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Interesting Aspects of the Empirical Wall Design Equation

By K. M. KRIPANARAYANAN

Chapter 14 of the ACI Building Code (318-71) indicates an equation for estimating the capacity of "concentrically" loaded walls having minimum amount of reinforcement. This equation is reviewed historically and evaluated critically with regard to reinforcement, slenderness, and eccentricity considerations. It is shown that: (a) the minimum amount of reinforcement required by code has negligible effect on wall capacity, and (b) eccentricity effects are satisfactorily accounted for by the design equation. However, the slenderness provisions of this design equation should be modified to conform to current experimental data.

Keywords: **bearing walls; building codes;** compressive strength; **eccentricity;** loads (forces); **reinforced concrete;** reinforcing steels; **slenderness ratio;** stresses; structural analysis; walls.

■ A REVIEW OF THE ACI BUILDING CODES[1] indicates that the development of design procedures for load-bearing concrete walls has been at a much slower pace than for other reinforced concrete members.

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Representatives from four national societies (the American Society for Testing and Materials, the American Society of Civil Engineers, the American Railway Engineering and Maintenance of Way Association, and the Association of Portland Cement Manufacturers) formed the Joint Committee on Reinforced Concrete in 1908. This joint committee reported that incompatibility existed between theory and tests of reinforced concrete members. Several interim reports were produced (based on research at universities, governmental agencies, etc.) from 1909 to 1916. Their final report in 1916 was the first code, a forerunner of the form and content of the reinforced concrete building code in use today.[2]

The 1928 ACI Building Code[3] limited compressive stresses in bearing walls to 0.125 f_c' for walls with an l_u/h of 15. A linear reduction was applied beyond this point up to a minimum of $0.0625 f_c'$ when the height of the wall was 25 times its thickness.

In the 1936 version of the code,[4] the allowable service load compressive stress was increased to $0.20 f_c'$ for walls with an l_u/h of 10 or less and decreasing proportionately to $0.11 f_c'$ for walls with an l_u/h of 25.

Section 1112 of the 1941 ACI Building Code[5] increased the allowable working stress under compressive loads to $0.25 f_c'$ for walls with a height-to-thickness ratio of 10 or less, and reduced linearly to $0.15 f_c'$ for walls having a height-to-thickness ratio of 25. The 1956 ACI Building Code[6] retained the same allowable stresses and height-to-thickness requirements as the 1941 ACI Building

Code. The 1963 ACI Building Code[7] presented the first major change in the design procedure since the original 1928 provisions by introducing the following equation for allowable compressive stress:

$$f_c = 0.225 f_c' [1 - (l_u/40h)^3] \quad (1)$$

where

f_c = allowable stress
f_c' = specified compressive strength of concrete
l_u/h = height-to-thickness ratio of the wall

The above equation resulted from the recommendation that the equation in the Uniform Building Code for allowable compressive stress be used.* That equation, $0.2f_c' [1.0 - (l_u/30h)^3]$, which was intended for walls with minimum reinforcement, appeared in the 1943 edition of the UBC published by the Pacific Coast Building Conference (now the International Conference of Building Officials). ACI Committee 318 adjusted the UBC equation to yield results fairly consistent with what had been used by ACI since 1941. The coefficient 0.225 was chosen originally to agree with the coefficient being considered for columns. When the column coefficient was later changed to 0.25, the coefficient for the wall equation was left unchanged.

The reduction of the allowable stress in the 1963 ACI Building Code for short walls from $0.25f_c'$ (which had been used since 1941) to a value of $0.225f_c'$ caused an extensive discussion in the profession. Also controversy existed over the term "reasonably concentric loads" in the 1963 ACI Building Code for the position of the load.

The 1971 ACI Building Code[2] adopted the "strength design method" as the principal design procedure with the wall design equation given as:

$$P_u = 0.55 \varphi f_c' A_g [1.0 - (l_u/40h)^2] \quad (2)$$

where

P_u = factored vertical load on the wall
f_c' = specified compressive strength of concrete
A_g = cross-sectional area
l_u/h = height-to-thickness ratio of the wall
φ = capacity reduction factor

The 1971 ACI Building Code defined "reasonably concentric loads" as those applied within the middle third of the cross section. In lieu of Eq. (2), the code allowed the design of wall elements as columns.

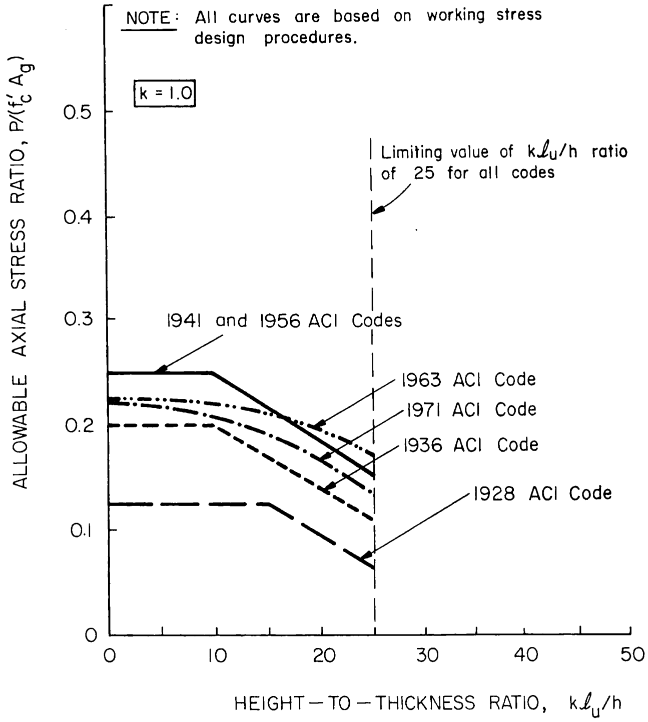

Fig. 1—Empirical wall design equation by various ACI building codes

Fig. 1 shows the development of the empirical wall design equation in the ACI Building Code from 1928 to 1971.

EVALUATION

Eq. (2) can be considered as a product of two functions as follows:

$$\frac{P_u}{\varphi f_c' b h} = F_1 \times F_2 \quad (3)$$

where

F_1 = 0.55, and is a function of eccentricity
F_2 = $[1 - (l_u/40h)^2]$, and is a function of slenderness

Eccentricity and reinforcement considerations

To evaluate F_1, interaction diagrams were drawn for walls with thicknesses of 8, 10, and 12 in. The walls had various percentages of reinforcement. The results are shown in Fig. 2 which were obtained using the PCA computer program.[8]

It is interesting to note that the provision of the minimum wall reinforcement as required by Section 14.2 of the ACI Building Code does not substantially increase the capacity of the walls. Also, it is shown in Table 1 that the use of 0.55 for F_1 in Eq. (2) is not only rational but realistic. Note that the increase in the axial load-carrying ca-

*Gibbons, A. T., "Summary of Evolution of Wall Provisions of ACI Code," Letter, Portland Cement Association, February 1973.

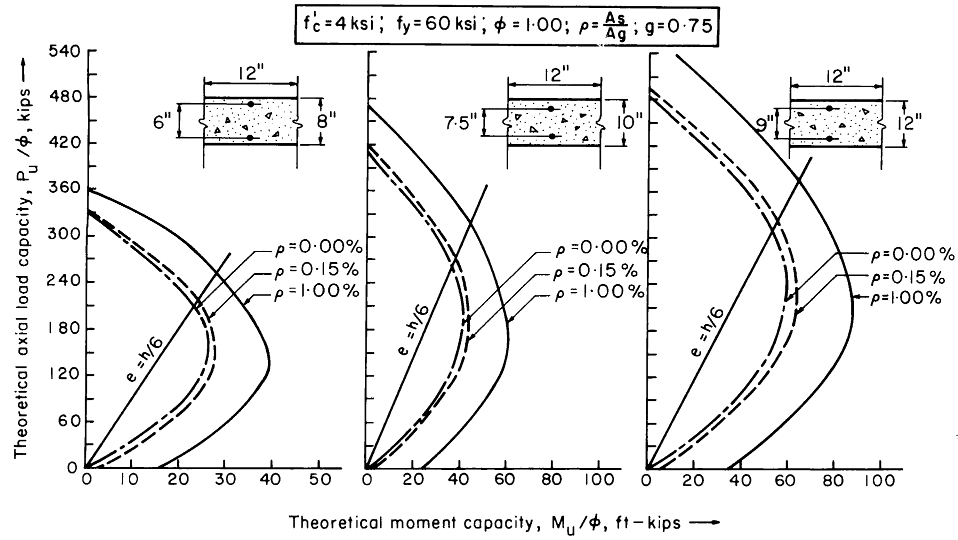

Fig. 2—Typical interaction charts for 8, 10, and 12 in. walls

pacity for reasonably concentric loads due to the presence of the minimum amount of reinforcement is of the order of 3 to 4 percent. To create a substantial increase in wall capacity, it is essential to have at least 1.0 percent of the gross cross-sectional area as reinforcement.

It is also obvious that the value of 0.55 for F_1 is valid only if lateral loads (perpendicular to the plane of the wall) are absent.

Slenderness considerations

Eq. (2) approximates the slenderness effects by the function F_2. In Fig. 3, this equation is compared with the elaborate slenderness analysis procedure of ACI Building Code, Section 10.11. In Fig. 3, the Salse-Fintel stress-strain relationship for concrete[9] was used in generating the curve for the slenderness procedure of Section 10.11. Note that the approximation is reasonable in the low slenderness ranges (i.e., $kl_u/h < 12$). However, in the medium-to-high slenderness ranges (i.e., $kl_u/h < 15$), the use of the empirical design equation tends to overestimate the wall capacities, especially for the case of pin-ended walls. This has been confirmed experimentally by Oberlander.[1]

Hence, it is suggested that Eq. (2) [ACI Building Code Eq. (14-1)] be modified as follows:

$$P_u = 0.55\,\varphi f_c' bh\left[1 - \left(\frac{kl_u}{32h}\right)^2\right] \qquad (4)$$

Fig. 4 shows the comparison between the experimental data of Reference 1 and the computed results using Section 14.2 as well as the proposed Eq. (4). Note that Eq. (4) predicts the experimental values of load capacities conservatively in the medium-to-high slenderness ranges.

The incorporation of the effective length factor k into the design equation will extend the applicability of the equation to a wider range of support conditions. The determination of k for a

TABLE 1—AXIAL LOAD CAPACITIES UNDER REASONABLY CONCENTRIC LOADS

Wall thickness, in.	$P_u/(\varphi f_c' bh)$ at $e = h/6$		
	$A_s/A_g = 0.00$	$A_s/A_g = 0.0015$	$A_s/A_g = 0.0100$
8	0.547	0.562	0.656
10	0.550	0.568	0.662
12	0.542	0.563	0.656
Average	0.546	0.564	0.658

Note: A wall is considered as being under reasonably concentric loads if the resultant of the loads falls within the kern of the section.

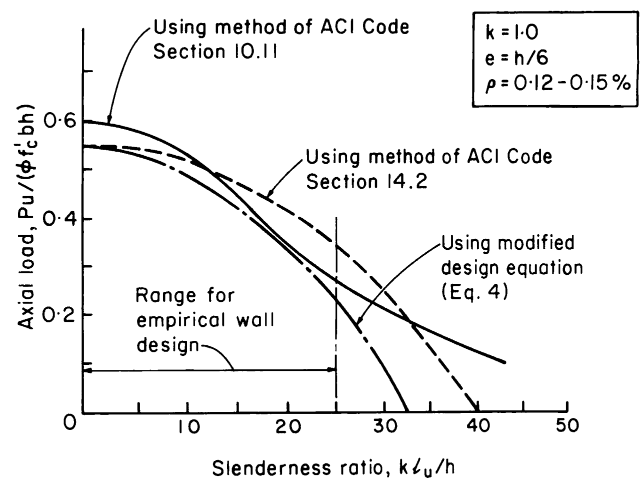

Fig. 3—Proposed and current methods of slenderness evaluation in empirical wall design

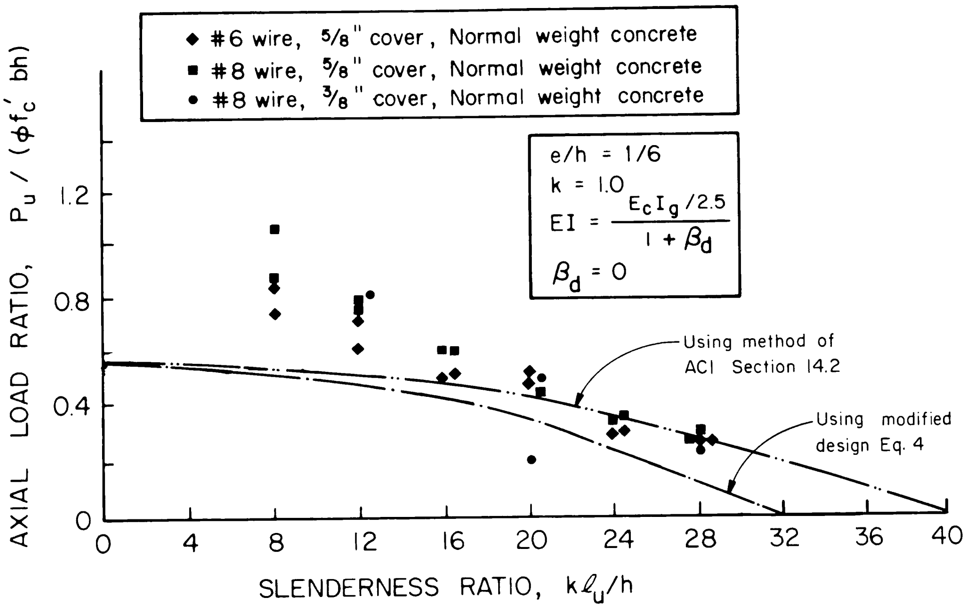

Fig. 4—Comparison of experimental results of Reference 1 with empirical wall design equations

particular set of support conditions can be left to engineering judgment.

SUMMARY AND CONCLUSIONS

An historical review of the empirical wall design equation indicates that the allowable compressive stress under reasonably concentric loads for service conditions has increased from $0.125f_c'$ to $0.22f_c'$ from 1928 to 1971. This increase was based partly on research and partly on experience.

Additional conclusions regarding the empirical wall design equation are:

1. The strength part of the design equation [i.e., F_1 in Eq. (2)] gives a satisfactory estimation of short wall capacities for both plain and minimally reinforced wall elements under reasonably concentric loads. Its application, however, is not valid for walls having area loads (i.e., loads perpendicular to the plane of the wall).

2. A substantial increase in wall capacity can be obtained only if the amount of vertical reinforcement is of the order of 0.75 to 1.0 percent of the gross cross-sectional area of the wall.

3. The slenderness part of the design equation [i.e., F_2 in Eq. (2)] does not give a realistic estimate of capacities for walls with pin-ended supports. It is proposed that F_2 be modified as:

$$F_2 = \left[1 - \left(\frac{kl_u}{32h} \right)^2 \right]$$

to allow for a wider range of application as well as for conformance with current experimental evidence.

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