TORSION DESIGN OF PRESTRESSED CONCRETE

Paul Zia
Professor and Associate Head
of Civil Engineering
North Carolina State University
Raleigh, North Carolina

W. Denis McGee
Assistant Professor of
Civil Engineering
Washington State University
Pullman, Washington

Design provisions for torsion in reinforced concrete members have been developed and included in the ACI 318-71 Building Code. These provisions are not applicable to prestressed concrete.
Based on a thorough review of available research data on torsion in prestressed concrete, this paper proposes an extension of the ACI design procedures to cover torsion in prestressed concrete. The basic approach consists of determining the concrete contribution to the ultimate strength of a member and then proportioning reinforcement for the remaining portion of the required ultimate strength.
Formulas for the nominal torsional stress and the basic equation for torsional strength of reinforced members are presented, and followed by a discussion of the torsion-shear interaction relation, taking into account the effect of the prestressing force. The requirements for both web and longitudinal reinforcements for torsion are explained.
The proposed procedures are illustrated by a design example of a precast prestressed L-girder in a roof framing subjected to combined torsion, bending, and shear. It is believed that the proposed procedure is reasonably conservative and future refinements and simplifications are possible when more complete research data become available.

The problem of torsion in concrete structures has received considerable attention of design engineers in recent years. In 1971, the ACI Building Code (ACI 318-71)[1] included for the first time explicit requirements of torsion design for reinforced concrete developed on the basis of substantial research data. However, the code provisions are not applicable to prestressed concrete members, for which research has been in progress but design criteria have yet to be developed.

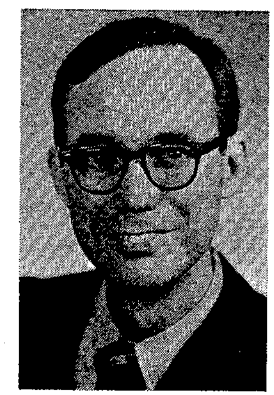
This paper presents a proposed design procedure for torsion in prestressed concrete, based on a thorough analysis of 394 test results available in the literature.[2] The proposal is in close parallel with the ACI Code procedure for reinforced concrete.

The basic approach consists of determining the concrete contribution to the ultimate strength of a member and then proportioning reinforcement for the remaining portion of the required ultimate strength. It is believed that the proposed procedure is reasonably conservative and future refinements and simplifications are possible when more complete research data become available.

In developing the proposed procedure, the authors have benefited from helpful discussions with several members of the ACI Committee 438 on Torsion. However, this proposal should not be regarded as a committee-endorsed document.

In the following discussion, the usual capacity reduction factor ϕ is not included for the sake of convenience.

Paul Zia



W. Denis McGee

TORSIONAL SHEAR STRESS

Rectangular section

A prestressed concrete member without web reinforcement fails, under torsion, in the form of skew bending. The torsional stress of a rectangular section can be expressed by:

$$\tau_u = \frac{T_u}{\alpha x^2 y} \qquad (1)$$

where
x = shorter side of rectangular section
y = longer side of rectangular section
α = torsion coefficient

It will be recalled that Eq. (1) is of the basic form for either the elastic the-

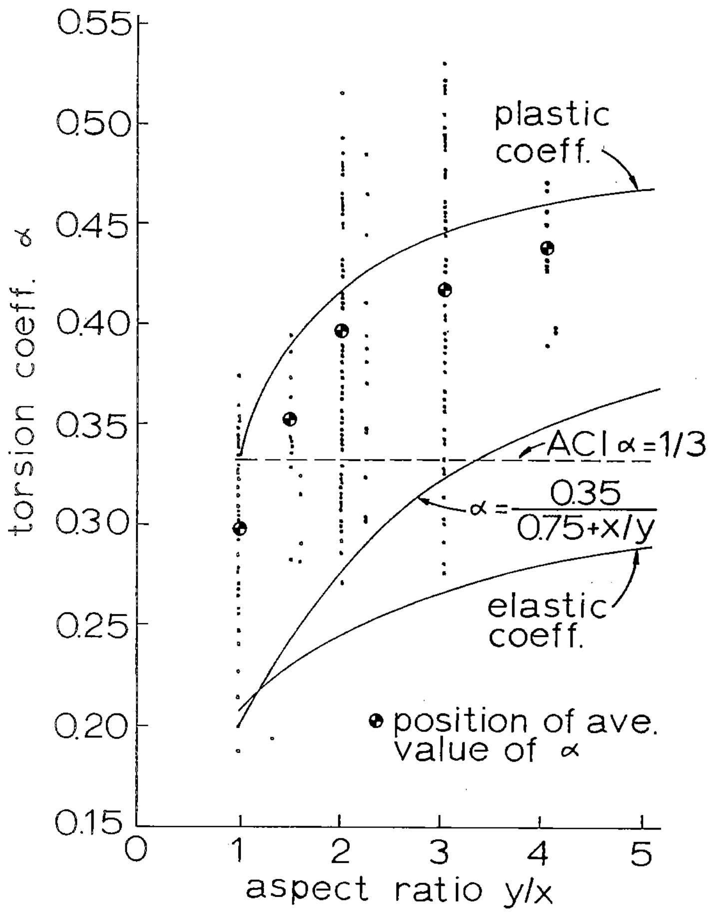


Fig. 1. Variation of torsion coefficients with aspect ratio.

ory, the plastic theory or the skew bending theory of torsion. The coefficient α varies from 0.208 to ⅓ in the elastic theory and from ⅓ to ½ in the plastic theory. For simplicity, the ACI Code adopted a value of ⅓ in accordance with the skew bending theory.[3]

Hsu[4] has shown that the torsional capacity of a prestressed section without reinforcement, or the cracking torque of a prestressed section with reinforcement is reached when:

$$\tau_u = (0.85\, f_r)\sqrt{1 + 10\,\sigma/f_c'} \qquad (2)$$

where

f_r = modulus of rupture

σ = average prestress on section

The factor $(0.85\, f_r)$ is nearly equal to the tensile strength of concrete f_t' and may be taken as $6\sqrt{f_c'}$. Hence, combining Eqs. (1) and (2):

$$\tau_u = 6\sqrt{f_c'}\,\sqrt{1 + 10\,\sigma/f_c'} = \frac{T_u}{\alpha x^2 y}$$

or

$$\alpha = \frac{T_u}{x^2 y\, 6\sqrt{f_c'}\,\sqrt{1 + 10\,\sigma/f_c'}} \qquad (3)$$

Using the test results of 218 concentrically or eccentrically prestressed rectangular beams, the value of α has been computed for each beam by Eq. (3) and these values are plotted against the aspect ratio y/x in Fig. 1. Also plotted for comparison in Fig. 1 are the curves for the elastic and plastic torsion coefficients as well as the value currently specified by the ACI Code.

It can be seen that for sections of low aspect ratio, the ACI 318-71 value of ⅓ could be quite unconservative. Since, as will be shown later, the contribution of the concrete to the ultimate torsional strength is a much larger portion of the cracking strength for prestressed members than for non-prestressed members, it is prudent and essential that the nominal torsional stress be evaluated more accurately. Therefore, it is proposed that the coefficient α in Eq. (1) be taken as:

$$\alpha = \frac{0.35}{0.75 + x/y} \qquad (4)$$

which represents a reasonably conservative lower bound to the experimental data.

Table 1. Comparison of experimental cracking torque of prestressed flanged sections with Eq. (5).

Investigator	Beam No.	$\Sigma(\alpha x^2 y)$ (in.^{3})	T_{ex} (in.-k)	T_{th}* (in.-k)	T_{ex}/T_{th}
Wyss, et. al. (I-beams) (5)	A1	294	253	234	1.08
	A2	294	256	234	1.10
	A3	294	300	233	1.29
	A4	294	316	233	1.35
	A5	294	329	233	1.42
	A6	294	313	233	1.35
	B1	294	203	185	1.10
	B2	294	216	185	1.17
	B3	294	222	188	1.19
	B4	294	222	188	1.19
	B5	294	209	188	1.12
	B6	294	200	188	1.06
Zia (6) (T-beams)	0.25T1	52.9 (41.5)**	51.12	40.3 (31.6)**	1.27 (1.62)**
	0.25T2	52.9 (41.5)	51.98	40.3 (31.6)	1.29 (1.64)
	2.25T1	52.9 (41.5)	35.20	39.9 (31.3)	0.89 (1.13)
	2.25T2	52.9 (41.5)	32.92	39.9 (31.3)	0.83 (1.06)
	2.75T1	52.9 (41.5)	33.96	39.8 (31.2)	0.88 (1.12)
	2.75T2	52.9 (41.5)	33.16	39.9 (31.3)	0.83 (1.06)
Zia (6) (I-beams)+	0.75I1	57.1 (49.4)	28.0	39.4 (34.1)	0.71 (0.82)
	0.75I2	57.1 (49.4)	35.0	39.4 (34.1)	0.89 (1.03)
	3.I1	57.1 (49.4)	30.0	39.7 (34.4)	0.75 (0.87)
	3.I2	57.1 (49.4)	33.0	39.7 (34.4)	0.83 (0.96)
	3.5I1	57.1 (49.4)	48.5	41.2 (35.6)	1.18 (1.36)
	3.5I2	57.1 (49.4)	46.0	41.2 (35.6)	1.11 (1.29)

*Computed from Equation (5).

**Neglecting outstanding flanges.

+Cracking torques for these members were not available; T_{ex} was taken as the elastic limit, which is less than the actual cracking load.

Table 2. Comparison of experimental cracking torque of prestressed box sections with theory.

Investigator	Beam No.	x, y (in.)	Core	Equiv. square core	4h/x	T_{ex} (in.-k)	T_{th}* (in.-k)	$\frac{T_{ex}}{T_{th}}$
	T	9	5" x 5"	5" x 5"	0.89	127.2	113.9	1.12
Johnston and Zia (9)	H-0-0-1	12	8.5"ϕ	7.5" x 7.5"	0.75	120	99.5	1.21
	H-0-3-1	12	8.5"ϕ	7.5" x 7.5"	0.75	120	73.7	1.63
	H-0-6-1	12	8.5"ϕ	7.5" x 7.5"	0.75	115	73.7	1.56

*Computed from Equation (1) multiplied by 4h/x.

Flanged section

For a flanged section, the same assumption implied by ACI 318-71 will be used. That is, the torsional strength of a flanged section can be expressed as the sum of the strengths of the individual components. Hence:

$$\tau_u = \frac{T_u}{\Sigma \alpha x^2 y} \qquad (5)$$

Eq. (5) is identical to the ACI 318-71 Eq. (11-16) except for the modification of torsional coefficient α as expressed by Eq. (4). The section may be divided into component rectangles such that the quantity $\Sigma \alpha x^2 y$ would be a maximum.

Comparison of Eq. (5) with test results reported in the literature indicates that the method is reasonably safe as shown in Table 1. However, it should be cautioned that only two investigations[5,6] have been reported, consisting of 24 test specimens, all being somewhat stocky in cross section. The margin of safety seems to reduce as the cross section becomes more stocky such as the specimens tested by Zia. For such disproportionate sections, it would seem logical to disregard the small outstanding flanges.

Box section

Based on test results, ACI 318-71 specifies that for reinforced concrete box sections with a wall thickness h not less than $x/4$, where x is the overall width of the box section, the torsional strength of the box section may be taken as that of a comparable solid rectangular section with the same overall dimension.

If the wall thickness h is less than $x/4$ but greater than $x/10$, the torsional strength of the box section is reduced and the strength reduction is accounted for by a reduction factor $4h/x$ multiplied to the denominator on the right-hand side of Eq. (1).

This design approach seems also applicable to prestressed concrete box sections. Although only four tests of prestressed concrete box sections under pure torsion have been reported in the literature that can be used for comparison, application of the ACI design rule for these test results indicates a considerable margin of safety in all cases as shown in Table 2.

For box sections with a very thin wall, h approaching $x/10$, caution should be exercised against possible wall buckling or crushing, particularly when high prestress is applied.

BASIC EQUATION FOR TORSIONAL STRENGTH

The basic equation for torsional strength has been developed from the test results of prestressed beams under pure torsion. Tests conducted by Hsu[7] show that the ultimate torsional strength can be expressed as the sum of the strength contributed by the concrete and the strength contributed by the web reinforcement, for both prestressed and non-prestressed beams (see Fig. 2). The effect of prestress is to increase the contribution of the concrete to the ultimate torsional strength, while the contribution of the reinforcement remains unchanged.

Referring to Fig. 3, it can be seen that for non-prestressed beams the contribution by concrete T_o is only a portion of the torsional strength T_{up} of a corresponding plain concrete section. Likewise, for prestressed beams, the contribution by concrete T_o' is also a portion, but a greater portion, of the torsional strength of a corresponding prestressed concrete section without web reinforcement. Accordingly, the basic strength equation for prestressed

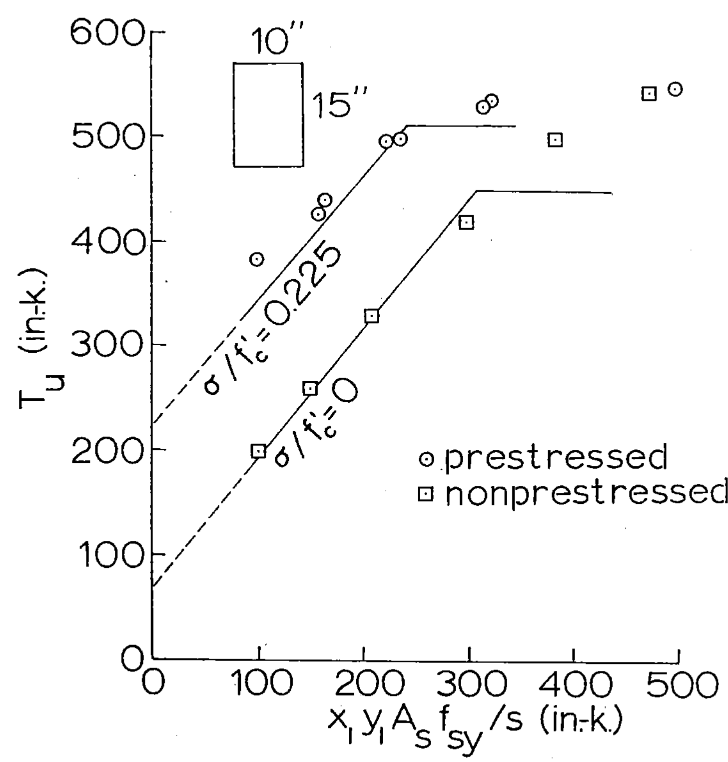


Fig. 2. Tests on prestressed and nonprestressed reinforced concrete beams (after Hsu).

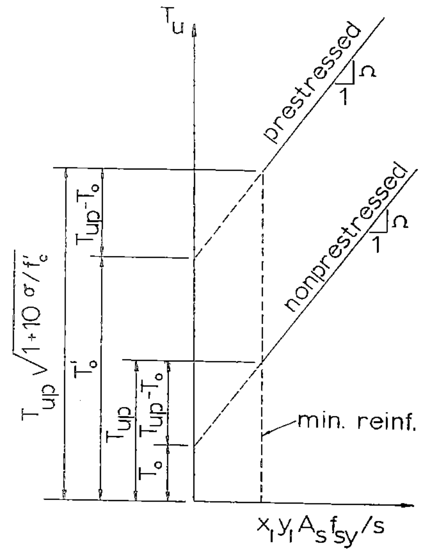


Fig. 3. Relation between ultimate torque for prestressed and nonprestressed reinforced, rectangular beams (after Hsu).

beams can be expressed as:

$$T_u = T_o' + \frac{\Omega x_1 y_1 A_t f_{sy}}{s} \qquad (6)$$

and

$$\Omega = 0.66\,m + 0.33\,y_1/x_1$$

where

T_o' = strength contributed by concrete
x_1 = shorter leg of a closed stirrup
y_1 = long leg of a closed stirrup
A_t = cross-sectional area of *one* leg of a stirrup
f_{sy} = yield strength of stirrup
s = spacing of stirrup
m = volume ratio of longitudinal reinforcement to web reinforcement

If the yield strength of the longitudinal reinforcement is different from that of the web reinforcement, then the parameter m should be replaced by mf_{ly}/f_{sy} in which f_{ly} is the yield strength of the longitudinal reinforcement. Hsu has recommended that:

$$0.7 \leq mf_{ly}/f_{sy} \leq 1.5 \qquad (7)$$

and

$$y_1/x_1 \leq 2.6 \qquad (8)$$

In Fig. 3, the cracking (or ultimate) torque of a plain concrete member can be obtained by substituting $6\sqrt{f_c'}$ for τ_u in Eq. (1). Thus:

$$T_{up} = (\alpha x^2 y)6\sqrt{f_c'} \qquad (9)$$

The ordinate T_o has been determined by Hsu experimentally as:

$$T_o = 0.4\left(\frac{1}{3}x^2y\right)6\sqrt{f_c'}$$
$$= 0.133(x^2y)6\sqrt{f_c'}$$

Hence:

$$T_o' = T_{up}\sqrt{1+10\,\sigma/f_c'} - (T_{up} - T_o)$$
$$= T_{up}\left[\sqrt{1+10\,\sigma/f_c'} - (1 - T_o/T_{up})\right]$$
$$= (\alpha x^2 y)6\sqrt{f_c'}\left(\sqrt{1+10\,\sigma/f_c'} - k\right) \qquad (10)$$

where

$$k = 1 - T_o/T_{up} = 1 - 0.133/\alpha \qquad (11)$$

Thus, by combining Eqs. (6) and (10), the basic strength equation becomes:

$$T_u = (\alpha x^2 y)6\sqrt{f_c'}\left(\sqrt{1+10\,\sigma/f_c'} - k\right) + \frac{\Omega x_1 y_1 A_t f_{sy}}{s} \qquad (12)$$

Likewise, for a box section:

$$T_u = \frac{4h}{x}\left[(\alpha x^2 y)6\sqrt{f_c'} \times \left(\sqrt{1+10\,\sigma/f_c'} - k\right) + \frac{\Omega x_1 y_1 A_t f_{sy}}{s}\right] \qquad (13)$$

and, for a flanged section:

$$T_u = (\Sigma\alpha x^2 y)6\sqrt{f_c'} \times \left(\sqrt{1+10\,\sigma/f_c'} - k\right) + \frac{\Omega x_1 y_1 A_t f_{sy}}{s} \qquad (14)$$

in which the value of k is determined

Table 3. Comparison of experimental ultimate torque of concentrically prestressed rectangular beams with Eq. (12).

Investigator	Beam No.	T_{ex} (in.-k)	T_{th}* (in.-k)	T_{ex}/T_{th}
Chandler, et. al. (10)	C1	59.32	54.67	1.09
	C3	79.00	62.07	1.27
	C5	75.60	66.94	1.23
	C9	85.10	72.47	1.17
	C11	94.50	78.86	1.20
	C15	106.40	85.64	1.24
	C7	54.77	58.68	0.93
	C13	59.10	49.18	1.20
	C17	41.08	42.54	0.97
	C19	51.60	62.00	0.83
	SP1261	164.90	173.74	0.95
	SP1262	134.65	130.15	1.03
	SP1269	151.15	155.25	0.97
	SP1270	129.90	112.80	1.15
	SP1241	82.90	67.71	1.22
	SP1242	81.40	67.71	1.20
	SPC1243	70.95	58.51	1.21
	SPC1245	103.00	84.67	1.22
	SP1247	93.70	77.99	1.20
	SPC941	52.20	46.05	1.13
	SP1263	175.15	172.89	1.01
	SP1264	136.15	129.31	1.05
	SP1271	162.40	155.39	1.05
	SP1272	121.15	112.94	1.07
	SPI	152.15	118.55	1.28
	SPII	142.65	118.58	1.20
	SP1267	169.25	170.91	0.99
Mukherjee Kemp (11)	SP1	190.8	129.79	1.47
	SP2	171.0	122.79	1.39
	SP3	233.8	204.82	1.14
	SP4	198.6	164.45	1.21
	SP5	200.8	153.85	1.31
	SP6	153.0	122.99	1.24
	SP7	156.0	110.51	1.41
	SP8	186.0	190.83	0.97
	SP9	185.0	155.62	1.19
	SP10	178.0	143.42	1.24
	SP11	207.0	163.39	1.27
	SP12	205.0	153.93	1.33
	SP13	232.0	243.96	0.95
	SP14	232.0	200.34	1.16
	SP15	203.0	184.46	1.10
Jacobsen (16)	306	177.9	122.09	1.46
Mukherjee and Warwaruk (12)	106	140.8	97.05	1.45
	206	146.4	121.24	1.21
Okada, et. al. (13)	R_bIII-1	47.0	25.05	1.88
	R_bIII-2	47.6	25.05	1.90
Superfesky (14)	SP17	213.0	141.19	1.51
	SP18	246.0	229.24	1.07
	SP19	245.0	188.70	1.30
	SP20	215.0	169.94	1.27
	4S1	63.5	35.63	1.78
	4S2	77.5	58.68	1.32
	4S3	92.7	61.50	1.51
	4S4	83.1	63.78	1.30
	4S5	95.8	68.91	1.39
	4S6	72.0	42.30	1.70
	4S7	80.6	65.75	1.23
	4S8	88.3	67.85	1.30
	4S9	99.4	71.14	1.40
	4S10	101.3	75.80	1.34
	4S11	49.0	40.00	1.23
	4S12	88.7	63.90	1.39
	4S13	90.7	66.02	1.37
	4S14	89.8	68.64	1.31
	4S15	94.7	72.82	1.30
Zia (6)	ORW1	50.64	40.25	1.26
	ORW2	57.44	40.25	1.43
GangaRao and Zia (15)	1-0	79.86	81.76	0.98
	2-0	111.68	89.50	1.25
	2-C	136.20	89.07	1.53
	3-0	104.70	104.29	1.00
	4-0	120.00	102.38	1.17
Hsu (7)	P1	382	318.76	1.20
	P2	440	384.67	1.14
	P3	500	468.25	1.07
	P4	534	563.47	0.95
	P5	424	349.29	1.21
	P6	496	414.17	1.20
	P7	530	504.05	1.05
	P8	546	687.99	0.79

*Computed from Equation (12)

on the basis of the largest component rectangle where the web reinforcement is usually placed.

By comparison with available test results, it has been shown that, with only a few exceptions, the above three equations are reasonably and consistently on the safe side (see Tables 3 through 5).[2] It would appear, therefore, that these equations are acceptable as the basis of design.

TORSION-SHEAR INTERACTION

Tests[2] have shown that the interactions between torsion and shear for prestressed beams without web reinforcement can be adequately represented by a circular curve. By following the same development adopted by ACI 318-71, it can be shown that the permissible shear stress for torsion and shear are, respectively:

Table 4. Comparison of experimental ultimate torque of eccentrically prestressed rectangular beams with Eq. (12).

Investigator	Beam No.	T_{ex} (in.-k)	T_{th}* (in.-k)	$\frac{T_{ex}}{T_{th}}$
Chandler, et. al. (10)	E2	52.82	54.90	0.96
	E4	64.70	62.94	1.03
	E6	70.20	66.94	1.05
	E10	79.70	72.47	1.10
	E12	92.20	78.86	1.17
	E16	107.82	85.64	1.26
	E8	50.44	58.68	0.86
	E14	61.90	49.18	1.26
	E18	41.35	42.54	0.97
	E20	53.10	62.00	0.87
	SPE1244	66.70	58.51	1.14
	SPE1246	97.00	84.67	1.15
	SPE942	44.70	46.05	0.97
Ganga Rao (15) and Zia	5-0	106.80	109.30	0.98
	6-0	119.50	107.96	1.11
Jacobsen (16)	326	154.2	143.50	1.07
Mukherjee and Warwaruk (12)	126	147.5	99.50	1.48
	226	152.3	121.23	1.26
Zia (6)	2RW1	48.44	40.33	1.20
	2RW2	52.24	40.09	1.30
	2.5RW1	53.44	39.81	1.34
	2.5RW2	52.44	39.81	1.32
McGee and Zia (2)	W2e-4.5-LL	105	63.99	1.64
Woodhead and McMullen (17)	II-1	105	70.89	1.48
	III-4	125	87.50	1.43

*Computed from Equation (12).

$$\tau_c = \frac{\tau_c'}{\sqrt{1+\left[\dfrac{v_u}{\beta\,\tau_u}\right]^2}} \qquad (15)$$

$$v_c = \frac{v_c'}{\sqrt{1+\left(\dfrac{\beta\,\tau_u}{v_u}\right)^2}} \qquad (16)$$

where

$$\tau_c' = 6\sqrt{f_c'}\left(\sqrt{1+10\,\sigma/f_c'} - k\right) \qquad (17)$$

v_c' = lesser of v_{ct} and v_{cw} as defined by ACI Eqs. (11-11) and 11-12), respectively.

$$\beta \approx \frac{1}{2}\left(\frac{v_c'}{\tau_c'}\right) \qquad (18)$$

It is noted that τ_c' and v_c' in Eqs. (15) and (16) are the permissible shear stresses for torsion and flexural shear, respectively, if torsion or shear is acting alone. The factor in the denominator of these equations accounts for the interaction between torsion and shear. For a discussion of Eq. (18), see Reference 2.

WEB REINFORCEMENT FOR TORSION

Recasting the basic equation for torsional strength discussed above and taking into account the torsion-shear interaction, we have:

$$T_u = \Sigma\alpha x^2 y \tau_c + \frac{\Omega x_1 y_1 A_t f_{sy}}{s}$$

$$= \Sigma\alpha x^2 y \tau_u$$

Transposing terms:

$$\Sigma\alpha x^2 y\,(\tau_u - \tau_c) = \frac{\Omega x_1 y_1 A_t f_{sy}}{s}$$

or

$$A_t = \frac{(\tau_u - \tau_c)\, s\, \Sigma\alpha x^2 y}{\Omega x_1 y_1 f_{sy}} \qquad (19)$$

in which, for simplicity, the coefficient Ω may be taken as:

$$\Omega = 0.66 + 0.33\, y_1/x_1$$

Table 5. Comparison of experimental ultimate torque of flanged and box sections with Eq. (14) and Eq. (13), respectively.

Investigator	Beam No.	T_{ex} (in.-k)	T_{th}* (in.-k)	$\frac{T_{ex}}{T_{th}}$
Wyss, et. al. (5)	A2	318.0	194.65	1.63
	A3	391.0	223.4	1.75
	A4	430.0	255.9	1.68
	A5	477.0	313.7	1.52
	A6	493.0	385.9	1.28
	B2	219.0	137.2	1.60
	B3	296.0	163.4	1.81
	B4	320.0	195.3	1.64
	B5	544.0	252.9	2.15
	B6	582.0	325.3	1.79
Zia (6) (T-beams)	0.25TW1	50.20	34.42	1.46
	0.25TW2	54.44	34.42	1.58
	2.25TW1	41.00	33.79	1.21
	2.25TW2	46.70	33.79	1.38
	2.75TW1	46.84	33.59	1.40
	2.75TW2	50.56	33.59	1.51
Zia (6) (I-beams)	0.75IW1	54.00	31.13	1.74
	0.75IW2	50.20	31.13	1.61
	3IW1	59.72	30.77	1.94
	3IW2	57.76	30.69	1.88
	3.5IW1	59.00	30.58	1.93
	3.5IW2	62.10	30.58	2.00
Johnston and Zia (9)	H-0-3-1	210	109	1.93
	H-0-6-1	176	74	2.38

*Computed from Equation (14) or Equation (13).

Thus, the web reinforcement is required for the torsional shear stress in excess of that carried by the concrete. The web reinforcement for torsion must be in the form of closed stirrups and it should be added to the web reinforcement for flexural shear. Note that the latter could be open stirrups.

MINIMUM AND MAXIMUM WEB REINFORCEMENT

To avoid brittle failure, a minimum amount of web reinforcement must be provided to resist both torsion and shear. A series of tests[2] with prestressed rectangular beams, containing a minimum amount of web reinforcement as specified by ACI 318-71 for shear, have shown that the post-cracking ductility of the beams was insufficient under high ratios of torsion and shear. In view of the sudden and explosive nature of the torsional failure, and the lack of sufficient research data, it is suggested that a beam should be reinforced for no less than its cracking torque.

At the other extreme, considerations should be given to the possible danger of over-reinforcing a member such that a compressive failure of the concrete might occur before the reinforcement yields. To avoid this type of failure, an upper limit of web reinforcement should be established by specifying a maximum permissible nominal torsional stress. If the design stress exceeds this limit, then a larger beam section would have to be used.

Since the prestressing force exerts a compressive load on a member before it is subjected to torque, it seems logical that the upper limit on the nominal torsional stress should be a function of the prestress ratio σ/f'_c. Based on this reasoning, it will be assumed that the maximum torque a rectangular section can carry is:

$$T_{u(max)} = \alpha x^2 y\, C \sqrt{f_c'} \sqrt{1 + 10\,\sigma/f_c'} \qquad (20)$$

in which C is an unknown coefficient yet to be determined. It is assumed that C is a function of σ/f_c'.

For the extreme case of $\sigma/f_c' = 0$, i.e., reinforced concrete members without prestress, ACI 318-71 specifies 12 $\sqrt{f_c'}$ as the limiting stress. Thus:

$$T_{u(max)} = \frac{1}{3} x^2 y\, 12 \sqrt{f_c'} = \alpha x^2 y\, C \sqrt{f_c'}$$

or $C = 4/\alpha$.

For a typical cross section with an aspect ratio of two:

$$\alpha = \frac{0.35}{0.75 + 0.5} = \frac{0.35}{1.25} = 0.28$$

and

$$C = 4/0.28 \approx 14$$

Hence, in Eq. (20), the coefficient C can be taken as 14 for $\sigma/f_c' = 0$.

For members subjected to high values of σ/f_c', Zia[6] has shown that a transition from tension failure to compression failure would take place at $\sigma/f_c' \geqq 0.6$. Since the tensile strength of the concrete can be taken as $6\sqrt{f_c'}$, it is then reasonable to limit the maximum nominal torsional stress as $6\sqrt{f_c'}$ for $\sigma/f_c' = 0.6$. Assuming a linear function of σ/f_c' between these two limiting values for C, one obtains:

$$C = 14 - 13.33\,(\sigma/f_c') \qquad (21)$$

In Fig. 4, test results are compared with Eq. (21) and it is noted that, with a few exceptions, all the data representing tension failure fall below the curve of Eq. (21) and the data for those beams reported to have failed in compression fall above the curve. It would seem that the coefficient C as represented by Eq. (21) is a very reasonable value to be used as the upper limit of the nominal shear stress for both rectangular and flanged sections.

Eq. (20) can be extended to the case of combined torsion and shear, again

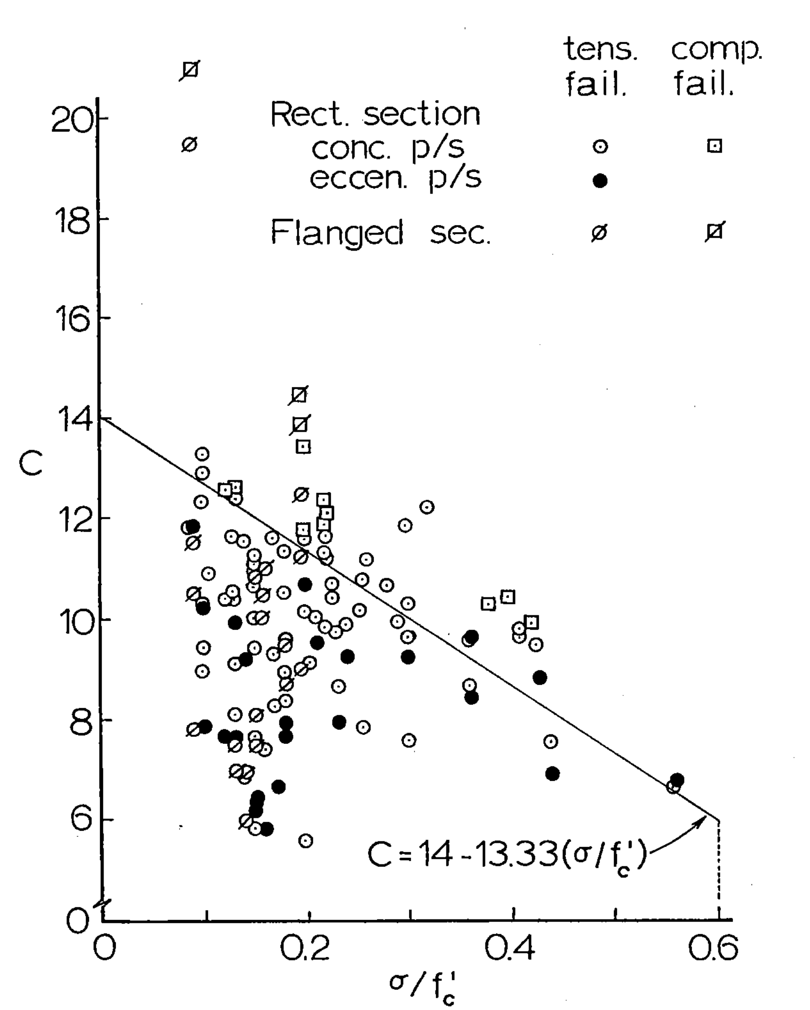


Fig. 4. Comparison of Eq. (21) with tests results for upper limit of nominal torsion stress.

based on the circular interaction relation. Thus:

$$T_{u(max)} = \frac{C'\sqrt{f_c'}}{\sqrt{1 + \left[\left(\frac{C'}{10}\right)\left(\frac{v_u}{\tau_u}\right)\right]^2}} \qquad (22)$$

$$v_{u(max)} = \frac{10\sqrt{f_c'}}{\sqrt{1 + \left[\left(\frac{10}{C'}\right)\left(\frac{\tau_u}{v_u}\right)\right]^2}} \qquad (23)$$

where

$$C' = C\sqrt{1 + 10\,\sigma/f_c'}$$

$$C = 14 - 13.33\,(\sigma/f_c')$$

The term $10\sqrt{f_c'}$ in Eq. (23) is based on the ACI Code upper limit for flexural shear stress and is obtained by taking $v_c = 2\sqrt{f_c'}$ in Section 11.6.4 of ACI 318-71.

LONGITUDINAL TORSION REINFORCEMENT

To resist the longitudinal component of the diagonal tension induced by torsion, longitudinal reinforcement must be provided in addition to that required by flexure. This longitudinal reinforcement should be approximately of equal volume as that of stirrups for torsion in order that both will yield at failure, i.e., $mf_{ly}/f_{sy} \approx 1$. Therefore:

$$A_l = \frac{2A_t(x_1 + y_1)}{s}\left(\frac{f_{sy}}{f_{ly}}\right) \qquad (24)$$

A_l is the total area of longitudinal torsion reinforcement and must be distributed around the inside perimeter of the closed stirrups.

The question of whether the prestressing steel can be considered as a part of longitudinal torsion reinforcement should be considered. The prestressing steel is provided primarily for resisting flexure and often is not effectively distributed around the perimeter of the closed stirrups.

Tests[2] have shown that without additional longitudinal reinforcement of mild steel, prestressed beams would fail abruptly under high torsion. In light of this observation, it is recommended that until more research data become available, only the prestressing steel in excess of that required for flexure and located around the perimeter of closed stirrups should be considered as a part of the longitudinal torsion steel, having an equivalent area of:

$$A_{(mild)eq} = A_{ps(eff)}\, f_{ps(eff)}/f_{ly}$$

in which $A_{ps(eff)}$ is the excess area of prestressing steel and $f_{ps(eff)}$ is the effective stress in prestressing steel.

OTHER LIMITATIONS

To ensure the development of ultimate torsional strength and to control cracking and stiffness at service load, the maximum yield strength of reinforcement should be limited to 60,000 psi and the maximum spacing of stirrups should not exceed $(x_1 + y_1)/4$ nor 12 in. If required, longitudinal torsion reinforcement of not less than a No. 3 bar should be distributed around the stirrup at no more than 12 in. apart. In any event, at least one No. 3 bar should be placed at each corner of the stirrup.

For reinforced concrete members, ACI 318-71 permits the torsional effect to be neglected if the torsional shear stress is less than 25 percent of the cracking stress. This same requirement would seem justified for prestressed beams. That is to say, the torsional effect may be neglected in design if the torsional stress τ_u is less than:

$$1.5\sqrt{f_c'}\sqrt{1 + 10\,\sigma/f_c'}$$

SUMMARY OF PROPOSED DESIGN PROCEDURE

The design procedure proposed herein may be summarized as follows:

1. Determine the nominal torsional stress according to Eq. (5). The stress τ_u should not exceed that specified in Eq. (22).

2. If $\tau_u < 1.5\sqrt{f_c'}\sqrt{1 + 10\,\sigma/f_c'}$, torsion may be neglected.

3. If $\tau_u > \tau_c$ according to Eq. (15), determine the required web reinforcement from Eq. (19). Check the minimum web reinforcement requirement.

4. Determine the required longitudinal torsion reinforcement according to Eq. (24). Check the minimum requirement.

DESIGN EXAMPLE

To illustrate the above design procedure, let us consider the design of torsional reinforcement in a prestressed concrete L-girder in a roof framing similar to that given in the *PCA Design Notes*[18] as shown in Fig. 5. The design live load plus insulation and roofing is 40 psf. For this loading, a double tee 8DT18A and a L-girder 18LB30 can be chosen according to the *PCI Design Handbook*.[19]

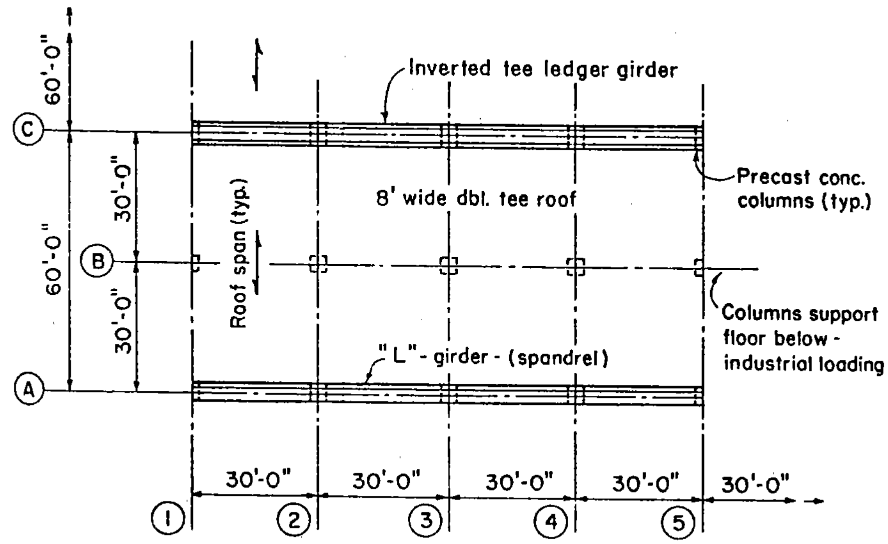


Fig. 5. Partial plan of precast concrete roof.

From an analysis for flexure, thirteen ½-in. diameter 7-wire strands (270 ksi grade) are required for the L-girder with the eccentricity of prestress at midspan being 7.3 in. and that at the support being 3.65 in. Assume that the loss of prestress is 22 percent; $f'_c = 5000$ psi; $f'_{ci} = 3500$ psi; $f_{sy} = f_{ly} = 40$ ksi. In the following calculations, the usual capacity reduction factor for shear and torsion $\phi = 0.85$ will be applied.

(a) Calculate T_u and V_u for spandrel

$W_{DL} = 30' \times 0.062 = 1.85$ kips per ft (double tees)
0.45 kips per ft (spandrel beam)
2.30 kips per ft

$W_{LL} = 0.040 \times 30' = 1.20$ kips per ft

Assume no continuity at columns.

$$DL = 1.4 \times 2.30 \times 30 \times \frac{1}{2} = 48.3 \text{ kips}$$

$$LL = 1.7 \times 1.20 \times 30 \times \frac{1}{2} = 30.6 \text{ kips}$$

V_u at centerline of support $= 78.9$ kips

Neglecting restraints from double tees and walls.

$DL = 1.4 \times 1.85 \times (9/12) \times 15' = 29.1$ ft-kips
$LL = 1.7 \times 1.20 \times (9/12) \times 15' = 22.9$ ft-kips
T_u at centerline of support = 52.0 ft-kips (or 625 in.-kips)

(b) **Calculate shear stress v_u**

V_u at "d = 22.7 in." from support $= 78.9 \times 13/15 = 68.4$ kips
$v_u = V_u/\phi b_w d = 68{,}400/0.85 \times 12 \times 22.7 = 295$ psi

(c) **Calculate torsion stress τ_u with Eq. (5)**

$\tau_u = T_u/\phi(\Sigma\alpha x^2 y)$
For maximum $\Sigma\alpha x^2 y$, consider beam web and outstanding ledge as component rectangles.

$$\alpha_1 = \frac{0.35}{0.75 + 12/30} = 0.304 \text{ (beam web)}$$
$$\alpha_2 = \frac{0.35}{0.75 + 6/12} = 0.280 \text{ (ledge)}$$
$$\Sigma\alpha x^2 y = 0.304 \times 12^2 \times 30 + 0.280 \times 6^2 \times 12 = 1313 + 121 = 1434 \text{ in.}^3$$

T_u at "d" from face of support $= 625 \times 13/15 = 541$ in.-kips
$\tau_u = 541/0.85 \times 1434 = 445$ psi
Effective prestress $= 0.78 \times 189 = 147$ ksi
$\sigma = 13 \times 0.153 \times 147/432 = 0.677$ ksi $= 677$ psi
$\sigma/f_c' = 677/5000 = 0.135$
$1.5\sqrt{f_c'}\sqrt{1 + 10\,\sigma/f_c'} = 1.5 \times 70.5 \times 1.53 = 162 \text{ psi} < 677 \text{ psi}$
Requires torsion design.

(d) **Check $\tau_{u(max)}$ and $v_{u(max)}$ with Eqs. (22) and (23)**

$C = 14 - 13.33(\sigma/f_c') = 14 - 13.33(0.135) = 12.2$
$C' = C\sqrt{1 + 10\,\sigma/f_c'} = 12.2 \times 1.53 = 18.7$

$$\tau_{u(max)} = \frac{C'\sqrt{f_c'}}{\sqrt{1 + \left(\frac{C'}{10}\cdot\frac{v_u}{\tau_u}\right)^2}} = \frac{18.7\sqrt{5000}}{\sqrt{1 + \left(\frac{18.7}{10}\cdot\frac{295}{445}\right)^2}}$$
$$= 1322/\sqrt{2.54} = 832 \text{ psi} > \tau_u \text{ (ok)}$$
$$v_{u(max)} = \frac{10\sqrt{f_c'}}{\sqrt{1 + \left(\frac{10}{C'}\cdot\frac{\tau_u}{v_u}\right)^2}} = \frac{10\sqrt{5000}}{\sqrt{1 + \left(\frac{10}{18.7}\cdot\frac{445}{295}\right)^2}}$$
$$= 707/\sqrt{1.65} = 550 \text{ psi} > v_u \text{ (ok)}$$

Section is adequate for torsion and shear.

(e) **Calculate torsion stress and shear stress carried by concrete τ_c and v_c with Eqs. (15) and (16)**

$\tau_c' = 6\sqrt{f_c'}\,(\sqrt{1 + 10\,\sigma/f_c'} - k_{web})$
$k_{web} = 1 - 0.133/\alpha_1 = 1 - 0.133/0.304 = 0.563$
$\tau_c' = 6\sqrt{5000}\,(\sqrt{1 + 10(0.135)} - 0.563) = 412$ psi

v_c' = lesser of v_{ci} and v_{cw} from ACI Eqs. (11-11) and (11-12)

$$= v_{cw} = 3.5\sqrt{f_c'} + 0.3 f_{pc} + \frac{V_p}{b_w d}$$
$$= 3.5\sqrt{5000} + 0.3(677) + 5930/12 \times 22.7$$
$$= 247 + 203 + 22 = 472 \text{ psi}$$
$$\beta = \frac{1}{2}\left(\frac{v_c'}{\tau_c'}\right) = \frac{1}{2}\left(\frac{472}{412}\right) = 0.573$$
$$\tau_c = \frac{\tau_c'}{\sqrt{1 + \left(\frac{1}{\beta}\cdot\frac{v_u}{\tau_u}\right)^2}} = \frac{412}{\sqrt{1 + \left(\frac{1}{0.573}\cdot\frac{295}{445}\right)^2}}$$
$$= \frac{412}{\sqrt{2.34}} = 270 \text{ psi}$$
$$v_c = \frac{v_c'}{\sqrt{1 + \left(\beta\frac{\tau_u}{v_u}\right)^2}} = \frac{472}{\sqrt{1 + \left(0.573\,\frac{445}{295}\right)^2}}$$
$$= \frac{472}{\sqrt{1.75}} = 358 \text{ psi}$$

(f) **Calculate web reinforcement for torsion with Eq. (19)**

$x_1 = 12 - 2 \times 1.5 = 9$ in.
$y_1 = 30 - 2 \times 1.5 = 27$ in.
$\Omega = 0.66 + 0.33\,(y_1/x_1)$
$= 0.66 + 0.33\,(27/9) = 1.65 > 1.5$
Use $\Omega = 1.5$

$$A_t = \frac{(\tau_u - \tau_c)s\,\Sigma\alpha x^2 y}{\Omega x_1 y_1 f_{sy}}$$
$$\frac{A_t}{s} = \frac{(445 - 270) \times 1434}{1.5 \times 9 \times 27 \times 40{,}000} = 0.0173 \text{ sq in. per in.}$$
$$\textit{Maximum } s = \frac{x_1 + y_1}{4} = \frac{9 + 27}{4} = 9 \text{ in.} < 12 \text{ in.}$$

$A_t = 0.0173 \times 9 = 0.155$ sq. in.
Use No. 4 closed stirrups at 9 in. on center

(g) **Check minimum web reinforcement for torsion**

Minimum web reinforcement should be what is required to develop cracking torque of section.
Torsional stress at cracking:
$\tau_{cr} = 6\sqrt{f_c'}\sqrt{1 + 10\,\sigma/f_c'}$
$= 6\sqrt{5000}\sqrt{1 + 10(0.135)} = 650$ psi
Torsional stress carried by concrete under pure torsion is $\tau_c' = 412$ psi.

$$\frac{A_t}{s} = \frac{(650 - 412) \times 1434}{1.5 \times 9 \times 27 \times 40{,}000} = 0.0234 \text{ sq in. per in.}$$

$A_t = 0.0234 \times 9 = 0.21$ sq in. ≈ 0.20 sq in.
Thus, No. 4 closed stirrups at 9 in. on center is adequate.

(h) Calculate web reinforcement for shear

$v_u = 295$ psi

$v_c = 358$ psi $> v_u$

But $v_u > v_c/2$; so minimum web reinforcement is required.

$$A_v = \frac{50 b_w s}{f_{sy}} = 50 \times 12 \times 9/40{,}000 = 0.135 \text{ sq in.}$$

$A_v/2 = 0.0675$ sq in. $<< 0.20$ sq in.

Thus, the minimum web reinforcement provided for torsion is more than enough for shear.

(i) Calculate longitudinal torsion reinforcement with Eq. (24)

$$A_l = 2(A_t/s)(x_1 + y_1)(f_{sy}/f_{ly})$$
$$= 2(0.0173)(9 + 27) = 1.24 \text{ sq in.}$$

Distribute A_l around perimeter of stirrups with maximum spacing of 12 in. Eight bars are required.

Use four No. 4 bars and four No. 3 bars.

Area provided $= 4 \times 0.20 + 4 \times 0.11 = 1.24$ sq in. (ok)

(j) Design reinforcement for ledge

Load on ledge:

DL $= 1.4 \times 1.85 = 2.59$ kips per ft

LL $= 1.7 \times 1.20 = 2.04$ kips per ft

$V_u = 4.63$ kips per ft

$d = 12 - 2 = 10$ in.

Check for beam action:

$$v_u = \frac{V_u}{\phi b_w d} = \frac{4630}{0.85 \times 12 \times 10} = 45.5 \text{ psi} < 2\sqrt{f_c'} = 141 \text{ psi (ok)}$$

Check for slab action:

$$v_u = \frac{V_u}{\phi b_o d} = \frac{4 \times 4630}{0.85 \times 21 \times 10} = 104 \text{ psi} < 4\sqrt{f_c'} = 282 \text{ psi (ok)}$$

$M_u = 4.63 \times 3 = 13.89$ in.-kips per ft $\qquad jd \approx 9.5$ in.

$$A_s = \frac{M_u}{\phi j d f_y} = \frac{13890}{0.9 \times 9.5 \times 40{,}000} = 0.0405 \text{ sq in. per ft}$$

Even if one-third more than the above steel area is provided, the design is controlled by ACI Code Section 10.5.2.

$A_s = 0.002 \times 12 \times 10 = 0.24$ sq in. per ft

Use No. 4 bars at 10 in. on center and one No. 4 bar in each corner.

Check shear friction:

$$A_{vf} = \frac{V_u}{\phi f_y \mu} = \frac{4630}{0.85 \times 40{,}000 \times 1.4} = 0.098 \text{ sq in. per ft} < 0.24 \text{ sq in. per ft}$$

The reinforcement details for the L-girder are shown in Fig. 6.

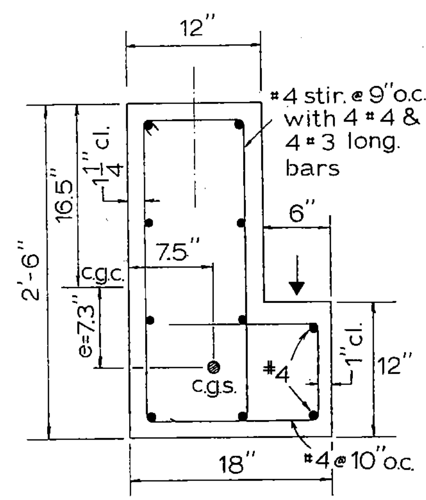


Fig. 6. Reinforcement details for L-girder.

CONCLUSIONS

A procedure for design of torsion in prestressed concrete members has been proposed, which follows the same approach as ACI 318-71 for design of torsion in reinforced concrete members. Comparison[2] with available test results of 132 beams under combined loading as reported in the literature indicates that the proposed procedure is reasonably conservative for rectangular beams and quite conservative for flanged and box beams. The application of the design procedure has been illustrated by a design example.

When more complete research data becomes available, especially for prestressed flanged and box members, the procedure can be further refined and simplified. In the meantime, design tables and curves can be easily prepared to aid the designers in reducing the amount of computation.

ACKNOWLEDGMENTS

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NOTATION

A_l = area of longitudinal torsion reinforcement
A_t = cross-sectional area of *one* leg of stirrup
V_u = ultimate shear force
T_o = contribution by concrete to torsional strength of non-prestressed beam
T_o' = contribution by concrete to torsional strength of prestressed beam
T_u = ultimate torque
T_{up} = torsional strength of plain concrete beam
f_c' = concrete cylinder strength
f_{ly} = yield strength of longitudinal reinforcement
f_r = modulus of rupture of concrete
f_{sy} = yield strength of stirrup
h = wall thickness of box section
m = volume ratio of longitudinal reinforcement to web reinforcement
s = spacing of stirrup
v_c = permissible flexural shear stress under combined torsion and flexural shear
v_c' = permissible flexural shear stress without torsion as defined by ACI Eq. (11-11) or (11-12)
v_u = nominal flexural shear stress
x = shorter side of rectangular section
x_1 = shorter leg of closed stirrup
y = longer side of rectangular section
y_1 = long leg of closed stirrup
α = torsion constant, see Eq. (1)
Ω = reinforcement coefficient, see Eq. (6)
σ = average prestress on section
τ_c = permissible shear stress for torsion under combined torsion and flexural shear
τ_c' = permissible shear stress for torsion without flexural shear
τ_u = nominal torsional shear stress

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