# Rationale for Suggested Development, Splice, and Standard Hook Provisions for Deformed Bars in Tension

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*Provisions recommended by ACI Committee 408 for development and splice lengths, and hooked bar anchorages, for bars in tension, are discussed and explained. They are compared with current code provisions and illustrated with several design examples. The recommendations are published in the preceding article.*

**Keywords:** anchorage; bond; building codes; **development length; hooks;** reinforced concrete; reinforcing steel; **splices;** structural design.

The recommendations developed by ACI Committee *408* are published in the preceding article. This paper offers a discussion and explanation of the recommendations, together with several numerical examples. The current Building Code Requirements (ACI 318-77) are also examined and compared with the recommendations. These recommendations were submitted to ACI Committee 318 which has the responsibility for developing code provisions.

## Development Length and Splices in Tension

### ACI 318-77 Building Code Requirements

**Development.** In order to provide a comparison between design methods and test results, average bond stresses along the embedded bars are used. Because current ACI 318[1] provisions require a specified development length to ensure yield being reached at the critical section, the design provisions must be restated in terms of the bond stresses implied by the specified development length. Section 12.2 of ACI 318-77 gives the basic development length

$$l_{db} = 0.04\, A_b f_y / \sqrt{f_c'} \geqslant 0.0004\, d_b f_y \text{ for \#11 bars and smaller} \tag{1}$$

$$\left.\begin{aligned} l_{db} &= 0.085\, f_y / \sqrt{f_c'} \text{ for \#14} \\ l_{db} &= 0.11\, f_y / \sqrt{f_c'} \text{ for \#18} \end{aligned}\right\} \tag{2}$$

It should be noted that Eq. (1) was developed assuming that the bar stress must reach 1.25 $f_y$ in order for the anchorage to perform satisfactorily. Where the bar is top cast (more than 12 in. of concrete cast below the bar), $l_d$ must be increased by 40 percent. From equilibrium, the *average* bond stress along the development length at the nominal $f_y$ can be expressed as

$$u = \frac{A_b f_y}{\pi\, d_b l_d} = \frac{d_b f_y}{4\, l_d} \tag{3}$$

Combining Eq. (1) and (2) with Eq. (3) leads to the following average bond stress equations:

All italic lower case k symbols in the text and figures refer to kips.

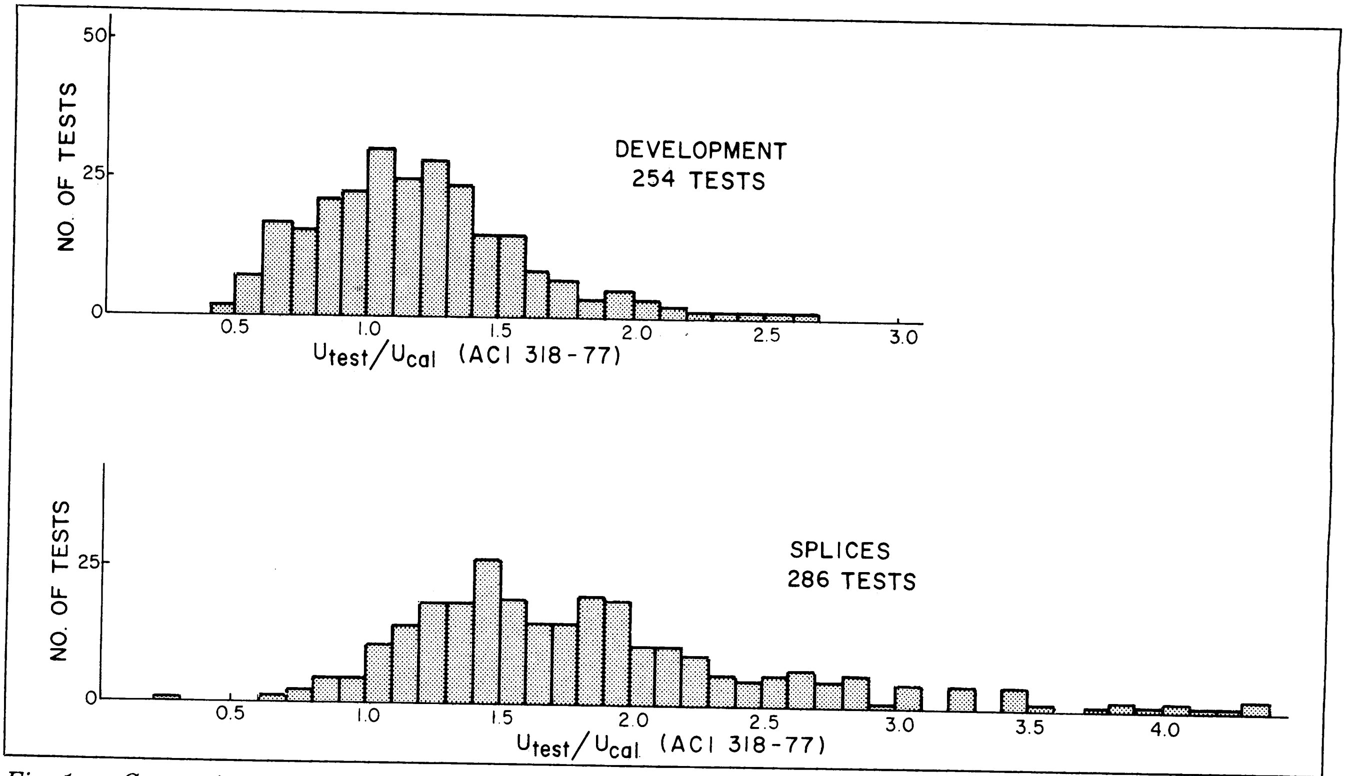


*Fig. 1 — Comparison of ACI 318-77 and test anchorage capacities*

$$u = \frac{7.95\sqrt{f_c'}}{d_b} \leqslant 625 \text{ psi for \#11 or smaller} \quad (4)$$

$$u = 5\sqrt{f_c'} \text{ for \#14 and \#18} \quad (5)$$

For top cast bars the average bond stresses must be divided by 1.4. Eq. (4) and (5) can be compared to test results to give an indication of the ability of current ACI provisions to estimate anchorage capacities of embedded bars. A report by Orangun, Jirsa, and Breen[2,3] summarizes experimental data on 254 development length tests reported in the literature. Only tests in which failure in bond occurred prior to yielding were considered. Using that data bank, a histogram showing the ratio of average bond stress from tests to calculated values using ACI 318-77 is given in Fig. 1. As can be seen, the variability is fairly large and a significant number of tests gave average bond stresses less than calculated.

**Splices.** For splices in tension, ACI 318-77 Section 12.16 provides for a lap splice length expressed in terms of $l_d$. Depending on the percent of steel spliced within a required lap length and the stress to be developed, the required lap splice length varies from 1.0 to 1.7 $l_d$. Therefore, the average bond stress developed along the splice for #11 bars or smaller can be expressed using Eq. (4) and divided by the appropriate factor for splice length. Lap splices are not permitted for bars larger than #11 (Section 12.15.2.1). References 2 and 3 summarize the results of 286 splice tests reported in the literature in which splice failure occurred prior to yielding. Since the tests studied involved all bars spliced at a given section, the implied average bond stress calculated using ACI 318-77 is

$$u = \frac{7.95\sqrt{f_c'}}{1.7\, d_b} = \frac{4.67\sqrt{f_c'}}{d_b} \leqslant 370 \text{ psi (2.55 MPa)} \quad (6)$$

A histogram showing the ratio of average bond stress from tests to calculated stresses is shown in Fig. 1. For splices, the variation is very great with many tests showing bond stresses more than twice as great as predicted using Eq. (6).

**Analysis of data.** An examination of the data indicates that development length data tend to be lower than predicted where concrete cover over the bars or spacing between bars is small. For large covers and spacings, and for cases where transverse reinforcement is present, the predicted values for both splices and development tend to be conservative. For splices, very few tests indicated stresses less than predicted, largely because the 1.7 factor is applied to splices. It is clear that the parameters (cover and bar spacing) excluded in current provisions may have an influence on development and splice characteristics and the current procedures may not always be conservative.[4]

**Reevaluation of data.** In the work reported by Orangun, Jirsa, and Breen,[2] available test data were evaluated to produce an empirical expression which would include all pertinent parameters. Drawing on work done by Tepfers,[5] the influence of cover or spacing was reduced to a single parameter $C$ which is the smaller of the clear cover or half the clear spacing between bars. The critical or lowest value of cover or spacing determines the direction splitting will occur and governs the mode of failure.

Using a regression analysis of a selected group of tests with reinforcement meeting current ASTM standards for deformations, an equation for average bond stress along an anchored bar or a splice was given by

$$u = \left[1.2 + \frac{3C}{d_b} + \frac{50 d_b}{l} + \frac{A_{tr} f_{yt}}{500\, s d_b}\right]\sqrt{f_c'} \qquad (7)$$

where $C$ is the cover parameter, $l$ is the splice or development length, and $A_{tr}$ represents the transverse reinforcement resisting splitting. The test results showed that where $C/d_b$ ratios were large, the mode of failure involved direct pullout rather than splitting of the cover. The transition between pullout and splitting appeared to be at a $C/d_b$ value of about 2.5. It was also found that increasing amounts of transverse reinforcement did not result in corresponding increase in average bond stress and a limit of $A_{tr} f_{yt}/sd_b \leq 1500$ was indicated. In most cases the transverse reinforcement was not highly stressed due to shear. The tests reported in Reference 6 were conducted with high bond and high shear occurring simultaneously and it appears that stirrups stressed in shear can be effective in resisting bond splitting.

Fig. 2 shows histograms of the data on splices and development compared with calculated values using Eq. (7). Where $C$ exceeded 2.5 $d_b$, the $C/d_b$ ratio in the equation was limited to 2.5. For values of $C$ greater than 2.5 $d_b$, the mode of failure was by pullout of the bar rather than cover splitting. Therefore, increases in cover do not increase bond capacity if a pullout failure occurs. Likewise, if large amounts of transverse reinforcement were present in the tests, $A_{tr} f_{yt}/500\, sd_b$ was limited to 3 in the calculations. As can be seen, the equation fits a large body of test results very well. Variability is substantially reduced over ACI 318-77 procedures and the fit is equally good for splices and development. Using this equation, ACI 408 developed the design recommendations. It should be noted that no distinction is made according to classes of splices, but the definition of $C_s$ reflects the number of bars spliced at a location and the $A_{sr}/A_{sp}$ ratio reflects the effect of stress level.

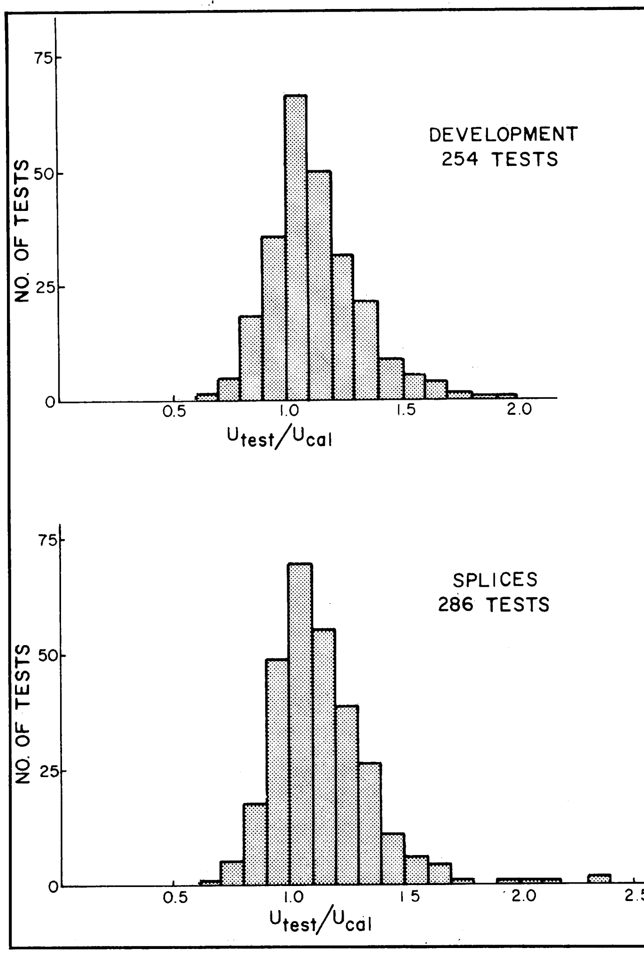


*Fig. 2 — Comparison of proposed and test anchorage capacities (Reference 2)*

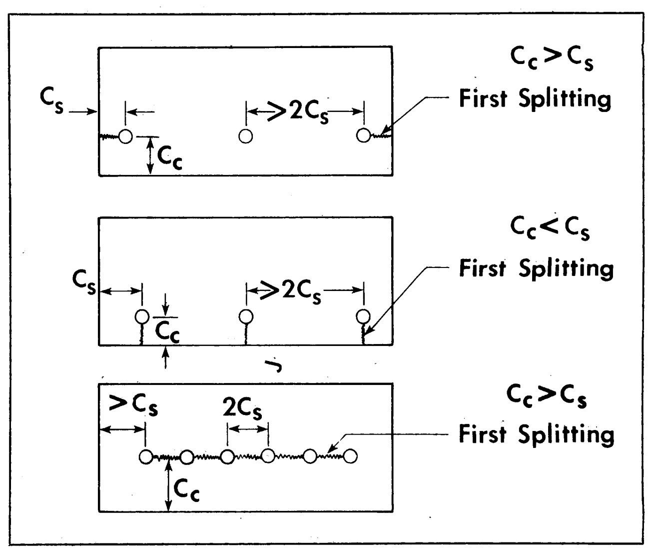


*Fig. 3 — Definition of cover parameters*

Eq. (B)* in the proposed design recommendations is a slightly modified version of Eq. (7). By rear-

*See the preceding Article.

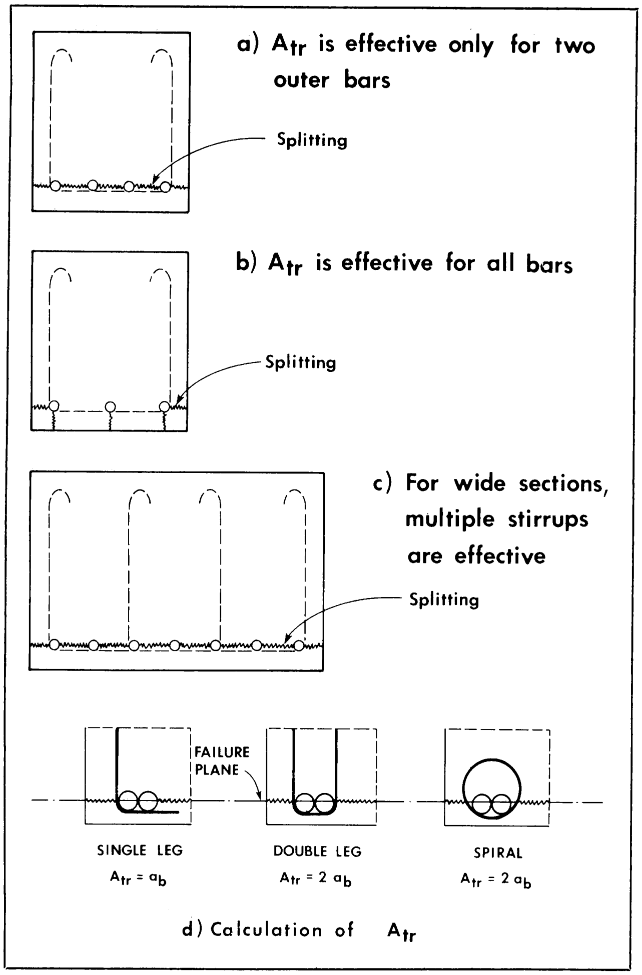


*Fig. 4 — Transverse reinforcement,* $A_{tr}$

ranging Eq. (7), the following equation is obtained for basic development length:

$$l_{db} = \frac{d_b\,(f_s/4\sqrt{f_c'} - 50)}{(1.2 + 3\,C/d_b + A_{tr}f_{yt}/500\,sd_b)} \quad (8)$$

The numerator can be rewritten as $(f_s - 200\sqrt{f_c'})$ $4\sqrt{f_c'}$ and $(f_s - 200\sqrt{f_c'})$ can be approximated by $(f_s - 11{,}000)$, or for Grade 60 reinforcement $(60{,}000 - 11{,}000) = 49{,}000$. For reinforcement with other values of yield strength, the numerator can be modified by the factor $f_y/50{,}000 - 0.2$. Factoring out $3/d_b$ from the denominator and inserting area of the bar leads to the equation

$$l_{db} = \frac{49000\,d_b\,[A_b/(\pi\,d_b^2/4)]}{4\sqrt{f_c'}\left(\dfrac{3}{d_b}\right)\left(0.4d_b + C + \dfrac{A_{tr}f_{yt}}{1500s}\right)} = \frac{5200A_b}{\sqrt{f_c'}\,(0.4d_b + C + K_{tr})} \quad (9)$$

where $K_{tr} = A_{tr}f_{yt}/1500s$. To further simplify the equation, the terms in the denominator $(0.4d_b + C)$ were rounded to $(0.5d_b + C)$, which is equivalent to the smaller of (1) $C_s$ the cover to the center of the bar measured along the line through the layer of the bars, or half the center-to-center distance of the bars in a layer, or (2) $C_c$ the thickness of concrete cover measured from extreme fiber to the center of the bar. Because this modification slightly reduces $l_d$, the constant in the numerator was increased from 5200 to 5500. The manner in which the critical cover parameter is determined is illustrated in Fig. 3. The smaller of $C_c + K_{tr}$ or $C_s + K_{tr}$ is denoted as $K$ in Eq. (B).

To be fully effective the transverse reinforcement must be adjacent to and on the outside of the bar in tension and must cross the potential plane of splitting causing failure; that is, $A_{tr}$ must be normal to the splitting crack. The effectiveness of the transverse reinforcement is shown in Fig. 4. In Case (a), $A_{tr}$ is effective only for the outer bars. The designer would have several choices in this case. A different $l_{db}$ could be calculated for the inner and outer bars, the effect of transverse reinforcement could be ignored, or $K_{tr}$ could be included as an average for the bars using $\Sigma\, K_{tr}/n$. The last approach was checked using data reported by Untrauer and Warren[6] and gave a reasonable estimate of measured values. This approach is incorporated into the proposed design recommendations in the definition of $A_{tr}$. At least three transverse bars are required along the development length; one should be near each end. In slabs, transverse bars outside the bar being anchored and crossing splitting cracks may be included in $K$.

For cases where the cover to the center of the bars is not less than 2.5 in. and the center-to-center spacing is not less than 5 in., Eq. (A1) and (A2) of the proposal offers the designer a simplified approach. The effect of transverse reinforcement is neglected in Eq. (A1) and (A2). (The value of $l_{db}$ could be adjusted by designers by multiplying by $2.5/C_s$ for smaller bar spacings. It should be noted that with this adjustment, Eq. (A1) and (A2) give the same values as Eq. (B) for bar spacings less than or equal to 5 in. and cover to center of bar at least 2.5 in. Similar factors could be introduced in design offices for other spacings to simplify calculations). Because no limit on the cover to diameter ratio is included, $l_{db} = 23d_b/\phi$ governs for bar sizes #6 and smaller.

Fig. 5 and 6 show the required $l_{db}$ for different bar sizes and cover or transverse reinforcement values using the proposed design recommendations and those in ACI 318-77. As can be seen, for small spacing (Fig. 5) $l_{db}$ is increased over ACI 318-77 values, but with larger spacing and with transverse steel (Fig. 6) the difference becomes smaller.

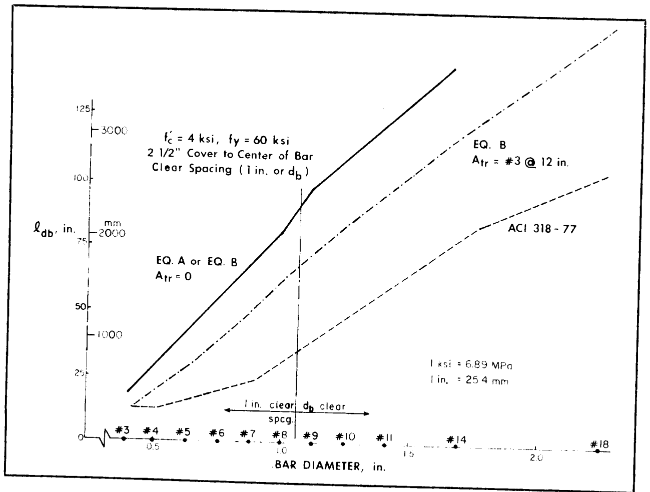


Fig. 5 — Comparison of development length for minimum spacing

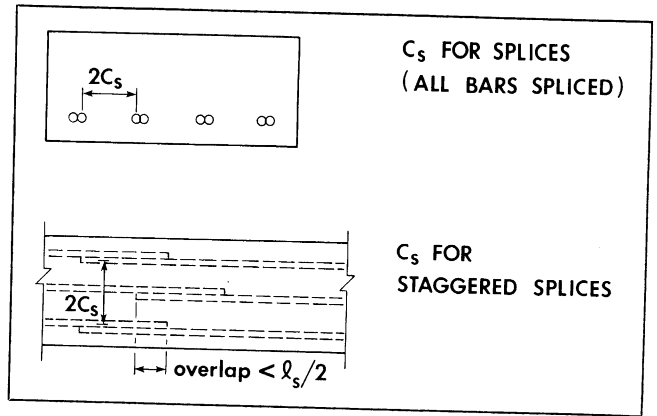


Fig. 7 — Definition of $C_s$ for splices

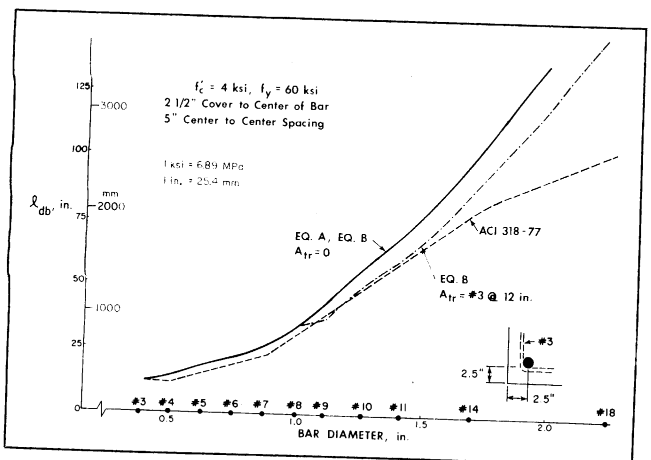


Fig. 6 — Comparison of proposed and ACI 318-77 development length

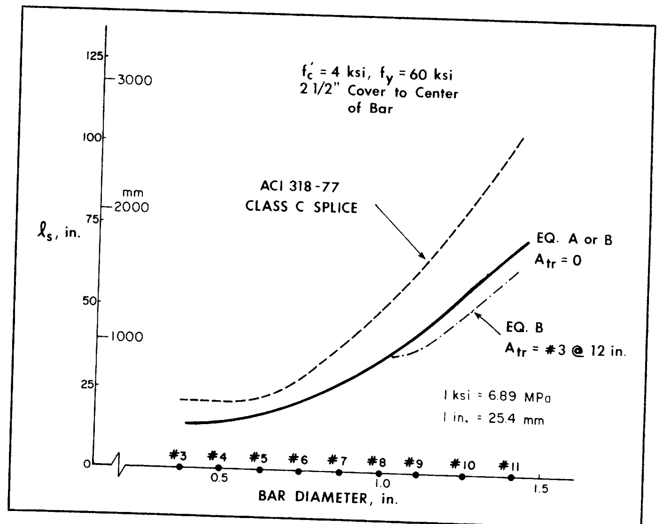


Fig. 8 — Comparison of recommended and ACI 318-77 splice provisions

**Modifications to $l_{db}$.** The modification in Section 1.1.3.1 for development length of bars with yield values other than 60 ksi has been discussed in the derivation of Eq. (9). Where the stress will not reach yield, e.g., where $A_{sr}$ is less than $A_{sp}$, the splice length or development length may be reduced as indicated in Section 1.1.3.4 of the proposal. This reduction should not be applied where large deformations may be imposed on the structure, such as seismic, blast, settlement or temperature effects, or when minimum flexural reinforcement is used.

It is well documented that the position of casting has an adverse affect on anchorage strength. A comparison of 68 tests (34 top cast with more than 12 in. of concrete below the bar) indicated that the ratio of the strength of top cast to bottom cast bars was 0.84 with a standard deviation of 0.12. The increase in required length is 20 percent based on strength and 40 percent based on slip. Considering the data available, an increase in $l_d$ of 30 percent for top cast bars is recommended. For bars embedded in lightweight aggregate concrete, an increase in $l_d$ of 25 percent is recommended as a simplification over the procedure in ACI 318-77 in which the increase varies from 18 percent to 33 percent, depending on the amount of lightweight aggregate used. It should be noted that research is needed in both areas, position of casting and lightweight aggregate, to improve the understanding of behavior of anchored bars under such conditions.

**Lap splices.** Section 1.2 of the recommendations is a major departure from ACI 318-77 because proposed splice length and development length are the same. The histograms shown in Fig. 2 indicate that Eq. (7) is equally reliable for both splices and development and no additional factors are needed.

Tension lap splices for #14 and #18 bars are not recommended. Although it may be possible to develop the strength of a splice in a large bar by requiring sufficient cover and transverse reinforcement, cracking at the ends of the splice may be excessive under service loads.

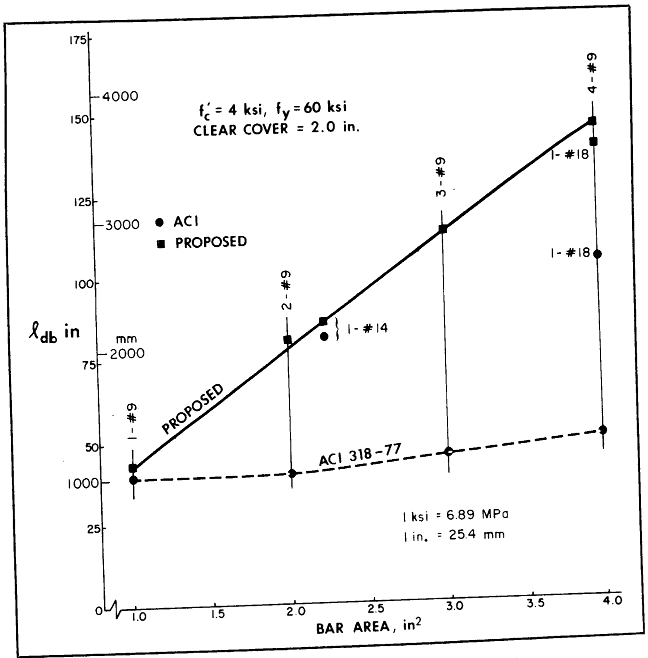


*Fig. 9 — Bundled bar development length*

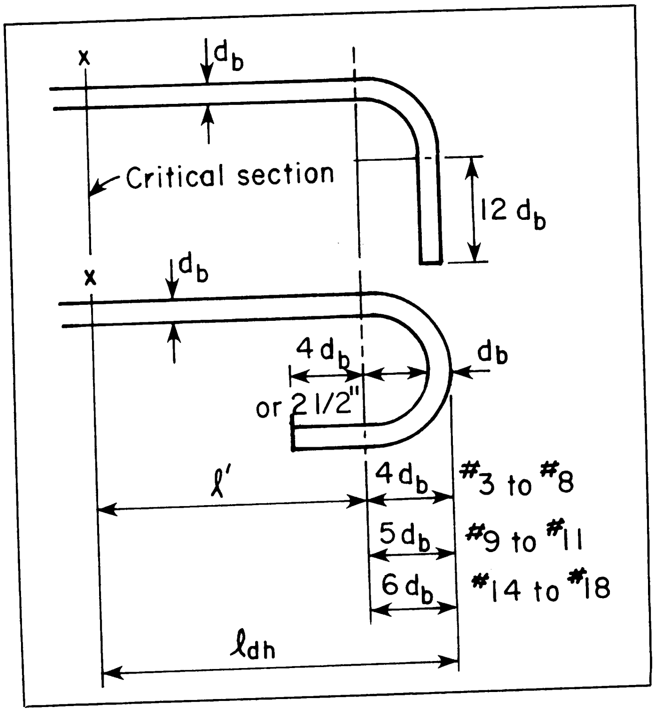


*Fig. 10 — Hook geometry*

In determining the required splice length, $l_s$, the distance $C_s$ to be used is illustrated in Fig. 7. Where all bars are spliced at the same location, $C_s$ is half the center-to-center distance between bars. Where the splices are staggered and the overlap is less than $l_s/2$, $C_s$ may be taken as half the distance between alternate splices. Staggering of splices has been shown to improve the behavior and the increase in $C_s$ reflects this improvement. With staggered splices, the spacing between bars generally will not be as critical as is the cover to the center of the bar.

Fig. 8 shows a comparison of proposed and current ACI 318-77 splice length requirements. For all bar diameter and transverse steel values, the splice length is less than required by ACI 318-77, except for Class A splices.

**Splices and development of bundled bars.** The provision for development or splice length of individual bars in a bundle remains unchanged from ACI 318-77. A 20 percent increase in $l_d$ is required for a 3-bar bundle, and a 33 percent increase for a 4-bar bundle.

In ACI 318-77, the development length for the entire bundle is presumably taken as the value for the individual bar increased by 20 or 33 percent, depending on the number of bars in a bundle. With the incorporation of factors such as cover and transverse reinforcement in the equation for development of reinforcement, the adjustments for splices and development length of bundled bars used in ACI 318-77 are no longer appropriate. The splitting action of a bundle of bars is similar to that of a single bar. The area of the bar or the bundle is much more important than its perimeter. Since the force $A_b f_y$ creates the splitting which controls the development length, 4 — #9 bars in a bundle should behave as one #18 bar and 2 — #9 bars as a #14 bar.

As illustrated in Fig. 9, if four #9 bars are bundled the required development length using the proposed approach is about 145 in., while it is only 50 in. using ACI 318-77. A large difference is also noted for #14 bars relative to two #9 bars in a bundle. The proposed approach for bundled bars eliminates these discrepancies and provides approximately the same development length for individual bars or bundled bars with the same total area.

Consistent with treating the bundle as a single bar, lap splicing of the entire bundle is permitted by Section 1.3.2. At present Section 12.15.2.2 of ACI 318-77 prohibits splicing of the entire bundle. Where the equivalent diameter of the bundle ($d_{be} = \sqrt{A_b}$) is at least 1.5 in., splices are not permitted.

## Hooked-Bar Anchorages

### ACI 318-77

Current design provisions for hooked bars in tension are a combination of special factors for hooked bars and standard development length provisions as expressed in Eq. (1). The procedure in ACI 318-77 for standard hooks in tension requires that the tensile stress developed by the hook (Section 12.5.1) be not greater than

$$f_h = \xi \sqrt{f_c'} \tag{10}$$

where $\xi$ is selected from Table 12.5.1 and is a function of bar diameter, $f_y$, and casting position. The value of $f_h$ is generally less than $f_y/2$ and the stress at the critical section must be developed along the straight bar between the standard hook and the critical section. The value of $\xi$ may be increased by 30 percent if enclosure in the form of "external concrete, or internal closed ties, spirals, or stirrups" is provided perpendicular to the plane of the hook. The amount of "enclosure" is not specified. The equivalent length of a standard hook, $l_e$, may be computed using provisions for $l_d$ and substituting $f_h$ for $f_y$:

$$l_e = 0.04\ A_b f_h / \sqrt{f_c'} \tag{11}$$

Eq. (10) and (11) can be rewrittten to determine the required straight bar development between the standard hook and the critical section,

$$l' = 0.004\ A_b (f_y / \sqrt{f_c'} - \xi) \tag{12}$$

The application of the provisions is made somewhat more difficult because the adjustments in standard hook stress $f_h$ for top bars, lightweight concrete, etc., are not clearly defined. In addition, there are inconsistencies in the values of $f_h$ obtained using ACI 318-77. For example, $f_h$ for a #6, Grade 60 bar is 50 percent greater than a similar bar of Grade 40 reinforcement. If the geometry and bar deformation are identical, it is not reasonable that a Grade 60 bar will develop 34 ksi and a Grade 40 bar only 23 ksi.

**Development length for a hooked bar, $l_{dh}$.** To provide data regarding hooked-bar anchorages, Marques and Jirsa[7] reported the results of a series of tests and developed an alternative to the standard hook strength $f_h$, Eq. (10); however, Eq. (11) was still used to determine the straight bar length required between the critical section and standard hook. Recent work reported by Pinc, Watkins, and Jirsa[8] provided additional data and resulted in the design approach embodied in the proposed hook provisions.

The proposed approach is a major departure from ACI 318-77 in that it uncouples hooked-bar anchorages from straight bar development provisions and gives the total embedment length $l_{dh}$ as indicated in Fig. 10. A study of the failures of hooked bars indicates that splitting of the cover parallel to the plane of the hook is the primary cause of failure and that the splitting originates at the inside of the hook where the local stress concentrations are very high. For this reason, Eq. (C) is a function of $d_b$ which governs the magnitude of compressive stresses on the inside of the hook. Only standard ACI hooked bars were tested and the influence of larger radius of bend can not be evaluated. The test results indicate that as the straight lead length increases, the lateral splitting force which can be

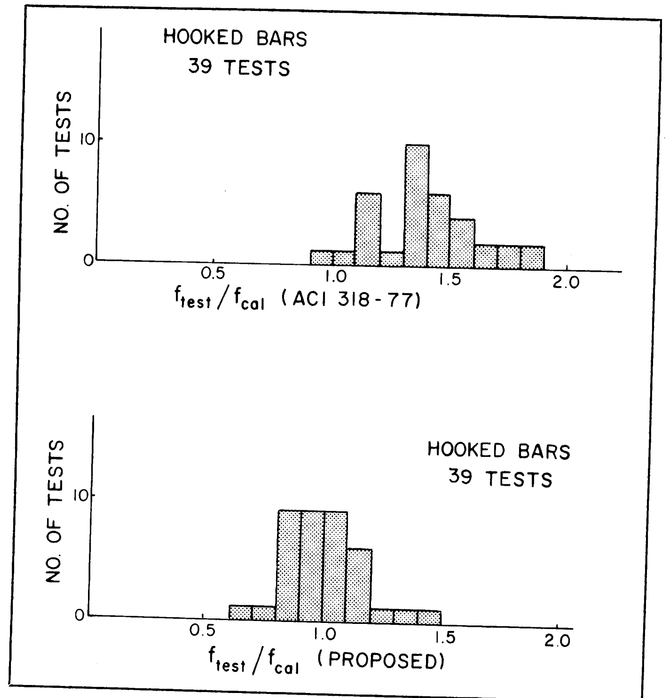


*Fig. 11 — Comparison of design and test hooked bar capacities*

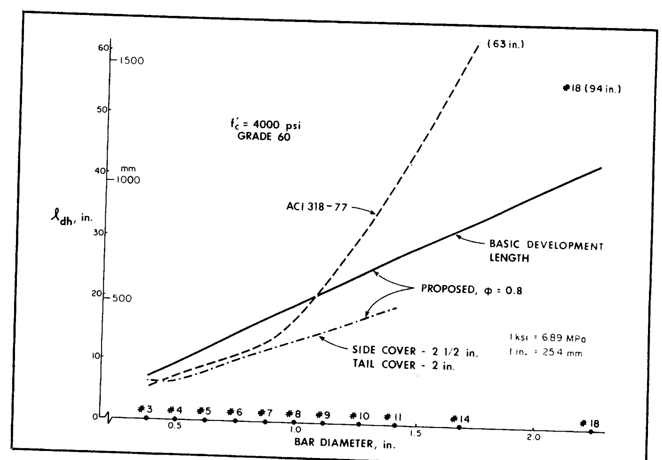


*Fig. 12 — Comparison of proposed and ACI 318-77 hook provisions*

developed in the side cover increases; this is reflected in an improvement in hook capacity.

The recommended provisions include adjustments to reflect the resistance to splitting provided by enclosure in transverse reinforcement. If the side cover is large so that side splitting is effectively eliminated, as in mass concrete, Section 1.4.3.2 (both factors $0.7 \times 0.8$) may be used. An adjustment for the reduction in tensile splitting capacity of lightweight aggregate is clearly defined. Minimum values of $l_{dh}$ are indicated to prevent failure by direct pullout in cases where the standard hook may be located very near the critical section. No distinction is made between top bars and other bars. Such a distinction is difficult for hooked bars in any case. As in ACI 318-77, hooks should not be considered effective in compression.

Fig. 11 is a histogram showing a comparison of test results with the proposed design procedure and with ACI 318-77. It should be noted that "other" bar values were used. Had "top" bar values been used, the ratio of $f_{test}/f_{cal}$ for ACI 318-77 would have been larger. Although the accuracy is improved with proposed procedure, of more importance is the simplification of calculations required for hooked-bar anchorages, and the incorporation of factors which influence the strength of the anchorage. Fig. 12 shows a comparison of required length $l_{dh}$ using the proposed approach [Eq. (C) and applicable factors] and ACI 318-77. The reduction in $l_{dh}$ afforded by the new provisions is most apparent for bars of larger diameter.

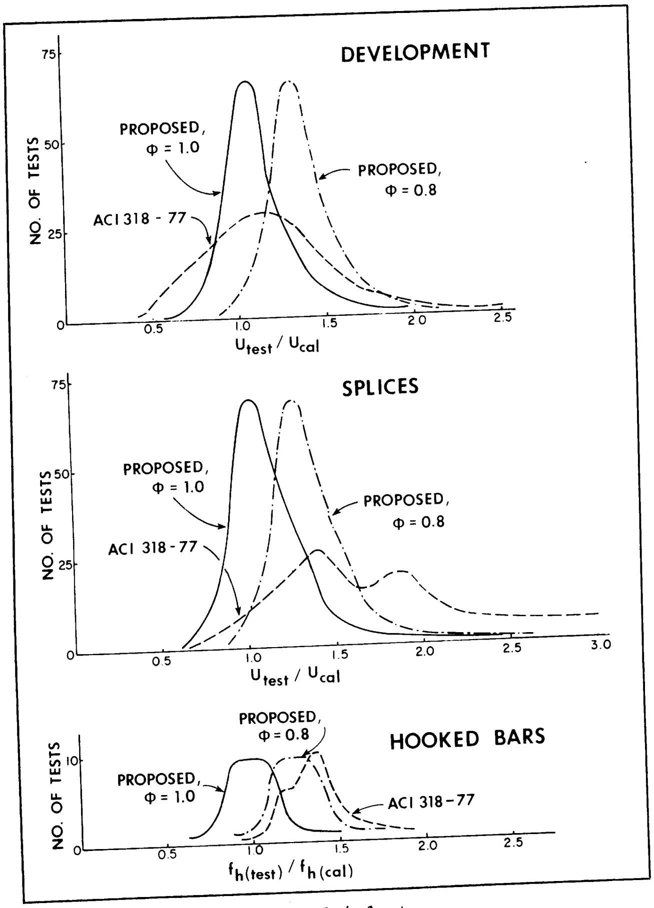


*Fig. 13 — Recommended $\phi$ factor*

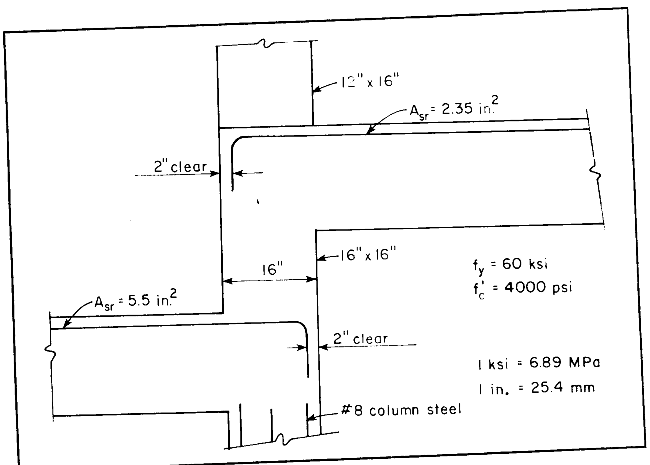


*Fig. 14 — Example 1: Beam-column joint*

**Introduction of $\phi$ Factor for Anchorage Design**

In ACI 318-77, no $\phi$ factors are introduced directly into the design recommendations. An understrength factor is indirectly specified because $l_d$ as derived in Eq. (1) is based on achieving 1.25 $f_y$ in the bar and the area of steel is based on $\phi$ = 0.9 in any flexural calculations. However, for consistency of approach, it is recommended that $\phi$ for anchorage be introduced directly into Code specifications. Fig. 13 shows the variation in test results with computed values using proposed Eqs. (A) and (B) and ACI 318-77. With $\phi$ = 0.8, virtually all test/calculated values lie above a ratio of 1.0. Note that using ACI 318-77 a substantial number of development length tests lie below a ratio of 1.0 and for both splices and development the variability is very high. For hooked bars, Eq. (C) with $\phi$ = 0.8 compares well with ACI 318-77.

In comparing test to calculated values, all test values considered in this report are based on monotonic tests to failure. Tests in which cyclic loads were applied or in which the anchored bars were subjected to reversals of load producing inelastic strains were not yet evaluated. Research is needed in these areas and under such loadings it may be necessary to reduce the value of $\phi$.

**Design Examples**

**Example 1: Development length of top reinforcement**

Let us evaluate the development lengths of the top bars into the two beams of Fig. 14 for $f_y$ = 60 ksi (414 MPa) and $f_c'$ = 4000 psi (27.6 MPa).

**(a) Lower beam**

With 5.5 in.² (3550 mm²) of reinforcement required, select 4 — #11 bars to be developed together in a 16 in. (406 mm) beam width. Consider #4 stirrups and 1.5 in. (38 mm) clear cover so that

$$C_c = 1.5 + 0.5 + 1.41/2 = 2.70 \text{ in. (68 mm).}$$

Between bars

$$C_s = \frac{16 - 2\,(2) - 1.41}{3\,(2)} = 1.76 \text{ in. (45 mm)}$$

$$K = 1.76 < 3d_b$$

The basic development length is given by Eq. (B) since the simple equation cannot be used

$$l_{db} = \frac{5500\,(1.56)}{0.8\,(1.76)\sqrt{4000}} = 96 \text{ in. (2440 mm)}$$

With modifying factors for the top bar effect and excess steel in non-seismic design

$$l_d = 1.3 \frac{5.5}{6.24} 96 = 110 \text{ in. } (2800 \text{ mm})$$

If the #4 ties were spaced at 10 in. (254 mm), the value of $K_{tr}$ could be calculated as follows

$$A_{tr} = \frac{2\,(0.2)}{4} = 0.10 \text{ in.}^2 \; (65 \text{ mm}^2)$$

$$K_{tr} = \frac{60{,}000\,(0.10)}{1500\,(10)} = 0.40 < d_b$$

and

$$K = 1.76 + 0.40 = 2.16 \text{ in. } (55 \text{ mm}) < 3\,d_b.$$

With lateral reinforcement considered, the basic development length would decrease to

$$l_{db} = \frac{5500\,(1.56)}{0.8\,(2.16)\sqrt{4000}} = 79 \text{ in. } (2000 \text{ mm})$$

and

$$l_d = 1.3 \frac{5.5}{6.24} 79 = 91 \text{ in. } (2310 \text{ mm})$$

With the present provisions in ACI 318-77, the basic development length of a #11 bar is

$$\frac{0.04\,(1.56)\,(60{,}000)}{\sqrt{4000}} = 59.2 \text{ in. } (1500 \text{ mm}).$$

Applying the modifying factors for the top bar effect and excess steel used,

$$l_d = 1.4 \frac{5.5}{6.24} (59.2) = 73.0 \text{ in. } (1850 \text{ mm})$$

which is 18 in. (460 mm) less than the length obtained from the recommended provisions. No benefit can be taken for the stirrup reinforcement in the beam using ACI 318-77 provisions.

**(b) Upper beam**

With 2.35 in.$^2$ (1520 mm$^2$) of reinforcement required, select 2 — #7 and 2 — #8 bars to be developed together in the 16 in. wide beam. Consider #3 stirrups at 9 in. (230 mm) spacing adjacent to the support. The bars are placed as shown in Fig. 15.

$$\text{Between bars } C_s = \frac{16 - 1.87\,(2) - 1}{3\,(2)}$$

$$= 1.87 \text{ in. } (47 \text{ mm})$$

$$\text{Side cover} \quad C_s = 1.87 + 1.00/2 = 2.37 \text{ in. } (60 \text{ mm})$$

$$\text{Top cover} \quad C_c = 1.87 + 0.87/2 = 2.31 \text{ in. } (58 \text{ mm})$$

Vertical transverse reinforcement per bar

$$A_{tr} = \frac{2\,(0.11)}{4} = 0.055 \text{ in.}^2 \; (35 \text{ mm}^2)$$

$$K_{tr} = \frac{60{,}000\,(0.055)}{1500\,(9)} = 0.24 \text{ in. } (6 \text{ mm}) < d_b$$

The horizontal transverse reinforcement parameter is

$$K_{tr} = \frac{60{,}000\,(0.11)}{1500\,(9)} = 0.49 \text{ in. } (12 \text{ mm}) < d_b$$

Therefore

$$K = C_c + K_{tr} = 2.31 + 0.49$$
$$= 2.80 \text{ in. } (71 \text{ mm}) \text{ for top cover splitting}$$

$$K = C_s + K_{tr} = 1.87 + 0.24$$
$$= 2.11 \text{ in. } (54 \text{ mm}) < 3d_b \text{ controls.}$$

Considering the top bar effect the development lengths are:

for #7 bars

$$l_d = 1.3 \frac{5500\,(0.60)}{0.8\,(2.11)\sqrt{4000}}$$
$$= 40 \text{ in. } (1015 \text{ mm})$$

for #8 bars

$$l_d = 1.3 \frac{5500\,(0.79)}{0.8\,(2.11)\sqrt{4000}}$$
$$= 53 \text{ in. } (1345 \text{ mm})$$

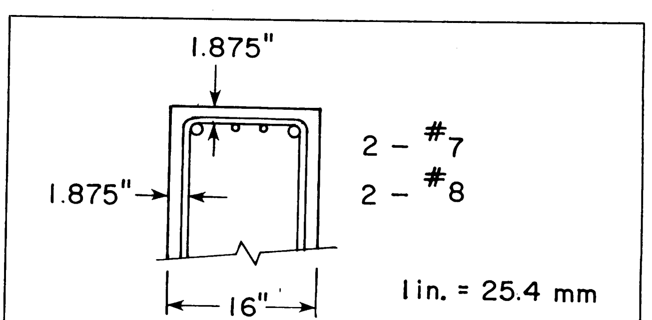


*Fig. 15 — Example 2*

but since the #8 hooks cannot fully develop (see Ex. 2):

$$l_d = \frac{14}{19}(53) = 39 \text{ in. } (990 \text{ mm})$$

With the present provision of ACI 318-77, the development lengths are

$$\#7 \text{ bars} \quad \frac{1.4\,(0.04)\,(0.60)\,60{,}000}{\sqrt{4000}}$$

$$= 31.9 \text{ in. } (810 \text{ mm})$$

$$\#8 \text{ bars} \quad \frac{1.4\,(0.04)\,(0.79)\,60{,}000}{\sqrt{4000}}$$

$$= 42.0 \text{ in. } (1070 \text{ mm})$$

The recommended provisions require a longer development length, reflecting the adverse effect of the relatively close spacing of reinforcing bars.

**Example 2: Hooks**

Let us design the hooked beam reinforcement for the two beam-column joints shown in Fig. 14.

**(a) Upper joint**

The column reinforcement has been tapered to accommodate the change in column size; thus the two outer hooked bars are placed outside of the column cage. (Note, however, that normally hooks should be inside column bars and transverse reinforcement.) The clear side cover on the outer hooked bars is thus 2 in. (51 mm) for #4 stirrup reinforcement in the beams. Hence none of the modifying factors in Section 1.4.3 are applicable for the outer bars. The 0.7 factor will be applicable for the inner bars.

First, since the capacity of the individual bars will differ, let us compute the tensile force required by 2.35 in.$^2$ (1516 mm$^2$) steel: 2.35 (60) = 141 $k$ (627 kN). Since the tensile force required is used, the factor in Section 1.4.3.4 will not be applied.

For the 3 — #8 bars

$$l_{dh} = \frac{960\,(1)}{0.8\sqrt{4000}}$$

$$= 19 \text{ in. } (483 \text{ mm}) \text{ for outer bars}$$

$$l_{dh} = 0.7\,(19)$$

$$= 13 \text{ in. } (330 \text{ mm}) \text{ for inner bars}$$

The available development length is 16 − 2 = 14 in. (356 mm). The inner bar develops 0.79 (60) = 47 $k$ (209 kN) and the outer bars develop 2 (0.79) (60) (14)/19 = 70 $k$ (311 kN) for a total of 117 $k$ (520 kN) which is less than 141 $k$ (627 kN). Thus 4 — #8 instead of 3 — #8 is a possible selection.

Let us consider the use of 4 — #7 bars with $A_s$ = 2.40 in.$^2$ (1550 mm$^2$)

$$l_{dh} = \frac{960\,(0.88)}{0.8\sqrt{4000}} = 17 \text{ in. } (430 \text{ mm})$$

hence 0.7 (17) = 12 in. (305 mm) < 14 in. (356 mm).

The inner two bars develop 2 (0.6) (60) = 72 $k$ (320 kN) and the outer bars develop 2 (0.6) (60) (14)/17 = 59 $k$ (262 kN) for a total of 131 $k$ (583 kN) which is less than 141 $k$ (627 kN). Thus 4 — #7 is not satisfactory.

Let us consider the use of 2 — #7 and 2 — #8 bars. From above

| | |
|---|---|
| 2 — #8 outer bars | 70 $k$ |
| 2 — #7 inner bars | 72 $k$ |
| | 142 $k$ > 141 $k$ |

Therefore we may use either 4 — #8 bars or 2 — #7 plus 2 — #8 bars with either size on the outside. The use of 6 — #6 bars is a possible alternative selection in this 16 in. (406 mm) wide beam.

Using the present provisions of ACI 318-77, $f_h = 360\sqrt{f_c'}$ = 22,770 psi (157 MPa) for the #8 hooked bars. The remaining straight length is 16-2-4 (1) = 10 in. (254 mm). This length develops an additional $10\sqrt{4000}/(0.04 \times 0.79)$ = 20,010 psi (138 MPa) in the bar. One of the outer #8 bars can develop a force of 0.79 (22.77 + 20.0) = 33.8 $k$ (150 kN). If the center #8 bar is confined, it can develop 1.3 (33.8) = 43.9$k$ (195 kN). So the 3 — #8 can develop 111.5$k$ (496 kN). Again, one will find that 4 — #8 will work by providing more than 141$k$ (627 kN) capacity.

Checking the #7 bar, which has $f_h$ = 22.77 ksi (157 MPa) and an additional $10.5\sqrt{4000}/(0.04 \times 0.6)$ = 27,670 psi (191 MPa) capacity for the straight portion, a 30.26$k$ (135 kN) capacity is determined. With 2 — #7 bars confined and 2 — #8 unconfined, the total capacity is 1.3 (2) (30.26) + 2 (33.8) = 146.3$k$ > 141$k$ (651 kN > 627 kN), which provides a satisfactory anchorage.

**(b) Lower joint**

Here all hooked bars are placed inside the column reinforcement. With 1.5 in. (38 mm) clear cover on the column reinforcement consisting of #4 ties and #8 bars, the outer hooked bars have the required 2.5 in. (64 mm) side cover for use of the 0.7 modifying factor. The design drawings should specify that the outer hooked bars be placed inside the column bars.

The obvious initial selection is 4 — #11 bars providing $A_s$ = 6.24 in.$^2$ (4026 mm$^2$).

$$l_{dh} = 0.7\,\frac{960\,(1.41)}{0.8\sqrt{4000}}$$

$$= 19 \text{ in. } (480 \text{ mm})$$

This is greater than the 14 in. (356 mm) available. Since there are few other alternatives, enclose the hooks with closed hoops such that the 0.8 modifying factor can also be used. It will also be necessary to employ the $A_{sr}/A_{sp}$ factor if seismic actions are not considered.

$$l_{dh} = 0.8\,(0.7)\,\frac{5.5}{6.24}\,\frac{960\,(1.41)}{0.8\sqrt{4000}}$$

$$= 13 \text{ in. } (330 \text{ mm}) < 14 \text{ in. } (356 \text{ mm})$$

The closed looped ties are detailed as shown in Fig. 16. To be consistent with the column ties, #4 ties are selected. The use of 6 ties at 5 equal spaces of 3 $d_b$ satisfies the requirements of Section 1.4.3.2.2 for 90 degree hooks. Four ties would be required for 180 degree hooks.

Using the current provisions of ACI 318-77, $f_h = 360\sqrt{4000} = 22{,}700$ psi (157 MPa) for the #11 hooked bars. With appropriate confinement of these hooks, this stress can be increased by 30 percent to 29,600 psi (204 MPa). The hook takes 5 $d_b$ = 7.05 in. (180 mm) of space, leaving 6.95 in. (176 mm) of straight bar to the column face, assuming 2 in. (51 mm) clear cover to the back of the hook. Using the basic development length expression

$$\Delta f_s = \frac{6.95\sqrt{4000}}{0.04\,(1.56)} = 7040 \text{ psi } (48.6 \text{ MPa})$$

Therefore, the stress capacities of the hooks are 29,600 + 7040 = 36,640 psi (253 MPa). Considering the $A_{sr}/A_{sp}$ factor, the required stress for the bars is 5.5 (60,000)/6.24 = 52,900 psi (365 MPa). Four #11 bars do not work according to ACI 318-77, but six #11 bars (two in a second layer) should work. However, two of the upper bars should be hooked horizontally, if possible, to enable positioning all #11 hooks with 2 in. (51 mm) back clear cover.

**Example 3: Development length**

Let us select the reinforcement for the beam shown in Fig. 17, based on bond and anchorage provisions. A 16 in. x 28 in. (406 x 711 mm) rectangular beam is selected and 4 – #11 bars are chosen for consideration based on a required area of 6.04 in.$^2$. Stirrups are #4 at 12 in. (305 mm) at each end. Given $f_y$ = 60 ksi (414 MPa) and $f_c'$ = 3000 psi (20.7 MPa).

Bond evaluation consists of checking $l_d$ versus the value of $M_n/V_u + l_a$. For 4 – #11 bars $M_n$ = 8180 in.-$k$ (924 m·kN). At the left end of the beam $V_u$ = 85.3 $k$ (380 kN) and the available additional length $l_a$ = 6 – 2 = 4 in. (102 mm). Therefore

$$\frac{M_n}{V_u} + l_a = \frac{8180}{85.3} + 4 = 100 \text{ in. } (2540 \text{ mm})$$

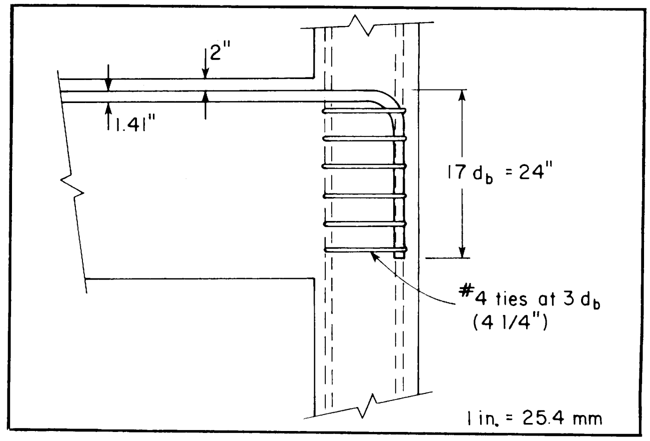


*Fig. 16 — Example 1: Lower joint*

With the inclusion of the 30 percent increase allowed at the support by Section 12.12.3 of ACI 318-77, the development length must be less than 130 in. (3300 mm).

The clear cover is 2 in. (51 mm), however, the spacing of bars is smaller than 5 in. (127 mm) and therefore Eq. (B) must be used.

Without transverse reinforcement

$$C_c = 2 + 1.41/2 = 2.70 \text{ in. } (68 \text{ mm})$$

$$C_s = \frac{16 - 4 - 1.41}{2\,(3)} = 1.76 \text{ in. } (45 \text{ mm})$$

$$K = C_s < 3d_b$$

$$l_{db} = \frac{5500\,(1.56)}{0.8\,(1.76)\sqrt{4000}}$$

$$= 96 \text{ in. } (2440 \text{ mm})$$

$$l_d = \frac{6.04}{6.24}\,(96) = 93 \text{ in. } (2360 \text{ mm})$$

$$< 130 \text{ in. } (3300 \text{ mm})$$

With #4 transverse reinforcement, for bottom splitting

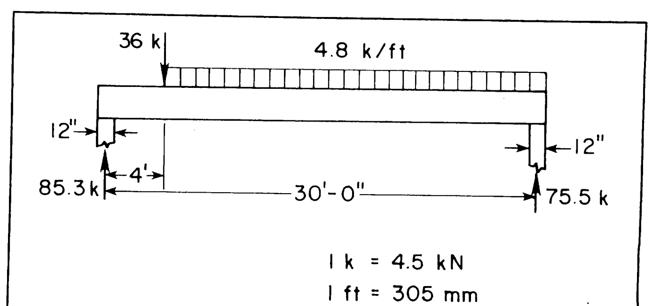


*Fig. 17 — Example 3*

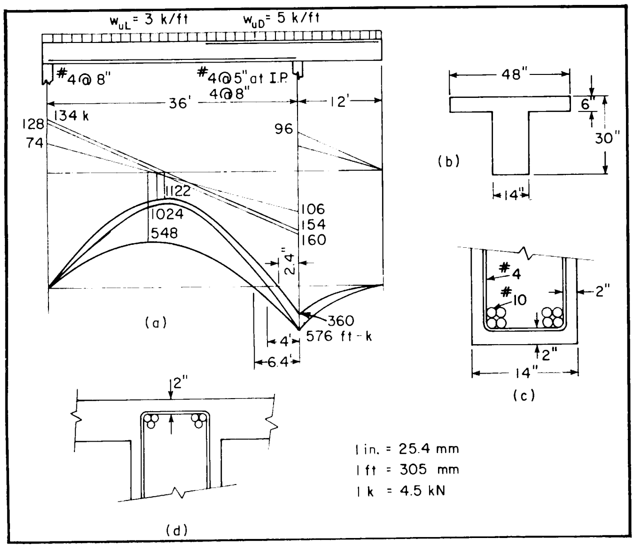


*Fig. 18 — Example 4*

$$K = C_c + K_{tr} = 2.70 + \frac{0.2\,(60{,}000)}{1500\,(12)}$$

$$= 3.37 \text{ in. (86 mm)}$$

For splitting between bars

$$A_{tr} = \frac{2\,(0.2)}{4} = 0.10 \text{ in.}^2 \text{ (65 mm}^2\text{)}$$

$$K_{tr} = \frac{0.10\,(60{,}000)}{1500\,(12)}$$

$$= 0.33 \text{ in. (8 mm)} < d_b$$

$$K = C_s + K_{tr} = 2.10 \text{ in. (53 mm)} < 3\,d_b$$

Therefore

$$l_d = \frac{6.04}{6.24}\;\frac{5500\,(1.56)}{0.8\,(2.10)\sqrt{4000}}$$

$$= 78 \text{ in. (1980 mm)} < 130 \text{ in. (3300 mm)}$$

instead of the 93 in. (2360 mm) without transverse steel. The transverse reinforcement is therefore not required to meet Section 12.14. Note that transverse reinforcement would have to be used over the entire development length to be included in *K*.

The development length for 5 — #10 bars would be 79 in. (2000 mm), which is not much different from the above value. The 4 — #11 design is preferable because of the larger clear spacing between bars.

Using the current provisions of ACI 318-77, the development length of the #11 bars is

$$l_d = \frac{6.04}{6.24}\,(0.04)\,(1.56)\,\frac{60{,}000}{\sqrt{4000}}$$

$$= 57.3 \text{ in. (1450 mm)}$$

which is significantly less than the 78 in. (1980 mm) determined above. If #10 bars were used instead of #11 bars, the difference would be even greater: $l_d$ = 46.6 in. (1185 mm) versus 79 in. (2000 mm) obtained using the recommended provisions. These differences are caused by the close spacing of 5 — #10 bars.

**Example 4: Bundled bars**

Let us evaluate the bundled bars used in the beam of Fig. 18 in terms of bond integrity for the given loading. Given $f_y$ = 60 ksi (414 MPa) and $f_c'$ = 4800 psi (33.1 MPa).

**(a) Bottom reinforcement**

Consider the use of 8 — #10 bars in two four-bar bundles. This reinforcement provides $M_n$ = 15,350 in.-*k* (1730 m·kN) with $A_s$ = 10.16 in.$^2$ (6550 mm$^2$). Using the required area of 9.90 in.$^2$, $A_{sr}/A_{sp}$ = 0.975.

Although it is frequently easy to determine the centroid of a bundle, it is perhaps more consistent to work with an equivalent bar. In the present case the equivalent bar diameter is $d_{be} = \sqrt{5.08} = 2.25$ in. (57 mm). The cover to the center of this bar is greater than 2 in. (51 mm) and the maximum bar spacing is greater than 5 in. (127 mm); therefore the simple approach, Eq. (A), could be used. However, for this case it gives much larger development length than Eq. (B) because it omits the effect of transverse reinforcement.

The side and bottom covers are 2 + 1.12 = 3.12 in. (80 mm) and $C_s$ = 3.88 in. (98 mm) between bars. For the left end of the beam, for bottom splitting $A_{tr}$ = 0.2 in.$^2$ (129 mm$^2$), $K_{tr}$ = 1.0 in. (25 mm) < $d_{be}$, and $K$ = 4.12 in. (105 mm) < 3 $d_{be}$. For side splitting $K$ is the same.

The basic development length for $f_c'$ = 4800 psi (33 MPa) is

$$l_{db} = \frac{5500\,(5.08)}{0.8\,(4.12)\sqrt{4800}} = 122 \text{ in. (3100 mm)}$$

and

$$l_d = 0.975\,(122) = 119 \text{ in. (3020 mm)}$$

The maximum permitted $l_d$ at the left end per Section 12.12.3 of ACI 318-77 is

$$1.3\left(\frac{M_n}{V_u} + l_a\right) = 1.3\left(\frac{15{,}350}{134} + 4\right)$$

$$= 154 \text{ in. (3910 mm)}$$

Thus the 4 — #10 bundles are satisfactory in bond at the left end.

At the inflection point $l_a$ is the lesser of 12 (2.25) = 27 in. (685 mm) or the effective depth 26.7 in. (680 mm). So $l_d$ must be less than

$$\frac{M_n}{V_u} + l_a = \frac{15{,}350}{134} + 26.7$$

$$= 141 \text{ in. (3580 mm)}$$

and

$$l_d = 0.975 \frac{5500\,(5.08)}{0.8\,(3.12 + 1.6)\sqrt{4800}}$$

$$= 104 \text{ in. (2640 mm)}$$

The 4 — #10 bundles are satisfactory considering that all four are carried 27 in. (685 mm) past the inflection point which means, for all practical purposes, that all four bars in each bundle would be carried 6 in. (150 mm) past the face of the support.

Under the provisions in ACI 318-77, it can only be presumed that Section 12.4, which defines the development length of individual bars in a bundle, is applicable if each bar in the bundle develops at the same location in the beam. This being the case

$$l_d = 0.975\,(1.33) \frac{(0.04)\,(1.27)\,60{,}000}{\sqrt{4000}}$$

$$= 57.0 \text{ in. (1450 mm)}$$

for each of the #10 bars. This is considerably less than the length obtained from the recommendations. A better value might be calculated using the basic development length equations with $A_b = 4\,(1.27) = 5.08$ in.$^2$ (3280 mm$^2$), which would give $l_d = 172$ in. (4360 mm).

Consider now two of the four bars in the bundle to be cut at the inflection point. A two bar bundle has an equivalent diameter of $d_{be} = \sqrt{2(1.27)} = 1.59$ in. (40 mm).

$M_n$ is taken as 15,350/2 = 7675 in.-$k$ (867 m·kN) and $l_a = 12\,(1.59) = 19$ in. (483 mm).

$$\frac{M_n}{V_u} + l_a = \frac{7675}{134} + 19 = 76 \text{ in. (1930 mm)}$$

The bottom and side covers to the center of equivalent bars are 2.80 in. (70 mm) which control in comparison with the bar spacing. In this region of the beam $K_{tr} = 1.6$ in. (40 mm) and $K = 4.40$ in. (110 mm) $< 3\,d_{be} = 4.77$ in. (120 mm). Thus

$$l_d = 0.975 \frac{5500\,(2.54)}{0.8\,(4.40)\sqrt{4800}}$$

$$= 56 \text{ in. (1422 mm)} < 76 \text{ in. (1930 mm)}$$

Note that the development of the bar must also be checked.

**(b) Top reinforcement**

Let us consider the use of 6 — #9 bars in two three-bar bundles as top reinforcement. This reinforcement with $A_s = 6$ in.$^2$ (3870 mm$^2$) provides $M_n = 8600$ in.-$k$ (970 m·kN). The required area is 5.35 in.$^2$ (3450 mm$^2$). Therefore

$$\frac{A_{sr}}{A_{sp}} = 0.892$$

$$d_{be} = \sqrt{3} = 1.73 \text{ in. (44 mm)}$$

$$C_c = 2 + 0.87 = 2.87 \text{ in. (73 mm)}$$

$$C_s = \frac{14 - 4 - 1.73}{2} = 4.13 \text{ in. (105 mm)}$$

$$A_{tr} = 0.2 \text{ in.}^2 \text{ (129 mm}^2\text{)}$$

For a transverse reinforcement spacing of 5 in. (127 mm)

$$K_{tr} = 1.6 \text{ in. (40 mm) and}$$
$$K = 4.47 \text{ in. (114 mm)} < 3\,d_{be}$$

$$l_d = 0.892\,(1.3) \frac{5500\,(3)}{0.8\,(4.47)\sqrt{4800}}$$

$$= 77 \text{ in. (1950 mm)}$$

From Section 12.13.3 of ACI 318-77 the negative reinforcement must be carried not less than $d = 27.0$ in. (685 mm), 12 (1.73) = 20.8 in. (530 mm) or 35 (12)/16 = 26.3 in. (670 mm) past the inflection point, which is 6.4 (12) + 27.0 = 104 in. (2640 mm) past the center of the support. Thus the three-bar bundles can easily be developed in the 104 in. (2640 mm) length.

Now consider the length for one of the bars in the bundle to be developed, if it is desired to cut one bar earlier than the other two in each bundle. Since it will be difficult to insure that a particular bar will be cut earlier, presume that the one with the lowest value of $K$ will be cut. So $C_c = 2 + 1.13/2 = 2.56$ in. (65 mm), $K = 2.56 + 1.6 = 4.16$ in. (106 mm) which is greater than the limit of 3 (1.13) = 3.39 in. (86 mm).

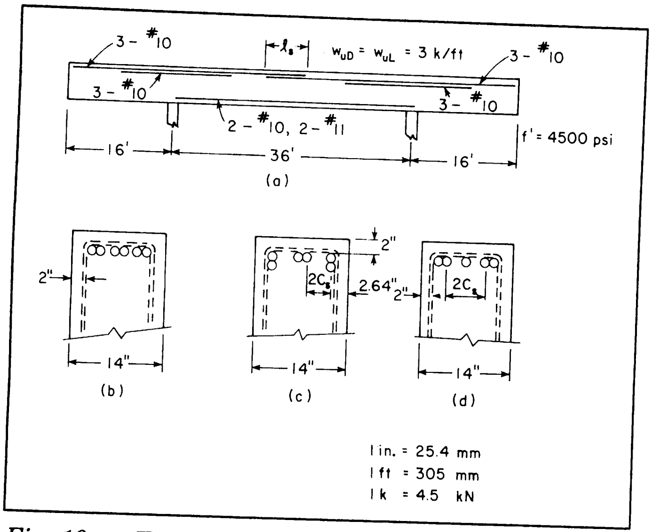


*Fig. 19 — Example 5*

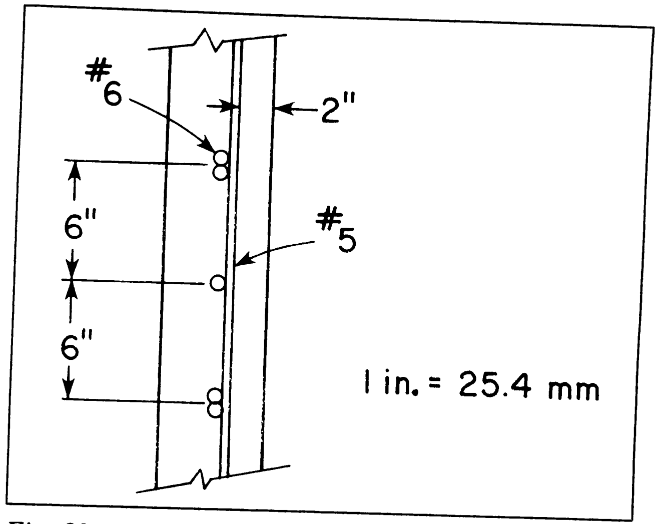


*Fig. 20 — Example 6*

Thus

$$l_d = 1.20\,(1.3)\,\frac{5500\,(1.00)}{0.8\,(3.39)\sqrt{4800}}$$

$$= 46 \text{ in. } (1170 \text{ mm})$$

The 1.2 factor is based on the 20 percent required increase for an individual bar in a three-bar bundle. This bar will be cut 46 in. (1170 mm) from the center of support or at a longer distance based on the provisions of Section 12.11.5 of ACI 318-77 covering bars cut in the tension zone.

Using the provisions of ACI 318-77, the development length is

$$l_d = 1.2\,(1.4)\,\frac{0.04\,(1.0)\,60{,}000}{\sqrt{4800}}$$

$$= 58.2 \text{ in. } (1480 \text{ mm})$$

This is more than that obtained for an individual bar under the recommended provisions, but less than the length resulting for the entire bundle being developed at one location.

**Example 5: Splices**

Let us evaluate the splice length required for the beam shown in Fig. 19a for two situations: (a) All 3 — #10 bars are spliced at the top center of the beam, (b) two of the 3 — #10 bars are spliced at the top center and the third is continuous.

(a) $A_{sr}$ = 2.48 in.$^2$ (1600 mm$^2$) at the beam centerline resulting from a moment of 280 ft-$k$ (380 m·kN). $A_{sp}$ = 3 (1.27) = 3.81 in.$^2$ (2460 mm$^2$).

$$\text{Clear spacing (Fig. 19b)}\ \frac{14 - 2\,(2) - 6\,(1.27)}{2}$$

$$= 1.19 \text{ in. } (30 \text{ mm})$$

The clear spacing is less than $d_b$ and therefore unacceptable. Normally the splice length should be based on the orientation shown in Fig. 19b which is critical. Since the clear spacing is too small, an alternative orientation is necessary which would have to be indicated clearly on the design drawings (Fig. 19c).

$$C_s = \frac{14 - 2\,(2.64) - 1.27}{2\,(2)}$$

$$= 1.86 \text{ in. } (47 \text{ mm})$$

Since there is no transverse reinforcement in the center region of the beam, $K_{tr}$ = 0 and $K$ = 1.86 in. (47 mm).

$$l_{db} = \frac{5500\,(1.27)}{0.8\,(1.86)\sqrt{4500}}$$

$$= 70 \text{ in. } (1780 \text{ mm})$$

The use of $A_{sr}$ = 2.48 in.$^2$ is not appropriate inasmuch as the moment increases away from the centerline of the beam. The moment at 3 ft (915 mm) off the centerline, near the end of the splice, is 295 ft-$k$ (400 m·kN).

$$A_{sr} \approx \frac{295}{282}\,(2.48) = 2.60 \text{ in.}^2 \ (1680 \text{ mm}^2)$$

$$\frac{A_{sr}}{A_{sp}} = \frac{2.60}{3.81} = 0.682$$

$$l_s = 1.3\,(0.682)\,(70) = 62 \text{ in. } (1570 \text{ mm})$$

Following the current provisions of ACI 318-77, the splice is considered a Class C splice since all bars are spliced and $A_{sp}/A_{sr} < 2$. Therefore, the splice length is

$$l_s = 1.7\, l_d = 1.7\,(1.4)\,\frac{0.04\,(1.27)\,60{,}000}{\sqrt{4500}}$$

$$= 108 \text{ in. } (2750 \text{ mm})$$

which is considerably more than the 62 in. (1575 mm) calculated under the recommended provisions.

(b) See Fig. 19d. Now $C_s$ = 3.1 in. (79 mm) and $C_c$ = 2.64 in. (67 mm).

Since the center bar is fully effective, the correct steel area from Part (a) is $A_{sr} = 2.60 - 1.27 = 1.33$ in.$^2$ (860 mm$^2$) for the two splices.

$$A_{sp} = 2\,(1.27) = 2.54 \text{ in.}^2\ (1640 \text{ mm}^2)$$

$$K = 2.64 \text{ in. } (67 \text{ mm})$$

$$l_{db} = \frac{5500\,(1.27)}{0.8\,(2.64)\sqrt{4500}}$$

$$= 49 \text{ in. } (1250 \text{ mm})$$

$$l_s = 1.3\left(\frac{1.33}{2.54}\right)49$$

$$= 34 \text{ in. } (860 \text{ mm})$$

Under the present provisions of ACI 318-77, the splice length $l_s$ is 108 in. (2750 mm) as calculated in Part (a) above since it is a Class C splice.

### Example 6: Splice in a tank wall

Evaluate the lap splice length required in a circular tank wall where alternate #6 bars are being lapped at a particular location. Grade 40 reinforcement is used ($f_y$ = 276 MPa) and $f_c'$ = 4000 psi (27.6 MPa). See Fig. 20.

$$\frac{40{,}000}{50{,}000} - 0.2 = 0.6$$

$$l_{db} = \frac{23\,(0.75)}{0.8} = 22 \text{ in. } (560 \text{ mm})$$

$$l_s = 0.6\,(22) = 13 \text{ in. } (330 \text{ mm})$$

According to Section 16.5.2 in the Commentary of ACI 318-77, a Class C splice is considered necessary; therefore

$$l_s = 1.7\,(0.0004)\,(0.75)(40{,}000)$$

$$= 20.4 \text{ in. } (518 \text{ mm})$$


### Acknowledgments

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### Notations

Refer to the notations and formulas in the preceding ACI Committee Report 408.1R-79.

### Additional notation

$l'$ = straight bar length between standard hook and critical section, in.

$l_e$ = equivalent embedment length of hook

$l_s$ = splice length, in.

$\xi$ = constant for standard hook, ACI 318-77, Chapter 12

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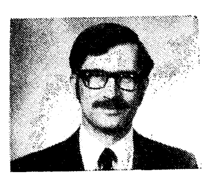

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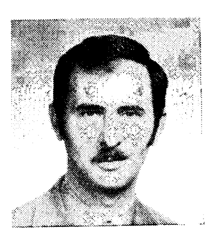

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