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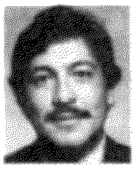
Detailing of Stirrup Reinforcement

by Neal S. Anderson and Julio A. Ramirez

This paper addresses the detailing of stirrup reinforcement for shear. First, the function of the stirrups in members under high shear stresses is explained by means of truss models. Next, the results of an experimental program evaluating stirrup details used in current practice are presented. Based on this experimental program, recommendations for detailing of stirrups in members under high shear stresses are given.

Keywords: anchorage (structural); **detailing;** models; **reinforced concrete; reinforcing steels;** shear stress; **stirrups;** trusses.

Successful performance of reinforced and prestressed concrete members requires an effective interaction between concrete and reinforcing steel. Not only is an adequate amount of reinforcement needed, but it must also be properly detailed to insure satisfactory member behavior under all loading conditions.

Current American design practice[1-3] for shear in reinforced and prestressed concrete beams with web reinforcement envisions a parallel chord truss with 45-deg compression diagonals as the fundamental behavioral model. Detailing of concrete members for shear would be more rational and simple if this behavioral model was brought to the foreground in current design specifications.

RESEARCH SIGNIFICANCE

American design practice for web reinforcement in concrete beams assumes that the maximum stirrup force requires development at or near middepth of the member. Anchorage of the required force in the stirrup bar is accomplished through straight embedment length, or a combination of straight embedment and hooked anchorages above and below the member middepth.[3,4] Modeling of the concrete beam as a truss where the stirrup reinforcement constitutes the vertical tension members demands a full-strength mobilization throughout the entire stirrup height, not only at or near middepth of the beam. Hence, some stirrup details used in concrete design[4] become questionable for members under high shear stresses.

Current detailing practices for shear in wide beams where the stirrup legs are concentrated around the outermost longitudinal bars are also questionable. Since forces can be developed only at the truss joints, lack of stirrup legs in the interior of the member web would force truss joints to form only at the exterior longitudinal bars. This could result in overloading of the truss joint, and inefficient use of the interior longitudinal tension reinforcement under high shear stresses.

Therefore, an examination of the performance of different stirrup detailings under high shear stresses used in American practice was undertaken. The experimental and analytical investigation[5] included 16 reinforced concrete beams tested to failure where the stirrup detailing was the main variable. The performance of the stirrup detail, the behavior, and the ultimate strength of the member were evaluated with the aid of truss models.

STIRRUPS IN THE TRUSS MODEL

Truss models are very useful for detailing. They represent the distribution of internal forces in the member at failure. Once this distribution is known, structural systems (truss models) comprised of concrete and steel can be furnished to satisfy equilibrium between applied loads and supports.

The concept of truss models in reinforced concrete members was first introduced at the turn of the last century by Ritter[6] and Mörsch.[7] This pioneering work was later refined by Rausch,[8] Kupfer,[9] and Leonhardt.[10] Lampert and Thürlimann and several others have provided a foundation for truss models by means of the theory of plasticity.[11-17] Mitchell and Collins[18] introduced compatibility equations incorporating deformations of the truss model and derived a rational design concept for shear and torsion in beams. Additional contributions have been provided by Rogowsky and MacGregor,[19] Ramirez and Breen,[20] and Hsu and

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Mo.[21] Recent work by Marti[22-24] applied the concepts of detailing concrete beams using truss models. Schlaich, Schafer, and Jennewein[25] extended the truss model approach to overall structures in the form of strut and tie systems.

The truss model shown in Fig. 1 of a reinforced concrete beam under bending and shear illustrates the function of vertical stirrups. The stirrups form the vertical tension tie(s) of the truss. Under the truss assumption that forces can only be equilibrated at the joints, stirrups must be capable of developing the required force over their entire height. Obviously, anchorage plates, as shown in Fig. 1, are not practical. Likewise, straight bar embedment beyond the nodal zone (truss joint)[22,25] is not feasible because of clear cover limitations. Thus hooks are preferred for stirrup anchorage. Stirrup hooks anchored into the flexural compression zone are preferable since they benefit from the normal pressure. Also, bending stirrup hooks around large compression reinforcement would result in a better distribution of bearing stresses in the hook interior. Anchorage of stirrup hooks in the flexural tension zone is questionable because of the reduced confinement provided by cracked concrete. Continuation of the stirrup leg would seem to be the only feasible alternative for stirrup anchorage in the flexural tension region for members under shear stress $v_u = V_u/b_w d > 6\sqrt{f'_c}$.

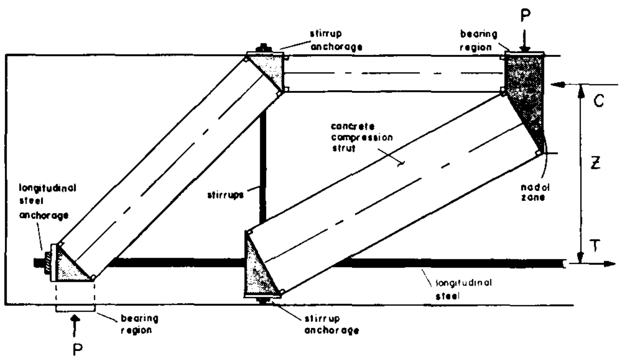


Fig. 1 — Truss model for beam under bending and shear

Adequate stirrup spacing is also critical; large spacings in the longitudinal direction of the beam create a concentration of diagonal compression stresses at the truss joints.[26] This practice results in overloading of the nodal zones or the diagonal struts themselves leading to premature failures due to concrete crushing. Spacing of stirrup legs in the transverse direction can also be a critical factor in wide beams when the legs are concentrated around the outer longitudinal bars.[27]

The plane truss model shown in Fig. 1 is actually the superposition of two trusses located on each side of the concrete member, as illustrated in Fig. 2(a). In narrow beams the longitudinal reinforcement is concentrated at the corners of the stirrup bends and an adequate two-truss system is developed. The diagonal compression truss members are equilibrated at the truss joint formed by the stirrup and the longitudinal reinforcement. The horizontal force component of the diagonals is balanced by the longitudinal reinforcement. The vertical force component is equilibrated by the stirrup reinforcement.

In wide beams with several longitudinal bars in a layer, as shown in Fig. 2(b), the lack of well-distributed

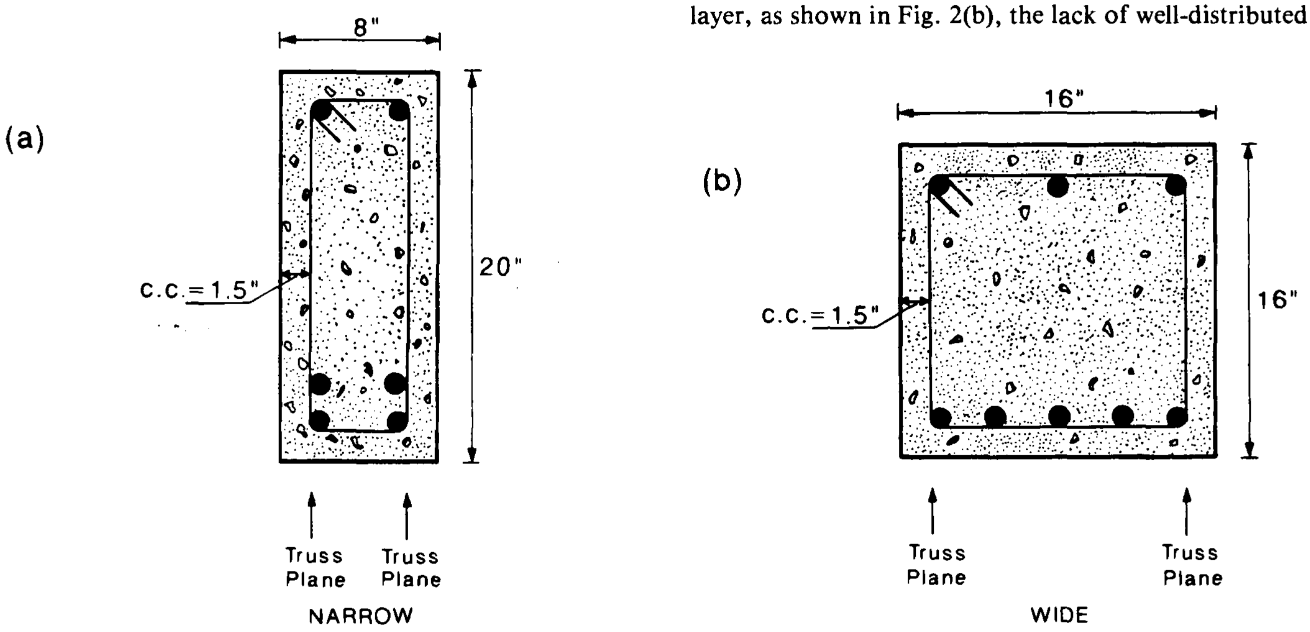


Fig. 2 — Truss planes for beam under bending and shear

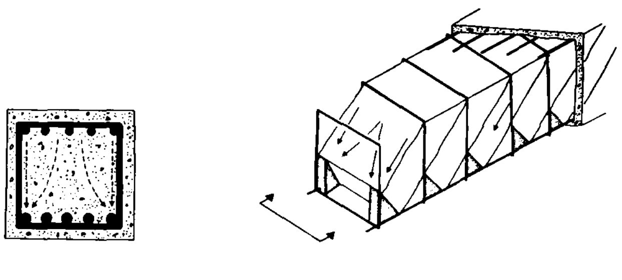


Fig. 3 — Concentration of diagonal struts on the outside longitudinal bars

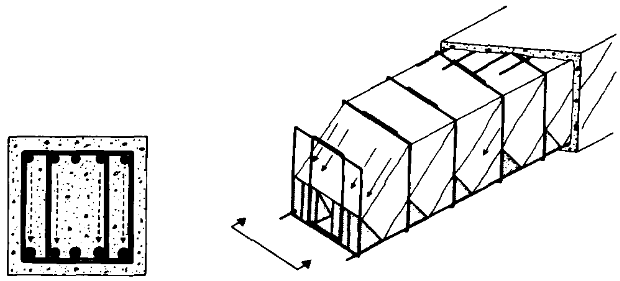


Fig. 4 — Diagonal struts in beam with distributed stirrup legs

stirrup legs across the web of the member could lead to a concentration of diagonal compression stresses at the joint of the stirrup leg and outside longitudinal bar. This is illustrated in Fig. 3. This situation could result in premature failures due to concrete crushing in these nodal zones and inefficient use of the interior longitudinal reinforcement. To correct this situation, interior stirrup legs can be placed to furnish the necessary vertical equilibrium resultants, thus creating additional interior truss joints as shown in Fig. 4.

EXPERIMENTAL PROGRAM

Two series of beams were tested in this investigation.[5] The first series consisted of twelve 8 in. wide x 20 in. deep specimens, referred to as narrow beams. The second series contained 16 x 16-in. specimens, referred to as wide beams. For each series, the a/d ratio, longitudinal reinforcement ratio ρ_w, and stirrup reinforcement index rf_{yv} were held constant. The concrete compressive strength f'_c ranged between 4000 and 6000 psi for both series. All specimens were loaded at third points on a 9 ft simple span. Beams were designed to fail in shear at levels of shear stress $V_{test}/b_wd > 6\ \sqrt{f'_c}$.

The main variable in the narrow beams was the stirrup detailing scheme. The different details were evaluated and a summary of the results of this series are shown in Table 1. The test setup and typical detailing are shown in Fig. 5.

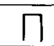
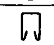
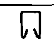
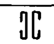
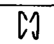
The transverse spacing of stirrup legs was the primary variable in the wide beam series. Test setup and specimen detailing are shown in Fig. 6. A summary of test results and specimen properties is given in Table 2.

Critical section for stirrup development

Earlier work by Kani[28] pointed out the critical stirrup anchorage condition that exists where the failure crack intersects it. Stirrups in Beams 9, 10, and 11 of the narrow specimen series were instrumented following the direction of the potential failure crack. Fig. 7 shows the failure crack pattern for Beam 10. The main failure crack is indicated by a heavier line. The vertical segments represent the stirrup locations, and targets indicate strain gage locations.

The measured strain readings are graphed versus the applied shear in Fig. 8 and 9 for Beam 10. These read-

Table 1 — Narrow beam test series

Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7	Col. 8
Beam	Detail	f'_c, psi	V_{test}, kips	K	V_{ACI}, kips	(4)/(6)	Failure mode
1		5660	107.6	10.7	78.1	1.38	S-C
2		6000	110.1	10.6	78.7	1.40	S-C
3		6200	114.9	10.9	79.0	1.45	S-C
4		3990	98.9	11.7	75.1	1.32	Flexure
5		4160	95.9	11.1	75.4	1.27	S-C
6		4290	82.9	9.5	75.7	1.10	S-C
7		4650	87.9	9.6	76.3	1.15	S-C
8		4910	80.9	8.6	76.8	1.05	S-C (bar buckling)
9		4990	88.9	9.4	77.0	1.16	S-C
10		4490	86.9	9.7	76.0	1.14	S-C
11		4680	82.9	9.1	76.4	1.09	S-C
12		4820	74.4	8.0	76.7	0.97	S-C (stirrup anchorage)

Notes: $\rho_w = 0.0265$; $a/d = 2.15$, $rf_{yv} = 410$ psi, $f_{yv} = 77$ ksi.
Col. 5 — $K = V_{test} / [(b_w)(d)\sqrt{f'_c}]$.
Col. 6 — ACI 318 Eq. 11-6 and 11-17 at d from support.
Col. 8 — Failure mode: S-C = shear-compression.

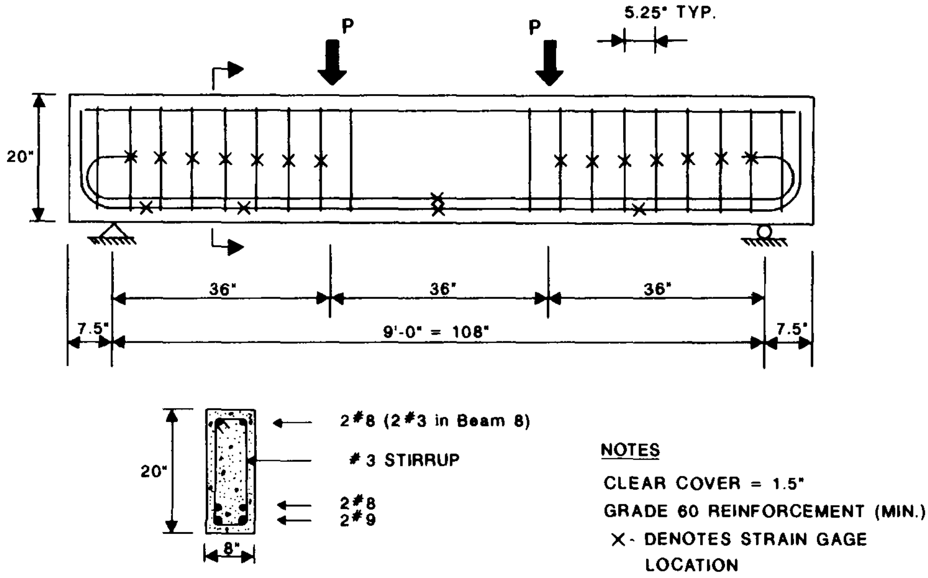


Fig. 5 — Narrow beam test series

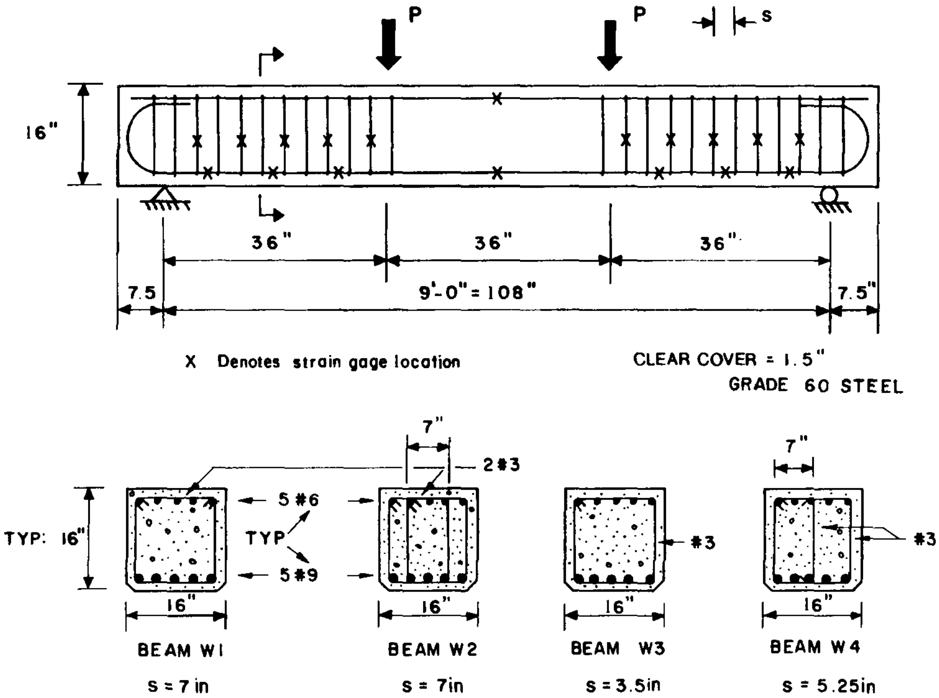


Fig. 6 — Wide beam test series

Table 2 — Wide beam test series

Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7	Col. 8
Beam	Detail	f'_c, psi	V_{test}, kips	K	V_{ACI}, kips	(4)/(6)	Mode of failure
W1		4230	103.4	7.3	98.8	1.05	S-C
W2		4670	123.4	8.2	100.2	1.23	S-C
W3		4690	113.4	7.6	100.2	1.13	S-C
W4		4900	131.4	8.7	100.8	1.30	Flexure

$\rho_w = 0.0231$, $rf_{yv} = 310$ psi, $f_y = 78.9$ ksi, $a/d = 2.65$.
Col. 6 — ACI 318 Eq. (11-6) and (11-17).

Col. 5 — $K = \frac{V_{test}}{[(b_w)(d)\sqrt{f'_c}]}$

Mode of failure: S-C = shear-compression.

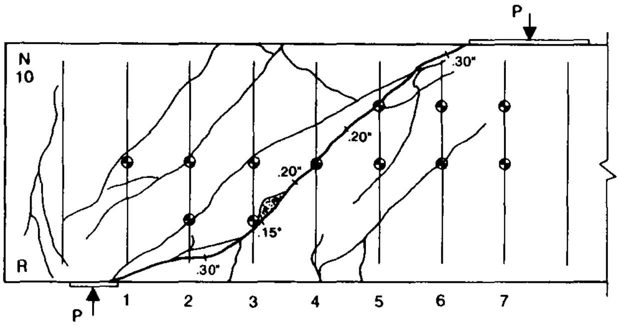


Fig. 7 — Failure crack pattern in Beam 10

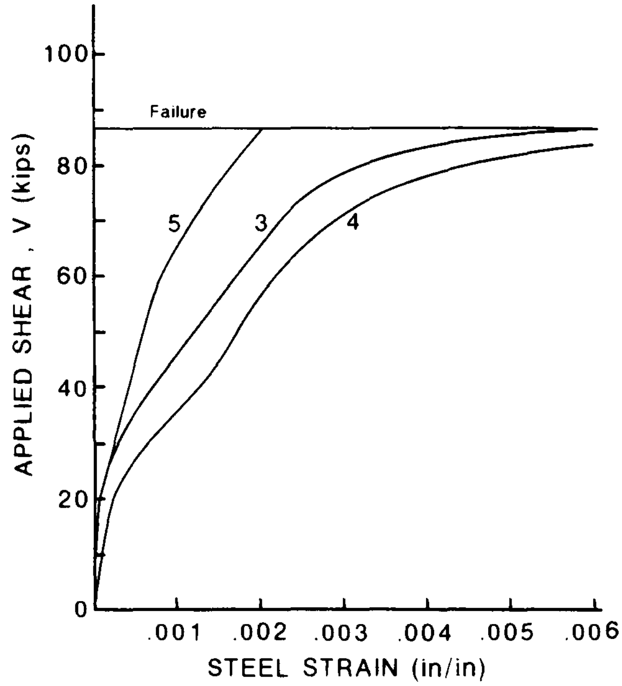


Fig. 8 — Midheight stirrup strains in Beam 10

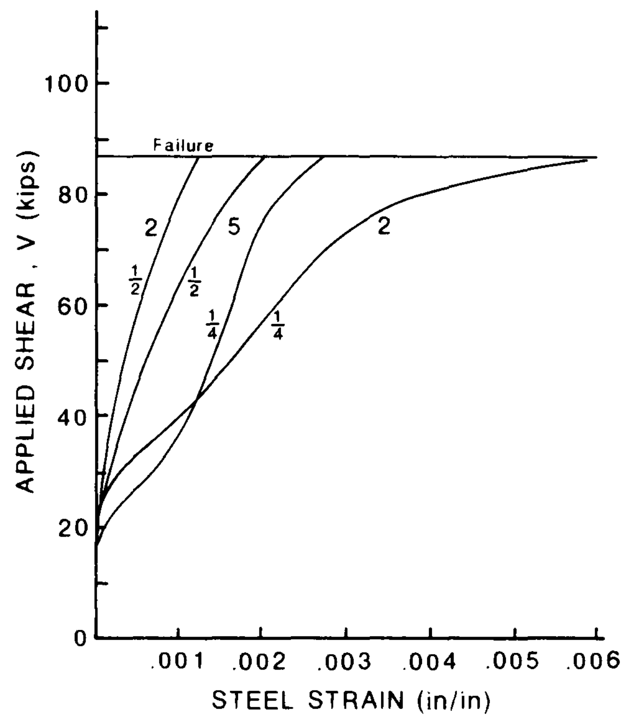


Fig. 9 — Comparison of midheight and quarter-height stirrup strains in Beam 10

ings indicate that the quarter-height gages showed more strain than the midheight gages on Stirrups 2 and 5. Examination of the crack pattern shown in Fig. 7 shows that the larger strain readings took place at the intersection of failure crack and stirrup. Similar observations were made in Beams 9 and 11. This indicates that stirrups may be required to mobilize yield under high shear stresses anywhere along their height due to the inclined nature of shear cracking. Since such crack locations are unknown prior to failure, proper stirrup anchorage must be available throughout the entire height. This anchorage constraint for stirrups is illustrated in the truss model shown in Fig. 1, where the vertical stirrups represent the vertical tension truss members. Since forces in a truss system can be equilibrated only at the joints, a constant force exists in the truss members between joints.

Evaluation of stirrup details

The schemes evaluated in the narrow beam series are shown in Column 2 of Table 1. These include closed stirrups, single-bar U-stirrups, and single-legged stirrups. The different anchorages of stirrup free ends consisted of ninety 135-deg standard hooks and 180-degree hooks anchored in the flexural tension and/or compression zones.[5]

The results shown in Column 4 of Table 1 indicate that U- and closed stirrups performed better than single-legged stirrups (Beams 6, 7, and 11). Beam 4 had lapped U-stirrups for shear, forming a closed unit. The force/leg was 8525 lb, and the stirrup legs extended the available full depth of the beam, thus meeting current ACI Building Code[3] requirements. Beam 4 failed in flexure at a load over the predicted ACI shear capacity. Fig. 10 shows that at failure some of the individual stirrup legs were not stressed to yield. The shear strength provided by the lapped closed unit exceeded the capacity of two legs alone; hence, the lap was adequate under high shear stresses.

The stirrup detailing in Beam 8 consisted of U-stirrups with 135-deg hooks anchored in the flexural compression zone. However, two No. 3 compression bars were used instead of two No. 8 bars, as shown in Fig. 5. The ACI Building Code[3] in the section for development of web reinforcement includes, as an alternative for anchorage of the free ends of stirrups No. 5 and smaller, a 135-deg standard hook around a longitudinal bar. When the stirrup design stress exceeds 40,000 psi, an additional straight embedment length of 0.33 l_d must be provided. Previous work by Müller[29] showed that anchorage of a bar is improved when the bar is bent around a transverse bar, as is the case for some stirrup anchorages. In practice, this benefit is obtained only if direct contact exists between the bars.

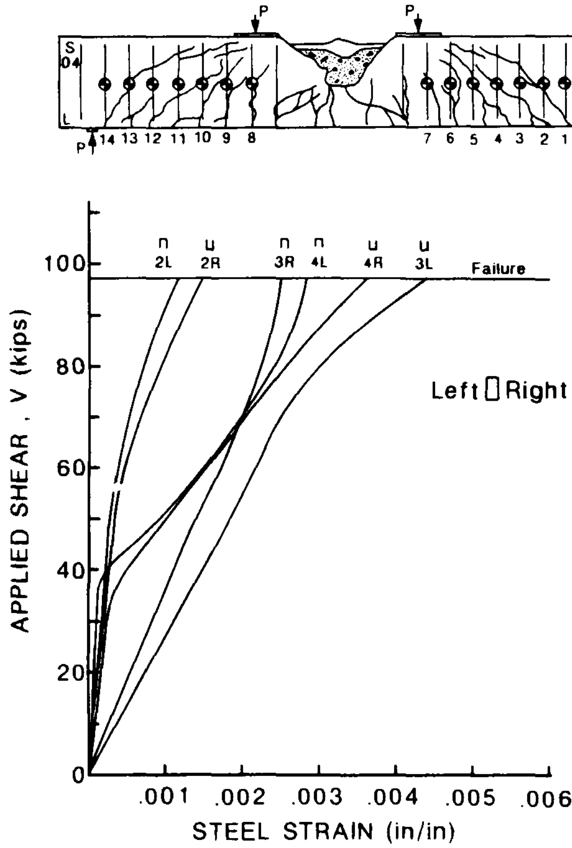


Fig. 10 — Stirrup strains in Beam 4

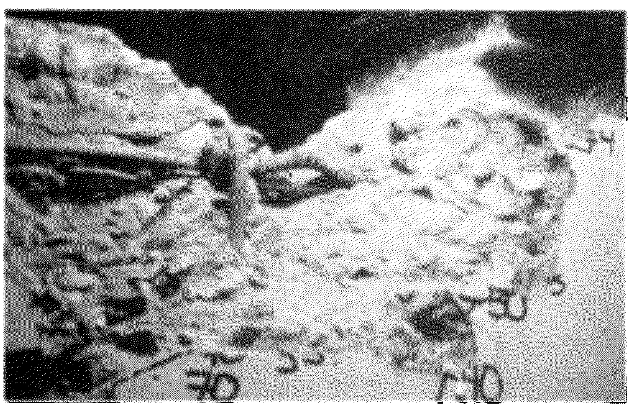


Fig. 11 — Buckling of No. 3 compression bar in Beam 8

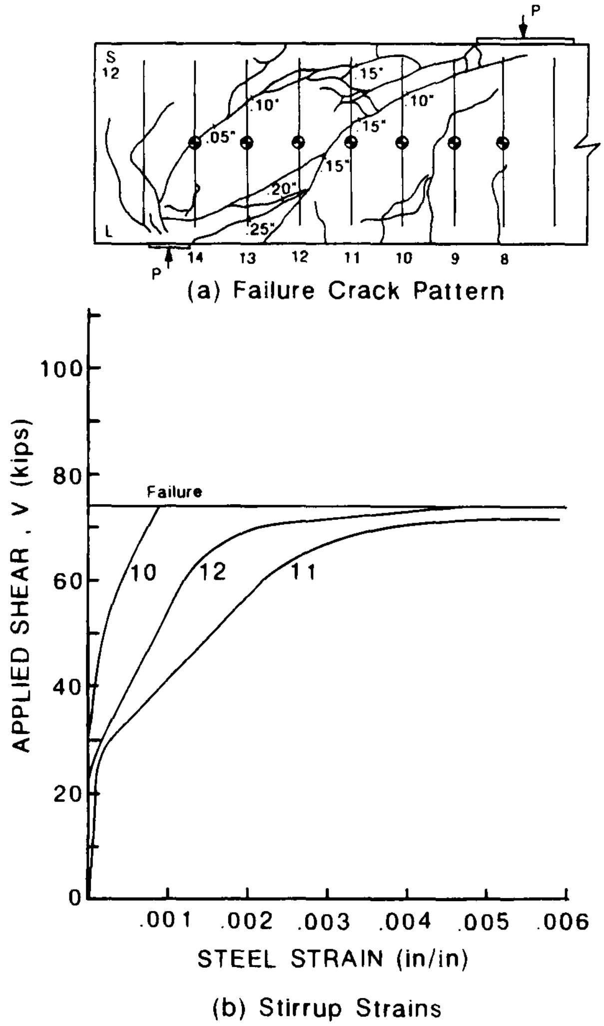


Fig. 12 — Improper anchorage of Stirrup 10 in Beam 12

Under normal construction conditions and with Grade 60 steel it is almost impossible to bend stirrups tightly around longitudinal bars. For this reason, it is assumed in current design recommendations[3] that in stirrups where such detailing is utilized, anchorage depends primarily on the hook and whatever lead length is provided. Therefore, the stirrup detailing in Beams 8 and 10 would be considered the same. The difference in ultimate shear between these two beams was the strength of the uncracked flexural compression zone, as both beams failed in shear compression. The penetration of shear cracks into the flexural compression zone increased the stress in the longitudinal compression reinforcement as the concrete weakened. In Beam 8, with the smaller compression bars, the increase in stress led to buckling of the No. 3 bar at a lower load level, as can be seen from Fig. 11.

The contribution of the uncracked flexural compression zone to the shear strength of beams with stirrups failing in shear compression can be illustrated further by comparing Beams 1 and 10. Beam 1 had a higher f_c' and was able to carry a larger ultimate shear.

Analysis of Beams 1 and 10 indicated that hooked ends anchored in the flexural compression zone following current design requirements[3] performed satisfactorily. However, anchorage in the flexural compression zone can be critical if a proper truss joint is not established. In Beam 12, the free ends of the U-stirrups were anchored in the flexural compression zone by means of straight embedment length only. Beam 12 experienced a premature failure due to improper anchorage of Stirrup 10. It can be seen from Fig. 12(a) that the failure crack crossed Stirrup 10 above midheight. The force generated led to stirrup anchorage failure. The anchorage was adequate for Stirrups 11 and 12, as can be seen from Fig. 12(b). This observation confirms the location of the critical section for stirrup development at the intersection of crack and stirrup bar. It must be noted that the stirrup detailing in Beam 12 did not meet the current ACI requirement[3] of a minimum straight embedment length of 12 in.

The results of this study indicated that the stirrup details with free ends anchored in the flexural tension zone were adequate. All the specimens with this detailing had a failure load larger than predicted (Column 7, Table 1). However, at failure in Beam 11 with 90-deg hooks, the free ends of the stirrup near the support region pushed out. This behavior could be critical in negative moment regions due to the combination of high shear and high flexural stresses. As this was not the case for the specimens in this study, it is felt that further evaluation of this detail is needed. It can be expected that the longitudinal tension reinforcement will play a significant role in crack control, thus improving anchorage conditions for stirrup free ends in the flexural tension zone.

The performance of stirrups with free ends anchored outside the member's confined concrete core (surrounded by the stirrup reinforcement) was studied in Beam 7 to compare with Beam 6. The stirrup detailing in Beam 6 was similar to Beam 7, but with free ends anchored inside the core (Table 1). Based on the ratio of tested to predicted capacity shown in Column 7 of Table 1, both details seemed to be adequate. The comparison of failure crack patterns for both beams shown in Fig. 13 illustrates the difference in performance. In Beam 7 at failure, the hooks of the stirrup anchored outside the core pushed out.

Transverse spacing of stirrup legs

This detailing aspect was evaluated in the wide beam series shown in Fig. 6. In Beams W1 and W3, the stirrup legs were concentrated around the corner longitudinal bars. The maximum transverse spacing of stirrup legs of 7 in. in Beam W2 and W4 met the limiting value of 20 cm (7.9 in.) proposed by Leonhardt and Walther[27] for beams under high shear stresses. At failure in Beams W1 and W3, the diagonal struts were forced to concentrate on the outside joints formed by the stirrup leg and the longitudinal steel, as shown in Fig. 3. As a result, the interior longitudinal reinforcement in Beams W1 and W3 was not utilized as well as in Beams W2 and W4.

Fig. 14 shows a comparison of the distribution of longitudinal bar strains in Beams W1 and W2. In Beams W2 and W4, the interior stirrup legs allowed the formation of additional truss joints in the interior of the core leading to an improved ultimate load behavior, as shown in Fig. 15. A comparison between Beams W1 and W3 illustrates that smaller stirrup spacings in the longitudinal direction also reduce the stress concentration on the outside joints.

In general, these tests showed the benefits of interior stirrup legs. Greater utilization of interior longitudinal bars can be afforded by reducing the transverse stirrup leg spacing in beams containing multiple bars per layer.

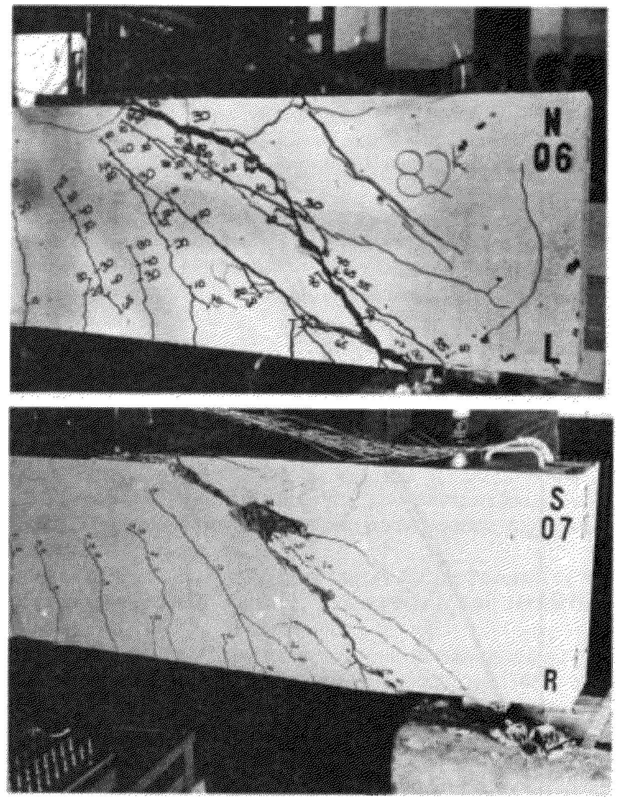


Fig. 13 — Failure crack patterns in Beams 6 and 7

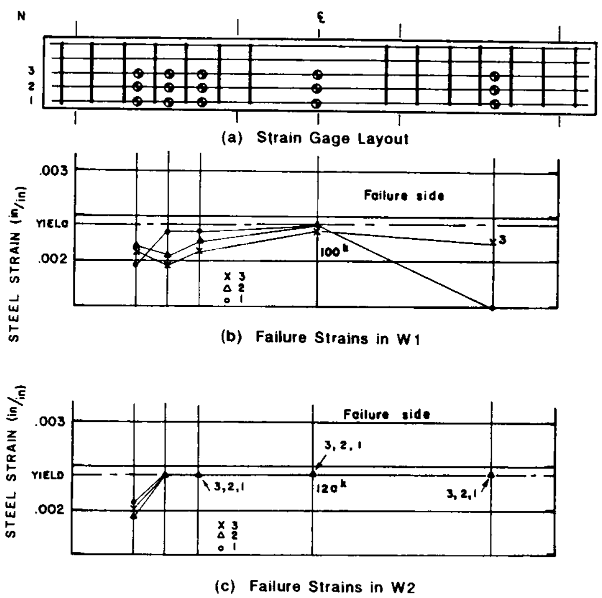


Fig. 14 — Distribution of strains in longitudinal tension bars

For the wide beam sizes tested in this study, the maximum transverse stirrup leg spacing should not exceed 7 in.

CONCLUSIONS AND RECOMMENDATIONS

An experimental evaluation of some of the detailing schemes used in current practice was presented. The truss model approach illustrated how detailing of rein-

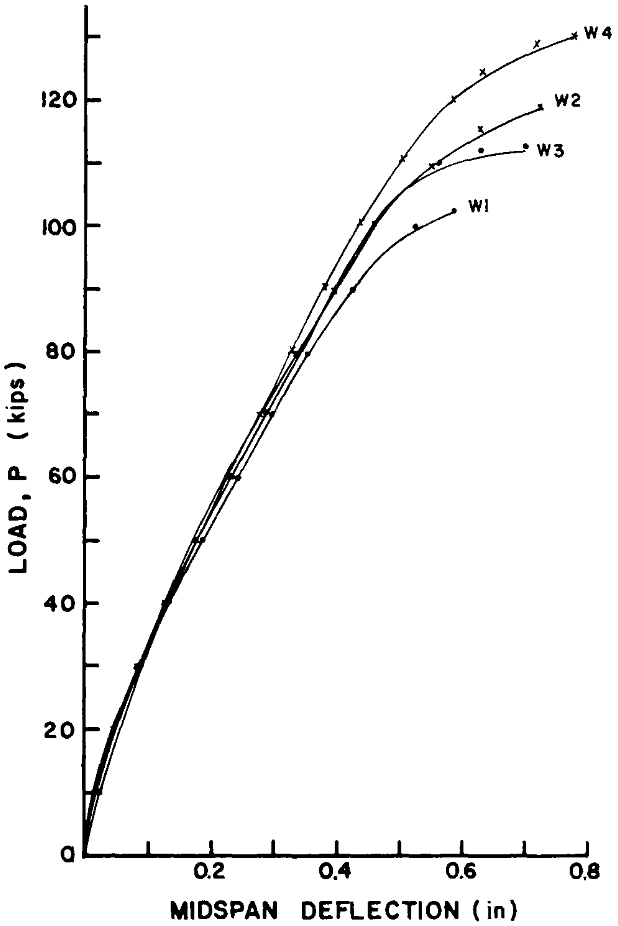


Fig. 15 — Load-deflection curve for wide beams

forced concrete members can be improved with the aid of behavioral models. Based on results of this study, the following recommendations are suggested to improve the performance of vertical stirrups in beams subjected to shear stresses exceeding $6\sqrt{f'_c}$:

1. Avoid the use of single-legged stirrups.
2. The free end of a continuous U-stirrup should be anchored by means of a standard hook bent within the confined concrete core.
3. Prevent anchorage of the free ends of a U-stirrup by means of a straight embedment length only.
4. In wide beams with multiple longitudinal bars per layer, stirrup legs should be distributed transversely across the member web.

FUTURE WORK

Further evaluation is needed on stirrups with free ends anchored in the flexural tension zone in negative moment regions, where the combination of high-flexural and shear stresses could lead to anchorage failures. The maximum transverse spacing of stirrup legs in wide beams needs further evaluation for beam sizes other than those evaluated in this study.

ACKNOWLEDGMENTS

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NOTATION

A_v = area of shear reinforcement located within spacing s, in.^{2}
A_s = area of tension reinforcement, in.^{2}
b_w = width of web, in.
d = effective depth of concrete beam, in.
f_{yv} = yield strength of stirrup reinforcement, psi
s = stirrup spacing in the longitudinal direction, in.
f'_c = concrete compressive strength, psi
l_d = development length of tension reinforcement, in.
r = $A_v/b_w s$
V_{ACI} = ACI predicted shear strength, kips
V_{test} = failure shear force, kips
v_u = factored shear stress, psi
ρ_w = $A_s/b_w d$

CONVERSION FACTORS

1 in. = 25.4 mm
1 lb(mass) = 0.4536 kg
1 lb(force) = 4.4482 N
1 psi = 6.895 Pa
1 kip = 4448.2 N
1 ksi = 6.895 MPa
1 kip-in. = 0.113 kN − m

Yield strength for stirrup reinforcement was determined by 0.2 percent offset in this study.

REFERENCES

1. *Standard Specifications for Highway Bridges,* 13th Edition, American Association of State Highway and Transportation Officials, Washington, D.C., 1983, 394 pp.
2. *Interim Specifications, Bridges,* American Association of State Highway and Transportation Officials, Washington, D.C., 1985.
3. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-83) (Revised 1986)," American Concrete Institute, Detroit, 1983, 111 pp.
4. ACI Committee 315, *ACI Detailing Manual,* SP-66, American Concrete Institute, Detroit, 1980, 212 pp.
5. Anderson, N. S., and Ramirez, J. A., "Effect of the Detailing of Stirrup Reinforcement on the Ultimate Strength and Behavior of Reinforced Concrete Members Failing in Shear," *Structural Engineering Research Report* No. CE-STR-87-2, Purdue University, W. Lafayette, 1987, 367 pp.
6. Ritter, W., "Die Bauweise Hennebique," *Schweizerische Bauzeitung* (Zurich), V. 33, No. 7, Feb. 1899, pp. 59-61.
7. Mörsch, E., "Der Eisenbetonbau—Seine Theorie und Anwendung," 5th Edition, Wittwev, Stuttgart, V. 1, Part 1, 1920, and Part 2, 1922, 112 pp.
8. Rausch, E., *Drillung, Schub und Scheren im Stahlbetonbau,* Deutscher Ingenieur-Verlag, Berlin, 1953, 168 pp.
9. Kupfer, H., "Erweiterung der Mörsch'schen Fachwerrkanalogie mit Hilfe des Prinzips vom Minimum der Formanderungsarbeit," *Bulletin d'Information* No. 40, Comité Européen du Béton, Paris, 1964, pp. 44-57.
10. Leonhardt, F., Die Verminderte Schudeckung bei Stahlbetontragwerken," *Der Baungenieur* (Heidelberg), V. 40, No. 1, Jan. 1965, pp. 1-15.
11. "Lampert, P., and Thürlimann, B., "Ultimate Strength and Design of Reinforced Concrete Beams in Torsion and Bending," *Publication* No. 31-1, IABSE, Zurich, 1971, pp. 107-131.
12. Grob, J., and Thürlimann, B., "Ultimate Strength and Design of Reinforced Concrete Beams under Bending and Shear," *Publication* No. 36-II, IABSE, Zurich, 1976, pp. 105-119.

13. Muller, P., "Failure Mechanisms for Reinforced Concrete Beams in Torsion and Bending," *IABSE Publications,* V. 36-II, International Association for Bridge and Structural Engineering, Zürich, 1976, pp. 147-163.

14. Marti, P., "Strength and Deformation of Reinforced Concrete Members Under Shear and Torsion and Combined Actions," *Bulletin d'Information* No. 146, Comité Euro-International du Béton, Paris, Jan. 1982, pp. 97-138.

15. Thürlimann, B.; Marti, P.; Pralong, J.; Ritz, P.; and Zimmerli, B., "Application of the Theory of Plasticity to Reinforced Concrete (Anwendung der Plastizitaetstheorie auf Stahlbeton)," Institut fur Bautechnik und Konstruktion, ETH Zurich, 1983, 252 pp.

16. Nielsen, M. P., and Braestrup, M. W., "Plastic Shear Strength of Reinforced Concrete Beams," *Byningsstatiske Meddelelser* (Lyngby), V. 46, No. 3, Sept. 1975, pp. 61-99.

17. Nielsen, M. P.; Braestrup, N. W.; and Bach, F., *Rational Analysis of Shear in Reinforced Concrete Beams,* International Association for Bridge and Structural Engineering, Zürich, May 1978, 15 pp.

18. Collins, Michael P., and Mitchell, Denis, "Shear and Torsion Design of Prestressed and Non-Prestressed Concrete Beams," *Journal,* Prestressed Concrete Institute, V. 25, No. 5, Sept.-Oct. 1980, pp. 32-100.

19. Rogowsky, D. M., and MacGregor, J. G., "Design of Reinforced Concrete Deep Beams," *Concrete International: Design & Construction,* V. 8, No. 8, Aug. 1986, pp. 49-58.

20. Ramirez, J. A., and Breen, J. E., "Experimental Verification of Design Procedures for Shear and Torsion in Reinforced and Prestressed Concrete," *Research Report* No. 248-3, Center for Transportation Research, University of Texas, Austin, Nov. 1983, 322 pp.

21. Hsu, Thomas T. C., and Mo, Y. L., "Softening of Concrete in Torsional Members," *Research Report* No. ST-TH-001-83, University of Houston, Mar. 1983, 107 pp.

22. Marti, Peter, "Basic Tools of Reinforced Concrete Beam Design," ACI JOURNAL, *Proceedings* V. 82, No. 1, Jan.-Feb. 1985, pp. 46-56.

23. Marti, Peter, "Truss Models in Detailing," *Concrete International: Design & Construction,* V. 7, No. 12, Dec. 1985, pp. 66-73.

24. Marti, Peter, "Staggered Shear Design of Simply Supported Concrete Beams," ACI JOURNAL, *Proceedings* V. 83, No. 1, Jan.-Feb. 1986, pp. 36-42.

25. Schlaich, Jörg; Schäfer, Kurt; and Jennewein, Mattias, "Toward a Consistent Design of Structural Concrete," *Journal,* Prestressed Concrete Institute, V. 32, No. 3, May-June 1987, pp. 74-150.

26. Mitchell, Denis, and Collins, Michael P., "Detailing for Torsion," ACI JOURNAL, *Proceedings* V. 73, No. 9, Sept. 1976, pp. 506-511.

27. Leonhardt, F., and Walther, R., "The Stuttgart Shear Tests," *C&CA Translation* No. 111, Cement and Concrete Association, London, 1964, 134 pp.

28. Kani, G. N. J., "A Rational Theory for the Function of Web Reinforcement," ACI JOURNAL, *Proceedings* V. 66, No. 3, Mar. 1969, pp. 185-197.

29. Müller, H. H., "Ausziehversuche mit Betonstahlhaken," Aus unseren Forrschungsarbeiten, *Bericht* No. 80, Dec. 1968, Technischen Hochschule, München, pp. 70-75.