TITLE NO. 66-24

CODE PREVIEW PAPER

Background to material being considered for the next *ACI Building Code*

Proposed Changes in Shear Provisions for Reinforced and Prestressed Concrete Beams*

By JAMES G. MACGREGOR and JOHN M. HANSON

Major additions and changes in the design provisions for shear and diagonal tension, proposed for incorporation in the revised (1970) ACI Building Code, are presented. To serve as background, the basic assumptions of the shear design provisions of the ACI Building Code (ACI 318-63) are reviewed. The proposals include a minimum web reinforcement provision, a revised design procedure for reinforced concrete beams under axial tension and axial compression, and a simplified alternate design procedure for prestressed concrete beams. Several design illustrations which use this alternate procedure for prestressed concrete beams are included.

Keywords: axial loads; beams (supports); diagonal tension; prestressed concrete; reinforced concrete; shear strength; structural analysis; structural design; web reinforcement.

■ HOGNESTAD† HAS PRESENTED and discussed the proposed ultimate strength shear design provisions for the 1970 ACI Building Code. These provisions have been reviewed by ACI-ASCE Committee 426. They have not yet been adopted, and revisions will probably take place during further review by ACI Committee 318.

The more important additions and changes in these proposed provisions for shear design of reinforced and prestressed concrete beams are presented and explained in this paper. The principal assumptions on which the shear design provisions of the 1963 ACI Building Code[1] are based remain essentially unchanged in the proposed provisions for the 1970 Code. These assumptions are reviewed.

PROPOSED ADDITIONS AND CHANGES IN SHEAR DESIGN PROVISIONS‡

1.1—General reinforcement requirements

1.1.1—A minimum area of shear reinforcement A_v not less than (50 psi) $b's/f_y$ shall be provided in all reinforced and prestressed concrete flexural members except:

1. Slabs and footings.
2. Concrete joist floor construction.
3. Beams where the total depth does not exceed 2.5 times the thickness of the flange or one-half the width of the web.
4. Where v_u is less than one-half of v_c.

This requirement may be waived if it is shown by test, including realistic longitudinal tensile forces simulating effects of restrained shrinkage

*This paper was prepared in conjunction with the work of ACI-ASCE Committee 426, Shear and Diagonal Tension.

†Hognestad, Eivind, "Proposed Provisions for 1970 ACI Building Code, Chapter 17 — Shear and Torsion," Third Draft, Committee Correspondence, June 14, 1968.

‡Only those paragraphs have been taken from the above footnote which present important additions or changes, or which are necessary for the clarity of this paper. The majority of the paragraphs pertaining to shear design, which are unchanged or only slightly changed from the 1963 Code, are omitted. Notation is listed at the end of this paper.

and creep, that the required ultimate flexural and shear capacity can be developed when shear reinforcement is omitted.

1.1.2—Shear reinforcement may consist of:

1.1.2.1—Welded wire fabric with principal wires located perpendicular to the axis of the member and spaced not further apart than one-half the depth $0.5t$ of the member.

1.2—Ultimate shear strength

1.2.1—The nominal ultimate shear stress v_u shall be computed by:

$$v_u = \frac{V_u}{b'd} \qquad (1\text{-}1)$$

For reinforced concrete members, d is the distance from the extreme compression fiber to the centroid of the longitudinal tension reinforcement. For prestressed concrete members, d is the same distance or $0.8\,t$, whichever is greater.

1.3—Stress v_c for normal weight reinforced concrete members

1.3.1—The nominal shear stress carried by the concrete v_c shall not exceed $2\phi\sqrt{f_c'}$ unless a more detailed analysis is made in accordance with Section 1.3.2. For members subjected to axial load, v_c shall not exceed values given in Sections 1.3.3 - 1.3.4.

1.3.2—The nominal shear stress v_c shall not exceed:

$$v_c = \phi\left(1.9\sqrt{f_c'} + 2500\,p_w\frac{Vd}{M}\right) \qquad (1\text{-}2)$$

nor shall v_c exceed $3.5\phi\sqrt{f_c'}$. V and M are the total shear and bending moment at the section considered, but M shall not be less than Vd in flexural members.

1.3.3—For members subjected to axial compression, M' shall be substituted for M in Eq. (1-2), and M' shall be permitted to have values less than Vd:

$$M' = M - N\left(\frac{4t - d}{8}\right) \qquad (1\text{-}3)$$

Alternatively, v_c may be computed by:

$$v_c = \frac{2\phi\sqrt{f_c'}}{1 - 0.0008\,N/A_g} \qquad (1\text{-}4)$$

However, v_c shall not exceed:

$$v_c = 3.5\phi\sqrt{f_c'}\,\sqrt{1 + 0.002\,N/A_g} \qquad (1\text{-}5)$$

The quantity N/A_g shall be expressed in psi.

1.3.4—For members subjected to significant axial tension, web reinforcement shall be designed to carry the total shear, unless a more detailed analysis is made using:

$$v_c = 2\phi\,(1 + 0.002\,N/A_g)\,\sqrt{f_c'} \qquad (1\text{-}6)$$

where N is negative for tension.

ACI member **James G. MacGregor** is professor of civil engineering, at the University of Alberta, Edmonton, Canada. Currently he is a member of the control groups of ACI-ASCE Committee 426, Shear and Diagonal Tension; and ACI-ASCE Committee 441, Reinforced Concrete Columns. Professor MacGregor is one of the two Canadian delegates to CEB and is a member of the CEB Commission on buckling.

ACI member **John M. Hanson** is principal research engineer, Structural Research Section, Research and Development Division, Portland Cement Association, Skokie, Illinois. He is a member of ACI Committee 215, Fatigue of Concrete, and ACI-ASCE Committee 426, Shear and Diagonal Tension. Dr. Hanson did his undergraduate work at South Dakota State University, and has received graduate degrees from Iowa State University and Lehigh University. He has several years of experience with consulting engineering firms, and is a registered engineer in Colorado. He joined PCA in 1965.

1.4—Stress v_c for normal weight prestressed concrete members

1.4.1—For members having an effective prestress at least equal to 40 percent of the tensile strength of the flexural tension reinforcement, unless a more detailed analysis is made in accordance with Section 1.4.2, the nominal shear stress carried by the concrete v_c shall not exceed:

$$v_c = \phi\left(0.6\sqrt{f_c'} + 700\,\frac{Vd}{M}\right) \qquad (1\text{-}7)$$

but v_c need not be taken as less than $2\phi\sqrt{f_c'}$ nor shall v_c be greater than $5\phi\sqrt{f_c}$.

1.4.1.1—When applying Eq. (1-7), d shall be taken as the distance from the extreme compression fiber to the centroid of the prestressing tendons.

1.4.2—The shear stress v_c shall be taken as the shear causing inclined cracking, which may be computed as the lesser of v_{ci} and v_{cw}:

$$v_{ci} = \phi\left[0.6\sqrt{f_c'} + \frac{V_d + (V_l\,M_{cr}/M_l)}{b'd}\right] \qquad (1\text{-}8)$$

but not less than $1.7\phi\sqrt{f_c'}$,
where $M_{cr} = (I/y_t)\,(6\sqrt{f_c'} + f_{pe} - f_d)$.

$$v_{cw} = \phi\left[3.5\sqrt{f_c'} + 0.3f_{pc} + \frac{V_p}{b'd}\right] \qquad (1\text{-}9)$$

1.4.2.1—When applying Eq. (1-8) and (1-9), d shall be taken as the distance from the extreme compression fiber to the centroid of the prestressing tendons, or $0.8t$, whichever is greater.

1.5—Design of shear reinforcement

1.5.1—For both reinforced and prestressed members, the required area of shear reinforcement perpendicular to the longitudinal axis shall be computed by:

$$A_v = \frac{(v_u - v_c)\,b's}{\phi\,f_y} \qquad (1\text{-}10)$$

REVIEW OF 1963 ACI SHEAR DESIGN PROVISIONS

Two important assumptions were made in the derivation of the 1963 ACI Code provisions for the shear capacity of reinforced and prestressed concrete beams. The first assumption is that the ultimate shear capacity v_u of a beam *without* web reinforcement can be taken equal to the inclined cracking shear, v_c. In tests, the difference between v_u and v_c depends on several variables. The most significant of these is probably the ratio of flexural stress to shear stress. This ratio is a function of the M/Vd ratio. In laboratory tests on relatively small beams, the M/Vd ratio is equal to the distance a between the support and the closest load point divided by the effective beam depth, d.

Fig. 1 shows the variation in inclined cracking shear and ultimate shear of beams without web reinforcement as the shear span to effective depth ratio a/d is varied. This figure is an idealization of similar figures presented by several authors.[2-5] For a/d ratios greater than about 6, flexural failures occur prior to inclined cracking. For a/d ratios less than about 3, V_u exceeds V_c. In this range, test beams have some reserve capacity after inclined cracking. However, for the practical range of a/d ratios from about 3 to 6, V_u can be assumed equal to V_c. In this range, beams without web reinforcement fail suddenly when inclined cracking develops.

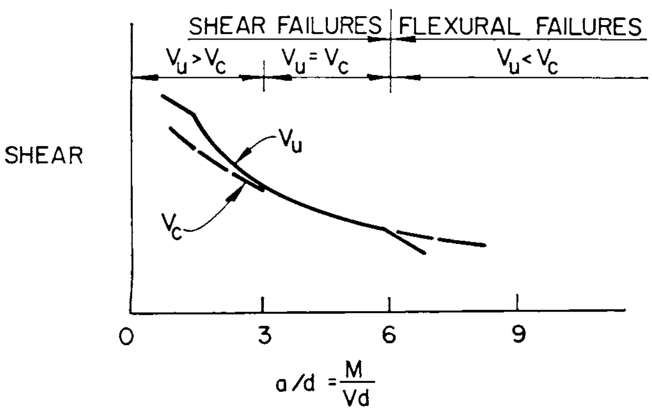


Fig. 1—Comparison of inclined cracking shear and ultimate shear for reinforced concrete beams without web reinforcement

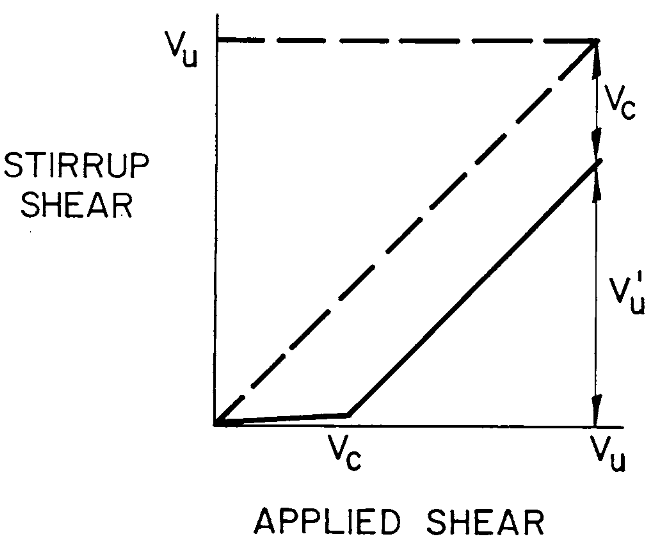


Fig. 2—Relationship between shear carried by stirrups and applied shear

The second assumption is that, in a beam *with* web reinforcement, the shear carried by the web reinforcement V_u' is equal to the ultimate shear V_u minus the inclined cracking shear V_c as given by:

$$V_u' = V_u - V_c \tag{1}$$

This assumption is illustrated in Fig. 2, where the shear carried by vertical stirrups in a reinforced concrete beam is compared to the applied shear. If all the applied shear were carried by stirrups, it would be represented by the dashed line. In tests of reinforced and prestressed concrete beams, however, the shear carried by the stirrups is less than the applied shear, as shown by the solid line. There is little or no stress in the stirrups until inclined cracking develops at the shear V_c. Following inclined cracking, the stress in the stirrups increases with further increase in the applied shear, such that a line representing the shear carried by the stirrups is essentially parallel to the applied shear line, but below it. Thus, Eq. (1) closely approximates the distribution of shear at ultimate load. Comparisons similar to Fig. 2 have been presented by several authors.[5-7]

These two assumptions, which form the basis of the shear design provisions in the 1963 Code, also apply to changes proposed for the 1970 Code. In the design of a beam for shear, the following three steps are therefore required:

1. Compute V_c, a measure of both the inclined cracking shear and the shear carried by the concrete after inclined cracking.

2. Compute the shear carried by the stirrups using Eq. (1) and proportion the stirrups for this shear. This is done using the traditional truss analogy equations in the 1963 Code.

3. Check whether the area of the stirrups exceeds minimum requirements. If not, provide minimum stirrup reinforcement.

These same three steps are followed in the design of both ordinary reinforced and prestressed concrete members. The primary difference between designs for these types of members is the way in which the inclined cracking shear V_c is computed.

Inclined cracking in concrete beams

Inclined cracking which develops in reinforced or prestressed concrete beams can be classified as either web-shear cracking or flexural-shear cracking, as shown in Fig. 3. Web-shear cracks occur before flexural cracking has occurred in their vicinity. Flexure-shear cracks occur after flexural cracking has occurred, often as a continuation of one of the flexural cracks. These two types of cracks and the variables affecting their development are discussed more fully in Reference 8.

Web-shear cracking is fairly unusual. However, it may occur near the supports of highly prestressed beams with thin webs. It may also occur near the inflection points or bar cut-off points of continuous reinforced concrete beams subjected to axial tension.

Shear cracking in reinforced and prestressed concrete beams generally falls into the flexure-shear classification. Flexure-shear cracks begin as flexural cracks, extending more or less vertically into the beam. When a critical combination of flexural and shear stresses develops in an element close to the top of the crack, the inclined crack forms.

The rate of transformation of the initiating flexural crack into a flexure-shear crack is affected by several things which, for simplicity, will be combined into the following two factors:

(a) The rate of growth and the height of flexural cracks in a region where shear exists.

(b) The magnitude of the shear stresses acting near the tops of flexural cracks.

To consider the first factor, Fig. 4 shows the results of an analytical study of the development of flexural cracking in a prestressed beam and in three reinforced concrete beams subjected to axial tension, axial compression, and no axial force. All four beams were assumed to have the same 6 x 12-in. (15.2 x 30.5-cm) cross section. The properties given to these beams are listed in Table 1. The average compressive stress on the axially loaded beam was the same as that of the prestressed concrete beam. Both the prestressed concrete beam and the reinforced concrete beam without axial load have the same ultimate moment capacities.

The reinforced concrete beam with no axial force cracks at a moment of about 5.5 ft-kips (5.3 m-kgf). When the cracks develop, they immediately grow to about 15 percent of the beam height. As the applied moment increases, the cracks continue to grow in height, rapidly at first, and then more slowly. After the reinforcement

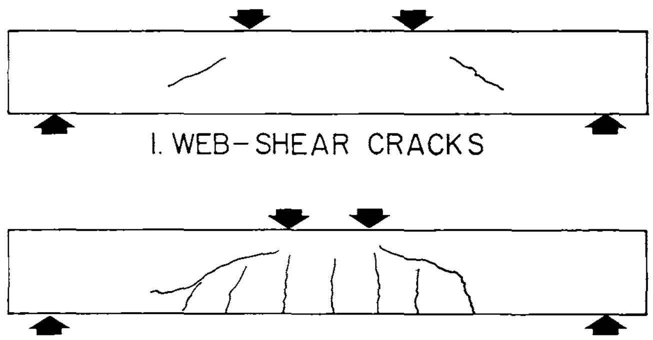


Fig. 3—Types of inclined cracks

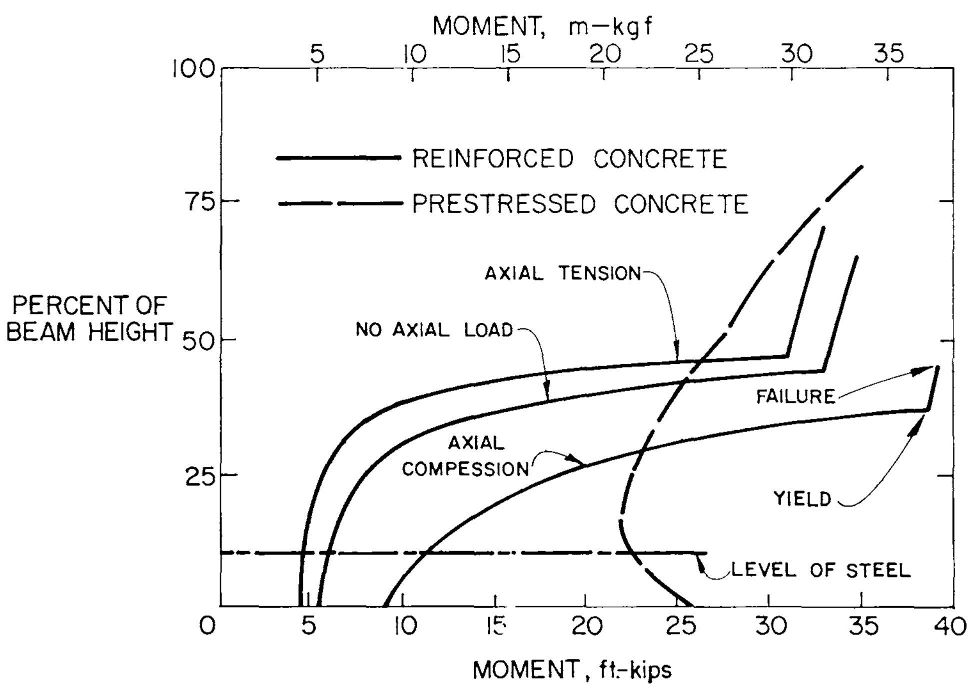


Fig. 4—Relationship between moment and flexural crack height

TABLE I — PROPERTIES OF BEAMS PLOTTED IN FIG. 4

Type of beam*	Reinforcement ratio p	Concrete cylinder strength f_c', psi	Steel yield stress f_y, ksi	Axial force N, kips	Effective prestress force F_{se}, kips	Calculated ultimate moment M_{fu}, ft-kips
Reinforced concrete	0.015	3000	45	0	—	34.8
Reinforced concrete — Axial tension	0.015	3000	45	−7.2	—	33.0
Reinforced concrete — Axial compression	0.015	3000	45	+27.0	—	39.3
Prestressed concrete	0.0028	5000	250†	—	27.0	34.8

*The effective depth d of all beams was 10.8 in. (27.4 cm)

†Ultimate stress f_{su} rather than yield stress.

Note: To convert psi to kgf/cm^2, multiply by 0.0703.
To convert ksi to kgf/cm^2, multiply by 70.3.
To convert kips to kgf, multiply by 453.6.
To convert ft-kips to m-kgf, multiply by 0.958.

yields, the cracks grow with little change in moment.

In the compression beam, the axial force acts to delay the formation of flexural cracks. At failure the cracks have not extended quite as far into the beam, since the compression zone must be larger to provide the additional compression force needed for equilibrium. In the beam subjected to axial tension, the cracks have extended somewhat higher at failure. Otherwise, the behavior is similar to that of the other two beams.

In the case of the prestressed concrete beam, the prestressing force acts to delay flexural cracking. Consequently, the moment at cracking is about five times that required to crack the reinforced concrete beam with no axial force. Once the crack forms, however, it immediately grows to almost 50 percent of the beam height. At this height, the beam attains equilibrium under the cracking moment.

The differences in the rate of growth and in the height of the flexural cracking in these four beams is affected by both the axial force or prestress and by the percentage of longitudinal reinforcement. Of these two, the latter is more important. Prior to flexural cracking, a tensile force is developed in the concrete at the bottom of the beam. When the beam cracks, the tension force in the reinforcement must increase by the amount that was carried by the concrete. With a relatively high steel percentage as normally found in a reinforced concrete beam, the crack does not have to progress very far before the steel strains develop the required tension force. In a prestressed concrete beam, the steel prcentage may be much lower. Therefore, a much larger steel strain and hence a much larger crack height is required to reach equilibrium. This variable is discussed more fully elsewhere.[9,10]

The second general factor affecting the rate at which the initiating flexural crack develops into an inclined crack is the magnitude of the shear stresses acting in the vicinity of the top of the initiating flexural crack. These shear stresses will tend to be high if the shear force is large when the flexural crack starts. This situation will occur in beams with small a/d ratios and thin webs. These stresses will also tend to be high if the flexural crack extends deeply into the beam so that very little uncracked concrete remains above the crack. This will occur in beams with low steel percentages.

As a result of the flexural cracking behavior just described, the shear and flexural stresses at the top of the initiating flexural crack rapidly become critical in a prestressed concrete beam. Consequently, in these beams the inclined crack develops soon after the flexural crack has formed. In a reinforced concrete beam, the initiating flexural crack starts at a lower applied shear and grows much more slowly than in the prestressed concrete beam. Largely as a result of this, a considerable increment of load is required to transform a flexural crack in a reinforced concrete beam into an inclined crack. This is true even when axial loads are present.

Code equations for V_c—Reinforced concrete beams

Because of the difference in inclined cracking behavior for prestressed concrete and reinforced concrete beams, investigators have approached the inclined cracking problem in different ways.

For reinforced concrete beams, the large load increment between flexural cracking and inclined cracking led Morrow and Viest[2] to estimate the principal tensile stresses in an element located at the top of a flexural crack in a region of combined bending and shear. As shown in Fig. 5(a), an element at this location is acted on by a shear stress v given by:

$$v = C_1 \left(\frac{V}{b'd}\right) \tag{2}$$

where C_1 is a constant, V is the shear force, b' is the web width, and d is the effective depth. The element is also subjected to a flexural tension f given by:

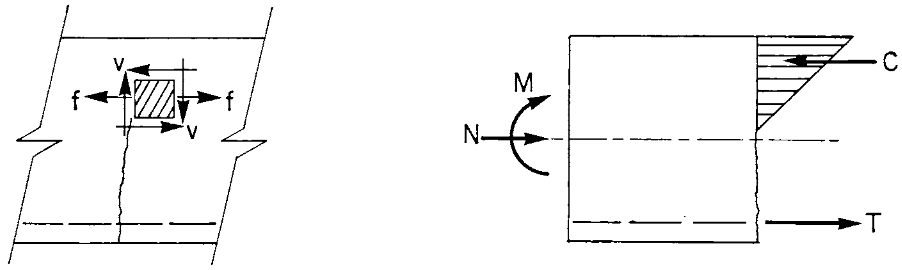


Fig. 5—Force systems used in derivation of inclined cracking equations for reinforced concrete beams

$$f = C_2 \left(\frac{M}{pjbd^2}\right) \qquad (3)$$

where C_2 is another constant, M is the moment, p is the longitudinal reinforcement ratio, j is the ratio of the distance between the centroid of compression and the centroid of tension to d, and b is the flange width. In Eq. (3) the tension in the concrete is assumed to be a function of the parameters that are used in computing the stress in the longitudinal reinforcement. The shear stress at inclined cracking v_c was determined from the expression for the principal tension stress in this element, which was semi-empirically reduced[11] to Eq. (1-2); it is identical to Eq. (17-2) of the 1963 ACI Code.

There are three significant variables in Eq. (1-2). These are the tensile strength of the concrete represented by $\sqrt{f_c'}$ the steel percentage p_w, and the moment to shear ratio M/Vd.

When an axial force is superimposed on a reinforced concrete beam, the axial force is assumed to affect the tension stress f in the element in the same manner as it affected the stress in the tension reinforcement.[11] If the equilibrium of the beam section shown in Fig. 5(b) is considered, the internal and external moments may be summed about the resultant of the compression force C to give:

$$M\text{-}N \left(\frac{t}{2} - d + jd\right) = pjbd^2 f_s \qquad (4)$$

where N is the axial force, t is the over-all height, and f_s is the stress in the steel. From Eq. (3) and (4), ACI Committee 326 (now 426) assumed that:

$$f = C_3 f_s = C_3 \left[\frac{M\text{-}N\ \frac{t}{2} - d + jd}{pjbd^2}\right] \qquad (5)$$

where C_3 is another constant and all other terms are as previously defined. For simplicity of notation, an equivalent moment M' was derived, where:

$$M' = M\text{-}N \left(\frac{t}{2} - d + jd\right) \qquad (6)$$

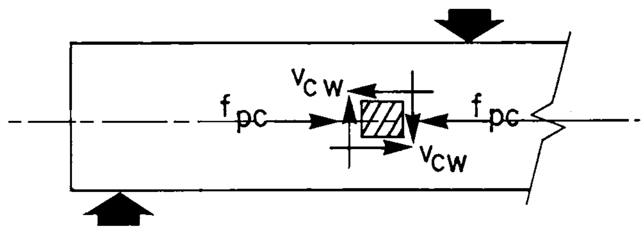


Fig. 6—Element at center of gravity of cross section of prestressed concrete beam

For $j = 7/8$, Eq. (6) reduces to Eq. (1-3), which is Eq. (17-3) of the 1963 ACI Code. In addition, ACI Committee 326 suggested that v_c should not exceed the value given by Eq. (1-5), which also is the same as in the 1963 ACI Code.

Recent analyses of the inclined cracking load[9,10] suggest that the tension stress f in the element shown in Fig. 5(a) will remain relatively unchanged when axial loads are applied, since it must be close to the tensile strength of the concrete. These analyses suggest that the primary effect of axial load is to change the shearing stresses in this element by changing the height of the flexural cracks in the member.

Code equations for V_c—Prestressed concrete beams

As previously discussed, for prestressed concrete beams the load increment between flexural cracking and inclined cracking is relatively small. As a result, investigators have assumed that the flexure-shear cracking capacity V_{ci} is equal to the sum of the live load shear required to cause an initiating flexural crack at a distance $d/2$ from the section under consideration, the additional shear required to transform the flexural crack into an inclined crack, and the dead load shear. These shears are given, respectively, by the three terms in the following equation:

$$V_{ci} = \frac{M_{cr}}{\frac{M_l}{V_l} - \frac{d}{2}} + 0.6 b'd\sqrt{f_c'} + V_d \qquad (7)$$

However, V_{ci} need not be taken less than $1.7b'd\sqrt{f_c'}$. M_{cr} is the flexural cracking moment and M_l and V_l are the live load moment and shear. Eq. (7) is the same as Eq. (26-12) of the 1963 ACI Code. It was developed by Sozen and Hawkins[12] from tests of prestressed concrete beams[13,14] carried out at the University of Illinois.

Web-shear cracking occurs when the principal tensile stresses reach the tensile strength of the concrete in some element near the neutral axis, as the one shown in Fig. 6. This element is acted on by a shear stress v_{cw} and a normal stress due to prestressing f_{pc}. From a Mohr's circle it can be shown that the tensile strength of the concrete f_t' will exist in the element when v_{cw}, the shear stress causing web-shear cracking, reaches the value given by the expression:

$$v_{cw} = f_t'\sqrt{1 + f_{pc}/f_t'} \qquad (8)$$

where f_t' is the tensile strength of the concrete and f_{pc} is the stress at the centroid of the cross section due to prestressing. If f_t' is assumed equal to $3.5\sqrt{f_c'}$, Eq. (8) becomes:

$$v_{cw} = 3.5\sqrt{f_c'}\sqrt{1 + f_{pc}/3.5\sqrt{f_c'}} \qquad (9)$$

Mattock simplified this to obtain the web-shear cracking capacity V_{cw} as given by:

$$V_{cw} = (3.5\sqrt{f_c'} + 0.3f_{pc})\,b'd + V_p \qquad (10)$$

In this expression, the term V_p accounts for any component of the prestress acting transversely to the axis of the beam. Eq. (10) is the same as Eq. (26-13) of the 1963 ACI Code. It is also similar to Eq. (1-5) in this paper. Both Eq. (10) and Eq. (1-5) form the upper limit to the shear strength of beams subjected to axial loads, and presumably both apply to inclined cracking which develops before the member cracks in flexure.

DISCUSSION OF PROPOSED ADDITIONS AND CHANGES

In the preceding section it is shown that the 1963 ACI shear design provisions are similar for reinforced and prestressed concrete beams, except for the way in which the inclined cracking shear is computed. The various equations for inclined cracking shear differ because the behavior of these types of beams is different. The revisions proposed for the 1970 ACI Code do not change the basic design procedures. There are, however, several additions and simplifying changes, primarily in the way that the various inclined cracking shears are computed. These revisions are discussed in the balance of this paper.

Provision for minimum web reinforcement

In the 1963 ACI Code there is no minimum web reinforcement provision for reinforced concrete beams when v_u is less than v_c. However, there is a minimum web reinforcement provision for prestressed concrete beams given by Eq. (26-11). This equation is related to the flexural capacity and the geometry of the prestressed concrete member by A_sf_s' and $\sqrt{d/b'}$, respectively. Thus a designer, proportioning a beam to carry a given ultimate moment, can increase the web width and thereby decrease the minimum amount of stirrup reinforcement.

Minimum web reinforcement restrains the growth of inclined cracking, thereby increasing ductility. This provides a warning in situations where the sudden formation of inclined cracking may otherwise lead directly to failure. Such reinforcement is of great value if a member is subjected to an unexpected tensile force or to other unforeseen loadings.

The proposed provision for minimum web reinforcement in Section 1.1.1:

$$A_v = (50\text{ psi})\ b's/f_y$$
$$[A_v = (3.5\text{ kgf/cm}^2)\,b's/f_y]$$

applies to both reinforced and prestressed concrete beams. This provision is similar to a former requirement for prestressed concrete beams which appeared in the 1958 ACI-ASCE Committee 323 report.[18] In this report, the minimum area of shear reinforcement was equal to 0.0025 $b's$. Assuming the yield strength of the stirrup reinforcement varies between 40,000 and 60,000 psi (nom. 1200 and 1800 kgf/cm^2), it may be seen that Section 1.1.1 requires from one-half to one-third of the minimum shear reinforcement suggested by the Committee 323 report. In general, Section 1.1.1 will require more minimum shear reinforcement for building-type prestressed concrete beams, and less for bridge-type beams, than Eq. (26-11) of the 1963 ACI Code.

When minimum web reinforcement is used, Eq. (1-10) can be written as:

$$v_u = v_c + \phi\,50\text{ psi}\ (v_u = v_c + \phi\,3.5\text{ kgf/cm}^2) \qquad (11)$$

No further web reinforcement is required unless the magnitude of v_u exceeds that given by Eq. (11).

Provisions for axially loaded reinforced concrete beams

In the 1963 ACI Code, the design of beams subjected to shear and moment combined with axial tension or compression was based on Eq. (1-2), (1-3), and (1-5). An illustration of the application of Eq. (1-2) and (1-3) to a 6 x 12-in. (15.2 x 30.5-cm) beam with an effective depth of 10.8 in. (27.4 cm) is shown in Fig. 7. The lower solid line corresponds to a condition where the concrete strength equals 5,000 psi (nom. 350 kgf/cm²), the a/d ratio equals 5, and the longitudinal steel ratio equals 0.005. The upper solid line corresponds to a condition where the concrete strength equals 2,500 psi (nom. 175 kgf/cm²), the a/d ratio equals 2, and the longitudinal steel ratio equals 0.03.

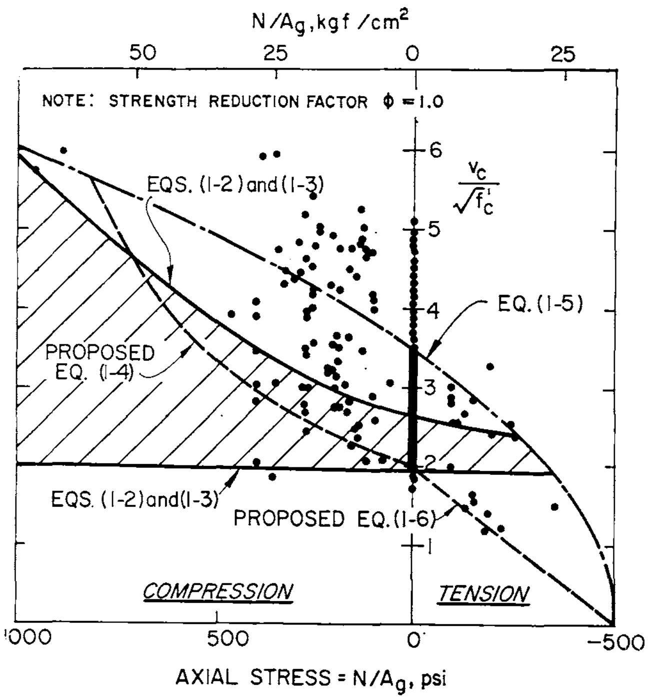


Fig. 7—Effect of axial load on inclined cracking shear stress

Thus, the shaded band between these lines approximately represents the range of Eqs. (1-2) and (1-3) for axially loaded reinforced concrete beams. Data from tests of beams and frames[1,2,11,15-17]* have also been plotted in Fig. 7.

In the tension region, Eq. (1-2) and (1-3) are unconservative compared to some of the test data plotted in this figure. In addition, it should be noted that there is virtually no reduction in the allowable shear stress until the average tensile stress exceeds about 300 psi (nom. 20 kgf/cm). For this reason, and because Eq. (1-2) and (1-3) are difficult to apply to beams subjected to combined shear and axial tension, it is proposed that they be replaced in the 1970 ACI Code by Eq. (1-6). This equation is plotted with a dashed line in Fig. 7.

In the compression region, Eq. (1-2), (1-3) and (1-5) form a lower bound on the test data and no change is proposed in these equations. However, again because these equations are difficult to apply, it is proposed that designers be allowed the option of computing v_c for members subjected to combined shear and axial compression from Eq. (1-4). This equation is also plotted with a dashed line in Fig. 7 and forms a reasonable lower bound

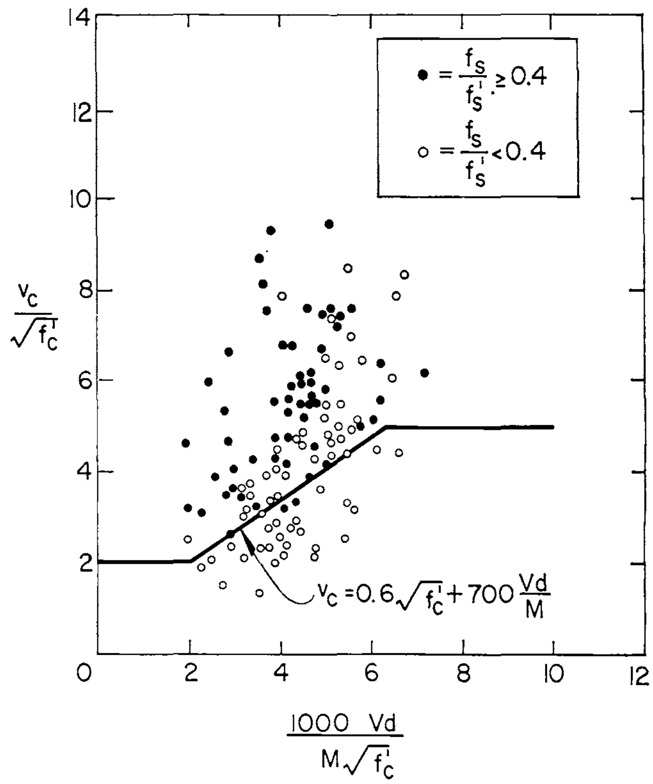


Fig. 8—Alternate equation for computing v_c for prestressed beams

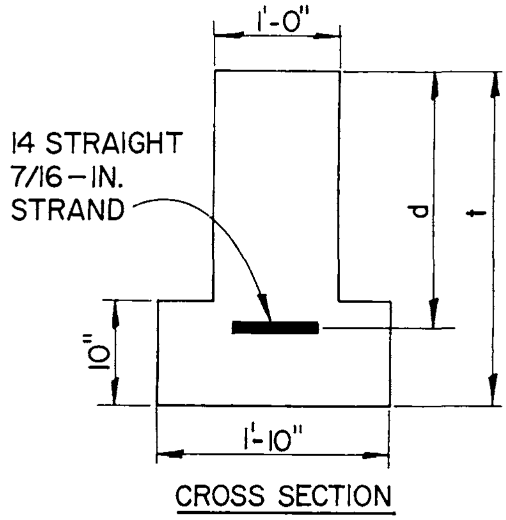
to the data shown. However, Eq. (1-4) may be unconservative for small values of p and large M/Vd ratios. This matter is under further investigation. The value of v_c computed from Eq. (1-4) should not exceed the value given by Eq. (1-5).

Provisions for prestressed concrete beams

In the 1963 ACI Code provisions for prestressed concrete beams, the shear carried by the concrete V_c is taken equal to the lesser of the two values given by Eqs. (7) and (10) for flexure-shear and web-shear cracking, respectively. In the proposed revisions, Eq. (7) is converted to a nominal stress basis and modified to Eq. (1-8).

The only change of Eq. (1-8) from Eq. (7) is a simplification of the term which predicts the

*Personal communication from A. H. Mattock, Mar. 1968.



NOTE: To convert in. to cm, multiply by 2.54
To convert ft to cm, multiply by 30.48
To convert ft-kips to m-kgf, multiply by 0.958

PRESTRESS	t in.	d in	$\frac{A_s}{A_g}$ percent	M_u ft-kips
LIGHT	48	37	0.16	878
MODERATE	32	24	0.25	514
HEAVY	24	18	0.36	350

Fig. 9—Shear design of inverted T-beams, $\phi = 0.85$

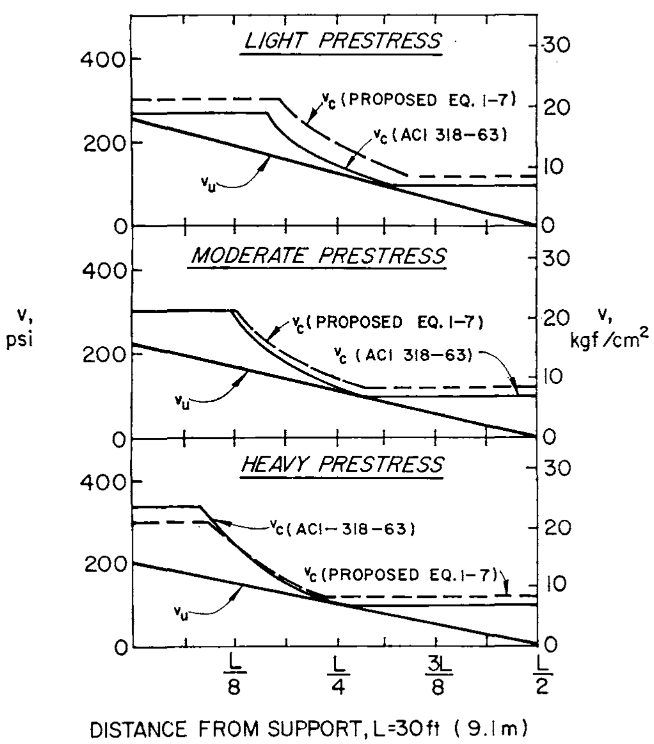


Fig. 9 (cont.)—Shear design of inverted T-beams, $\phi = 0.85$

shear causing the formation of the initiating flexural crack, by removal of the $d/2$ term. In Eq. (7) the shear carried by the concrete was estimated by computing the flexural cracking load at a point $d/2$ toward the reaction from the section being investigated. In Eq. (1-8), the shear carried by the concrete is more conservatively estimated by computing the flexural cracking load at the section being investigated.

There is no change proposed in Eq. (10), other than conversion to a nominal stress basis, which leads directly to Eq. (1-9).

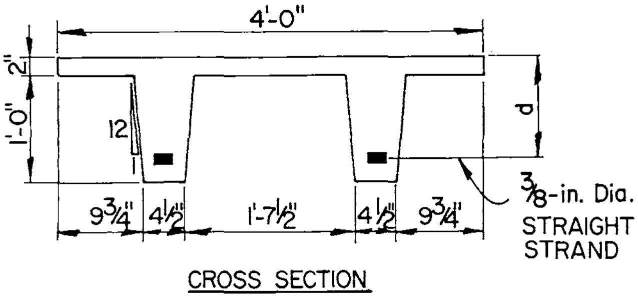
Alternate procedure for computing v_c—As an alternate to Eq. (1-8) and (1-9), a simplified method of computing v_c is proposed, as given by Eq. (1-7). The use of this equation is limited to members having prestress equal to 40 percent of the tensile strength of all of the flexural reinforcement. When using this equation, v_c need not be taken less than $2\phi\sqrt{f_c'}$ and shall not be greater than $5\phi\sqrt{f_c'}$.

Eq. (1-7) is a simplified and generally conservative approximation of Eq. (1-8). The lower limit of $2\phi\sqrt{f_c'}$ is greater than the lower limit of $1.7\phi\sqrt{f_c'}$ for Eq. (1-8), but Eq. (1-8) has no restriction on the amount of prestress in the reinforcement.

The upper limit of $5\phi\sqrt{f_c'}$ serves as a restriction on v_{cw}. In a member with straight strand, the proposed upper limit is less than v_{cw}, assuming that f_c' equals 5,000 psi (nom. 350 kgf/cm^2) and f_{se} equals 100 ksi (nom. 7000 kgf/cm^2), or 0.4 of 250 ksi (nom. 17,500 kgf/cm^2), the tensile strength of ASTM grade strand, whenever the ratio of steel area to cross concrete area is greater than 0.35 percent. Thus it should be conservative in most designs. Assuming an extremely low reinforcement ratio of 0.10 percent, the upper limit is 27 percent greater than v_{cw}.

Comparison of Eq. (1-7) with test data—Eq. (1-7) is compared in Fig. 8 with test data of prestressed beams failing in shear and containing no web reinforcement. The majority of the data was obtained from tests conducted at the University of Illinois,[13] with additional data from tests conducted at Lehigh University.[19-21] These data include members with a wide range of prestress and concrete strengths. In comparing Eq. (1-7) with test data, ϕ was assumed equal to one.

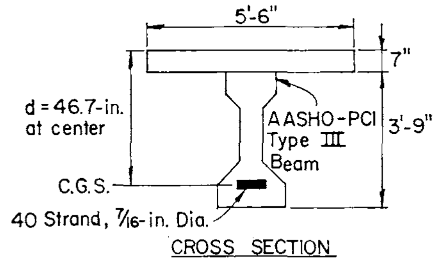
In Fig. 8, the solid points represent test results for members with an effective prestress equal to or greater than 0.4 f_s'. The open points are for a prestress less than 0.4 f_s'. These data include several tests where there was no prestress. It may be seen that only two beams with an effective prestress of more than 0.4 f_s' failed at a shear stress less than that predicted by Eq. (1-7).



NOTE: To convert in. to cm, multiply by 2.54
To convert ft to cm, multiply by 30.48
To convert ft-kips to m-kgf, multiply by 0.958

PRESTRESS	NO. OF STRAND	d in.	A_s/A_g percent	M_u ft-kips
LIGHT	8	11	0.28	137
MODERATE	12	10	0.42	178
HEAVY	16	9	0.56	205

Fig. 10—Shear design of double T-beams, $\phi = 0.85$



NOTE: C.G.S. Deflected upward at 10 ft either side centerline to d=40.3 in. at the supports.
To convert in. to cm, multiply by 2.54
To convert ft to cm, multiply by 30.48

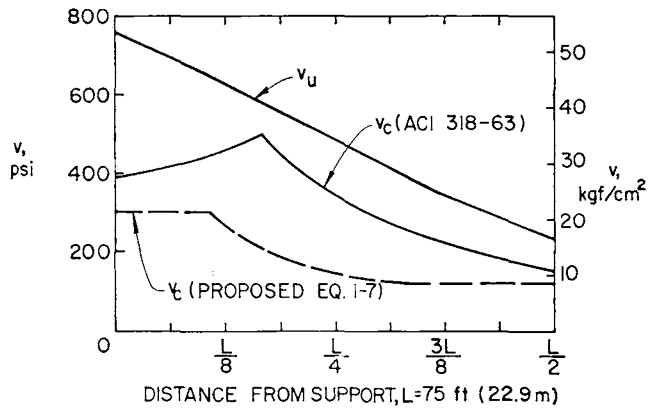


Fig. 11—Shear design of AASHO-PCI girder spanning 75 ft, $\phi = 0.85$

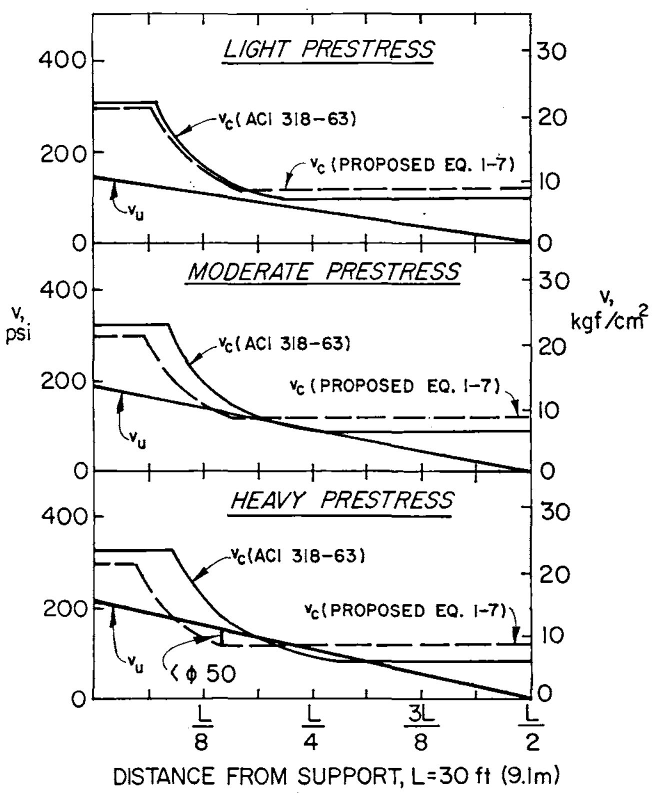


Fig. 10 (cont.)—Shear design of double T-beams, $\phi = 0.85$

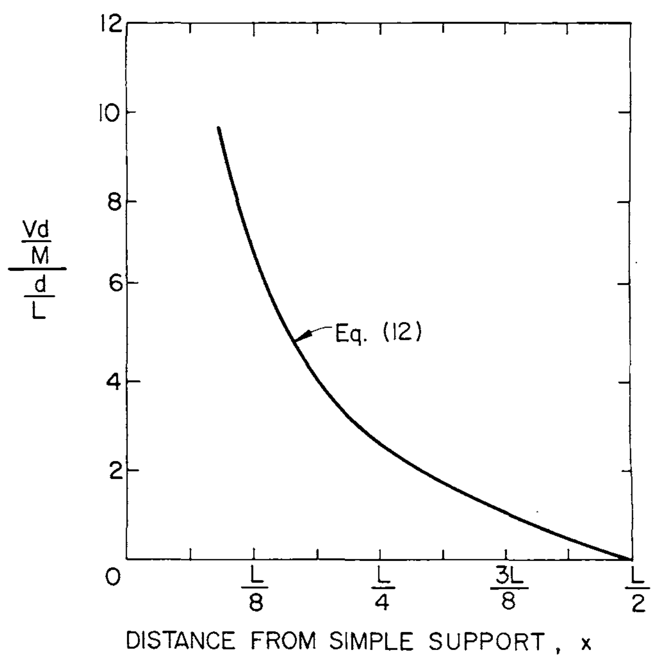


Fig. 12—Vd/M for uniformly loaded beams

Design illustrations using Eq. (1-7)

The proposed Eq. (1-7) has been applied to designs of inverted T-beams, double T-beams, and I-section bridge girders. These design examples are illustrated in Fig. 9, 10, and 11, respectively. Comparisons are made between v_c as computed from Eq. (1-7) and an equivalent v_c computed from Eqs. (7) and (10), which are the provisions of the 1963 Code.

For the inverted T-beams and double T-beams, three different cases, corresponding to light, moderate, and heavy prestress, were considered. In the inverted T-beam, these different cases were obtained by varying the depth of the section while holding the number of prestressing strand constant. This design was taken from Reference 22. In the double T-beam, the section was held constant and the number of strand was varied. Effective depth d; reinforcement ratio, A_s/A_g; and ultimate flexural capacity M_{fu} are given for each case. The concrete strength f_c' was assumed equal to 5000 psi (nom. 350 kgf/cm^2) in all of the examples.

In Fig. 9, the inverted T-beam was considered to be uniformly loaded and simply supported over a span of 30 ft (9.1 m). For the moderate and heavy prestress case, v_c computed from Eq. (1-7), as shown by the dashed line, is close to v_c according to the 1963 Code, as shown by the lighter solid line. For the case of light prestress, the proposed equation is somewhat less conservative than the 1963 Code. However, in all three cases v_c is greater than the shear stress v_u required to develop the flexural capacity M_{fu} at midspan. Thus in every case only minimum web reinforcement is required.

The double T-beam was also considered as uniformly loaded and simply supported over a span of 30 ft (9.1 m). As shown in Fig. 10, the comparison between the proposed v_c and that calculated according to the 1963 Code is good for the light prestress case. The proposed value of v_c becomes increasingly conservative for the moderate and heavy prestress case. It is significant, however, that even for the heavy prestress case, only minimum web reinforcement is required.

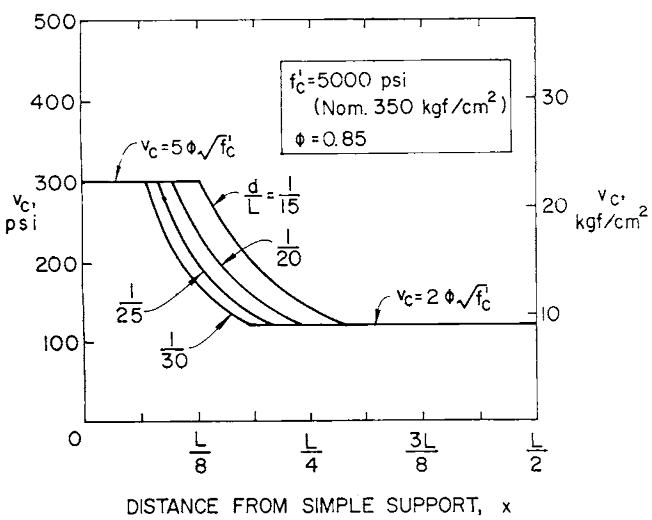


Fig. 13—Shear stress v_c for uniformly loaded prestressed concrete beams as calculated by Eq. (1-7)

For the I-section bridge girder in Fig. 11, a Type III AASHO-PCI composite girder spanning 75 ft (22.9 m) and carrying an H20-S16-44 highway loading was considered. This same girder was used as a design example by Preston.[23] The ultimate shear stress v_u was based on a system of loads which would develop the ultimate flexural capacity of the member, and is therefore substantially higher than produced by the H20-S16-44 loading multiplied by the standard AASHO load factor. It may be seen that the proposed value of v_c is much less than the value of v_c based on the 1963 Code, and thus very conservative.

It therefore appears that the proposed Eq. (1-7) has the greatest application to typical building members. It will be less applicable to bridge girders and other special members where a complete design is generally required.

Design aid for Eq. (1-7)

For simply supported uniformly loaded beams, the Vd/M ratio in Eq. (1-7) becomes a simple function of the span length L, and may be determined from:

$$\frac{Vd}{M} = \frac{d(L-2x)}{x(L-x)} \tag{12}$$

where x is the distance from the section being investigated to the support. Values of Vd/M as a function of d/L and x are shown in Fig. 12. Consequently, Eq. (1-7) and its upper and lower limits can be presented as shown in Fig. 13 for uniformly loaded beams with a concrete compressive strength of 5000 psi (nom. 350 kgf/cm^2). Since minimum web reinforcement will be required in most designs, shear design by Eq. (1-7) simply involves showing that v_u less $\phi\,50 = 42.5$ psi (2.98 kgf/cm^2) for the minimum web reinforcement, is less than the value of v_c obtained from Fig. 13.

SUMMARY

The ACI design provisions for the shear capacity of reinforced and prestressed concrete beams are discussed in this paper.

Additions and changes proposed for the 1970 ACI Building Code include a minimum web reinforcement provision, a revised design procedure for reinforced concrete beams under axial tension and compression, and a simplified alternate design procedure for prestressed concrete beams. Several design illustrations are included. In general, the proposed changes lead to simpler but slightly more conservative designs.

ACKNOWLEDGMENTS

The authors wish to acknowledge the helpful comments of Dr. Alan Mattock, Dr. Frederic Roll, and several other members of ACI-ASCE Committee 426.

REFERENCES

1. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-63)," American Concrete Institute, Detroit, 1963, 144 pp.

2. Morrow, J., and Viest, I. M., "Shear Strength of Reinforced Concrete Frame Members Without Web Reinforcement," ACI JOURNAL, *Proceedings* V. 53, Mar. 1957, pp. 833-869.

3. Mattock, A. H., Discussion of "Shear Strength of Reinforced Concrete Frame Members Without Web Reinforcement" by J. Morrow and I. M. Viest, ACI JOURNAL, *Proceedings* V. 53, Dec. 1957, pp. 1352-1354.

4. Kani, G. N. J., "Basic Facts Concerning Shear Failure," ACI JOURNAL, *Proceedings* V. 64, June 1966, pp. 675-692.

5. Leonhart, F., and Walther, R., "The Stuttgart Shear Tests, 1961," *Beton-und Stahlbetonbau* (Berlin-Wilmersdorf), V. 56, No. 12, 1961, and V. 57, No. 2, 3, 6, 7, and 8, 1962. Also, *Translation* No. 111, Cement and Concrete Association, London, 1961.

6. Richart, F. E., "An Investigation of Web Stresses in Reinforced Concrete Beams," *Bulletin* No. 166, University of Illinois Engineering Experiment Station, Urbana, June 1927.

7. Bruce, R. H., "An Experimental Study of the Action of Web Reinforcement in Prestressed Concrete Beams," PhD Thesis, University of Illinois, Urbana, Sept. 1962.

8. Bresler, B., and MacGregor, J. G., "Review of Concrete Beams Failing in Shear," *Proceedings*, ASCE, V. 93, ST1, Feb. 1967, pp. 343-372.

9. Krahl, N. W.; Khachaturian, N.; and Siess, C. P., "Stability of Tensile Cracks in Concrete Beams," *Proceedings*, ASCE, V. 93, ST1, Feb. 1967, pp. 235-254.

10. MacGregor, J. G., and Walters, J. R. V., "Analysis of Inclined Cracking Shear in Slender Reinforced Concrete Beams," ACI JOURNAL, *Proceedings* V. 64, Oct. 1967, pp. 644-653.

11. ACI-ASCE Committee 326, "Shear and Diagonal Tension," ACI JOURNAL, *Proceedings* V. 59, No. 1, Jan. 1962, pp. 1-30; No. 2, Feb. 1962, pp. 277-333; and No. 3, Mar. 1962, pp. 353-395.

12. Sozen, M. A., and Hawkins, N. M., Discussion of "Shear and Diagonal Tension" by ACI-ASCE Committee 326, ACI JOURNAL, *Proceedings* V. 59, Sept. 1962, pp. 1341-1347.

13. Sozen, M. A.; Zwoyer, E. M.; and Siess, C. P., "Strength in Shear of Beams Without Web Reinforcement," *Bulletin* No. 452, University of Illinois Engineering Experiment Station, Urbana, 1959.

14. MacGregor, J. G.; Sozen, M. A.; and Siess, C. P., "Strength of Prestressed Concrete Beams with Web Reinforcement," ACI JOURNAL, *Proceedings* V. 62, Dec. 1965, pp. 1503-1519.

15. Baldwin, J. W. Jr., and Viest, I. M., "Effect of Axial Compression on Shear Strength of Reinforced Concrete Frame Members," ACI JOURNAL, *Proceedings* V. 55, Nov. 1958, pp. 635-654.

16. Diaz de Cossio, R., and Siess, C. P., "Behavior and Strength in Shear of Beams and Frames without Web Reinforcement," ACI JOURNAL, *Proceedings* V. 56, Feb. 1960, pp. 695-735.

17. Elstner, R. C., and Hognestad, E., "Laboratory Investigation of Rigid Frame Failures," ACI JOURNAL, *Proceedings* V. 53, Jan. 1957, pp. 637-668.

18. ACI-ASCE Committee 323, "Tentative Recommendations for Prestressed Concrete," ACI JOURNAL, *Proceedings* V. 54, Jan. 1968, pp. 546-578.

19. Walther, R. E., and Warner, R. F., "Ultimate Strength Tests of Prestressed and Conventionally Reinforced Concrete Beams in Combined Bending and Shear," *Report* No. 223.18, Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania, Sept. 1958.

20. McClarnon, F. M.; Wakabayashi, M.; and Ekberg, C. C. Jr., "Further Investigation Into the Shear Strength of Prestressed Concrete Beams Without Web Reinforcement," *Report* No. 223.22, Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania, Jan. 1962.

21. Hanson, J. M., and Hulsbos, C. L., "Overload Behavior of Pretensioned Prestressed Concrete I-Beams with Web Reinforcement," *Highway Research Record* 76, Highway Research Board, 1965, pp. 1-31.

22. "Design Data for Simply Supported Prestressed Beams in One-Story Buildings," Portland Cement Association, Chicago, 1965.

23. Preston, H. K., *Practical Prestressed Concrete*, McGraw-Hill, New York, 1960, 340 pp.

APPENDIX

NOTATION

a = distance from beam reaction to concentrated load = M/V

A_g = gross area of section

A_s = area of longitudinal tension reinforcement

A_v = total area of web reinforcement within a distance s measured parallel to the longitudinal reinforcement

b = width of beam compression flange

b' = width of beam web

C = compression force in concrete

C_1, C_2, C_3 = constants

d = effective depth of beam

f = flexural stress on concrete element

f_c' = 28 day compressive strength determined from 6 x 12-in. (nom. 15 x 30-cm) cylinders

f_{pc} = stress at centroid due to prestressing

f_s = stress in steel

f_{su} = stress in prestressing reinforcement at ultimate

f_s' = tensile strength of prestressing reinforcement

f_t' = tensile strength of concrete

f_y = yield strength of reinforcement

F_{se} = effective prestress force

j = ratio of distance between centroid of compression and centroid of tension to the depth d in working stress design

L = span length

M = moment at a section

M_{cr} = flexural cracking moment

M_{fu} = ultimate moment in flexural failure

M_l = moment due to live load

M' = effective moment in an axially loaded section

N = axial load in a section, positive if compression

p = longitudinal reinforcement ratio, A_s/bd

p_w = longitudinal reinforcement ratio, $A_s/b'd$

s = spacing of web reinforcement measured parallel to longitudinal reinforcement

t = overall beam depth

T = tension force in steel reinforcement

v = nominal shear stress = $V/b'd$

v_c = nominal shear stress at inclined cracking = $V_c/b'd$

v_{ci} = shear stress at flexure-shear cracking

v_{cw} = shear stress at web-shear cracking

v_u = nominal ultimate shear stress

v_u' = nominal shear stress carried by web reinforcement at ultimate load = $V_u'/b'd$

V = shear at a section

V_c = shear at inclined cracking

V_d = dead load shear

V_l = live load shear

V_p = vertical component of the effective prestress force at the section considered

V_u = shear at ultimate load

V_u' = shear carried by web reinforcement at ultimate load

x = distance from simple support to section being investigated

ϕ = capacity reduction factor

Received by the Institute Nov. 3, 1967.

PCA R/D Ser. 1384-1

Sinopsis—Résumé—Zusammenfassung

Cambios Propuestos en las Provisiones de Cortante para Vigas de Concreto Reforzado y Presforzado

Se presentan adiciones y cambios notables en las disposiciones de diseño para cortante y tensión diagonal, propuestas para ser incorporadas en el Reglamento de las Construcciones del ACI (1970) revisado. Para servir como antecedente, se revisan las suposiciones básicas de disposiciones para el diseño a cortante del Reglamento de las Construcciones del ACI (318-63). Las propuestas incluyen una disposición mínima para refuerzo en el alma, un procedimiento de diseño revisado para vigas de concreto reforzado bajo tensión y compresión axiales, y un procedimiento simplificado de diseño alterno para vigas de concreto presforzado. Se incluyen varios ejemplos de diseño los cuales usan este procedimiento alterno para vigas de concreto preforzado.

Propositions de Changement dans les Prévisions de Cisaillement pour Poutres en Béton Armé et Précontraint

Les additifs et changements majeurs dans les dessins prévisionnels pour des tensions de cisaillement et diagonales proposés pour incorporation dans le standard de construction de l'ACI (1970) sont présentés. Pour servir de support les prévisions de base au cisaillement du code de construction de l'ACI (ACI 318-63) sont passées en revue. Cette proposition inclus un minimum d'armature par nervurage, une révision du procédé de dessin pour poutres en béton armé sous tension axiale et compression axiale, et un autre procédé de sélection simplifié pour poutres en béton précontraint. Plusieurs dessins illustrant se procédé simplifié pour poutres en beton précontraint sont inclus à cet exposé.

Vorschläge für eine Anderung der Vorschriften zur Bemessung von Stahlbeton- und Spannbeton-Balken gegen Schub

Wesentliche Zusätze und Änderungen für die Bemessung gegen Schub und Hauptzugspannungen, die in einer überarbeiteten Form der ACI Vorschriften 1970 Berücksichtigung finden sollen, werden diskutiert. Als Einführung werden die Grundannahmen, die zu den derseitigen Bemessungsvorschriften gegen Schub in der ACI Vorschrift (ACI 318-63) führten, zusammengefasst. Die Vorschläge schliessen auch einen Minimalwert für die Bügelbewehrung ein; ebenso wird eine revidierte Entwurfsmethode für Stahlbetonbalken unter axialem Zug und axialem Druck und eine vereinfachende Entwurfsmethode für vorgespannte Balken diskutiert. Verschiedene Entwurfsbeispiele werden gegeben, bei welchen diese neuen Vorschläge auf die Bemessung von vorgespannten Balken angewendet werden.