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Effect of Concrete Strength and Minimum Stirrups on Shear Strength of Large Members

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There is concern that current ACI shear design procedures can be unconservative if applied to thick one-way slabs or large beams containing only minimum stirrups. This paper discusses the results of 21 large beams tested to investigate these concerns. Based on the experimental results, the paper concludes that until the current ACI shear provisions are modified, it would be prudent to use the recent shear provisions of the AASHTO LRFD specifications as these provide a more consistent level of safety. A simple spreadsheet is described that enables these provisions to be conveniently applied.

Keywords: beams; ductility; shear reinforcement; shear strength; slab.

INTRODUCTION

Since 1963,[1] the ACI Building Code has specified that the shear strength of reinforced concrete members not containing stirrups can be taken as $2\sqrt{f_c'}\,b_w d$ (psi units). This simple equation, which was based[2] on experiments of small rectangular beams containing relatively large amounts of longitudinal reinforcement, was intended to represent a conservative estimate of the shear at which diagonal cracks would form. Figure 1 shows a typical set of experimental results obtained by some of the engineers[3] involved in developing the ACI shear equation. It can be seen that for these 16 beams the shear strength increases by a factor of approximately 2, as the concrete strength is increased by a factor of approximately 4. Note that the ACI shear equation is consistently conservative with the average ratio of experimental-to-calculated shear capacities being 1.38 with a coefficient of variation of only 7.3%. Also shown in Fig. 1 are the calculated shear capacities of these beams according to the recently updated shear provisions of the AASHTO LRFD Bridge Design Specifications.[4] For the 16 beams, these provisions, which are based on the modified compression field theory (MCFT),[5,6] give an average ratio of experimental-to-calculated shear capacity of 1.22 with a coefficient of variation of 7.2%.

The simplified ACI shear equation assumes that, for beams without stirrups, the shear stress at failure will depend only on the cylinder strength of the concrete. Thus, in Fig. 2, it can be seen that the ACI expression predicts the same failure shear stress for small, heavily reinforced beams, similar to those tested in the development of the code equation, and for large, lightly reinforced beams. The AASHTO LRFD shear provisions, on the other hand, predict that the large, lightly reinforced beams will fail at much lower shear stresses than the small beams (Fig. 2). Further, for the large beams, the shear stress at failure is predicted to increase rather slowly as the concrete cylinder strength increases.

A recent paper by Collins and Kuchma[7] described an extensive experimental investigation aimed at evaluating the size effect in shear. The results of two series of beams from

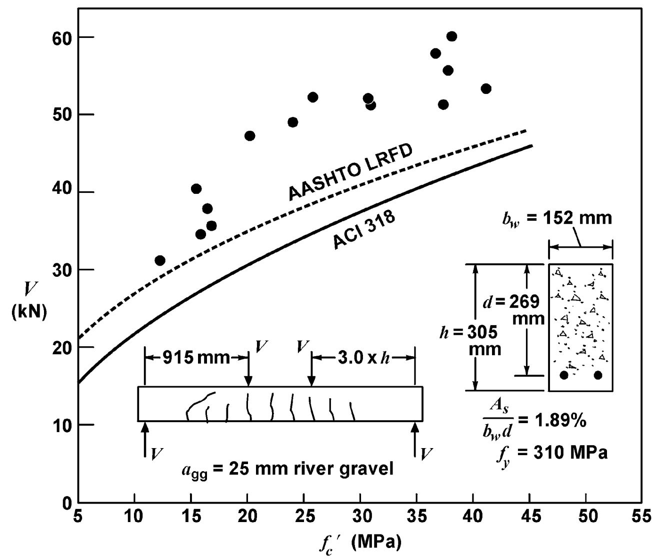

Fig. 1—Experiments by Moody, Viest, Elstner, and Hognestad to investigate influence of concrete strength on shear capacity.

this investigation are shown in Fig. 3. It can be seen that as the depth of the beams increased, the shear stress at failure diminished in a manner similar to that predicted by the AASHTO LRFD shear provisions. The shear capacities of the beams made from high-strength concrete (the BH series) differed very little from the shear capacities of the beams made from normal-strength concrete (the BN series). For Beam BH100, the shear stress at failure was only 0.695 MPa (101 psi). The 1963 ACI equation would overestimate the failure shear stress for this beam by a factor of approximately 2.4. Because of concerns with the applicability of the traditional empirical equation to beams made from high-strength concrete, the ACI codes since 1989[8] have placed an upper limit of 200 psi (1.38 MPa) on the failure shear stress calculated for such beams. Unfortunately, this reduced value of failure shear stress, which is plotted in Fig. 3, still overestimates the shear capacity of Beam BH100 by a factor of 2.

The current ACI Code requires that a minimum area of stirrups be provided in beams if the factored shear force exceeds $0.5\phi V_c$. For large beams, the safety issue then becomes whether such a member containing the minimum amount of stirrups specified by the ACI Code will fail at shears signif-

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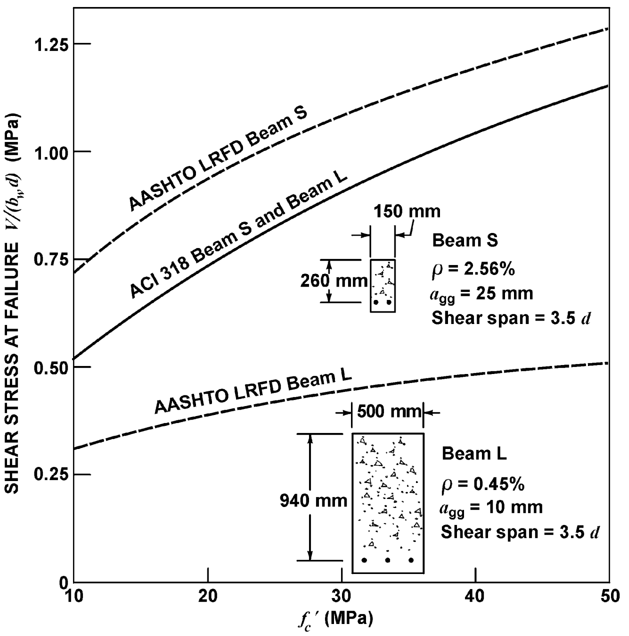


Fig. 2—Predicted influence of concrete strength on shear capacity for two series of beams.

icantly less than those predicted by the traditional equations. Unfortunately, very few experiments are available that can help to answer this question. To investigate these issues in more detail, a series of twelve 1 m deep beams consisting of six pairs of beams with concrete strengths varying from 21 to 80 MPa (3050 to 11,600 psi) were designed and tested. For five of these pairs, one beam contained approximately the minimum quantity of stirrups specified by the ACI Code while the other contained no stirrups. The results of these experimental investigations are summarized in this paper.

Concrete shear design provisions, based on the modified compression field theory, were introduced into the first edition of the AASHTO LRFD Bridge Design Specifications in 1994.[9] Based on experience with these early provisions and the results of further research, a number of changes were made to the AASHTO shear provisions in the annual update issued in 2000. This paper will briefly describe the current AASHTO LRFD shear provisions and will introduce a simple spreadsheet that enables the shear strength predicted by these provisions to be calculated conveniently. As the AASHTO code does not explicitly deal with strength predictions for beams with stirrups that contain less than the minimum amount, a method to assess the strength of such beams is also provided.

RESEARCH SIGNIFICANCE

Recent tests[7] have shown that the ACI Code equations for the shear strength of large, lightly reinforced concrete beams

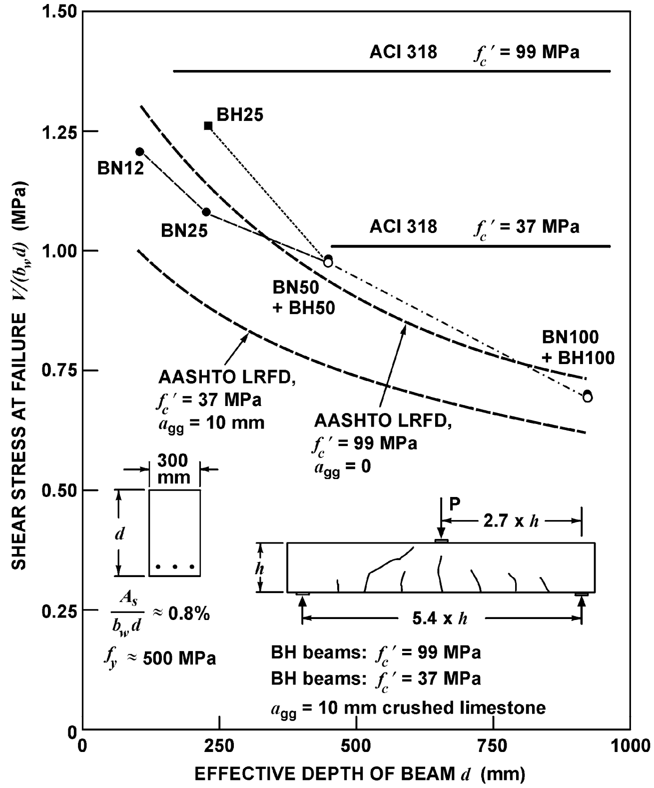


Fig. 3—Influence of member size and concrete strength on shear stress at failure.

and one-way slabs can be very unconservative. Further, a small number of tests had shown that concrete strength had little effect on the observed shear failure loads for such members. The research reported in this paper significantly increases the available experimental data on the shear strength of large, lightly reinforced members, particularly for members containing a minimum amount of stirrups. The experimental and analytical results reported herein indicate that it will be necessary to change the current ACI shear provisions for large, lightly reinforced members.

AASHTO LRFD shear design provisions

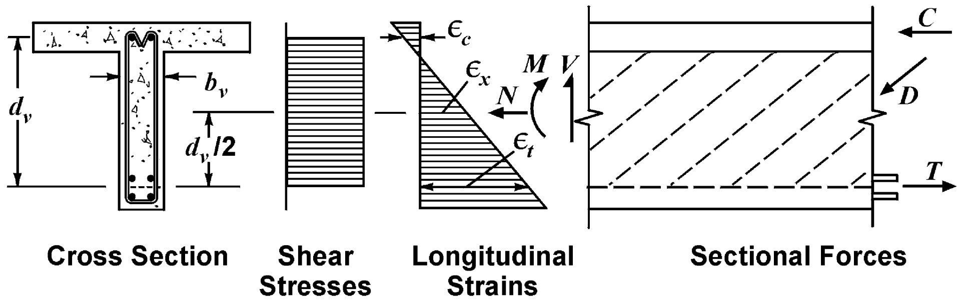
In the AASHTO LRFD specifications, the shear strength of a reinforced concrete section is expressed as

$$V_n = 0.083\beta\sqrt{f_c'}b_v d_v + \frac{A_v f_y}{s} d_v \cot\theta \tag{1}$$

Values of β and θ determined from the MCFT are given in Fig. 4 for sections containing at least the minimum amount of transverse reinforcement, and in Fig. 5 for sections without transverse reinforcement. The minimum amount of transverse reinforcement required in the AASHTO code is a function of the strength of the concrete, with higher strength concrete requiring more transverse reinforcement as follows

$$\frac{A_v f_y}{b_v s} = \sqrt{f_c'} \text{ (psi units)} = 0.083\sqrt{f_c'} \text{ (MPa units)} \tag{2}$$

This contrasts with the minimum specified by the ACI Code, which is 0.35 MPa (50 psi).



(a)

$\frac{v}{f_c'}$		$\varepsilon_x \times 1000$							
		≤0	≤0.125	≤0.25	≤0.50	≤0.75	≤1.00	≤1.50	≤2.00
≤0.075	θ, degrees	21.8	24.3	26.6	30.5	33.7	36.4	40.8	43.9
	β	3.75	3.24	2.94	2.59	2.38	2.23	1.95	1.67
≤0.100	θ, degrees	22.5	24.9	27.1	30.8	34.0	36.7	40.8	43.1
	β	3.14	2.91	2.75	2.50	2.32	2.18	1.93	1.69
≤0.125	θ, degrees	23.7	25.9	27.9	31.4	34.4	37.0	41.0	43.2
	β	2.87	2.74	2.62	2.42	2.26	2.13	1.90	1.67
≤0.150	θ, degrees	25.0	26.9	28.8	32.1	34.9	37.3	40.5	42.8
	β	2.72	2.60	2.52	2.36	2.21	2.08	1.82	1.62
≤0.175	θ, degrees	26.2	28.0	29.7	32.7	35.2	36.8	39.7	42.2
	β	2.60	2.52	2.44	2.28	2.14	1.96	1.71	1.54
≤0.200	θ, degrees	27.4	29.0	30.6	32.8	34.5	36.1	39.2	41.7
	β	2.51	2.43	2.37	2.14	1.94	1.79	1.61	1.47
≤0.250	θ, degrees	28.5	30.0	30.8	32.3	34.0	35.7	38.8	41.4
	β	2.40	2.34	2.14	1.86	1.73	1.64	1.51	1.39
≤0.250	θ, degrees	29.7	30.6	31.6	32.8	34.3	35.8	38.6	41.2
	β	2.33	2.12	1.93	1.70	1.58	1.50	1.38	1.29

(b)

Fig. 4—AASHTO provisions for beams with more than minimum stirrups: (a) location of e_x for members with stirrups and illustration of parameters; and (b) values of q and b for sections containing at least minimum amount of transverse reinforcement.

The values of both θ and β are related to the longitudinal strain ε_x occurring in the web (Fig. 4). As a simple procedure for calculating ε_x it can be related to the strain ε_t of the flexural tension chord of an equivalent truss. Thus, for nonprestressed members with no axial load

$$\varepsilon_t = \frac{(M_u/d_v) + 0.5V_u \cot\theta}{A_s E_s} \quad (3)$$

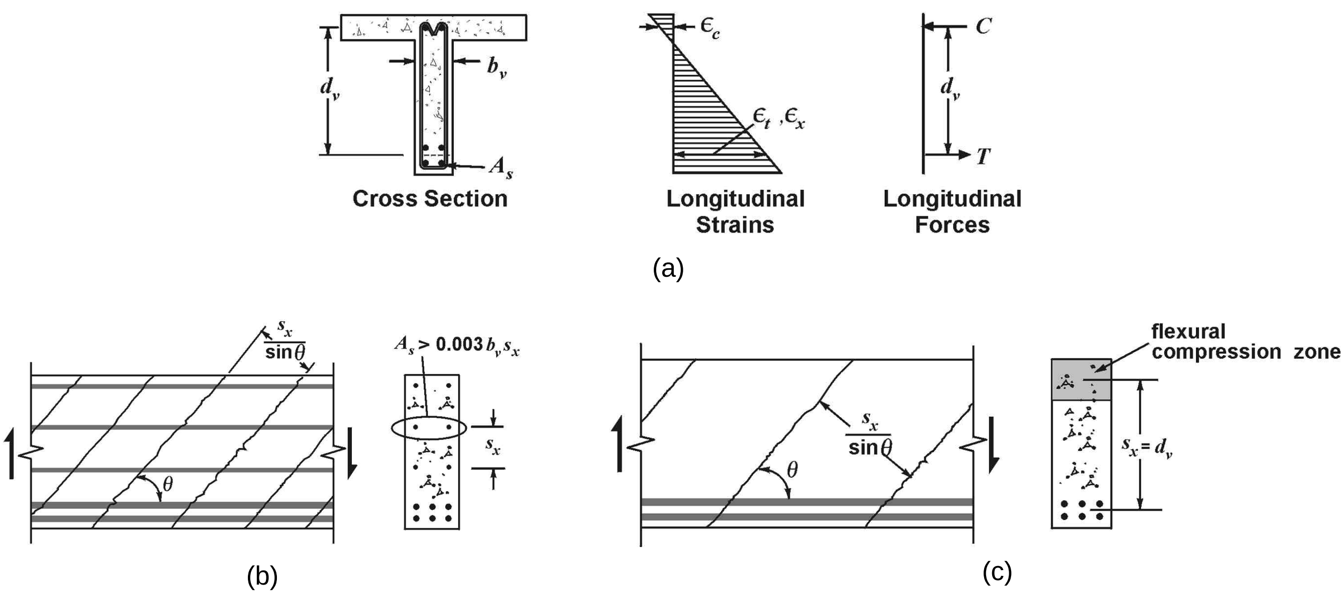
where A_s is the area of longitudinal reinforcement on the flexural tension side of the member and M_u and V_u are the coincident moment and shear values. For members with at least minimum web reinforcement, the average longitudinal strain over the depth of the web can be used for ε_x. Because the strain ε_c on the flexural compression side of the equivalent truss is usually quite small in comparison to ε_t, it is appropriate to take ε_x as 0.5 ε_t (Fig. 4).

For members without stirrups, the predicted shear strength is a function of the spacing of the diagonal cracks in the web (Fig. 5). The crack spacing when θ equals 90 degrees is called s_x, and is primarily a function of the maximum distance between the longitudinal reinforcing bars. As s_x increases, β decreases and hence, the shear strength decreases. The β values in Fig. 5 were derived assuming that the maximum aggregate size a_{gg} was 19 mm (3/4 in.); however, the tabulated values can be used for other aggregate sizes by using an equivalent crack spacing parameter s_{xe} where

$$s_{xe} = \frac{35 s_x}{a_{gg} + 16} \text{ (mm units)} \quad (4)$$

In beams made from concrete with high compressive strengths, for example, greater than 60 MPa, the cracks tend to pass through the aggregate, rather than going around the aggregate. As a consequence, it is recommended that a_{gg} be taken as zero for such members. To avoid a discontinuity in predicted strengths, a_{gg} can be linearly reduced to zero as f_c' goes from 60 to 70 MPa. Because members without stirrups are relatively brittle, it is prudent to use the highest longitudinal strain that occurs in the web in determining the β values (Fig. 5). Thus, ε_x can be taken as equal to ε_t for members without stirrups. Note that beams with stirrups that do not have at least the minimum amount of stirrups are not directly addressed by the AASHTO code, and a method to rate the strength of such beams is described in this paper.

Shear causes tensile stresses in the longitudinal reinforcement as well as in the stirrups. If a member contains an insufficient amount of longitudinal reinforcement, its shear strength will be limited by the yielding of this reinforcement. To avoid this type of failure, the longitudinal reinforcement



s_{xe}, mm		$\varepsilon_x \times 1000$ ≤0	≤0.125	≤0.25	≤0.50	≤0.75	≤1.00	≤1.50	≤2.00
≤127	θ, degrees	26.4	27.7	28.9	30.9	32.4	33.7	35.6	37.2
	β	5.15	4.41	3.91	3.26	2.86	2.58	2.21	1.96
≤254	θ, degrees	29.3	31.6	33.5	36.3	38.4	40.1	42.7	44.7
	β	4.89	4.05	3.52	2.88	2.50	2.23	1.88	1.65
≤381	θ, degrees	31.1	34.1	36.5	39.9	42.4	44.4	47.4	49.7
	β	4.73	3.82	3.28	2.64	2.26	2.01	1.68	1.46
≤510	θ, degrees	32.3	36.0	38.8	42.7	45.5	47.6	50.9	53.4
	β	4.61	3.65	3.09	2.46	2.09	1.85	1.52	1.31
≤760	θ, degrees	34.2	38.9	42.3	46.9	50.1	52.6	56.3	59.0
	β	4.43	3.39	2.82	2.19	1.84	1.60	1.30	1.10
≤1020	θ, degrees	36.6	41.1	45.0	50.2	53.7	56.3	60.2	63.0
	β	4.06	3.20	2.62	2.00	1.66	1.43	1.14	0.95
≤1520	θ, degrees	40.8	44.5	49.2	55.1	58.9	61.8	65.8	68.6
	β	3.50	2.92	2.32	1.72	1.40	1.18	0.92	0.75
≤2030	θ, degrees	44.3	47.1	52.3	58.7	62.8	65.7	69.7	72.4
	β	3.10	2.71	2.11	1.52	1.21	1.01	0.76	0.62

(d)

Fig. 5—AASHTO provisions for beams with less than minimum stirrups: (a) location of e_x *for members without stirrups; (b) member without stirrups and with concentrated longitudinal reinforcement; (c) member without stirrups but with well-distributed longitudinal reinforcement; (d) values of* **q** *and* **b** *for sections without transverse reinforcement.*

on the flexural tension side of the member must satisfy the following requirement

$$A_s f_y \geq M_u / d_v + (V_u - 0.5 V_s)\cot\theta \tag{5}$$

In using both Eq. (3) and (5) to predict the strength of a beam, a location in the beam must be selected that is critical for shear. This critical location for shear is generally a distance d_v from the face of the support, or d_v from the edge of the loading plate. The effective shear depth d_v can be taken as 0.9*d*. Thus, for Beam BH100, shown in Fig. 6, d_v is 0.9 × 925 = 833 mm. As the shear span for this beam is 2700 mm and the loading plate is 152 mm wide, the critical section is 2700 – 0.5 × 152 – 833 = 1792 mm from the support and hence, the *M*/*V* ratio at this location is 1.792 m. As this beam is made from 99 MPa concrete, a_{gg} can be taken as zero and from Eq. (4)

$$s_{xe} = \frac{35 \times 833}{0 + 16} = 1822 \text{ mm}$$

Before the values of θ and β can be read from Fig. 5, the failure value of ε_x is needed. For hand calculations, this requires some trial and error. As a first guess, it may be assumed that ε_x is 1.0×10^{-3}, meaning that the stress in the longitudinal steel would be approximately 200 MPa (29 ksi)

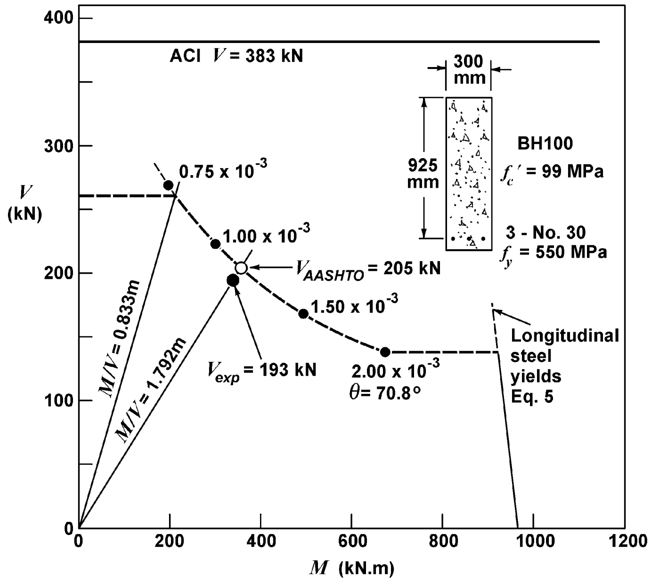


Fig. 6—Shear-moment interaction diagram for Beam BH100.

at shear failure. For an ε_x of 1.0×10^{-3}, Fig. 5 shows $\theta = 64.1$ degrees and $\beta = 1.08$. The predicted shear capacity from Eq. (1) will then be

$$V = 0.083 \cdot 1.08\sqrt{99} \times 300 \times 833 = 224 \text{ kN}$$

At the critical section, this shear will be associated with a moment of $1.792 \times 224 = 401$ kNm and hence, from Eq. (3), ε_x can be calculated as

$$\varepsilon_x = \varepsilon_t = \frac{(401 \times 10^6/833) + 0.5 \times 224 \times 10^3 \cot 64.1°}{3 \times 700 \times 200{,}000}$$

$$= 1.28 \times 10^{-3}$$

A second estimate of ε_x as 1.16×10^{-3} results in $\theta = 65.4$ degrees and $\beta = 1.00$, which gives a predicted shear strength of 207 kN and a calculated value of ε_x of 1.17×10^{-3}, which is close enough to the estimated value. Thus, the calculated shear capacity of Beam BH100 is 207 kN, which agrees well with the experimental shear failure load of 193 kN. For this beam, yielding of the longitudinal reinforcement did not govern the failure as Eq. (5) gave

$$3 \times 700 \times 550 \geq \frac{1.79 \times 207 \times 10^6}{833} + 207 \times 10^3 \cot 65.4°$$

$$1155 \times 10^3 \geq 540 \times 10^3$$

Spreadsheet for AASHTO LRFD shear provisions

To calculate the shear capacity of a given section from the AASHTO LRFD shear provisions, a convenient spreadsheet is available via the Internet from the address listed as follows. Also available at this address are the original spreadsheets used to calculate the tabulated values of β and θ: http://www.ecf.utoronto.ca/~bentz/aashto.htm.

From the entered value of s_{xe}, for members without stirrups, the spreadsheet interpolates between the rows of the table in Fig. 5 to find the θ and β values for each value of ε_x. From these values and Eq. (3) and (5), the spreadsheet calculates the combinations of shear and moment that will cause failure of the section. Thus, for Beam BH100 at ε_x value of 1.0×10^{-3}, the shear capacity is calculated to be 224 kN with θ being equal to 64.1 degrees. Knowing ε_x, V, and θ, Eq. (3) can be used to determine that the corresponding moment is 305 kNm. This gives one point of the interaction diagram (Fig. 6). When the calculations are repeated for the other values ε_x, the interaction diagram is obtained. The intersection of the loading line (which is defined by the moment-to-shear ratio at the critical section) with the failure envelope gives the predicted shear capacity of the section; in the case of Beam BH100, this equals 205 kN. Note that while the simplified ACI shear expression assumes that the shear failure load is independent of the magnitude of the moment, the AASHTO provisions indicate a substantial reduction in shear capacity as the magnitude of the moment increases.

For members with at least the minimum amount of stirrups, the values of θ and β depend on the longitudinal strain ε_x and the concrete shear stress v where

$$v = \frac{V}{b_v d_v} \tag{6}$$

To draw the moment-shear interaction diagram, it is necessary to have the values of θ and β for each value of ε_x. This can be derived from the values in the table. The failure shear stress of the section can be derived from Eq. (1) and (6) as

$$v = 0.083\beta\sqrt{f_c'} + \frac{A_v f_y}{b_v s}\cot\theta$$

For a given value of f_c', this equation can be used to determine the required amount of stirrups for each of the cells in the θ, β table given in Fig. 4. For example, if f_c' equals 25 MPa, then for v/f_c' to be equal to 0.100 and ε_x to be equal to 0.001, the stirrups quantity $A_v f_y/(b_v s)$ would need to be 1.186 MPa. If the shear stress was reduced to 0.075 f_c', the required stirrups would be reduced to 0.697 MPa. If the actual amount of stirrups lies between these calculated values, then the level of stirrups itself can be used to interpolate the values of β and θ. For example, if $A_v f_y/(b_v s)$ equalled 0.8 MPa for this case, β and θ would be found by interpolating between the values for $v/f_c' = 0.075$ and $v/f_c' = 0.1$.

Once the θ and β values have been determined for each of the values of ε_x, then the shear-moment interaction diagram is constructed in the same manner as that previously described. A typical shear-moment interaction diagram calculated by this procedure is shown in Fig. 7 for Beam BM100. Beam BM100 was a large, lightly reinforced beam tested in the same investigation[7] as Beam BH100. It was made from 47 MPa concrete and contained 16% more than the minimum quantity of stirrups specified by the ACI Code, but 30% less stirrups than the minimum required by the AASHTO LRFD specifications. Even though the provided amount of stirrups was less than the minimum requirement, the provisions of Fig. 4 were applied in drawing the shear-moment interaction diagram shown in Fig. 7. In predicting the failure load of lightly reinforced members such as Beam BM100, it must be recognized that the beam may fail in flexure at midspan before it fails in combined shear and flexure at the crit-

Table 1—Summary of experimental results

	Reinforcement		Concrete				Experimental observations						ACI		AASHTO	
Beam	ρ, %	$A_vf_y/(b_ws)$, MPa	f_c', MPa	Cast date	Test age, days	ε_{shrink}, mm/m	V_{exp}, kN	Δ, mm	w, mm	γ, mm/m	ε_{long}, mm/m	ε_{stirr}, mm/m	V_{ACI}, kN	V_{exp}/V_{ACI}	V_{AASHTO}, kN	V_{exp}/V_{AASHTO}
DB120	1.01	0.000	21	2-19-98	75	n/a	179	5.6	0.25	0.38	0.94	—	226	0.79	158	1.13
DB130	1.01	0.000	32	11-6-97	26	n/a	185	4.9	0.30	0.34	1.02	—	273	0.68	182	1.02
DB140	1.01	0.000	38	6-15-98	32	0.18	180	4.6	0.15	0.50	1.00	—	295	0.61	193	0.93
DB165	1.01	0.000	65	7-10-98	48	0.19	185	4.5	0.15	0.37	0.95	—	378	0.49	217	0.85
DB180	1.01	0.000	80	8-13-98	55	0.35	172	5.2	0.15	0.54	1.14	—	389	0.44	214	0.80
DB230	2.09	0.000	32	11-6-97	27	n/a	257	5.4	0.90	1.18	0.82	—	288	0.89	220	1.17
DB0.530	0.50	0.000	32	5-29-98	39	0.14	165	7.5	0.15	0.55	1.53	—	261	0.63	144	1.15
DB0.530M	0.50	0.401	32	5-29-98	34	0.14	263	20.2	1.50	3.70	2.45	10.8	372	0.71	258	1.02
DB120M	1.01	0.401	21	2-19-98	70	n/a	282	14.8	3.50	2.72	1.49	6.2	337	0.84	358	0.79
DB140M	1.01	0.401	38	6-15-98	26	0.18	277	13.2	1.80	2.50	4.59	7.3	406	0.68	384	0.72
DB165M	1.01	0.401	65	7-10-98	39	0.19	452	22.2	3.00	3.85	3.40	8.0	489	0.92	394	1.15
DB180M	1.01	0.401	80	8-13-96	48	0.35	395	20.8	2.50	4.15	3.15	8.5	500	0.79	375	1.05
B100	1.01	0.000	36	8-30-95	54	n/a	225	5.7	0.20	0.20	1.15	—	288	0.78	189	1.19
B100H	1.01	0.000	98	12-21-95	27	n/a	193	7.7	n/a	2.00	1.05	—	389	0.50	227	0.85
B100HE	1.01	0.000	98	12-21-95	29	n/a	217	6.1	n/a	1.23	1.11	—	289	0.56	227	0.96
B100L	1.01	0.000	39	2-2-95	13	n/a	223	5.4	n/a	0.33	1.26	—	299	0.75	195	1.14
B100B	1.01	0.000	39	2-2-95	18	n/a	204	5.4	n/a	0.27	1.24	—	299	0.68	195	1.05
BN100	0.76	0.000	37	6-20-96	42	n/a	192	5.9	0.25	0.60	1.20	—	285	0.67	173	1.11
BH100	0.76	0.000	99	5-17-97	41	0.27	193	n/a	0.65	0.75	1.25	—	384	0.50	206	0.94
BRL100	0.50	0.000	94	7-5-96	38	n/a	163	6.8	0.10	0.75	1.85	—	384	0.42	172	0.95
BM100	0.76	0.401	47	11-4-96	119	0.16	342	20.8	2.50	3.65	2.69	15.5	431	0.79	344	0.99
													Average	0.67	—	1.00
													Coefficient of variation	21.7	—	13.8

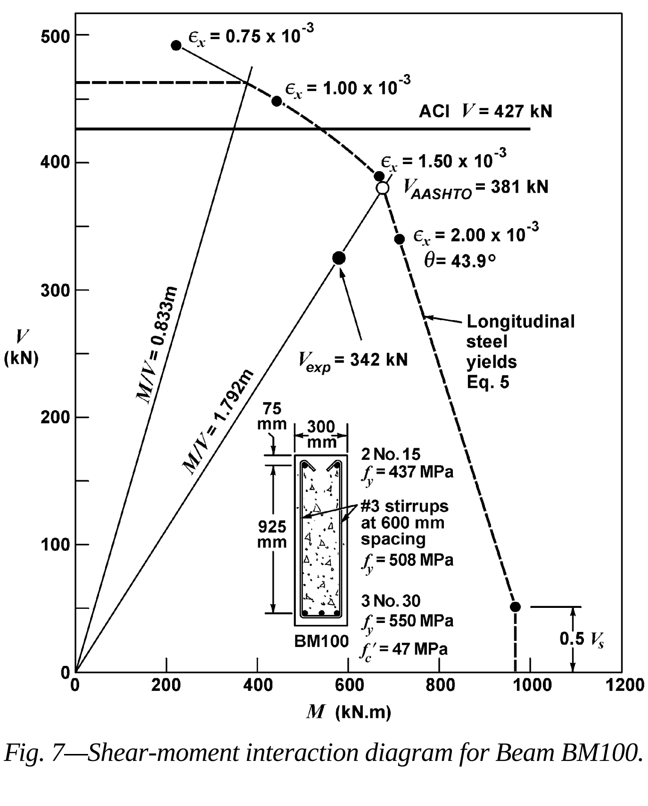


Fig. 7—Shear-moment interaction diagram for Beam BM100.

ical shear location. The calculated flexural capacity of Beam BM100, using the ACI procedures, is approximately 1010 kNm. Thus, it would be estimated that Beam BM100 would fail in flexure at midspan at a shear force of 374 kN before reaching the calculated shear failure load of 381 kN (Fig. 7). In the experiment, the beam failed in shear at 342 kN.

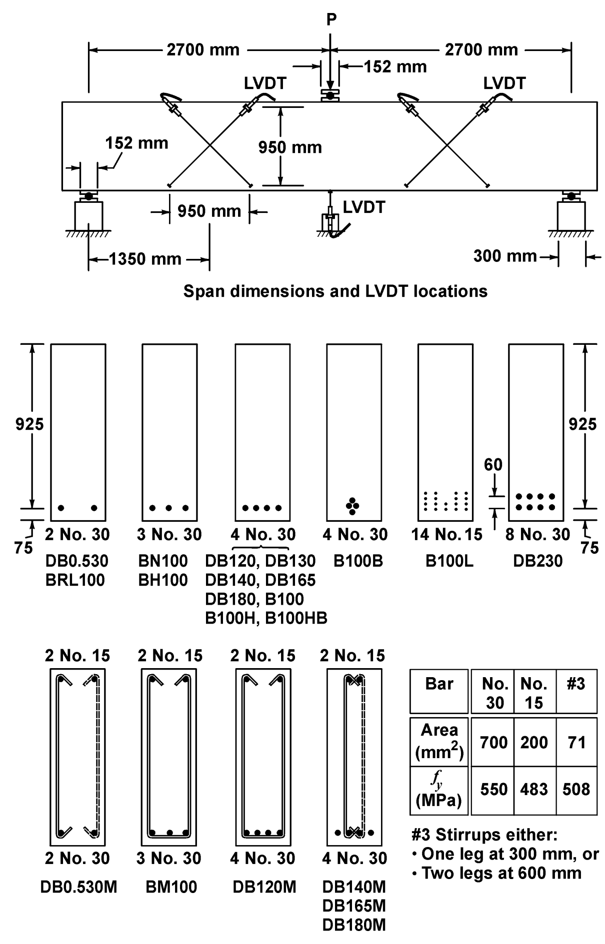


Fig. 8—Details of 21 large, lightly reinforced beams.

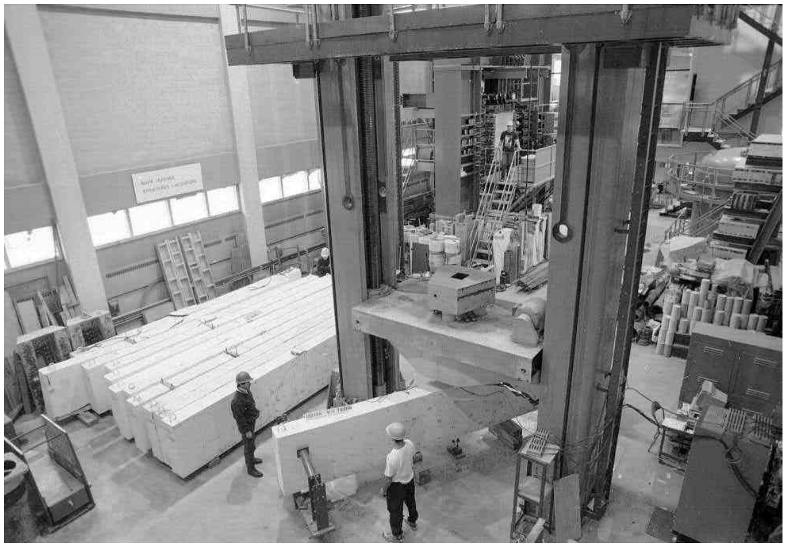


Fig. 9—Testing of 1 m beam in progress.

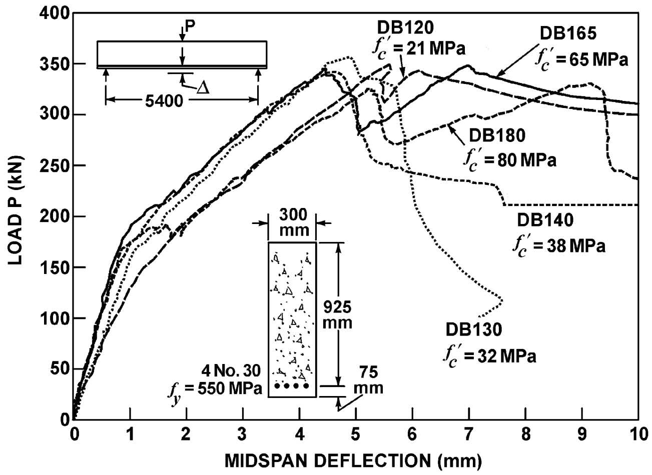


Fig. 10—Load-deflection relationships for five beams with no stirrups.

EXPERIMENTAL PROGRAM

To learn more about the influence of concrete compressive strength and minimum stirrups on the shear response of large, lightly reinforced concrete members, twelve 1 m deep beams with concrete strengths ranging from 21 to 80 MPa were constructed. These specimens were designed so that comparisons could be made with the results from nine 1 m deep beams tested in previous studies at the University of Toronto.[7] The details of the 21 beams are summarized in Fig. 8 and Table 1. The beam names for the 12 new beams start with the letters DB, while for the other nine beams the names start with the letter B. Fourteen of the 21 beams contained 1.01% of longitudinal reinforcement, three contained 0.76%, another three contained 0.50%, and one beam was relatively heavily reinforced, containing 2.09% of longitudinal steel. Six of the beams contained stirrups with the amount being such that $A_v f_y/(b_v s)$ equalled 0.401 MPa (58 psi), which is 16% more than the minimum amount specified by the ACI Code. All of the beams were made from concretes with crushed limestone aggregate with a maximum size of 10 mm (0.39 in.).

The beams were loaded by a point load applied at the middle of a 5.4 m (17 ft 8.6 in.) simply supported span (Fig. 8 and 9). The deflection at midspan was recorded, as were the shear strains at the quarter points of the span (measured by pairs of linear variable displacement transducers [LVDTs] inclined at 45 degrees). Continuous readings were also taken of the strain at midspan in the longitudinal reinforcement and, if the beam contained stirrups, the strains on approximately six of the stirrups. At approximately four load stages during each experiment, the displacement of the beam was held nearly constant while the crack patterns were recorded, crack widths were measured with a crack comparator, and surface strains were measured with demountable displacement gages using targets on a 300 x 300 mm grid.

The most important experimental observations are summarized in Table 1. The tabulated failure shears V_{exp} are one-half of the highest point load applied to the beam plus an allowance for the shear due to self-weight of the beam taken as 7 kN. The midspan deflection Δ, the shear strain γ, the highest strain in the longitudinal reinforcement ε_{long}, and the highest strain in the stirrups ε_{stirr} were all measured at the time the point load reached its highest value and are listed in Table 1. Also given are the largest crack widths measured at the last load stage prior to failure. More details of the experimental results can be found in Angelakos.[10]

The influence of the concrete strength on the load-deformation response of the five DB beams that contained 1.01% of longitudinal reinforcement and no stirrups is shown in Fig. 10, and photographs of the beams at failure are shown in Fig. 11. It was somewhat surprising to observe that varying the concrete cylinder strength from 21 to 80 MPa (3050 to 11,600 psi) had almost no effect on the load at which a brittle shear failure occurred. In examining the beams after failure, it was observed that the crack roughness decreased noticeably as the concrete strength increased. It seemed probable that the crushed limestone aggregates used in making the concretes were rather weak as even for Beams DB130 and DB140 the cracks cleaved most of the aggregate particles. Note also that the 80 MPa concrete developed noticeably higher shrinkage strains prior to testing.

While changing the concrete strength by a factor of 4 had almost no influence on the shear strength of these large beams, changing the longitudinal reinforcement ratio from 0.50 to 2.09% increased the observed shear strengths by 62% (Fig. 12). Note that all of the beams without stirrups failed at very

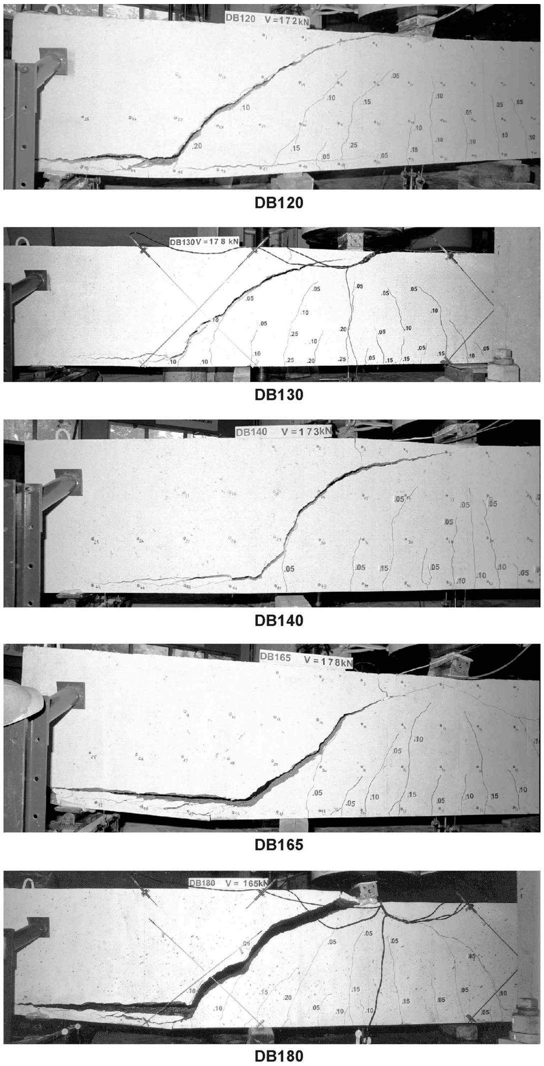


Fig. 11—Photos of failure of beams without stirrups.

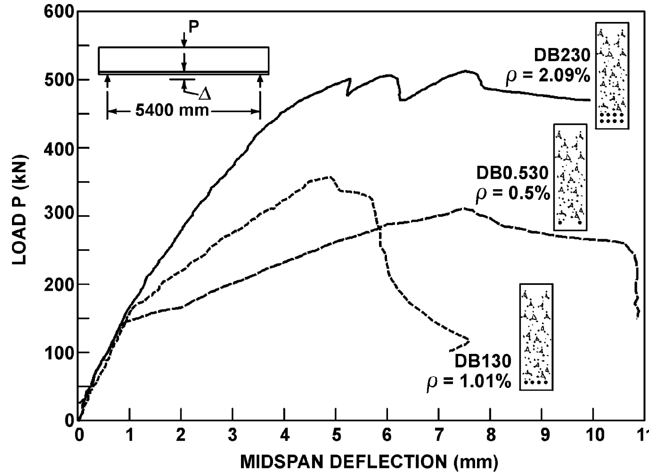


Fig. 12—Influence of amount of longitudinal reinforcement on load-deformation response of three large beams.

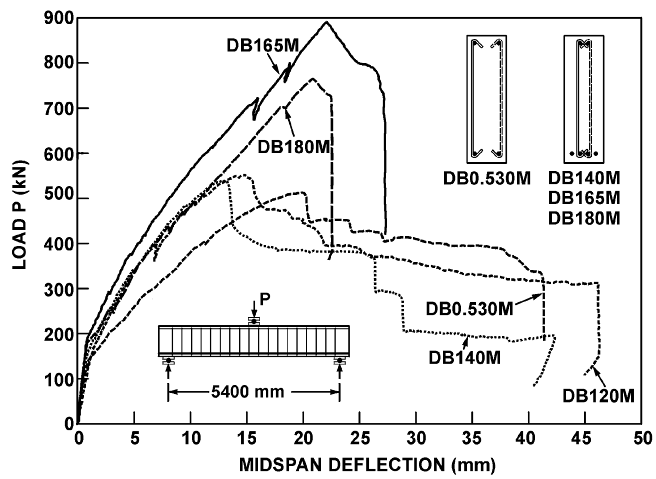


Fig. 13—Load-deflection relationships for beams containing ACI minimum amount of stirrups.

small deflections with the midspan deflection at failure corresponding to only approximately 1/1000 of the span length.

The presence of ACI minimum stirrups increased shear strength of the beams by factors ranging from 1.54 to 2.44 and increased the midspan deflections at failure by a factor of between 2 and 5 (Fig. 13 and Table 1). For the lower-strength concretes, the postpeak response was relatively ductile, while for the high-strength members the drop in capacity at failure was very abrupt, indicating that for such members, the ACI minimum level of stirrups should be increased.

Comparisons with ACI and AASHTO predicted shear capacities

The influence of concrete strength and the amount of longitudinal reinforcement on the shear strength of large, lightly reinforced concrete members that do not contain stirrups is illustrated in Fig. 14. In preparing this figure and Table 1, the more complex and presumably more accurate expression for V_c was used, namely[1,8]

$$V_c = (1.9\sqrt{f_c'} + 2500\rho Vd/M)b_w d \qquad (7)$$

As can be seen in Fig 14, this equation predicts only a very small reduction in shear strength as the longitudinal reinforcement ratio is reduced from 1 to 0.5%, but predicts a very large increase in shear capacity as the concrete strength is increased. In contrast, the AASHTO provisions predict substantial reductions in shear capacity as the reinforcement ratio is reduced and a much smaller increase in shear capacity as the concrete cylinder strength is increased. None of the experimental series shown in Fig. 14 indicate any significant increase in shear capacity as the concrete strength increases. The experiments do, however, indicate a substantial change in shear capacity as the longitudinal reinforcement ratio changes.

Figure 15 is primarily intended to compare the calculated and observed capacities of the four beams that contained stirrups and 1.01% of longitudinal reinforcement. For convenience, however, the experimental failure shear for the beam with 0.75% longitudinal reinforcement (Beam BM100) is also plotted. It can be seen that traditional ACI procedures significantly overestimate the shear capacity of these beams, even though the beams contain 16% more than the minimum quantity of stirrups specified by the ACI Code. It can be seen from Table 1 that, for these six beams, the ratio of observed failure shear to calculated ACI shear capacity ranges from 0.68 to 0.92, with an average of 0.79.

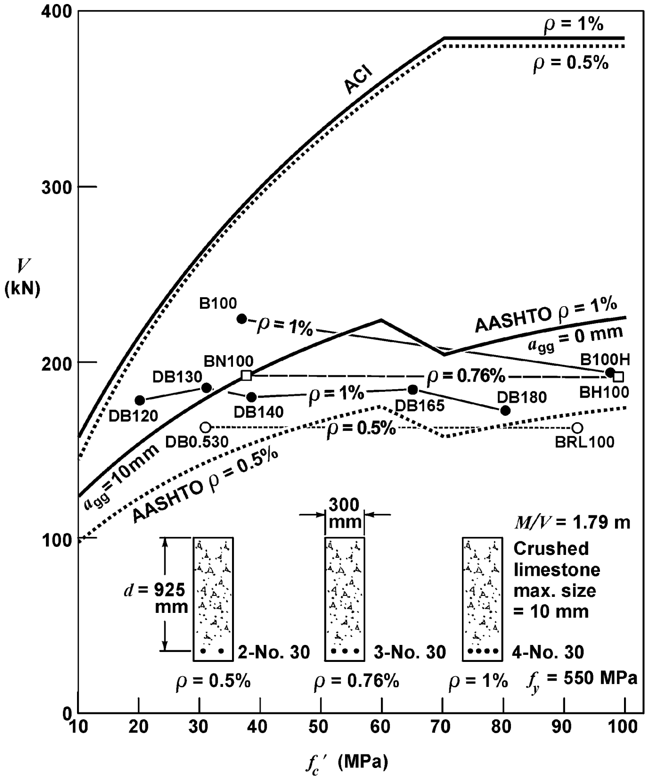


Fig. 14—Influence of concrete strength and amount of longitudinal reinforcement on shear capacity of 14 large beams without stirrups.

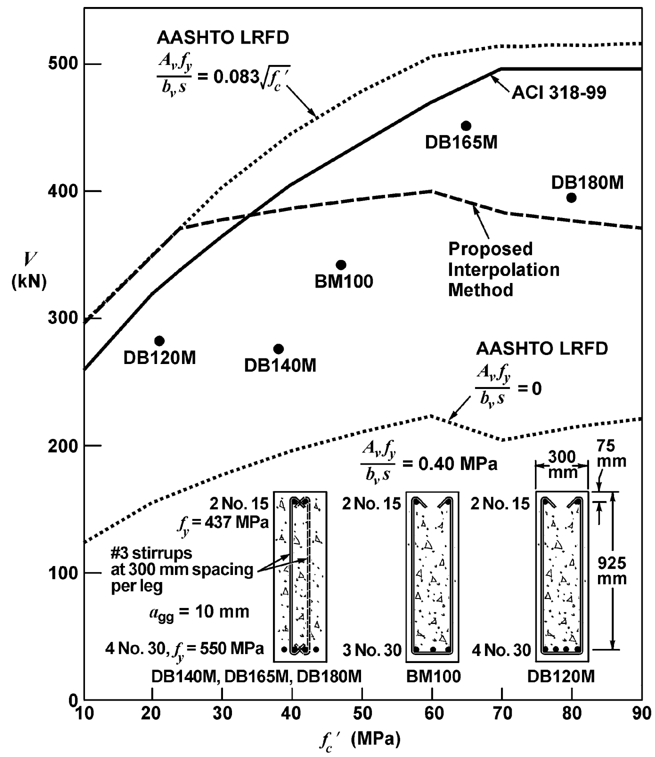


Fig. 15—Comparisons of shear capacities for five large beams with stirrups.

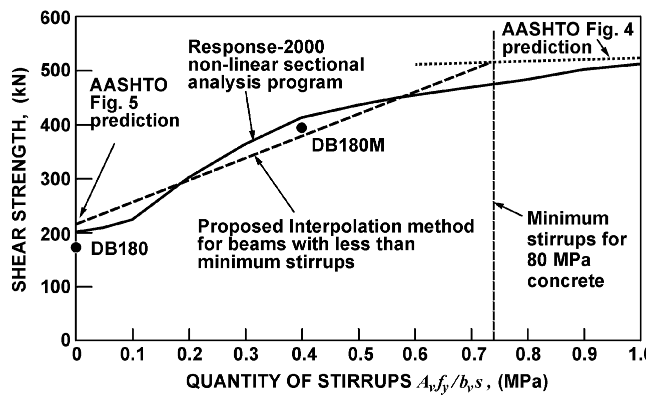


Fig. 16—Proposed interpolation technique to estimate strength of beams with less than AASHTO specified minimum stirrups.

Both the ACI Code and the AASHTO provisions require that if stirrups are needed to resist the applied shear, a minimum quantity must be provided. The Code does not explicitly provide guidance on how to evaluate the shear strength of beams with stirrups that have less than the specified minimum. It is suggested that a good estimate of the shear strength of members with less than minimum stirrups can be obtained if the shear strength is calculated first by ignoring the stirrups as in Fig. 5, and second, by assuming the beam contains the full AASHTO minimum amount of stirrups using Fig. 4. The shear strength estimate can then be obtained by interpolating between the two calculated values in accordance with how close the actual amount of stirrups is to the specified minimum.

The bottom line in Fig. 15 shows the strength with no stirrups, the top line shows the strength with full AASHTO minimum stirrups, and the middle line shows the suggested interpolation. Figure 16 shows how the predicted shear strength of beams made from 80 MPa concrete, such as DB180 and DB180M, increases as the quantity of stirrups increases. Also shown on the figure are the predictions from a nonlinear sectional analysis program based on the MCFT, called Response-2000.[11] It can be seen that the proposed interpolation method is a reasonably good approximation of both the Response-2000 predictions, as well as the experimental results. The shear strength estimates for the five beams with less than the AASHTO specified minimum stirrups listed in Table 1 were obtained using this proposed linear interpolation method.

If the ratios of experimental failure shears V_{exp} to the calculated shear capacities from the ACI provisions V_{ACI} for the 21 beams listed in Table 1 are examined, it will be recognized that the current ACI shear provisions are very unconservative for these large, lightly reinforced sections. None of the beams reached the calculated ACI shear capacity, and three of the beams failed in shear at less than one half of the calculated ACI shear capacity. For the 21 beams, the average value of the V_{exp}/V_{ACI} ratio is 0.67 with a coefficient of variation of 21.7%. When these values are compared with the 1.38 and 7.3% obtained for the 16 small, highly reinforced beams shown in Fig. 1, it seems unlikely that the ACI shear equations would have their present form if the engineers involved in their development had tested several series of large, lightly reinforced beams. It can be seen from Table 1 that the AASHTO LRFD shear provisions give a much more consistent estimate of the shear capacity of these large, lightly reinforced members with an experimental over predicted shear strength ratio of 1.00 and a coefficient of variation of 13.8%.

Example of large one-way slab

To illustrate the consequences of the fact that the ACI provisions overestimate the shear capacity of large, lightly reinforced concrete members, the one-way slab described in Fig. 17 will be used. This slab spans 50 ft (15.2 m) and must safely support a uniform loading of 2.5 kips/ft^2 (120 kN/m^2) caused by a substantial depth of overburden. Because of durability considerations, the engineer has specified a concrete strength of 5000 psi (34 MPa). As is the usual practice for one-way slabs, the engineer has decided not to use shear reinforcement but rather to rely solely on the concrete contribution V_c to provide the required shear resistance.

From the ACI equation, it has been found that a 7 ft (2.13 m) thick slab will provide adequate shear strength. Knowing the slab thickness, the total factored loading, which the slab must resist, is calculated to be 5.72 kips/ft^2 (274 kN/m^2). This means that the slab must resist a factored moment at midspan of 1788 kip-ft/ft (7950 kNm/m) and a factored shear d from the face of the support of 104 kips/ft (1520 kN/m) (Fig. 17). To resist the moment, No. 11 (36 mm) bars at 3.5 in. (90 mm) spacing have been chosen. An engineer evaluating this slab using the ACI Code would conclude that the design was satisfactory, and that if for some reason the slab was overloaded, it would fail in flexure and, being lightly reinforced (ρ = 0.54%), would exhibit very large deflections prior to failure. The nominal flexural capacity would be reached when the applied loading was 6.72 kips/ft^2 (322 kN/m^2).

To evaluate the shear capacity of a uniformly loaded member using the AASHTO LRFD shear provisions, it is necessary to check a few points along the span. Usually the critical section for members without stirrups occurs at approximately 20% of the span from the support. The AASHTO capacities shown in Fig. 16 were calculated at $0.1L$, $0.2L$, and $0.3L$ from the support by changing the M/V values in the spreadsheet. From this figure, it can be seen that the AASHTO provisions predict that the large slab can safely resist a factored load of 3.60 kips/ft^2 (172 kN/m^2), which is only 63% of the required factored capacity. Further, these calculations indicate that the slab would suffer a brittle shear failure similar to those shown in Fig. 11 at a very low deflection. The nominal shear capacity would be reached when the applied loading was 4.24 kips/ft^2 (203 kN/m^2). Note that this value is only 19% greater than the service load applied to the slab, indicating a factor of safety of only 1.19, a value that is dangerously low.

The AASHTO LRFD shear provisions indicate that for this large, lightly reinforced slab, it would be appropriate to provide a minimum quantity of shear reinforcement. With an $A_v f_y/(b_v s)$ value of 71 psi (0.49 MPa), as required by Eq. (2), the shear failure would be suppressed and the nominal flexural capacity would be reached at 6.72 kips/ft^2 (322 kN/m^2). That is, this small quantity of stirrups would change the failure mode of the slab from a brittle shear failure to a ductile flexural failure and would increase the overall factor of safety from 1.19 to 1.89.

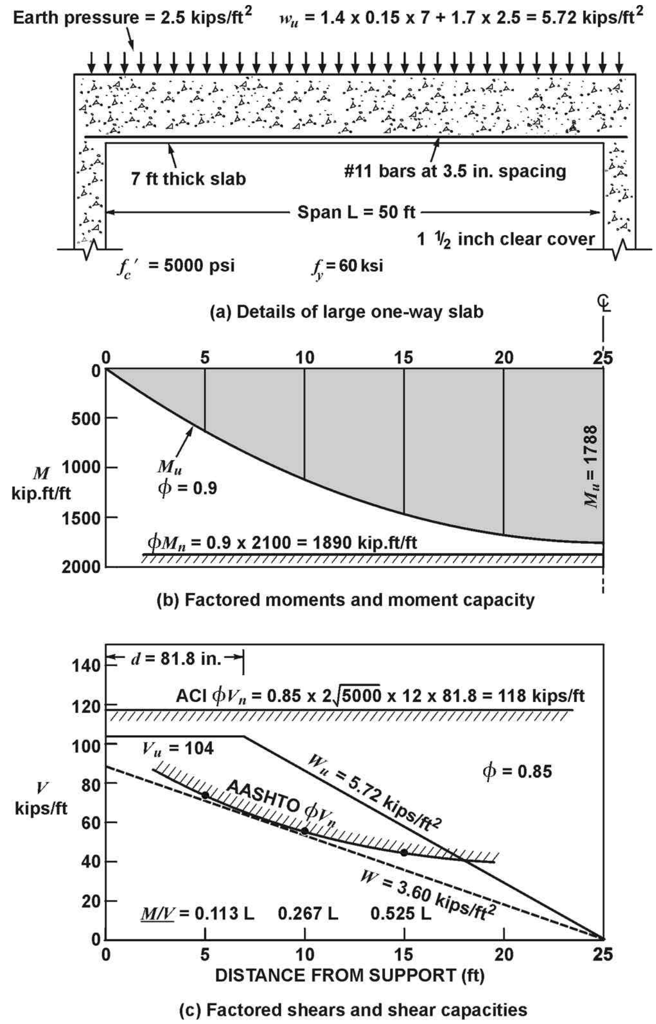


Fig. 17—Predicted shear capacity of large, lightly reinforced one-way slab: (a) details of large one-way slab; (b) factored moments and moment capacity; and (c) factored shears and shear capacities.

CONCLUSIONS

The experiments and analytical studies described in this paper indicate that large, lightly reinforced concrete members designed using the shear provisions of the current ACI Code can have inadequate margins of safety. For members without stirrups, these margins will become smaller as the concrete strength increases, as the member size increases, and as the longitudinal reinforcement ratio decreases. The experiments indicate that even members containing 16% more than the minimum quantity of stirrups specified by the ACI Code will still have inadequate margins of safety.

The basic shear provisions of the current ACI Code were derived approximately 40 years ago using experimental results from small, heavily reinforced beams. From the results of these tests, it was concluded that concrete cylinder strength was the most important parameter influencing failure shear stress of members without stirrups and that the amount of longitudinal reinforcement had only a small effect.

In the ensuing years, significant changes have occurred in the concrete construction industry and considerable research has been conducted on the shear response of concrete members. The typical yield strength of reinforcement used in practice has increased from 40 to 60 ksi (275 to 414 MPa), causing longitudinal reinforcement ratios for similar structures to be substantially reduced. At the same time, advances in concrete materials technology have made it practical to use a much wider range of concrete strengths, including concretes made from potentially lower quality aggregates that can still have high concrete cylinder strengths. As was seen in this investigation, these high cylinder strengths do not necessarily result in high values of failure shear stress. Recent shear research,[12-16] including that reported herein, has shown that for members without stirrups, the shear stress at failure can decrease substantially as the members become larger and as the longitudinal reinforcement ratio becomes lower. The tests reported in this paper have revealed that even large, lightly reinforced members containing the ACI minimum level of stirrups can fail at approximately 70% of the ACI predicted shear strength. More research is needed to find the reasons and extent of this unconservative behavior.

Until the ACI shear provisions are modified to provide a more consistent level of safety, it is suggested that it would be prudent for engineers designing large, lightly reinforced concrete members to use the shear provision of the AASHTO LRFD Bridge Design Specifications. It has been shown that these give more consistent estimates of the shear capacity of such members. While these provisions are certainly more complex than the traditional ACI equation, the spreadsheet described herein makes the evaluation of shear strength according to these provisions very simple.

ACKNOWLEDGMENTS

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