Preventing Progressive Collapse of Slab Structures

By Denis Mitchell,[1] M. ASCE and William D. Cook[2]

Abstract: The response of slab structures after initial failure is investigated in order to determine a means of preventing progressive collapse. Analytical models for predicting the post-failure response of slabs are presented and the predictions are compared with experimental results. These analytical models along with an experimental investigation enabled the development of simple design and detailing guidelines for bottom slab reinforcement which is capable of hanging the slab from the columns after initial failures due to punching shear and flexure.

Introduction

Collapses of slab structures at Bailey's Crossroads (16), Boston (17), Kalamazoo (10) and Cocoa Beach (15) indicate the need to carefully design and detail reinforced concrete slabs to prevent progressive collapse in the event of a local failure. A description of initiation of failures and propagation of collapse in flat plate structures can be found in Ref. 11.

The key in preventing progressive collapse may be to design and detail slabs such that they are able to develop secondary load carrying mechanisms after initial failures have occurred. Fig. 1(*a*) shows an idealized load-deflection response for a slab fully restrained at its edges and containing continuous reinforcement well anchored into the supports. If the continuous reinforcement is properly anchored, then the slab, after initial failure and after experiencing extremely large deflections, starts to develop tensile membrane action. In this region of response the reinforcing mat acts as a "hanging net" and provided the reinforcement is well anchored, final collapse will occur due to rupture of the reinforcement. Fig. 1(*b*) gives the idealized load-deflection response of an interior panel of a two-way slab structure. After initial failure has occurred the reinforcement must be designed and detailed such that the slab will be able to hang off of the columns. The secondary load resisting mechanism that forms must be capable of developing after punching shear failures have occurred around the columns and general flexural yielding has occurred in the slab. If the final collapse load is higher than the initial failure load then a means of preventing progressive collapse has been provided. The behavior of these slab systems together with the special design and detailing requirements for the reinforcement will be discussed in this paper.

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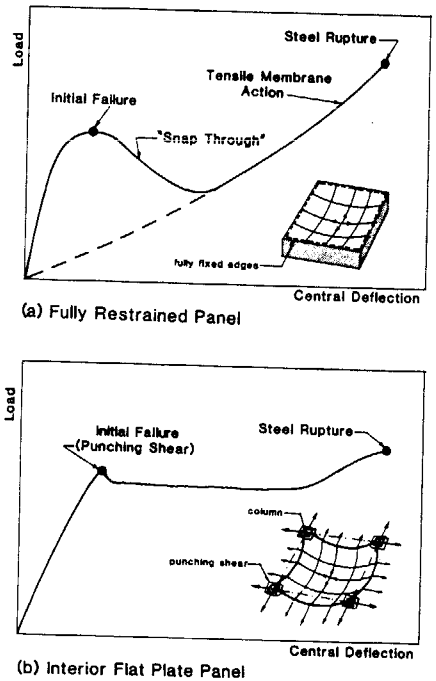
Note.—Discussion open until December 1, 1984. To extend the closing date one month, a written request must be filed with the ASCE Manager of Technical and Professional Publications. The manuscript for this paper was submitted for review and possible publication on June 10, 1983. This paper is part of the ***Journal of Structural Engineering***, Vol. 110, No. 7, July, 1984. ©ASCE, ISSN 0733-9445/84/0007-1513/$01.00. Paper No. 19010.



(a) Fully Restrained Panel

(b) Interior Flat Plate Panel

FIG. 1.—Idealized Response of Slab Panels

Analytical Models for Predicting Tensile Membrane Response

Membrane Equation.—Park (18) derived an equation describing the load versus central deflection response of a uniformly loaded rectangular reinforced slab fully restrained at its edges. This relationship is given:

$$\frac{wl_x^2}{T_x\delta} = \frac{\pi^3}{4\sum\limits_{n=1,3,\ldots}^{\infty}\left(\frac{1}{n^3}\right)(-1)^{0.5(n-1)}\left\{1-\left[\cosh\left(\frac{n\pi l_y}{2l_x}\sqrt{\frac{T_x}{T_y}}\right)\right]^{-1}\right\}} \quad \ldots\ldots\ldots (1)$$

in which w = uniformly distributed load, per unit area of the slab; δ = central deflection corresponding to load, w; l_x, l_y = the clear span in the short and long directions respectively; and T_x, T_y = yield force of the reinforcement per unit width, in the l_x and l_y directions respectively.

This equation assumes that all the reinforcement has yielded, that the reinforcement has a bilinear, elastic, fully plastic stress-strain relationship and that the concrete carries no tension.

For an isotropically reinforced square panel the dimensionless parameter $(wl_x^2/(T_x\delta))$ in Eq. 1 becomes 13.56. Park suggested that the use of Eq. 1 together with an assumed deflection equal to 0.10 times the shorter clear span would give a conservative estimate of the tensile membrane strength. Black (4) in 1975 suggested that a more accurate prediction of strength could be obtained by using Park's equation with a dimension-

less parameter of 20.0 rather than 13.56 together with a limiting deflection of 0.15 times the shorter clear span.

Simplified Iterative Method.—Hawkins and Mitchell (11) reported on a simplified iterative method for determining the tensile membrane response of slabs having in-plane restraint at their edges. The method assumes that the membrane takes a circular deformed shape and that the concrete carries no tension. The complete load deflection response can be predicted by using Eqs. 2 and 3, are given, together with the stress-strain relationship of the reinforcement:

$$w = \frac{2T_x \sin\sqrt{6\epsilon_x}}{l_x} + \frac{2T_y \sin\sqrt{6\epsilon_y}}{l_y} \quad \ldots\ldots\ldots (2)$$

in which w = predicted load, per unit area; l_x, l_y = the clear span in the short and long directions, respectively; T_x = the force in the reinforcement per unit length in the x-direction corresponding to a strain, ϵ_x; T_y = the force in the reinforcement per unit length in the y-direction corresponding to a strain, ϵ_y; ϵ_x = the strain in the x-direction; and ϵ_y = the strain in the y-direction assumed to be equal to $\epsilon_x l_x^2/l_y^2$. The relationship between the central deflection, δ, the geometry of the panel and the strain in the reinforcement is:

$$\delta = \frac{3l_x\epsilon_x}{2\sin\sqrt{6\epsilon_x}} \quad \ldots\ldots\ldots (3)$$

The complete load versus central deflection response can be obtained by using the following solution technique: (1) Choose a value of ϵ_x; (2) calculate ϵ_y; (3) determine T_x and T_y corresponding to ϵ_x and ϵ_y respectively, using the actual stress-strain relationship for the reinforcement; (4) calculate the load, w, using Eq. 2; and (5) calculate the deflection, δ, using Eq. 3. These steps give one point on the predicted load-deflection response. By repeating the steps for different values of ϵ_x, the complete tensile membrane response up to the rupture of the reinforcement can be predicted.

Nonlinear Computer Program.—A computer model was developed (7) to predict the post-failure response of slabs. The slab reinforcement mesh is modeled by a grid of truss elements. In order to predict the response of the grid which experiences large deflections, the program accounts for the nonlinearities arising from the changes in geometry (nonlinear geometry) as well as the nonlinearities arising from changes in element stiffnesses (nonlinear material). The computer program which utilizes the stiffness method of analysis takes account of the nonlinearities by employing a step-wise iterative solution technique.

Three different types of stress-strain relationships are available for the steel reinforcement. These include: (1) A linear relationship; (2) a line segment model capable of modeling the true stress-strain relationship of typical reinforcing steel; and (3) an Inverse Ramberg-Osgood Polynomial to enable curved stress-strain relationships to be modeled.

Since the elements possess axial stiffness only and are horizontal when the applied load is zero, the idealized slab has no initial vertical stiffness. In order to overcome the numerical instability this creates, a vertical stiffness is formed by one of two methods: (1) By imposing an estimated deflection configuration corresponding to the initial load increment; or (2) by utilizing fictitious elasto-plastic vertical springs at all nodes. If the area of each spring is chosen to be proportional to the tributary area of each node then, provided all of the springs yield at an early stage, the applied vertical load, w, can be corrected by subtracting the fictitious uniform load created by the "yielded springs."

The complete tensile membrane response can be predicted by solving for the displacements for a number of load increments.

Responses of Fully Restrained Single Panels

Tests on single panel slabs have been reported by Wood (22), Park (18), Powell (20), Brotchie and Holley (5), Keenan (14), Black (4), and Ritz, Marti and Thurlimann (21). A summary of the membrane response of slab panels can be found in Ref. 19.

Fig. 2(a) compares the load versus central deflection response of Park's Specimen A3 with the predictions from the three methods of analysis described in the previous section. This slab was 40 in. (916 mm) by 60 in. (1,524 mm) by 2 in. (50.8 mm) thick and contained continuous bottom steel percentages of 0.590 and 0.161 in the short and long directions, respectively. This specimen also contained top reinforcement around the edges of the slab.

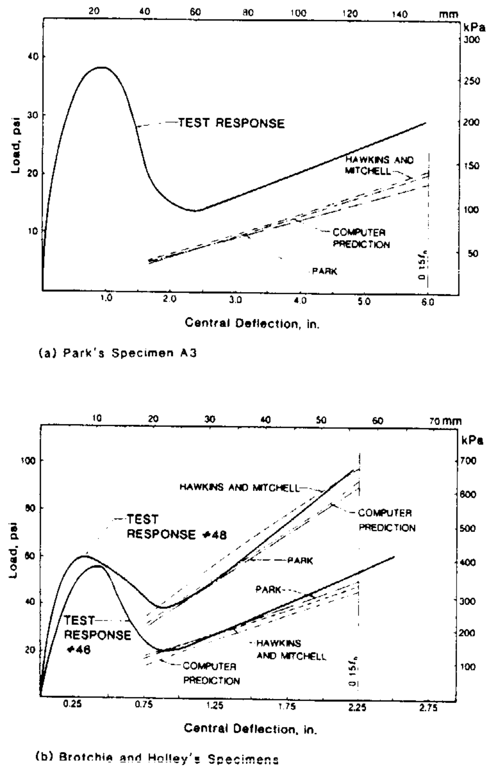


FIG. 2.—Comparisons of Predicted and Observed Responses for Fully Restrained Single Panels Tested by Park (Ref. 18) and Brotchie and Holley (Ref. 5)

Fig. 2(b) compares the predicted and measured responses of Specimens 46 and 48 tested by Brotchie and Holley. Specimen 48 was 15 in. (371 mm) by 15 in. (371 mm) by 0.75 in. (19.1 mm) thick and contained a mat of bottom reinforcement only, with a steel percentage of 1.49 in each direction. Specimen 46 was 15 in. (371 mm) square by 1.5 in. (38.1 mm) thick and contained a bottom mat of reinforcing with a steel percentage of 0.747.

The estimated strengths corresponding to deflections of 0.15 times the clear span, l_n, are also shown in Fig. 2(a) and 2(b). It can be seen that the predictions of the three methods are very similar. The conservative predictions shown in Fig. 2(a) for Park's specimen are probably due to the considerable amount of residual flexural stiffness and resistance around the fully restrained perimeter of the slabs due to the negative moment reinforcement which was neglected in making the predictions. All three predictions agree reasonably well (see Fig. 2(b)) with the response of Brotchie and Holley's slabs which did not contain negative moment reinforcement around the perimeter.

Role of Bottom Reinforcement Anchored into Supports

In order to prevent progressive collapse typical two-way slabs without beams must be capable of providing post-failure resistance in the presence of punching shear failures and severe distress around the columns. Fig. 3 demonstrates the marked difference in post-failure resistance of slab-column connections with and without continuous bottom reinforcement properly anchored into the columns. After shear failure has occurred the top reinforcement "rips-out" of the top surface of the slab (9,11) and becomes ineffective in carrying the load. A slab-column con-

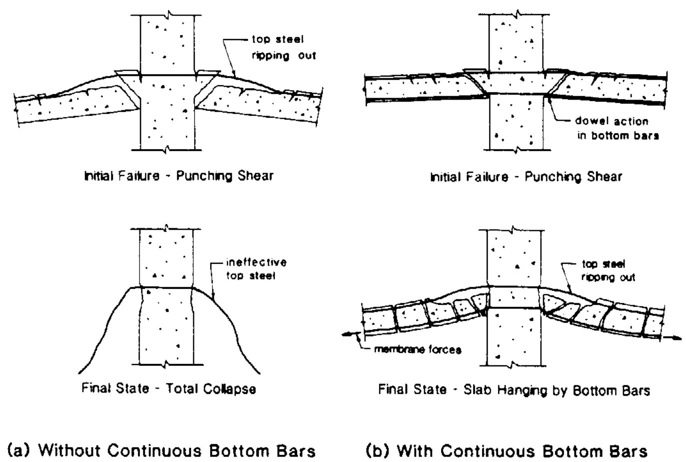


FIG. 3.—The Role of Bottom Reinforcement Anchored into Supports

(a) Plan View

(b) Elevation View

FIG. 4.—Post-Failure Condition of a Slab Hanging from an Edge Column by Anchored Bottom Bars (Ref. 6)

nection without bottom reinforcement properly anchored into the column would therefore have negligible post punching shear resistance which would result in the collapse of the slab and would likely cause progressive collapse of the structure (see Fig. 3(*a*)). In contrast bottom reinforcement well anchored into supports does not rip-out of the slab and thus provides significant dowel action, providing some post-punching shear resistance. If these bottom bars are both well anchored and effectively continuous then they will be capable of hanging the damaged slab off of the columns as tensile membrane action develops (see Fig. 3(*b*)). The photograph (6) shown in Fig. 4 of an edge slab-column connection having well anchored bottom bars illustrates the role of these bars in hanging up the slab. The severity of distress in this region after final collapse emphasizes the need to carefully detail these bottom bars.

Post Failure Response of Slab Structures

The response of a slab structure after initial failure depends on the amount and details of the steel reinforcement, the vertical support conditions and the horizontal restraint conditions at the panel edges. The post-failure responses of various slab structures are described below.

Slabs Simply Supported on all Sides without External Horizontal Edge Restraint.—Tests done by Brotchie and Holley (5) demonstrated the ability

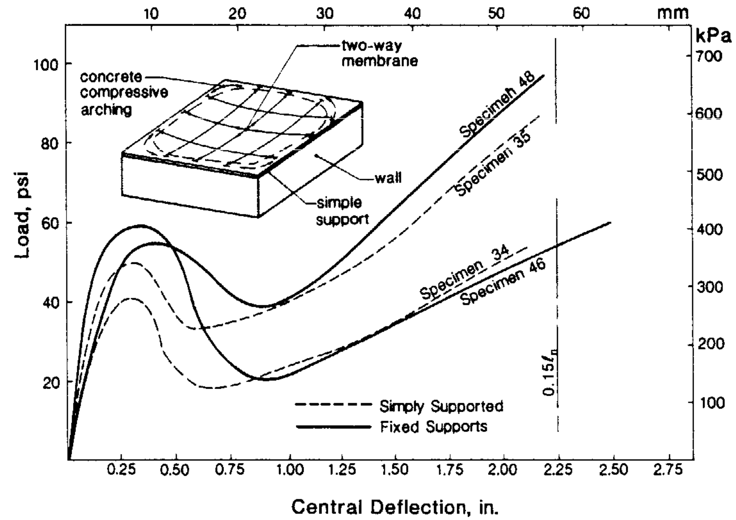


FIG. 5.—Effect of Edge Restraint on Response of Single Panel Slabs Tested by Brotchie and Holley (Ref. 5)

of a slab, simply supported around its edges but with no external horizontal restraint to develop tensile membrane action. The responses of their specimens 34 and 35 (without horizontal edge restraint) are compared in Fig. 5 with the responses of their specimens 46 and 48 (fully restrained at their edges). Although the responses of similarly reinforced specimens are different before initial failure due to the boundary conditions, they are essentially the same after large deflections have been attained. This effect has also been observed by Ritz, Marti and Thurlimann (21) in the testing of prestressed slabs which had a variety of edge support conditions. This is due to the ability of a slab which is vertically supported around its edges to provide its own in-plane edge restraint by forming a compressive ring of concrete around its perimeter. This compressive ring of concrete equilibrates the tensions in the tensile membrane. It is noted that if a single panel were allowed to deflect vertically around its edges (i.e., supported on four corner columns) then this compressive arching action could not develop at large deflections and therefore the internal horizontal edge restraint would not form. It is important to provide continuous bottom reinforcement well anchored around the perimeter of the slab in order to develop tensile membrane action.

Two-Way Slabs Supported on Extremely Stiff Beams.—A two-way slab panel supported on four sides by extremely stiff beams develops a two-way tensile membrane which "hangs-off" the beams as shown in Fig. 6(*a*). This type of structure is not sensitive to the region that is overloaded because of the ability of each panel to form its own tensile membrane. An interior panel, if overloaded would be capable of forming a tensile membrane restrained by adjacent regions of the structure. An edge or corner panel is capable of providing its own horizontal in-plane restraint provided that the free edges are supported by beams that are

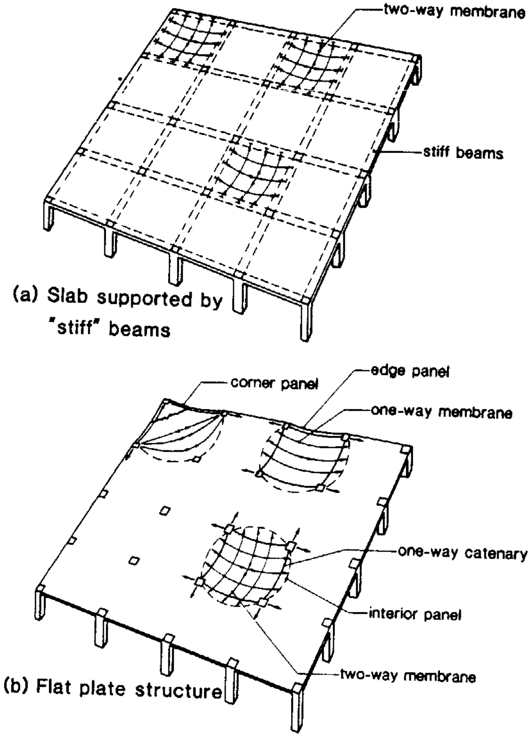
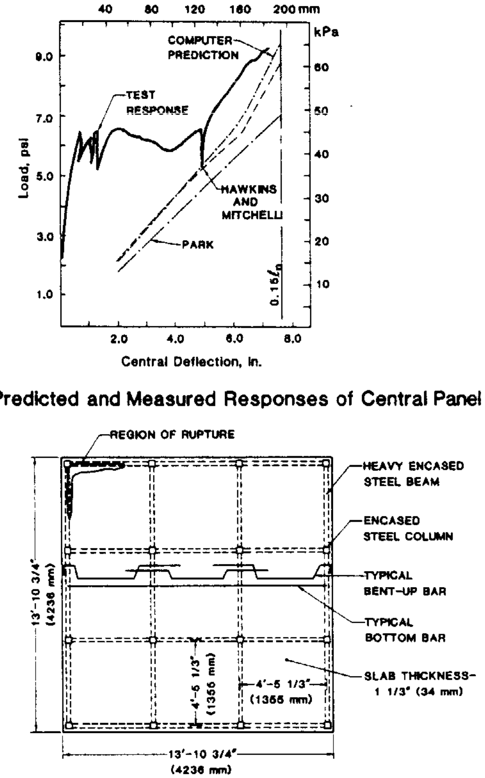


FIG. 6.—Development of "Hanging Nets" in Panels of Two-Way Slab Structures

capable of preventing significant vertical displacements throughout the entire tensile membrane response. It is essential to provide sufficient bottom reinforcement well anchored into the stiff supporting regions.

The behavior of a nine panel slab structure supported on encased steel beams which was tested by Huff (13) demonstrates the ability of structures with extremely stiff beams to develop tensile membrane action. This heavily reinforced structure was designed for blast resistance. Fig. 7(*a*) gives the predicted and measured load versus central deflection responses for the central panel while Fig. 7(*b*) shows the typical reinforcement details on the plan view of the structure. The slab was loaded over its entire area.

In predicting the tensile membrane response, it was assumed that the edges of the central panel were fully restrained. Fig. 7(*a*) shows the significant tensile membrane response that was obtained. The tensile membrane predictions, which are conservative, agree reasonably well with the measured response. The loading of the slab was discontinued due to rupturing of the reinforcing bars in a corner panel which contained a smaller amount of reinforcement than the central panel. The nonlinear computer analysis and simplified iterative analysis give reasonably good predictions of the load corresponding to a deflection of $0.15\ l_n$. The membrane equation gives more conservative results since no strain hardening of reinforcement was assumed.



(a) Predicted and Measured Responses of Central Panel

(b) Details of Slab Structure

FIG. 7.—Two-Way Slab Structure Supported on Stiff Beams Tested by Huff (Ref. 13)

Typical Two-Way Slab Structures.—If a two-way slab structure is overloaded over its entire surface it would probably not be capable of developing the necessary in-plane horizontal restraint required to develop tensile membranes even if it contained well-anchored, continuous bottom reinforcement. The vertical displacements along column lines do not allow significant internal horizontal restraint to develop. However, if only a portion of the structure is overloaded, then the remaining regions of the structure which do not experience significant vertical displacements and therefore remain essentially in a horizontal plane could offer horizontal in-plane restraint. An overloaded well detailed interior panel would form a two-way membrane which is supported by one-way catenaries along column lines which must in turn "hang-off" the columns (see Fig. 6(*b*)).

Fig. 6(*b*) also shows the likely post-failure resisting mechanism for an overloaded edge panel. The edge panel forms a one-way membrane which spans between adjacent edge panels taking advantage of the available horizontal in-plane restraint of these panels. This one-way membrane is supported by one-way catenaries which hang-off of the columns and are

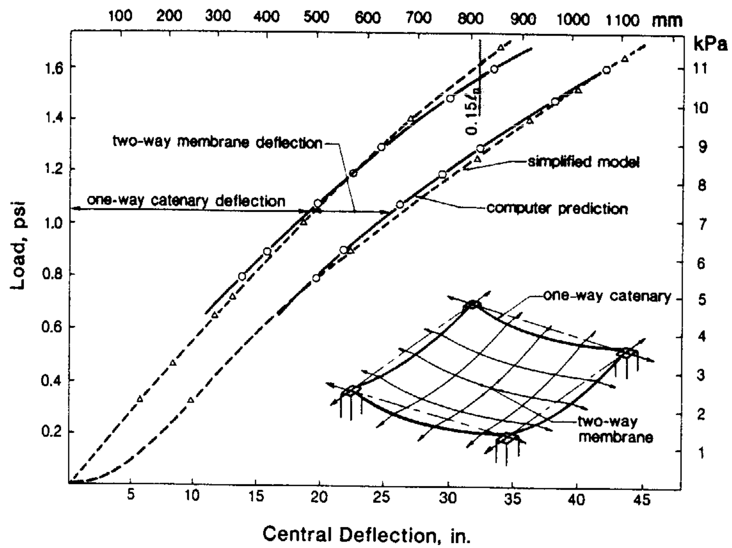


FIG. 8.—Predicted Load versus Central Deflection Response for a Typical Interior Flat Plate Panel

perpendicular to the free edge. The post-failure resisting mechanism for a corner panel (see Fig. 6(*b*)) involves folding of the slab across the corner and the development of one-way catenaries diagonally across the corner of the slab seeking the edge restraint offered by adjacent panels.

The nonlinear computer program presented in this paper is capable of predicting the tensile membrane response of slabs having a wide variety of boundary conditions. The computer program was used to analyze the tensile membrane response of a typical interior square panel of a flat plate structure. The span length, center-to-center of the 16 in. (406 mm) square columns, is 19′-0″ (5.8 m), and the slab thickness is 8 in. (203 mm). The additional dead load is 10 psf (0.48 kPa) and the live load is 40 psf (1.95 kPa). The reinforcement assumed to be effective in the tensile membrane was 50% of the reinforcement required for temperature and shrinkage over the entire panel. In addition 3-#4 (12.7 mm diam) continuous bottom bars along each column line were assumed to be well anchored into the columns. The predicted response is shown in Fig. 8.

Fig. 8 also shows a prediction using a model which assumes that the load is carried by a two-way tensile membrane which "hangs-off" one-way catenaries which, in turn, hang from the columns. This simplified model requires that both the membrane and the catenary be capable of resisting the loads. The two-way membrane response for an interior panel can be obtained by using Eqs. 2 and 3.

It is assumed that the one-way catenary is subjected to a uniform load equal to 0.5 wl_2 per unit length. This arises from the vertical components of the forces in the bars of the two-way membrane(s) which are hanging off of the one-way catenary. Thus, from Eq. 2 the one-way catenary response can be described as:

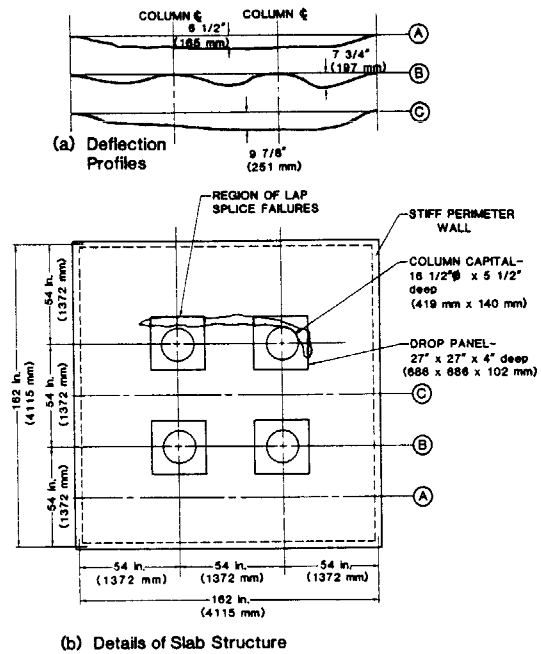


FIG. 9.—Flat Slab Structure Tested by Criswell (Refs. 8 and 9)

$$w = \frac{4T_c \sin \sqrt{6\epsilon}}{l_n l_2} \qquad (4)$$

in which T_c = force in the anchored bottom reinforcement in the one-way catenary; l_n = clear span of the one-way catenary; and l_2 = distance measured from the centerline of the panel on one side of the catenary to the centerline of the panel on the other side of the catenary.

The deflection in the one-way catenary corresponding to this w is given by Eq. 3. For a given value of w, the panel central deflection can be approximated by adding the calculated maximum deflection of the one-way catenary to the calculated maximum deflection of the two-way membrane. As can be seen from Fig. 8, the simplified model agrees reasonably well with the computer prediction. Although practical designs result in relatively small amounts of reinforcement in the middle regions of a panel, it can be shown that the post-failure capacity for typical structures is governed by the capacities of the one-way catenaries.

The nine panel flat slab structure tested by Criswell (8,9) offers some insight into the likely response of such structures. The structure shown in Fig. 9 contained both drop panels and column capitals. Uniform load was applied to the entire surface of the slab by water pressure. Tension lap splices were used to provide continuity of the top and bottom reinforcement. However at some drop panel edges all the top and bottom reinforcement was lap spliced. Although these lap splices performed well up to and beyond the initial failure load they failed before the full tensile membrane capacity could be reached. The large deflections obtained before final collapse (see deflection profiles of slab in Fig. 9) indicate that tensile membrane action was beginning to develop. The results of this test emphasize the need to carefully choose the location of tension lap splices in order to achieve effectively continuous bottom reinforcement. Criswell reported (9) on the presence of "portions of the slab . . . which acted, along with the top of the wall as deep horizontally-placed beams spanning the side lengths of the slab structure." The presence of the perimeter wall which prevented vertical edge displacements of the slab enabled the slab to develop its own horizontal in-plane restraint in the form of these deep ring beams.

Hopkins and Park (12) reported the results of testing a nine panel two-way slab supported on beams. The slab structure had continuous bottom reinforcement in the slab and the beams. This experiment illustrated the ability of such a structure to fully develop tensile membrane action.

An experimental program was carried out at McGill University (3,6,7) to study the post-failure response of flat plate corner panels. Corner panels were tested in order to determine whether or not post-failure resistance could be obtained in a region of a structure having the least amount of horizontal restraint. The one-quarter scale test structure, shown in Fig. 10, consisted of four panels (59 in. (1,500 mm) square) supported on nine 4 in. (100 mm) square columns. The slab thickness was 1.9 in. (47.5 mm). One panel at a time was loaded with up to 36 point loads to collapse while the remaining three panels had a uniform loading applied by bricks to correct for the modeling of the self weight of the slab. The uniform pressure versus central deflection response for three of the corner panel tests are given in Fig. 10. All three specimens had identical spans, column sizes and slab thicknesses and similar concrete material properties. The slabs were detailed in accordance with the ACI Standard 318-77 (1) with the exception of the bottom reinforcement. The bottom bars were extended to within the clear cover distance at the free edges of the slab and every second bottom bar was lapped with the bottom bars of adjacent panels in order to fully develop the capacity of these bars. In addition the bottom bars were lap spliced over columns to hang the slab from the columns after initial punching shear failure.

Corner panels CP1 and CP2 contained 2-D2 and 4-D2 [area of one D2 is 0.02 $in.^2$ (12.9 mm^2)] continuous bottom bars respectively, which were anchored into the columns. All other reinforcement was identical. The additional anchored bottom bars in panel CP2 resulted in a larger initial failure load and a larger collapse load than was obtained in panel CP1. Corner panel CP3 which had a 4 in. × 4 in. (101.6 mm × 101.6 mm) spandrel beam had 3-D4 [area of one D4 is 0.04 $in.^2$ (25.8 mm^2)] bottom bars well-anchored into the columns. The beneficial effects of strengthening and stiffening the free edges of a corner panel are evident if the responses given in Fig. 10 are compared.

Fig. 11 shows panels CP2 and CP3 after final collapse. The corner columns in both structures did not offer significant in-plane horizontal restraint. After initial failure of CP2 the slab folded across the corner (see

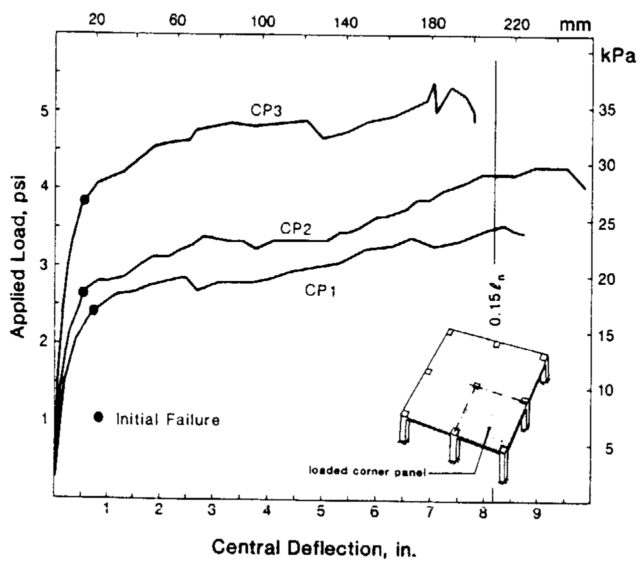
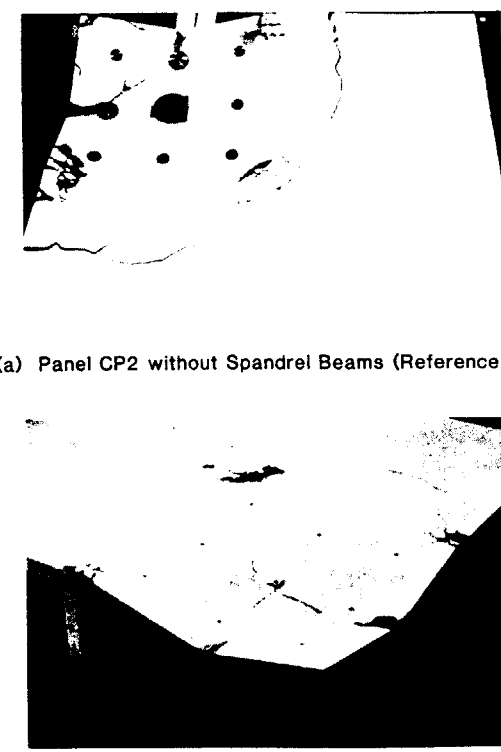


FIG. 10.—Flat Plate Corner Panels Tested at McGill University (Refs. 3 and 6)



(a) Panel CP2 without Spandrel Beams (Reference 6)

(b) Panel CP3 with Spandrel Beams (Reference 3)

FIG. 11.—Photographs of Flat Plate Corner Panels at Final Collapse

Fig. 11(a)) and developed a tensile membrane diagonally across the corner panel as shown in Fig. 6(b). Maximum capacity was reached when the bottom bars ruptured at the edge of the panel at the locations of folding. The cracking which propagated through the entire thickness of the slab in the middle regions of the panel is evidence of tensile membrane action. The severe cracks in adjacent edge panels (see Fig. 11(a)) are evidence of the transfer of tension into the region of the structure which is providing the horizontal in-plane restraint. Figs. 4 and 11 show the severe distress around the columns as the top reinforcement rips-out of the slab surface and the slab hangs-off of the columns by the anchored bottom bars.

Fig. 11(b) shows the large deformations of corner panel CP3 having spandrel beams. This panel also showed signs of tensile membrane action in the central regions of the corner panel. Maximum capacity was reached as the bottom bars ruptured in the spandrel beams.

Design and Detailing Recommendations.—An initial failure in a slab structure may be caused by an overload, a design error, a construction error, the loss of a supporting member or a combination of these. The ability of a structure to develop post-failure resistance and prevent progressive collapse depends on the extent of the initial failure region, the type of slab structure, and the amount and details of the reinforcing steel.

Typical two-way slab structures cannot develop sufficient horizontal restraint at the edges to enable full tensile membrane action when the entire slab is overloaded. These structures can, however, form a secondary load resisting mechanism if only localized regions of the structure are overloaded (see Fig. 6(b)).

In order to design and detail for tensile membrane action it is useful to first prepare a sketch of the structural system showing the expected membranes and catenaries in order to appreciate the flow of forces. In general the membranes form a path towards vertical supporting members and regions of horizontal restraint (See Fig. 6).

Analyses indicate that a portion of the bottom reinforcement required for flexural design must extend into and be well anchored in each column such that the slab is capable of hanging off of the column after a punching shear failure. If a limiting deflection of 0.15 l_n is used for the one-way catenary, then from Eqs. 3 and 4, the area of continuous bottom steel, A_{sb}, required in the direction l_n can be written in the form of a design equation as:

$$A_{sb} = \frac{0.5\, w_s l_n l_2}{\phi f_y} \qquad (5)$$

in which A_{sb} = minimum area of effectively continuous bottom bars in

TABLE 1.—Determination of A_{sb} for Various Slab Systems

Type of structure and span (1)	Live load, in pounds per square foot (kilonewtons per square meter) (2)	Additional dead load, in pounds per square foot (kilonewtons per square meter) (3)	Thickness h, in inches (millimeters) (4)	w_s, in pounds per square foot (kilonewtons per square meter) (5)	Dimensions of reaction area, in inches (millimeters) (6)	A_{sb} ANCHORED INTO REACTION AREA: Interior Span: A_{sb} (7)	Interior Span: Bar choice (8)	Edge Span: A_{sb} (9)	Edge Span: Bar choice (10)
(a) Flat plate 18 ft × 18 ft (5.49 m × 5.49 m)	40 (1.92)	10 (0.48)	7.5 (191)	187.5 (8.98)	16 (406) square columns	0.52 in.^{2} (335 mm^2)	3-#4 (2-#15M)	0.28 in.^{2} (181 mm^2)	2-#4 2-#10M
(b) Flat slab with drop panel 18 ft × 18 ft (5.49 m × 5.59 m)	100 (4.79)	10 (0.48	8 (203) plus 3 (76) drop panel	214 (10.25)	16 (406) diam circular columns	0.60 in.^{2} (387 mm^2)	3-#4 (2-#15M)	0.32 in.^{2} (208 mm^2)	2-#4 (2-#10M)
(c) Flat slab with drop panel plus capital 25 ft × 20 ft (7.62 m × 6.10 m)	100 (4.79)	20 (0.96)	7.5 (1.90) plus 3 (76) drop panel	218 (10.43)	60 (1,524) diam interior capital 54 (1,372) × 35 (889) exterior capital	0.83 in.^{2}[a] (536 mm^2)	5-#4 (3-#15M)	0.44 in.^{2}[a] (285 mm^2)	3-#4 (3-#10M)
(d) Flat plate post-tensioned unbonded 25 ft × 20 ft (7.62 m × 6.10 m)	40 (1.92)	15 (0.72)	6.5 (165)	162.5 (7.78)	18 (457) square columns	0.25 in.^{2}[a] (161 mm^2)	2-0.5 in. (13 mm) strands	0.13 in.^{2}[a] (87 mm^2)	1-0.5 in. (13 mm) strand

[a]Calculations given for larger span direction only.
Note: f_y assumed to be 60 ksi (414 MPa); f_{ps} assumed to be 170 ksi (1,172 MPa).

direction l_n placed through reaction area of supports; w_s = the load to be carried after initial failure, assumed to be the larger of the total service load acting on the slab or twice the slab dead load; l_n = clear span, in the direction being considered, measured face-to-face of supports; l_2 = distance measured from the centerline of the panel on one side of the catenary to the centerline of the panel on the other side of the catenary; f_y = specified yield strength of non-pretressed reinforcement; and ϕ = capacity reduction factor, 0.9 for tension.

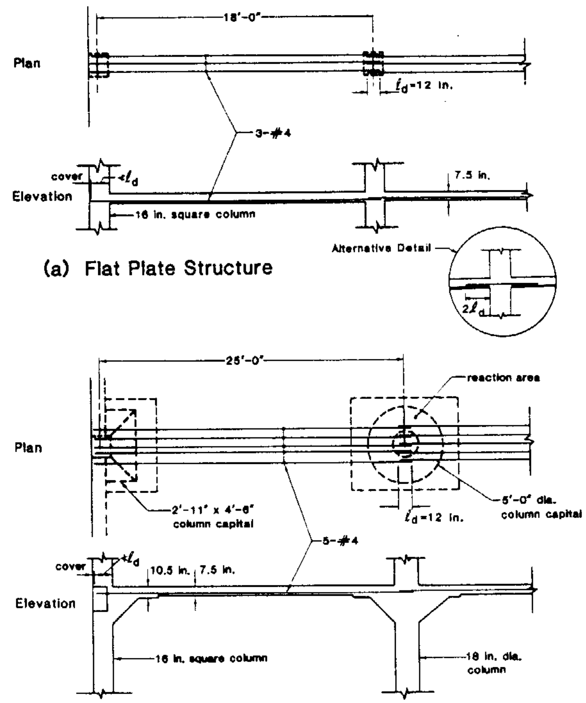
The bottom bars having an area of A_{sb} may be considered effectively continuous if: (1) They are lap spliced within a support reaction area with reinforcement in adjacent spans with a minimum lap length of l_d; or (2) they are lap spliced immediately outside of a support reaction area with a minimum lap splice of $2l_d$ provided the lap splice occurs within a region containing top reinforcement; or (3) are bent, hooked or otherwise anchored at discontinuous edges to develop the yield stress at the support face. When the calculated values of A_{sb} differ for adjacent spans (due to unequal spans or unequal loading) then the larger value of A_{sb} must be used in each of these spans to enable the transfer of tension across the supporting member.

The loading in Eq. 5 should not be taken less than the total unfactored service loading for the intended use of the structure. However, since most progressive collapses have occurred during construction it is suggested that the loading not be taken less than twice the slab dead load which corresponds to typical load levels during construction (2). It is emphasized that these design recommendations are not intended to prevent progressive collapses of typical two-way slab structures that are grossly overloaded over a large portion of the structure.

In providing the required area of steel, A_{sb}, the area of continuous draped prestressing tendons passing through the columns or reaction areas can be considered effective. In order to calculate the required area of prestressing, f_y should be replaced with f_{ps} in Eq. 5. In lieu of more detailed calculations the stress in the prestressing steel may be assumed to be equal to the value of f_{ps} used in flexural strength calculations [e.g. Section 18.7.2 of ACI 318-77 (1)].

The top reinforcement in regions close to vertical supports is considered ineffective since it rips out of the top surface of the slab. However in regions away from supports the top reinforcement does not rip-out of the slab. In addition this reinforcement has a significant overlap with the bottom bars and therefore participates in the transfer of tension between bottom reinforcing bars in adjacent panels. Analyses indicate that standard slab reinforcement details in regions away from supports, with a minimum amount of bottom steel in the middle regions of the slab, together with the proposed requirements for effectively continuous bottom bars anchored into the columns should provide a secondary defense mechanism capable of preventing progressive collapse.

Bottom bars which are well anchored and effectively continuous provide not only a means of arresting progressive collapse but also provide a means for preventing initiation of punching shear failures in slab-column connections transferring significant moment (11). A connection transferring significant moment can develop tensile cracking on the bottom surface of the slab adjacent to the column which would lead to a



(a) Flat Plate Structure

(b) Flat Slab with Drop Panels and Column Capitals

FIG. 12.—Details Showing Bottom Bars Designed to Provide Structural Integrity (Bottom Bars Shown in One Direction Only)

brittle failure without the presence of anchored bottom reinforcement.

Table 1 summarizes the calculations of A_{sb} for a variety of structural slab systems. Fig. 12 shows the special details of the bottom bars, of area A_{sb} in one direction only for cases (a) and (c) from Table 1.

Summary and Conclusions

Existing methods of predicting the tensile membrane response of slab panels having fully restrained edges are discussed and the predictions of these methods are compared with single panel test results. A nonlinear computer program capable of simulating both geometric and material nonlinearities was developed to predict the post-failure response of different types of typical slab structures.

Experimental results of slab structures having improved reinforcement details which were loaded well beyond initial failure and up to total collapse are described. Resisting mechanisms which develop after the occurrence of initial failures are described along with simple design and detailing recommendations necessary to enable the damaged slab to hang from its supports. These design and detailing requirements result in effectively continuous bottom bars along column lines that are well anchored into the column or support regions. The result of applying these requirements to a variety of two-way slab structures is presented. It is felt that the secondary load carrying mechanism which can develop is the key in preventing progressive collapse of slab structures.

Acknowledgments

The financial support of the Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged. The flat plate structures were tested by Eddy K. Chu and John M. Balazic in the Jamieson Structures Laboratory of the Department of Civil Engineering and Applied Mechanics at McGill University. Thanks are also extended to John Balazic for his helpful suggestions during the preparation of this paper and to Peter Bischoff for his valuable assistance in preparing the drawings.

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Appendix II.—Notation

The following symbols are used in this paper:

A_{sb} = minimum area of effectively continuous bottom bars in direction l_n anchored into reaction area of support;
f_y = specified yield strength of nonprestressed reinforcement;
f_{ps} = stress in prestressed reinforcement;
l_d = tension development length;
l_n = clear span;
l_x, l_y = clear span in the long and short directions respectively;
l_2 = distance measured from the centerline of the panel on one side of the one-way catenary to the centerline of the panel on the other side of the catenary;
T = force per unit length in the reinforcement;
T_c = force in the reinforcement in the one-way catenary;
T_x, T_y = force per unit length in the reinforcement in the x and y directions respectively;
w = uniformly distributed load per unit area of the slab;
w_s = unfactored service loading per unit area of the slab, but not less than twice the slab dead load;

δ = central deflection of the slab;
ϵ = strain in the reinforcement of the one-way catenary;
ϵ_x, ϵ_y = strain in the reinforcement in the x and y directions, respectively; and
ϕ = capacity reduction factor.