as shown in Fig. 3(b). This force ranged from $A_s f_y$ for beams without stirrups to about 80 percent of $A_s f_y$ for beams with minimum stirrups.

Beams with horizontal web reinforcement

Fig. 8 presents an equilibrium truss model for a beam with one layer of horizontal web reinforcement at middepth. Here there are two trusses and, for an ideal plastic material, the shear capacity of the beam would be the sum of the shear capacities of the two trusses. For reinforced concrete this is not necessarily the case. The lower chord would reach yield first. The additional deformations required for the upper layer of steel to yield so that the upper truss can reach its capacity will generally be large enough to destroy the bottom truss. In the tests, horizontal web reinforcement was found to be ineffective in the quantities tested. These and other tests[13] indicate that horizontal web reinforcement should be neglected in the truss model when determining the strength of a beam.

Continuous beams

The two span beams with heavy stirrups were ductile and developed yielding of the top and bottom reinforcement producing full plastic mechanisms before failure. This is shown by the close comparison of the measured and calculated bar forces for the test specimen shown in Fig. 6. The truss model in Fig. 6 consists of two compression fan regions separated by a region of parallel compression diagonals, referred to as a compression field.

On the other hand, the results for the two-span test specimens without heavy stirrups were rather variable but consistently indicated that the top reinforcement did not yield before failure. The test specimens were proportioned for an elastic distribution of bending moments ignoring shear deformations. The experimental data, crack patterns, support reactions and strain measurements, all indicated that the negative moment at the interior support was less than the positive moment at midspan. The ratio between experimental and elastic interior support moment was typically 60 to 70 percent prior to yielding of longitudinal reinforcement. In the beam shown in Fig. 9, for example, the equilibrium truss model correctly predicted the bottom chord force but overestimated the top chord force. In this beam the top chord did not yield at failure.

In the continuous beams without stirrups and those with minimum stirrups, the full plastic moment capacities were not attained.

Continuous deep beams are very sensitive to support movements and, without heavy stirrup reinforcement, they may not have enough ductility to permit one to

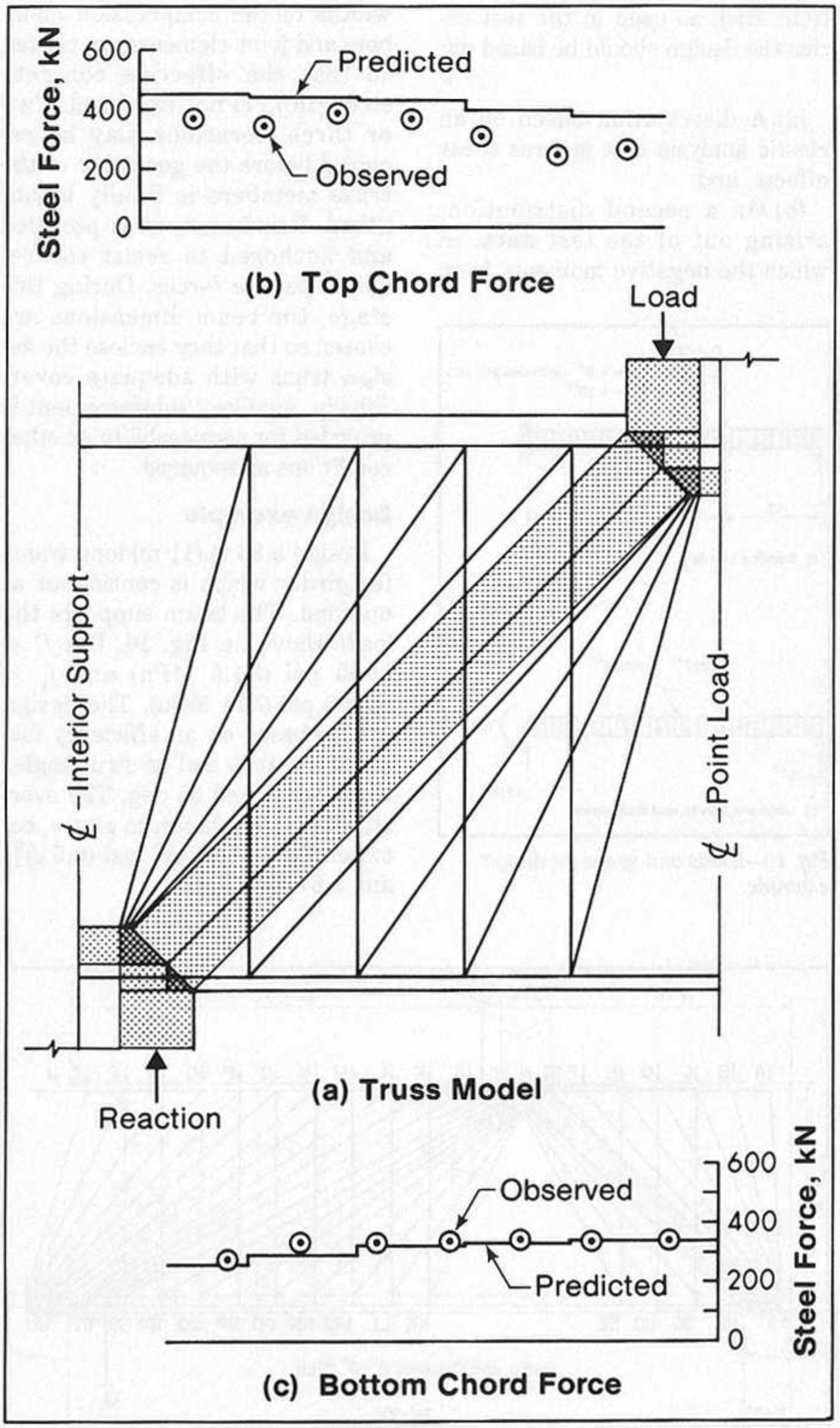

Fig. 9—Truss model for interior shear span of a two-span continuous deep beam with light web reinforcement.

design for any given distribution of moments and support reactions. The designer should use reinforcement which can accommodate all reasonable distributions of bending moments and support reactions. The distributions chosen by the designer will depend on the specifics of the structure: foundation settlements, column shortening, etc. For ideal support conditions such as used in the test series the design should be based on:

(a) A distribution based on an elastic analysis that ignores shear effects, and

(b) On a second distribution, arising out of the test data, in which the negative moments from the first distribution are reduced by 40 percent and the remaining moments adjusted accordingly.

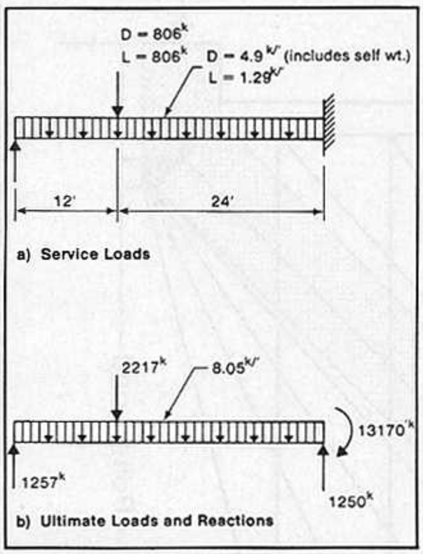

Fig. 10—Loads and spans for design example.

Design by equilibrium truss model

The design of a deep beam is very similar to the design of a truss. The design process consists of drawing to scale a truss capable of resisting the factored loads. The widths of the compression members and joint elements are chosen so that the effective concrete strength f_{ce} is not exceeded. Two or three iterations may be required before the geometry of the truss members is finally established. Reinforcement is provided and anchored to resist the required tensile forces. During this stage, the beam dimensions are chosen so that they enclose the design truss with adequate cover. Finally, auxiliary reinforcement is provided for serviceability or other conditions as required.

Design example

Design a 36 ft (11 m) long transfer girder which is continuous at one end. The beam supports the loads shown in Fig. 10. Use f'_c = 5000 psi (34.5 MPa) and f_y = 60000 psi (414 MPa). The design will be based on an efficiency factor of $\nu = 0.60$ and on strut angles between 25 and 65 deg. The overall size will be chosen to give v_u between $6\sqrt{f'_c}$ and $8\sqrt{f'_c}$ psi ($0.5\sqrt{f'_c}$ and $0.67\sqrt{f'_c}$ MPa).

For simplicity in this design example, only one load case will be considered. Normally, one would consider a series of loading cases and a series of support settlement cases.

The design for shear and flexure or other forces is done simultaneously with one model and hence different strength reduction factors, ϕ, for shear and flexure are awkward. In the design example ϕ = 0.85 has been used throughout. This was done by dividing the applied loads and reactions by 0.85.

In the design the uniform loading will be idealized as a series of concentrated loads at 2 ft (600 mm) on centers along the beam. In a similar fashion, the initial design will consider stirrups at a hypothetical spacing of 2 ft on centers coinciding with the concentrated load points. Once the area of stirrups per 2 ft space has been calculated, it will be provided by stirrups at the spacing required to provide that area.

The truss finally chosen in the design is shown in Fig. 11. The beam and columns all have a thickness of 28 in. (660 mm). This is the result of an initial choice and several iterations in which the slopes and widths of the struts and the sizes of the nodal zones are adjusted to satisfy the assumptions made earlier, particularly assumptions 1, 2, and 4. It should be noted that there is no unique final design. Any one of several will prove satisfactory, provided that the detailing of the structure allows the truss to carry the loads in the manner assumed.

In the left shear span, possible designs vary all the way from having all the shear carried by the concrete strut, on one hand, to having all the shear carried by stirrups on the other. The stirrup reinforcement in the left shear span was selected on the basis of having 30-35 percent of the shear carried by stirrups to reduce the size of the major compression strut and to give a minimum amount of ductility. In the right shear span the stirrups were selected so that all the shear across

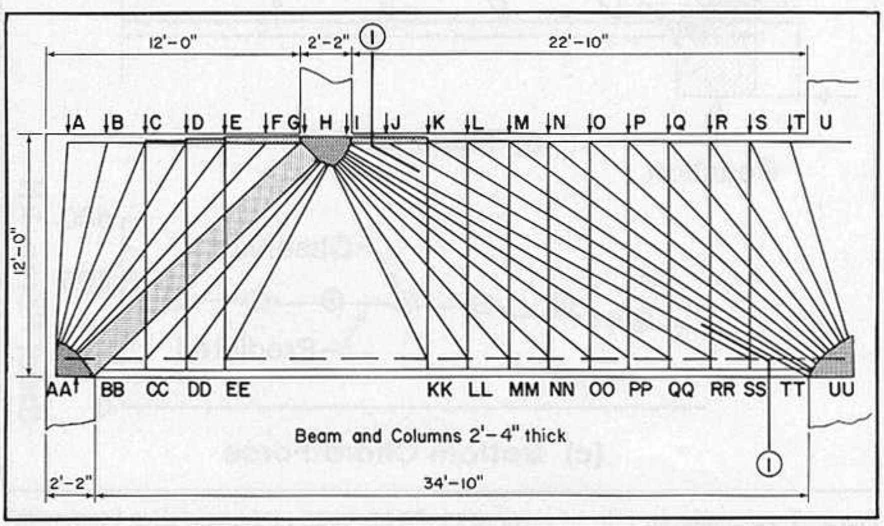

Fig. 11—Truss selected in design example.

Section 1-1 in Fig. 11 was carried by stirrups. Stirrups loaded by struts steeper than 65 deg were ignored in the model since they cannot be counted on to yield before failure.

The longitudinal steel at midspan and the right support was computed from the moments at those locations assuming the steel yielded at both locations. The calculation of the forces at other locations in the top chord is illustrated in Fig. 12. At the support U, the bar force is 1476 kips (6565 kN). At joint T, the vertical applied load is equilibrated by a steep inclined strut T-UU. Horizontal equilibrium of the joint shows that the top chord force drops to 1470 kips (6539 kN). At joint S, the inclined strut equilibrates the vertical force applied at S plus the force in stirrup S-SS which is assumed to have yielded. Joint equilibrium shows that the top chord force drops to 1396 kips (6120 kN) and so on.

The shaded area in Fig. 13(a) shows the top chord bar force calculated in this way. The outer envelope in this figure shows the tensile capacity of the steel provided in this chord. The sloping portions of this envelope and the envelope in Fig. 13(c) were drawn assuming the force in a bar can increase linearly from zero to $A_s f_y$ over the development length, ℓ_d. The steel chosen is shown in Fig. 13(b). Fig. 13(c) is similar for the lower chord.

It is important to note that the lower chord is in tension from column to column and has a high tensile force in it at the left support. This must be anchored in that support. The detailing and distribution of the bars must be such that the resultants of all the compression forces coincide with the tension force and loads or reactions at points such as AA.

Conclusions, recommendations

This article has presented an overview of the design of deep beams using an equilibrium truss model which is shown to give good agreement with tests of beams, particularly for beams with large amounts of stirrups.

For the design of deep beams it is recommended that the equilibrium truss model be used, based on the assumptions presented earlier in this article and:

(a) Strut angles between 25 and 65 deg to the axis of the member. Major struts should be within ± 15 deg of the elastic compressive stress trajectories.

(b) An effective concrete strength of $0.6f'_c$.

(c) In shear spans where a major strut exists, the stirrups crossing the diagonal of the span should have a shear capacity not less than 30 percent of the applied shear force.

(d) The effects of support settlements should be assessed and the resulting moment and shear envelopes considered in the design.

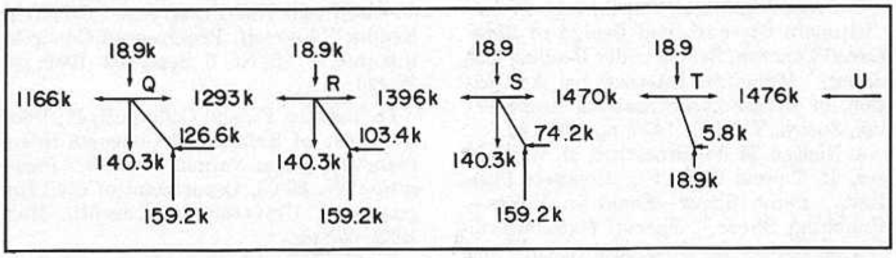

Fig. 12—Forces on upper chord · joints Q, R, S, T, and U.

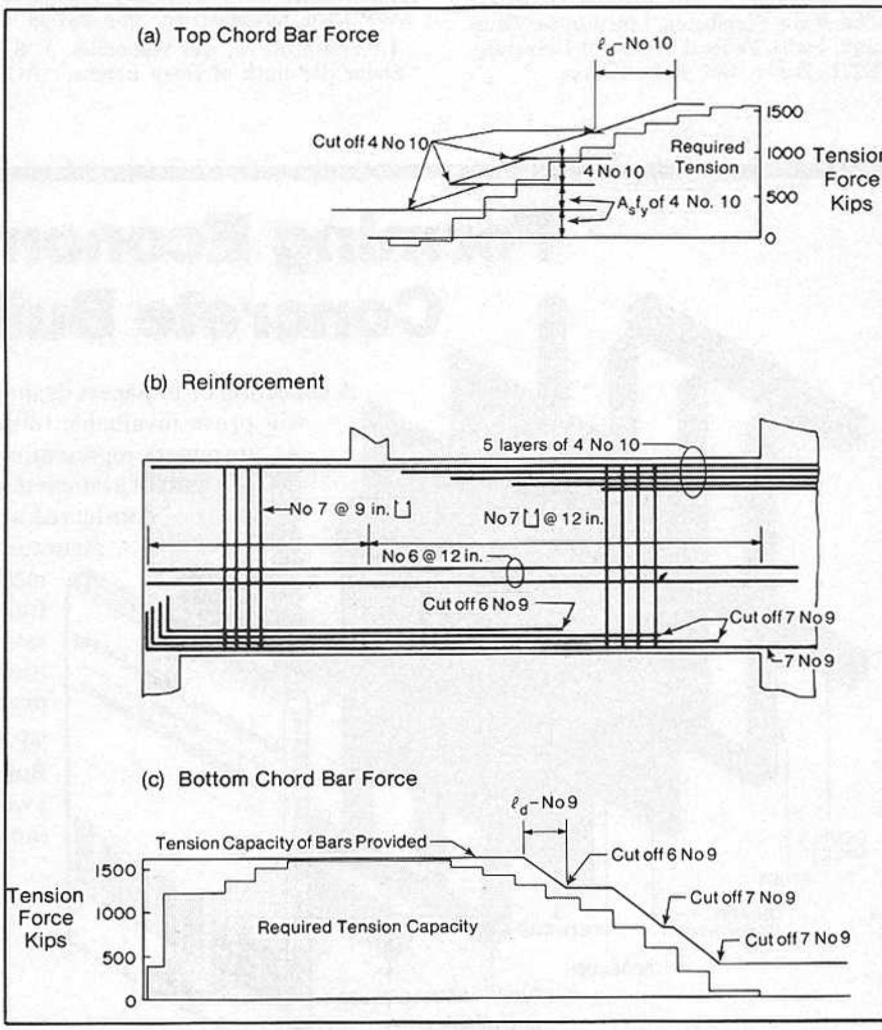

Fig. 13—Forces in top and bottom chords and reinforcement selected in design example.

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