Title No. 58-33

Experimental Study of Lateral Stability of Reinforced Concrete Beams

By JAGADISH K. SANT and RICHARD W. BLETZACKER

The study, which involves both experimental and theoretical phases, provides some basis for the formulation of design provisions for the lateral stability of reinforced concrete beams. Stability criteria, reduced to simplified formulas involving the ratios of L/b and d/b, and based on conservative assumptions, are suggested for three types of loading commonly met in practice. The usefulness of these formulas is limited to the under-reinforced rectangular concrete beams. The experimental study consisted of casting and testing to destruction four groups of identical specimens all having an L/b ratio of 96 and a tensile steel content of approximately 3.85 with d/b ratios varying from 3.78 to 12.45. For the given strength of steel and concrete there exists a critical slenderness ratio, Ld/b^2, beyond which instability is the primary mode of failure reducing the ultimate flexural strength. Experimental results verified the theoretical predictions for the test specimens

■ LATERAL STABILITY is a secondary problem in structural concrete members designed by the working strength method. Because of its secondary nature within the conservative design method, the stability of concrete structures has remained relatively unexplored until recently. Consequently, concrete codes do not include specific provisions for lateral stability. The only restriction to be found in Section 704 of the ACI Building Code (ACI 318-56) against lateral instability does not appear to have a rational or experimental basis. Recent progress in concrete technology, especially the advance of new design methods and techniques, viz., ultimate strength design, limit design, and the prestressing techniques, make it imperative to review and to put on a sound basis the design provisions for lateral stability. The present study is an attempt to provide the necessary basis, theoretical as well as experimental, for the formulation of adequate design provisions for lateral stability.

The instability phenomenon is a unique type of failure, caused by compression in a member. Compressive stresses in a member may be generated either by axial compression, bending compression, shear compression, or any combination of two or more of these. The present

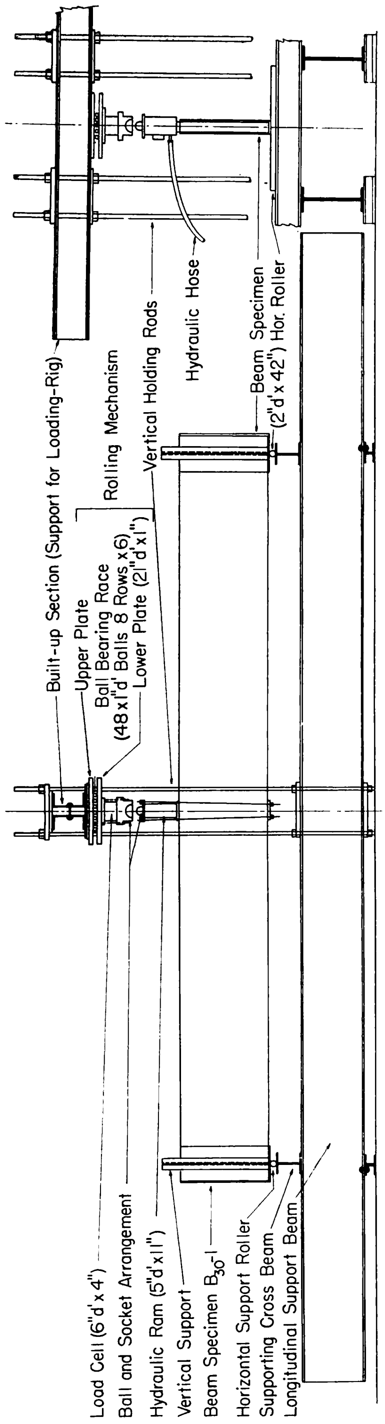

Fig. 1—Apparatus: front view (left); side view (right)

analysis will be limited to the case of instability in a reinforced concrete beam due to bending compression obtained by either transverse loads or end moments. Timoshenko[4] and many others have developed basic buckling formulas for a beam with various load and end conditions from the fundamental equations of stability. These formulas show that a member may become unstable before reaching the ultimate flexural capacity, depending on the dimensional proportions.

In the past few years there have been attempts to solve, experimentally as well as theoretically, the stability problem.[1-3] Marshall,[1] presented a sound theoretical study, but no experimental verification was reported. The study by Vasarhelyi and Turkalp,[2] is an experimental study but demands more research for definite conclusions.

Hansell and Winter[3] reported a study, both experimental and theoretical, of the lateral stability problem suggesting a buckling formula which involves only the L/b ratio. R. B. L. Smith, in discussing the Hansell and Winter paper, reported an experimental study involving tests of twelve concrete "micro-beams" which resulted in the only certain instance of lateral instability found in the literature study.

The purpose of this study was to analyze theoretically the problem of the lateral stability of reinforced concrete beams and verify the analysis with experimental evidence

ACI member **Jagadish K. Sant** is structural designer for DeLeuw, Cather and Brill, consulting engineers, Columbus, Ohio. This paper is a condensation of a research study while Mr. Sant was a graduate research assistant and project supervisor on the Building Research Laboratory staff, Engineering Experiment Station, Ohio State University (1958-1960).

ACI member **Richard W. Bletzacker**, assistant professor of civil engineering, has been supervisor of Building Research, Engineering Experiment Station, Ohio State University, since 1958. From 1957 to 1958 Mr. Bletzacker was a research associate at the University. He has had wide experience in highway bridge design and construction, and also in structural steel design and fabrication. This paper was prepared under his supervision.

of such instability. The objectives included evolving criteria for stability of reinforced concrete beams and to develop information on which a satisfactory design for stability can be based. The scope was limited to a theoretical analysis involving under-reinforced rectangular concrete beams, restrained from rotation about the longitudinal axis, and an experimental phase which involved casting and testing of 11 reinforced concrete beams with L/b ratios of 96, and approximately the same tensile steel content of 3.85 percent but with varying d/b ratios from 3.78 to 12.45.

EXPERIMENTATION

General

The experimental phase of the study consisted of subjecting four groups of identical specimens to a single concentrated load applied at centerspan. The development of the design of the members was directed to producing a range of specimens which would transcend the line demarking primary flexure failure and primary buckling failure. Based on the work of Marshall and others, the general area of demarkation was at least tentatively known. The object of the experimentation was, first of all, to determine if reinforced concrete beams would buckle laterally and, if so, to define more precisely the geometric configuration which contributes to this phenomenon. The design calculations for the reinforced concrete specimens indicated that depths varying from 12 to 36 in. would be required for a 20 ft span length and a 2½ in. width to assure the range of geometrics to provide both flexural and buckling failure modes. In all, 11 reinforced concrete beams were tested. All the test beams had L/b ratios of 96. The four d/b ratios used were 12.45, 10.20, 8.13, and 3.78. The average yield stress for the reinforcing steel was 46,000 psi. The average f_c', ultimate concrete fiber stress in axial compression, was 5860 psi.

Materials

The concrete mix selected was proportioned by weight with one part cement, one part fine aggregate, and 2.08 parts coarse aggregate, and a water-cement ratio of 0.5. The cement used was a Type III high early strength portland cement meeting the requirements of ASTM C 150-56. The coarse aggregate used was a crushed limestone meeting the requirements of the Ohio Department of

Highways Specifications Section M-3.1 and graded to standard size No. 6. The fine aggregate was a river sand available locally. The reinforcing steel used was standard deformed bars of intermediate grade meeting the requirements of ASTM A 305-56T. The bar sizes used were # 4, 8, and 9.

Load and support arrangements

The load frame consisted of two 21 in. longitudinal beams supporting two 8 in. transverse beams spaced 20 ft center to center to provide a bearing for the concrete specimens. The details of the load and support arrangement are shown in Fig. 1, 2, and 3. The concrete beams were provided with special end plates built into each beam so as to facilitate end bearing and rotational restraint. The concrete beams were placed on the end bearing beams in a support rig consisting of a round horizontal bar to permit vertical rotation and between two round vertical bars welded to T-sections cut from a standard I-beam. The vertical bars permitted lateral buckling of the specimen but restrained the ends from rotation. The loads were applied with a hydraulic ram having a capacity of 50 tons. A built-up channel section was tied down at midspan with four round steel bars to each 21 in. longitudinal beam providing a frame against which the loads were applied. Because of the stringent requirement against transverse restraint to the concrete specimen at the point of load application, a specially designed load system was developed. On the top and bottom of each specimen a steel plate was imbedded using a mixture of portland cement, plaster of paris, and water. The hydraulic ram was seated on the top plate and tied to the specimen with four rods from the plate on the bottom of the beam to a plate on top of the ram which was provided with a hole to permit cylinder extension. On the head of the ram was placed a large ball and socket and a calibrated electronic load cell. Above the load cell was placed a rolling mechanism consisting of a ball bearing race composed of 48 one in. diameter hardened steel balls between two 21 in. diameter machined surface hardened steel plates An auxiliary test performed to measure the restraining force offered at the load point showed that the maximum

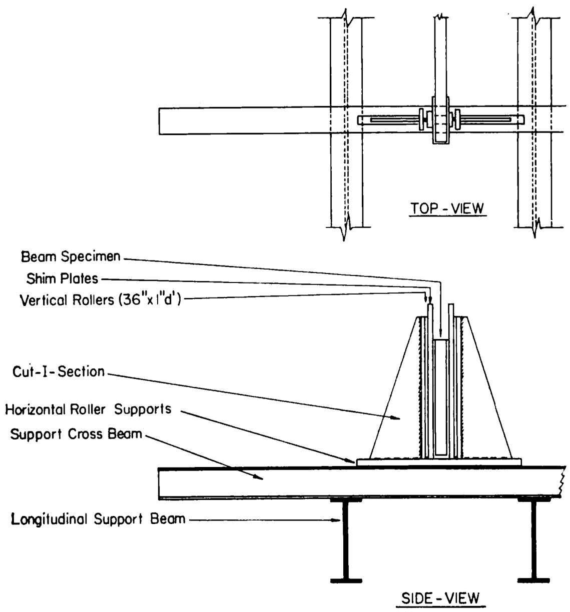

Fig. 2—End support rigging

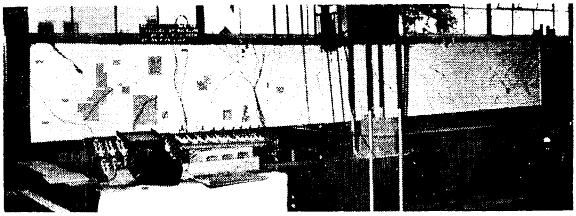

Fig. 3—Test setup

horizontal force required to overcome the rolling friction of the ball bearing was about 0.2 percent of the vertical force.

Specimen instrumentation

SR-4 strain gages were mounted on both the tensile steel and concrete surface. Dial indicators calibrated to the thousandths of an inch were used to measure the lateral and vertical deflections at midspan. The locations of the strain gages and dial indicators are shown in Fig. 4. SR-4 indicators and an automatic two-channel recorder were used to measure strain. The gages were mounted with epoxy cement and waterproofed.

Test procedure

All the specimens were positioned in the testing machine with an electric bridge crane. Accuracy of the beam locations with respect to load frame was determined with a plumb bob, levels, and metal tape. Centering under the load was obtained with a machinist's level. Each specimen was preloaded three times to set the strain gages and eliminate shrinkage strains in the concrete. The specimens were then loaded to failure. The initial increments in the load for the

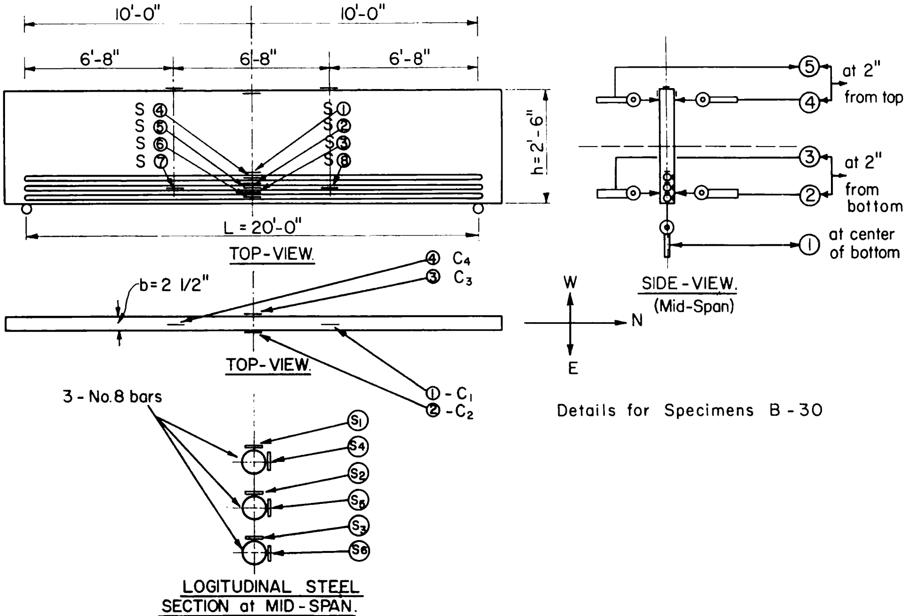

Fig. 4—Typical location of strain gages and dial indicators

TABLE 1—RESULTS OF BEAM TESTS

Group	Beam specimen	d/b ratio	Test moment, in.-kips M_{test}	Predicted moment, in.-kips M_{cr}	Theoretical flexural moment capacity, in.-kips	M_u/M_{test}	Steel strain, microin. per in.	Types of primary failure	
								Test	Predicted
I	B_{36}-1	12.45	1620	1335	3483.75	2.155	970	Instability	Instability
	B_{36}-2	12.45	1845	1335	3483.75	1.890	940	Instability	Instability
	B_{36}-3	12.45	1350	1335	3483.75	2.580	585	Instability	Instability
						Avg = 2.21			
II	B_{30}-1	10.20	2040	1162	2250.8	1.105	1480	Instability	Instability
	B_{30}-2	10.20	2160	1162	2250.8	1.041	1580	Instability	Instability
	B_{30}-3	10.20	1402	1162	2250.8	1.600	890	Instability	Instability
						Avg = 1.25			
III	B_{24}-1	8.13	1260	974	1492.5	1.185	1360	Instability	Instability
	B_{24}-2	8.13	1350	974	1492.5	1.105	1408	Instability	Instability
	B_{24}-3	8.13	1440	974	1492.5	1.037	1560	Instability	Instability
						Avg = 1.11	Avg = 1443		
IV	B_{12}-1	3.78	300		330.0		1545	Flexure	Flexure
	B_{12}-2	3.78	210		330.0		1590	Flexure	Flexure

Note: $L/b = 96$ for all the specimens. Tensile test on a sample steel bar showed an average yield strain of 1600 microin. per in. The average yield strain in the tests of Beams B_{12}-1 and B_{12}-2 was 1568 microin. per in. due to live load. Additional unrecorded strain was developed in steel due to the dead weight of the beam and loading arrangement. This is true of all specimens but is of more significance in the shallower beams.

specimens of the first two groups, B_{36} and B_{30}, were 5000 lb up to a total load of 15,000 lb. The increments for the specimens of the third group, B_{24}, were 2000 lb up to a total load of 14,000 lb and for the specimens of the fourth group, B_{12}, were 500 lb. The increments were then applied at a diminishing rate until failure of the specimen.

Initial readings were taken prior to application of the superimposed loads and a complete set of strain and deflection readings was taken at each load increment.

Experimental results

The test results are given in Table 1. All the test specimens of the first three groups, B_{36}, B_{30}, and B_{24}, failed due to lateral instability. The average values of the ratio M_u/M_{test} for the above mentioned three groups are 2.21, 1.25, and 1.11. Comparison of the values of M_u and M_{test} indicate that the ultimate flexural capacity of the specimens was not realized prior to instability failure, and this is supported by the fact that the tensile strain in the steel, as shown in Table 1, did not reach the initial yield value at failure. The comparison of the ratio of M_u/M_{test} also indicates that the potential reserve of flexural strength decreases as the d/b ratio diminishes. Both specimens of the fourth group, B_{12}, failed primarily in flexure.

Typical crack patterns characteristic of instability and flexural failures are shown in Fig. 5 and 6, respectively. Typical load-strain curves of the tensile steel strain in specimens which failed due to instability and due to flexure are shown in Fig. 7a and 7b. Typical load-strain curves of the compressive concrete strain in specimens which failed due to instability and due to flexure are shown in Fig. 8.

Excessive lateral deflection followed by the diagonal tension cracks on the convex side with little or no diagonal tension cracking on the

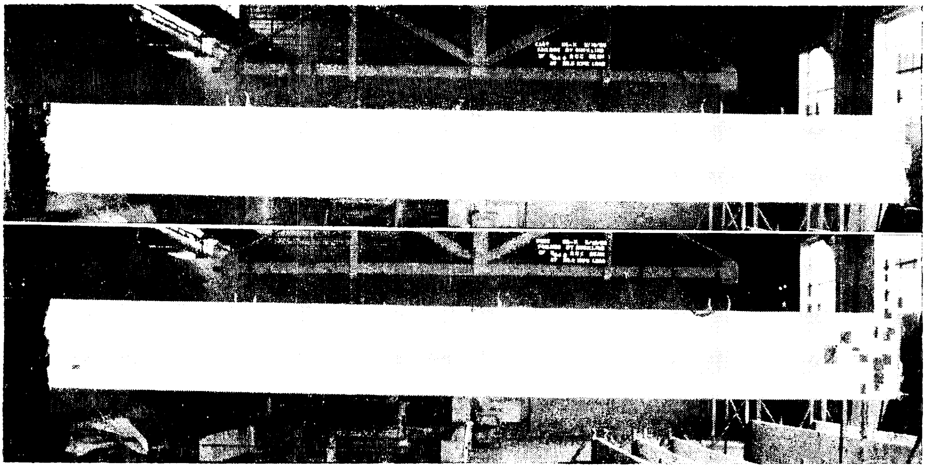

Fig. 5—Characteristic crack pattern attending lateral instability failures; concave side (top); convex side (bottom)

concave side was characteristic of the instability failure. Moreover, strain reversals in the concrete top convex fiber and tensile steel were noticed. Lateral movement and rotation which attends beams which fail in buckling is illustrated in Fig. 9.

Excessive vertical deflection and steel strain, followed by the opening of tensile cracks at midspan, were characteristic of the flexural failure. No reversal was noticed in either the steel strain or the concrete strain in the top convex fiber at the time of flexural failure. Lateral deflections were recorded from the first increment of load and these deflections continued to increase until the specimen reached failure regardless of the mode as shown in Fig. 10 and 11.

ELASTIC BUCKLING ANALYSES

Analyses of the lateral buckling phenomenon are based on the fundamental conditions of stability. As indicated previously, Timoshenko and others have presented the basic equations for various loading and end conditions in homogeneous elastic materials. The analysis presented here as well as those recently presented by Hensel and Winter[3] and by

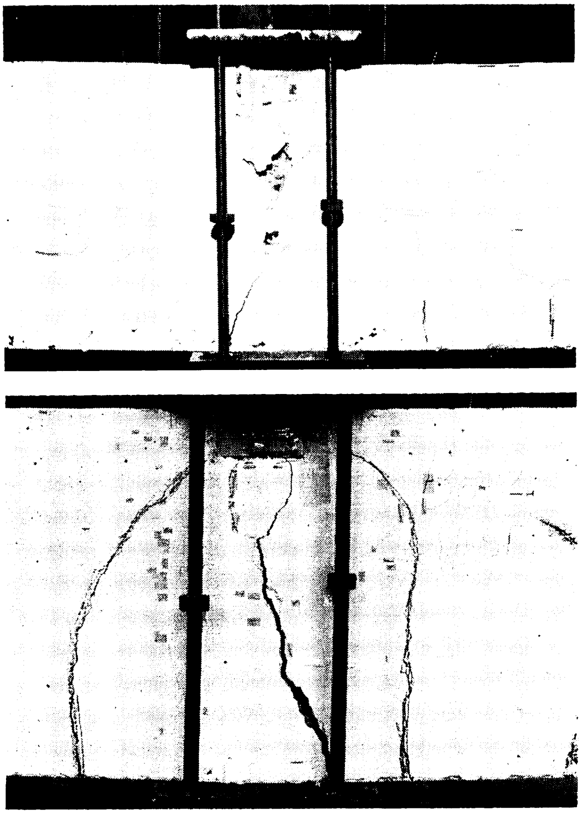

Fig. 6 — Typical flexural cracks at midspan attended by concrete crushing in compressive area of Specimen B_{12}-2; convex side (top); concave side (bottom)

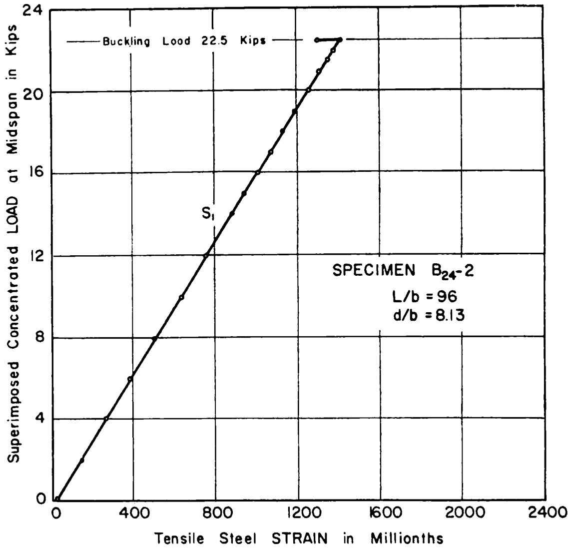

Fig. 7a — Typical tensile steel load-strain curve for lateral instability failures

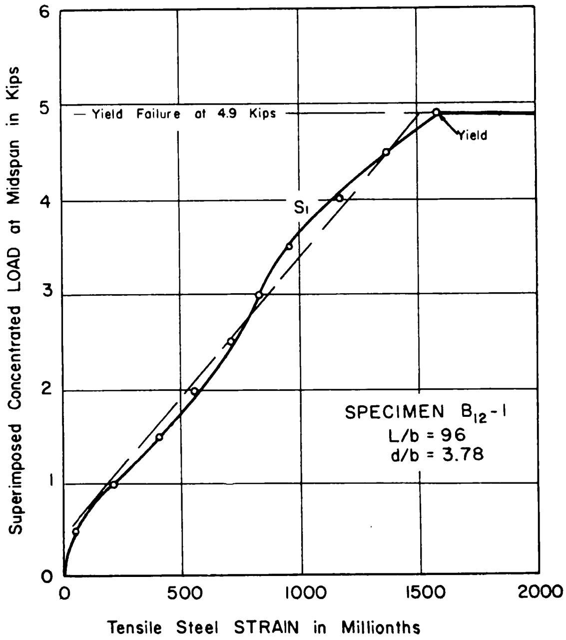

Fig. 7b — Typical tensile steel load-strain curve for flexural failures

Marshall[1] are extensions and modifications of these basic equations. Hensel and Winters proceeded from the equation for pure bending and applied modified elastic constants based on the tangent modulus, discounting the concrete in the tensile area of the beam. Marshall proceeded from three load and end condition equations and made modifications based on a linear stress-strain relationship.

The present analysis uses the reduced modulus theory for column buckling[11] to modify the basic equations for stability.

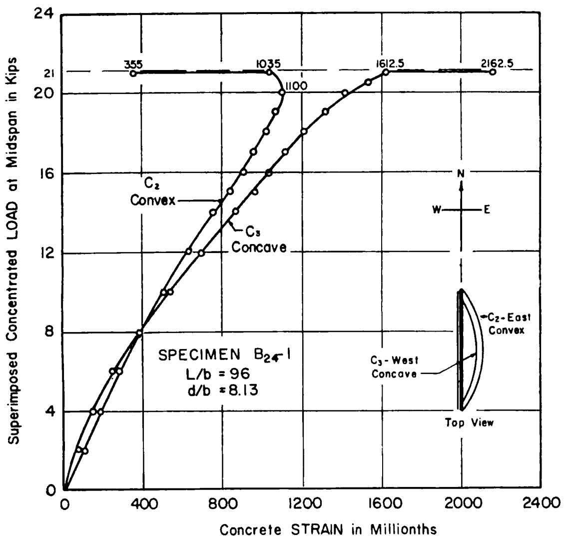

Fig. 8a — Typical compressive concrete load-strain curves for lateral instability failure

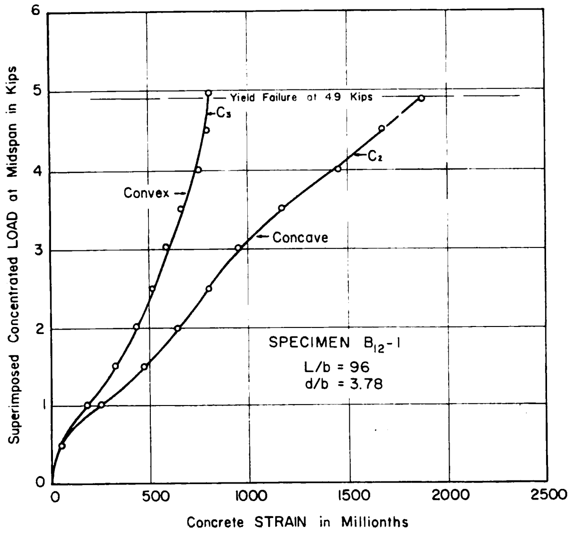

Fig. 8b — Typical compressive concrete load-strain curves for flexural failures

Notation

a = depth of the compressive stress block
a' = distance of the point of application of load from the centroid of the section
b = width of the concrete beam
B_2 = EI_y = flexural rigidity about vertical plane
C = GI_t = torsional rigidity of a section
d = effective depth of the concrete beam
e = strain in any fiber
e_o = strain in the extreme fiber
E = modulus of elasticity
E_c = instantaneous concrete modulus
E_r = reduced modulus
E_{sec} = secant modulus
E_{tan} = tangent modulus
f_c' = ultimate fiber stress
f_{sy} = yield stress in tensile steel
G = shear modulus
I_y = $db^3/12$
I_t = $db^3/3$
j = lever arm factor
K_1 = load factor
K_2 = modulus factor
L = unsupported span length
M_{cr} = theoretical critical buckling moment
M_{test} = test failure moment
M_u = ultimate flexural moment
p = A_s/bd = percentage of tensile steel content
f = stress at any fiber
f_o = stress in the extreme fiber
ν = Poisson's ratio

The critical buckling moment for a simply supported, concentrically loaded, rectangular, homogenous beam is given by

$$M_{cr} = 4.234 \leq \frac{(B_2C)^{1/2}}{L}\left[1 - \frac{3.48\,a'}{L}\left(\frac{B_2}{C}\right)^{1/2}\right] \qquad (1)$$

This formula cannot be used directly to find the buckling moment of reinforced concrete beams because concrete is a nonhomogeneous material that is not perfectly elastic. The following assumptions must be made to provide applicability to reinforced concrete beams:

1. An under-reinforced concrete beam is assumed "elastic" until the tensile steel reaches the initial yield strain. Specifically:

   (a) The range of linear elasticity is assumed to extend to approximately 0.5 f_c', and

   (b) The beam is assumed to be nonlinearly elastic to 0.85 f_c' and has a reversible stress-strain curve for the loading and unloading process.

2. At a given stress level, concrete in flexure strains more than concrete in axial compression.

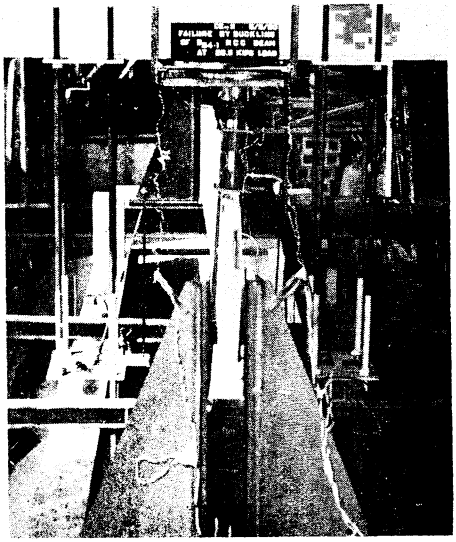

Fig. 9—Typical lateral movement and torsional rotation of laterally unstable specimen

3. Shear modulus for concrete is taken as $G = E/2(1+\nu)$ where $\nu = 0.25$; therefore, G is $0.4E$.

4. The concrete area below the centroid of tensile steel is neglected for the elastic buckling analysis and the cross-sectional area resisting torsional and lateral bending is taken as the product of bd.

Using these assumptions, Eq. (1) becomes

$$M_{cr} = 4.234 \frac{\left[\left(\frac{db^3}{12}\right)\left(\frac{db^3}{3}\right)(0.4)(E_r^2)\right]^{1/2}}{L} \left\{ 1 - \frac{3.48\,a'}{L} \left(\frac{E_r \frac{db^3}{12}}{0.4\,E_r \frac{db^3}{3}} \right)^{1/2} \right\} \quad \ldots\ldots (2)$$

Combining terms, Eq. (2) becomes

$$M_{cr} = (0.447)(E_r)\left(\frac{db^3}{L}\right)\left(1 - \frac{2.75\,a'}{L}\right) \quad \ldots\ldots (3)$$

where E_r is the reduced modulus.

The evaluation of the critical buckling moment is dependent primarily on a proper determination of E_r. A rigorous derivation of E_r for a column is given in Reference 11. In a beam, however, each element of the various cross sections has different stresses; therefore, E_{tan} will vary throughout the depth and length of the beam. Consequently, a rational solution of E_r for a beam is so complicated as to be impractical for direct application to this problem.[7] It is possible however, to establish the lower limit of the critical buckling load by assuming that E_r, calculated for the maximum stressed fiber in a beam, is valid anywhere in the beam. Applying the column analogy for uniform compression to a

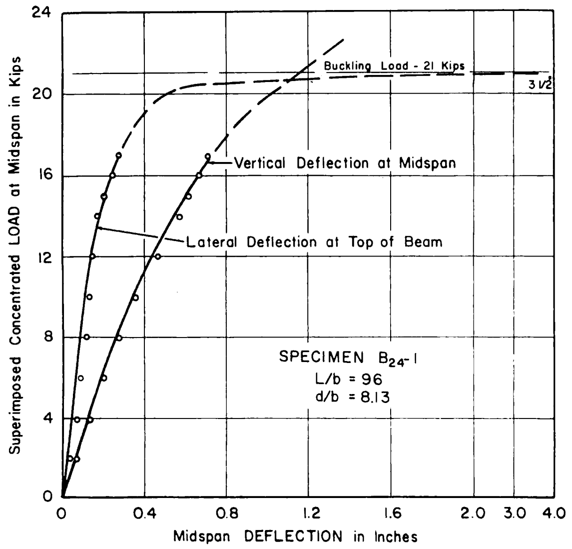

Fig. 10 — Typical load-deflection relation of vertical and lateral deflection in laterally unstable section

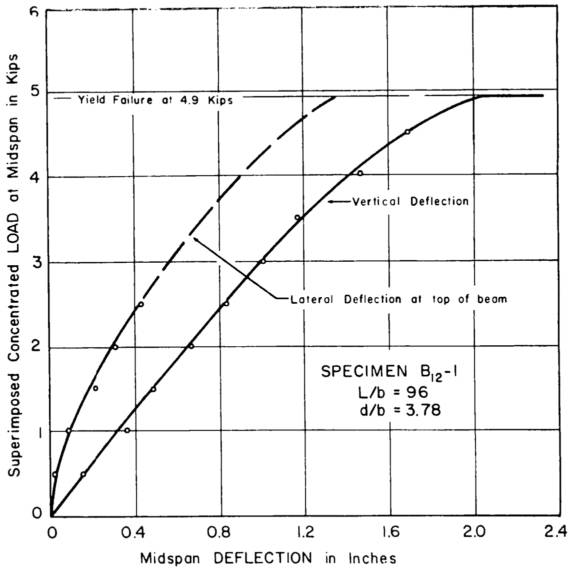

Fig. 11 — Typical load-deflection curves of vertical and lateral deflection in flexural failure specimens

rectangular beam with the assumed nonlinear stress-strain properties the reduced modulus has a value[12]

$$E_r = \frac{4\,E_c\,E_{tan}}{(\sqrt{E_c} + \sqrt{E_{tan}})^2} \qquad (4)$$

The actual evaluation of E_r for a given set of conditions is a difficult matter unless complete experimental data is available on the physical properties of the concrete and the composite steel and concrete member. Invoking the two stipulations given in Assumption 1, E_r would equal $E_c = E_{tan} = E_{sec}$ under Condition (a), but under Condition (b) knowledge of the equation of the nonlinear stress-strain curve is required. To overcome these difficulties the following simplifying assumption is made to approximate the form of the stress-strain curve in the region near $0.85f_c'$.

For elastic buckling

$$E_{tan} = \tfrac{1}{2}\,E_c$$

and substituting in Eq. (4)

$$E_r = 0.687\,E_c$$

Many equations have been suggested to express E_c, the instantaneous modulus for concrete based on various experimental results. It is proposed here to use Lyse's[10] equation[6] which is

$$E_c = 1{,}800{,}000 + 460\,f_c' \qquad (5)$$

With slight modification for the flexural computations,

$$E_c = 1{,}800{,}000 + 460(0.85 f_c') \qquad (6)$$

where f_c' is the ultimate fiber stress for concrete under uniform compression. Then the simplified equation for critical buckling moment for a given strength of concrete is

$$M_{cr} = 0.447(0.687\,E_c)\left(\frac{db^3}{L}\right)\left(1 - \frac{2.72\,a'}{L}\right) \qquad (7)$$

Calculation of M_{cr} for the test specimens

Eq. (7) is applied to the simply supported, rectangular, under-reinforced concentrically loaded experimental beams. The average of f_c' for these beams was 5860 psi and, using Eq. (6), E_c was 4.1×10^6 psi.

Then Eq. (7) becomes

$$M_{cr} = 0.447(0.687)(4.1 \times 10^6)\,\frac{db^3}{L}\,(1 - 11.52 \times 10^{-3}\,a')$$

Rearranging terms and dividing both sides by b^3j,

$$\frac{M_{cr}}{b^3j} = 1{,}310{,}000\,\frac{d/b}{L/b}\,(1 - 11.52 \times 10^{-3}\,a') \qquad (8)$$

Eq. (8) is the general theoretical elastic buckling formula for the test specimens.

The theoretical flexural capacity for the test specimens was calculated from the formula

$$\frac{M_u}{b^3j} = p\,f_{sy}(d/b)^2 \qquad (9)$$

Eq. (8) and (9) are plotted as shown in Fig. 12. The intersection point of the two curves marks the critical d/b ratio beyond which elastic buckling failures in beams will occur for the given tensile steel strength and L/b ratio. The experimental results verify the theoretical predictions that Specimens B_{36}, B_{30}, and B_{24} should fail in buckling whereas Specimen B_{12} should fail in flexure. Quantitative agreement between the experimental and theoretical results was not apparent, indicating that the theoretical elastic buckling curve is based on conservative assumptions. The lateral torsional rigidity offered by the tensile and web reinforcement, offsetting the counter effect of eccentricities, may have caused the experimental buckling values to be higher than the predicted theoretical values. The predicted load carrying capacity of the test beams is indicated by the solid segments of the two curves shown in Fig. 12.

All the specimens of Group B_{36} failed in lateral torsional bending at values of load higher than the theoretical predicted buckling loads but well below the predicted flexure failure loads. The tensile steel strains at failure were well below the initial yield strain. The typical tensile crack which attends flexural failure was not present at the failure of these specimens. All the specimens of Group B_{30} likewise failed in lateral torsional bending. Although the failure loads for Specimens B_{30}-1 and B_{30}-2 were quite near to the predicted flexural failure loads, the failure was attended by lateral buckling and not by opening of tensile crack. In Specimen B_{30}—2, which failed at 36 kips, one of the three tensile rods had just entered into the inelastic strain range, but the average tensile steel strain at the time of failure was below the initial yield strain. None of the tensile cracks had opened at the time of failure. The specimens of Group B_{24} also failed in lateral torsional bending.

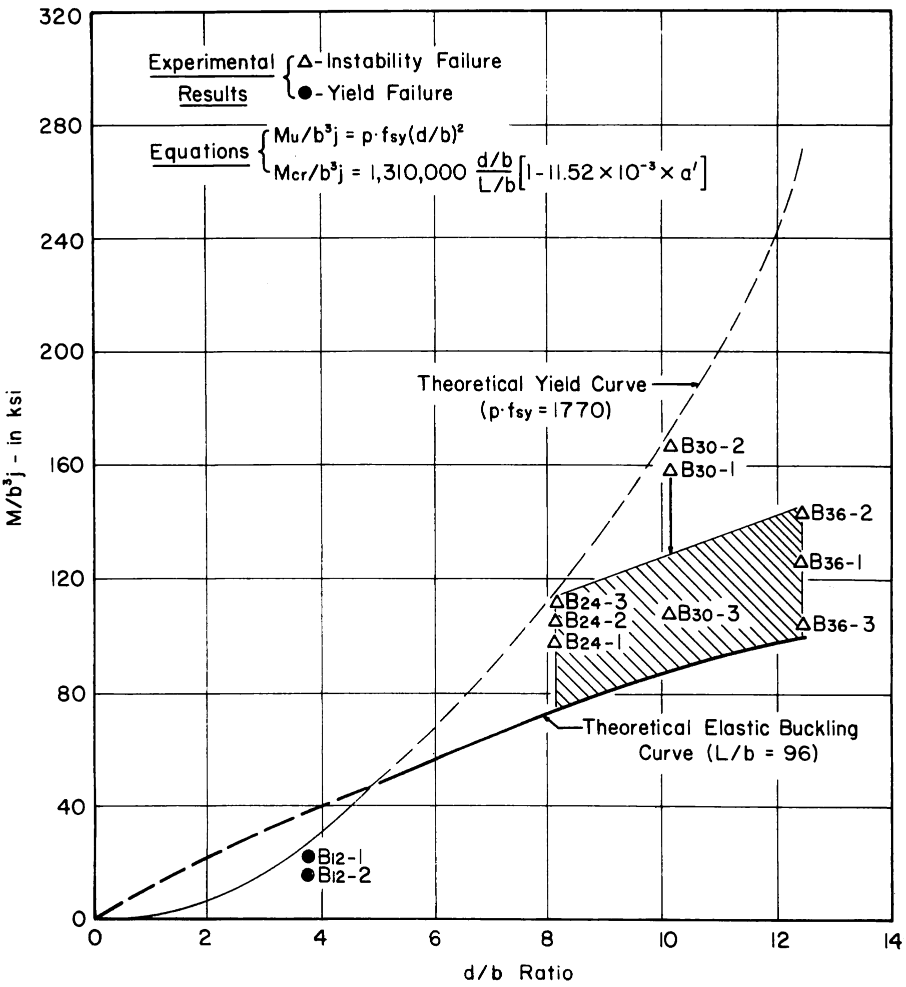

Fig. 12—Elastic flexure-instability relationship for experimental specimens

Test loads for the specimens of this group were in close agreement with each other. The average tensile steel strains for these specimens at the time of failure were below the initial yield strain. None of the tensile cracks were opened at the time of failure.

Both specimens in the fourth group, B_{12}, failed primarily in flexure due to yield of the tensile steel as noted on the strain indicator connected to the gage on the tensile steel. Secondary and tertiary failures immediately followed the primary failure without any increase in load. The secondary failure was lateral instability. As a consequence of the yield and instability failures, the concrete fiber stress at the top concave side of the beam exceeded the ultimate fiber stress f_c', and the tertiary failure resulted by crushing of the concrete. The theoretical prediction that the specimens of the fourth group, B_{12}, would fail primarily in flexure was therefore substantiated by the experimental results. The discrepancy in the failure loads of Specimens B_{12}-1 and B_{12}-2, which might be mistaken for the effect of lateral deflection on the flexural strength, is explained as follows:

> The dead weight of the loading arrangement acting on top of the specimen at the midspan was 0.25 kips. If this additional weight is taken into consideration then the test failure load for Specimen B_{12}-1 is within 5 percent of the predicted failure load. Specimen B_{12}-2 was loaded twice. First, the load was applied with lateral side supports provided at midspan, preventing any laterial deflection during the load application. The yield strain was reached, as recorded by the strain indicators, at the 3.5 kips level. When yield was noticed, load was quickly removed. The second load application was made with the lateral side supports removed. The specimen was then loaded to failure. On the second application lateral deflection was observed as the load increased. The yield of the tensile steel started at 3.5 kips which indicated that with or without the lateral side supports at the midspan, the yield load remained the same for Specimen B_{12}-2. Moreover, this behavior established the fact that small lateral deflections occurring in a specimen do not impair its flexural strength. The only noticeable effect of these small lateral deflections was the introduction of additional bending.

APPLICATION TO THE DESIGN OF REINFORCED CONCRETE BEAMS

A myriad of factors enter into the design of reinforced concrete structures and it is rare indeed, in present practice, when these factors would produce flexural members of such proportions as to be subject to the limitations of lateral instability. Nevertheless, the phenomenon exists and with the advanced design techniques presently proposed and in the offing it is essential that the subject be treated in codes and specifications. In general, it may suffice that restrictions be stated such that flexural members proportioned by conventional design considerations may be compared to limitations on the allowable stress or the allowable moment or load. Proportions which transcend these limits must either be revised or an analysis performed to determine the buckling tendency.

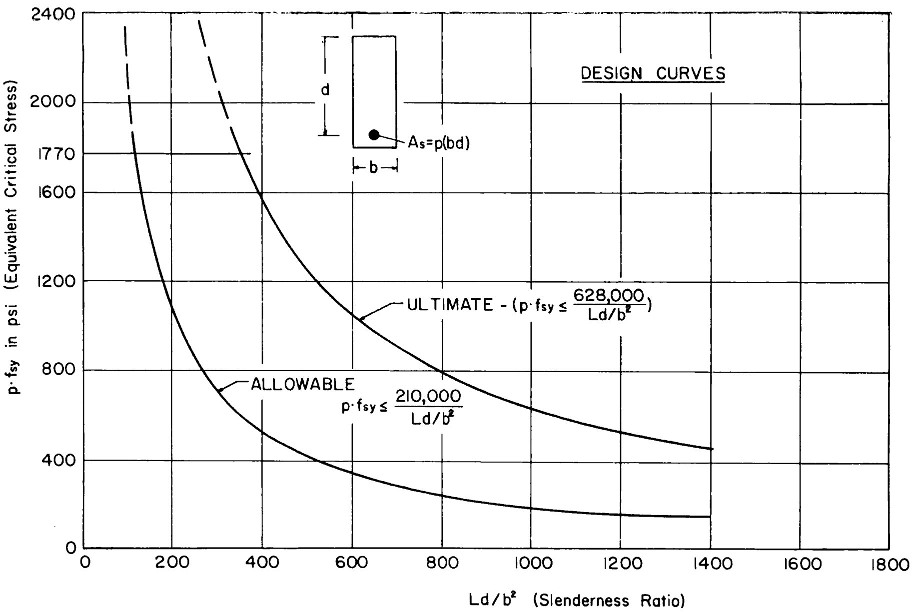

Fig. 13—Relationship of critical stress versus slenderness ratio at ultimate failure and working load level for under-reinforced rectangular concrete beams under pure bending

In the case of an under-reinforced rectangular beam restrained at the ends against rotation about the longitudinal axis, a general expression for such limitation may be developed.

Taking an under-reinforced rectangular beam simply supported and concentrically loaded (at centroid of the midspan section) from Eq. (3)

$$M_{cr} = (0.447)\,(E_r)\left(\frac{d/b}{L/b}\right)\,(b^3)$$

and dividing by $b^3 j$

$$\frac{M_{cr}}{b^3 j} = (0.447)\left(\frac{E_r}{j}\right)\left(\frac{d/b}{L/b}\right) = K_1\,\frac{(0.1052)\,(E_r)}{j}\left(\frac{d/b}{L/b}\right)$$

where K_1 is a load factor, equal to 4.234 for a concentric load at the centroid of the midspan section.

Using $K_2 = (0.1052)\ (E_r)/j$, the buckling formula applicable to any type of loading and end condition is

$$\frac{M_{cr}}{b^3 j} = (K_1)\,(K_2)\left(\frac{d/b}{L/b}\right) \qquad (10)$$

To provide the maximum limit of geometric proportions wherein the primary ultimate failure would be flexure, the condition $M_u \leqq M_{cr}$ must be satisfied.

Then using Eq. (9) and (10)

$$p\, f_{sy} (d/b)^2 \leq K_1\, K_2 \left(\frac{d/b}{L/b} \right)$$

therefore

$$p\, f_{sy} \leq \frac{K_1\, K_2}{Ld/b^2} \qquad (11)$$

This is the general form of the stability criteria proposed here for rectangular under-reinforced concrete beams.

The left-hand side of Eq. (11) represents equivalent flexural stress in psi, while the right-hand side of the equation represents equivalent critical stress. So long as the left-hand side is equal to or less than the right-hand side the beam is laterally stable up to the flexural yield moment capacity.

To be of practical applicability the general equation is considered for three types of loading commonly met in practice. These are (1) pure bending, (2) uniformly distributed load, and (3) concentric load at midspan. The load factor K_1 will have values 3.142, 3.538, and 4.234, respectively, for these loading conditions.[4] The factor K_2 may be assumed constant, conservatively at the lowest value for a given strength of concrete, by using $E_{tan} = 1/4E_c$ and substituting into Eq. (4), $E_r = 0.445E_c$. Then for the case of the test beams where $f_c' = 5860$ psi, $E_c = 4.1 \times 10^6$ psi, and taking $j = 0.96$ to keep K_2 to a minimum

$$K_2 = \frac{(0.1052)\,(0.445)\,(4.1)\,(10^6)}{0.96} \qquad (12)$$

$$K_2 = 2 \times 10^5 \text{ psi}$$

With this value of K_2, Eq. (11) becomes for

Pure bending

$$p\, f_{sy} = \frac{628{,}000}{Ld/b^2} \qquad (13)$$

Uniform load along the center line

$$p\, f_{sy} = \frac{708{,}000}{Ld/b^2} \qquad (14)$$

Concentric load at centroid of the midspan section

$$p\, f_{sy} = \frac{847{,}000}{Ld/b^2} \qquad (15)$$

In the case of a single concentrated load the factor K_1 increases as the load position shifts either from the midspan location toward the supports or from the centroid to the bottom of the section. If the point of load application moves above the centroid, the factor K_1 decreases. In the case of uniform loading, the factor K_1 decreases as the load line rises above the center line and vice versa. In the case of pure bending, K_1 is a constant. If any restraint is encountered on the compression side of the beam, within the span length, then the factor K_1 will have values higher than the values given for the respective types of loading. In practice some degree of restraint is nearly always provided, either intentionally or incidentally, and the values of K_1 given herein will provide conservative estimates of the equivalent flexural stress. T-beam

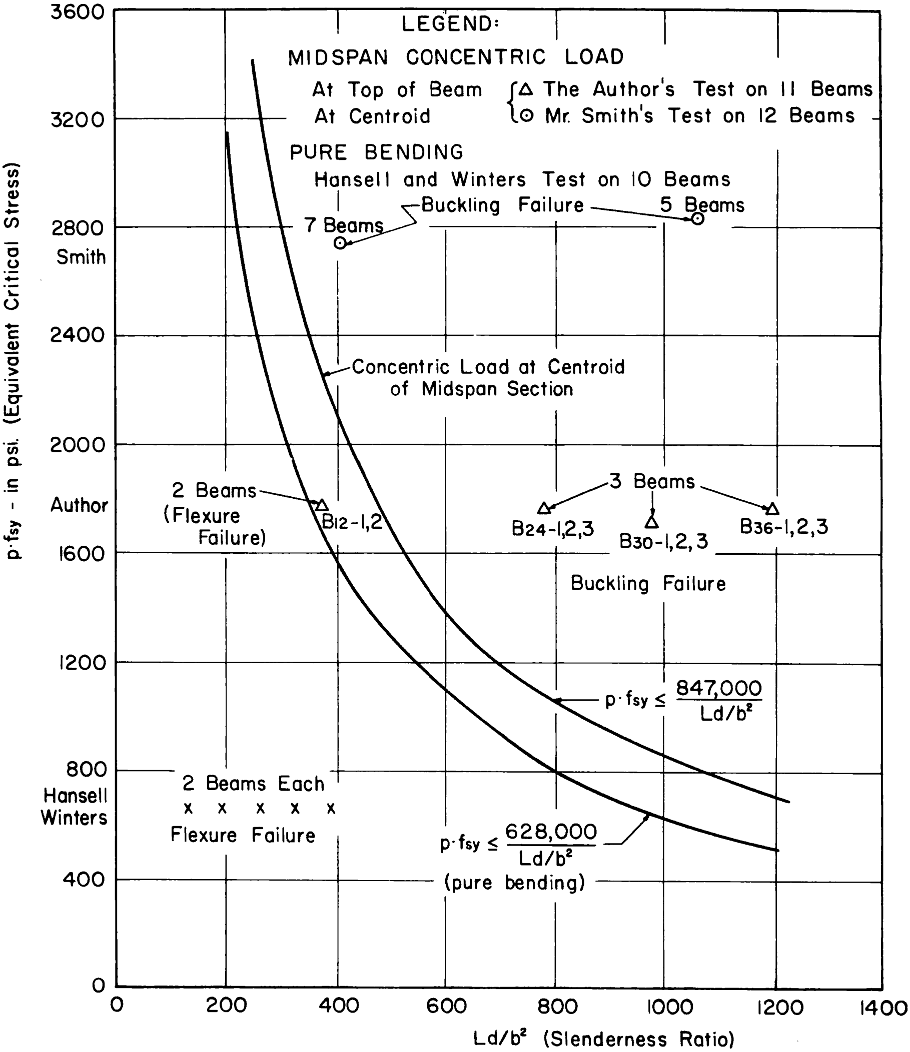

Fig. 14—Critical stress versus slenderness ratio for test specimens with comparative results by other recent investigators

construction and monolithic joists and slabs subjected to positive moments are extreme examples of such restraint and only those areas subjected to negative moments need be studied for stability considerations.

Eq. (13), (14), and (15) were developed on the ultimate strength basis, using the properties of the concrete involved in the test specimens. For practical application it is necessary to reduce these equations to provide a margin of safety and especially to allow for initial bow and warpage of the beam, small eccentricities of the applied load, nonhomogeneous nature and inelastic behavior of the composite concrete and steel and cracking tendency of the concrete on the tension side.

A factor of approximately three was chosen to reduce these equations to a level considered safe.

The reduced equations are for

Pure bending

$$p f_{sy} = \frac{210{,}000}{Ld/b^2} \qquad (16)$$

Uniform load along center line

$$p f_{sy} = \frac{236{,}000}{Ld/b^2} \qquad (17)$$

Concentric load at centroid of midspan section

$$p f_{sy} = \frac{282{,}000}{Ld/b^2} \qquad (18)$$

For the case of pure bending in a beam, design curves based on the ultimate and allowable basis are drawn as shown in Fig. 13. From these curves the critical slenderness ratios can be obtained for the different values of the product pf_{sy}.

Fig. 14 shows the theoretical curves for the pure bending and concentric load situation. Test data of specimens used by various investigators are also plotted for comparison.

SUMMARY

Eleven under-reinforced, simply supported, concentrically loaded, rectangular concrete beams were subjected to destructive tests. The load was applied at midspan on top of the specimens, through a hydraulic ram. All the test specimens had an L/b ratio of 96, a 20 ft span length, and a 2½-in. width. Four d/b ratios, 12.45, 10.20, 8.13, and 3.78 were used. The tensile steel percentage used was approximately 3.85. The average yield stress for the reinforcing steel was 46,000 psi. The average f_c' was 5860 psi. The loading arrangement was such that there would be no appreciable horizontal restraint at the load point between the supports. The test specimens were designed by the ultimate strength method. Buckling formulas were derived from the fundamental equations of stability. Necessary modifications were made to these formulas to provide application to rectangular under-reinforced concrete beams. Nine test specimens comprising three d/b ratios, 12.45, 10.20, and 8.13, failed as predicted, i.e., in lateral instability. Both specimens having a d/b ratio of 3.78 failed as predicted, i.e., in primary flexure. The experimental results verified, qualitatively and, to a major extent, quantitatively, the theoretical results. The problem of lateral stability does exist in slender concrete beams. The flexural capacity of such beams is reduced due to such instability. Based on the buckling analysis presented, a general criterion for lateral stability incorporating the ratios, L/b and d/b, has been suggested, which is of the form

$$p f_{sy} = \frac{K_1 K_2}{Ld/b^2} \qquad (19)$$

Design formulas for three types of loading commonly met in practice are suggested.

CONCLUSIONS

1. The specimens which failed in lateral instability had failure moments higher than the theoretically predicted values, indicating that the buckling analysis used was based on conservative assumptions.

2. The small lateral deflections which occur prior to instability or flexural failure do not reduce the actual flexural capacity of beams.

3. The yield strength was also the buckling strength for the shallow beams. This raises a serious question as to whether slender reinforced concrete beams with high strength steel under bending compression can ever reach a stress above the tensile yield strength of steel without buckling. Additional research study is needed to investigate this problem.

4. The experimental results obtained by previous investigators compare well with the findings of this study.

5. The L/b limitations specified in Section 704 of the ACI Building Code (ACI 318-56) are extremely conservative for ordinary strength steel and relatively small d/b ratios.

6. Both the ratios d/b and L/b are concurrent criteria for lateral stability of rectangular beams. Stability provisions based on the L/b ratio alone is insufficient to completely define the required physical dimensions.

ACKNOWLEDGMENTS

This paper represents a condensation of a research study done at Ohio State University, Engineering Experiment Station by Mr. J. K. Sant, Research Assistant, under the supervision of Mr. R. W. Bletzacker, Supervisor of Building Research. The authors are indebted to G. E. Large for suggesting the subject and providing useful advice throughout the work; Robert F. Baker, for administrative assistance in obtaining financial support of the project; John G. Birle, for supervision and assistance during the experimental phase of the study, and to various members of the Building Research staff for their ready assistance in execution of the project.

REFERENCES

1. Marshall, W. T., "The Lateral Stability of Reinforced Concrete Beams," *Journal, Institution of Civil Engineers* (London), V. 30, No. 6, Apr. 1948, pp. 194-196.

2. Vasarhelyi, D., and Turkalp, I., "Lateral Buckling of Slender Reinforced Concrete Beams," *The Trends in Engineering at the University of Washington* (Seattle), V. 6, No. 3, July, 1954, pp. 8-10.

3. Hansell, William, and Winter, George, "Lateral Stability of Reinforced Concrete Beams," ACI JOURNAL, *Proceedings* V. 56, No. 3, Sept. 1959, pp. 193-213.

4. Timoshenko, S., *Theory of Elastic Stability*, McGraw-Hill Book Co., Inc., New York, 1936, pp. 239-279.

5. Winter, George, "Strength of Slender Beams," *Transactions*, ASCE, V. 109, Paper No. 2232, 1944, p. 1321.

6. Hognestad, E., "Study of Combined Bending and Axial Load in Reinforced Concrete Members, *Bulletin*, V. 49, No. 22, University of Illinois, Nov. 1951.

7. Bleich, F., *Buckling Strength of Metal Structures*, McGraw-Hill Book Co., Inc., New York, 1952.

8. Sant, J., "Experimental Investigation of the Lateral Stability of Thin Deep Reinforced Concrete Beams," MS Thesis, Ohio State University, Columbus, Ohio, Dec. 1959.

9. Large, G., *Basic Reinforced Concrete Design*, Ronald Press Co., New York, 2nd Edition, 1957, pp. 357-381.

10. Lyse, Inge, and Godfrey, M., "Investigation of Web Buckling in Steel Beams," *Transactions*, ASCE, V. 100, Paper No. 1907, 1935, p. 675.

11. Timoshenko, S., *Strength of Materials, Part II*, D. Van Nostrand Co., Inc., 3rd Edition, 1956.

12. Wolford, D., "Significance of the Secant and Tangent Moduli of Elastisity in Structural Design," *Journal of the Aeronautical Sciences*, V. 10, No. 6,June, 1943, p. 169.

Received by the Institute Feb. 13, 1961. Title No. 58-33 is a part of copyrighted Journal of the American Concrete Institute, Proceedings V. 58, No. 6, Dec. 1961. Separate prints are available at 60 cents each.

American Concrete Institute, P. O. Box 4754, Redford Station, Detroit 19, Mich.

Discussion of this paper should reach ACI headquarters in triplicate by Mar. 1, 1962, for publication in Part 2, June 1962 JOURNAL.

Estudio Experimental de la Estabilidad Lateral de Vigas de Hormigón Armado

Este estudio que incluye fases experimentales y teóricas, provee de alguna base para formular especificationes para la estabilidad lateral de vigas de hormigón armado. Se sugiere para tres clases de carga comunes en la práctica, una norma de estabilidad reducida a fórmulas simplificadas involucrando las razones L/b y d/b basadas en suposiciones conservadoras. La utilidad de estas fórmulas se limita a vigas rectangulares de hormigón armado y reforzadas por la parte inferior. El estudio experimental consistió en fundir y probar hasta la destrucción cuatro grupos de muestras idénticas teniendo todas una razón L/b de 96 y un porcentaje de acero de tensión de aproximadamente 3.85 con razones d/b variables de 3.78 hasta 12.45.Para una resistencia dada de acero y concreto existe una tolerancia crítica de esbeltez Ld/b^2 más allá de la cual, la inestabilidad es el modo primario de falla, reduciendo la resistencia última de flexión. Los resultados de los experimentos comprobaron las predicciones teóricas para las muestras probadas.

Etude Expérimentale sur la Stabilité Latérale de Poutres en Béton Armé

Une étude, comprenant les phases tant expérimentales que théoriques, fournit une base pour la formulation de prévisions de calcul assurant la stabilité latérale de poutres en béton armé. Quelques critères de la stabilité, ramenés à des formules simplifiées contenant les rapports L/b et d/b, et basées sur des suppositions prudentes, sont présentées pour trois types de charge qu'on rencontre habituellement dans la pratique. L'utilité de ces formules se limite au cas de la poutre rectangulaire non suffisament armé. L'étude expérimentale comprenait la coulée et l'essai à la rupture de quatre séries de spécimens identiques, tous ayant un rapport L/b de 96 et une teneur en acier extensible de 3.85 environ, leurs rapports d/b variant de 3.78 jusqu'à 12.45. Pour une résistance donnée de l'acier et du béton il y a un rapport critique de tenuité, Ld/b^2, au delà duquel l'instabilité est la mode primaire d'effondrement, réduisant la résistance limite au plissement. Les résultats expérimentaux vérifient les prédictions théoriques pour les échantillons.

Experimentelle Untersuchung der Kippsicherheit von Stahlbetanträgern

Sowohl versuchsmässige als auch theoretische Untersuchungen einschliesst, gibt in etwa eine Grundlage für die Aufstellung von Entwurfsunterlagen für die vorliegende Arbeit, die Kippsicherheit von Stahlbetonträgern. Fur 3 allgemein in der Praxis vorkommended Belastungsarten wird ein Stabilitatskriterium vorgeschlagen, das zu vereinfachten Formeln, die die Verhältnisse L/b und d/b einschliessen, reduziert ist und auf konservativen Voraussetzungen beruht. Die Anwendbarkeit dieser Formeln ist auf rechteckige Stahlbetonträger mit Bewehrung Armierung santeil beschränkt. Das bestand daraus, dass 4 Gruppen identischer Versuchsbalken, alle mit einem L/b-Verhältnis von 96 und einem Zugbewehrungs von 3.85 und d/b-Verhaltnisse, die zwischen 3.78 und 12.45 variieren,

betoniert und bis zum Bruch geprueft wurden. Fur eine gegebene Festigkeit von Stahl und Beton besteht ein kritischer Schlankheitsgrad Ld/b^2, uber den hinaus Instabilität die hauptsächlichste Bruchursache ist, wodurch die Biegebruchfestigkeit reduziert wird. Die experimentellen Ergebnisse bestätigten die theoretischen Ergebnisse fur die Versuchsbalken.