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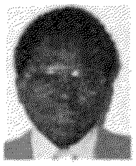
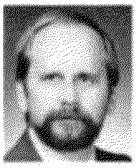
Shear Tests of High- and Low-Strength Concrete Beams Without Stirrups





by Andrew G. Mphonde and Gregory C. Frantz

Three series of reinforced concrete beams without shear reinforcement were tested to determine their diagonal cracking strengths and ultimate shear capacities. Within each series the shear span-depth ratio was held constant at 3.6, 2.5, or 1.5, while nominal concrete compressive strength was varied from 3000 to 15,000 psi (21 to 103 MPa) in otherwise identical specimens. Test results indicate that for slender beams the ACI beam shear strength equations are conservative, but their accuracy varies greatly with concrete strength. A new regression equation is presented to more accurately predict ultimate shear capacity of slender beams over the entire range of concrete strengths tested. The effect of concrete strength on shear capacity becomes more significant as the beams become shorter. For the shorter beams, the ACI equation underestimates the actual shear strength by 71 percent at high concrete strengths.

Keywords: beams (supports); compressive strength; cracking (fracturing); **high-strength concretes**; regression analysis; **reinforced concrete**; **research**; **shear strength**; shear tests; statistical analysis; structural design.

Current specifications in the ACI Building Code[1] for shear strength of reinforced concrete beams are based on the results of thousands of beam tests using concretes with relatively low compressive strengths, varying mostly from 2000 to 6000 psi (14 to 41 MPa). Because of the recent use of concretes with strengths up to 11,000 psi (76 MPa),[2] it has become necessary to check the validity of present shear design methods when applied to these higher concrete strengths.

There has been much discussion on the correct relationship between concrete compressive strength and concrete shear capacity, even for the lower range of concrete strengths. The current ACI Building Code assumes that the nominal shear capacity is essentially proportional to $f_c'^{\,0.5}$ while some investigators have concluded it is proportional to $f_c'^{\,0.33}$.[3,4] For compression strengths up to about 10,000 psi (69 MPa), experimental studies[5,6] indicate, that as the concrete strength increases, shear capacity increases at a slower rate than the $f_c'^{\,0.5}$ proportionality would indicate. Previous work[7-9] with low-strength concretes has shown that the effect of compressive strength on shear capacity is also dependent on the shear span-depth ratio of the beam.

This paper summarizes results of an experimental program that examined the effect of a very wide range of concrete strengths [from 3000 to 15,000 psi (21 to 103 MPa)] on diagonal cracking capacity and ultimate shear capacity of 19 reinforced concrete beams without shear reinforcement.

Significance of research

The research described in this paper is important to designers using high-strength concrete and to writers of building codes for reinforced concrete, because use of high-strength concrete is proceeding faster than applicable design methods are being developed. Because current design provisions were found to be conservative in predicting the shear capacity of slender beams without stirrups for concrete strengths up to 15,000 psi (103 MPa), a new equation is presented to allow a more accurate estimate of the shear capacity of such beams. Also, as the shear span-depth ratio became lower and the concrete strength became higher, measured shear capacities became much higher than the current predicted values.

EXPERIMENTAL PROGRAM

Specimen details

Three series (19 total specimens) of reinforced concrete beams without shear reinforcement were tested. In each series the shear span-depth ratio a/d was held constant at 1.5, 2.5, or 3.6, while the nominal concrete compressive strength was varied from 3000 to 7000, to 11,000 to 15,000 psi (21 to 48, to 76 to 103 MPa) in otherwise identical specimens.

Fig. 1 and Tables 1 and 2 give specimen details. In the specimen identification for AO-7-3a, "AO" means no stirrups; "7," a nominal concrete strength of 7000

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Table 1 — Specimen details and test results
a/d = 3.6

			Measured shear stress, psi		Predicted shear stress, psi**					
					Cracking			Ultimate		
Specimen	f'_c*, psi	Equivalent f'_c**, psi	Cracking	Ultimate	ACI 11-3 Eq. (1)	ACI 11-6 Eq. (2)	Zsutty Eq. (3)	Zsutty Eq. (4)	$\sqrt{f'_c}$ Eq. (5)	$\sqrt[3]{f'_c}$ Eq. (6)
AO-3-3b	3273	3011	181	206	110	128	179	193	219	217
AO-3-3c†	4277	3935	121	213	125	135	173	186	230	230
AO-7-3a	5938	5463	213	262	148	164	219	235	248	249
AO-7-3b	6562	6037	199	264	155	171	226	243	254	255
AO-11-3a	11812	10867	213	286	208	221	275	296	294	295
AO-11-3b	11766	10825	213	285	208	221	275	295	293	294
AO-15-3a	12823	11797	270	298	217	230	283	304	300	301
AO-15-3b	14768	13587	—	319	233	245	296	319	313	312
AO-15-3c	14477	13319	305	312	231	243	294	316	311	310

*From 3 × 6 in. (76 × 152 mm) cylinders.
**Using equivalent 6 × 12 in. (152 × 305 mm) cylinder strength, assumed to be 92 percent of 3 × 6 in. strength.
†This specimen has ρ = 2.32 percent; all other specimens have ρ = 3.36 percent.
1000 psi = 6.895 MPa.
All beams failed by diagonal tension.

Table 2 — Specimen details and test results
a/d = 2.5, 1.5

			Measured shear stress, psi		Predicted shear stress @ ultimate, psi**		
Specimen	f'_c*, psi	Equivalent f'_c**, psi	Cracking	Ultimate	ACI 11-29 Eq. (7)	Zsutty Eq. (8)	Failure mode†
a/d = 2.5							
AO-3-2	3246	2986	198	248	188	202	Shear compression
AO-7-2	7119	6549	255	376	238	262	Shear compression
AO-11-2	12498	11498	284	355	288	317	Diagonal tension
AO-15-2a	13204	12148	340	567	293	322	Shear compression
AO-15-2b	10940	10065	255	656	275	303	Shear compression
a/d = 1.5							
AO-3-1	3637	3346	227	370	361	415	Shear flexure
AO-7-1	6593	6066	340	993	422	506	Arch rib
AO-11-1	10364	9535	369	1379	483	588	Arch rib
AO-15-1a	12526	11524	454	879	513	626	Shear flexure
AO-15-1b	12810	11785	426	1578	517	631	Arch rib

*From 3 × 6 in. (76 × 152 mm) cylinders.
**Using equivalent 6 × 12 in. (152 × 305 mm) cylinder strengths, assumed to be 92 percent of 3 × 6 in. strength.
†Failure mode: *Shear compression* — progress of inclined crack stopped at load point, crack pattern stabilizes, and crack extends through top of beam; *Arch rib* — crushing of compression strut; *Shear flexure* — crack extends through top of beam, no crushing of strut.
1000 psi = 6.895 MPa.

psi (48 MPa); "3," a shear span-depth ratio of 3.6; and "a," one of several identical specimens. All beams were 6 × 13.25 × 96 in. (152 x 337 x 2438 mm) with an effective depth of 11.75 in. (298 mm) and a cover of 1 in. (25 mm). Beams were simply supported and loaded at midspan with the distance between the supports varied to produce the desired a/d ratio. All but one beam had tension reinforcement consisting of 3 #8 bars (ρ = 3.36 percent), insuring a shear failure rather than a flexure failure in all beams. Specimen AO-3-3c had tension reinforcement of 2 #7 bars plus 1 #6 bar (ρ = 2.32 percent) to check the effect of steel reinforcement ratio on shear capacity. In these tests the steel reinforcement ratio was not a main variable.

Materials

Reinforcement consisted of Grade 60 steel. Table 3 lists the concrete mix details. Each beam and 18 3 x 6 in. (76 x 152 mm) test cylinders required two batches of concrete. Mixing was done in a rotating pan and paddle-type mixer. For each batch, one-half of the coarse and fine aggregate was first mixed, followed by one-half of the cement, slag, and fly ash, followed by one-half of the water and admixture. The rest of the batch

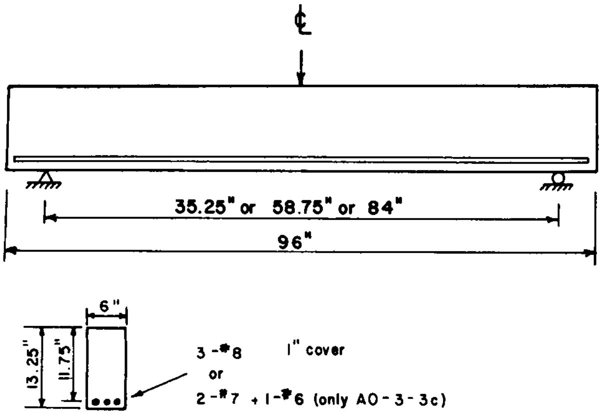

Fig. 1 — Test specimen (1 in. = 25.4 mm)

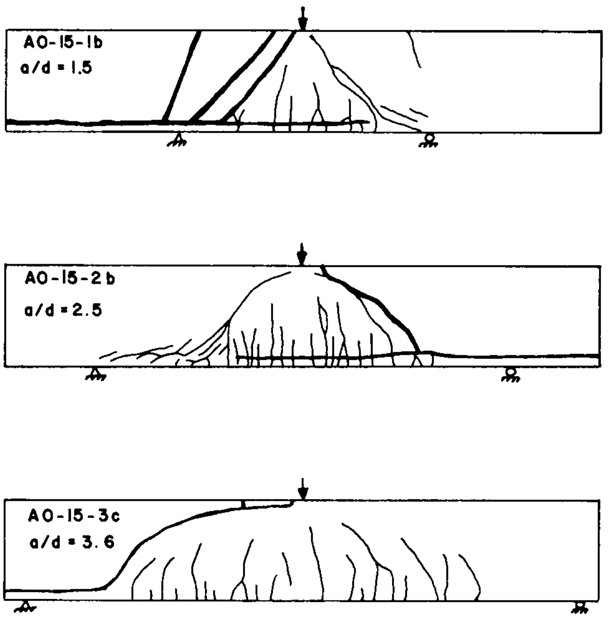

Fig. 2 — Typical crack patterns for each series

was then added and mixed in the same order. The two batches were thoroughly mixed together in a separate pan before casting. The slump for all mixes was from 6 to 7 in. (150 to 180 mm).

The 3000 and 7000 psi (21 and 48 MPa) beams were stripped after two days and stored uncovered in the laboratory until testing at an age of about one month. The 11,000 and 15,000 psi (76 and 103 MPa) beams were stripped after one day, placed in a saturated lime water bath for one or three months, then removed and tested about one week later.

The measured concrete strengths were based on the average of at least six cylinders. Various investigators[10,11] have reported that standard 6 × 12 in. (152 × 305 mm) cylinders yield compressive strengths 94 to 90 percent of those measured by the smaller cylinders used here. This correction factor was not investigated in this study; however, a value of 92 percent was

Table 3 — Concrete mix proportions

	Amounts per cubic yard*			
	Nominal concrete strength, psi			
	3000	7000	11000	15000
Water-cement ratio (by weight)	0.64	0.42	0.30**	0.27
Cement type I-II	551	810	950	522.5
Granulated slag	—	—	—	522.5
Fly ash	—	—	95	—
Water	355	337	305	286
⅜ in. trap rock (SSD)	1258	1520	1818	1874
Sand (SSD)	1588	1180	855	882
Type D superplasticizer, oz.	—	—	110	110

*All amounts are in pounds except for the admixture, which is in ounces.
**Counting ⅔ of the fly ash as cementious material.
1000 psi = 6.895 MPa; 1 lb/yd^3 = 0.5933 kg/m^3; 1 in. = 25.4 mm; and 1 oz. = 2957 cm^3.

assumed. While the nominal concrete strengths refer to the smaller cylinder strengths, all calculations and all plotted data involving compressive strength use the equivalent standard cylinder strengths.

In the preliminary mix proportioning work, the 15,000 psi (103 MPa) mix consistently gave strengths in the 14,000 to 15,000 psi (97 to 103 MPa) range. However, the test beam concrete strengths for this level, although consistent within each casting, varied from 12,500 to 14,800 psi (86.1 to 102 MPa). In future work it is suggested that the water-cement ratio be further reduced to gain additional strength. It did not appear that aggregate strength limited the concrete compressive strength.

Test procedure

Beams were tested in a 300 kip (1330 kN) hydraulic testing machine and loaded at midspan in load intervals of 2 or 4 kips (8.9 or 17.8 kN) until failure. At each load stage, beam deflection was measured, and the developing crack pattern was marked on the beam surface. The final failure was carefully observed.

Test results

Tables 1 and 2 list the main test results from all three series of tests. Included are concrete strengths and the measured shear stress V/bd at inclined cracking and at failure.

DISCUSSION OF TEST RESULTS

General behavior

All beams failed in shear, although the type of shear failure varied with the a/d ratio. At a/d 3.6, failure was always by diagonal tension, while it was generally by shear compression at a/d of 2.5 and by crushing of the arch rib at 1.5.

Fig. 2 shows typical failure crack patterns for each a/d series. Vertical flexure cracks initially developed near midspan. Further loading produced more flexure cracks and also diagonal cracks. As these crack patterns show, the load-carrying mechanism changed significantly as a/d changed. At a/d of 3.6, the cracks indicate a predominantly flexural behavior. At a/d of 1.5 the typical deep beam crack pattern shows the "tied arch" behavior with much less vertical flexural cracking.

At a/d 3.6, failure was sudden and occurred soon after inclined cracking. However, at a/d values of 2.5 and 1.5, there was significant reserve strength in most of the beams due to arching action after the crack pattern was fully developed. While inclined cracks formed gradually from a flexure crack for beams with a/d of 3.6, in the beams with a/d of 2.5 or 1.5 they usually developed very suddenly and were often not associated with any particular flexure crack. Their formation was accompanied by a momentary drop in load as the load-carrying mechanism changed from beam to arch action. In the deeper beams tension cracks usually extended down from the "compression" face near the supports at high loads.

At failure longitudinal splitting along main reinforcement was prevalent in all beams. Very careful observation of the actual failure sequence showed that the longitudinal splitting did not initiate the final failure.

Beams with a/d = 3.6, slender beams

Table 1 and Fig. 3 present the measured test results and four predicted values of shear capacity expressed in terms of shear stress V/bd, two from the ACI Code and two from Zsutty's[3] statistical analysis of shear strength data. The shear stress and concrete strength are in units of psi. All four of these equations are based on beam tests with relatively low strength concrete. At inclined cracking

ACI Eq. (11-3)

$$v_{cr} = 2\sqrt{f'_c} \qquad (1)$$

ACI Eq. (11-6)

$$v_{cr} = 1.9\sqrt{f'_c} + 2500\rho\frac{Vd}{M} \qquad (2)$$

Zsutty equation

$$v_{cr} = 59\left(f'_c\ \rho\ \frac{d}{a}\right)^{0.333} \qquad (3)$$

At ultimate:
Zsutty equation

$$v_u = 63.4\left(f'_c\ \rho\ \frac{d}{a}\right)^{0.333} \qquad (4)$$

Although the ACI Code states that Eq. (11-3) and (11-6) define nominal concrete shear strength (i.e., ultimate), the equations are based on the shear causing inclined cracking and are listed in that category.

In this study, diagonal cracking stress was defined as the shear stress at the time when critical diagonal crack (the one that caused failure) became inclined and crossed middepth. Fig. 3 shows that while the inclined cracking shear stress tends to increase with increasing concrete strength, the variation is quite erratic. The

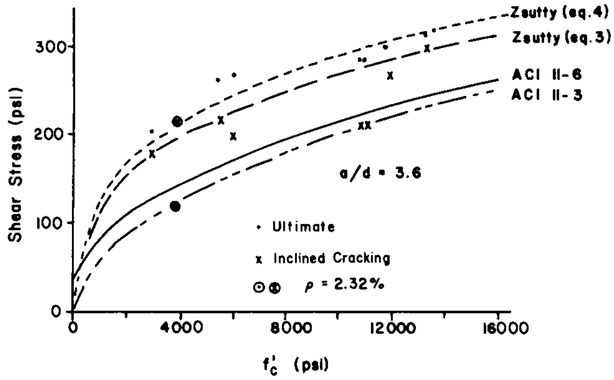

Fig. 3 — Test results, a/d = *3.6 series (1000 psi = 6.895 MPa)*

"measured" inclined cracking stress is obviously affected by the observer's judgement and is also sensitive to the actual location of the initiating flexural crack.

In contrast to the inclined cracking data, the measured ultimate shear data follows a well-defined trend reasonably close to values predicted by the Zsutty Eq. (4). Notice that similar pairs of beams, like specimens AO-11-3a and 3b, and AO-15-3b and 3c, have similar measured ultimate strengths, indicating good reproducibility in the data.

Since all beams except one had the same amount of flexural reinforcement (ρ = 3.36 percent), the steel stress at failure varied for different concrete strengths and may have affected the failure loads. To check this possibility, Specimen AO-3-3c had a lower amount of reinforcement (ρ = 2.32 percent) and was designed to have approximately the same flexural reinforcement stress at failure as Specimen AO-15-3b, about 39 ksi (262 MPa). Table 1 and Fig. 3 indicate that the data (especially the ultimate strength) from this specimen fits in very well with the other data. It appears that this varying steel stress was not a significant variable in these tests. Other tests[12] have shown this variable is significant only for very low amounts of reinforcement, i.e., for very high flexural stresses. It has been suggested[13] that ACI code provisions should reflect a reduced shear capacity for low amounts of flexural reinforcement (ρ < 1.25 percent), i.e., for high values of flexural reinforcement stress. However, as concrete strength increases, beam shear strength and applied flexural moment at shear failure also increases. Thus, it is likely that when applied to high-strength concrete this critical steel ratio should increase with increasing concrete strength.

Fig. 4 examines the effect of concrete strength on the ratio of the measured ultimate/predicted shear stress. When using ACI Eq. (11-6) as the predicted value, this ratio, which indicates part of the overall factor of safety against shear failure, is significantly affected by concrete strength, decreasing from about 1.64 at f'_c of 3000 psi (20.7 MPa) to 1.20 at f'_c of 15,000 psi (103.4 MPa). While ACI Eq. (11-6) is a conservative

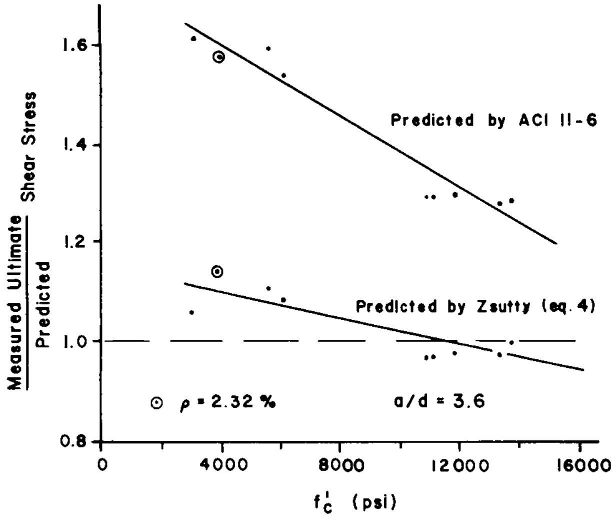

Fig. 4 — Relative accuracy of ACI and Zsutty equations (1000 psi = 6.895 MPa)

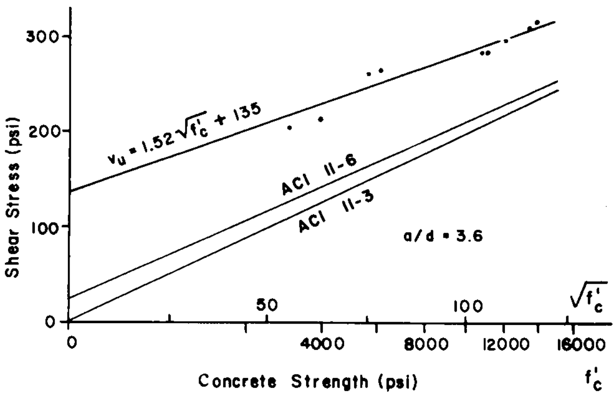

Fig. 5 — Shear strength as function of $\sqrt{f_c'}$ *(1000 psi = 6.895 MPa)*

estimate of the ultimate shear capacities of these beams, it does not yield a constant factor of safety against shear failure. Using the Zsutty equation for ultimate strength [Eq. (4)] as the predicted value yields ratios of measured/predicted stress at failure quite close to 1.0 but still showing a definite decrease with increasing f_c'. Although not shown in these figures, the semiempirical shear strength equations recently developed by Heger and McGrath[14] also yield conservative values for the predicted shear strength of these slender beams, with the predicted strengths lying between the Zsutty and the ACI strengths.

Plotting the test data as a function of $(f_c')^{0.5}$ (Fig. 5) shows that a reasonable regression equation (not including AO-3-3c) and the corresponding standard error of estimate S is

$$v_u = 1.52\sqrt{f_c'} + 135, \quad \text{with } S = 10.5 \text{ psi without AO-3-3c} \qquad (5)$$

where v_u and f_c' are in units of psi.

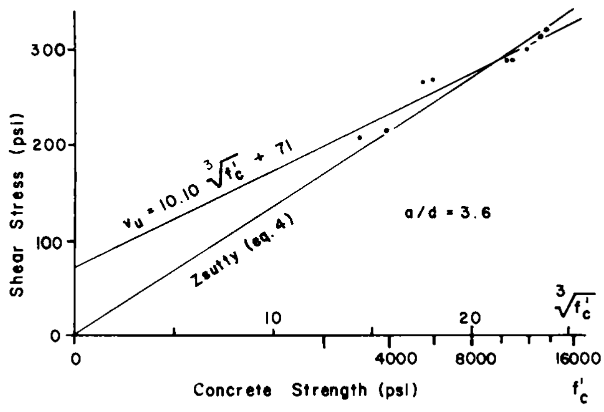

Fig. 6 — Shear strength as function of $\sqrt[3]{f_c'}$ *(1000 psi = 6.895 MPa)*

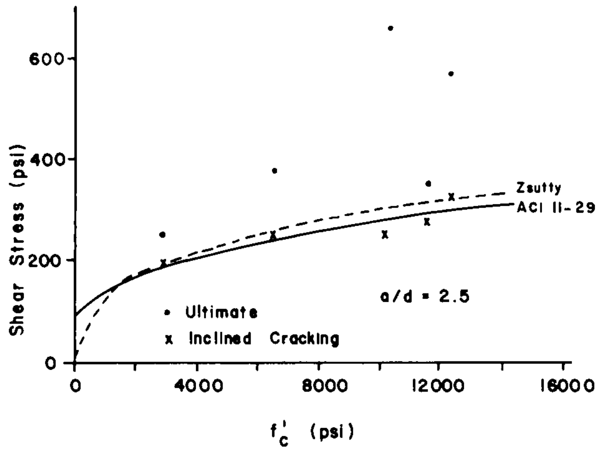

Fig. 7 — Test results, a/d *= 2.5 series (1000 psi = 6.895 MPa)*

If plotted using $(f_c')^{0.333}$ as the variable (Fig. 6), the regression equation is

$$v_u = 10.10\sqrt[3]{f_c'} + 71, \quad \text{with } S = 10.0 \text{ psi without AO-3-3c} \qquad (6)$$

Zsutty's Eq. (4) also fits the data fairly well with an S value of 16.4 psi for all beams.

Beams with *a/d* = 2.5 or 1.5, short beams

Table 2 and Fig. 7 and 8 present the measured test results and two predicted values of shear capacity, one from the ACI Code and one from Zsutty.[9] These equations, shown below, predict the ultimate shear capacity of beams loaded on the top face and supported on the bottom face.

ACI Eq. (11-29)

$$v_u = \left(3.5 - 2.5\frac{M}{Vd}\right)[\text{Eq. (2)}] \qquad (7)$$

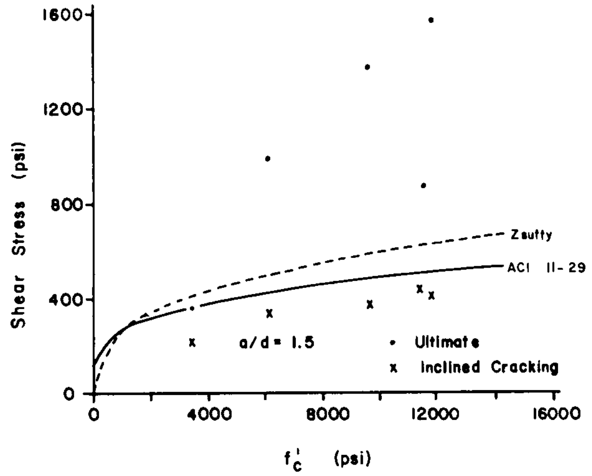

Fig. 8 — Test results, a/d *= 1.5 series (1000 psi = 6.895 MPa)*

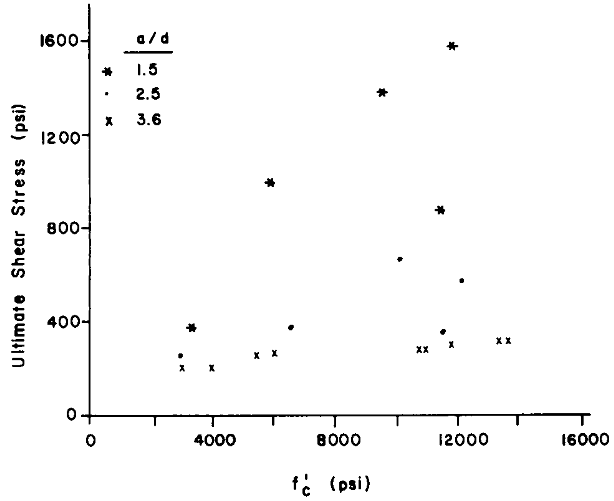

Fig. 9 — Comparison of results of all series (1000 psi = 6.895 MPa)

Zsutty equation

$$v_u = \left(\frac{2.5}{a/d}\right)[\text{Eq. (3)}] \qquad (8)$$

Arching action allows such short beams to develop ultimate shear strengths substantially above the diagonal cracking strengths. These equations use a multiplier (the first term) to account for most of the increase in shear strength due to arching action in beams with a/d less than 2.5. The two equations both predict approximately the same shear strength for these test beams, the ACI equation being slightly lower.

Compared to slender beams with a/d of 3.6, these short beams showed a much more clearly defined relationship between the inclined cracking stress and the concrete strength (Fig. 7 and 8). At the lower a/d values, (1) the diagonal crack formed suddenly, not gradually as in the slender beams, and thus the cracking stage was not subject to the observer's judgement very much, and (2) the diagonal crack was not dependent on a flexural crack to initiate its formation, as it was in the slender beams.

At these lower a/d values, there is a possibility of a significant increase in load capacity beyond inclined cracking due to arching action. However, as Fig. 7 and 8 show, for deep beams there can be a much greater variation in the measured ultimate shear capacities than exists for slender beams (Fig. 3). Zsutty[9] also observed this difference in behavior. These tests show that this scatter is due mainly to the way inclined cracking develops at the different a/d values.

In a slender beam with a long shear span, the exact location of the flexure crack that initiates the inclined crack is not too important, because the inclined crack can adjust its path as it extends towards the load; thus, a low variation in ultimate shear capacity exists between similar beams. In a deep beam with a/d of 1.5, the initial inclined crack develops suddenly along almost its entire length. The initial crack location is critical and helps determine if much arching action can develop, leading ultimately to crushing of the arch rib (as in Beams AO-7-1, AO-11-1, and AO-15-lb), or if the capacity is increased only slightly as the inclined crack punches through the top face, very little arching action develops (as in Beams AO-3-1 and AO-15-1a).

Complete arching action cannot be guaranteed in all deep beams. Therefore, design values of shear strength for deep beams without shear reinforcement must be a lower bound of the test results. At a/d 2.5, the ACI, Zsutty, and Heger and McGrath equations are lower bounds and seem to predict the inclined cracking strength quite well. They should be suitable as design equations throughout the range of concrete strengths tested here.

At a/d of 1.5, the ACI and Zsutty equations both significantly underestimate even the lowest measured shear strength at high concrete strengths by 71 and 40 percent, respectively. These results show that the full arching action shear capacity at a/d of 1.5 seems to be approximately proportional to the concrete compressive strength. At a/d of 1.5, the Heger and McGrath equation predicts the inclined cracking shear quite well. Although more test data on deep beam shear strength is needed, it is apparent that the ACI Eq. (11-29) should be modified to more accurately predict even the lower-bound shear strength of deep beams made with high-strength concrete.

Since one of the functions of shear reinforcement is to support the developing concrete arches,[15] the addition of sufficient web reinforcement in deep beams may eliminate these lower-bound strengths and permit all beams to fully develop the high shear capacity possible with arching action. The effect of shear reinforcement on deep beam shear strength was not studied in this work.

Results from all series are shown in Fig. 9. At a/d of 2.5, the lower-bound strengths lie close to the strengths for a/d of 3.6, while some are much higher, indicating these beams lie at the transition point between slender and deep beam behavior. However, at a/d 1.5, the significant effect of concrete strength on even the lower-bound strengths is very evident.

CONCLUSIONS

This paper summarized the results of an experimental study of the shear strength of reinforced concrete beams without shear reinforcement and with nominal concrete strengths from 3000 to 15,000 psi (21 to 103 MPa). Within the scope of this study, the following conclusions are valid:

1. At an a/d ratio of 3.6, the current ACI equations for shear design, Eq. (11-3) and (11-6), are conservative. However, the ratio of measured/predicted capacity using Eq. (11-6) decreases from 1.64 to 1.20 as the concrete strength increases from 3000 to 15,000 psi (21 to 103 MPa).

2. The regression equation

$$v_u = 10.10\sqrt[3]{f'_c} + 71 \qquad (6)$$

best describes ultimate strength test results at a/d of 3.6, with a standard error of estimate of 10.0 psi.

3. At an a/d of 2.5, ACI Eq. (11-29) is a reasonable estimate of the lower-bound measured shear capacity.

4. At an a/d of 1.5, ACI Eq. (11-29) underestimates even the lower-bound measured shear capacity by 71 percent for high-concrete strengths.

5. The effect of concrete strength on shear capacity becomes more significant as the a/d ratio decreases.

6. There is much more scatter in the ultimate shear strengths as the a/d ratio decreases due to possible variation of failure modes. At a/d of 3.6, all failures were by diagonal tension; at 1.5 failure was either by crushing of the arch rib (high capacity) or by extension of the inclined crack through the top surface (lower capacity).

7. Failures become more sudden and explosive as the concrete strength increases, especially at lower a/d values.

RECOMMENDATIONS

It is recommended that a form of Eq. (6) be used to best predict the ultimate shear capacity of slender beams. Further work is needed to study the shear strength of deep beams with high-strength concrete and to study the interaction between shear reinforcement and the concrete contribution to beam shear strength for high-strength concretes.

ACKNOWLEDGMENTS

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NOTATION

a = shear span
b = beam width
d = effective beam depth
f'_c = concrete compressive strength
M = bending moment
S = standard error of estimate
v_{cr} = shear stress at inclined cracking
v_u = shear stress at ultimate
V = shear force
ρ = ratio of tension reinforcement

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APPENDIX

SI versions of equations in this paper, shear stress f'_c and S are in units of MPa.

$$v_{cr} = 0.17\sqrt{f'_c} \qquad (A1)$$

$$v_{cr} = 0.16\sqrt{f'_c} + 17.2\,\rho\,\frac{Vd}{M} \qquad (A2)$$

$$v_b = 2.1 \left(f'_c \, \rho \frac{d}{a} \right)^{[illegible]} \tag{A3}$$

$$v_u = 2.3 \left(f'_c \, \rho \frac{d}{a} \right)^{[illegible]} \tag{A4}$$

$$v_u = 0.126 \sqrt{f'_c} + 0.931, \qquad \text{with } S = 0.072 \tag{A5}$$

$$v_c = 0.366 \sqrt[3]{f'_c} + 0.49, \qquad \text{with } S = 0.069 \tag{A6}$$

$$v_c = \left(3.5 - 2.5 \frac{M}{Vd} \right) \text{[Eq. (2)]} \tag{A7}$$

$$v_c = \left(\frac{2.5}{a/d} \right) \text{[Eq. (3)]} \tag{A8}$$