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Tensile Strength and Diagonal Tension Resistance of Structural Lightweight Concrete

By J. A. HANSON

Describes the tests employed and the results obtained in an extension of a previous study of diagonal tension resistance reported by the author. This extension of the original program involves lightweight concrete beams of longer span and lower steel percentages. An important conclusion, that diagonal cracking load should be considered as the ultimate load for non-web-reinforced beams, has been confirmed.

A large number of 6 x 12-in. cylinders from the beam concretes were broken by the "split-cylinder" tension test. Good correlation was established between this indirect tension measurement and the shear resistance of the beams at diagonal cracking. This correlation shows that the diagonal tension resistance of lightweight concretes varies from approximately 60 percent of that of the similar normal weight concrete to nearly 100 percent, depending on the particular lightweight aggregates used.

Proposed ultimate load design recommendations are made for structural lightweight concrete. These are in general accord with the recommendations of the ACI-ASCE Committee 326 on shear and diagonal tension for normal weight concrete. It has been found that diagonal tension strength of the lightweight concretes is affected by the same variables as affect the resistance of normal weight concrete. The difference between the two types of materials is one of magnitude of diagonal tension resistance and not of fundamental difference in behavior.

The proposed design recommendations also provide for the fundamental differences in tensile resistance that exist between the various lightweight aggregates. A combination of compressive strength and split-cylinder tension testing provides a convenient and safe measure of the ultimate diagonal tension resistance to be associated with each of the various aggregates.

Several years ago, Subcommittee 1 of ACI Committee 213, "Properties of Lightweight Aggregates and Lightweight Aggregate Concrete," Truman R. Jones, Jr., Acting Chairman, was assigned the task of collecting material on the properties of structural lightweight concrete and of preparing recommended design procedures for the consideration of other committees of the Institute. This subcommittee is preparing a proposed guide for structural lightweight concrete, but much work remains to be done before the guide can be submitted as a report for approval of the committee as a whole.

One of the more important strength characteristics of structural lightweight concrete is the resistance to diagonal tension of beams without web reinforcement. In recent years, a few structures of both lightweight and normal weight materials have suffered damage due to lack of sufficiently precise design knowledge of this property. ACI-ASCE Committee 326 has completed recom-

mendations for ultimate strength design for shear in normal weight concrete members, and their report urges that ACI Committee 213 complete similar recommendations for structural lightweight concrete. Subcommittee 1 of Committee 213 has gathered sufficient data for formulation of such recommendations.*

This paper presents the information gathered and includes recommended design procedures for diagonal tension in lightweight concrete beams without web reinforcement. These design procedures were developed by the combination of the reported test analyses with Eq. (9) of this paper. Eq. (9) was originally formulated by Committee 326, and the details of its development will be presented in a complete Committee 326 report which is now nearing completion. In addition, the format of the specific design recommendations in the final section of this paper follows that used by the Committee 326.

In the interest of early dissemination of the information, and of obtaining early additional discussion of the subject, particularly as an aid to the ACI Committee 318, Standard Building Code, this paper is published prior to the basic reference report. The paper was prepared under the direction of Subcommittee 1, chiefly from material furnished by the Portland Cement Association. During the 1961 annual ACI convention at St. Louis, Committee 213 unanimously approved sponsorship of this paper and its recommendations for diagonal tension design of lightweight concrete structures.

A complete list of Committee 213 members follows:

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■ FIVE YEARS AGO, as part of a comprehensive study of the characteristics of structural lightweight concrete, the Portland Cement Association tested a series of beams designed to fail in diagonal tension. The results of these tests were reported in 1958.[1] The beams of this earlier study were characterized by high percentages of steel and rather short spans, and the important variable, in addition to type of aggregate, was the compresive strength of the concrete. For several years investigators of diagonal tension have recognized that shear span and steel percentage also have an important bearing on the resistance of beams to shear. In their discussion of the previous paper, Ferguson and Thompson[2] of the University of Texas presented test data on the ultimate shear strength of 12 lightweight beams made with a single ag-

*Report by ACI-ASCE Committee 326 on Shear and Diagonal Tension, Part 2, Chapter 5, unpublished.

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gregate, but with lower steel percentages and longer spans. The indicated shear strengths of the University of Texas beams were lower than those reported in the original paper, and Ferguson and Thompson suggested that the effects of steel percentage and shear span may be more pronounced in lightweight than in normal weight concrete. At a later time, Ferguson furnished the unpublished results[3] of 15 additional beams using the same lightweight aggregate. These later findings corroborated the indicated low diagonal tension resistance of long span, low steel percentage lightweight beams for the particular aggregate used.

With the increasing general demand for inclusion in building codes of provisions for design of lightweight concrete, the original diagonal tension beam program at the PCA laboratories was extended to include the variables of span and percentage of tension steel. The effect of these important variables in the extended program has been studied mainly with concretes having a nominal compressive strength of 4500 psi. Two additional commercially available lightweight aggregates, as well as six of those employed in the original study, have also been included. With these eight lightweight aggregates and the two variables of span and steel percentage, it was not practical to make duplicate beam tests. Thus, close controls on mix proportioning and mix duplication became fundamentally important. In addition to those of lightweight concrete, normal weight beams containing Elgin sand-and-gravel aggregate were tested for comparison of performance. This comparison was made on the basis of equal compressive strengths. For the readers' greater understanding, it is emphasized that this comparison normal weight material is designated as Aggregate 8 throughout this report.

It has been recognized for some time that diagonal tension resistance is fundamentally a function of the tensile strength of the concrete. Previous attempts to establish this relationship by modulus of rupture tests have met with meager success. This is probably due to the sensitivity of the plain concrete beams to shrinkage, temperature, minor specimen defects or other accidental variations. In the present program a measure of the tensile strength of the various concretes was obtained by the "Brazilian" or "split-cylinder" test, in which the cylinder is tested to failure under diametral compression. The cylinders for the splitting test were cast from the beam concretes and cured in the same manner. The concretes used in the 1958 series were reproduced to obtain cylinders for correlation with the early beam diagonal tension strengths. These early tests have been reanalyzed and incorporated into this report.

Both of the shear investigations are integral parts of a general study of the properties of lightweight aggregate concrete. Many basic design properties, including compressive and flexural strength, modulus of elasticity, bond, creep, and drying shrinkage of concretes containing these same aggregates, have been reported by Shideler.[4] He has also reported results[5] of a study on the effects of steam curing on creep of concrete. Values of ultimate strength design coefficients for lightweight concretes were given by Hognestad, *et al.*[6] These studies have demonstrated a large variation of properties between different lightweight aggregates, even between those of the same type and manufacture. This indicates the need for individual producers to develop design data through sound investigations of their own products. Fortunately, a simple means of establishing the diagonal tension resistance to be associated with a particular aggregate is suggested by the analysis of this new investigation.

TESTING PROCEDURE

The new investigation was directed toward test specimens with longer spans and more practical amounts of tension steel, and was planned around four types of beams as shown by Fig. 1. All beams were 6 x 12 in. in gross cross section and the spans were either 6½ or 10 ft. Type 1 beams were identical with those of the first investigation. Type 2 beams contained the same tension steel as the Type 1, but were lengthened to provide twice the shear-span length. Type 3 beams retained the shear span of the Type 1 beams but contained only half of the tensile steel area. Finally, the Type 4 beams combined the changes of both the variables, shear span and tensile steel area, into one beam, with twice

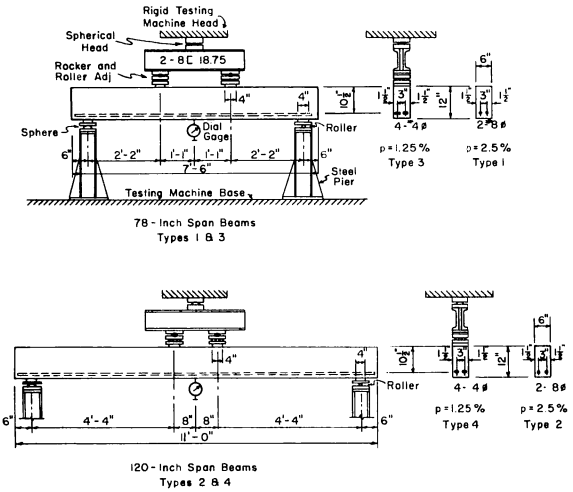

Fig. 1 — Details of beams and testing arrangement

TABLE I — PROGRAM OF BEAM TESTS

Beam type	No. of beams	Span, ft	Shear span, a, in.	a/d	p, percent	Nominal f_c', psi
1	9	6.5	26	2.5	2.50	3000
1	10	6.5	26	2.5	2.50	4500
1	3	6.5	26	2.5	5.00	7000
1	2	6.5	26	2.5	5.00	9000
2	9	10.0	52	5.0	2.50	4500
3	9	6.5	26	2.5	1.25	4500
4	4	10.0	52	5.0	1.25	3000
4	11	10.0	52	5.0	1.25	4500
Total	57					

the shear span and half the tensile steel of the Type 1 beams. Web stirrups and compression steel were not used. Table 1 gives the detailed outline of all diagonal tension test beams in both of the programs. There were 36 beams tested during the recent investigation.

Most of the Type 1 beams were cast and tested during the 1958 study. However, three Type 1 beams were added in the recent program using the nominal 4500 psi compressive strength concrete. Two of these contained the two new aggregates, No. 10 and 13, and the third beam was tested to confirm that the high diagonal tension resistance of Aggregate 7 as found in the original study could also be obtained from a later shipment of that aggregate. The four Type 4 beams at $f_c' = 3000$ psi were tested to verify the analysis of the other beams, particularly in regard to the effect of tensile strength on the diagonal tension resistance. Ten beams containing sand-and-gravel aggregate are included in Table 1. These beams were tested to obtain a comparison with the performance of the lightweight beams. Since only one rectangular beam cross section was employed, and also since only symmetrical concentrated loading was employed, the reported performance of the lightweight concrete beams as compared to normal weight beams may then be qualified by the particular dimensions and loading chosen for the test.

Aggregates and concrete mixes

The aggregate identification numbers used here are the same as in the previous reports.[1,4,5] No. 10 and 13 represent the new lightweight aggregates added to the test series. These aggregates, as well as the new shipment of Aggregate 7, were used with their commercial grading. The older aggregates were screened and recombined to the standard grading presented by Shideler,[4] who furnished a complete description of the aggregates. For convenience, an abbreviated description is given below:

Aggregate 2 is an expanded shale produced in a rotary kiln. Material passing No. 4 sieve is obtained by crushing.

Aggregate 3 is an expanded shale produced in a rotary kiln. All particles are rounded and have a smooth shell.

Aggregate 4 is an expanded clay produced in a rotary kiln. Most of the material is crushed but is not harshly angular.

Aggregate 5 is an expanded slate produced in a rotary kiln. The coarser particles are somewhat angular and porous. Material passing No. 4 sieve is obtained by crushing.

Aggregate 6 is an expanded blast furnace slag produced by spraying a controlled amount of water on a thin layer of molten slag. All particles are angular and porous and most of the finer sizes are obtained by crushing.

Aggregate 7 is produced by burning on a sintering grate a carbonaceous shale from anthracite coal processing. All sizes are obtained by crushing and particles are sharp and porous.

Aggregate 9 is an experimental expanded shale made in a rotary kiln. Particles are rounded and have a smooth shell.

Aggregate 10 is an expanded shale produced in a rotary kiln. The particles are generally rounded and sealed.

Aggregate 13 is an expanded shale produced in a rotary kiln. All particles are rounded and have a smooth shell.

Aggregate 8 is *normal weight* Elgin sand and gravel. The gravel is well rounded.

The concrete placing and proportioning procedures for the new beams were also similar to those for the plain concrete tests and for the original shear study. Details may be found in Reference 4. For comparison purposes the final mix quantities, plastic unit weights, and slumps of the new concrete mixes are given in Table 2.

TABLE 2 — MIX PROPORTIONS OF CONCRETES FOR DIAGONAL TENSION BEAMS

Aggregate No.	Fine aggregate, percent by volume	Quantities per cu yd of concrete					Percent air (Rollometer)	Plastic unit weight of concrete, lb per cu ft	Slump, in.
		Water, lb	Cement		Aggregate				
			Sacks	lb	Fine, lb	Coarse, lb			
3000 psi Series A									
2	50	442	6.06	570	785	638	7.0	90.2	2⅛
3X	45	332	4.41	415	980	754	7.0	91.9	2
3	45	346	5.41	509	991	766	6.4	96.7	2⅛
4	55	498	4.22	397	1048	750	5.9	99.7	2⅛
5	55	423	5.62	528	1098	606	7.2	98.3	1½
6	55	398	6.53	614	1187	650	8.2	105.5	1⅛
7	55	424	5.98	562	1136	630	7.0	101.9	1⅛
10	39	350	4.04	380	764	918	6.2	89.3	2
8	48	196*	3.63	341	1588*	1720*	5.9	142.4	3¼
4500 psi Series B									
2	45	450	8.82	829	653	650	5.9	95.6	2¼
3	40	360	8.50	799	751	713	5.3	97.2	2⅛
4	50	472	5.72	538	927	811	5.9	101.8	2⅛
5	50	418	8.02	754	969	655	6.0	103.0	2⅛
6	50	402	8.96	842	1045	698	6.1	110.6	2
7	50	443	8.82	829	961	649	5.9	106.7	2⅜
10	36	346	5.71	537	678	926	6.0	92.1	2⅜
13	55	372	8.53	802	855	518	5.9	94.3	2⅛
8	39	205*	5.31	499	1241*	1942*	5.9	143.9	2⅝
7000 psi Series C									
4	35	454	6.79	638	705	1100	2.0	107.3	½
8	25	224*	5.85	550	843*	2528*	1.1	153.5	⅝
9000 psi Series D									
4	35	475	10.06	946	622	974	2.0	111.7	¾
8	25	276*	9.93	933	727*	2182*	1.1	152.5	⅞

*Net water—Saturated surface dry aggregate.

TABLE 3 — AVERAGE YIELD STRENGTH AND MODULUS OF ELASTICITY

Grade of steel	Bar size	Yield point		Modulus of elasticity	
		No. of samples	f_y, psi	No. of samples	E, psi
Intermediate	#8	5	47,300	4	28,200,000
	#4	31	48,450	5	27,750,000
High strength	#7*	1	93,500	—	—
	#4	10	88,580	—	—

*Two pairs of one #7 and one #4 bundled reinforcement were used as equivalent to two #8 bars in Beam 8B2.

The mix quantities shown for the lightweight concretes have been computed on the cement content, dry aggregate basis in accordance with ACI 613A-59.* Most of the Series A and all of the Series C and D mixes were cast into 6 x 12-in. cylinders only. The 1958 report should be consulted for mix details of the original beam series. The Series A mix designated 3X was cast to produce split cylinders for Aggregate 3 at a lower strength level than was obtained with the beam mix labeled Aggregate 3.

Reinforcing steel

Preliminary estimates prior to the performance of the tests indicated that the intermediate grade reinforcing steel used in the lightweight beams would have a sufficiently high yield point to prevent flexural failure prior to diagonal tension cracking. These same estimates indicated that high strength steel should be employed in the sand-and-gravel beams. This same precaution should have been observed in the Type 4 beams containing two of the lightweight aggregates, these beams indicating flexural rather than diagonal tension failure. The deformations of all bars were in accordance with ASTM A 305. Samples of the reinforcement were tested at intervals throughout the duration of the program. Average yield strengths and moduli of elasticity are given in Table 3.

Fabrication and curing of specimens

All beams were cast in plywood forms. The reinforcing bars were supported from the bottom form by metal chairs at a height to provide an effective depth of 10½ in. for all beams. The four #4 bars used in the Types 3 and 4 beams were bundled[8] in pairs to provide clearance for the larger size aggregate during concreting.

The concrete was batched by weight and mixed in a 1¾ cu ft pan-type mixer. In accordance with ACI 613A-59 the aggregate and two-thirds of the mixing water were first introduced into the mixer and mixed for 2 min; then the cement, air-entraining agent and the remaining water were added and mixing was continued for an additional 3 min. A blend of four commercial Type I cements was used.

The concrete was placed in the beam form in four 3-in. lifts. Immediately after casting each beam, additional 1½-cu ft batches of concrete were mixed for fabrication of 6 x 12-in. cylinders for compressive strength and tension splitting. The concrete in all beams and cylinders was consolidated by internal

*ACI Committee 613, Subcommittee on Proportioning Lightweight Aggregate Concrete, "Recommended Practice for Selecting Proportions for Structural Lightweight Concrete (ACI-613A-59)," American Concrete Institute, Detroit, 1959, 10 pp.

vibration, with the beam concrete being vibrated after placement of each second lift. The steel cylinder molds were covered with ground steel plates for the first 24 hr after casting and the beams were covered with a plastic film. At the end of 1 day, the forms and molds were stripped and the specimens were placed in a moist room (73 F) until 7 days. At this time, all beams, all cylinders for compression testing, and one-half the cylinders for tension splitting were stored in laboratory space controlled at 73 F, 50 percent relative humidity, until tested. The other half of the tension splitting cylinders remained in the moist room until tested in the wet condition. All tests were performed at 28 days.

Testing procedure

The beams were tested in a 1,000,000 lb hydraulic compression machine. Fig. 1 shows the details of the beam fabrication and the testing arrangement. The 4-in. wide reaction and load bearing plates were seated and leveled in thickened polyester resin to assure full bearing. A dial gage located at center-span indicated maximum deflection of the simply supported beams. The light-weight concrete beams were loaded in increments of 1000 lb and the normal weight beams in 2000-lb increments. At each increment the load was maintained for a period sufficient to record deflections and mark the progression of cracks. About 45 min were required for each beam test.

A 400,000 lb universal hydraulic testing machine was employed for the split-cylinder test. This machine was fitted with a 12 in. wide rectangular head on a spherical bearing. The testing machine center line was carefully marked on each side of the 12-in. dimension of this head. The concrete cylinder was placed on the testing machine platen on top of a plywood strip, as shown by the upper view of Fig. 2. The machine crosshead was then adjusted close to the concrete cylinder. After determining that the ends of the cylinder coincided with the sides of the bearing block, the cylinder was adjusted sideways until the machine centerline was in alignment with the vertical diameter. No premarking of the vertical diameter was made, but the alignment was judged by eye when the machine center line passed through the point of horizontal tangency to the cylinder. Another plywood pad was then inserted between the cylinder and the testing machine head. These plywood pads were strips ⅛ in. thick, 2 in. wide, and a full 12 in. long, cut from sheets to which a kraft paper was bonded on both sides.

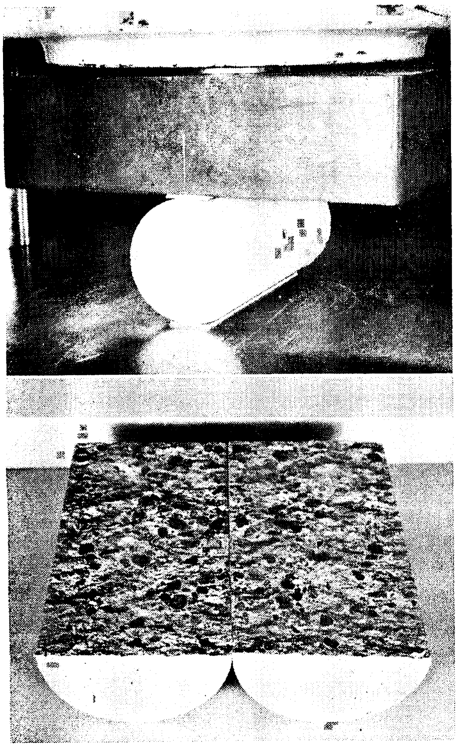

Fig. 2—Cylinder splitting test in progress (top); lightweight concrete cylinder after tensile failure (bottom)

One of the European authorities, Thaulow,[9] has recommended that the tension cylinder be loaded at approximately one-fourth the rate of a corresponding compression cylinder. Consequently, a loading rate of 14,000 lb per min was established for these tests. This rate was applied immediately after contact with the head was established, and the loading was continued until needle drop and vertical splitting occurred simultaneously. It is of interest to note that examination of the pads showed that approximately a ½ in. length of arc was in contact with the cylinder.

TEST RESULTS AND DISCUSSION

Split-cylinder tension tests

The Brazilian or split-cylinder test for tensile strength of concrete may have been first introduced by Lobo B. Carneiro and Barcellos.[10] This particular test has found considerable favor in other parts of the world, but is not widely used in the United States. However, a few investigations of the adequacy of this simple measure of tensile strength are continuing in this country, one of which was discussed in the Research Session of the 1960 ACI annual convention. Thaulow presents a good bibliography of foreign theoretical and experimental studies. Investigators have studied such test variables as water-cement ratio, size of aggregate, age of concrete, size of specimen, rate of loading, and size and thickness of loading pads. All of these variables will affect the test results to a greater or smaller degree.

Rüsch and Vigerust[11] have presented perhaps the most effective discussion of the reliability and usefulness of the tensile splitting test. In addition to the practical advantage of ease of testing, they discuss several important advantages of this method over the flexure test as a measure of tension. They point out the sensitivity of the outer fibers of the flexure specimen, which sustain the maximum stress, to incidental moisture changes, while the maximum tension stresses in the split-cylinder test are applied to the interior of the cylinder. A similar effect results from minor local defects such as large pieces of aggregate near the surface. They also discuss the increased mathematical probability that weak points with low tensile resistance will fall within an area of maximum stress in the flexure test, while the split-cylinder test is virtually independent of the dimensions of the test specimen. These two authorities conclude that the split cylinder is easier to test; that the dispersion of the test data will be less for the tensile splitting test; and that the indicated tensile strength obtained from the cylinder splitting test will not deviate far from the true tensile strength of the material.

There is one major conclusion that may be drawn from all the various investigations reported on this indirect measurement of tensile strength. This is that the split cylinder can furnish a reliable measure of the *relative* tensile strength of concrete, provided that several samples are employed.

The theory of elasticity indicates that a uniform tensile stress at right angles to the direction of load application must exist over a substantial part of the interior of a cylinder subjected to concentrated loads at the opposite ends of a diameter. This stress is given by the formula

$$f'_{sp} = \frac{2P}{\pi DL} \qquad (1)$$

where f'_{sp} is the uniform tensile stress, P is the magnitude of load, and D and L are the cylinder diameter and length, respectively.

The tensile strength determined in this manner from measured values of P may be subject to influence from test conditions such as cylinder size, loading rate, and pad dimensions. The stress condition in the cylinder is obviously biaxial. The theory shows that the compressive stress at the center of the cylinder, and parallel to the loaded diameter, is equal to three times the tensile stress. The average compressive stress over the loaded diameter may equal five or six times the uniform tensile stress, depending on the conditions of load distribution and plasticity under the concentrated loads.

For the above reasons, the stress computed by Eq. (1) should probably not be considered as the true tensile strength of concrete. However, a study of the various investigations show that the split-cylinder test is a good measure of the relative tensile strength and that this test will reliably reflect such variables as compressive strength, water-cement ratio, age of concrete, and type of curing. Some advantages of the split-cylinder test over the modulus of rupture test have already been discussed but perhaps the most important advantages of the split cylinder are the following two considerations. First, the tensile stress distribution must approach uniformity over a large diametral area of the cylinder, and thus stress concentrations from specimen defects or other causes will be reduced by plasticity. Secondly, the test procedure is extremely simple and affords the opportunity to economically test a large number of specimens, thus offsetting the rather large variation of results that must always be expected in tensile investigations.

In connection with the extended diagonal tension program, 225 lightweight concrete cylinders and 57 normal weight concrete cylinders were broken by the splitting test. Photographs of this test, in progress and after failure, are shown in Fig. 2. Approximately one-half of the specimens of each type of concrete were given 7 days moist curing followed by 21 days drying at 73 F and 50 percent relative humidity. The other half of the specimens were continuously moist cured at 73 F. The break of the cylinder was always an approximate plane located on or near to the loaded diameter. In all lightweight concrete cylinders, regardless of compressive strength, this failure plane passed directly through nearly all pieces of aggregate. In the lower strength sand-and-gravel specimens (Series A and B), only 10 to 20 percent of the aggregate pieces were broken. In the extremely high strength sand-and-gravel cylinders (Series C and D) practically all aggregate was ruptured. It would appear then that tensile strength of lightweight concrete is largely determined by the tensile strength of the particular aggregate, while the tensile resistance for the usual ranges of strength of normal weight concrete is determined by bond of the paste to the aggregate. Only after the compressive strength reaches approximately 7000 psi or above, will the tensile strength of this particular sand-and-gravel concrete be primarily affected by tensile strength of the aggregate.

The measured split-cylinder tensile strengths of all the concretes are presented in Table 4. These results include cylinders of approximately the same compressive strength as the concretes in the original shear beam series[1] as well as from the present study. The table also presents the number of cylinders tested and the coefficients of variation of the measurements for each concrete. The coefficient of variation has been computed by an approximate method[12] for a small number of samples (10 or less). Examination of the table shows, with only a single excep-

TABLE 4 — TENSILE STRENGTH OF LIGHTWEIGHT CONCRETES AS MEASURED BY THE SPLIT-CYLINDER TEST

		Dry concrete*							Moist concrete*			
Aggregate No. and strength series	f_c', psi	No. of specimens	Avg f'_{sp}, psi	Coefficient of variation, percent	$\frac{f'_{sp}}{f_c'}$, percent	a_1†	Calculated f'_{sp}	$\frac{f'_{sp\ test}}{f'_{sp\ calc}}$	No. of specimens	Avg f'_{sp}, psi	Coefficient of variation, percent	$\frac{f'_{sp}}{f_c'}$, percent
2 A	3210	6	265	8.0	8.3	4.28	243	1.09	6	365	6.0	11.4
B	4840	6	271	6.3	5.6		298	0.91	6	434	9.9	9.0
3 AX	3000	6	269	8.9	9.0	4.84	265	1.02	6	357	10.4	11.9
A	3850	5	299	6.9	7.8		300	1.00	4	428	4.8	11.1
B	4320	5	314	6.9	7.3		318	0.98	4	452	7.5	10.5
4 A	3100	5	294	13.5	9.5	4.80	267	1.10	5	359	6.7	11.6
B	5090	6	307	3.5	6.0		342	0.90	5	394	21.2	7.7
C	7100	6	348	13.8	4.9		405	0.86	6	593	8.1	8.4
D	8100	6	372	11.8	4.6		432	0.86	6	598	10.2	7.4
5 A	3260	6	323	10.2	9.9	5.28	301	1.07	5	361	4.3	11.1
B	4940	6	345	9.4	7.0		371	0.93	6	471	11.8	9.5
6 A	3320	6	302	2.7	9.1	4.96	286	1.06	6	362	10.9	10.9
B	4890	8	327	6.5	6.7		346	0.94	7	471	11.3	9.6
7 A	3280	6	348	10.6	10.6	5.96	342	1.02	6	351	8.4	10.7
B	4840	8	408	7.9	8.4		415	0.98	9	479	8.6	9.9
10 A	3270	6	273	6.2	8.4	4.55	260	1.05	5	394	5.8	12.0
B	4700	9	297	5.7	6.3		312	0.95	10	472	10.2	10.0
13 B	5040	8	276	10.8	5.5	3.89	276	—	9	439	12.7	8.7
8 A‡	3030	7	380	3.2	12.5	6.88	378	1.00	6	324	6.5	10.7
B	4380	10	454	7.5	10.4		455	1.00	10	415	9.4	9.5
C	7300	6	687	9.1	9.4		588	1.17	6	618	5.0	8.5
D	8600	6	663	6.5	7.7		638	1.04	6	672	10.6	7.8

*Dry concrete indicates 7 days moist curing followed by 21 days drying at 73 F, 50 percent relative humidity. Moist concrete indicates 28 days moist curing.

† $a_1 = f'_{sp}/\sqrt{f_c'}$

‡ **Aggregate 8 is Elgin sand and gravel.**

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tion, that the coefficient of variation of the different concretes varies from 3 to 14 percent, with many values between 6 and 10 percent. These values may appear to be rather large, but it must be remembered that tension tests always have a large variation.

Relation of tensile strength to compressive strength

Table 4 also shows the split-cylinder tensile strength as a percentage of the compressive strength. The program did not include 28 day moist-cured compression tests, so the assumption was made that the 28 day compressive strengths of the dry and moist cured concretes were equal. This assumption is well justified from an examination of Shideler's[4] data on similar concretes. For the dry concretes (7 days moist curing, 21 days drying at 73 F, and 50 percent relative humidity) the tensile strengths of the lightweight concretes range from approximately 9.5 percent of 3000 psi compressive strengths down to some 7 percent at the 4500-psi level. These percentages approach 4.5 percent of the extremely high strengths of the Aggregate 4 concrete. Corresponding approximate percentages for the sand-and-gravel concrete are 12.5, 10.5, and 7.5. The ratio of the moist split-cylinder tensile strengths to the compressive strengths for lightweight concrete averages about 2 to 3 percentage units higher than the corresponding ratio with dry tensile strengths. This comparison is reversed for the normal weight concretes where the "wet" percentages are one to two units lower than the dry. Certainly the data indicate that tensile strength is not a constant proportional part of the compressive strength, but that the proportionality ratio decreases as compressive strength increases. This trend agrees with the findings of Rüsch and Vigerust[11] who tested concrete that had dried for 3 weeks. Similar verification is furnished by Gruenwald[13] for continuously moist cured concrete.

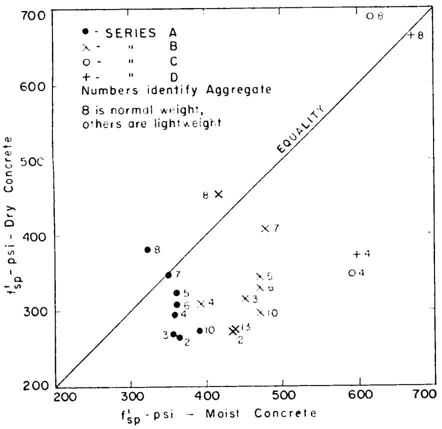


Fig. 3—Relationship of moist and dry split-cylinder tensile strength

Since tensile strength then cannot be considered as a linear proportion of compressive strength, many investigators have related tensile resistance to the square root of the compressive strength, such as:

$$f'_{sp} = \alpha_1 \sqrt{f_c'} \quad \text{............(2)}$$

Table 4 presents values of α_1 for the dry concretes containing the various aggregates together with calculated values of f'_{sp} for each compressive strength. The ratio of measured f'_{sp} to calculated f'_{sp} reveals that the average α_1 computes a tensile strength that is too low for the lower compressive strength lightweight concretes and too high for the higher strengths. However, this variation averages only $\pm$ 6 percent. Eq. (2) is then not entirely unsatisfactory for lightweight concretes provided that the value of α_1 is considered as a function of the aggregate. The relation appears quite satisfactory for the normal weight concretes of usual compressive strengths.

A most important observation from Table 4 is that the relative level of tensile resistance of each concrete is a definite characteristic of each particular aggregate, when the dry concretes are compared on an equal compressive strength basis.

Effect of drying on tensile strength

The dry split-cylinder tensile strengths of Table 4 are plotted in Fig. 3 as a function of the corresponding tensile strengths for continuous moist curing. It will be noted that the moist tensile strengths of each of the four series fall in a rather narrow band; the dry tensile strengths, however, vary widely within each series. Also, the dry tensile strengths of the concretes containing each aggregate fall in the same order of progression. This again reflects that the magnitude of dry tensile strength is a characteristic peculiar to the concrete containing each particular aggregate.

Examination of Fig. 3 also shows that drying of the lightweight concrete decreases the tensile strength by amounts up to 40 percent depending on the aggregate and the compressive strength. It might be surmised that this effect is due to tensile stresses imposed by drying shrinkage, but attempts to correlate this with shrinkage measurements on similar concretes were not successful.

In opposition to the effect on the lightweight concretes, drying of the sand-and-gravel concretes (Aggregate 8) generally increased the split-cylinder tensile strength by amounts up to 15 percent. No explanation is offered for this, unless it might be that stresses induced by drying act to increase the bond of paste to aggregate.

Split-cylinder strength and modulus of rupture

The relationship of tensile strength data obtained by plain concrete beams and by cylinder splitting tests is shown in Fig. 4. The modulus of rupture data were gathered by Shideler[4] on similar concretes. A generally linear relationship between the two test methods is suggested by the data, but the scatter of the points is large, probably because of the sensitivity of the modulus of rupture of partially dried beams to extraneous test conditions. Evidence of this erratic test condition is found in Fig. 4 where it is shown that the tensile splitting values of the *dry* lightweight concretes are larger than the corresponding measured moduli of rupture. Theoretical considerations have led most investigators to regard the modulus of rupture as somewhat larger in magnitude than either the simple or split-cylinder tensile strength. The anomaly can then

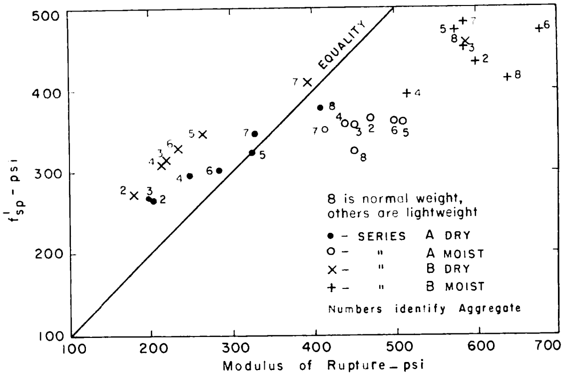
TABLE 5 — RESULTS OF DIAGONAL TENSION BEAM TESTS

Beam No.	f_c', psi	f'_{sp}, psi	P_{c_1}, kips	P_{c_2}, kips	P_u, kips	v_c, psi
Type 4 beams, $p = 1.25$ percent — $M_{max}/Vd = 5.0$						
3A4	3850	299	12.00	—	12.00	95
4A4	3100	294	11.70	—	11.70	93
10A4	3270	273	12.90	—	12.90	102
8A4	3030	380	15.20	—	15.20	121
2B4	4940	271	10.90	11.50	11.50	87
3B4	4400	314	11.55	—	11.55	92
4B4	5120	307	12.00	—	13.15	95
5B4	5010	345	—	—	13.80*	—
6B4	4850	327	13.00	—	13.00	103
7B4	4860	408	—	—	15.10*	—
10B4	4850	297	12.40	—	12.40	98
13B4	5150	276	13.20	—	13.20	105
8B4	4490	454	19.25	—	19.25	153
10BW4	4530	—	11.60	—	11.60	92
8BW4	4300	—	18.00	—	18.00	143
Type 2 beams, $p = 2.5$ percent — $M_{max}/Vd = 5.0$						
2B2	4880	271	15.00	—	15.00	119
3B2	4540	314	16.00	—	16.00	127
4B2	5100	307	16.00	—	16.00	127
5B2	4930	345	19.80	—	19.80	157
6B2	5000	327	17.00	—	17.00	135
7B2	4960	408	22.00	—	22.00	175
10B2	4430	297	16.65	—	16.65	132
13B2	5060	276	15.00	17.00	17.25	119
8B2	4470	454	23.56	—	23.56	187
Type 3 beams, $p = 1.25$ percent — $M_{max}/Vd = 2.5$						
2B3	4780	271	14.90	14.90	26.95	118
3B3	4170	314	16.00	16.94	30.02*	127
4B3	5020	307	13.90	17.00	29.70*	110
5B3	4900	345	20.00	20.00	29.70*	159
6B3	4820	327	16.80	16.80	19.00	133
7B3	4870	408	21.85	24.10	25.00	173
10B3	5020	297	17.00	18.35	31.40*	135
13B3	5020	276	16.40	17.00	26.00	130
8B3	4370	454	19.00	20.70	20.70	151

Beam No.	f_c', psi	f'_{sp}, psi	P_{c_1}, kips	P_{c_2}, kips	P_u, kips	v_c, psi
Type 1 beams, $p = 2.5$ percent — $M_{max}/Vd = 2.5$						
2A1	3680	267	19.20	19.20	37.00	152
3A1	3310	280	19.80	—	19.80	157
4A1	2980	293	18.45	20.81	24.71	146
5A1	3490	327	21.71	—	21.71	172
6A1	3670	313	19.75	20.00	20.05	157
7A1X	3210	345	25.46	—	25.46	202
7A1	4240	386	27.72	27.90	39.75	220
8A1X	3700	419	24.06	28.05	36.16	191
8A1	4020	436	25.95	—	25.95	206
2B1	5350	273	17.97	21.18	43.22	143
3B1	4090	300	19.82	20.25	22.00	157
4B1	4890	305	21.56	22.00	37.69	171
5B1	4790	344	21.90	—	21.90	174
6B1	4870	322	19.47	21.56	31.55	155
7B1X	4680	408	28.50	29.65	35.50	226
7B1	5200	417	28.04	29.80	39.40	223
10B1	4860	297	20.00	23.00	43.00	159
13B1	4940	276	18.00	21.10	39.90	143
8B1	5380	502	28.50	33.70	40.64	226
Type 1 beams, $p = 5.0$ percent — $M_{max}/Vd = 2.5$						
9C1	6910	340	26.00	28.40	50.80	206
4C1	7000	348	24.40	28.00	49.10	194
8C1	8410	663	37.74	40.00	57.24	300
4D1	8160	372	28.25	28.25	53.50	224
8D1	10,680	625	39.30	40.00	74.40	312

f_c' = compressive strength of 6 x 12-in. cylinders
f'_{sp} = split-cylinder tensile strength
P_{c_1} = load causing diagonal crack at one end of beam
P_{c_2} = load causing diagonal crack at other end of beam
P_u = ultimate load capacity of beam
$v_c = V/bd$ = unit shear based on first diagonal cracking load
*Denotes failure by yielding of tensile reinforcement

Fig. 4—Relationship of modulus of rupture and split-cylinder tensile strength



be interpreted as further evidence of the deleterious effect of drying on the extreme fibers of the plain concrete beam.

Diagonal tension beam tests

Table 5 presents the test data on diagonal cracking, ultimate load, and unit shear of the diagonal tension test beams. The table includes the 21 beams that were tested in the original program[1] with the unit shears of these beams recomputed according to the following definition:

$$v_c = \frac{V}{bd} \qquad (3)$$

where v_c is the nominal unit shear at diagonal tension cracking, V is the total external shear, and b and d are the width and effective depth of the beam.

This definition, which eliminates the factor j from the denominator, has been proposed by Bower and Viest[14] and Committee 326. The identification numbers of the beams have been coded as follows: the first number is the aggregate identification number described previously (8 being the normal weight aggregate), the following letter shows the nominal compressive strength of the concrete as given by Table 2, and the last number is the beam type defined in Fig. 1. Thus the Beam 4B3 indicates a beam containing a concrete with a rotary kiln expanded shale aggregate, of 4500 psi nominal compressive strength, and a beam containing 1.25 percent steel area over a shear span of 26 in. It will be noted that all of the 21 original beams are Type 1, thus this number has been merely added to the original identification number. The only new Type 1 beams cast in the present investigation were 7B1X, 10B1, and 13B1. Any numbers labeled with an "X" indicates a recasting and retesting of a beam to verify previous results or to adjust compressive strength. The two beams labeled 10BW4 and 8BW4 were tested in the moist condition at 28 days.

The compressive strengths of Table 5 are the average strengths measured by 6 × 12-in. cylinders that accompanied each particular beam.

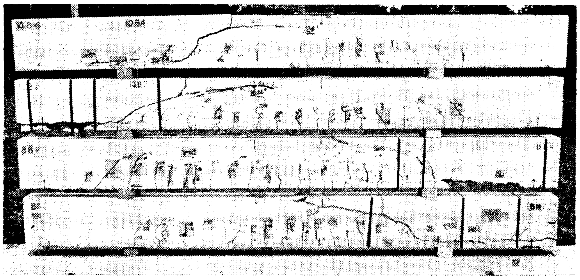


Fig. 5 — Typical cracking patterns of longer span beams

The dry split-cylinder tensile strengths, f'_{sp}, are the average of 5 to 10 tests of cylinders that accompanied several beams. The tensile strength values for all new beams are identical with Table 4, while, for the early beams, the values of Table 4 have been adjusted to compensate for changes in compressive strength. The values of P_{c1}, P_{c2}, and P_u are the total loads on each of the beams at first diagonal cracking, second diagonal cracking (if such occurred), and at ultimate failure of the beam. These loads were indicated in every case by a drop of the testing machine load-indicator needle.

Ultimate strength of test beams

The flexural cracking of all beams developed in a normal manner. These vertical cracks first appeared in the zone of pure moment, and as the load was increased to higher levels they spread to the areas outside the symmetrically applied loads. In many beams, the flexural cracks outside the pure moment zone inclined toward the loading points just prior to formation of the diagonal crack. The diagonal cracks formed suddenly and resulted in either complete failure of the beam or in sizable loss of applied load. Typical cracking patterns of both long and short span beams are shown in Fig. 5 and 6.

Measured ultimate load capacities of all beams are listed in Table 5. Comparison of the ultimate load, P_u, with P_{c1}, the load at first diagonal cracking, indicates that 19 out of the 24 longer span beams (Types 2 and 4) failed completely at diagonal tension cracking. All of the Type 3 beams ($p = 1.25$ percent) achieved redistribution of forces and absorbed loads of variable amounts above that causing diagonal cracking. As noted in the earlier paper,[1] the diagonal cracking load was ultimate for five of the Type 1 beams ($p = 2.5$ percent). This somewhat erratic ultimate behavior of the beams confirms the conclusion of the earlier paper and of ACI-ASCE Committee 326 that only the load at diagonal cracking should be considered as the ultimate load in the design of non-web-reinforced beams.

Two of the Type 4 beams, those containing Aggregates 5 and 7, failed by yielding of the longitudinal reinforcement prior to formation of diagonal cracks. The high resistance to diagonal tension of concretes containing these aggregates will be discussed later in this report. Four of

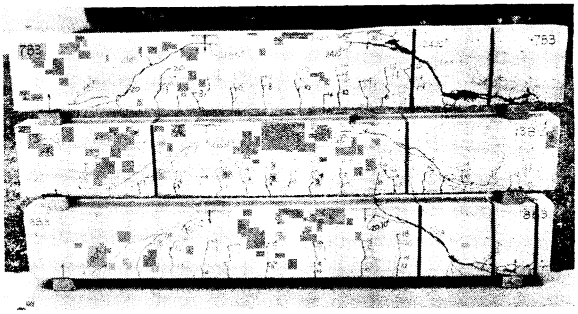


Fig. 6 — Typical cracking patterns of short span beams, $p = 1.25$ percent

the Type 3 beams also failed ultimately by yielding of the steel *after* formation of the diagonal cracks. These six beams are indicated by an asterisk accompanying the ultimate load value. A comparison of the measured ultimate flexural moments of these beams with the moments computed prior to testing showed that measured moments vary from approximately 0 to 12 percent higher than the computed, indicating the good reliability of the ultimate design coefficients used in the computation.

Location of diagonal cracks

Both Bower and Viest[14] and the Committee 326 have assumed that the section of potential initial diagonal cracking is located at a distance equal to the beam effective depth d, outside of the point of load application. For short shear spans, this distance is limited to no greater than one-half the shear span. Such an assumption is necessary to the complete functioning of their proposed design formulas. Fig. 7 indicates this assumed location relative to the diagonal crack location for the various beams of nominal 4500 psi compressive strength. The crack locations were traced from photographs after completion of each test. It would appear that this design assumption might agree with an over-all average crack location with a rather large variation in either direction for individual beams. This situation is not surprising since past experience has shown that diagonal crack location is highly variable and in many ways a matter of chance.

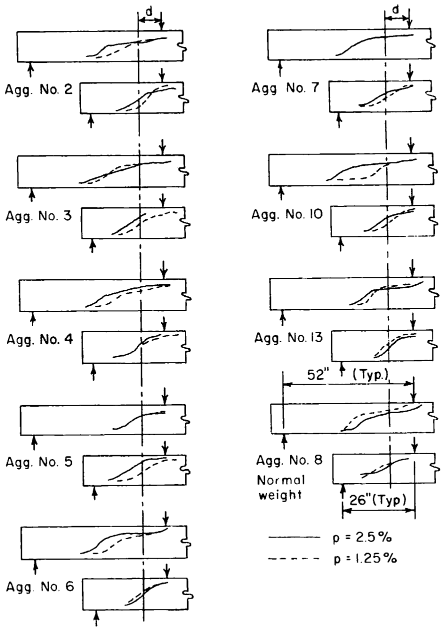


Fig. 7—Location of diagonal cracks in Series B beams

Nominal shear strength

The present ACI Building Code (ACI 318-56) directs that unit shear stresses, v, shall be computed by the formula

$$v = \frac{V}{b\,j\,d} \qquad (4)$$

where V is the total external shear at any section and j is the ratio of the internal resisting-moment arm to the effective depth.

The Code also provides that the allowable working stress in shear in non-web-reinforced beams shall be $0.03 f_c'$ with an upper limit of 90 psi. Fig. 8 presents a comparison of the unit shears of the test beams computed by Eq. (4) with the code allowable stresses. For convenience, the factor j has been assumed as $7/8$. This value is somewhat high for most of the lightweight beams, particularly those with larger steel percentages, and thus the plotted values of unit shear of these beams are somewhat low. These unit shears represent *ultimate* values (at diagonal cracking) and the figure shows that many beams have safety factors with respect to the Code allowable approaching 1.0. Even the normal weight beam (Aggregate 8) with the lowest steel area has a safety factor of only 1.5. All of the Type 4 beams and most of the beams in the Types 2 and 3 categories have measured nominal shears of less than twice the Code allowable. The need for more reliable code provisions for non-web-reinforced beams is obvious. This has been recognized for some time, and Committee 326 is recommending ultimate design procedures for

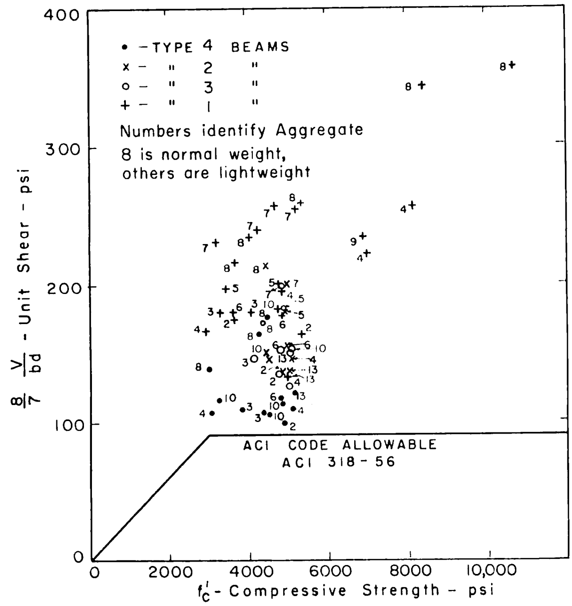


Fig. 8—Measured unit shear as compared with code allowable stresses

diagonal tension in normal weight concrete. A similar approach for lightweight concrete will be discussed in the final section of this paper.

Correlation of unit shear with tensile splitting strength

The Committee 326 report* presents an equation that relates shearing stress to the diagonal tensile strength of concrete. This equation was derived by Morrow and Viest in working toward a rational relationship for the diagonal cracking load starting from the formula for principle stress at a point. The derivation is also based on the theoretical linear stress distribution for reinforced concrete. The resulting equation is quite complex and simplification is obtained by rationalizing that the resistance of concrete to principal tensile stress is comparable to the tensile strength of the concrete. The tensile strength of the concrete in turn is assumed to be directly proportional to the square root of the compressive strength. In the present investigation, the relative tensile strength has been measured by the cylinder splitting test. The detailed results of these tests were presented in Table 4 and the tensile resistance, f'_{sp}, to be associated with each particular beam was given by Table 5.

Detailed examination of Table 5 in regard to both f'_{sp} and v_c, the unit shear computed by Eq. (3), reveals that these quantities fall into a typical sequence with respect to the various aggregates. Further comparison of the two values indicates that the relationship between v_c and f'_{sp} is linear for each of the beam types. This direct relationship and typical sequence of tensile resistance is illustrated in Fig. 9. The dispersion of data shown in this figure appears quite small, particularly when compared to diagrams such as Fig. 8, where unit shear is plotted against compressive strength. This reduction in the spread of diagonal tension data definitely indicates the utility of the split-cylinder tension test as related to the basic cause of diagonal tension failure, i.e., tensile stress that exceeds the tensile strength of the concrete.

The five diagrams of Fig. 9 show the relationship between v_c and f'_{sp} for each of the five values of $pd/(M_{max}/V)$ available in the test program. The term, M_{max}/V, is identical to the shear span length, a, for simple beams with a single concentrated load or with two symmetrical concentrated loads. Each plotted point is labeled with the particular aggregate identification number involved. The typical sequence of points in each diagram indicates the characteristic value of tensile (and thus shear) strength associated with each aggregate. Particular attention is directed to the relative position of the points for Aggregates 6, 5, and 7 which fall in the intermediate to high range, respectively. The sand-and-gravel concrete (Aggregate 8) shows the highest resistance but is often approached in magnitude by Aggregate 7. Pertinent evidence of the high tensile resistance of some lightweight aggregates is indicated by the

*Unpublished.

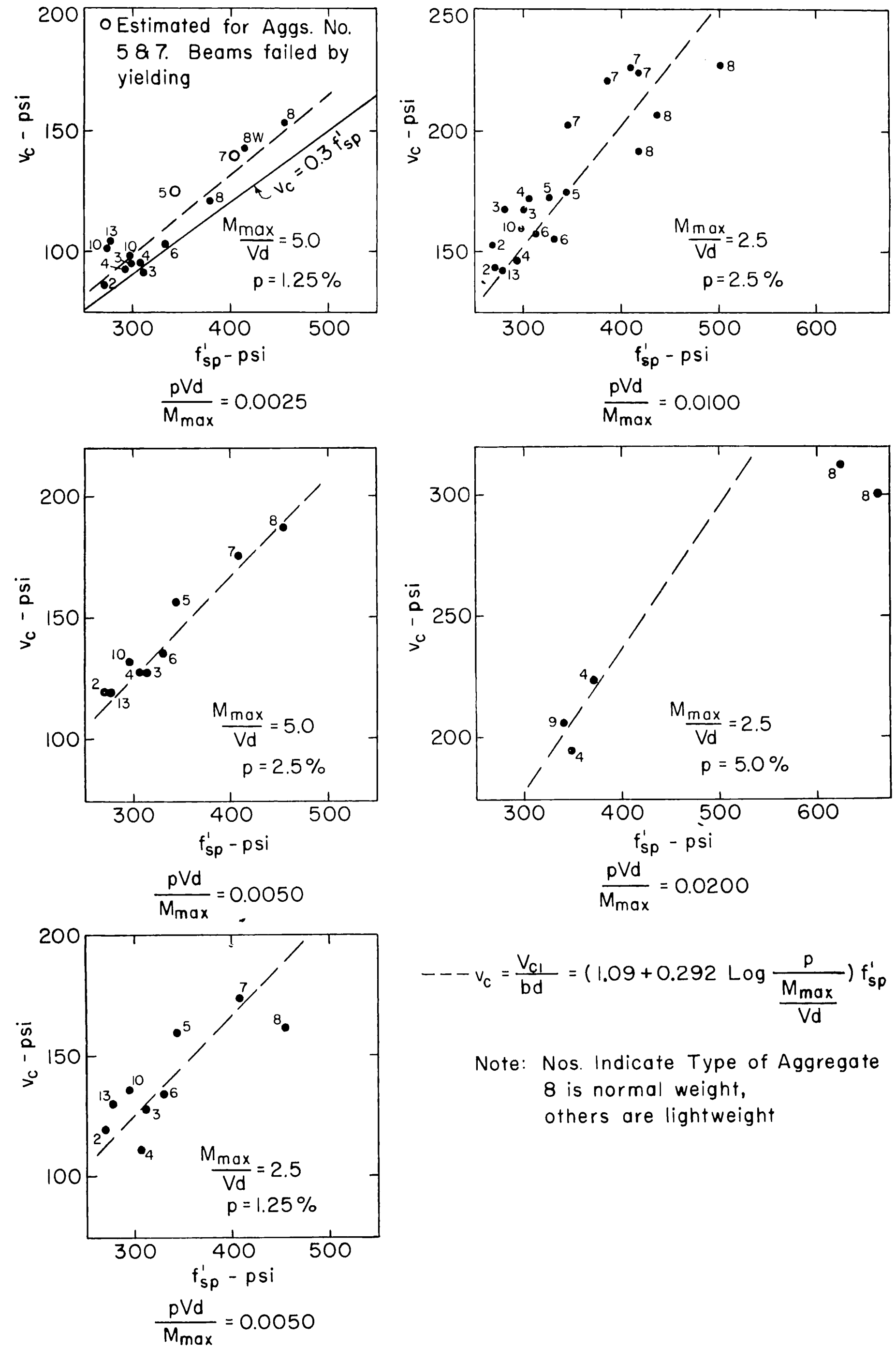


Fig. 9—Diagonal tension strength of beams versus split 6 x 12 in. cylinder tensile strength

yielding failure of the two long span, low steel beams ($pVd/M_{max}=$ 0.0025) containing Aggregates 5 and 7. Estimated values of v_c for these beams are shown in the upper left diagram. The other aggregates included in the study show similar sequence of tensile resistance ranging from low to intermediate magnitudes.

The data of each diagram of Fig. 9 appear to be quite linear in the relationship of unit shear to the split-cylinder tensile strength and further, by extrapolation, the relationship could quite easily pass through the point of zero values of v_c and f'_{sp}. Consequently, a function of the type

$$v_c = \alpha_2 f'_{sp} \qquad (5)$$

was assigned to each of the five diagrams. Examination of the five different slopes, α_2, indicated that the change of slope was a function of the beam characteristics, pVd/M_{max}, and that this function could be fitted almost exactly by a semilogarithmic function. Consequently, a least squares computation was made of the data from 54 beams and the following relation was established:

$$v_c = \frac{V_{c_1}}{bd} = \left(1.09 + 0.292 \text{ Log } \frac{pVd}{M_{max}}\right) f'_{sp} \qquad (6)$$

This function is plotted as the dashed lines on Fig. 9. It should be noted that the parameter of beam characteristics, pVd/M_{max}, in this equation differs from a similar parameter of the Committee 326 formula presented in the last section of this report [see Eq. (10)]. From the rather precise agreement of the diagonal tension strengths of Types 2 and 3 beams, shown in Table 5 and the lack of definite location of the diagonal cracks of Fig. 7, it appears that term, M_{max}/V, reflecting the complete shear span, more aptly reflects the behavior of the beams of this program. The term, M/V, of the Committee 326 formula presumes the location of the section of diagonal cracking. Such an assumption is necessary for the analysis of beams with other types of loading and conditions of restraint.

The fit of the semilogarithmic function, Eq. (6), is fortunately best for the longer span beams. These are more representative of beams in service than are the shorter beams. Such closer fit for the longer span beams may reflect a larger proportional effect of principal tensile resistance in these beams. It may be that the rather wide dispersion of points for the Type 1 beams, $pVd/M_{max} = 0.01$ and 0.02, is the result of greater shear stress effect on the diagonal tension resistance.

The over-all coefficient of variation of V_{test}/V_{calc} from Eq. (6) was 8.4 percent. The coefficient of variation for the longest beams with lowest amount of steel, $pVd/M_{max} = 0.0025$, was reduced to 6.7 percent. Thus rather good correlation is shown between the average split-cylinder tensile strength and the unit shear strength of the beams at diagonal cracking. Such good correlation over the wide range of beam characteristics (pVd/M_{max} varying from 0.0025 to 0.02) and the wide range of compressive strengths (3000 to 9000 psi) strongly indicates that the split-cylinder test may be used as a reliable measure of unit shear strength.

Deflections

The midspan deflection of all beams was measured at each increment of load up to the point of initial diagonal cracking. The measured deflections of all new beams prior to diagonal cracking are reported in Table 6. Computed deflection at the same loading are also given in the table along with the moduli of elasticity of steel and concrete and the

TABLE 6 — BEAM PROPERTIES AND DEFLECTION COMPARISON

Beam No.	E_c,* 10^6 psi	E_s, 10^6 psi	n	p	Computed k	I,† in.[4]	Computed center deflection, in.	Measured center deflection, in.
Applied load = 10 kips								
3A4	2.00	27.75	13.9	0.0125	0.44	575	0.305	0.305
4A4	1.54	27.75	15.4	0.0125	0.46	616	0.368	0.350
10A4	1.90	27.75	14.6	0.0125	0.45	595	0.308	0.309
8A4	3.21	27.75	8.6	0.0125	0.37	415	0.264	0.294
2B4	1.69	27.75	16.4	0.0125	0.47	641	0.325	0.318
3B4	1.80	27.75	15.4	0.0125	0.46	616	0.318	0.300
4B4	1.80	27.75	15.4	0.0125	0.46	616	0.318	0.332
5B4	2.30	27.75	12.1	0.0125	0.42	524	0.293	0.286
6B4	2.70	27.75	10.0	0.0125	0.39	460	0.278	0.292
7B4	2.16	27.75	12.8	0.0125	0.43	546	0.298	0.288
10B4	2.06	27.75	13.5	0.0125	0.44	564	0.304	0.290
13B4	1.97	27.75	14.1	0.0125	0.44	580	0.308	0.279
8B4	3.48	27.75	8.0	0.0125	0.36	391	0.257	0.257
10BW4	2.15	27.75	12.9	0.0125	0.43	548	0.297	0.277
8BW4	3.46	27.75	8.0	0.0125	0.36	393	0.257	0.219
Applied load = 12 kips								
2B3	1.60	27.75	17.3	0.0125	0.48	663	0.096	0.096
3B3	1.79	27.75	15.5	0.0125	0.46	618	0.092	0.087
4B3	1.80	27.75	15.4	0.0125	0.46	616	0.092	0.090
5B3	2.20	27.75	12.6	0.0125	0.43	540	0.086	0.081
6B3	2.78	27.75	10.0	0.0125	0.39	460	0.080	0.087
7B3	2.13	27.75	13.0	0.0125	0.43	551	0.086	0.072
10B3	2.14	27.75	13.0	0.0125	0.43	550	0.086	0.080
13B3	1.94	27.75	14.3	0.0125	0.45	587	0.090	0.091
8B3	3.43	27.75	8.1	0.0125	0.36	396	0.075	0.075
Applied load = 14 kips								
2B2	1.62	28.2	17.4	0.025	0.59	984	0.310	0.330
3B2	1.87	28.2	15.1	0.025	0.57	913	0.328	0.335
4B2	1.80	28.2	15.7	0.025	0.58	932	0.293	0.313
5B2	2.21	28.2	12.8	0.025	0.54	833	0.268	0.273
6B2	2.68	28.2	10.5	0.025	0.51	746	0.247	0.261
7B2	2.18	28.2	12.9	0.025	0.54	840	0.268	0.275
10B2	2.04	28.2	13.8	0.025	0.55	872	0.278	0.292
13B2	1.92	28.2	14.7	0.025	0.57	900	0.285	0.287
8B2	3.51	28.2	8.0	0.025	0.46	632	0.225	0.228
Applied load = 16 kips								
7B1X	2.20	27.25	12.4	0.025	0.54	820	0.075	0.075
10B1	2.07	28.2	13.6	0.025	0.55	864	0.075	0.085
13B1	1.88	28.2	15.0	0.025	0.57	910	0.079	0.081

*Secant modulus of elasticity at $0.3 f_c'$.
†Moment of inertia of cracked transformed cross section

ratios n, p, and k. The deflections were computed by the "cracked transformed cross section" method. The ratio of depth to neutral axis to effective depth, k, was computed by the standard formula

$$k = [2np + (np)^2]^{1/2} - np \qquad (7)$$

With this formula and the parallel axis theorem, a convenient expression can be derived for the moment of inertia, I, of the cracked cross section. This expression is:

$$I = \frac{bd^3}{6} k^2 (3 - k) \qquad (8)$$

Examination of Table 6 and of the corresponding data of the earlier report[1] shows that the cracked cross section method of deflection computation is quite adequate for the beams containing 2.5 percent of tension reinforcement. The measured deflections of these beams agree quite closely with that computed for all loadings nearly to the diagonal cracking load. The measured load deflection diagrams for the beams containing 1.25 percent reinforcing steel were considerably more curvilinear than the diagrams for the beams with heavier steel. The computed deflections of the beams with lighter steel were generally larger than the measured deflections in the working stress ranges, and the curve of measured deflection crossed the computed deflection line at 70 to 100 percent of the diagonal cracking load. At 50 percent of the cracking load, the measured deflections were approximately 90 percent of the computed deflections. The comparison of computed and measured deflections indicates that the cracked cross section method offers an adequate and conservative method for prediction of deflection in lightweight structures.

Comparison of the measured deflections of the lightweight beams and the normal weight beams of the same compressive strength, span and tensile reinforcement confirms the findings of the earlier study. For example, if the deflections of beams containing Aggregate 4 are compared to those of the corresponding beams containing Aggregate 8, it is found that the deflections of the lightweight beams are only 15 to 35 percent greater than those of the normal weight beams, even though the modulus of elasticity of Aggregate 4 concrete is only 50 percent of that for the concrete containing the sand and gravel, Aggregate 8.

PROPOSED ULTIMATE LOAD DESIGN RECOMMENDATIONS FOR SHEAR IN LIGHTWEIGHT CONCRETE BEAMS

In addition to the beams of this Portland Cement Association investigation, the University of Texas[3] has made the results of an excellent series of tests on lightweight concrete available to ACI Committee 213. The following considerations of diagonal tension resistance of lightweight concrete are based on the Texas study as well as that at the PCA labo-

TABLE 7 — PROGRAM OF BEAM TESTS AT THE UNIVERSITY OF TEXAS

No. of Beams	Shear span, in.	a/d	Tension steel, percent	Compression steel	Nominal f_c', psi
4	24 to 43.2	3.0 to 5.4	0.75	1-#4	3800
4*	18 to 42	3.0 to 7.0	1.48	1-#4	3800
4	10.8 to 40	1.35 to 5.0	3.03†	1-#6	3500
6	32	4.0	0.75 to 5.38	1-#6	3500
5	32	4.0	3.03	1-#6	2800 to 6800
3	24 to 40	3.0 to 5.0	0.91	2-#4	4500
1	32	4.0	2.73	2-#7	4500
27 = total lightweight beams					

All beams unless noted: $b = 5.5$ in., $d = 8.0$ in.
All beams tested at 7 days after 6 days moist curing, except the last four which were moist cured 2 or 3 days and tested dry at 135 to 142 days.
*Beams where $b = 4.5$ in., $d = 6$ in.
†Hard grade steel.

ratories. Examination of the test results of two other investigations have been made and these studies provide good corroboration of the design recommendations made in this report.

The 27 lightweight concrete beams tested at the University of Texas were fabricated with concrete containing a single aggregate and generally with compressive strengths in the range of 3500 to 4000 psi. All beams contained compression steel. The program was designed to investigate the effects of varying shear spans and percentages of tension steel. Particular emphasis was placed on large shear spans and low steel percentages. The total program was separated into six parts as follows:

Three parts studied the effect of variable shear span, each at a different level of tensile reinforcement. A fourth portion held the shear span constant but varied the longitudinal steel over a wide range. The fifth employed beams of the same span and steel to study the effect of variable compressive strength. The sixth was particularly concerned with the effect of long-time drying on diagonal tension resistance. The program of tests of lightweight beams at the University of Texas is given in Table 7.

Design equation for lightweight concrete

Comparison of lightweight beams and normal weight beams of similar design shows that the general behavior of the lightweight material is similar to that of normal weight concrete. The differences that develop are those in magnitude of diagonal tension resistance and not in stress distribution. As with stone concrete, beam collapse may occur at the formation of the diagonal crack, or redistribution of forces may be accomplished with ultimate failure occurring under shear-compression. The lightweight tests give further justification to the principle expressed by the Committee 326 report (i.e., only diagonal cracking load should be considered as the ultimate load in design of non-web-reinforced members). This compatible behavior of normal weight and lightweight beams justifies the development at this time of an ultimate load design formula for the diagonal tension resistance of lightweight concrete.

As Bower and Viest[14] have pointed out, several earlier studies show that the magnitude of shear at initial diagonal tension cracking is a function of several variables. The most important of these are strength of concrete, percentage of tension reinforcement, beam dimensions, and ratio of moment to shear. The above authors have suggested that the shear in non-web-reinforced, normal weight concrete beams may be related to the above variables by the formula

$$\frac{v_o}{\sqrt{f_c'}} = \frac{V}{bd\sqrt{f_c'}} = 1.9 + 2500 \frac{pd}{\frac{M}{V}\sqrt{f_c'}} \leqq 3.5 \quad (9)$$

where V is the total external shear and M/V is the ratio of moment to shear defined by the following equation:

$$\frac{M}{V} = \frac{M_{max}}{V} - d \geqq \frac{M_{max}}{V} - \frac{a}{2} \quad (10)$$

ACI-ASCE Committee 326, Shear and Diagonal Tension, has concurred in this empirical formula, and their notation and numerical values of constants are as shown. For convenient reference below, the term $V/bd\sqrt{f_c'}$ is called Parameter A, and the term $pVd/M\sqrt{f_c'}$ is referred to as Parameter B. It should be noted that this definition of Parameter B is the reciprocal of that adopted by the Committee 326 report.

A study of the diagonal tension tests of lightweight concrete from the two sources clearly indicates that the variables of Eq. (9) are also the important variables in design of lightweight concrete. Further, this study shows that these variables can be grouped into the same parameters as for normal weight concrete.

It is logical then to assume that the trend of lightweight concrete data may be expressed in an empirical form similar to Eq. (9). The numerical

TABLE 8 — CONSTANTS DETERMINED BY MULTIPLE CORRELATION FOR LIGHTWEIGHT CONCRETE DIAGONAL TENSION FORMULA*

Test series	C_1	C_2	No. of beams	Coefficient of variation, percent
University of Texas	1.11	6940	26	14.7
PCA Aggregate 2	1.15	4360	5	8.5
PCA Aggregate 3	1.30	4820	6	6.8
PCA Aggregate 10	1.37	4130	6	9.9
PCA Aggregate 6	1.39	4090	5	6.9
PCA Aggregate 13	1.41	2800	4	5.4
PCA Aggregate 4	1.42	2870	8	10.7
PCA Aggregate 7	1.90	5560	6	2.4
PCA Aggregate 5	1.93	2970	4	4.9

* $v_c = C_1\sqrt{f_c'} + C_2 \frac{pVd}{M}$

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TABLE 9 — PCA LIGHTWEIGHT SIMPLY SUPPORTED BEAMS WITH TWO SYMMETRICAL CONCENTRATED LOADS

Beam No.	$\frac{1000pVd}{M\sqrt{f_c'}}$	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{calc}}$
Aggregate 2—Expanded shale				
2B4	0.04439	1.266	1.231	0.972
2B2	0.08946	1.435	1.703	1.187
2B3	0.12048	1.552	1.711	1.102
2A1	0.27481	2.131	2.512	1.179
2B1	0.22791	1.955	1.948	0.996
	$n = 5$		Avg	1.087
			C.V.*	8.7
Aggregate 3—Expanded shale				
3A4	0.05028	1.289	1.534	1.190
3B4	0.04704	1.276	1.382	1.083
3B2	0.09276	1.448	1.885	1.302
3B3	0.12899	1.584	1.967	1.242
3A1	0.28925	2.185	2.731	1.250
3B1	0.26067	2.078	2.460	1.184
	$n = 6$		Avg	1.209
			C.V.*	7.2
Aggregate 4—Expanded clay				
4A4	0.05603	1.310	1.668	1.273
4B4	0.04361	1.264	1.331	1.053
4B2	0.08752	1.428	1.778	1.245
4B3	0.11757	1.541	1.557	1.010
4A1	0.30536	2.245	2.680	1.194
4B1	0.23940	1.998	2.447	1.225
4C1	0.39835	2.594	2.315	0.892
4D1	0.36898	2.484	2.481	0.999
	$n = 8$		Avg	1.111
			C.V.*	8.9
Aggregate 5—Expanded slate				
5B4	0.04408	1.265	—	—
5B2	0.08902	1.434	2.238	1.561
5B3	0.11900	1.546	2.267	1.466
5A1	0.28216	2.158	2.918	1.352
5B1	0.24086	2.003	2.511	1.254
	$n = 4$		Avg	1.408
			C.V.*	10.7
Aggregate 6—Expanded slag				
6B4	0.04480	1.268	1.482	1.169
6B2	0.08839	1.431	1.908	1.333
6B3	0.11998	1.550	1.920	1.239
6A1	0.27517	2.132	2.588	1.214
6B1	0.23886	1.996	2.215	1.108
	$n = 5$		Avg	1.213
			C.V.*	8.0
Aggregate 7—Expanded shale				
7B4	0.04476	1.268	—	—
7B2	0.08874	1.433	2.479	1.730
7B3	0.11936	1.548	2.485	1.605
7A1X	0.29421	2.203	3.567	1.619
7A1	0.25599	2.060	3.378	1.640
7B1X	0.24368	2.014	3.306	1.642
7B1	0.23117	1.967	3.086	1.569
	$n = 6$		Avg	1.634
			C.V.*	3.9
Aggregate 9—Experimental Expanded shale				
9C1	0.40094	2.604	2.482	0.953
Aggregate 10—Expanded shale				
10A4	0.05456	1.305	1.791	1.372
10B4	0.04480	1.268	1.413	1.114
10BW4	0.04635	1.274	1.368	1.074
10B2	0.09390	1.452	1.985	1.367
10B3	0.11757	1.541	1.904	1.236
10B1	0.23913	1.997	2.277	1.140
	$n = 6$		Avg	1.217
			C.V.*	9.8
Aggregate 13—Expanded shale				
13B4	0.04348	1.263	1.460	1.157
13B2	0.08787	1.430	1.673	1.170
13B3	0.11757	1.541	1.838	1.193
13B1	0.23716	1.989	2.033	1.022
	$n = 4$		Avg	1.136
			C.V.*	7.4

$$\frac{V_{calc}}{bd\sqrt{f_c'}} = 1.1 + 3750 \frac{pVd}{M\sqrt{f_c'}} \leq 3.5$$

$$\frac{M}{V} = \frac{M_{max}}{V} - d \geq \frac{M_{max}}{V} - \frac{a}{2}$$

*Coefficient of variation, percent.

constants, however, will be changed to reflect the lowered tensile resistance of some lightweight concretes. It is also assumed that the location of the diagonal crack and consequently the proper ratio of M/V for analysis of lightweight beams will be defined identically with that for normal weight concrete as in Eq. (10). The type form of the design equation for lightweight concrete is then

$$\frac{v_c}{\sqrt{f_c'}} = \frac{V}{bd\sqrt{f_c'}} = C_1 + C_2 \frac{pVd}{M\sqrt{f_c'}} \qquad (11)$$

In view of the characteristic levels of tensile resistance discussed previously, values of the constants C_1 and C_2 have been determined on a multiple correlation basis for each of the concretes containing the various aggregates. It was found that for the aggregates included in the two test sources, C_1 varied from 1.11 to 1.93. In other words C_1 varied from a low value up to the value 1.9 assigned by Committee 326 to normal weight concrete, depending on the aggregate considered. Meanwhile, the constant C_2 varied in a haphazard manner from a low value of 2800 to a high of 6940. For most of the aggregates, C_2 was 4000 or higher. It appears then that lightweight concrete, in relation to normal weight concrete, tends to have a lower initial point (at a value of Parameter B equal to zero) and a more rapid increase in Parameter A, as Parameter B increases. These "best fit" values of C_1 and C_2 are shown in Table 8.

The original PCA test series demonstrated that lightweight concrete fails in the same range of unit shear strength as does the normal weight concrete when the value of Parameter B is quite large. This conclusion is also confirmed by the appropriate tests included in the University of Texas investigation. In view of these data and the higher trend of constant C_2 for lightweight concrete, it is logical, and convenient, to assume an upper limit of Parameter A at 3.5, the same limitation imposed by Committee 326 for stone concrete. By the same reasoning, this limitation will also be assumed to occur at the same point as for normal weight concrete, that is at the value of Parameter B equal to 0.00064. This value is obtained by equating Eq. (9) to 3.5 and solving for Parameter B. Thus, one point on the design function for lightweight diagonal tension has been obtained.

Certainly the design formula chosen for lightweight beams should be conservative. Magnitudes of unit shear derived from the formula should reflect that the characteristics of particular lightweight aggregates vary widely and that only a few of the aggregates available on the market have been used in tests for diagonal tension. On the other hand, examination of the existing test data reveals that some aggregates develop diagonal tension strength approaching that of the stone concretes. In the interest of economical design these aggregates should not be unduly penalized.

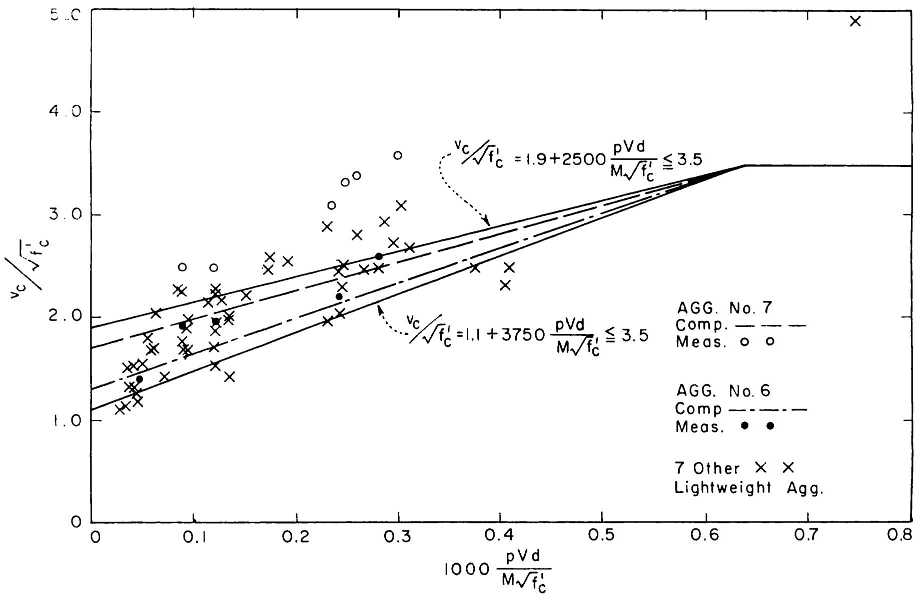

Fig. 10—Design equations for lightweight beams without web reinforcement

It appears from the tabulation of C_1 in Table 8 that a value of Parameter A, for Parameter B equal to zero, may be chosen as 1.1. A second point for a *minimum* lightweight design formula is now available along with that chosen above at Parameter A equal to 3.5 when Parameter B equals 0.00064. Substitution of these simultaneous values in the typical form Eq. (11), results in the following design formula:

$$\frac{v_c}{\sqrt{f_c'}} = \frac{V}{bd\sqrt{f_c'}} = 1.1 + 3750 \frac{pVd}{M\sqrt{f_c'}} \leq 3.5 \quad \text{.................(12)}$$

Provided that

$$\frac{M}{V} = \frac{M_{max}}{V} - d \geq \frac{M_{max}}{V} - \frac{a}{2} \quad \text{..............................(10)}$$

Fig. 10 shows the trend of the data from the 72 lightweight beams available and the relation of this trend to Eq. (12). Detailed computations, Tables 9 and 10, have been made of the relative safety for each of the beams tested when compared to Eq. (12). Table 11 summarizes the average values of V_{test}/V_{calc} and the coefficients of variation for each aggregate series from these detailed computations. Examination of the individual values of V_{test}/V_{calc} shows that 14 percent of the measured values fall on or below the line defined by Eq. (12). This is practically the same as the percentage of those normal weight beams having the same range of Parameter B, which fall on or below the line defined by Eq. (9), as reported by Committee 326.

Examination of Fig. 10 indicates the excessive safety of design Eq. (12) in regard to several aggregates. This equation then should be considered applicable only to those aggregates that exhibit low diagonal tension resistance or in cases where sufficient laboratory data are lacking for upgrading the coefficients of the design formula. A convenient method for providing safe but reasonable values for the unit shear for any particular aggregate now follows.

Design equations for concretes containing a particular lightweight aggregate

The limitation of 3.5 for Parameter A provides a conservative magnitude of unit shear for beams of the short, deep variety. Further, the requirement that this limitation for lightweight concrete should occur at the same point (Parameter $B = 0.00064$) as specified for normal weight concrete, provides further assurance of safe values. This point, then, should remain constant in the design formula associated with any particular aggregate. A second point established for concrete made with this particular aggregate would allow computation of the empirical constants associated with Parameters A and B by simple solution of simultaneous equations. This second point should be established for small values of Parameter B such as occur in most service beams. The recent investigations provide a simple but safe test procedure for determination of this second point and the specific constants for the special design formula. The relationship between unit shear stress and split-cylinder tension, exemplified by Eq. (6), provides the "tool" for obtaining the special formula.

The upper left diagram of Fig. 9 represents the relationship between v_c and f'_{sp} for the longest beams with the smallest amount of tensile steel for which adequate tension tests are available. It is reasonable, therefore, to specify an empirical relationship between these two variables for this particular ratio of beam physical characteristics. The dashed line on the figure, Eq. (6), represents a "best fit" of all the data. By examination of this function for all 54 beams, it was found that if the slope of the function was reduced by 10 percent, all lightweight beam tests except one would exceed the reduced function. The unit shear of this particular beam falls below the reduced function by only 5 psi. The solid line of Fig. 9 (upper left diagram) represents the reduced function and is expressed by the empirical relationship

$$v_c = 0.3\, f'_{sp} \qquad \text{(13)}$$

provided $pVd/M = 0.00312$, where $pVd/M_{max} = 0.0025$ [see Eq. (10)].

Application of the limitation of Parameter A to 3.5 and of Eq. (13) to the type formula, Eq. (11), results in two simultaneous equations as follows:

$$C_3 + 0.00064\, C_4 = 3.5$$

and

$$C_3 + 0.00312 \frac{C_4}{\sqrt{f_c'}} = 0.3 \frac{f'_{sp}}{\sqrt{f_c'}}$$

Subtraction of the lower equation from the upper yields

$$C_4 \left(0.00064 - \frac{0.00312}{\sqrt{f_c'}} \right) = 3.5 - \frac{0.3\, f'_{sp}}{\sqrt{f_c'}}$$

TABLE 10 — UNIVERSITY OF TEXAS LIGHTWEIGHT BEAMS WITH TWO SYMMETRICAL CONCENTRATED LOADS

$$\frac{V_{calc}}{bd\sqrt{f_c'}} = 1.1 + 3750 \frac{pVd}{M\sqrt{f_c'}} \leq 3.5$$

Beam No.	$\frac{1000\, pVd}{M\sqrt{f_c'}}$	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{calc}}$
Expanded shale aggregate				
A4	0.02791	1.205	1.120	0.929
A1	0.03577	1.234	1.329	1.077
T3	0.03376	1.227	1.145	0.933
Gu1	0.03816	1.243	1.511	1.216
Go5	0.04132	1.255	1.514	1.206
A2	0.04645	1.274	1.513	1.188
T2	0.04355	1.263	1.176	0.931
Gu2	0.06471	1.343	2.027	1.509
A3	0.06100	1.329	1.700	1.279
Go4	0.08298	1.411	2.273	1.611
T1	0.07063	1.365	1.447	1.060
Gu3	0.09654	1.462	1.722	1.178
Go3	0.11420	1.528	2.167	1.418
Gu4	0.11941	1.548	2.175	1.405
Ma3	0.12830	1.581	2.173	1.374
T4	0.13462	1.605	1.413	0.880
Ma1	0.17295	1.749	2.587	1.479
E11	0.19191	1.820	2.548	1.400
E12	0.17271	1.748	2.449	1.401
E13	0.14956	1.661	2.221	1.337
E14	0.13521	1.607	2.008	1.250
E15	0.12222	1.558	1.540	0.988
Go6	0.17100	1.741	2.464	1.415
Go1	0.22973	1.961	2.889	1.473
Ma2	0.25906	2.071	2.798	1.351
Go2	0.30308	2.237	3.081	1.377
Ma4	0.74605	3.500	4.873	1.392

$n = 27$ Average = 1.261

Coefficient of variation, percent = 15.90

For the extreme ranges of compressive strength from 2000 to 9000 psi, the second term inside the parentheses varies only from 0.00007 to 0.00003. Therefore, the value inside the parentheses can be closely approximated as 0.0006 and the above formula may be expressed as:

$$\frac{f'_{sp}}{\sqrt{f_c'}} = 11.67 - 0.002\,C_4 \quad \text{...(14a)}$$

or, if preferred,

$$\frac{f'_{sp}}{\sqrt{f_c'}} = 0.729 + 3.125\,C_3 \quad \text{...(14b)}$$

TABLE 11—SUMMARY OF AVERAGE V_{test}/V_{calc} FOR VARIOUS LIGHTWEIGHT AGGREGATES

From formula $v_c = 1.1\sqrt{f_c'} + 3750\,\frac{pVd}{M}$

Test series, aggregate	No. of beams	Average $\frac{V_{test}}{V_{calc}}$	Coefficient of variation, percent
PCA 2	5	1.087	8.5
PCA 4	8	1.111	8.9
PCA 13	4	1.136	7.4
PCA 3	6	1.209	7.2
PCA 6	5	1.213	8.0
PCA 10	6	1.217	9.8
Univ. of Texas	27	1.261	15.9
PCA 5	4	1.408	10.7
PCA 7	6	1.634	3.9
All beams	71*	1.253	15.9

*PCA Aggregate 9 beam not included in tabulation.

These are simultaneous linear relationships between $f'_{sp}/\sqrt{f_c'}$ and the constants associated with Parameters A and B for concrete with any particular aggregate.

The design formula for any particular aggregate may thus be established on the basis of adequate compression and split-cylinder tests. The cylinders for these tests should be fabricated, cured, and tested in accordance with the provisions presented later in this report under "Recommendations for Diagonal Tension Ultimate Load Design of Lightweight Concrete Beams." The test results then allow the determination of the ratio $f'_{sp}/\sqrt{f_c'}$ and consequently, from Eq. (14a) and (14b), the values of C_3 and C_4 for the particular aggregate. These values have been tabulated in the recommendations over a range of this ratio from that corresponding to the minimum formula, Eq. (12), to that assigned to normal weight concrete, Eq. (9).

Examples of upgraded design formula

Two examples have been computed to demonstrate the proposed method by which the design formula for particular lightweight beams may be established by test. The pertinent data for these computations are presented in Table 12 together with the resulting design formula constants taken from the recommended table.

Fig. 10 shows these upgraded design formulas relative to the test data and also relative to the minimum lightweight concrete design formula and to that for normal weight concrete from Eq. (9). Table 13 summarizes the new average values of V_{test}/V_{calc} for the example aggregates and compares them with the averages computed by the minimum design Eq. (12).

TABLE 12 — DATA FOR DESIGN CONSTANTS OF EXAMPLE AGGREGATES

Aggregate No.	f_c', psi	f'_{sp}, psi	$f_c'{}_{avg}$, psi	$f'_{sp\ avg}$, psi	$\dfrac{f'_{sp\ avg}}{\sqrt{f_c'{}_{avg}}}$	C_3	C_4
6	3320 4890	302 327	4105	314	4.90	1.3	3440
7	3280 4840	348 408	4060	378	5.93	1.7	2810

TABLE 13 — COMPARISON OF V_{test}/V_{calc} AVERAGES WITH THOSE COMPUTED BY MINIMUM DESIGN FORMULA

Test series	No. of beams	Upgraded formula		Minimum formula	
		Average $\dfrac{V_{test}}{V_{calc}}$	Coefficient of variation, percent	Average $\dfrac{V_{test}}{V_{calc}}$	Coefficient of variation, percent
PCA Aggregate 6	5	1.105	6.7	1.213	8.0
PCA Aggregate 7	6	1.333	5.7	1.634	3.9

TABLE 14 — DESIGN FORMULAS FOR PARTICULAR AGGREGATES RESULTING FROM CYLINDER SPLITTING AND COMPRESSION TESTS

Beam No.	$\dfrac{1000\,pVd}{M\sqrt{f_c'}}$	$\dfrac{V_{calc}}{bd\sqrt{f_c'}}$	$\dfrac{V_{test}}{bd\sqrt{f_c'}}$	$\dfrac{V_{test}}{V_{calc}}$
Aggregate 6—Expanded slag				
$\dfrac{V_{calc}}{bd\sqrt{f_c'}} = 1.30 + 3438\,\dfrac{pVd}{M\sqrt{f_c'}} \leqq 3.5 \quad \dfrac{M}{V} = \dfrac{M_{max}}{V} - d \geqq \dfrac{M_{max}}{V} - \dfrac{a}{2}$				
6B4	0.04480	1.454	1.482	1.019
6B2	0.08839	1.604	1.908	1.190
6B3	0.11998	1.712	1.920	1.121
6A1	0.27517	2.246	2.588	1.152
6B1	0.23886	2.121	2.215	1.044
$n = 5$				Average 1.105
				Coefficient of variation, percent 6.7
Aggregate 7—Expanded shale				
$\dfrac{V_{calc}}{bd\sqrt{f_c'}} = 1.70 + 2812\,\dfrac{pVd}{M\sqrt{f_c'}} \leqq 3.5 \quad \dfrac{M}{V} = \dfrac{M_{max}}{V} - d \geqq \dfrac{M_{max}}{V} - \dfrac{a}{2}$				
7B4	0.04476	1.826	—	—
7B2	0.08874	1.950	2.479	1.271
7B3	0.11936	2.036	2.485	1.221
7A1X	0.29421	2.527	3.567	1.412
7A1	0.25599	2.420	3.378	1.396
7B1X	0.24368	2.385	3.306	1.386
7B1	0.23117	2.350	3.086	1.313
$n = 6$				Average 1.333
				Coefficient of variation, percent 5.7

TABLE 15 — DESIGN CONSTANTS

Range of ratio $f'_{sp}/\sqrt{f_c'}$	Constant, C_3	Constant, C_4	Constant, C_5
4.01-4.31	1.1	3750	4280
4.32-4.63	1.2	3590	4110
4.64-4.94	1.3	3440	3930
4.95-5.25	1.4	3280	3750
5.26-5.56	1.5	3120	3570
5.57-5.88	1.6	2970	3390
5.89-6.19	1.7	2810	3210
6.20-6.50	1.8	2660	3040
6.51-6.67	1.9	2500	2850

The excessive safety factors for these two aggregates have been reduced to reasonable but still safe magnitudes. The detailed computation of V_{test}/V_{calc} for the example beams is given in Table 14.

Two major considerations which influence the choice of a diagonal tension design formula are: first, the equation should be simple to facilitate everyday design work; secondly, and more importantly, the equation should be such that the ultimate strength of beams resulting from practical design will be governed by flexure rather than by shear. These important considerations are satisfied by the proposed method of diagonal tension design for lightweight concrete. The formula is the same type as that for normal weight and the constants in the lightweight concrete method have been chosen by an even more conservative method. The slopes for the lightweight concretes have been deliberately lowered by setting the upper limit of unit shear at $3.5\sqrt{f_c'}$ and the magnitudes of the constants, C_3 and C_4, have been effectively reduced by expressing the unit shear stress as $0.3f'_{sp}$ at $pVd/M = 0.00312$. This highly conservative nature of the lightweight concrete method should adequately take care of such elusive effects as shrinkage and continuity on diagonal tension resistance.

As a final stage in the development of the proposed ultimate load design formula for normal weight concrete, Committee 326 transformed Eq. (9) into a form considered more convenient for everyday design practice. This same transformation can be easily applied to the proposed lightweight concrete design formulas by multiplying the constant C_4 by the factor 8/7.

SUMMARY AND CONCLUSIONS

1. This investigation was limited to simply supported beams with two symmetrical concentrated loads. With combinations of two spans, 10 ft and 6 ft 6 in., two amounts of longitudinal reinforcement (1.25 and 2.5 percent), ten different aggregates and, for most conditions, two levels

of compressive strength, single beam tests were generally used for each condition.

2. The longer span beams generally failed completely at formation of the initial diagonal crack. The ability of the shorter span beams to achieve stress redistribution after diagonal cracking was materially affected by chance location of the crack. Therefore, the load at diagonal cracking should be considered the ultimate load for non-web-reinforced concrete beams.

3. Each beam test was accompanied with 6 x 12-in. cylinders for compressive and tensile splitting tests. The tensile splitting test provides an adequate measure of the relative tensile strength of concrete provided that sufficient samples are tested to give a reliable average.

4. Drying at 50 percent relative humidity will reduce the split cylinder tensile strength of lightweight concretes by magnitudes of 0 to 40 percent. The amount of tensile strength reduction will depend on the particular aggregate and on the compressve strength of the concrete.

5. A good correlation was found between the nominal unit shear strength of the beams and the accompanying split-cylinder tensile strengths of dry concretes (cured and dried according to ASTM C 330-59T). This is to be expected since it has been recognized that diagonal tension resistance is a primary function of tensile strength. Therefore, the split-cylinder tension test may be used as a reliable measure of unit shear strength.

6. The split-cylinder tests and the beam tests have shown that the nominal unit shear strength of concrete containing a particular lightweight aggregate is determined by the characteristic level of tensile strength associated with that aggregate. The unit shear strengths of the lightweight concrete beams varied from approximately 60 to 100 percent of the unit shear strength of comparable beams containing normal weight aggregate, depending on the lightweight aggregate considered and on the beam characteristics.

7. Comparison of the unit shear strengths at diagonal cracking with the ACI Building Code working stresses reveals that inadequate factors of safety existed for the lightweight concrete beams with longer spans and lower steel percentages. Thus, code provisions in regard to diagonal tension resistance of structural lightweight concrete should be established at the earliest opportunity. These provisions can probably be best established through ultimate load design criteria. These criteria should be conservative to reflect the increased chance of higher shrinkage stresses existing in restrained beams.

8. Deflections of both the normal weight and the lightweight beams after flexural cracking under working loads were computed by the transformed cross section method with acceptable and conservative accuracy. The measured deflections have shown that consideration of

deflection as inversely proportional to modulus of elasticity is in error on the conservative side by large magnitudes.

RECOMMENDATIONS FOR DIAGONAL TENSION ULTIMATE LOAD DESIGN OF LIGHTWEIGHT CONCRETE BEAMS

In accordance with the proposed design provisions established by ACI-ASCE Committee 326 for normal weight concrete, ACI Committee 213 recommends the following ultimate strength design procedures for structural lightweight concrete beams without web reinforcement.

1. The diagonal tension strength shall be determined on the basis of the average unit shearing stress v computed by the formula $v = V/bd$, where $V =$ shear force, $b =$ width of member, and $d =$ effective depth to centroid of tension steel.

2. For beams of I- and T-section, or for floor joist construction the web width b' shall be substituted for the width b.

3. The ultimate diagonal tension strength v_c of a lightweight concrete section of an unreinforced web shall be computed by either of the formulas

$$v_c = C_3\sqrt{f_c'} + C_4\frac{pVd}{M}, \qquad \text{or} \qquad v_c = C_3\sqrt{f_c'}\left(\frac{f_s}{f_s - C_5}\right),$$

where the constants, C_3 and C_4, or C_3 and C_5, shall be selected from Table 15 and

f_c' = concrete compressive strength, psi
f_s = longitudinal steel stress at the section considered, psi
p = ratio of area of flexural tension reinforcement to the effective cross-sectional area of the beam
M/V = the ratio of moment to shear at the section considered

(a) For purposes of using Table 15, the ratio of the split-cylinder tensile strength, f'_{sp}, to the square root of the compressive strength, f_c', shall be determined as follows:

The tests for compressive strength shall be made in accordance with ASTM C 330-59T, which provides for 7 days moist curing followed by storage at 73 F and 50 percent relative humidity until time of test. To conform with Table 15, the test shall be at 28 days. The test cylinders shall be 6 x 12 in. and shall be cast from the same batches of concrete as the cylinders that are to be tested by splitting. A total of twenty-four 6 x 12 in. concrete cylinders shall be cast for both compressive strength and split-cylinder tension, one-half of these at the compressive strength level of approximately 3000 psi and the other half at approximately 5000 psi. The cylinders at each strength level shall be produced from two or more batches. After 7 days moist curing followed by 21 days drying at 73 F, 50 percent relative humidity, four of the test cylinders at each of the two nominal strength levels shall be tested for compressive strength and eight cylinders at each of the two strength levels shall be tested for tension splitting in the manner recommended below. For the purpose of establishing the constants of the design equation from Table 15, the results of all 16 individual split-cylinder tests shall be averaged to obtain f'_{sp} and all eight of compression test to obtain f_c' and consequently the ratio of $f'_{sp}/\sqrt{f_c'}$.

To conform with the data from which Table 15 was established, the split-cylinder tests should be performed as follows:

The testing machine shall have an attachment for the head at least 12 in. long to accommodate the 12 in. length of the specimen. Fir plywood pads, also 12 in. long, ⅛-in. thick, shall be inserted between the testing machine platen and the specimen, and between the head and the specimen. The vertical diameter at each end of the cylinder shall be in alignment with the testing machine head center line. The specimen is to be loaded at a rate of 10,000 to 15,000 lb per min until splitting occurs. The split-cylinder tensile strength is then determined from equation $f'_{sp} = P/36\pi$, in which P is the testing machine load at point of vertical diameter splitting.

(b) In the event that adequate test data are lacking, either of the following minimum formulas shall apply:

$$v_c = 1.1\sqrt{f_c'} + 3750\,\frac{pVd}{M} \qquad \text{or} \qquad v_c = 1.1\sqrt{f_c'}\left(\frac{f_s}{f_s - 4280}\right)$$

(c) The limitation, v_c shall not exceed $3.5\sqrt{f_c'}$, and shall apply to the formulas of either of Paragraphs 3(a) or 3(b).

4. If the ultimate load on the member produces a unit shearing stress v, which exceeds the calculated ultimate diagonal tension strength v_c, web reinforcement must be provided, or the cross section must be increased.

5. The provision of Paragraph 4 is subject to the following limitations depending on the length a of any portion of the member within which the shear diagram retains the same sign: *

(a) If $a > 2d$, the provision of Paragraph 4 shall not apply within the distance d at either end of the length a.

(b) If $2d \geqq a \geqq \frac{3}{4}d$, the provision of Paragraph 4 shall apply only at the section located in the middle of the length a.

(c) If $a < \frac{3}{4}d$, the provision of Paragraph 4 shall not apply.

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Resistencia a la tensión y resistencia a la tensión diagonal del concreto estructural liviano

Se describen los ensayos empleados y los resultados obtenidos en una extensión de un estudio anterior sobre la resistencia a la tensión diagonal dado por el autor. Esta extensión del programa original se refiere a las vigas de concreto liviano de tramo más largo y de porcentajes inferiores de acero. Una conclusión importante, es decir, que la carga de agrietamiento diagonal debe considerarse como la carga final para las vigas reforzadas sin alma, ha sido confirmada.

Un gran número de los cilindros de 6 x 12 pulgadas de los concretos de viga fueron rotos por el ensayo de la tensión del "cilíndro partido." Una buena correlación fué establecida entre esta medida indirecta de la tensión y la resistencia al corte de las vigas al agrietamiento diagonal. Esta correlación muestra que la resistencia a la tensión diagonal de los concretos livianos varía de aproximadamente 60 por ciento de la resistencia del concreto semejante de peso normal a casi 100 por ciento, dependiendo de los agregados livianos particulares empleados.

Las recomendaciones propuestas para la carga final de diseño se formulan para el concreto estructural liviano. Estas están de acuerdo general con las recomendaciones del Comisión 326 de ACI-ASCE sobre la tensión al corte y la tensión diagonal para el concreto de peso normal. Se ha verificado que la resistencia a la tensión diagonal de los concretos livianos se ve afectada por las mismas variables que afectan la resistencia del concreto de peso normal. La diferencia entre los dos tipos de materiales es la de la magnitud de la resistencia a la tensión diagonal y no es una diferencia fundamental en el comportamiento.

Las recomendaciones propuestas del diseño también tienen en cuenta las diferencias fundamentales en la resistencia a la tensión, las cuales existen entre los varios agregados livianos. Una combinación de la resistencia a la compresión y el ensayo de la tensión del cilindro partido proporciona una medida conveniente y segura de la resistencia a la tensión diagonal final para ser asociada con cada uno de los varios agregados.

Résistance à la tension et résistance à la tension diagonale de béton léger de construction

Décrit les épreuves utilisées et les résultats obtenus par extension d'une étude antérieure de la résistance á la tension diagonale rapportée par l'auteur. Cette extension du programme initial concerne des poutres de béton léger, de portée plus longue et contenant moins d'acier. Une conclusion intéressante, que la charge de rupture diagonale doit être considerée comme la charge définitive pour les poutres armées sans âme pleine, est confirmêe.

Un très grand nombre de cylindres de 6 x 12 pouces, coulés des bétons dont venaient les poutres, étaient cassés par l'essai de tension dit "Cylindre fendu". Une bonne corrélation était établie, entre cette mesure indirecte de la tension et la résistance au cisaillement des poutres sous la fissuration diagonale. Cette corrélation démontre que la résistance à la tension diagonale de béton léger varie entre 60 pour cent, à peu près, de celle d'un béton analogue à poids normal, et presque 100 pour cent, selon les agrégats légers utilisés.

On présente des essais de préconisations pour la conception définitive des

charges pour le béton léger de construction. Ceux-ci sont en accord général avec les préconisations du Comité 326 de l'ACI-ASCE au sujet de tensions diagonales et de cisaillement pour le béton de poids normal. On a découvert que la résistance à la tension diagonale des bétons légers est gouverné par les mêmes variables qui gouvernent la résistance de béton de poids normal. La différence entre les deux types de matières est une différence d'amplitude de la résistance à la tension diagonale et non pas une différence fondamentale de comportement.

Les essais de préconisation tiennent également en considération les différences fondamentales de résistance à la tension qui existent entre les divers agrégats légers. Une combinaison d'épreuves de la résistance à l'écrasement et de la tension du cylindre fendu fournit une mesure convenable et sûre des résistances définitives à la tension diagonale qui doit être associées à chacun des divers agrégats.

Zugfestigkeit und diagonaler Spannungswiderstand von Leichtbaubeton

Beschreibt die Versuche und Resultate, die in einer Fortsetzung einer frueheren, vom Verfasser berichteten Untersuchung des diagonalen Spannungswiderstandes erhalten wurden. Diese Fortsetzung des urspruenglichen Programms betrifft Leichtbetontraeger mit laengerer Spannweite und nierigerem Stahlgehalt. Eine wichtige Schlussfolgerung, naemlich dass die diagonale Risslast als die aeusserste Last fuer nicht durch Stege verstaerkte Traeger angesehen werden sollten, wurde bestaetigt.

Eine grosse Anzahl von 6 x 12 Zoll-Zylindern aus dem Traegerbeton wurden durch Spannungsversuche mit "Spaltzylindern" gebrochen. Eine gute Beziehung wurde zwischen dieser indirekten Spannungsmessung und der Scherfestigkeit der Traeger bei diagonaler Rissbildung hergestellt. Diese Beziehung zeight, dass der diagonale Spannungswiderstand von Leichtbaubeton von ungefaehr / 60 Prozent des Widerstandes von Normalbeton bis zu fast 100 Prozent reicht, je nach den besonderen Zuschlagstoffen des Leichtbetons, die verwendet werden.

Die vorgeschlagenen aeussersten Belastungen gelten fuer Leichtbaubeton. Diese sind gewoehnlich in Uebereinstimmung den Empfehlungen der ACI ASCE-Auschusses 326 fuer Scher- und Diagonal-Spannung fuer Normalbeton. Es hat sich gezeigt, dass die diagonale Spannungsfestigkeit von Leichtbaubeton von denselben Variablen beeinflusst wird, die auch den Widerstand von Normalbeton beeinflussen. Der Unterschied zwischen den beiden Arten von Material ist ein Groessenunterschied des diagonalen Spannungswiderstandes, und nicht ein fundamentaler Unterschied im Verhalten.

Die vorgeschlagenen Empfehlungen beruecksichtigen also die fundamentalen Unterschiede, die zwischen den verschiedenen Zuschlagstoffen des Leichtbetons bestehen. Eine Verbindung von Druckfestigkeit-und Spaltzylinder-Probe stellt daher eine bequemes und sicheres Mass des aeussersten diagonalen Spannungswiderstandes dar, der jedem der verschiedenen Zuschlagstoffe zugeordnet werden muss.