TITLE NO. 75-24

Proposed Revisions By Committee 435 to ACI Building Code and Commentary Provisions on Deflections*

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Presents a proposal for the control of deflections in which both minimum thickness and computed versus allowable deflection approaches are included. The format is that of a code, a commentary, and an example. The principal effects taken into account are those of cracking, continuity, compression steel in nonprestressed members, creep and shrinkage, the use of different weight concrete, and the degree of vulnerability of nonstructural elements.

Keywords: beams (supports); camber; concrete slabs; cracking (fracturing); creep properties; **deflection**; lightweight concrete; modulus of elasticity; moments of inertia; **prestressed concrete**; prestressing steels; **reinforced concrete**; reinforcing steels; **structural analysis**; **thickness.**

*This paper is based on a report by ACI Committee 435 dated November 1975.

†Chairman of ACI Committee 435 from 1973 to 1977 during which time the report was developed.

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CODE

9.5—Control of deflections

9.5.1 *General* — Reinforced concrete members subject to bending shall be designed to have adequate stiffness to limit deflections or any deformation which may adversely affect the strength or serviceability of the structure at service loads (same as ACI 318-71).

In long span construction and in the case of large long-time deflections due to sustained loads, special consideration shall be given to providing sliding joints, flexible connections, or other measures to prevent damage.

The approximate procedures for determining minimum thickness or computing representative deflections under service loads specified in Section 9.5 are deemed adequate in most cases for the conditions specified therein. A more comprehensive analysis may be made in lieu of using the provision of Section 9.5.

9.5.2 *Nonprestressed one-way construction*

9.5.2.1 *Minimum thickness.* Although the general requirements of Section 9.5.1 must be satisfied, deflections under uniform loads commonly encountered in buildings will normally be satisfactory when the minimum thicknesses specified in Table 9.5(a) for one-way members are met or exceeded. Lesser thicknesses may be used when the provisions of Section 9.5.2.4 are satisfied.

9.5.2.2 *Computation of immediately deflection.* Unless a more comprehensive analysis is made, representative deflections which occur immediately on application of load may be computed by the usual methods or formulas for elastic deflections, using the modulus of elasticity for concrete as specified in Section 8.3.1, and the following average effective moment of inertia for prismatic members, but not greater than I_g.

$$I_e = (M_{cr}/M_a)^3 I_g + [1 - (M_{cr}/M_a)^3] I_{cr} \quad (9\text{-}4)$$

$$M_{cr} = f_r I_g / y_t \quad (9\text{-}5)$$

$$f_r = 0.65\sqrt{w f_c'},\ 90 \text{ pcf} \leq w \leq 145 \text{ pcf} \quad (9\text{-}6)$$

where I_e is determined at the support section for cantilevers and the midspan section for simple and continuous spans. For different load levels, the deflection should be computed in each case using Eq. (9-4) for the total load level being considered, such as dead load or dead plus live load. The incremental deflection, such as for live load, is then computed as the difference in these values. For normal weight concrete, f_r may be considered as $7.5\sqrt{f_c'}$. When lightweight aggregate concretes are used, either Eq. (9-6) or the following may be used: modify f_r by substituting $f_{ct}/6.7$ for $\sqrt{f_c'}$, but the value of $f_{ct}/6.7$ used shall not exceed $\sqrt{f_c'}$. The value of f_{ct} shall be specified and the concrete proportioned in accordance with Section 4.2.

9.5.2.3 *Computation of long-time deflection.* Unless a more comprehensive analysis is made, representative values of the additional long-time

TABLE 9.5(a)—MINIMUM THICKNESS OF BEAMS OR ONE-WAY SLABS USED IN ROOF AND FLOOR CONSTRUCTION, AS DESCRIBED IN SECTIONS 9.5.2.1 AND 9.5.5*†

Member	Members not supporting or not attached to nonstructural elements likely to be damaged by large deflections				Members supporting or attached to nonstructural elements likely to be damaged by large deflections			
	Simply supported	One end continuous	Both ends continuous	Cantilever	Simply supported	One end continuous	Both ends continuous	Cantilever
Roof slab‡	$l/22$	$l/28$	$l/35$	$l/9$	$l/14$	$l/18$	$l/22$	$l/5.5$
Floor slab, and roof beam‡ or ribbed roof slab	$l/18$	$l/23$	$l/28$	$l/7$	$l/12$	$l/15$	$l/19$	$l/5$
Floor beam or ribbed floor slab	$l/14$	$l/18$	$l/21$	$l/5.5$	$l/10$	$l/13$	$l/16$	$l/4$

*The span length l is in inches.

†The values given in this table shall be used directly for nonprestressed reinforced concrete members made with normal weight concrete (w = 145 pcf) and Grade 60 reinforcement. For other conditions, the values shall be modified as follows:

For structural lightweight concrete having unit weights in the range 90-120 pcf, the values in the table shall be multiplied by $1.65 - 0.005\,w$, but not less than 1.09, where w is the unit weight in pcf.

For nonprestressed reinforcement having yield strengths other than 60,000 psi, the values in the table shall be multiplied by $0.4 + f_y/100{,}000$.

‡Refers to roofs subjected to normal snow or construction live loads only, and with minimal ponding problems.

deflection for both normal weight and lightweight concrete flexural members may be determined by multiplying the immediate deflection caused by the sustained load considered by the factor $k_r T$, where

$$k_r = 1/(1 + 50\rho') \qquad (9\text{-}7)$$

in which the compressive steel ratio ρ' is determined at the support section for cantilevers and the midspan section for simple and continuous spans. Under normal conditions the value of T may be assumed to be 2.5. Where creep and/or shrinkage are expected to be abnormally high, a larger value of T may be required.

9.5.2.4 *Allowable deflection.* The deflection computed in accordance with Sections 9.5.2.2 and 9.5.2.3 shall not exceed the limits stipulated in Table 9.5(b). Add in Table 9.5(b) the word "not," as *not* supporting and *not* attached, etc., the same as in Table 9.5(a) herein.

9.5.3 *Nonprestressed two-way construction*

9.5.3.1, 9.5.3.2, 9.5.3.3 *Minimum thickness, drop panels, edge beams.* Essentially the same as ACI 318-71. Delete the words "for floors" from Line 3 of Section 9.5.3.1, as it has been determined that the procedure is applicable to roofs as well. Add statement that these minimum thicknesses do not apply to cantilever portions of a two-way floor system. Cantilevered portions should be considered one-way construction.

9.5.3.4 *Computation of immediate and long-time deflection.* Essentially the same as ACI 318-71, with the use of Eq. (9-4) and (9-7) herein.

9.5.4 *Prestressed concrete*

9.5.4.1 *Computation of immediate camber and deflection.* For prestressed concrete flexural members designed in accordance with the requirements of Chapter 18, camber and deflection shall be calculated by the usual methods and formulas using the moment of inertia of the gross concrete section for uncracked members. When members are cracked, a bilinear moment-deflection method [one such method is the use of I_e in Eq. (9-4)] shall be used.

9.5.4.2 *Computation of long-time camber and deflection.* The additional long-time camber and deflection of prestressed concrete members shall be computed taking into account the stresses and strains in the concrete and steel under the sustained loads, and including the effects of creep and shrinkage of the concrete and relaxation of the steel.

9.5.4.3 *Allowable deflection.* The deflection computed in accordance with Sections 9.5.4.1 and 9.5.4.2 shall not exceed the limits stipulated in Table 9.5(b).

9.5.5 *Composite members*
Essentially the same as ACI 318-71.

COMMENTARY

9.5—Control of deflections

9.5.1 *General*—This section is concerned with the immediate or short-time and long-time deflection or deformation that occurs at service load levels, including the effects of elastic deformation, cracking, creep, shrinkage, and temperature. Such deflection is measured below the level of supports for horizontal members, with consideration also being given to camber of both nonprestressed and prestressed members.

Where excessive deflections may cause damage to nonstructural or other structural elements, only that part of the deflection that occurs after the construction of the element that might be damaged should be included. Where excessive deflections result in either esthetic or functional problems, such as objectionable visual sagging, ponding of water, vibration, and improper operation of machinery or sliding doors, the total deflection should be included. Procedures and suggestions for computing and controlling deflections may be found in various ACI publications (References 9.1 through 9.7), and elsewhere.

The provisions of Section 9.5 do not apply to the estimation of deflections from loads with strong dynamic characteristics such as from earthquakes, transient winds, machinery, moving vehicles, or humans where response can be experienced from the frequency of footfalls or from dynamic loading patterns. Such loadings require an analysis of dynamic response for the particular time-dependent loading. Particular care should be taken with flexible structures with a low mass and low damping such as lightweight, high strength prestressed concrete. Unusual and undesirable complex response patterns can be excited by such loads, regardless of strength considerations.

9.5.2 *Nonprestressed one-way construction*

9.5.2.1 *Minimum thickness.* Although the general requirements of Section 9.5.1 must be satisfied, deflections are not likely to cause problems when the provisions of Table 9.5(a) are met or exceeded for uniform loads commonly used in the design of buildings. Lesser thicknesses may be used when the allowable deflections of Section 9.5.2.4 are satisfied. Such practices as the use of wide beams and/or large amounts of compressive reinforcement, and keeping shores in place for longer periods, for example, may allow for the use of reduced thicknesses. Table 9.5(a) is based primarily on experience and is not intended to apply for special cases such as heavy wall or point loads, etc.

TABLE 1—CORRECTION FACTORS

f_y, ksi	40	50	60	75	w, pcf =	90	100	110	120
Materials correction factor =	0.80	0.90	1.00	1.15		1.20	1.15	1.10	1.09

The modification in Table 9.5 (a) for lightweight concrete in the second footnote is based on studies of the results and discussions in Reference 9.8. No correction is given for concretes weighing between 120 and 145 lb/ft^3 since the correction term would be close to unity in this range. The modification in Table 9.5 (a) for yield strength in the second footnote is based on judgment, experience, and studies of the results of tests. The simple expression is considered satisfactory for values of f_y between 40 and 80 ksi. Sample values of these correction factors are given in Table 1.

Alternatively, normal weight structural members will usually be of sufficient size so that deflections will be within acceptable limits when the tension steel ratio used in the positive moment zone does not exceed the following percentages of that in the balanced condition:

For members not supporting or not attached to nonstructural elements likely to be damaged by large deflections

Rectangular beams: 35 percent of ρ_b;

T-beams or box beams: 40 percent of ρ_b

For members supporting or attached to nonstructural elements likely to be damaged by large deflections

Rectangular beams: 25 percent of ρ_b;

T-beams or box beams: 30 percent of ρ_b

For lightweight members use values 5 percent less in each case (20 to 35 percent).

9.5.2.2, 9.5.2.3 *Computation of immediate and long-time deflection.* Relatively simple methods are presented for the computation of deflections. Because of the degree of uncertainty involved,[9.5] undue reliance should not be placed on the computed results using these methods, or even more comprehensive methods.

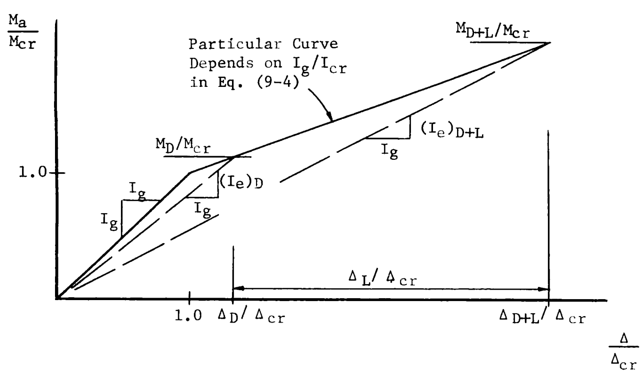

Fig. 9.2—Idealized immediate moment versus deflection diagram using Eq. (9-4)

Some of the difficulties in estimating deflections of structures include the following: The level of cracking may be controlled by construction loads, not design loads. Considerable creep and shrinkage deflection may take place before the design live load (or even any significant part of the live load) is applied, and thus before the increased cracking due to live load. Although the envelope of repeated load cycles tends to approximate a single load-deflection curve, the effect of additional cracking under repeated live loading on both immediate and long-time deformation (creep and shrinkage) is not easily determined. Other pertinent factors include the concrete age when forms are removed and nonstructural elements are installed, and magnitude of creep and shrinkage. Notwithstanding these complications, the approximate procedure given by Eq. (9-4) through (9-7) is deemed adequate in most cases for computing representative deflections under service loads for the purpose of comparison with the limiting values in Table 9.5 (b).

Eq. (9-4) for an average I_e[9.2, 9.12] between inflection points was developed in Reference 9.9 to provide a transition between the upper and lower bounds of I_g and I_{cr}, as a function of the level of cracking in the form of M_{cr}/M_a. For different load or moment levels, the deflection should be computed in each case using Eq. (9-4) for the total load level being considered, such as dead load or dead plus live load. The incremental deflection, such as due to live load deflection, is then computed as the difference in these values. These various moment levels, moments of inertia, and deflections are shown in Fig. 9.2, which represents a typical idealized load or moment versus deflection relation as found in the literature.[9.10] This method compares favorably with other methods and experimental results in Reference 9.11 and elsewhere.

With regard to the effect of repeated loading, since the upper envelope of repeated load-deflection curves approximates the single loading curve for both reinforced and prestressed members, even though increasing residual deflections (due to creep and cracking effects) occur, it appears reasonable to compute immediate deflections using I_e and the residual deflections by an appropriate factor in terms of the total time-dependent or separate creep and shrinkage deflections.

The use of the midspan section properties for continuous prismatic members is considered satisfactory in approximate calculations primarily because the midspan rigidity (including the effect of cracking) has the dominant effect on deflections, as shown by ACI Committee 435[9.6] and Reference 9.7a. A somewhat improved result may be obtained for continuous members by using the following weighted average section properties:[9.6, 9.12]

Beams with both ends continuous:

$$\text{Average } I_e = 0.70 I_m + 0.15 (I_{e1} + I_{e2}) \quad (9C\text{-}1)$$

Beams with one end continuous:

$$\text{Average } I_e = 0.85 I_m + 0.15 (I_{cont.\ end}) \quad (9C\text{-}2)$$

where I_m refers to the midspan section I_e, and I_{e1}, I_{e2}, to the I_e's at the beam ends. Moment envelopes should be used in computing both positive and negative values of I_e. For a single heavy concentrated load, only the midspan effective moment of inertia I_e should be used.

Eq. (9-6) for f_r is used from the ACI Committee 209 paper[9.4a] and more recent work.[9.12] From the Code equations for f_r and E_c:

Different weight concrete	$f_r = 0.65\sqrt{w f_c'}$	$E_c = 33\sqrt{w^3 f_c'}$
Normal weight (w = 145 pcf)	$= 7.83\sqrt{f_c'}$	$= 57{,}600\sqrt{f_c'}$
Sand-lightweight (w = 120 pcf)	$= 7.12\sqrt{f_c'}$	$= 43{,}400\sqrt{f_c'}$
All-lightweight (w = 100 pcf)	$= 6.50\sqrt{f_c'}$	$= 33{,}000\sqrt{f_c'}$

Eq. (9-6) for f_r is similar in form to that for E_c in the Code, and is thought to be more realistic than the values currently in use. However, for normal weight concrete a value of $7.5\sqrt{f_c'}$ may also be used as before.

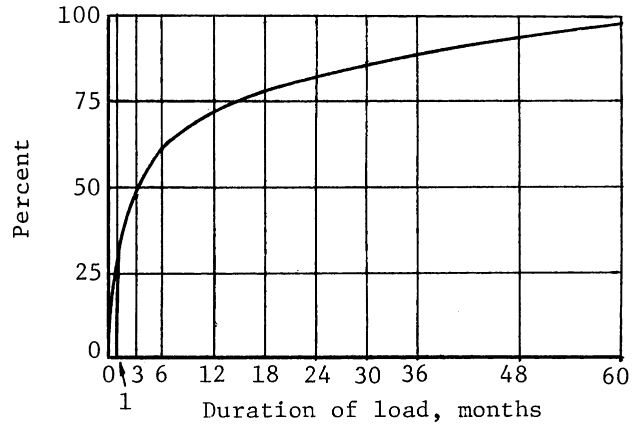

Fig. 9.3—Variation in long-time deflection

The immediate deflection Δ_i may be computed using the following elastic equation for the maximum deflection of cantilevers and the midspan deflection of simple and continuous beams. In the case of continuous beams, the midspan deflection may normally be used as an approximation of the maximum deflection.

$$\Delta_i = K(5/48) Ml^2/E_c I_e \qquad (9C\text{-}3)$$

where M is the support moment for cantilevers and the midspan moment for simple and contin-

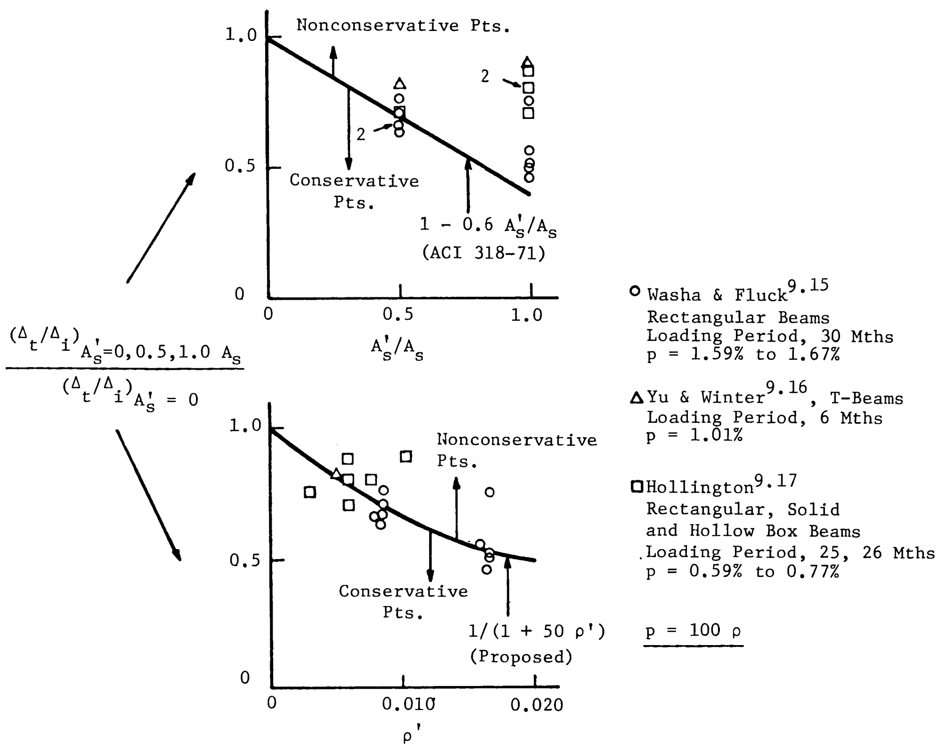

Fig. 9.4—Comparison between experimental results and computed values of k_r by ACI 318-71 and the proposed Eq. (9-7) from References 9.12 and 9.14. These comparisons isolate the compression steel effect. For the loading periods shown, T for the above data varied from 0.98 to 2.27, versus 2.0 in ACI 318-71 and 2.5 recommended herein.

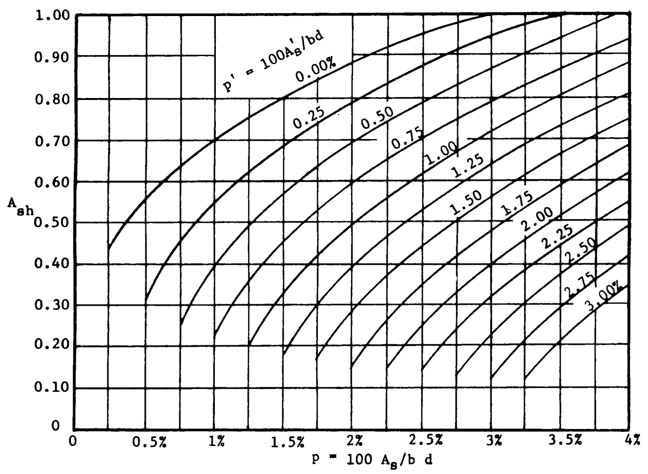

Fig. 9.5—Values of A_{sh} for calculating shrinkage curvature

uous beams. For uniform loads, $K = 12/5$ for cantilevers, 1 for simple spans, and $1.20 - 0.20\ M_o/M_m$ for continuous spans[9.12] (M_o is the simple span moment at midspan and M_m is the net midspan moment). In the case of cantilevers, the deflection due to the rotation at the support must also be included. For other types of loading, see Reference 9.7b and various other references.[9.12, 9.13]

Eq. (9-7) was developed in Reference 9.14 based on the deflection data from References 9.15 to 9.17 for rectangular, T, and box beams. In Eq. (9-7), k_r accounts for the effect of compression steel in reducing time-dependent deflections, and $T = 2.5$ represents a nominal time-dependent factor for average conditions. For loading periods less than permanent, the curve[9.16] in Fig. 9-3 may be used in estimating the additional long-time deflection. For example, for a loading period of 3 months, $T =$ (0.50 from Fig. 9-3) (2.5) $= 1.25$.

TABLE 9.5(c)—VALUES OF A_{sh} FOR CALCULATING SHRINKAGE CURVATURE*

p \ p'	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
0.25	0.44	0.00	—	—	—	—	—	—	—	—	—	—	—
0.50	0.56	0.31	0.00	—	—	—	—	—	—	—	—	—	—
0.75	0.64	0.45	0.26	0.00	—	—	—	—	—	—	—	—	—
1.00	0.70	0.55	0.39	0.22	0.00	—	—	—	—	—	—	—	—
1.25	0.75	0.63	0.49	0.35	0.20	0.00	—	—	—	—	—	—	—
1.50	0.80	0.69	0.57	0.45	0.32	0.18	0.00	—	—	—	—	—	—
1.75	0.84	0.74	0.64	0.53	0.42	0.30	0.17	0.00	—	—	—	—	—
2.00	0.88	0.79	0.69	0.60	0.50	0.39	0.28	0.16	0.00	—	—	—	—
2.25	0.92	0.83	0.74	0.65	0.56	0.47	0.37	0.26	0.15	0.00	—	—	—
2.50	0.95	0.87	0.79	0.71	0.62	0.53	0.44	0.35	0.25	0.14	0.00	—	—
2.75	0.98	0.91	0.83	0.75	0.67	0.59	0.51	0.42	0.33	0.24	0.13	0.00	—
3.00	1.00	0.94	0.87	0.79	0.72	0.64	0.57	0.49	0.40	0.32	0.23	0.13	0.00
3.25	1.00	0.97	0.90	0.83	0.76	0.69	0.62	0.54	0.47	0.39	0.31	0.22	0.12
3.50	1.00	1.00	0.94	0.87	0.80	0.74	0.67	0.60	0.52	0.45	0.37	0.29	0.21
3.75	1.00	1.00	1.00	0.90	0.84	0.78	0.71	0.64	0.58	0.51	0.44	0.36	0.28
4.00	1.00	1.00	1.00	1.00	0.88	0.81	0.75	0.69	0.62	0.56	0.49	0.42	0.35

*$p = 100\ A_s/bd$, $p' = 100\ A_s'/bd$. When $p' > p$, interchange p' and p to obtain corresponding solution in opposite direction.

Eq. (9-7) is presented in the form of two factors, k_r (a section property), and T (a material property), the same as EI. This equation was derived empirically. However, conceptually it would seem that the effect of compression steel in reducing time-dependent (or creep alone) deflection might more appropriately be related to the bulk of the concrete in compression, such as by means of the parameter ρ' than to the ratio of compression to tension steel, A_s'/A_s. A comparison of the current and proposed expressions for the effect of compression steel in Fig. 9-4 indicates that Eq. (9-7) should be preferable. As shown by tests in Reference 9.17, the current Code method can significantly overpredict (nonconservative side) the effect of compression steel for typical lightly reinforced precast concrete floor units, for example, when A_s'/A_s is relatively high.

A value of $T = 2.5$, instead of the current Code value of $T = 2$, is thought to be more realistic. Other codes, for example, use values of 3 (Australian), 3.5 (AASHTO for low humidity), and higher.

When it is desired to consider creep and shrinkage deflections separately, the following approximate equations may be used.[9.2, 9.9, 9.12, 9.14]

Creep deflection,

$$\Delta_{cp} = k_r\ C_t\ \Delta_i \tag{9C-4}$$

where $k_r = 0.85/(1 + 50\ \rho')$. For average conditions, Ultimate $C_t = 1.6$ may be used.

Shrinkage deflection,

$$\Delta_{sh} = K_{sh}\ \phi_{sh}\ l^2 \tag{9C-5}$$

$$\phi_{sh} = A_{sh}\ \varepsilon_{sh}/h \tag{9C-6}$$

where A_{sh} (based on shrinkage curvature equations from Reference 9.9) is obtained from Table 9.5(c) or Fig. 9-5. For average conditions, Ultimate $\varepsilon_{sh} = 400 \times 10^{-6}$ in./in. (mm/mm) may be used. The steel ratios or percentages refer to the support section of cantilevers and the midspan section of simple and continuous beams. For T-beams, use an average of p and p_w for p in determining A_{sh}. From Reference 9.2, $K_{sh} = 0.500$ for cantilevers, 0.125 for simple beams, 0.086 for beams continuous at one end only, and 0.063 for beams continuous at both ends (similar values based on a typical steel profile are derived in Reference 9.12). Eq. (9C-6) and Table 9.5(c) are also used in the British Standard Code of Practice (1972), with d instead of h.

This approach for calculating deflections is further discussed in Reference 9.7c, and in Reference 9.12. More detailed information on the creep and shrinkage factors may be found in References 9.4 and 9.12. A similar procedure for computing deflections is described in Reference 9.13.

Eq. (9-7) is used to compute the combined creep and shrinkage deflection, and Eq. (9C-4) to (9C-6) are used to compute the separate creep and shrinkage deflection. One case in which the latter method may be preferable is when part of the live load is considered as a sustained load, as illustrated in the example herein.

The method of computing immediate and long-time deflection in the Code and Commentary is provided as a consistent approach for determining representative deflections, as opposed to predicting deflections (for which probabilistic methods are required), for the purpose of comparison with allowable values. Alternative methods are also referenced. The procedures are referred to as approximations. However, it is suggested that more complex deterministic methods for calculating deflections may improve the results for test specimens under prescribed loading conditions in some cases, but such may not generally be the case for actual structures under imprecise or unknown loading histories (including construction loads), variable creep and shrinkage behavior for different structures, etc.

9.5.2.4 *Allowable deflections.* Same as Table 9.5(b) of ACI 318-71. A more extensive list of allowable deflections is given by ACI Committee 435.[9.3]

9.5.3 *Nonprestressed two-way construction*

9.5.3.1, 9.5.3.2, 9.5.3.3 *Minimum thickness, drop panels, edge beams.* Similar to ACI 318-71 Commentary.

9.5.3.4 *Computation of immediate and long-time deflection.* The calculation of deflections for slab systems is complicated even if linear elastic behavior can be assumed. Realistic estimates [ACI Committee 435 (Reference 9.7d), and Reference 9.18] of the immediate deflection may be obtained using E_c and I_e as specified in Section 9.5.2.2. However, other procedures and other values of stiffness, EI, may be used when they result in the prediction of deflections that are in reasonable agreement with test results, such as those of References 9.19 through 9.22. Such a procedure, for example, is described in Reference 9.20.

As a guide with regard to the effect of cracking at service load levels, the following section properties have been found to apply in certain typical cases:[9.12]

Flat plates and flat slabs—For all dead load deflections: I_g. For dead plus live load deflections: I_g for the middle strips in both directions and I_e for the column strips in both directions.

Two-way slabs—For all dead load deflections. I_g. For dead plus live load deflections: I_g for the column strips in both directions and I_e for the middle strips in both directions.

This appears to be consistent with the principal crack patterns found in References 9.19, 9.21, and 9.22, for example, which are along column lines for flat plates and flat slabs and diagonally for two-way slabs.

Since the available data on long-time deflections of slabs are too limited to justify more elaborate procedures, this section requires the additional long-time deflection to be computed using the multiplier given in Section 9.5.2.3.

9.5.4 *Prestressed concrete.*

9.5.4.1 *Computation of immediate camber and deflection.* Immediate camber and deflection of prestressed concrete members may be calculated by the usual methods or formulas for elastic deflections using the moment of inertia of the gross (uncracked) concrete section and the modulus of elasticity for concrete specified in Section 8.3.1. Since this method assumes that the concrete is uncracked, it may be unconservative for members having a relatively large tension stress in the concrete, as permitted by Section 18.4.2.3. Hence, Section 18.4.2.3 requires the calculation of deflections in the cracked range based on a bilinear moment-deflection relation. Two different procedures are presented in Reference 9.7e and Reference 9.12 using I_e by Eq. (9-4) for computing such deflections for both noncomposite and composite members. This approach is used in the PCI Handbook (1971 and 1977).

9.5.4.2 *Computation of long-time camber and deflection.* The calculation of long-time camber and deflection of prestressed concrete flexural members is relatively complicated. Any suitable method, such as that described in Reference 9.1 for noncomposite beams and References 9.7e, 9.12, and 9.23 for noncomposite and composite beams, may be used. The following equation[9.24] may be used to approximate the effect of nonprestressed tension steel in reducing time-dependent camber (similar to the effect of compression steel in reinforced members to reduce time-dependent deflection):

$$\text{Time-dependent camber} = [\text{Time-dependent camber when } A_s = 0]\,[1/(1 + A_s/A_{ps})] \qquad \text{(9C-7)}$$

9.5.4.3 *Allowable deflection.* The camber and deflection of prestressed concrete flexural members must be computed and compared with the allowable values in Table 9.5(b)

9.5.5 *Composite members*—Since few tests have been made to study the immediate and long-time deflections of reinforced composite members (one such study is Reference 9.7f), the rules given in Sections 9.5.5.1 and 9.5.5.2 are based on the judgment of the committee and on experience.

If any portion of a composite member is prestressed or if the member is prestressed after the components have been cast, the provisions of Section 9.5.4 apply and deflections shall always be calculated. For nonprestressed members, deflections need be calculated and compared with the limiting values in Table 9.5(b) only when the thickness of the member or the precast part of the member is less than the minimum thickness given in Table 9.5(a). In unshored construction the thickness of concern depends on whether the deflection before or after the attainment of effective composite action is being considered.

CONCLUDING REMARKS

The values in Tables 9.5(a) and 9.5(b) are based primarily on experience, and are considered by the committee to be acceptable for the purpose of this document.

It will be argued that the ACI Building Code is a legal document, and therefore one cannot couch it in such terms as "normally satisfactory," "in most cases," etc. However, it might also be argued that Section 9.5 on deflections is somewhat unique in the Code, and that one cannot afford not to use such qualifications, provided the general performance criteria in Section 9.5.1, first paragraph, takes precedence in any conflict with other sections of Section 9.5. This is the context in which this report was written. The British Standard Code of Practice (1972), for example, explicitly states its deflection provision as a guide only.

With regard to the use of an unqualified Code provision on deflections, it is not impossible to specify simple methods (or even complicated ones for that matter) that are feasible for controlling deflections by means of minimum thicknesses and allowable versus computed deflections with reasonable assurance of satisfactory performance in all (or even most) cases. As has been pointed out, concrete deformation is probabilistic, and our knowledge is imperfect to even provide mean value functions and variance.

The best alternative to the approach of this proposal may be to include only a general section or performance critera in the Code and everything else in the Commentary.

A detailed review of approximate procedures, with numerical examples, for determining the deformation of nonprestressed noncomposite one-way and two-way members, noncomposite prestressed members, and composite nonprestressed and prestressed members may be found in Reference 9.12.

NOTATION

C_t = creep coefficient defined as the ratio of creep strain to initial strain at any time
C_u = ultimate creep coefficient
E_c = modulus of elasticity of concrete
f_{ct} = average splitting tensile strength of lightweight aggregate concrete
f_r = modulus of rupture of concrete
h = overall thickness of member
I_{cr} = moment of inertia of cracked section transformed to concrete
I_e = effective moment of inertia for computation of deflection
I_g = moment of inertia of gross concrete section about the centroidal axis, neglecting the reinforcement
K = deflection coefficient
K_{sh} = shrinkage deflection coefficient
k_r = reduction factor
l = span length, such as clear span for deflections
M_a = maximum service load moment at the stage for which deflections are being considered
M_{cr} = cracking moment
M_m = net moment at midspan
M_o = statical moment at midspan, as $M_o = wl^2/8$ for uniform loading
p = $100\ (A_s/bd) = 100\rho$. $p' = 100(A_s'/bd) = 100\rho'$
T = multiplier for additional long-time deflection due to time- dependent effects, such as creep and shrinkage
w = uniformly distributed load
w = unit weight of concrete, pcf
y_t = distance from centroidal axis of gross section, neglecting the reinforcement, to extreme fiber in tension
Δ = deflection
ε_{sh} = shrinkage strain
$(\varepsilon_{sh})_u$ = ultimate shrinkage strain
ρ = A_s/bd = ratio of nonprestressed tension reinforcement ($p = 100\rho$)
ρ_w = $A_s/b_w d$
ρ' = A_s'/bd = ratio of compression reinforcement ($p' = 100\rho'$)
ϕ_{sh} = shrinkage curvature

Subscripts

i = initial value
cp = creep
D = dead load
L = live load
sh = shrinkage
t = time-dependent
u = ultimate value

EXAMPLE

End-span of continuous nonprestressed T-beam, normal weight concrete. Clear span = 40 ft (12.2 m), beam thickness h = 30 in. (76 cm), f_c' = 4000 psi (28 N/mm^2 = 28 MPa), f_y = 50 ksi (345 N/mm^2 = 345 MPa), w_D = 850 lb/ft (12,400 N/m), w_L = 800 lb/ft (11,700 N/m), assume 30 percent of the live load to be sustained.

Midspan—y_t = 20 in. (50.8 cm), ρ = 0.0025, ρ_w = 0.0125, ρ' = 0, I_g = 50,000 in.^{4} (2,081,000 cm^4), I_{cr} = 20,000 in.^{4} (832,000 cm^4)

h_{min} = (l/28,23,18,15,13) (0.90 for f_y = 50 ksi) for the different one-end continuous cases in Table 9.5(a) = (480/28) (0.90) to (480/13) (0.90)

= 15.4 in. (39 cm) to 33.2 in. (84 cm) versus h = 30 in. (76 cm)

Hence, deflections are likely to be satisfactory, except possibly for the most critical cases in Table 9.5(a) or the most stringent limitation in Table 9.5(b) of l/480.

$$E_c = 33\sqrt{w^3 f_c'} = 33\sqrt{(145)^3\ (4000)}$$
$$= 3.64 \times 10^6 \text{ psi } (25.1 \text{ kN/mm}^2 = 25{,}100 \text{ MPa})$$
$$f_r = 0.65\sqrt{w f_c'} = 0.65\sqrt{(145)\ (4000)}$$
$$= 495 \text{ psi } (3.41 \text{ N/mm}^2 = 3410 \text{ kPa}) \quad \text{(Eq. 9-6)}$$

Midspan

$$M_{cr} = f_r I_g / y_t = (495)\ (50{,}000)/(20)\ (12{,}000)$$
$$= 103 \text{ ft-k } (140 \text{ kN-m}) \quad \text{(Eq. 9-5)}$$

Using the ACI Building Code positive moment equation for the case of a discontinuous end built integrally with the support,

$$M_D = w_D l^2/14 = (0.850)\ (40)^2/14$$
$$= 97 \text{ ft-k } (132 \text{ kN-m}) < M_{cr}$$

Hence,

$$(I_e)_D = I_g = 50{,}000 \text{ in.}^4\ (2{,}081{,}000 \text{ cm}^4)$$
$$M_{D+L} = w_{D+L}\, l^2/14 = (1.650)\ (40)^2/14$$
$$= 189 \text{ ft-k } (256 \text{ kN-m}) > M_{cr}$$
$$(M_{cr}/M_{D+L})^3 = (103/189)^3 = 0.162$$
$$(I_e)_{D+L} = (M_{cr}/M_{D+L})^3\ I_g + [1 - (M_{cr}/M_{D+L})^3]\, I_{cr} \quad \text{(Eq. 9-4)}$$
$$= (0.162)\ (50{,}000) + (1 - 0.162)\ (20{,}000)$$
$$= 24{,}900 \text{ in.}^4\ (1{,}035{,}000 \text{ cm}^4)$$

$$K = 1.20 - 0.20\, M_o/M_m = 1.20 - (0.20)\ (wl^2/8)/(wl^2/14)$$
$$= 0.850$$
$$(\Delta_i)_D = K\ (5/48)\ M_D l^2/E_c I_e \quad \text{(Eq. 9C-3)}$$
$$= (0.850)\ (5/48)\ (97{,}000)\ (40)^2 (12)^3/(3.64 \times 10^6)\ (50{,}000)$$
$$= 0.13 \text{ in. } (3.3 \text{ mm})$$
$$(\Delta_i)_{D+L} = (0.850)\ (5/48)\ (189{,}000)\ (40)^2 (12)^3/(3.64 \times 10^6)\ (24{,}900)$$
$$= 0.51 \text{ in. } (13.0 \text{ mm})$$
$$(\Delta_i)_L = (\Delta_i)_{D+L} - (\Delta_i)_D = 0.51 - 0.13 = 0.38 \text{ in. } (9.7 \text{ mm})$$

Comparing this deflection with the ACI Building Code allowable values in Table 9.5(b): Flat roofs not supporting or not attached to nonstructural elements likely to be damaged by large deflections: $\Delta_L \leqq l/180$ = 480/180 = 2.67 in. (68 mm). Floors not supporting or not attached to nonstructural elements likely to be damaged by large deflections: $\Delta_L \leqq l/360$ = 480/360 = 1.33 in. (34 mm).

Hence, both of these conditions are satisfied.

Since

$$A_s'/A_s = \rho' = 0,\ k_r T = 2.5 \quad \text{(Eq. 9-7)}$$
$$\Delta_t = 2.5\ \Delta_i = (2.5)\ (0.13 + 0.30 \times 0.38)$$
$$= 0.61 \text{ in. } (15.5 \text{ mm})$$
$$\Delta_t + \Delta_L = 0.61 + 0.38 = 0.99 \text{ in. } (25.2 \text{ mm})$$

Alternatively,

$$k_r = 0.85 \text{ for } \rho' = 0 \text{ to be used in Eq. (9C-4)}$$
$$\Delta_{cp} = k_r C_u \Delta_i \quad \text{(Eq. 9C-4)}$$
$$= (0.85)\ (1.6)\ (0.13 + 0.30 \times 0.38)$$
$$= 0.33 \text{ in. } (8.4 \text{ mm})$$

For

$$p = 100\ (\rho + \rho_w)/2 = 0.750 \text{ percent},\ \rho' = 0,$$
$$A_{sh} = 0.64 \text{ (Table 9.5c or Fig. 9-5)}.$$

For beam continuous at one end only, K_{sh} =

$$\phi_{sh} = A_{sh}\ (\varepsilon_{sh})_u/h \quad \text{(Eq. 9C-6)}$$
$$= (0.64)\ (400 \times 10^{-6})/30 = 8.53 \times 10^{-6} \text{ 1/in. } (3.36 \times 10^{-6} \text{ 1/cm})$$
$$\Delta_{sh} = K_{sh}\, \phi_{sh}\, l^2 \quad \text{(Eq. 9C-5)}$$
$$= (0.086)\ (8.53 \times 10^{-6})\ (40)^2 (12)^2$$
$$= 0.17 \text{ in. } (4.3 \text{ mm})$$
$$\Delta_t = \Delta_{cp} + \Delta_{sh} = 0.33 + 0.17$$
$$= 0.50 \text{ in. } (12.7 \text{ mm})$$
$$\Delta_t + \Delta_L = 0.50 + 0.38 = 0.88 \text{ in. } (22.4 \text{ mm})$$

0.086 versus 0.99 in. (25.2 mm) above. The primary (in general) reason for this difference is the effect of the shrinkage plus creep factor of 2.5, instead of the creep factor alone of 1.6, being applied to the sustained live load deflection: (2.5) (0.30 × 0.38) = 0.29 in. versus (1.6) (0.30 × 0.38) = 0.18 in., or a difference of 0.11 in. (2.8 mm).

Comparing these deflections with the ACI Building Code allowable values in Table 9.5(b): Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections: $\Delta_t + \Delta_L \leqq l/480$ = 480/480 = 1.00 in. (25 mm). Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections: $\Delta_t + \Delta_L \leqq l/240 = 480/240 = 2.00$ in. (51 mm).

Hence, both of these conditions are satisfied by the two methods above.

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In the balloting of the eight members of the Building Code Subcommittee, ACI Committee 435, all eight voted affirmatively. In the balloting of the entire Committee 435 consisting of 22 members, 21 returned their ballots, of whom 20 voted affirmatively and none negatively. One member who returned his ballot abstained.