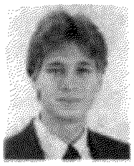
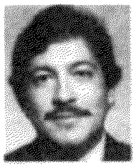
Title no. 86-S33

Minimum Shear Reinforcement in Beams with Higher Strength Concrete

by Mark K. Johnson and Julio A. Ramirez

The adequacy of the minimum shear reinforcement provision in higher strength reinforced concrete beams was investigated. Eight rectangular beams with concrete compressive strengths in the range of 5000 to 10,500 psi and with web reinforcement indexes in the range of 0 to 100 psi designed to fail in shear were tested. Truss models were used to illustrate the ultimate shear behavior of such members.

From an evaluation of the results of this experimental investigation and previous studies, it was concluded that the overall reserve shear strength after diagonal tension cracking diminished with the increase in f_c' for beams with the current minimum amount of shear reinforcement.

Keywords: beams (supports); detailing; failure; **high-strength concretes**; models; reinforced concrete; **reinforcing steels**; **shear strength**; trusses.

Current design provisions in the ACI Building Code[1] for shear in reinforced concrete beams are based on the results of numerous beam tests using concrete with compressive strengths mostly in the range of 2000 to 6000 psi. The extrapolation of these same provisions to beams with higher concrete strengths might be unjustified.

One such provision is the minimum amount of shear reinforcement in reinforced concrete beams. Under the current ACI Building Code,[1] a constant minimum amount of shear reinforcement, equivalent to a 50 psi shear stress, is required if the factored shear force exceeds one-half of the shear strength provided by the concrete. Web reinforcement in this design situation is intended to impart reserve shear strength by preventing sudden shear failures upon first diagonal tension cracking as a result of unexpected tensile forces or catastrophic loading.

RESEARCH SIGNIFICANCE

In the Building Code,[1] the shear strength of a reinforced concrete beam is taken as the sum of the shear that is carried by the concrete V_c and the web reinforcement V_s. The term V_c in a diagonally cracked beam with web reinforcement represents the sum of at least three separate components: (a) dowel action of the longitudinal reinforcement, (b) aggregate interlock, and (c) shear carried across the uncracked concrete in the flexural compression zone. The term V_s represents the vertical component of the shear reinforcement across an assumed 45 deg failure crack.

The ability of the current minimum amount of shear reinforcement to provide adequate reserve strength to reinforced concrete beams with higher compressive strength concretes is questionable. As the compressive strength of the concrete increases, the diagonal tension cracking load also increases, enhancing the amount of distress to be accommodated by the previously mentioned shear transfer mechanisms after diagonal tension cracking. Furthermore, in contrast with the rough crack surfaces typical of lower strength concretes, the crack surfaces in higher strength concretes tend to be smooth. This difference in crack surfaces may result in a reduction in the shear carried by aggregate interlock, and thus in the shear carried by V_c. It would then seem reasonable to expect that the reserve shear strength of beams with the current minimum amount of web reinforcement would decrease as the concrete strength increases and that more shear reinforcement may be needed to provide a comparable reserve shear strength. However, current design provisions call for a constant minimum amount, independent of the concrete strength.

In this study, the reserve shear strength provided by the current minimum shear reinforcement provision[1] in reinforced concrete beams with concrete compressive strengths greater than 6000 psi is evaluated. The purpose of this evaluation is to study the adequacy of the current minimum amount of shear reinforcement in beams with higher strength concrete.

ACI Structural Journal, V. 86, No. 4, July-August 1989.
Received Mar. 7, 1988, and reviewed under Institute publication policies. Pertinent discussion will be published in the May-June 1990 *ACI Structural Journal* if received by Jan. 1, 1990.

ACI member Mark K. Johnson is a research civil engineer with the National Bureau of Standards, Washington, D.C. He was formerly a research assistant and received his BSCE and MSCE degrees from Purdue University, West Lafayette, Ind. Mr. Johnson was a recipient of an ACI Fellowship Award and a CRSI Cameron Scholarship for his graduate studies.

ACI member Julio A. Ramirez is Assistant Professor of Structural Engineering at Purdue University. He received his BSCE degree from Universidad Autonoma de Mexico in 1977, his MS from the University of Texas at El Paso, and his PhD from the University of Texas at Austin. Dr. Ramirez is a member of joint ASCE-ACI Committee 445, Shear and Torsion.

RESERVE SHEAR STRENGTH

Reserve shear strength at first inclined cracking of reinforced concrete beams with minimum amount of shear reinforcement indicates the ability to delay shear failures upon initial diagonal tension cracking.

Present ACI code[1] philosophy assumes that in beams with web reinforcement the amount of shear stress carried by the concrete at ultimate v_c is at least equal to the amount of shear stress that would cause diagonal tension cracking. A relationship between the variables affecting the diagonal cracking strength of beams was determined from test results of beams without web reinforcement and concrete compressive strengths up to 6000 psi.[2] The term v_c is defined as the nominal shear strength of such beams

$$v_c = 1.9\sqrt{f_c'} + 2500\,\rho\,\frac{Vd}{M}\ \text{(psi)} \qquad (1)$$

Reserve shear strength can be defined then in quantitative terms as the relationship between the diagonal cracking strength v_c and the failure shear stress v_f. If expressed in the form $(v_f - v_c)$, it is a measure of the amount of remaining shear strength of the beam at diagonal cracking. It can also be expressed as an index in the form of v_f/v_c.

In subsequent sections, the reserve shear strength of reinforced concrete beams with small amounts of web reinforcement is evaluated with results of previous studies[3-8] and beams tested during this investigation.[9]

EXPERIMENTAL PROGRAM

All the specimens in this research study[9] were 15.5 ft in length and had rectangular 12 by 24 in. cross sections. The specimen details are given in Table 1 and Fig. 1. The test setup is shown in Fig. 2. Longitudinal tension reinforcement consisted of five No. 10 Grade 60 deformed bars arranged in two layers. The bottom layer had three No. 10 bars with 90-deg standard hooks at both ends, while the top layer had two No. 10 bars with 180-deg standard hooks at both ends. The yield stress of the No. 10 bar was 76.1 ksi. Longitudinal compression reinforcement consisted of two No. 9 straight Grade 60 deformed bars with a yield strength of 78.3 ksi.

The web reinforcement consisted of No. 2 deformed bars with a yield strength of 69.5 ksi. The stirrup detail was a closed loop with 135-deg standard hooks at the free ends. Stirrup spacings for all the specimens are shown in Fig. 1.

Table 1 — Test specimens*

Beam	f_c', psi	rf_y, psi
1	5280	100
2	5280	50
3	10,490	50
4	10,490	50
5	8100	100
6	8100	0
7	7440	50
8	7440	50

*All specimens had:
$a/d = 3.1$ ($a = 65.75$ in., $d = 21.21$ in.)
$\rho_w = 2.49$ percent
$\rho' = 0.79$ percent
12 x 24 in. cross sections

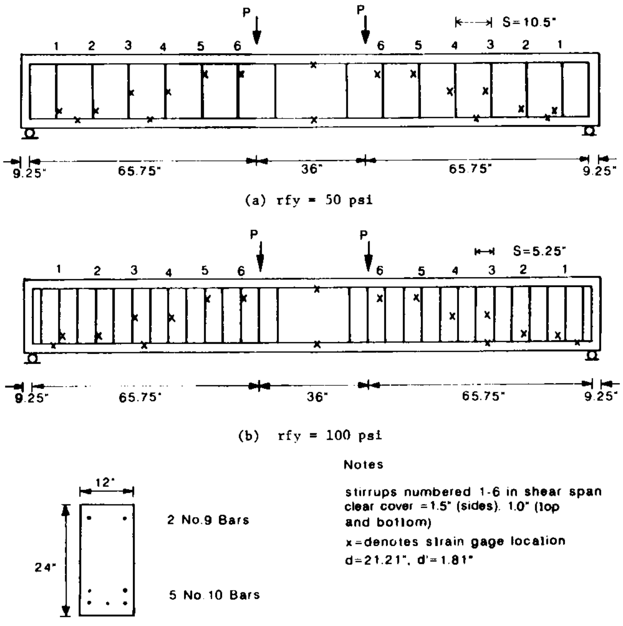


Fig. 1 — Test specimen details

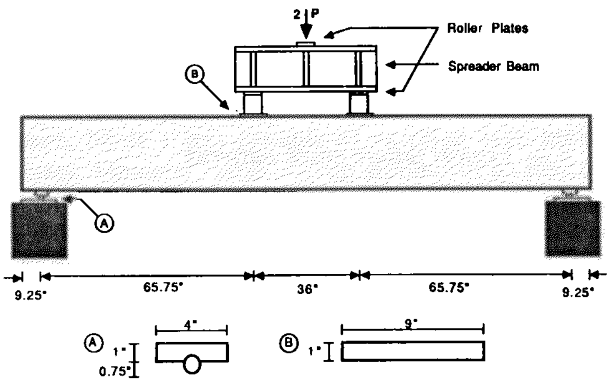


Fig. 2 — Test setup

Load was applied using a 600 kip universal testing machine. The load increment in the shear span was initially 10 kips and was reduced to 5 kips after flexural cracking occurred. When the anticipated diagonal tension cracking load was approached, the load increment was reduced further to 2 kips. Both stirrup strains and crack patterns were used to determine the inclined cracking load.

Table 2 — Test results

Col. 1 Beam	Col. 2 v_{cr}, psi	Col. 3 v_f, psi	Col. 4 v_f/v_{cr}	Col. 5 v_c*, psi	Col. 6 v_n†, psi	Col. 7 $\frac{v_{cr}}{v_c}$	Col. 8 $\frac{v_f}{v_c}$	Col. 9 $\frac{v_f}{v_n}$	Col. 10 Failure mode‡
6	169	169	1.00	191	191	0.88	0.88	0.88	DT
2	157	196	1.25	158	209	0.99	1.24	0.94	SC
7	157	248	1.58	184	235	0.85	1.35	1.06	SC
8	173	228	1.32	184	235	0.94	1.24	0.97	SC
3	196	232	1.18	215	265	0.91	1.08	0.88	SF
4	189	279	1.48	215	265	0.88	1.30	1.05	SC
1	157	299	1.90	158	260	0.99	1.89	1.15	SC
5	157	338	2.15	191	292	0.82	1.77	1.16	SC

*$v_c = (1.9\sqrt{f_c'}) + 2500\,\rho_w\,Vd/M$.
†$v_n = v_c + rf_y$.
‡DT = diagonal tension; SC = shear compression; SF = stirrup fracture.

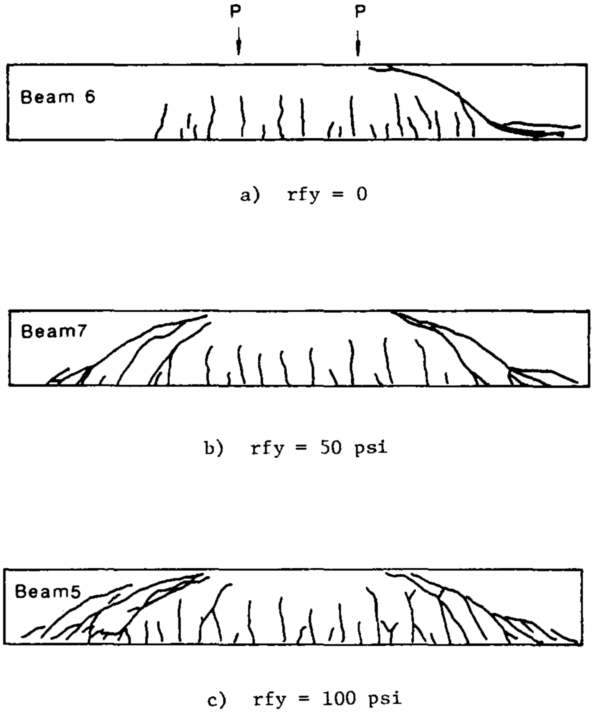


Fig. 3 — Typical failure crack patterns

At each load increment, readings of all instruments were taken. The deflection was monitored on both sides of the beam at the centerline and quarter points. Strains in the longitudinal and stirrup reinforcement were measured by electrical resistance strain gages. Stirrup gages were placed at the location of the anticipated diagonal tension crack, as shown in Fig. 1. After diagonal tension cracking, widths of the main inclined cracks were recorded. Each beam test was followed by standard cylinder tests to determine the compressive strength of the concrete.

Results and discussion

A summary of the test results is shown in Table 2. As can be seen from Column 8, Beam 6 without web reinforcement failed in diagonal tension with a reserve shear strength index v_f/v_c of 0.88. All five beams with the minimum amount of web reinforcement failed in shear compression, except for Beam 3, which failed following the fracture of one of the stirrups. For those specimens failing in shear compression, the reserve shear strength index was between 1.24 and 1.35 as the concrete strength increased from 5000 to 10,500 psi. Beam 3 failed immediately after fracture of one of the stirrups and had the lowest index (v_f/v_c = 1.08) of all five beams with the minimum amount of shear reinforcement. The reserve capacity increased significantly as the amount of web reinforcement increased from 50 to 100 psi. For Beam 1 with f_c' = 5280 psi, v_f/v_c was 1.89, and for Beam 5 with f_c' = 8100 psi, v_f/v_c was 1.77.

The ratio of measured inclined cracking shear stress to predicted concrete contribution v_{cr}/v_c shown in Column 7 of Table 2 was less than 1.0 for all the beams. The ratio of observed to predicted ultimate capacity v_f/v_n shown in Column 9 increased with increasing amounts of web reinforcement from 0.88 for rf_y of 0 psi to 1.16 for rf_y of 100 psi, showing that the ACI code equations became more conservative as the amount of web reinforcement increased. For the beams with rf_y of 50 psi, the ratio had an average value of 1.01 for all the specimens failing in shear compression. The lowest value of 0.88 was also observed in Beam 3, which failed immediately following stirrup fracture.

Typical failure crack patterns for the beams are shown in Fig. 3. The number of inclined cracks increased with increasing amount of web reinforcement, indicating an enhanced redistribution of internal forces in the beams with rf_y of 100 psi. The beams with the minimum amount of web reinforcement typically had two inclined cracks, whereas the beams with rf_y of 100 psi had several inclined cracks. The beam without web reinforcement had only a single diagonal crack on one end of the beam extending throughout the shear span.

The widths of the main inclined cracks at failure for different stirrup locations are shown in Table 3. The crack widths, and hence stirrup strains, tended to increase with higher concrete strengths at failure for the beams with minimum amount of web reinforcement. In

Table 3 — Widths of main inclined failure cracks

Col. 1	Col. 2	Col. 3	Col. 4	Col. 5		
				Crack width (in.)* stirrup locations		
Beam	rf_y, psi	f'_c, psi	Failure load P, kips	No. 3	No. 4	No. 5
6	0	8100	43	0.50	0.45	0.40
2	50	5280	50	0.05	0.06	0.06
7	50	7440	63	0.18	0.39	0.15
8	50	7440	58	0.24	0.23	0.31
3	50	10,490	59	1.05	0.39	0.41
4	50	10,490	71	0.25	0.27	0.25
5	100	8100	86	0.10	0.15	0.03

*Measured at intersection of main crack and stirrup leg 3, 4, and 5.
Note: Crack width of Beam 1 was not measured.

the beams with rf_y of 100 psi, the crack widths were comparatively small at failure.

The stirrup strain readings correlated well with the crack width observations. In the beams with the minimum amount of web reinforcement, stirrup strains increased as the concrete strength increased. In all five beams, at least one stirrup yielded at the formation of the inclined crack. In Beams 2, 4, 7, and 8, all the stirrups crossed by the main diagonal crack showed yielding when the applied load had been increased 18 percent over the diagonal cracking load. In Beam 3, all the stirrups reached yield immediately at the formation of the first diagonal crack. Increased stirrup strains in the higher strength concrete beams resulted in fracture of Stirrup 3 and necking of Stirrup 4 at failure in Beam 3. In Beam 4, Stirrups 4 and 5 showed signs of necking at failure. Stirrup 4 of Beam 7 also showed necking at failure.

The stirrup strains decreased substantially as rf_y increased from 50 to 100 psi. In Beams 1 and 5, none of the stirrups showed yield at diagonal tension cracking. It was not until the load had been increased about 18 percent over the cracking load that yielding began in any of the stirrups. At failure, yielding of several of the stirrups was observed, but there were no signs of necking.

The crack surfaces were observed to be much smoother in the higher strength concrete beams, indicating that the contribution of aggregate interlock to the shear strength of such beams was probably diminished. The diminished contribution of aggregate interlock resulted in an increase on the share of the load of the remaining components of the shear failure mechanism such as dowel action, shear carried across the flexural compression zone, and web reinforcement.

Truss models

Truss models were used to illustrate qualitatively the ultimate behavior of the beams tested in this study and helped explain the role of the minimum amount of web reinforcement. Due to the similarity in failure crack patterns, amount of longitudinal and web reinforcement, and measured stirrup forces, all five beams with the minimum amount of web reinforcement were represented by the same basic truss model shown in Fig. 4(a). The truss had two panels and three inclined concrete struts. There was a single vertical tie representing the resultant of the stirrup forces in the shear span. Examination of this truss model shows that failure could have resulted from crushing or shearing in any of the four nodes, the inclined struts could have crushed, or the ties could have fractured or lost anchorage.

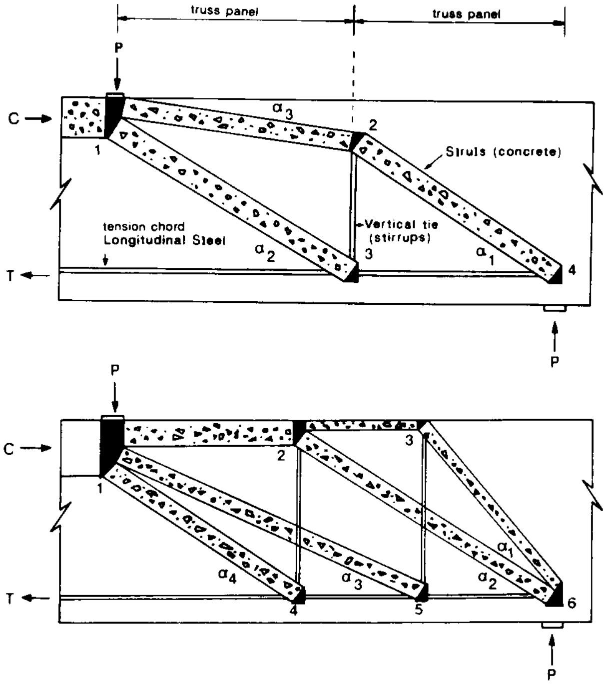


Fig. 4 — Truss models

In Beam 2, the weak link of the truss was the node under the point load (Node 1), which sheared at failure. The higher concrete strength of Beams 7 and 8 strengthened Node 1, and these members were able to sustain a higher failure load.

In Beam 3, because of the higher f'_c, the concrete truss elements were stronger than in Beams 2, 7, and 8. Hence, Node 1 was capable of carrying even higher stresses. However, large stirrup strains led to fracture of the third stirrup, initiating failure. It can be concluded that Node 1, the weak link in Beams 2, 7, and 8, had been sufficiently strengthened in Beam 3 so that the web reinforcement, represented by the vertical tie, became the weaker link of the truss.

Beam 4, on the other hand, which was identical to Beam 3, failed in shear compression; stirrup strains were only about half as large as in Beam 3. Failure in Beam 4 was controlled by the shearing of Node 1 and took place at a much higher load than in Beam 3. As indicated by previous studies,[5,10] higher concrete compressive strengths allow further redistribution of internal forces by strengthening the concrete components of the truss model. This redistribution permits increased mobilization of the stirrups and may lead to larger shear strengths if an adequate amount and detailing of the longitudinal and web reinforcement is provided.

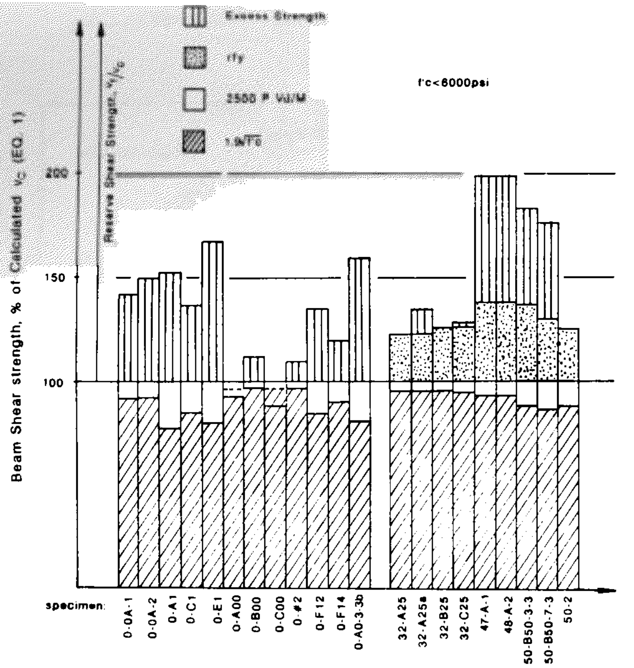


Fig. 5 — Reserve shear strength of beams with $rf_y \leq 50$ *psi and* $f'_c < 6000$ *psi (test results from References 4 through 9)*

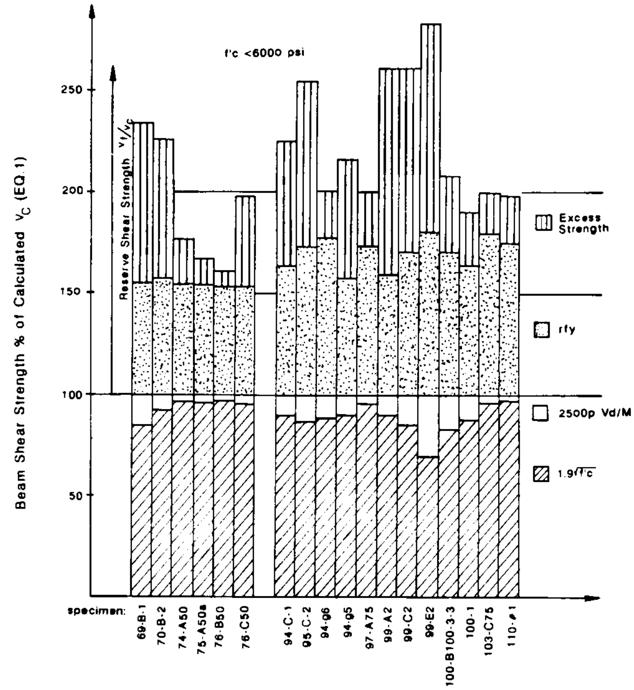


Fig. 6 — Reserve shear strength of beams with $rf_y > 50$ *psi and* $f'_c < 6000$ *psi (test results from References 4 through 9)*

However, in beams with minimum amount of shear reinforcement, as the concrete compressive strength is increased, the transfer of forces at first inclined cracking could be such that the first mobilized stirrups are forced to yield and rupture. Hence, any potential redistribution of internal forces and load-carrying capacity could be dangerously diminished. The failure of Beam 3 confirms this observation. In this specimen, the transfer of forces at first inclined cracking led to yielding of the first mobilized stirrups, and as further load was applied, the redistribution of internal forces and load-carrying capacity was halted by the fracture of the first mobilized Stirrup 3. The fracture of this stirrup resulted in a premature failure.

The truss model of the 100 psi beams shown in Fig. 4(b) had four inclined struts and three panels. The two vertical ties together represented the total stirrup force. The differences between the truss models of the 50 and 100 psi beams were due to the increased redistribution of internal forces in the 100 psi beams, as shown by the increased number of inclined cracks at failure (see Fig. 3).

In summary, increasing the concrete compressive strength increases the diagonal tension cracking load, which results in larger shear stresses to be carried by the combination of aggregate interlock, dowel action of the longitudinal reinforcement, uncracked concrete, and web reinforcement. These larger shear stresses induce larger crack widths, which in combination with the smoother crack surface typical of higher strength concrete results in a diminished aggregate interlock contribution. The reduced aggregate interlock and the larger stirrup strains could make the stirrups the weak link in the load-carrying system in beams with the minimum amount of web reinforcement.

EVALUATION OF CURRENT DESIGN PROVISION

The reserve shear strength index v_f/v_c in reinforced concrete beams is evaluated in Fig. 5 through 7 with test results from References 4 through 9. The first number in all the specimens (shown in the horizontal axis) indicates the amount of web reinforcement rf_y. All the specimens in Fig. 5 through 7 had an a/d ratio greater than 3.0 except for specimens 0-A1, 0-E1, 99-A2, and 99-E2, which had a/d ratios of 2.5.

As shown in Fig. 5, the reserve index v_f/v_c for beams with the minimum amount of web reinforcement (50 psi) and $f'_c < 6000$ psi was between 1.24 and 2.0. Fig. 6 shows that a small increase in the amount of shear reinforcement resulted in a substantial increase in the reserve shear strength in beams with $f'_c < 6000$ psi. For beams with rf_y of about 75 psi, the reserve shear strength index was between 1.52 and 2.32. For beams with rf_y of about 100 psi, the reserve index was between 1.89 and 2.82.

Fig. 7 evaluates the reserve shear strength index in beams with concrete compressive strengths between 6000 and 12,000 psi. For beams with the minimum amount of web reinforcement (50 psi), the reserve shear strength index was between 1.08 and 1.56. Beam 50-3 with f'_c of 10,490 psi tested during this investigation had the lowest index value. In this specimen, rupture of one of the stirrups was observed at failure. For beams with a web reinforcement index rf_y of 100 psi, the reserve shear strength index was between 1.60 and 2.25.

The reserve shear strength index in beams with the minimum amount of web reinforcement decreased with an increase in f_c'. Furthermore, the excess strength $[v_f - (v_c + rf_y)]/v_c$, shaded in Fig. 5 and 7 with vertical lines, also decreased for such beams as f_c' increased.

Since under the current design approach the shear strength provided by the minimum amount of web reinforcement is constant, any decrease in the excess strength with increasing f_c' could be attributed to a deficiency in the predicted concrete contribution to the ultimate shear strength of the member. This problem has been addressed by previous investigators[3,5] who found that, although the shear strength of beams without stirrups increased with increasing f_c', the ratio of tested to predicted shear strength decreased and was less than 1.0 for a significant number of tests. This deficiency could certainly be due in part to an overestimation of the benefit of increasing f_c' on the diagonal tensile strength of the concrete.

A possible solution to the deficiency in reserve shear strength would be to limit the contribution of the concrete compressive strength to the shear strength of the member by establishing an upper limit, such as 100 psi, on the term $\sqrt{f_c'}$. This provision would limit the increase in the shear strength due to the increase in f_c' for concrete with compressive strength greater than 10,000 psi.

However, this approach would unduly penalize beams with larger amounts of web reinforcement where the concrete contribution would become a smaller portion of the total shear strength. As was previously explained with the truss model approach, an increase of f_c' strengthens the concrete truss members, effectively increasing the shear that can be carried across the uncracked concrete in the flexural compression zone and delaying web crushing failures. Thus, in spite of the reduced aggregate interlock, if an adequate amount of longitudinal and web reinforcement is provided, an increase in f_c' should result in an increase in the shear strength of the member, as shown in Fig. 7 for the beams with rf_y of 100 psi. This observation has been confirmed by previous investigations.[5,7,10]

Another solution to the reduction in reserve shear strength would be to increase the minimum amount of shear reinforcement as the concrete compressive strength increases. This approach would address directly the problem of the transfer of forces upon first inclined cracking and at the same time would not penalize unduly those beams with shear reinforcement greater than the minimum amount.

Most of the specimens included in this evaluation had a/d ratios greater than 3.0. For members with smaller a/d ratios, the same reduction in reserve shear strength with the increase of f_c' in beams with minimum amount of shear reinforcement is not to be expected. Arch action in this type of member would increase the ultimate shear strength; however, special attention should be given to the anchorage and amount of longitudinal steel.

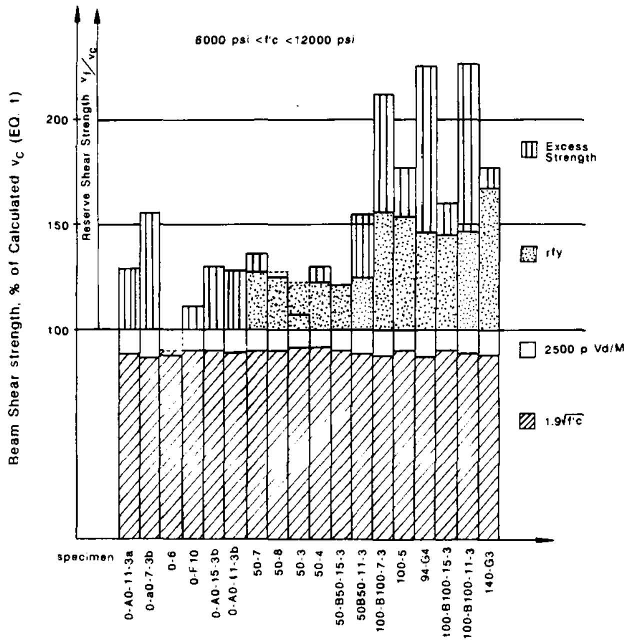


Fig. 7 — Reserve shear strength of beams with 6000 psi < f_c' < 12,000 psi (test results from References 5, 7, and 9)

CONCLUSIONS

The redistribution of forces at diagonal tension cracking in beams with higher concrete strengths can result in substantial reductions in the reserve and excess shear strengths if an adequate detailing and amount of longitudinal and web reinforcement is not provided.

The truss model analysis indicates that an increase in f_c' would increase the strength of the concrete components of the truss. For the truss mechanism to form, however, an adequate transfer of forces must take place at diagonal tension cracking. In higher strength concrete beams with small amounts of web reinforcement, because of the increased shear force to be transferred at the onset of diagonal tension cracking and the reduced aggregate interlock contribution, this transfer of forces may cause the first mobilized stirrups to yield and rupture. Stirrup rupture would stop any further redistribution of forces and could result in a diminished reserve capacity.

An evaluation of the current design provision indicated that the overall reserve shear strength after diagonal tension cracking diminished with increasing f_c' for beams with minimum amount of web reinforcement. This situation would be more critical for beams with larger a/d ratios and smaller amounts of longitudinal steel.

A solution to the reduction in reserve shear strength would be to increase the minimum amount of shear reinforcement as f_c' increases.

FUTURE WORK

Further work is needed to determine an adequate minimum amount of shear reinforcement for high-strength reinforced concrete beams. The required amount should be made a function of f'_c. The effect of the amount of longitudinal steel on the shear strength of high-strength concrete beams with small amounts of web reinforcement should also be investigated. As shown with the truss model approach, increasing the amount of web reinforcement and f'_c without insuring an adequate amount of longitudinal steel could make this reinforcement the weak link in the member strength. The effects of axial load, a/d ratio, and prestressing need to be included as well.

ACKNOWLEDGMENTS

The work described formed part of the MS thesis of the first author, at Purdue University, under the direction of the second author. The research was funded by scholarships from ACI and CRSI, and support of the CECO Corporation, Elkem Chemicals Inc., and the School of Civil Engineering at Purdue University. Any opinions, findings, and conclusions expressed in this paper are those of the authors and do not necessarily reflect the views of the sponsors.

NOTATION

a = shear span, distance between concentrated load and support centerline, in.
A_v = area of shear reinforcement located within spacing s, in.^{2}
A'_s = area of compression reinforcement, in.^{2}
A_s = area of tension reinforcement, in.^{2}
b = effective web width, in.
d = effective depth of the beam, in.
f'_c = specified concrete compressive strength, psi
f_y = specified yield strength of the reinforcement, psi
M = bending moment at the section
s = stirrup spacing in longitudinal direction, in.
r = A_v/bs
V_n = $V_c + V_s$, member shear strength
V_c = shear force carried by the concrete
V_s = shear force carried by the stirrups
V = applied shear force at the section
V_f = failure shear force
V_{cr} = cracking shear force
v_c = V_c/bd
v_{cr} = V_{cr}/bd
v_f = V_f/bd
v_n = V_n/bd
ρ_w = A_s/bd
ρ' = A'_s/bd

CONVERSION FACTORS

1 in.	=	25.4 mm
1 lb (mass)	=	0.4536 kg
1 lb (force)	=	4.4482 N
1 psi	=	6.895 Pa
1 kip	=	4448.2 N
1 ksi	=	6.895 MPa
2 kip-in.	=	0.113 kN-m

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