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Ductile Multiple-Anchor Steel-to-Concrete Connections

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Abstract: Multiple-anchor connections are commonly used in attaching steel members to concrete. A typical connection includes a steel attachment, the anchors that actually do the connecting, and an embedment of the anchors into the concrete. The behavior and design of these connections is not well-defined by existing design standards. Multiple-anchor connections can be divided into two categories, connections for which strength is controlled by the strength of the anchor-steel, and connections for which strength is controlled by the strength of the embedment. Based on experimental research, the behavior and design of connections for which strength is controlled by the strength of the anchor steel is addressed. A behavioral model for determining the distribution of loads to the individual anchors in a connection is presented. The model is based on limit design theory. Experimental results are reported for 28 tests of multiple-anchor connections loaded monotonically by various combinations of moment and shear. Test specimens included steel attachments with rigid and flexible base plates connected to concrete with threaded cast-in-place or retrofit (undercut and adhesive) anchors.

Introduction

The connection of steel members to concrete is a common structural feature, with applications in many types of construction. Fig. 1 shows a typical steel-to-concrete connection with a base plate mounted to hardened concrete; this was the type of connection considered in this study. The anchors used in this type of connection can be either cast-in-place or retrofit. The orientation of the connection shown in Fig. 1 and other figures in this paper is only for illustration. In practice, the connections are used in any orientation.

The design of multiple-anchor connections to concrete involves three steps:

1. Calculation of the loads on the connection.
2. Distribution of those loads to the anchors.
3. Design of each anchor for its loads.

This paper is concerned primarily with the second step. Existing information on the design of single ductile anchors, plus the experimental results of tests on multiple-anchor connections, are used to develop a model for the behavior and design of multiple-anchor steel-to-concrete connections whose strength is controlled by the strength of the anchor steel. Complete results of this study are provided in Cook and Klingner (1989), and have been incorporated into a design guide for steel-to-concrete connections (Cook et al. 1989).

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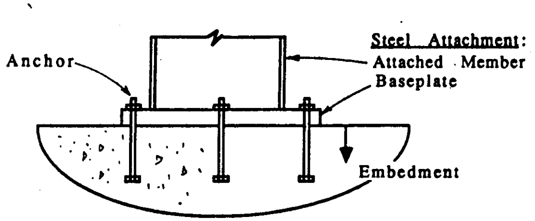


FIG. 1. Typical Steel-to-Concrete Connection: (*a*) Rigid Base Plate; and (*b*) Flexible Base Plate

For the purpose of this study, all anchors were designed to be ductile. An anchor is ductile if its ultimate strength is controlled by the strength of the steel. All nonductile failure modes were precluded based on existing design procedures for cast-in-place anchors ("Code Requirements" 1985), and on previous experimental testing of retrofit anchors (Collins et al. 1989).

BACKGROUND

The distribution of anchor loads in a ductile multiple-anchor connection with moment and shear is not obvious. Fig. 2 shows the possible forces on multiple-anchor connections with two rows of anchors in the tension zone loaded in shear, V, and moment, M ($V \times e$). In Fig. 2 the tensile forces, T_1 and T_2, and the compressive force, C, result from the internal couple required to resist the applied moment. The location of the compressive force, C, depends on the flexibility of the base plate. The frictional force, μC, is the compressive force, C, multiplied by the coefficient of friction between steel and concrete, μ. The anchor shear forces, V_1, V_2, and V_3, may or may not be present depending on the magnitude of the frictional force, μC, and the applied shear, V.

Testing has previously been conducted on multiple-anchor connections having one row of anchors in the tension zone and subjected to eccentric shear loads at high eccentricities (large M/V ratios) ("Anchorage to" 1979; Mahoney and Burdette 1978; "Welded Stud" 1979, 1980; "Eight-Bolt" 1984; Armstrong et al. 1985; Picard and Beaulieu 1985). In this situation, the compressive reaction from the internal couple is so large that the frictional shear strength, μC, exceeds the applied shear, V. The connection does not slip and the anchors transfer no shear load; that is V_1, V_2, and V_3, as shown in Fig. 2, are zero. The anchors in the tension zone fail in pure tension. This test is analogous to pulling a nail with a claw hammer. The hammer stays in one place as the nail is pulled in tension.

Limited tests on four-anchor connections in pure shear ("Anchorage to" 1979) show that shear forces redistribute in the connection prior to failure. These results indicate that sufficient inelastic deformation occurs in these connections so that each anchor achieves its single-anchor shear strength. These tests were performed on connections with welded studs, and on connections with threaded anchors.

The only previous study involving multiple-anchor connections subjected to various combinations of moment and shear was a study by Hawkins et al. (1980). This study investigated the behavior of multiple-anchor connections with welded studs. The test results indicated that sufficient inelastic deformation occurs in these connections so that each anchor achieves its single-anchor strength. The study concluded that a "plastic distribution"

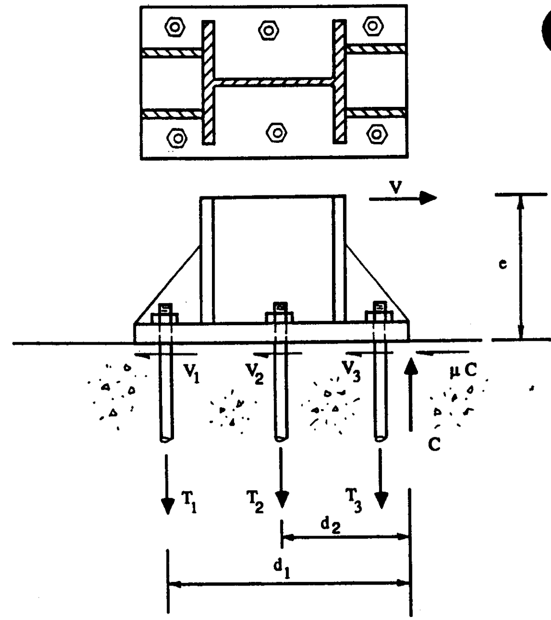


FIG. 2(*a*). Possible Distribution of Forces on Multiple-Anchor Connections: Rigid Base Plate

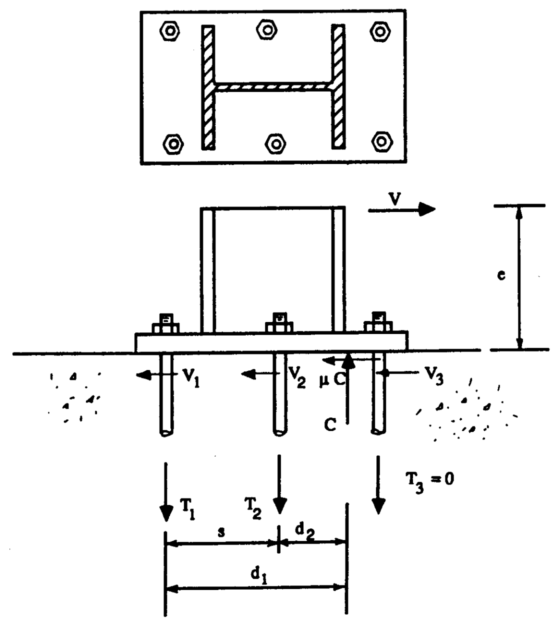


FIG. 2 (*b*). Possible Distribution of Forces on Multiple-Anchor Connections: Flexible Base Plate

metho[illegible]st predicted the strength of the welded stud connections. This study [illegible]ot include connections with more than one row of anchors in tension and did not consider the contribution of the frictional force between the base plate and the concrete.

Experimental Program

The behavior of a ductile multiple-anchor connection to concrete depends on a number of variables, including the following:

- Loading (axial load, moment, shear).
- Size of the steel attachment.
- Size, number, location, and type of anchors.
- Coefficient of friction between the base plate and the concrete.
- Tension/shear interaction for a single anchor.
- Distribution of shear among the anchors.
- Distribution of tension among the anchors.
- Flexibility of the base plate.

In a typical design situation only the loading is known. The job of the designer is to determine the size of the steel attachment and the size, number, location, and type of anchors. To complete this task, the designer must consider the effects of the last five variables.

The coefficient of friction between steel and concrete and the tension/shear interaction relationship for anchors play important roles in the strength of multiple-anchor connections. Although previous research and some existing design standards address these variables, their definition in an actual multiple-anchor connection is not well documented. For that reason, a portion of the overall experimental program associated with this research project was directed toward quantifying these variables in a multiple-anchor connection. That portion of the experimental program is discussed in detail in Cook and Klingner (1989) and is summarized as follows:

1. For base plates with clean mill scale (i.e., no special surface treatment) installed on hardened concrete, the nominal value for the coefficient of friction, μ, may be taken as 0.40. A coefficient of friction of 0.50 represents a reasonable upper limit for this type of base plate.
2. An elliptical tension/shear interaction is appropriate for anchors in steel-to-concrete connections. A linear interaction is conservative. The shear strength of cast-in-place and adhesive anchors in a multiple-anchor connection should be taken as 50% of the tensile strength ($\gamma = 0.50$). The shear strength of sleeved anchors (such as undercut anchors) in a multiple-anchor connection should be taken as 60% of the tensile strength ($\gamma = 0.60$).

The purpose of the testing program described in this paper was to quantify and define the last three variables for multiple-anchor connections to concrete. In order to define these variables it was necessary to control the variables not being investigated. The loading; the size of the steel attachment; and the size, number, location, and type of anchors were controlled in all tests. Since each of the variables being investigated could be studied in the absence of any externally applied axial load, the experimental program was limited to the study of multiple-anchor connections subjected to moment

and shear only. This was accomplished by applying an ecce[illegible]hear load to multiple-anchor connections at various load eccentricities.

The experimental program included four-anchor rigid base-plate tests, six-anchor rigid base-plate tests, and six-anchor flexible base-plate tests. In each test, measurements were made of the eccentric shear load, the eccentricity of the shear load, the individual anchor tensions, the base-plate slip, and the base-plate displacement normal to the concrete. All tests were loaded until failure. Connection failure was defined as the fracture of any anchor.

Four-Anchor Rigid Base-Plate Tests

The four-anchor rigid base-plate tests were developed to determine the distribution of shear among anchors. The four-anchor specimens lacked the center row of anchors shown in Fig. 2. Individual anchor shear was not measured. By using the coefficient of friction, the tension/shear interaction relationship, and the measured values of anchor tension, the amount of shear redistribution in the connection at failure could be evaluated. For example: If the total applied shear load at failure is equal to the sum of the frictional force between the concrete and the steel, plus the pure shear strength of the anchors on the compression side of the connection, plus the residual shear strength of the tension-side anchors based on their tension/shear interaction, then full redistribution of shear occurred in the connection.

Six-Anchor Rigid Base-Plate Tests

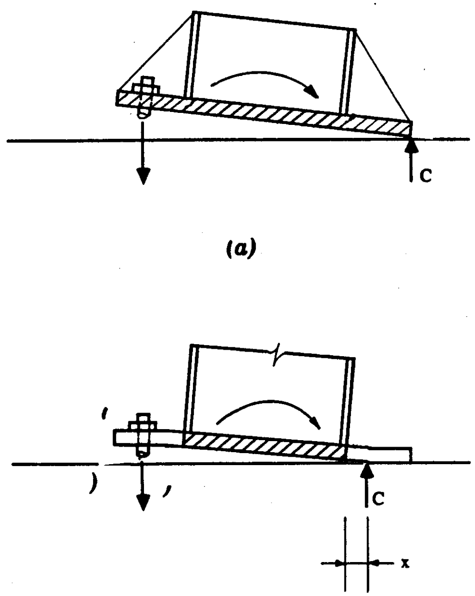
The six-anchor rigid base-plate tests were developed to determine the distribution of tension among the anchors, and to verify if the method of shear distribution determined from the four-anchor tests could be extended to a six-anchor configuration. Fig. 2 shows a free body diagram of a typical six-anchor rigid base-plate specimen.

The difference between the six-anchor tests and the four-anchor tests was the addition of a middle row of anchors located at the base-plate centerline. From a design viewpoint, this is a very inefficient location for additional anchors. For additional moment capacity, the anchors should be placed toward the tension side of the connection; for additional shear capacity, the anchors should be placed toward the compression side of the connection. Because the purpose of these tests was to determine the distribution of tension and shear in an extreme situation, the anchors were placed at the centerline of the connection. Since the anchor tension was measured for all anchors, the distribution of tensile forces in the connection was known throughout the test.

Six-Anchor Flexible Base-Plate Tests

The primary purpose of the six-anchor flexible base-plate tests was to evaluate the effects of base-plate flexibility on the location of the compressive resultant. A secondary purpose was to determine if the methods of predicting shear and tension distribution developed in the rigid base-plate tests could be extended to connections with flexible base plates.

Fig. 2 shows a free-body diagram of a typical six-anchor flexible base-plate specimen. Since the applied moment, ($V \times e$), and the anchor tensions were measured throughout the test, the location of the compressive reaction could be calculated at any load level from the condition of moment equilibrium.



(b)

FIG. 3. Probable Locations of Compressive Reaction

In a rigid base-plate test there is no flexibility in the steel attachment, and the compressive reaction from applied moment is located approximately at the leading edge of the plate. In a flexible base plate loaded with applied moment, the portion of the base plate extending beyond the attached member bends and causes the compressive reaction to shift inward from the leading edge. Fig. 3 shows the likely locations of the compressive reaction for a rigid base plate and a flexible base plate.

Development of Test Specimens

Anchors tested in this study included cast-in-place anchors, undercut anchors, and six types of adhesive anchors. Anchor material, anchor diameter, anchor patterns, and base-plate size were consistent with what might typically be used to connect a W12 steel beam to concrete. Load eccentricities were chosen to be in the range where connection behavior is least understood (between a claw-hammer type tension test with a high shear load eccentricity and a pure shear test with zero shear load eccentricity).

The following describes the design basis for the materials, anchor patterns, embedment lengths, base plates, and test setup used in the experimental program.

Materials

The design basis for selecting the particular anchor material, base-plate material, and concrete used in this study are given as:

1. Anchors: To produce a probable worst-case condition for redistribution of shear and tension, the anchor material chosen was a high-strength steel with no yield plateau. The material used for all types of anchors conformed to ASTM A193-B7. All anchors were 0.625 in. diameter with threads in the shear plane. The minimum length of the threaded portion of the anchors below the surface of the concrete was 0.50 in. The average tensile strength of the anchors as determined from 34 tests (Cook and Klingner 1989; Collins et al. 1989; Doerr et al. 1989) was 31.0 kips, or 137 ksi on the tensile stress area of 0.226 sq in.

2. Base plates: The base-plate material was ASTM A572 grade 50 for the rigid base-plate tests and ASTM A36 for the flexible base-plate tests. The bottom surface of the plates was clean mill scale.

3. Concrete: The concrete chosen for the experimental program was a ready-mix concrete. Minimum design compressive strength was 3,600 psi at 28 days. Actual 28-day compressive strengths ranged from 4,500 psi to 6,000 psi.

Choice of Anchor Pattern

The anchor pattern chosen for the experimental study was consistent with what is required to develop the plastic moment capacity of a W12 × 22 steel beam with a yield strength of 36 ksi using 0.625-in. diameter ASTM A193-B7 anchors.

Embedment Design Basis

The embedded length of the anchors was determined using the provisions of ACI 349-85 for cast-in-place and undercut anchors, and using the results of the study by Collins et al. (1989) for adhesive anchors. In both cases, the required embedded length necessary to fully develop the six-anchor pattern was determined to be 11 in.

Rigid Base-Plate Design Basis

Although the anchor patterns were developed to be consistent with connecting a W12 steel beam with stiffeners, it was not possible, using a W12 member, to provide an adequate interface with the test frame. Fig. 4 shows the steel attachment used for the rigid base-plate tests. Anchor load cell adapters were designed to allow the anchors to deform as in a connection without the anchor load cells. The anchor holes were larger than what is normally specified for steel members. The large oversize was to accommodate construction tolerances and to provide a probable worst case for redistribution of shear in the connection.

Flexible Base-Plate Design Basis

The flexible base-plate was designed to yield on the compression side of the base plate, and be at or just above yield on the tension side of the base plate at anchor failure. The particular design chosen was meant to represent a reasonable limit on plate flexibility. If the plate were more flexible (thinner), a plastic hinge would form on the tension side of the base plate, possibly causing prying forces in the anchors. Yielding on the compression side was not expected to degrade the performance of the attachment. As verified in the test program, this was in fact the case.

The six-anchor flexible base-plate dimensions were chosen based on using a 12-in. deep member with the same anchor pattern as the rigid base-plate tests. The flexible base plate was 2 in. longer than the rigid base plate. The extra length was provided to increase the flexibility of the base plate. Fig. 5 shows the steel attachment used for the flexible base-plate tests.

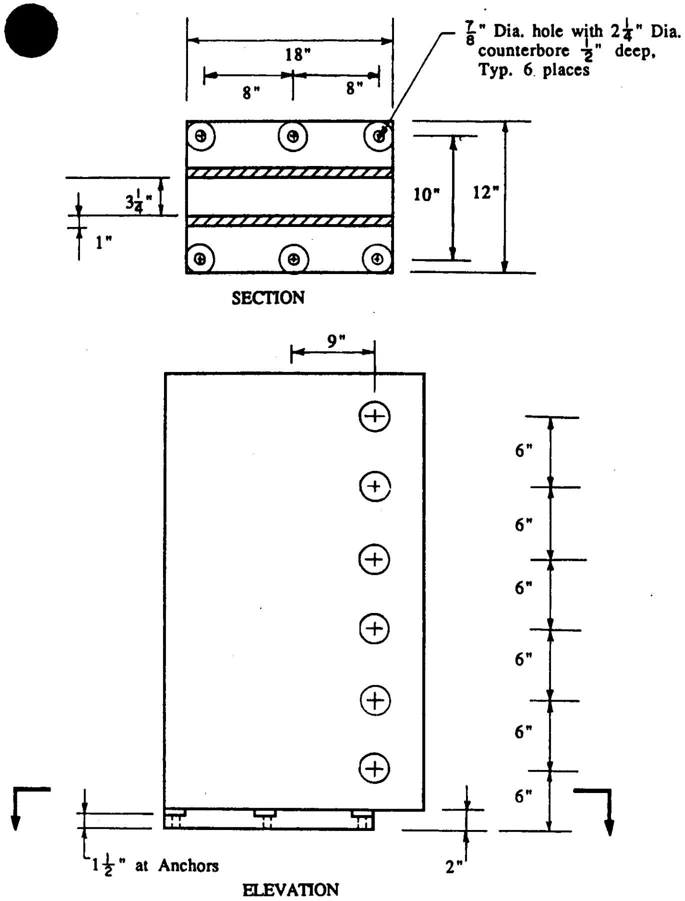


FIG. 4. Steel Attachment for Rigid Base Plate Tests

Test Setup

The test setup was developed to apply shear loads to the steel attachment at various eccentricities, and to be capable of failing the anchors at all eccentricities. The test setup is shown schematically in Fig. 6.

EXPERIMENTAL RESULTS

All test specimens failed by yielding and fracture of the anchors. The strength of the connection was limited by the strength of the steel in all tests. The most important observation from the tests was that the anchors underwent significant inelastic deformation prior to failure.

Typical load-displacement diagrams are presented in Fig. 7 and in Cook and Klingner (1989). Their principal purpose is to show the ductile behavior of connections dominated by anchor shear and also by anchor tension. The displacement shown in those diagrams is the total displacement at the location of the outer row of tension anchors. The total displacement was determined as the square root of the sum of the squares of the horizontal

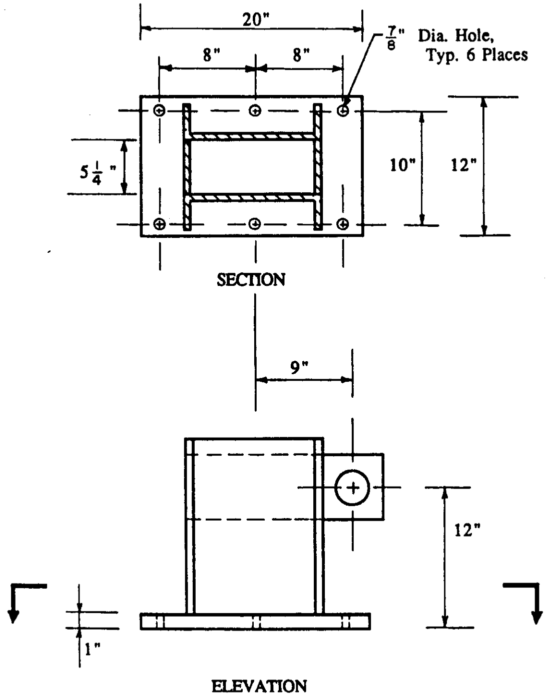


FIG. 5. Steel Attachment for Flexible Base Plate Tests

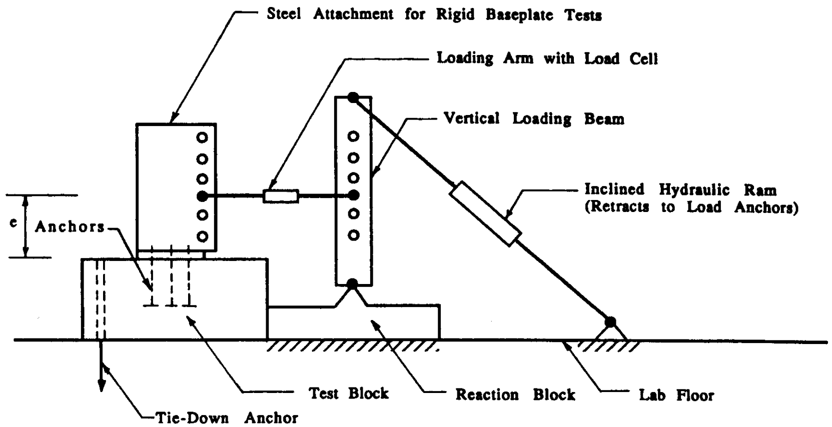


FIG. 6. Schematic Diagram of Test Setup

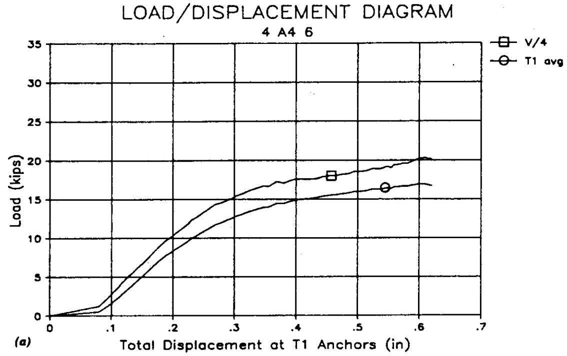


FIG. 7(*a*). Typical Load Displacement Diagram: Four-Anchor Test Dominated By Anchor Shear

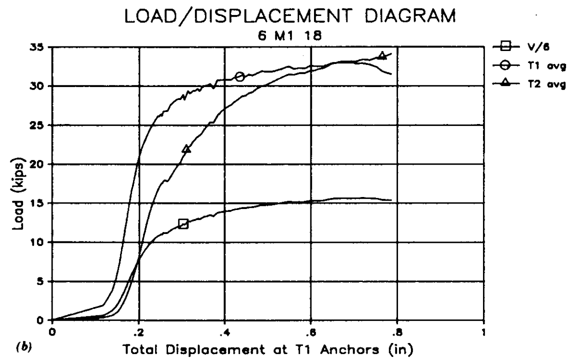


FIG. 7(*b*). Typical Load Displacement Diagram: Six-Anchor Test Dominated By Anchor Tension

slip and the vertical displacement at the location of the outer row of tension anchors. Ultimate loads and displacements are shown in Table 1.

Distribution of Tension and Shear among Anchors

Results of the tests indicated the following:

1. Tension and shear forces in the anchors redistribute inelastically as required to maintain equilibrium with the applied loading.
2. For connections dominated by moment (high eccentricity of the applied load), the anchors away from the toe of the baseplate attain their full tensile strength.
3. For connections dominated by shear (low eccentricity of the applied load), the ultimate strength of the connection is not sensitive to the distribution of tension in the anchors.
4. The initial distribution of anchor tension (prior to inelastic redistribution) has no effect on the ultimate strength of the connection.

Based on these observations, a limit-design approach appeared to be appropriate for ductile multiple-anchor connections. Limit design requires

TABLE 1. Test Results versus Predicted Strength

Test number[a] (1)	Maximum Displacement[b] δ_h (in.) (2)	Maximum Displacement[b] δ_v (in.) (3)	V_{test} (kips) (4)	$\mu = 0.40$ V_{ut} (kips) (5)	$\mu = 0.40$ V_{test}/V_{ut} (6)	$\mu = 0.50$ V_{ut} (kips) (7)	$\mu = 0.50$ V_{test}/V_{ut} (8)
4 CIP 6	0.23	0.07	74.4	69.3	1.07	72.0	1.03
4 A1 6	0.47	0.08	75.2	69.3	1.08	72.0	1.04
4 A4 6	0.60	0.14	81.8	69.3	1.18	72.0	1.14
4 M1 6	0.24	0.09	86.9	81.7	1.06	84.7	1.03
4 CIP 12	0.30	0.19	76.7	69.6	1.10	73.8	1.04
4 A1 12	0.49	0.14	80.5	69.6	1.16	73.8	1.09
4 A4 12	0.34	0.13	77.1	69.6	1.11	73.8	1.04
A M1 12	0.37	0.25	85.9	76.9	1.12	80.5	1.07
4 CIP 18	0.24	0.40	58.3	58.3	1.00	58.6	0.99
4 A1 18	0.21	0.33	59.9	58.3	1.03	58.6	1.02
4 A4 18	0.26	0.66	58.3	58.3	1.00	58.6	0.99
4 M1 18	0.21	0.79	63.9	58.6	1.09	58.6	1.09
4 CIP 24	0.07	0.20	40.5	43.9	0.92	43.9	0.92
6 CIP 6	0.39	0.11	107.8	107.7	1.00	113.3	0.95
6 M1 6	0.27	0.11	137.0	126.2	1.09	132.5	1.03
6 CIP 12	0.33	0.38	123.6	107.8	1.15	115.9	1.07
6 A1 12	0.30	0.28	110.6	107.8	1.03	115.9	0.95
6 A2 12	0.44	0.46	118.6	107.8	1.10	115.9	1.02
6 A3 12	0.28	0.27	125.3	107.8	1.16	115.9	1.08
6 A4 12	0.48	0.41	120.7	107.8	1.12	115.9	1.04
6 A5 12	0.40	0.21	104.7	107.8	0.97	115.9	0.90
6 A6 12	0.41	0.25	113.8	107.8	1.06	115.9	0.98
6 M1 12	0.35	0.36	130.5	117.0	1.12	123.5	1.06
6 CIP 18	0.22	0.46	86.8	88.7	0.98	89.6	0.97
6 M1 18	0.29	0.72	94.0	89.5	1.05	89.6	1.05
6 A1 12*x*	0.27	0.20	107.7	100.8	1.07	106.1	1.02
6 A4 12*x*	0.44	0.32	104.8	100.8	1.04	106.1	0.99
6 M1 12*x*	0.21	0.28	110.4	105.2	1.05	108.4	1.02

[a]Indicates number of anchors, type of anchor (CIP for cast in place, M1 for undercut, A1–A6 for adhesive), and eccentricity of load. *x* represents flexible base-plate test. For example: Test number 6 A1 12*x* was six-anchor test with adhesive anchors and flexible base-plate loaded at 12-in. eccentricity.

[b]δ_h = horizontal slip; and δ_v = vertical displacement at outer row of anchors in tension zone.

Note: 1 in. = 25.4 mm; and 1 kip = 4.45 kN.

that forces redistribute prior to failure and that the distribution of forces prior to redistribution not effect the ultimate strength.

Effect of Base-Plate Flexibility

As expected, in the rigid base-plate tests, the compressive reaction was concentrated at the toe of the base plate. The contact zone at the toe of the plate was typically a strip 0.25–0.50 in. wide, extending the full width of the base plate. There was no measurable difference in the width of the strip for the four-anchor, or six-anchor rigid base-plate tests. For the six-anchor rigid base-plate tests, the bearing stress at the toe of the plate was

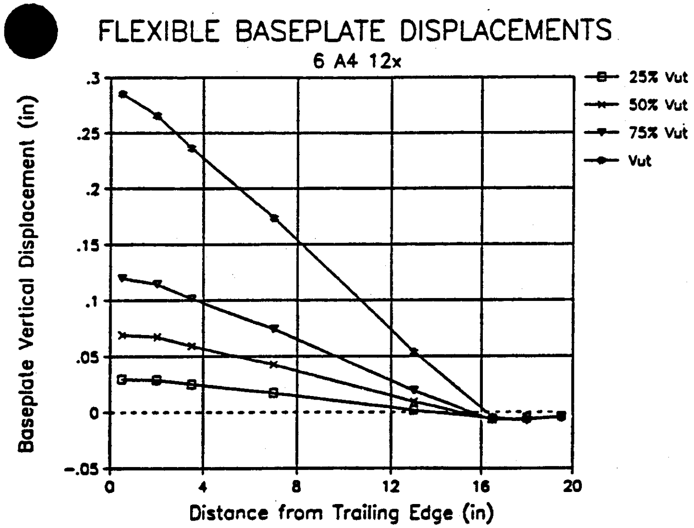


FIG. 8. Typical Vertical Displacements along Centerline of Flexible Base Plate

between four and eight times the actual 28-day compressive strength of the concrete. These high bearing stresses had no influence on the strength of the connection, since the base-plate was not close to any free edge of the supporting concrete.

Fig. 8 shows the vertical displacements along the centerline of the base plate for a typical flexible base-plate test. As shown in Fig. 8, the flexible base plates rotated about a point very near the compression flange of the attached member (the compression flange was 4 in. from the leading edge).

The effect of base-plate flexibility on the location of the compressive reaction can be explained if the relative stiffness of the base plate and the concrete are considered. For all practical purposes, the concrete can be assumed to be rigid, as can that portion of the base plate welded to the attached member. Any overhanging projection of the base plate beyond the compression element of the attached member can be considered flexible. This is shown in Fig. 3.

The behavior of a flexible base plate can be described as follows:

1. Initially, the base plate tries to rotate as a rigid body pivoting about the toe of the plate.
2. As the compressive load increases, the portion of the base plate adjacent to the compressive element of the attached member reaches the yield moment, M_y, of the base plate. This causes the compressive reaction, C, to move inward toward the compression element as indicated in Fig. 3. The smallest distance, $x_{\min}$, between the compressive reaction and the compression element of the attached member can be determined by:

$$x_{\min} = \frac{M_y}{C} \qquad (1)$$

3. With a further increase in the compressive reaction, the base plate begins to form a plastic hinge, and the compressive reaction moves away from the

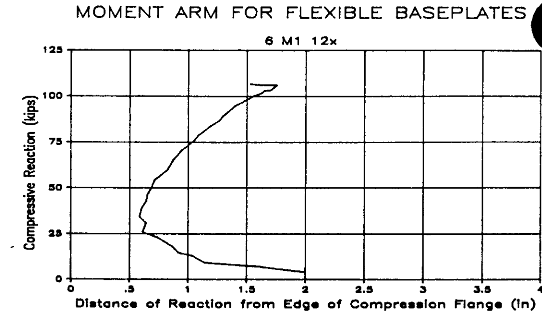


FIG. 9. Calculated Location of Compressive Reaction for Typical Flexible Base-Plate Test

compression element to a maximum distance determined by substituting the plastic moment of the plate for the yield moment of the plate in (1).

All the flexible base-plate tests exhibited this type of behavior. Typical graphical results showing the reaction moving in from the toe of the plate toward the compression element and back out again are shown in Fig. 9.

For the flexible base-plate tests, $x_{\min}$ can be determined from (1) by taking C as the sum of the tensile strength of the anchors in the tension zone to be 0.58 in. This agrees with the test results, which indicated that the nearest the compressive reaction was to the compressive element was 0.55 in., 0.49 in., and 0.60 in. for the three flexible base-plate tests.

Behavioral Model

Introduction to the Behavioral Model

The behavior of a ductile multiple-anchor connection can be separated into three distinct ranges:

1. If the shear strength provided by the frictional force is larger than the applied shear, then the connection does not slip and anchors are not required for shear. The anchors in the tension zone can be assumed to develop their full tensile strength for moment resistance.
2. If the shear strength provided by the frictional force and by the anchors in the compression zone exceeds the applied shear, the anchors in the tension zone can be assumed to develop their full tensile strength for moment resistance.
3. If the shear strength provided by the frictional force and by the anchors in the compression zone is less than the applied shear, the anchors in the tension zone must transfer the remaining shear load. The strength of the anchors in the tension zone is limited by their tension/shear interaction relationship.

The transitions between these three ranges of behavior can be determined by considering two critical values of shear load eccentricity, e. The shear load eccentricity, e, is equal to the moment to shear ratio, (M/V), of the applied loading at the surface of the concrete.

The first critical eccentricity, e', corresponds to the point at which the applied shear load is equal to the frictional force. For eccentricities larger than e', the connection does not slip and no shear anchors are required (i.e., t[illegible]aw hammer tensile test). For eccentricities smaller than e', the connection slips and shear anchors must be provided.

The second critical eccentricity, e'', corresponds to the point at which the applied shear load is equal to the sum of the frictional force and the shear strength of the anchors in the compression zone. For eccentricities larger than e'', the anchors in the tension zone can be assumed to develop their full tensile strength for moment resistance. For eccentricities smaller than e'', the anchors in the tension zone carry both tension and shear.

The critical eccentricities, e' and e'', can be determined by the conditions of force and moment equilibrium to be:

$$e' = \frac{d}{\mu} \quad \text{(2)}$$

$$e'' = \frac{nd}{n\mu + m\gamma} \quad \text{(3)}$$

The strength of a ductile multiple-anchor connection can be summarized by considering two distinct areas of connection strength:

1. Strength dominated by moment: For $e \geq e''$, the strength of the connection is controlled by the tensile strength of the anchors in the tension zone.
2. Strength dominated by shear: For $e < e''$, the strength of the connection is controlled by the shear strength of the anchors in the compression zone and the combined tensile and shear strength of the anchors in the tension zone.

Development of the Behavioral Model

The experimental results obtained in this study and other studies ("Anchorage to" 1979; "Anchorage test" 1975; Mahoney and Burdette 1978; "Welded Stud" 1979, 1980; Hawkins et al. 1980; "Eight-Bolt" 1984; Armstrong et al. 1985; Picard and Beaulieu 1985) indicate that a design procedure based on limit design is appropriate. The behavioral model for predicting the strength of ductile multiple-anchor connections is based on a lower-bound approach to limit design theory. In the application of a lower-bound approach to ductile multiple-anchor connections, the predicted strength of the connection is determined by assuming a distribution of tensile and shear forces in the anchors that satisfies the conditions of equilibrium and that does not exceed the strength of the individual anchors. The resulting predicted strength is less than, or at best equal to, the true strength of the connection.

The verification of a lower-bound approach for the design of ductile multiple-anchor connections requires that the upper limit to the lower bound be determined and compared to the experimental results. The purpose of the following is to determine the upper limit to the lower bound.

Maximum Predicted Strength for Connections Dominated by Moment ($e \geq e''$)

The strength of a connection dominated by moment is controlled by the tensile strength of the anchors in the tension zone. For this condition it is obvious that the maximum predicted strength occurs when all the anchors in the tension zone reach their tensile strength, T_0. The tensile forces in all the anchors in the tension zone can be considered as a single force acting

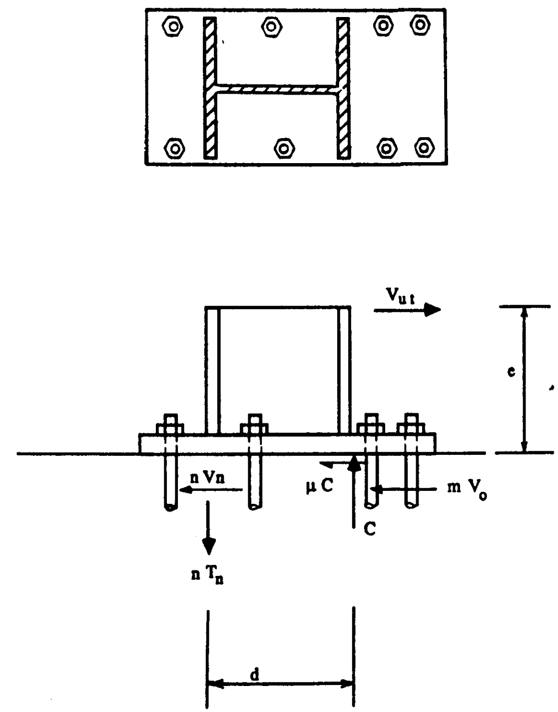


FIG. 10. Distribution of Forces on Multiple-Anchor Connection for Maximum Predicted Strength

at the centroid, d, of the anchors. The moment equilibrium condition for the connection shown in Fig. 10, with $e \geq e''$ ($T_n = T_0$), gives the maximum predicted strength of the connection:

$$V_{ut} = nT_0 \frac{d}{e} \quad \text{(4)}$$

In the application of limit design theory, the issue of available inelastic deformation capacity must be addressed. Limit design theory is based on the assumption that materials have infinite plastic deformation capacity after yield. This is not the case. In a connection with two or more rows of anchors, subjected to an applied moment, if the inner row of tension anchors is too close to the compressive reaction, anchors there will not be able to reach their tensile strength before the available deformation capacity is exceeded in the outer row of tension anchors. Anchor materials typically have a specified minimum elongation requirement of at least 10% in 2 in. This represents an ultimate strain of 0.10 or greater. To ensure that the tensile force in the inner row of anchors reaches the minimum specified tensile strength of the anchors, the distance between the inner row of anchors and the compressive reaction should not be less than about 10% of the distance from the outer row of anchors to the compressive reaction. The reason for this is as follows. When the inner row of anchors is so located, the tensile strain there will be at least 0.01 when the tensile strain in the outer row of anch[illegible]eaches its maximum value. Since a tensile strain of 0.01 is roughly two t[illegible]e times the yield strain for typical anchor materials, both rows of anchors will have yielded. This somewhat arbitrary limit ensures that the innermost row of tension will approach their tensile strength prior to tensile failure of the outermost row of tension anchors. This limit has little effect in a typical design situation, since greater flexural capacity of the connection is always achieved by locating the tension anchors as far as possible from the compressive reaction.

Maximum Predicted Strength for Connections Dominated by Shear ($e < e''$)

To properly assess the behavioral model in the area dominated by shear, it was necessary to determine the assumed distribution of tension and shear that would give the highest predicted strength of the connection. This was accomplished by considering a connection with two rows of anchors in the tension zone and no anchors in the compression zone. Note that e'' reduces to e' for this type of connection.

The strength of connections dominated by shear is dependent on the tension/shear interaction relationship of the anchors. An elliptical interaction best describes the strength of a single anchor in combined tension and shear. A linear interaction is more conservative and easier to apply in practice. An elliptical tension/shear interaction was used since it will better predict an upper limit to the lower-bound limit design approach.

The distribution of tension and shear, which produces the maximum predicted strength in the shear-dominated connection, was determined by the conditions of equilibrium. The tensile force in the inner row of anchors was assumed to vary between zero and the tensile force in the outer row of anchors. Additionally, the location of the inner row of anchors was assumed at various locations between the compressive reaction and the outer row of anchors. The details of this parameter study are provided in Cook and Klingner (1989). The results of the parameter study indicated that the maximum predicted strength of the connection occurs when the tensile force in the inner row of anchors is equal to the tensile force in the outer row of anchors. The assumption of equal tension also implies equal shear in all the anchors in the tension zone.

Analytically, the assumption of equal tension and shear in all the anchors in the tension zone is very convenient. The forces in all the anchors in the tension zone can be considered as a single force acting at the centroid, d, of the anchors in the tension zone, the same as in the moment-dominated area of behavior. This is shown in Fig. 10.

The conditions of equilibrium for the typical connection shown in Fig. 10 with $e < e''$ and with elliptical tension/shear interaction, give the maximum predicted strength of the connection:

$$V_{ut} = \gamma T_0 \frac{ma + \sqrt{n^2(a^2 + b^2) - m^2b^2}}{a^2 + b^2} \quad \text{(5)}$$

For the more conservative assumption of linear tension/shear interaction, the maximum predicted strength of the connection when $e < e''$ is given by:

$$V_{ut} = \gamma T_0 \frac{m + n}{1 + (\gamma - \mu)\left(\dfrac{e}{d}\right)} \quad \text{(6)}$$

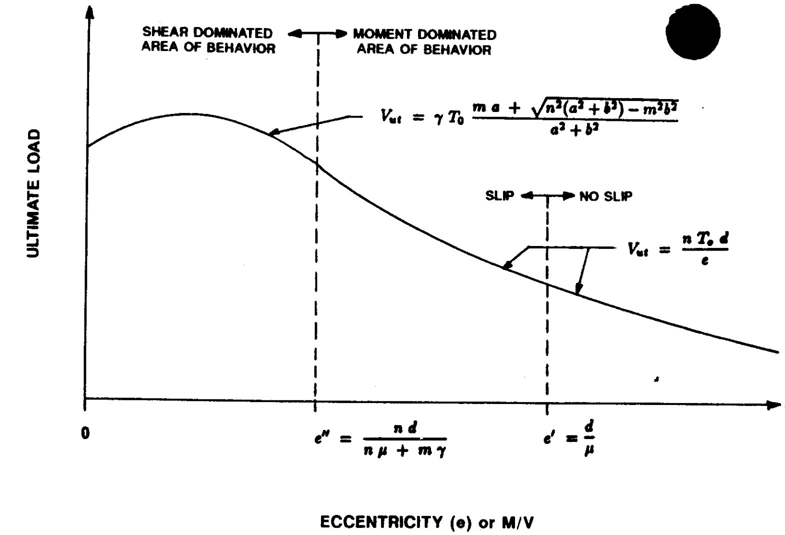


FIG. 11. Maximum Predicted Strength for Ductile Multiple-Anchor Connections

Fig. 11 shows the maximum predicted strength for ductile multiple-anchor connections over the full range of behavior.

Comparison of Behavioral Model with Experimental Results

Table 1 provides a comparison of the results of the tests to the connection strengths predicted by the behavioral model using elliptical tension/shear interaction. In Fig. 12, the predicted strengths for both elliptical and linear tension/shear interaction are graphically compared to the test results for the rigid base-plate specimens with cast-in-place and adhesive anchors. Graphical comparisons for undercut anchors were similar.

The ratio between the shear strength and the tensile strength of the anchor, γ, used in calculating the predicted strengths was taken as 0.50 for cast-in-place and adhesive anchors and 0.60 for undercut anchors. The compressive reaction is assumed to act at the toe of the base plate for the rigid base-plate tests, and at the distance determined by (1) for the flexible base-plate tests. For both graphical and tabular comparisons, the coefficient of friction, μ, used in calculating the predicted strengths, is 0.40. The tabular comparisons also include the predicted strengths calculated using a coefficient of friction, μ, of 0.50.

It should be noted that the comparisons are based on the actual tested tensile strength of the anchors and not on the minimum specified tensile strength (91%) of the tested strength) or yield strength (77% of the tested strength). For design purposes, either the minimum-specified tensile strength or the minimum-specified yield strength should be used with appropriate capacity reduction factors. Design recommendations are provided in Cook et al. (1989).

As indicated by Table 1, the predicted strengths agree closely with the test results. Table 1 also shows that the predicted strengths are not particularly sensitive to the assumed value of the coefficient of friction ($\mu = 0.40$ or $\mu = 0.50$).

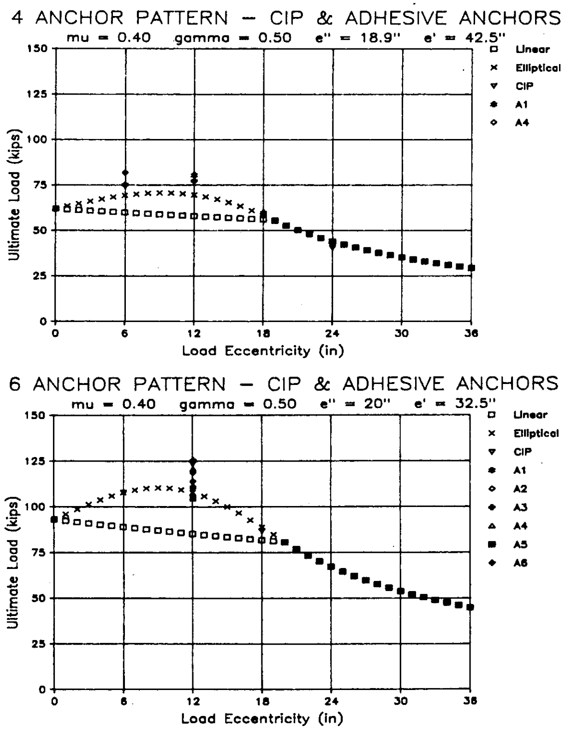


FIG. 12. Test Results versus Predicted Strengths for Rigid Base-Plate Specimens with Cast-in-Place and Adhesive Anchors: (*a*) Four-Anchor Pattern; and (*b*) Six-Anchor Pattern

CONCLUSIONS

1. A design procedure based on a lower-bound approach to limit design theory is appropriate for ductile multiple-anchor steel-to-concrete connections. In the application of a lower-bound approach, the designer assumes a distribution of forces to the anchors that satisfies equilibrium and does not exceed the individual anchor capacities. The resulting predicted strength is less than, or at best equal to, the true strength of the connection.

2. Ductile steel-to-concrete connections can be divided into two distinct areas of behavior depending on the moment-to-shear ratio of the applied loading:

   A. An area dominated by the applied moment. For connections in the moment-dominated area of behavior, the anchors in the tension zone can be assumed to attain their tensile strength prior to failure of the connection. In this case, the combined shear strength, which is provided by the frictional force at the steel/concrete interface (due to the compressive reaction from the applied moment) and by the shear strength of anchors in the compression zone, exceeds the applied shear. The strength of these connections is controlled by the tensile strength of the anchors in the tension zone.

   B. An area dominated by the applied shear. For connections in the shear-dominated area of behavior, the anchors in the tension zone can be assumed to act as a single composite anchor acting at the centroid of the anchors in the tension zone. The strength of this composite anchor is limited by the anchors' tension/shear interaction relationship. In this case, anchors in the compression zone can be assumed to be at their maximum shear strength. The strength of these connections is controlled by the shear strength of the anchors in the compression zone, coupled with the combined tensile and shear strength of the anchors in the tension zone.

3. Base-plate flexibility affects the assumed location of the compressive reaction from the applied moment. To locate the compressive reaction from the applied moment in a conservative manner, the reaction can be considered to be located at a distance, x_{min}, determined by (1), from the outer edge of the compression element of the attached member. If the base-plate thickness is unknown, it is conservative to consider the compressive reaction to be located directly under the outer edge of the outermost compression element of the attached member.

4. The design recommendations resulting from this study are incorporated into a design guide for steel-to-concrete connections (Cook et al. 1989).

ACKNOWLEDGMENTS

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APPENDIX I. CONVERSION TO SI UNITS

To convert	To	Multiply by
in.	mm	25.04
psi	MPa	6,890
ksi	Mpa	6.89
kip	kN	4.45

APPENDIX II. REFERENCES

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