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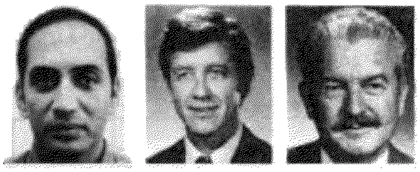
Shear Capacity of Reinforced Concrete Beams Using High-Strength Concrete



by Ashraf H. Elzanaty, Arthur H. Nilson, and Floyd O. Slate

Results of an experimental investigation of the shear strength of reinforced concrete beams made using concrete with compressive strength ranging from approximately 3000 to 12,000 psi (21 to 83 MPa) are summarized. A total of 18 beams was tested: 15 without web reinforcement and 3 with web reinforcement in the form of vertical stirrups. In addition to concrete strength, other variables included longitudinal steel ratio and shear span-to-depth ratio. Test results are compared with strengths predicted using the equations of ACI 318-83. For beams without web steel, ACI Building Code equations may overestimate shear strength by 10 to 30 percent, particularly for higher concrete strength combined with normal to high shear-span ratios and relatively low longitudinal-steel ratios. For beams with web steel, all test strengths exceeded those predicted following present ACI code procedures.

Keywords: beams (supports); compressive strength; cracking (fracturing); diagonal tension; **high-strength concretes; reinforced concrete; shear strength;** span-depth ratio; web reinforcement.

In the past decade there has been a rapid growth of interest in high-strength concrete with compressive strength f_c' ranging to about 12,000 psi (83 MPa). Concretes of this strength can be made with carefully selected but commonly available cement, sand, and stone, by using a very low water-cement ratio and careful quality control in production. The necessary workability is achieved by high-range water-reducing admixtures, the so-called superplasticizers. Strengths above 6000 psi (41 MPa) are no longer unusual in practice.

Extensive experimentation has greatly improved understanding of the fundamental behavior and basic engineering properties of the material. While most properties of concrete improve with increased compressive strength, some characteristics require special attention. To insure the safety and serviceability of structural concrete, certain essentially empirical design procedures and equations, based on concrete of much lower strength, must be re-examined.

The shear capacity of reinforced high-strength concrete beams is an important issue. Provisions for shear design in the ACI Building Code (ACI 318-83)[1] are based mainly on experimentally derived equations. Tests providing the basic data for these equations were conducted on members with concrete strength mostly below 6000 psi (41 MPa). A recent report by ACI Committee 363[2] points out that using such equations to extrapolate may be unsafe in some cases with concrete strengths more than twice that for which they were developed, although there is nothing in the present ACI Building Code to prohibit this extrapolation.

According to ACI 318-83, the total nominal shear strength V_n is taken equal to the sum of the contributions of the web reinforcement V_s and the concrete V_c. The contribution termed V_c in a member with web steel is actually the sum of at least three separate components: (a) shear resistance of the still-uncracked compression concrete above the top of the diagonal crack, (b) aggregate interlock along the diagonal crack, and (c) dowel resistance provided by the longitudinal reinforcement. By a well-known rationalization,[3] the value of V_c in a member with web reinforcement is taken equal to the shear that causes diagonal tension failure in an otherwise identical beam without web steel.

A characteristic of high-strength concrete loaded to failure in uniaxial compression is that it fractures suddenly, and in doing so forms a failure surface that is typically a smooth plane.[4,5] This contrasts with the rough surface typical of lower-strength concretes, with internal cracking following along the interface between stone and mortar, then branching out through the mortar in many directions. In the diagonal cracking region of shear-critical beams, the state of stress is biaxial, combining diagonal compression in the direction sloping upward from the support with diagonal tension in the perpendicular direction. It could be expected that the surface of a diagonal tension fracture in a high-strength concrete beam would be relatively smooth, as

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obtained in uniaxial compression, and that the smooth surface might be deficient in aggregate interlock, an important component of what is called V_c.

Experimental research is needed to establish the shear capacity of beams, both without and with web reinforcement, made with high-strength concrete. The tests described here included 15 beams without web reinforcement and 3 beams with web reinforcement in the form of vertical stirrups. A continuation of the research, in which 34 shear-critical prestressed beams were tested, will be described in a subsequent paper.

RESEARCH SIGNIFICANCE

Experimental results described herein show that the shear strength of high-strength concrete beams without web reinforcement may be from 10 to 30 percent below the predicted strength based on current ACI code equations. The Building Code does not require shear reinforcement for certain classes of members, including slabs, footings, joists, and shallow beams. The presence of web reinforcement did not modify the diagonal cracking loads, compared with otherwise identical members without web steel. Although the total shear strength of beams with web reinforcement, accounting for the contributions of both concrete and web steel, was greater than predicted by ACI code equations for all beams tested, there is an urgent need to develop improved design procedures for shear for both types of members that correctly account for the influence of concrete strength and other variables.

EXPERIMENTAL PROGRAM

A brief summary of the experimental investigation dealing with shear strength of reinforced concrete beams is given here. Reference 6 supplies full information.

Materials

ASTM Type I portland cement was used. Sand was from a glacial alluvial deposit near Ithaca, N.Y., and consisted mainly of quartz and (particularly in the larger particles) shale, sandstone, and limestone. Coarse aggregate was crushed limestone from a local quarry, with 0.5 in. maximum size. A high-range water-reducing admixture (superplasticizer) with retarder, ASTM C 494 Type F, was used in mixes with f_c' over 6000 psi (41 MPa) to give good workability with the very low water-cement ratios that were used. Reference 6 gives mix proportions.

Two types of reinforcing bars were used. Longitudinal bars were deformed ASTM Grade 60 with an actual yield stress of 63 ksi (434 MPa). The smaller steel area needed for stirrups was provided by smooth round bars of ¼ in. diameter (6.4 mm) with yield stress of 55 ksi (379 Mpa).

Test beams

The test specimens (shown in Fig. 1) consisted of 18 rectangular beams, 3 with stirrups and 15 without. All

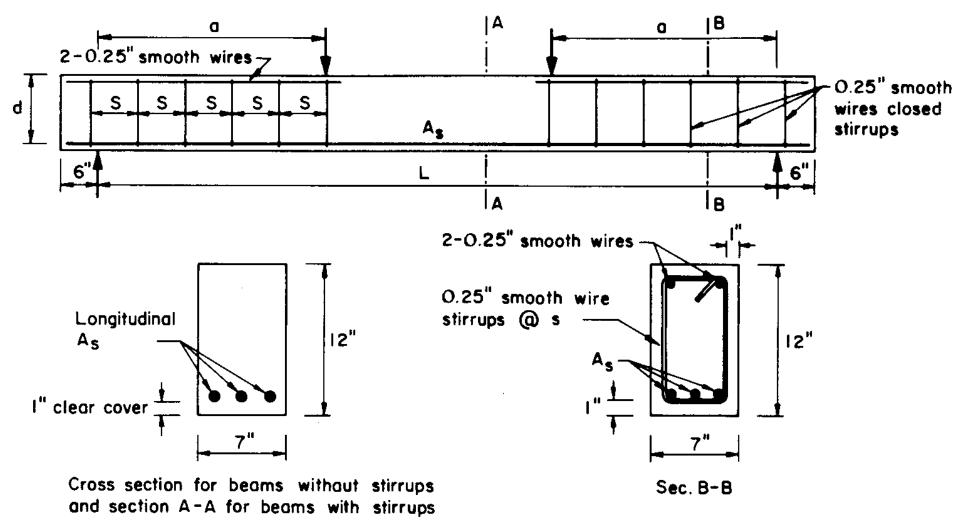

Fig. 1 — Details of test beams

Table 1—Shear strength of reinforced concrete beams without stirrups

Beam	Shear span ratio a/d	Concrete strength f'_c, psi	Steel ratio ρ_w	$V_{c,test}/V_{c,ACI}$
F7	4.0	3000	0.006	0.94
F11	"	"	0.012	1.20
F12	"	"	0.025	1.36
F8	"	5800	0.010	0.91
F13	"	"	0.012	0.91
F14	"	"	0.025	1.21
F1	"	9500	0.012	0.91
F2	"	"	0.025	1.00
F10	"	"	0.033	1.14
F9	"	11,500	0.016	0.89
F15	"	"	0.025	0.93
F3	2.0	10,000	0.012	1.18
F4	"	"	0.025	1.57
F5	6.0	9200	0.012	0.70
F6	"	"	0.025	0.96

Table 2—Shear strength of beams with stirrups

Beam	Shear Span ratio a/d	Concrete stength f'_c, psi	Steel ratio ρ_w	Web reinforcement index $\rho_v f_y$	$V_{n,test}/V_{n,ACI}$
G4	4.0	9100	0.033	94	1.54
G5	"	5800	0.025	"	1.34
G6	"	3000	"	"	1.11

beams had the same cross section, with 7 in. (178 mm) width and 12 in. (305 mm) total depth. The beams without stirrups were designed to investigate the effects of f'_c, a/d, and ρ_w. Those with stirrups had a constant a/d of 4.0 with varying f'_c. Tables 1 and 2 give summary details of all beams.

Test arrangement and procedure

All beams were loaded symmetrically with two equal concentrated loads. The load was applied in increments of about 4 kips (17.8 kN) up to approximately 70 percent of the estimated ultimate load, after which the increments were reduced to 2 kips (8.9 kN). At each load increment, all strain and displacement readings were recorded, the beam was carefully inspected, and all cracks were marked. Each beam test was followed by tests of control specimens to determine compressive strength of concrete and modulus of rupture.

RESULTS AND DISCUSSION

Crack patterns and general behavior

In the early stages of loading, the beams were free of cracks. Flexural cracks were observed in the region of pure bending at loads close to those predicted based on the modulus of rupture. With further increase of load, additional flexural cracks formed in the central region and new flexural cracks formed in the shear spans between load and support, curving toward the loading points. After cracking, the behavior of the beams was dependent upon f'_c, a/d, and ρ_w, and was influenced by the presence or absence of web steel.

For beams without stirrups with a/d of 6 (see Table 1), formation of inclined cracks corresponded to ultimate capacity. In Beam F5, with ρ_w = 0.012, the longitudinal steel yielded before the inclined crack formed, and flexural failure was expected as predicted by ACI code equations. However, the beam then failed suddenly in shear and formed a diagonal crack that propagated immediately in two directions: toward the load point in the compression zone and toward the support along the tension steel.

For beams without stirrups with a/d of 4, the ultimate capacity in shear slightly exceeded the inclined cracking load, while those with a/d of 2 exhibited considerable reserve strength after inclined cracks formed, as expected due to arch action.

Depending on the particular combination of f'_c, a/d, and ρ_w for these beams, the proximate cause of failure was either splitting along the flexural reinforcement or sudden extension of the critical inclined crack into the compression zone of the beam close to the load point. The failure was more abrupt and the failure crack surfaces were distinctly smoother for the beams with higher compressive strength concrete.

For beams with stirrups, behavior was generally the same as for those without stirrups up to the inclined cracking load. As the load was increased beyond that load, additional flexural and diagonal tension cracks formed, and existing cracks lengthened and widened. Failure occurred when one of the inclined cracks propagated suddenly into the compression zone near the load point.

Shear strength of beams without stirrups

The Building Code contains two equations for the shear strength of beams without stirrups, subject only to shear and flexure, ACI code Eq. (11-3)

$$V_c = 2\sqrt{f'_c}\, b_w d \qquad (1)$$

and Eq. (11-6)

$$V_c = \left[1.9\sqrt{f'_c} + 2500\,\rho_w \frac{V_u d}{M_u}\right] b_w d \qquad (2)$$

Eq. (11-6) is the basic expression for shear strength, while Eq. (11-3) is an approximation often used in design practice.* Both equations are based on the assumption that the useful shear strength of a beam without shear reinforcement is exhausted when inclined cracking first develops.

Measuring the inclined cracking load experimentally is not simple. Researchers have suggested different methods for determining that load, such as measuring the strain in stirrups if present, concrete strains in the compression face, and beam depth increase after cracking, as well as by direct observation of concrete cracking patterns. Depending on the observer's judgement and the definition used to establish the cracking load, the values obtained from the same test executed by different researchers may differ. This study reports the ultimate shear capacity, i.e., the failure load.

*S.I. translations of these equations are given in Reference 1.

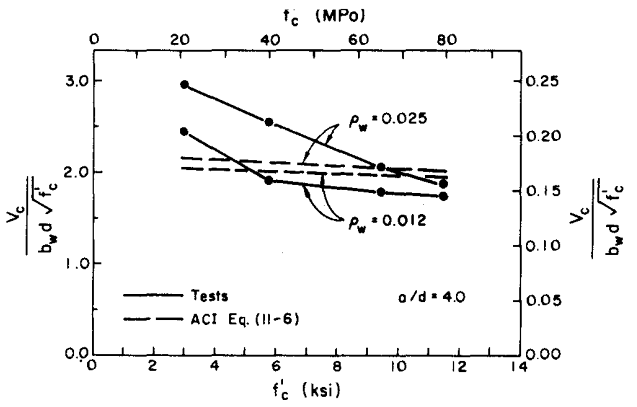

Fig. 2 — Effect on f$_c'$ *of shear strength of beams without stirrups*

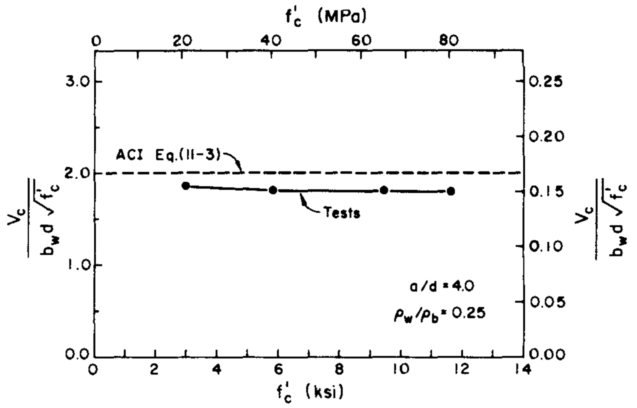

Fig. 3 — Effect of f$_c'$ *on shear strength of beams with constant* ρ_w/ρ_b

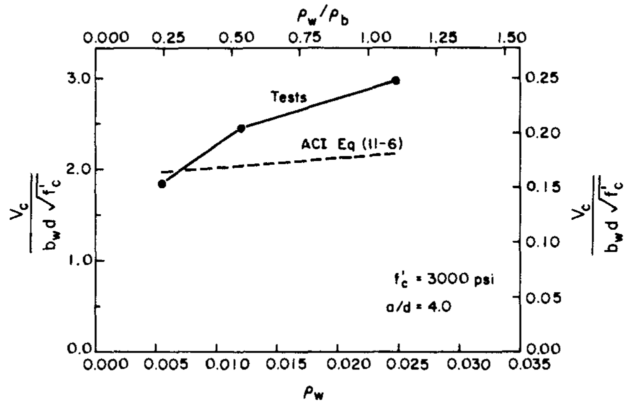

Fig. 4 — Effect of ρ_w *on shear strength of beams with* f$_c'$ *= 3000 psi*

Table 1 shows the ratio of the shear strengths of test beams to shear strengths predicted by Eq. (11-6) for beams without web steel. The effects of the different parameters on the shear strength are discussed as follows.

Effect of concrete strength

The shear strength V_c of the test beams increased as expected with increase in f_c'. However, the predictions of Eq. (11.6), while quite conservative for lower concrete strengths (see Fig. 2) became unconservative for medium-to-high concrete strengths. For beams with ρ_w of 0.025, the ratio of $V_{c,test}/V_{c,ACI}$ decreased from 1.36 to 0.93 as f_c' increased from 3000 to 11,500 psi (21 to 79 MPa), and for beams with ρ_w of 0.012 the ratio decreased from 1.20 to 0.89 for the same increase in f_c'.

The tensile steel ratio was varied for beams with different f_c' to obtain a constant value of tensile steel ratio to balanced steel ratio. Results are shown in Fig. 3. For beams with $a/d = 4$ and $\rho_w/\rho_b = 0.25$, the ratio of $V_{c,test}/V_{c,ACI}$ varied only from 0.94 to 0.89 over the full range of f_c'. Thus for a constant ratio of ρ_ω/ρ_b concrete strength has little effect on the ratio of test to predicted strength. However, for all beams plotted in Fig. 3, the ACI code equation overpredicted strength by about 10 percent.

Up to the cracking load, shear is resisted mostly by shear stresses in the concrete. After cracking, shear is resisted by aggregate interlock, dowel action of the main reinforcing bars, and resistance of the still uncracked concrete at the top of the beam.[7] The percentage carried by aggregate interlock depends strongly on the surface roughness at the crack and on the amount of displacement (opening) at the crack. Examination of the failure surfaces of the beams in this investigation confirmed that for higher concrete strengths, the crack surfaces were distinctly smoother, indicating that the shear force carried by aggregate interlock decreases with increase in f_c'.

After cracking, shear displacement along the crack is resisted in part by dowel action of the longitudinal steel, giving rise to vertical tension in the surrounding concrete. Concrete tensile strength generally correlates with $\sqrt{f_c'}$. The shear force resisted by the uncracked concrete also depends on f_c' as well as on the area of the compression zone above the crack.

These shear transfer mechanisms are interdependent and not easy to separate. Any deficiency in one will affect the other two, and which breaks down first to cause beam failure will depend on the relative magnitudes of the forces carried by each, and the resistance. For beams with higher concrete strengths, aggregate interlock is the first mechanism to break down, calling on dowel action and shear in the compression zone to resist additional shear forces. Failure may take one of two forms. If the state of stress meets the failure criterion for the concrete in the compression zone, an abrupt and sometimes explosive failure occurs. If dowel resistance controls, vertical tension in the concrete around the bars causes splitting cracks along the reinforcement. Both types of failure were observed.

Effect of longitudinal steel ratio

The effect of ρ_w on the shear strength of test beams with f_c' of 3000, 5800, 9500, and 11,500 psi (21, 40, 66 and 79 MPa) is shown in Fig. 4, 5, 6, and 7, respectively. For all concrete strengths, increasing ρ_w increased the shear strength of test beams without stirrups. But it is clear from these figures that the effect of ρ_w on shear strength is not adequately accounted for in ACI code Eq. (11-6). The ρ_w below which Eq. (11-6) becomes unconservative, with $V_{c,test}$ less than $V_{c,ACI}$, increases with the increase of f_c'. For constant a/d equal to 4.0, ρ_w below which the ACI code predictions were unconservative was about 0.007, 0.014, and 0.025 for 3000, 5800, and 9500 psi concrete, respectively. For

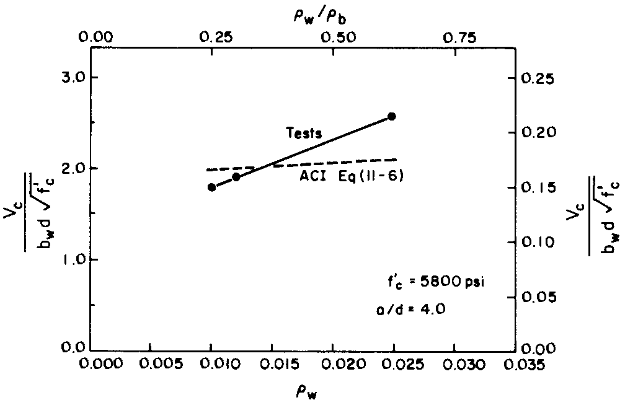

Fig. 5 — Effect of ρ_w on shear strength of beams with f_c' = 5800 psi

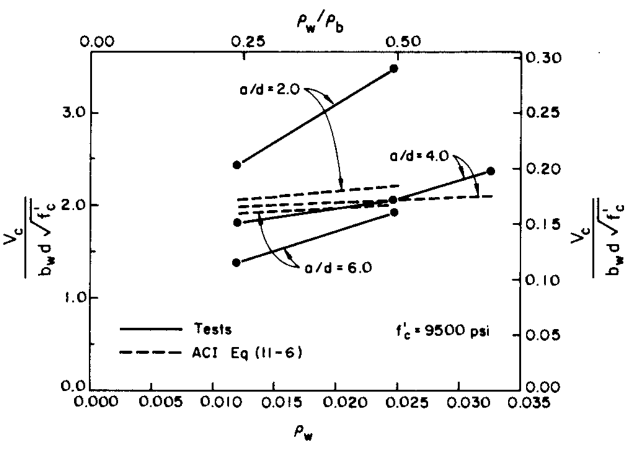

Fig. 6 — Effect of ρ_w on shear strength of beams with f_c' = 9500 psi

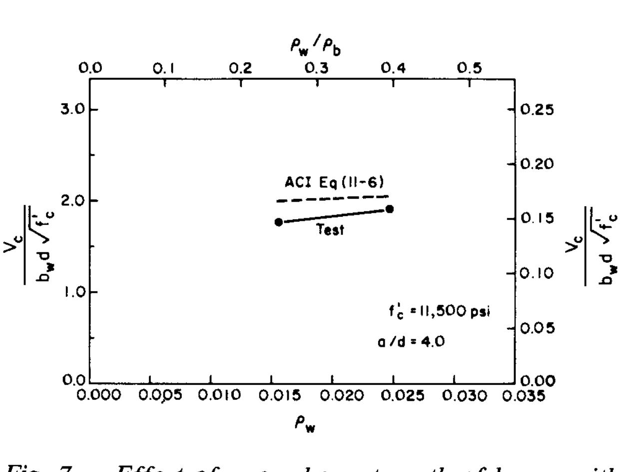

Fig. 7 — Effect of ρ_w on shear strength of beams with f_c' = 11,500 psi

beams with f_c' equal to 11,500 psi, the ratio of test to predicted shear strength was 0.93 for ρ_w equal to 0.025; no higher ratio of ρ_w was used.

The longitudinal reinforcement ratio has a pronounced effect on the basic shear transfer mechanisms. An important factor that affects the rate at which a flexural crack develops into an inclined one is the magnitude of shear stresses near the tip of that crack. The intensity of principal stresses above the flexural crack depends on the depth of penetration of the crack. The greater the value of ρ_w, the less the penetration of the flexural crack. The less the penetration of the flexural crack, the less the principal stresses for a given applied load, and consequently the greater must be the shear to cause the principal stresses that will result in diagonal tension cracking.

Increasing ρ_w also increases the dowel capacity of the member by increasing the dowel area and hence decreasing the tensile stresses induced in the surrounding concrete. Many researchers have pointed out that the steel stress at the time of shear failure is a significant factor in determining the dowel force carried by the reinforcing bars, and hence the shear capacity of the concrete beam. Test observations of Beam F5 confirm this observation. This beam showed substantial deficiency in its shear strength; the ratio of test to predicted strength was 0.70. Close to failure, the longitudinal steel began to yield, and splitting along the reinforcing bars occurred suddenly due to deficient dowel action, followed by shear failure. The steel stresses at failure for other beams were well below yield stress.

Increasing ρ_w also affects the aggregate interlock capacity. Beams with low ρ_w will have wide, long cracks in contrast to the shorter, narrow cracks found in beams with high ρ_w. Since the aggregate interlock mechanism depends on the crack width, an increase in the aggregate interlock force is to be expected with an increase in ρ_w.

Effect of *a/d* ratio

The effect of a/d on the shear strength of test beams f_c' equal to 9500 psi is shown in Fig. 8. The shear strength decreased sharply with increase of a/d. Eq.

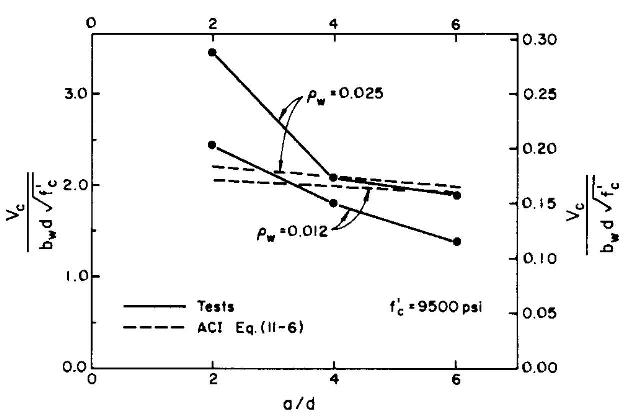

Fig. 8 — Effect of a/d on shear strength of beams with f_c' = 9500 psi

(11-6) was unconservative for beams with a/d equal to 6.0 for even high ρ_w, while it was conservative for beams having a/d of 2.0, even for low ρ_w. The ratio of test to predicted shear strength decreased from 1.19 to 0.70 for ρ_w equal to 0.012, and from 1.57 to 0.96 for ρ_w equal to 0.025, as a/d increased from 2.0 to 6.0.

The high shear strength of beams with a/d of 2.0 may be attributed to arch action. Many researchers have shown that for low a/d, part of the load is transmitted directly by diagonal compression to the supports. This is not a shear transfer mechanism, in the sense that it does not transmit a tangential force to a nearby parallel plane, but permits the transfer of a vertical concentrated force to a reaction and thereby reduces the demand on other types of load transfer. For beams with a/d of 2.0, ρ_w had a great effect on the shear strength. Increasing ρ_w from 0.012 to 0.025 increased the shear strength by 43 percent.

It is generally recognized that increasing a/d decreases the shear strength. For the same applied load, larger a/d means larger bending moment in the shear span; thus, the depth of penetration of the flexural cracks increases, and hence the flexural stresses near the crack tip increase. By increasing a/d, the probability grows that a flexural crack will develop into an inclined one. However, the effect on shear strength of increasing a/d was dependent on ρ_w. For high ρ_w of 0.025, increasing a/d from 4.0 to 6.0 resulted in a reduction in shear strength of only 9 percent. For low ρ_w of 0.012, increasing a/d from 4.0 to 6.0 reduced the shear strength by 35 percent. This significant reduction is attributed to yielding of the longitudinal reinforcement close to failure, which permitted the crack to increase in both length and width, and also adversely affected the dowel action.

It appears from the test results that the effect of a/d on shear strength is not adequately accounted for in the current ACI code provisions.

Shear strength of beams with stirrups

Five beams with stirrups were tested to study the effect of f'_c on shear strength. Two of these beams failed in flexure rather than shear, and so are not reported. The remaining three beams, listed in Table 2, had an a/d of 4.0, with f'_c varying from 3000 to 9100 psi (21 to 63 MPa). Smooth wires (0.25 in. diameter) were used in fabricating the stirrups, which had a 7.5 in. spacing, resulting in $\rho_v f_y = 94$ psi. Test results compared with predictions based on the ACI Building Code are reported in Table 2 and Fig. 9. The ACI code equations were conservative in predicting the shear strength for all test beams with stirrups.

Test results show that the shear strength of beams with stirrups increases as expected with the increase f'_c. However, and in contrast to what was observed in beams without stirrups, the ratio of test to predicted shear strength V_n (the sum of $V_c + V_s$) increased with increase of concrete strength. The limited number of test beams, with one fixed set of parameters, does not allow any generalization. Mphonde and Frantz[8] tested beams with different values of $\rho_v f_y$ and f'_c and concluded that the ratio of test to predicted shear strength decreased with the increase of concrete strength for beams with $\rho_v f_y$ equal to 50 psi (0.35 MPa), in contrast to the present indication. They reported that for beams with $\rho_v f_y$ of 100 psi (0.69 MPa), there was so much scatter in the results that there was no clear relationship. However, their test results show a definite increase in $V_{n,test}/V_{n,ACI}$ with the increase of concrete strength. Values of that ratio were 1.23, 1.35, and 1.53 for beams with f'_c of 4050, 6800, and 10,000 psi (28, 47, and 69 MPa), respectively. These values are close to those obtained from this study.

Haddadin, Hong, and Mattock[9] showed that the effectiveness of web reinforcement in increasing shear strength is greater in the case of diagonal tension failures than in shear compression failures. They also showed that for a particular amount of shear reinforcement, the mode of failure may change from shear compression to diagonal tension as the concrete strength increases, thus increasing the effectiveness of shear reinforcement. This may explain the increasing ratio of test to predicted shear strength as f'_c increases.

Test results indicate the stirrups had no effect on the diagonal cracking load. Stirrup strains measured during testing were negligible up to the diagonal cracking load, after which they increased rapidly. At failure, all stirrups crossing the critical diagonal crack were at yield.

It is clear from test results that stirrups not only carry shear themselves but also enhance the strength of the other shear transfer mechanisms. The stirrups provide support for the longitudinal steel and prevent the rebars from splitting from the surrounding concrete, hence greatly increase the strength of the dowel action. At the same time, the stirrups help to contain the crack, limiting its propagation and keeping its width small. These effects increase both the shear carried by aggregate interlock and the shear strength of the uncracked compression zone. Stirrups also increase the

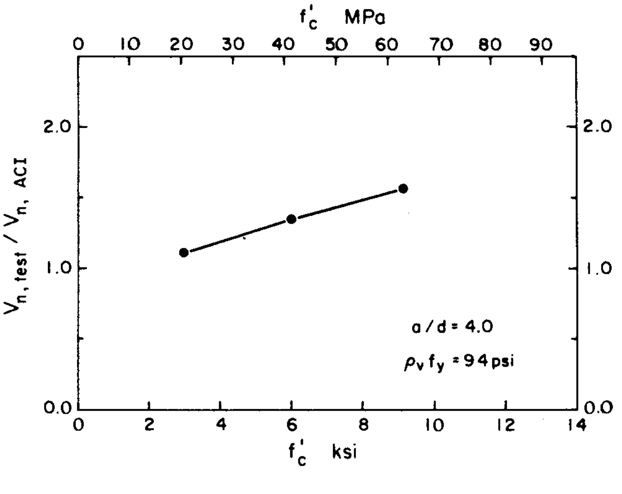

Fig. 9 — Effect of f'_c *on shear strength of beams with stirrups*

strength of compression concrete by providing confinement. Although stirrups do not affect the diagonal cracking load, they enhance the concrete contribution by increasing the capacity of the different shear transfer mechanisms.

CONCLUSIONS

The main purpose of this phase of research on high-strength concrete was to study the effect of concrete strength on the shear strength of reinforced concrete beams and to compare test results with the current ACI code provisions. The most important conclusions may be summarized as follows.

1. The shear strength of beams without stirrups increased with the increase of concrete strength. However, the ratio of test to predicted shear strength decreased with increase of concrete strength, and was less than 1.0 for a significant number of tests.

2. ACI code Eq. (11-6) was seriously unconservative for beams without stirrups having high f'_c and a/d, with low ρ_w.

3. The steel ratio below which ACI code Eq. (11-6) was unconservative was higher for high-strength than for low-strength concretes.

4. ACI code Eq. (11-6) underestimates the importance of both ρ_w and a/d, and overestimates the benefit of increasing f'_c.

5. For short beams without stirrups, ACI code provisions were conservative in predicting the shear strength of beams for all f'_c, even with low ρ_w.

6. Shear failure was more abrupt and failure surfaces were smoother for higher concrete strengths.

7. For beams with stirrups, both the shear strength and the ratio of test to predicted shear strength increased with the increase of concrete strength.

8. The ACI code approach to shear design was conservative for all beams with stirrups, regardless of strength.

9. For all test beams with stirrups, the concrete contribution to shear strength V_c was higher than that assumed by ACI code procedures.

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NOTATION

a = shear span from support to first concentrated load, in.
A_s = longitudinal steel area, in.^{2}
b_w = beam width, in.
d = effective depth of beam, in.
f'_c = compressive strength of concrete, psi
M_u = bending moment at section considered, in.-lb
V_c = shear strength provided by concrete, lb
V_n = total shear strength = $V_c + V_s$, lb
V_s = shear strength provided by shear reinforcement, lb
V_u = shear force at section considered, lb
$V_{c,ACI}$ = shear strength predicted by ACI Eq. (11-6), lb
$V_{c,test}$ = failure shear force obtained in test, lb
ρ_b = balanced steel ratio in flexure
ρ_w = longitudinal steel ratio = $A_s/b_w d$
ρ_u = ratio of shear reinforcement = $A_u/b_w s$

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