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Deflections of Prestressed Concrete Members

Reported by Subcommittee 5, ACI Committee 435
(Formerly Committee 335)

A. C. SCORDELIS
Subcommittee Chairman

DAN E. BRANSON

METE A. SOZEN

This report discusses the factors affecting the short-time and long-time deflection behavior of prestressed concrete members. Analytical methods are presented for calculating these deflections taking into account prestress, transverse loading, creep, shrinkage, and relaxation of steel stress.

Key words: ACI committee report; beam; creep, deflection; design; prestress; prestressed concrete; relaxation; shrinkage; slab; transverse load.

CONTENTS

CHAPTER 1 — INTRODUCTION

101 — Objectives

The objectives of this report are to discuss the factors affecting the short-time and long-time deflection behavior of prestressed concrete members and to recommend both approximate and more precise analytical methods for calculating these deflections.

In the design of prestressed concrete structures the deflections under short-time or long-time service loads may often be the governing criteria in the determination of the required member sizes and amounts of prestress. The variety of possible situations that can arise are too numerous to be covered by a single set of fixed rules for calculating deflections. However, it is felt that a thorough understanding of the basic factors contributing to these deflections, which are discussed in this report, will enable a competent designer to make a reasonable estimate of the deflections in most of the cases encountered in prestressed concrete design.

102 — Scope

Both short-time and long-time transverse deflections of beams, girders, and slabs involving prestressing with high strength steel are considered. Specific values of material constants given in this report, such as modulus of elasticity, creep coefficients, and shrinkage coefficients, refer to normal weight concrete. Bonded or unbonded prestressing steel is included, but the effect of unprestressed reinforcement is not considered.

103 — Notation

A = area of section
B, b = subscript denoting bottom fiber of section
C_t = creep coefficient defined as ratio of creep strain to initial strain at any time t
C_u = ratio of ultimate creep strain to initial strain
c = subscript denoting concrete, as f_c' and E_c
E = modulus of elasticity
e = eccentricity of prestress steel
H = relative humidity ($H = 70$ for 70 percent relative humidity)
h = over-all depth of cross section
I = moment of inertia (second moment of the area) of section
i = subscript denoting initial value (immediately following application of prestress or transverse load)
L = beam span
M = bending moment
P = prestress force
sh = subscript denoting shrinkage

ACI Committee 435 (335), Deflection of Concrete Building Structures, was formed in 1957 under the chairmanship of M. V. Pregnoff; Dan E. Branson became chairman in 1962. The committee's assigned task is to study available research on deflection of concrete flexural members in building structures under rapid and long-term loads to develop recommendations for the prediction of such deflection. Committee membership is: Professor Branson, chairman; J. R. Benjamin; A. H. Brownfield; C. D. Bullock; W. G. Corley; L. N. Larson; D. R. Peirce; M. V. Pregnoff; A. C. Scordelis; M. A. Sozen; and D. Watstein. This is the committee's first report.

T, t = subscript denoting top fiber of section

t = subscript denoting variable time measured from the time of application of prestress force

u = subscript denoting ultimate value

x = variable distance along beam

y = variable deflection along beam

y = distance from centroid of section to fiber under consideration

δ = unit creep strain defined as creep per unit stress

ε = unit strain, tensile strains are positive and compressive strains are negative

ϕ = curvature or angle change per unit length of beam $= d^2y/dx^2$

σ = unit stress

CHAPTER 2 — GENERAL FEATURES OF BEHAVIOR

201 — Introductory remarks

This chapter is concerned with the transverse deflections of prestressed concrete members caused by the application of prestressing and external forces. Especially in indeterminate structures, the axial changes in length of prestressed concrete members caused by the prestressing force or by external forces may also be critical. Although direct reference will be made only to transverse deflections, it is also possible in general to estimate the effects of axial displacements by means of the methods outlined in this report.

The definition of the deflection of a prestressed concrete beam can be ambiguous. In this report, the deflection will refer to the position of the beam before the prestressing operation unless specified otherwise. A simply supported beam deflects upwards under the action of the prestressing force and downward under the action of the transverse loads. The deflection may be defined with respect to two different reference lines: the position of the beam before the release of prestress or the position of the beam just before the application of load. The situation may become complicated if the load or prestress is applied in stages. Any definition may be justified in relation to the critical quantity being sought. For roof beams, the critical deflection would be that with respect to the original position of the beam in the forms. For bridges with cast-in-place riding surfaces, the critical deflection would be that occurring after the riding surface is placed.

Only uncracked prestressed concrete sections will be considered; cracked prestressed concrete sections can also be analyzed on the basis of the methods described here. Unless noted otherwise, the beams considered will be assumed to be cast in a single operation and have bonded reinforcement.

The deflections of prestressed concrete beams are considered under two engineering definitions: short-time and long-time deflections.

Short-time deflections are defined as those occurring instantaneously under the application of any internal or external force. The time element is assumed to be unimportant, no matter what the rate of loading, provided the load is applied within a matter of hours.

Long-time deflections refer to those existing at some time interval after the prestressing or loading operation.

202 — Short-time deflections

In general, the variables affecting the short-time deflections of a prestressed concrete beam are the magnitude and distribution of the load, the length of the span, the size and configuration of the cross section, and the quality of the concrete. More specifically, the effect of critical variables may be summarized by the magnitude of the strain or stress gradient or the curvature at a section and the variation of this quantity along the span.

The curvature at a particular section (see Fig. 1) is defined by

$$\phi_i = \frac{\varepsilon_{bi} - \varepsilon_{ti}}{h} = \frac{M}{E_c I} \qquad (1)$$

in which tensile strains are positive and compressive strains are negative.

In most cases the variations in the amount of the prestressing steel affect the short-time deflections due to transverse loads negligibly. As long as the beam remains uncracked, variations in the prestressing force do not affect the short-time changes in deflection at all, provided the concrete and the steel strains increase linearly with stress, as will be assumed throughout this chapter.

In effect, the short-time deflections of uncracked prestressed concrete elements can be calculated in accordance with the usual methods of calculating deflections applied to linearly elastic members. The modulus of elasticity to be assumed in these calculations is discussed in the following chapter. As indicated above, the calculations may be carried out on the basis of the gross concrete section.

203 — Long-time deflections

Even if the external loads remain constant, the deformations and displacements of a prestressed concrete beam will change with time as a result of creep and shrinkage of the concrete and relaxation of the prestressing reinforcement.

Shrinkage strain is defined as deformation of the concrete which occurs without stresses attributable to forces external to the concrete. Creep is defined as the time-dependent deformation occurring under stress over and above that which would have been caused by shrinkage.

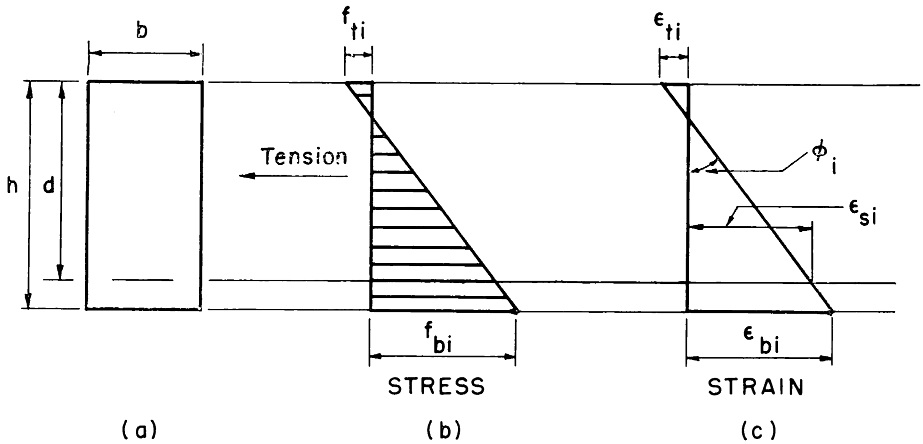

Fig. 1—Stress and strain distribution immediately after application of the prestressing force

Actually, these two types of strains are rather difficult to separate from each other and more elaborate definitions have to be stated if the quest of the analysis were these strains alone. Nevertheless, in considering the deflections of reinforced or prestressed concrete elements, the above simple definitions are satisfactory.

Relaxation loss of the prestressing steel is defined as the stress loss occurring at constant strain, a phenomenon related to creep.

In Chapter 3, all of these effects are discussed in detail.

To bring out the effects of the critical variables on time-dependent deflections, two simple cases will be considered in the following sections: a prestressed concrete beam without any transverse load and a prestressed concrete beam with all the transverse load applied at once.

203.1 — Prestressed concrete beam without transverse loads

Stress and strain distributions over the depth of a cross section of a rectangular bonded beam immediately after application of the prestressing force are shown in Fig. 1. It is assumed that there is a linear distribution of strain over the depth of the section and that there is a linear relationship between concrete stress and strain. Under ordinary conditions, both of these assumptions are reasonably correct. The stress at any level is given by the well-known relationship:

$$f = \pm \frac{P}{A} \pm \frac{Mc}{I} \qquad (2)$$

and the curvature can be expressed as

$$\phi_i = \frac{\varepsilon_{bi} - \varepsilon_{ti}}{h} = \frac{Pe}{E_c I} = \frac{M}{E_c I} \qquad (3)$$

where P is the prestressing force.

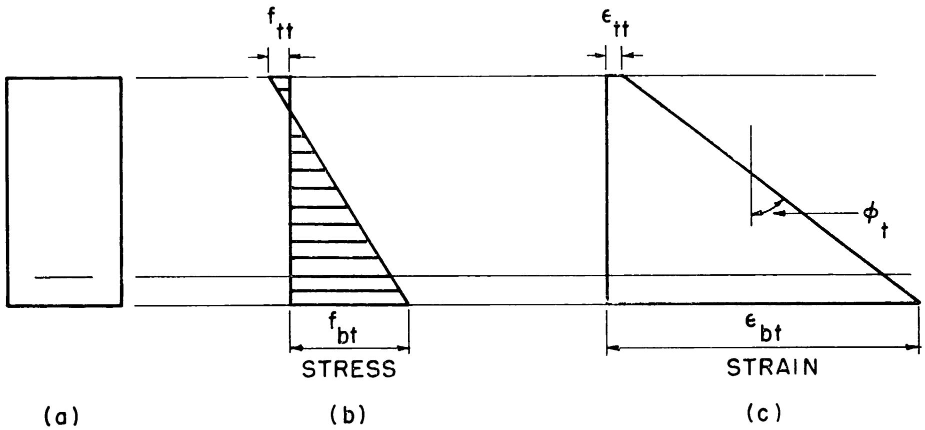

Fig. 2—Stress and strain distribution at a time t after initial application of prestressing force

The stress and strain distributions in Fig. 2 depict the conditions existing after a given time. The normal stresses on the section decrease as a result of a reduction in the prestressing force while there is a general shift to the right in the strain distribution accompanied by an increase in the strain gradient or angle change ϕ.

These changes are caused by an interaction between creep and shrinkage of the concrete and relaxation of the reinforcement. All of these effects occur continuously with time and affect each other continuously. However, it is preferable to treat these individually and as step-functions to simplify the discussion.

Consider first the effect of shrinkage strains. It is assumed that each element of concrete in the cross section shrinks equally. Thus, the shrinkage strain distribution after a time t is given in Fig. 3b. This distribution of shrinkage strain causes a reduction in the reinforcement strain which corresponds to a reduction in the prestress. The loss in prestress causes a change in the stress distribution over the depth of the section as indicated in Fig. 3c and the corresponding change in the strain distribution, Fig. 3d. Thus, the change in curvature is

$$\Delta\phi = \frac{\Delta\varepsilon_b - \Delta\varepsilon_t}{h} \qquad (4)$$

The effect of the relaxation losses in the steel is quite similar to that of shrinkage. At a time t there is a certain loss in the prestress force which creates a reduction in the curvature as explained above.

The effects of the creep of the concrete are not as simple, since the reduction in stress causes changes in the rate of creep strain.

It is assumed that the amount of creep strain at a given time is proportional to the stress. Thus, the change in strain caused by creep is directly

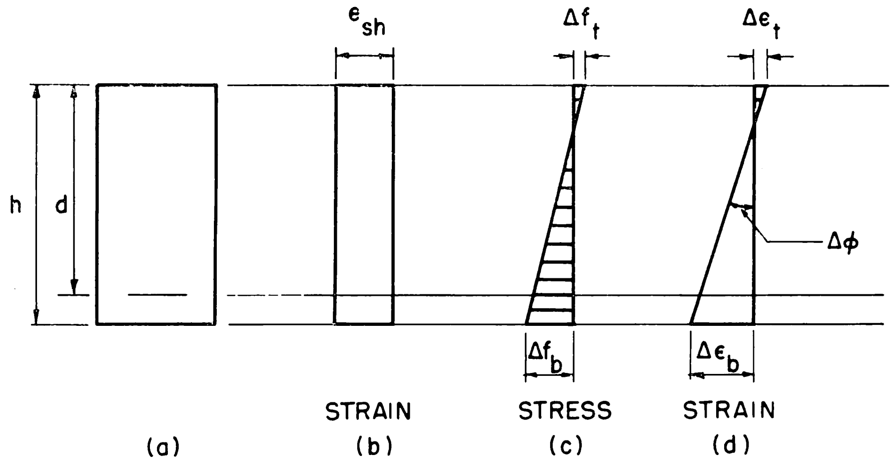

Fig. 3—Stress and strain distributions due to shrinkage

proportional to the instantaneous strain distribution (Fig. 1c), which is related directly to the stress distribution. This change in the strain distribution involves a contraction at the level of the steel and, therefore, a reduction in prestress. The reduction in prestress caused by creep, shrinkage, and relaxation decreases the normal stress. This decrease in normal stress reduces the rate of creep.

Thus, after a finite interval of time the increase in curvature caused by creep is a multiple of the instantaneous curvature. However, this multiple is less than the ratio of creep to instantaneous strain obtained for the same time interval from a comparable concrete subjected to constant stress.

The effects of shrinkage, creep, and relaxation add up to the stress and strain distributions shown in Fig. 2b and 2c. Although the stress in the top fiber remains in tension, the total strain may be a shortening if the shrinkage strain is large.

Creep strains affect the curvature almost directly while the shrinkage and relaxation affect it indirectly through losses in prestress. The beams subjected to prestress alone may be considered to be subjected to two different effects: the increase in curvature caused by creep and the decrease in curvature caused by the prestress loss due to relaxation, shrinkage, and creep. In general, these add up to an increase in the curvature.

A qualitative curvature versus time curve is shown in Fig. 4. The magnitude of the time-dependent deflection depends on the particular characteristics of the beam involved. The deflection of the beam depends on the distribution of the curvature which is influenced primarily by the profile of the steel.

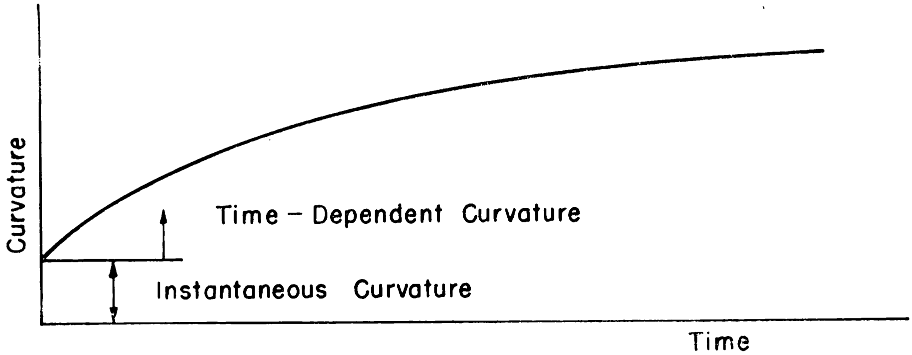

Fig. 4—Plot of curvature versus time for a beam subjected to prestressing force only

As in the case of short-time deflections, the magnitude of the deflection may be estimated by the magnitude of the stress gradient over the depth of the section after release of prestress. If the stress gradient is very small, then shrinkage and relaxation are bound to dominate, in which case the beam may deflect downward. However, under usual circumstances the stress gradient is large and creep dominates the deflection thus causing the beam to move upward in a simply supported case.

203.2 — Prestressed concrete beam subjected to transverse loading

If the beam considered in the preceding paragraphs is subjected to a transverse load, the stress distribution across the section at a given point along the span may be as indicated in Fig. 5d. Provided neither the concrete nor the steel is strained into the inelastic range, the stress distribution caused by the prestressing force (Fig. 5b) can be superposed on the stress distribution caused by the transverse load on the uncracked transformed section (Fig. 5c) to obtain the stress distribution shown in Fig. 5d.

Naturally, the magnitude and gradient of the stress distribution varies depending on the combinations of the stress distributions shown in Fig. 5b and 5c. For example, in a simply supported uniformly loaded beam with straight cables, the stress distribution over the end reaction may be assumed to be essentially that caused by the prestress. The stress distribution at midspan is the one influenced most by the transverse loads. While the curvature near the reaction may be increasing with time so that the curvature is concave downward, the curvature at midspan may be changing so that the additional curvature is concave upward.

The phenomena occurring at a section under the combined influence of the prestressing force and the transverse load may be described in a simpler manner if the effects of the prestress and the transverse load

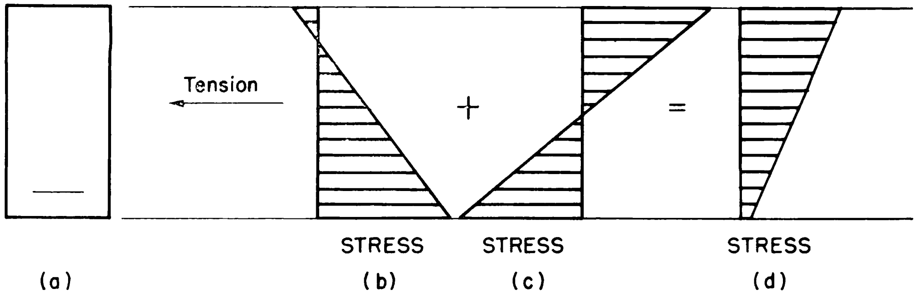

Fig. 5—Stress distributions due to prestressing force and transverse loading

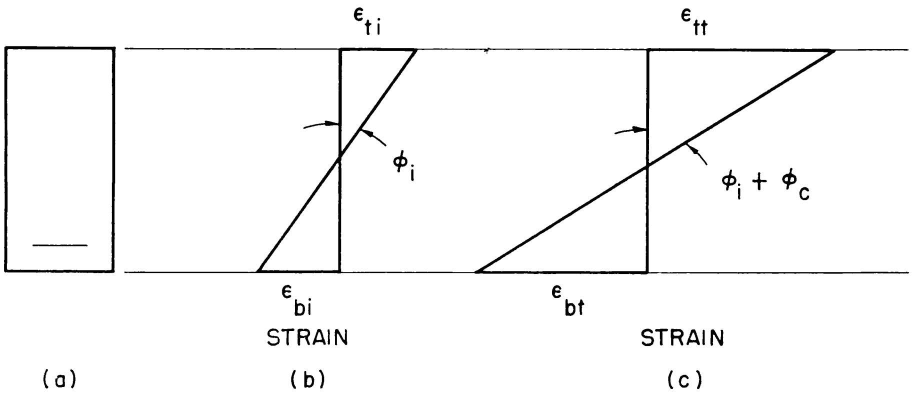

Fig. 6—Strain distribution due to transverse loading only

are treated separately. The effect of the prestressing force was discussed in the preceding section along with the effects of shrinkage and relaxation. The effect of the stresses caused by the transverse load are discussed below.

The strain distribution shown in Fig. 6b corresponds to the stress distribution in Fig. 5c. It depicts the strains that would occur in an uncracked section under the influence of only the transverse load. The short-time curvature is

$$\phi_i = \frac{\varepsilon_{bi} - \varepsilon_{ti}}{h} \qquad (5)$$

If it is assumed that the creep characteristics for this imaginary beam are equal in tension and compression, it follows that after a given time the strain distribution will be as shown in Fig. 6c. Depending on the amount of steel and creep strain at the level of the steel, the neutral axis will shift towards the steel and the stresses in the concrete will be reduced. However, this is a minor effect and may be neglected. Consequently, the relative increase in curvature after a given time will be

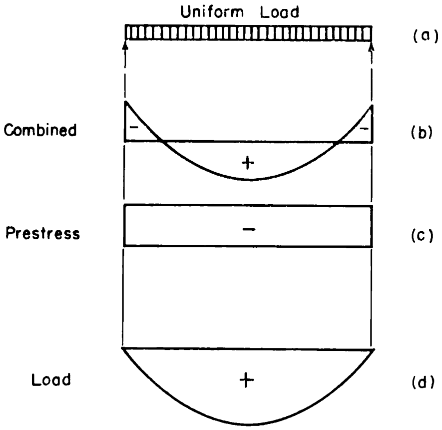

Fig. 7—Variation in curvature with span

the same as the ratio of the creep to instantaneous strain in a comparable specimen of concrete under constant stress.

The changes in the curvature or in the deflection of the beam caused by the prestress and the transverse load may be determined by superposition. For example, in the uniformly loaded beam considered, the curvature distribution on application of the load is as shown in Fig. 7b. Assuming that the prestress force is developed so that the corresponding strain distribution is constant throughout the span, the curvature distribution shown in Fig. 7b can be divided in two parts: (1) a rectangular distribution caused by the straight tendons (Fig. 7c) and (2) a parabolic distribution (Fig. 7d) caused by the uniform transverse load. Both of these curvature distributions will change with time. The deflections corresponding to these two imaginary systems are shown in Fig. 8. Curve A shows the variation with time of the deflection caused by the prestress while Curve B indicates the same variation for the load. It is assumed that the prestress is released simultaneously with the application of the load.

To get the net deflection, the imaginary deflections caused by the prestress and transverse load can be added as indicated by the broken

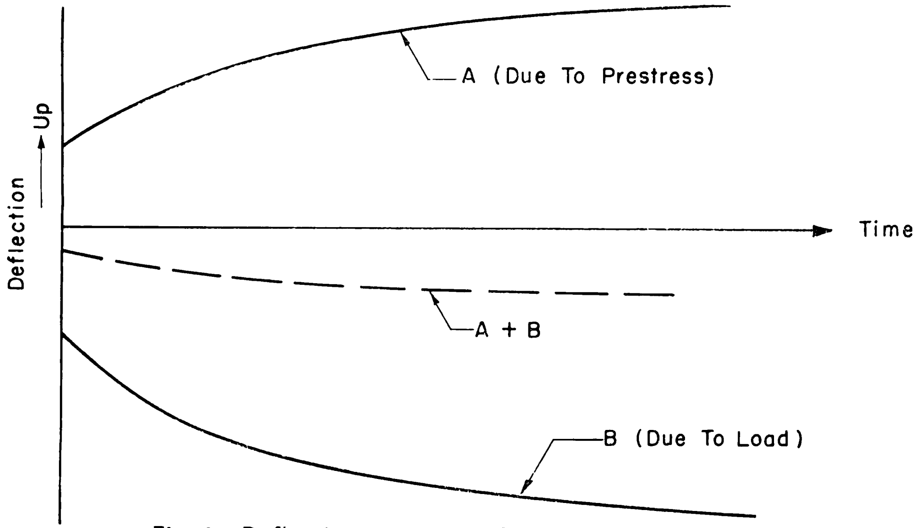

Fig. 8—Deflection versus time due to prestress and load

curve. It is seen that how much the beam deflects and whether it deflects upward or downward depends on the relative effect of the prestress and of the transverse loads. Ideally, a beam can be designed to have no midspan deflection at all.

The picture shown in Fig. 8 does not change drastically if the load is applied at a different time from the time of prestressing or if the load is applied in several increments. The total deflection can be found by superposing at the appropriate time the deflections caused by the prestress and the transverse load.

An important feature of the deflection of prestressed concrete beams is brought out by these considerations: if the short-time deflections caused by the prestress and the permanent load are comparable, the time-dependent deflection, with respect to the position of the beam before prestressing, is bound to be very small.

204 — Comparison of the deflection characteristics of prestressed and reinforced concrete beams

It is difficult to make a direct and fair comparison of the deflections of reinforced and prestressed concrete beams. It would be unfair to compare the relative deflections of two identical sections, one prestressed and the other nonprestressed, under the same load. The sections would be quite different if they were originally designed as reinforced or prestressed concrete sections for the same load. Nevertheless, some discussion of this problem is necessary because of the many misunderstandings that exist about the relative deflections of prestressed and reinforced concrete sections.

The mere act of prestressing does not reduce the deflections of a reinforced concrete element under live load. In fact, there would be no difference between the deflections of two sections, one prestressed and the other not prestressed, if both sections were uncracked. However, the act of prestressing does reduce the deflections at working loads because it maintains an uncracked section under large working loads.

Consider a particular example: a simply supported beam subjected to a uniform load. If the prestressed and reinforced concrete sections are proportioned to develop the maximum permissible stresses, the following comparison may be made.

Typical permissible stresses for prestressed concrete are 0.45 f_c' in compression and $6\sqrt{f_c'}$ in tension. Therefore, in an extreme case, the curvature over the reaction could be as large as

$$\phi_1 = \frac{0.45 f_c' + 6\sqrt{f_c'}}{E_c h} \text{ in.}^{-1} \text{ (concave down)} \quad \ldots\ldots\ldots\ldots \quad (6)$$

If $E_c = 1000 f_c'$ and if it is assumed for the sake of numerical simplicity that the tensile stress is $0.05\ f_c'$:

$$\phi_1 = \frac{0.5}{h} \times 10^{-3} \text{ in.}^{-1} \text{ (concave down)}$$

The curvature at midspan due to transverse load alone could be as large as

$$\phi_2 = \frac{6\sqrt{f_c'} + 0.45 f_c'}{E_c h/2} \text{ in.}^{-1} \text{ (concave up)} \qquad (7)$$

In accordance with the preceding considerations:

$$\phi_2 = \frac{1}{h} \times 10^{-3} \text{ in.}^{-1} \text{ (concave up)}$$

Assuming that ϕ_1 (due to prestress) is constant throughout the length of the beam and ϕ_2 (due to the transverse load) varies parabolically along the span, then the downward deflection of the beam at midspan:

$$\Delta = L^2 \left(\frac{5}{48} \phi_2 - \frac{1}{8} \phi_1 \right) \text{ in.} \qquad (8)$$

$$\frac{\Delta}{L} = \left(\frac{1}{24} \right) \left(\frac{L}{h} \right) 10^{-3} \qquad (9)$$

For a reinforced concrete beam with a "balanced" section according to ACI 318-56 working stress design:

$$\phi_2 = \frac{0.45 f_c'}{0.4d\ E_c} \text{ in.}^{-1} \text{ (concave up)} \qquad (10)$$

If it is assumed that $d = 0.8h$, the deflection at midspan:

$$\frac{\Delta}{L} = \left(\frac{1}{6.9} \right) \left(\frac{L}{h} \right) 10^{-3} \qquad (11)$$

Thus, the short-time deflection of the reinforced concrete beam may be about three times the deflection of the prestressed concrete beam, provided the two are designed for the maximum permissible stresses. Actually, the estimate of the deflection of the prestressed concrete beam may be quite accurate although the estimate of the deflection of the reinforced concrete beam may be on the conservative side since the distribution of curvature along the span would not be parabolic as assumed in calculating the deflection from the curvature. Furthermore, if a continuous beam is considered, the advantage given to the prestressed concrete beam, because of the beneficial distribution of curvature along the span, would exist also for the reinforced concrete beam. Consequently, the ratio between the deflections would not be as large.

It is interesting to note also the manner in which creep and shrinkage of the concrete affect the time-deflections of reinforced concrete beams.

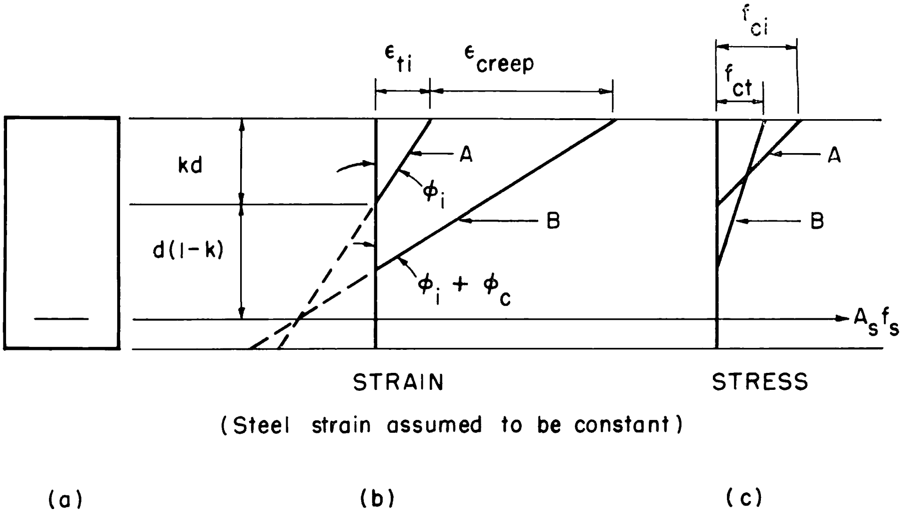

Fig. 9—Strain distribution in a reinforced concrete beam

Usually, the steel stresses in reinforced concrete beams are such that we may ignore the relaxation losses of the reinforcement.

Shrinkage generally causes a downward deflection in both ordinary reinforced and prestressed concrete simple span beams. The magnitude of the shrinkage curvature depends on the amount of nonsymmetry of reinforcement and on the relative areas of concrete and steel in the reinforced concrete beam. This effect is more or less similar to what occurs in prestressed concrete. If the centroid of the prestressing reinforcement is at the centroid of the section, shrinkage of the concrete would not cause any increases in curvature with time.

There is a distinct difference, however, between the way in which creep affects the deflections of reinforced and prestressed concrete beams. The strain distribution in a reinforced concrete beam immediately on application of load is shown by Line A in Fig. 9.

Considering creep only, the increase in strain will be as shown by Line B. Strain in the extreme fiber in compression increases considerably, while the strain at the level of the steel changes very little. As this occurs, the compressive stresses in the concrete are reduced since the neutral axis moves toward the reinforcement, and the steel stresses increase since the internal lever arm shortens.

It is seen from Fig. 9b that the relative increase in curvature caused by creep is less than the relative increase in strain such that

$$\frac{\phi_c}{\phi_i} = k \frac{\varepsilon_c}{\varepsilon_i} \qquad (12)$$

where k is less than 1. Thus, the relative increase in deflection is less than the increase in creep. However, no matter what the conditions, the reinforced concrete beam does undergo creep deflection. On the other hand, the prestressed concrete beam may be proportioned so that under sustained loads it will have practically no deflection due to sustained loads at a given point in the span.

CHAPTER 3 — MATERIALS

301 — Reinforcing steel

301.1 — Introductory remarks

It follows from the discussion of the general features of the short-time and long-time deflections of prestressed concrete members that two properties of the reinforcing steel are of importance in determining the deflections of the prestressed concrete beam: the modulus of elasticity and the relaxation losses of the reinforcement.

In discussing short-time deflections, it was mentioned that the cross-sectional area of the reinforcing steel in a beam is usually small enough so that the deflections may be based on the gross concrete area. In that case, the significance of a knowledge of the modulus of elasticity is not important. However, in considering time-dependent deflections resulting from shrinkage, and those resulting from creep at the level of the prestressing steel, it is important to have a fairly good estimate of the modulus of elasticity of the reinforcing steel.

The relaxation loss is of direct importance in estimating the change in deflection caused by this loss.

301.2 — Modulus of elasticity

In calculating deflections under working conditions, it is sufficient to refer to the modulus of elasticity of the prestressing reinforcement rather than to the whole stress-strain curve since this reinforcement is seldom stressed into the inelastic range.

The definition and determination of the modulus of elasticity for single wire or bar reinforcement is a simple matter. In most calculations, the assumption of this value as 30×10^6 psi, is of sufficient accuracy considering the unknowns relating to properties of the concrete which are more critical in the calculation of deflections.

In the case of strand reinforcement, both the definition and the determination of this parameter become problematic. During the prestressing operation, or during release of prestress if the strands are unbonded, the pertinent parameter is the change in deformation over a given length compared with the change in force. The apparent modulus of elasticity in this case may depend on the length of the wire and the type of the grips at the ends. Under these conditions, because of the tendency of the strand to untwist, the apparent modulus of elas-

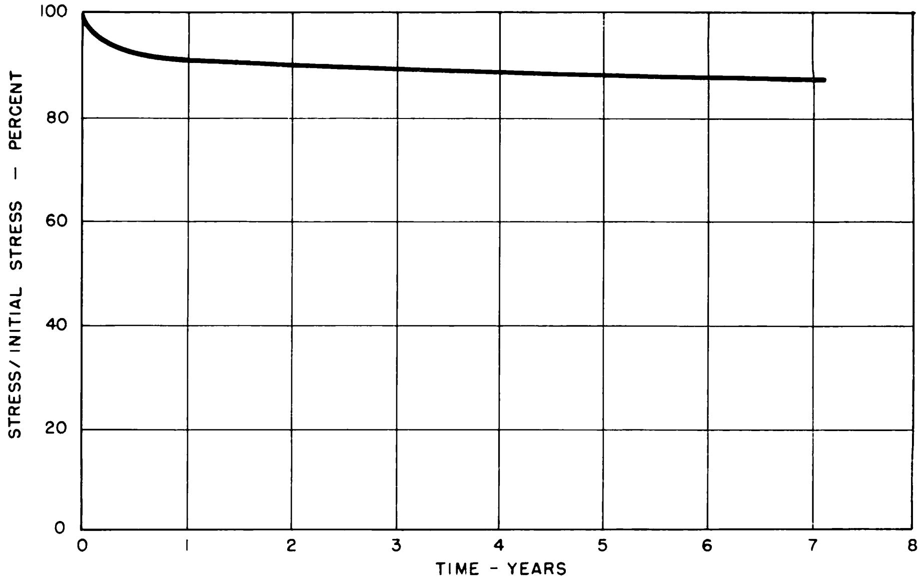

Fig. 10—Stress variation with time in a prestressing wire due to relaxation

ticity is less than that assumed for steel. This is usually on the order of 27×10^6 psi.

If the strand is embedded in concrete, the freedom to twist is lessened considerably. In that case, it seems unnecessary to differentiate the modulus of elasticity of the strand from that of single-wire reinforcement.

In calculations involving time-dependent deflections, it is generally unwarranted to differentiate the modulus of elasticity of strand from that of ordinary reinforcement.

301.3 — Relaxation characteristics of prestressing reinforcement

The relaxation loss is defined as that loss in stress occuring at constant strain.

In an ordinary prestressed concrete member, the strain of the steel does change with time as a result of shrinkage and creep of the concrete and also as a result of loss of prestress due to its own relaxation. However, this change in strain is a fraction of its initial strain. Thus although the type of loss occurring in a prestressed concrete member is not pure relaxation as defined, it is closer to relaxation than it is to the basic definition of creep, which is change in strain under constant stress.

In the early days of prestressed concrete construction, relaxation losses of the steel were thought to be short-lived. It was the general opinion that the wire would stop relaxing after a matter of weeks if not hours. Later information showed that the relaxation losses do not

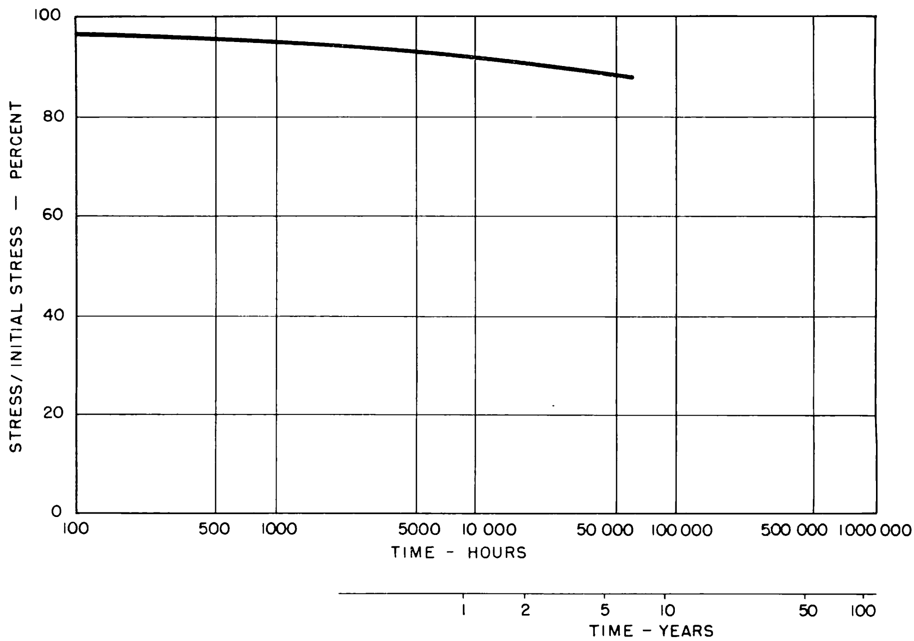

Fig. 11—Stress variation with time in a prestressing wire due to relaxation

stop, although, as in the case of creep and shrinkage of the concrete, the rate decreases drastically within the first year.

In Fig. 10, as an example, the stress variation in a wire prestressed to 86 percent of its 0.1 percent offset stress (or 79 percent of its 1 percent stress) is shown as a fraction of the initial stress. There is a relatively large loss within the first 6 months. After 2 years the loss rate becomes very small. In Fig. 11, the same data are plotted against the logarithm of time. This curve indicates that the relaxation loss may never stop. Nevertheless, the half-life for this wire would be on the order of 100,000 years. It can also be reasoned that given enough time the stress in the wire would approach zero. Although this condition is conceivable, it does not apply strictly to a prestressed member since there is a statical check on the lower bound of stress that can be reached.

To make generalizations about the amount of relaxation loss to be expected is even more difficult than making generalizations about the amount of creep strain to be expected. Relaxation losses may be critically affected by the manner in which a particular wire is manufactured. Thus, they may change not only from type to type of steel but also from manufacturer to manufacturer. Factors such as reduction in diameter of the wire and the heat treatment of the wire may be significant in fixing the rate and amount of relaxation loss that may be expected.

Nevertheless, sufficient data over a sufficient number of years have been obtained to make it possible for a statement to be made of the amount of relaxation loss to be expected in the ordinary types of prestressing wire or strand being used currently. A recent study[21] of 427 test results with test durations up to 9 years has shown that the following simple formula may be used for estimating the relaxation loss in prestressing reinforcement for time periods up to about 50 years. The expression which should apply to high tensile strength single wire and strand reinforcement is

$$\frac{f_s}{f_{si}} = 1 - \frac{\log t}{10}\left(\frac{f_{si}}{f_{sy}} - 0.55\right) \qquad (13)$$

in which f_s is the steel stress t hours after initially being stressed to f_{si}, and f_{sy} is the yield point stress. Log t is to the base 10.

The form of this expression, which should be used only as a guide, indicates that the critical parameter is the relation of the initial stress to the yield point stress which has been defined as the 0.1 percent offset stress.

302 — Concrete

302.1 — Modulus of elasticity

The modulus of elasticity of concrete has always been an elusive property because of its significant variation primarily with concrete quality, concrete age, stress level, and rate or duration of applied stress. These effects are all associated, at least in part, with the creep or plastic flow of the material, even for very rapid rates of loading. Evans[1] has stated, interestingly, that under instantaneous loading much more creep occurs in the first 0.01 sec than in the period from 0.01 sec to 1.0 min. Of primary interest with regard to the design aspects of the concrete modulus of elasticity are the relations between E_c and f_c', E_c with concrete age (under instantaneous loading), and E_c with concrete creep (creep and material stiffness are inherently related in a visco-elastic material).

The equation given by ACI-ASCE Committee 423 (323)[2] which is a slight modification of that originally suggested by Lyse:

$$E_c = 1{,}800{,}000 + 500\, f_c' \qquad (14)$$

or by Jensen:[3]

$$E_c = \frac{E_s}{5 + \dfrac{10{,}000}{f_c'}} \qquad (15)$$

or by ACI Committee 318:[22]

$$E_c = w^{1.5}\, 33 \sqrt{f_c'} \qquad (16)$$

All equations provide an adequate relation between E_c and f_c' for normal sand and stone concrete. It is well known that E_c increases significantly with time. The data of Davis and Troxell[4] show an increase of f_c' and E_c up to 1.3 times the 28-day value at the end of 3 years. The modulus of elasticity employed in the analysis of concrete structures must be used with caution and properly reduced for loads other than those of very short duration. The resulting reduced modulus is given by the equation:

$$E_{ct} = \frac{E_c}{1 + C_t} \qquad (17)$$

where C_t is the creep coefficient, which is a function mainly of relative humidity, concrete quality, duration of applied loading, and age of the concrete when loaded.

302.2 — Creep

Under the action of sustained stress, concrete exhibits a prolonged yielding or time-dependent creep strain. The true nature of creep mechanism is not yet fully understood. Twenty-year shrinkage and creep data of Troxell, Raphael and Davis[5] illustrate the fact that concrete shrinkage and creep continue for a very long time. Average curves show slightly less than 80 percent of their 20-year value at the end of 1 year for both shrinkage and creep. Approximate ultimate values for the creep coefficient for normal weight concrete under average design conditions are shown in Table 1, where, in each case, the larger of the values corresponds to an earlier loading age.

In considering the effects of creep in the deflection of concrete members, the use of a unit creep strain δ_t (creep per unit stress) or creep coefficient C_t (ratio of creep strain to initial strain) amounts to the same thing, since the concrete modulus E_c must be brought in in either case and

$$C_t = \delta_t E_c \qquad (18)$$

This is seen from the relation:

$$\text{Creep strain} = (\sigma_{constant})\ \delta_t = (\varepsilon_{initial})\ C_t \qquad (19)$$

where

$$E_c = \frac{\sigma_{constant}}{\varepsilon_{initial}}$$

TABLE 1 — C_u RATIO OF ULTIMATE CREEP STRAIN TO INITIAL STRAIN

Concrete strength	Average relative humidity		
	100 percent	70 percent	50 percent
Ordinary	1-2	1.5-3	2-4
high	0.7-1.5	1-2.5	1.5-3.5

Which to use is a matter of convenience depending on whether it is desired to apply it to stress or strain.

302.3 — Shrinkage

Among the more important factors that influence drying shrinkage are the water-cement ratio of the paste, the amount of paste in the concrete, the mix proportions, the curing conditions, the length of the drying period, the humidity of the surrounding air, the maximum size and composition of the aggregate, and the size and shape of the concrete mass.

The most important single factor affecting shrinkage is the amount of water placed in the mix per unit volume of concrete. The shrinkage of concrete is mainly due to the evaporation of the mixing water. Because of this, the humidity of the surrounding air, for a given concrete mix, affects to a large extent the magnitude of the resulting shrinkage.

Schorer's[6] formula is probably adequate for calculating shrinkage strains for most design purposes:

$$\varepsilon_{sh} = 12.5 \times 10^{-6}\,(90 - H) \qquad (20)$$

where H is relative humidity ($H = 70$ for 70 percent relative humidity). This formula gives an ultimate or design total shrinkage strain as a function of relative humidity, but other variables account for rather wide variations under certain conditions. Eq. (20) yields $\varepsilon_{sh} = 500 \times 10^{-6}$ for $H = 50$. Staley and Peabody[7] reported shrinkage strains as high as 870×10^{-6} for specimens cured at 50 percent relative humidity, but this variation from Schorer's value is extreme. Most shrinkage data agree with the above formula within 25 percent.

CHAPTER 4 — CALCULATION OF DEFLECTIONS

401 — Introduction

This chapter presents two methods for calculating long-time deflections of prestressed concrete members. The first is a general method which is applicable in almost all cases and involves a step by step procedure of calculation. The second method is shorter and approximate, but should yield sufficiently accurate answers for design purposes in most cases.

402 — Loading conditions

To calculate the deflections of a member at a particular time, its load history, including prestressing, up to that time should be known. The designer must make a reasonable prediction of the expected load history in advance to be able to predict the member deflections at various important stages with reasonable accuracy.

Listed below are the usual loadings which should be investigated. In each case both short-time and long-time deflections should be checked. Special conditions may require additional stages to be investigated.

(a) Prestress plus dead load.

(b) Prestress plus lightest expected total service load.

(c) Prestress plus heaviest expected total service load.

403 — Calculation of long-time deflections

The phenomenon of time-dependent deflecions in prestressed concrete beams was discussed in Section 203 of this report where it was shown that a prestressed concrete beam develops deformations under the influence of two usually opposing effects: the prestress and the transverse load. Thus, the net curvature at a section at a given time t, ϕ_t, is

$$\phi_t = \phi_{mt} + \phi_{pt} \qquad (21)$$

where ϕ_{mt} is the curvature caused by the transverse load and ϕ_{pt} is that caused by prestress.

The variation of ϕ_{mt} with time was discussed qualitatively in Section 203.2 and is illustrated in Fig. 6. On application of the transverse load:

$$\phi_i = \frac{\varepsilon_{bi} - \varepsilon_{ti}}{h} = \frac{M_x}{EI} \qquad (22)$$

where M_x is the bending moment acting on the particular section considered. As a result of creep, the strains and therefore the curvature increase. As this occurs, the neutral axis shifts towards the reinforcement since the reinforcement becomes relatively stiffer. Consequently, the compressive stress distribution on the concrete changes. However, in practical cases, this change in compressive stress is small and it can be assumed without appreciable error that the neutral axis remains stationary and that the concrete creeps under constant stress. Accordingly, the creep strain at any time t can be obtained by multiplying the instantaneous strain by the creep coefficient C_t for that particular time:

$$\phi_{mt} = (1 + C_t)\,\phi_i = (1 + C_t)\,\frac{M_x}{EI} \qquad (23)$$

The form of Eq. (23) indicates that the determination of ϕ_{mt} represents an ideal application of the "reduced modulus" method, the quantity $E/(1 + C_t)$ being the reduced modulus. However, it must be noted that Eq. (23) is a simplification. It refers to a fictitious condition (see Section 203.2). It ignores changes in actual stress caused by prestress as well as the shift of the neutral axis. It assumes tacitly that E and C_t do not vary over the cross section or for different loading conditions; the variations in these quantities may be critical if the permanent load is applied in several increments at different times.

The determination of ϕ_{pt}, the curvature caused by the prestress, is not as simple. As discussed in Section 203.1, it involves the effects of creep, shrinkage, and relaxation. The quantity ϕ_{pt} can be considered in three parts: (1) Instantaneous curvature occurring upon application of the prestress, (2) change in curvature corresponding to the loss in prestress as a result of creep, shrinkage, and relaxation, and (3) change in curvature resulting from creep under prestress. Since the loss in prestress is usually appreciable, Part 3 involves creep under varying stress. Furthermore, there is an interrelationship between Parts 2 and 3: Creep affects the loss in prestress while the prestress affects the amount of creep. Shrinkage and relaxation are also affected by the varying stress. However, this can be ignored in view of the fact that the possible errors in the basic shrinkage and relaxation coefficients are larger than any effect the varying stress may have on them.

Two different approaches can be used in calculating creep strain in concrete subjected to varying stress: the rate of creep and the superposition methods.

The rate-of-creep method, illustrated in Fig. 12, is straightforward. Consider an extreme case in which a concrete specimen is subjected to a compressive stress σ for a time interval t_1. At the end of this interval, the stress is removed completely.

According to the rate-of-creep method, the creep strain at time t_1 is $\sigma\delta_{t1}$, the product of the sustained stress and the unit creep strain for the

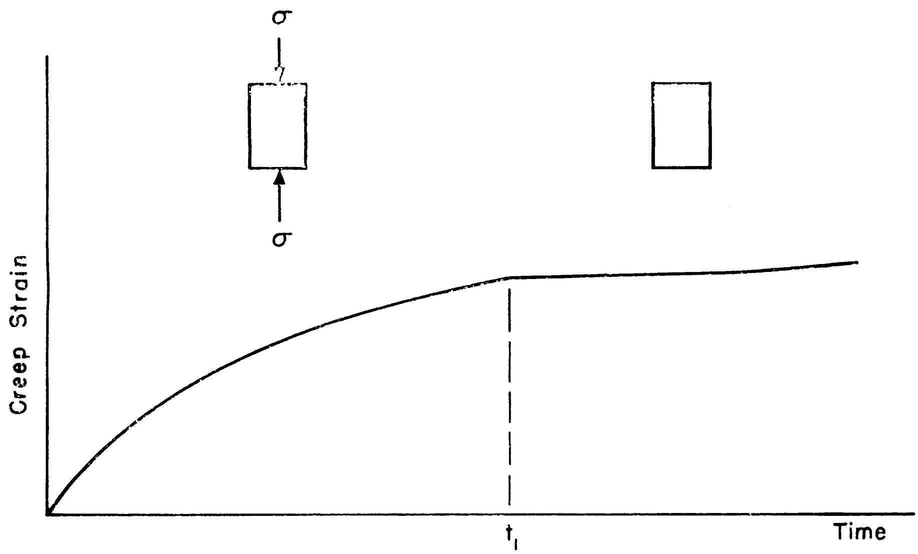

Fig. 12—Creep strains by the rate of creep method

time considered. Once the stress σ is removed, there is no further change in creep strain and at a time, say $2t_1$, the creep strain is still $\sigma\delta_{t1}$.

The superposition method,[13] illustrated in Fig. 13, predicts the same creep strain at time t_1, $\sigma\delta_{t1}$. However, rather than assuming directly that the compressive stress is removed at time t_1, it is assumed that the specimen is subjected to an additional stress of σ in tension and creeps under two opposing fictitious stresses. For example, if it is assumed that the creep characteristics of the concrete are the same in tension and compression and are independent of the concrete age when loaded, the compressive creep strain at time $2t_1$ is $\sigma\delta_{t2}$ while the tensile creep strain is $\sigma\delta_{t1}$ since the tensile stress is a new stress at time t_1. The total compressive creep strain at time $2t_1$ is thus $\sigma(\delta_{t2} - \delta_{t1})$ and represents a reduction with respect to the creep strain at time t_1, since $(\delta_{t2} - \delta_{t1}) < \delta_{t1}$.

In the particular case considered, the rate-of-creep and superposition methods give significantly different results. However, their results are comparable if the stress variation is not as drastic as the one assumed. Although the superposition method has been claimed to predict the time-dependent deflections caused by prestress better than the rate-of-creep method,[9] its use is laborious and worthwhile only if the basic creep, shrinkage, and relaxation data are known reliably and accurately. The method proposed here for the calculation of ϕ_{pt} is based on the rate-of-creep method. However, the calculation of ϕ_t, Eq. (21) does involve the superposition method since the two curvatures ϕ_{mt} and ϕ_{pt} are determined separately using unit creep curves with time zero defined as the time corresponding to the application of the particular effect considered.

In determining ϕ_{pt}, the changes in curvature caused by the loss in prestress and creep can be evaluated by a summation procedure which recognizes the changes in compressive stress. Thus:

$$\phi_{pt} = \underbrace{-\frac{P_i e_x}{EI}}_{\text{Part 1}} + \underbrace{\sum_0^t (P_{n-1} - P_n)\frac{e_x}{EI}}_{\text{Part 2}} - \underbrace{\sum_0^t (C_n - C_{n-1})\, P_{n-1}\frac{e_x}{EI}}_{\text{Part 3}} \tag{24}$$

In the above equation, the curvature caused by the initial prestress has been taken as negative. The subscripts $(n-1)$ and (n) define the beginning and end of a particular time increment. Parts 1, 2, and 3 refer to the parts into which ϕ_{pt} was divided in the preceding discussion. The prestress force at any time, P_n is determined by subtracting the losses caused by shrinkage, relaxation, and creep from the initial prestress force P_i. The losses caused by shrinkage and relaxation are obtained directly from the shrinkage and relaxation coefficients assumed. How-

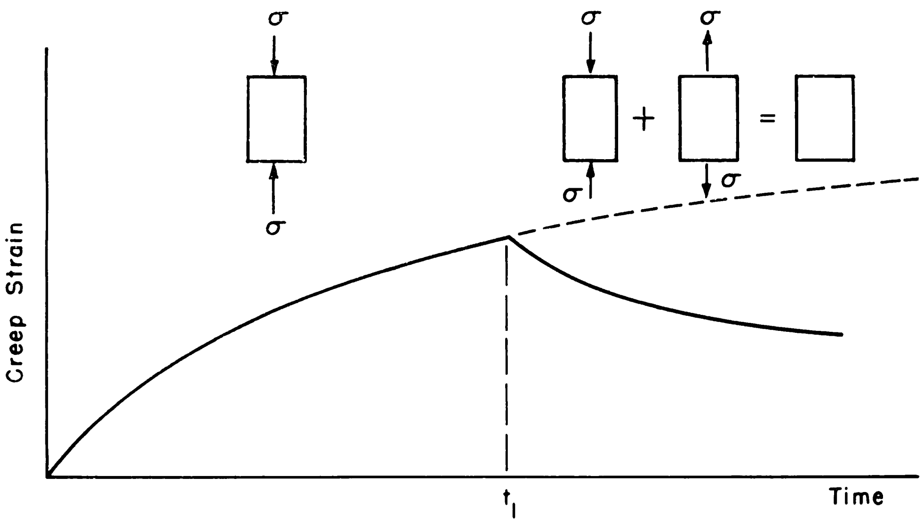

Fig. 13—Creep strains by the superposition method

ever, the effect of the creep must be determined from the summation:

$$\sum_{0}^{t} (C_n - C_{n-1})\, P_{n-1}$$

Parts 2 and 3 of Eq. (24) may be determined using the following sequence:

1. Obtain the gross increase in creep strain by multiplying the stress in each extreme fiber at the beginning of the interval considered by the increment of unit creep strain for that interval.

2. Determine the creep strain at the level of the center of gravity of the reinforcement.

3. Sum the creep strain determined in Step 2 and the shrinkage strain increment for the time interval considered to obtain the total change in strain at the level of reinforcement.

4. Determine the total loss in reinforcement stress for the interval considered by adding the loss caused by the shrinkage and creep (the strain in Step 3 times the modulus of elasticity of the reinforcement) to the relaxation loss for the corresponding time interval considered.

5. Find the change in stress in the extreme fibers corresponding to the loss in steel stress. Using the short-time modulus of elasticity E_c, of the concrete, determine the corresponding change in strain.

6. Determine the net change in creep strain in the extreme fibers by finding the algebraic difference between the gross change in strain (Step 1) and the change in strain caused by the change in stress in the reinforcement (Step 5).

7. Obtain the increase in curvature from the net strains determined in Step 6.

8. Find the stress in the extreme fibers at the end of the time interval by finding the algebraic difference between the initial stress and the change in stress determined in Step 5.

The procedure described above or any other summation technique to evaluate Eq. (24) can be programmed for a digital computer. Once such a program is available, it is feasible to use Eq. (21), (23), and (24) in preliminary design and even in cases where the basic information on creep, shrinkage, and relaxation is not well established. If the data on the time-dependent properties of the materials are not reliable, deflections should be calculated for critical combinations of the expected upper and lower bounds of the creep, shrinkage and relaxation versus time curves.

Eq. (24) may be simplified by making some approximations to Parts 2 and 3. First, consider Part 2. At any given time, the losses caused by relaxation and shrinkage are known directly from the assumed relationships. The summation is required to determine the effect of creep. If the creep coefficient is C_t, the creep loss in the prestress level may be written roughly as $C_t n f_{ci}$, where f_{ci} is the effective initial prestress in the concrete at the level of the prestressing steel and n is the modular ratio. The actual loss will be less than this quantity since creep strains will occur under a decaying rather than a constant prestress. Thus, assuming the creep loss to be a stress of $C_t n\ f_{ci}$ tends to underestimate ϕ_{pt} since it increases Part 2 in Eq. (24). An alternative is to admit the creep loss to be $C_t n\ f_{ci}\ (1 - C_t n\ f_{ci}/f_{se})$, where f_{se} is the initial effective prestress. This tends to overestimate ϕ_{pt}. A desirable solution would be to use $C_t n\ f_{ci}$ when considering the deflections under load and $C_t n\ f_{ci}\ (1 - C_t n\ f_{ci}/f_{se})$ when considering the deflections under prestress alone. A compromise is to assume that the loss in the prestress level is equal to a stress of $C_t n\ f_{ci}\ (1 - C_t n\ f_{ci}/2f_{se})$. Assuming that one of these approximations is adopted the loss in prestress force $P_t' = (P_i - P_t)$ due to relaxation, shrinkage and creep can be evaluated. If the variation in E with time is neglected Part 2 of Eq. (24) becomes simply:

$$(P_i - P_t)\ \frac{e_x}{EI} = \frac{P_t'\ e_x}{EI} \qquad (25)$$

Part 3 of Eq. (24) may be simplified by assuming that creep occurs under a constant prestressing force equal to the mean of the initial and final prestressing forces:

$$\frac{P_i + P_t}{2}\ C_t\ \frac{e_x}{EI} = \frac{2P_i - P_t'}{2}\ C_t\ \frac{e_x}{EI} \qquad (26)$$

The loss in the prestressing forces P_t', is determined in the evaluation of Part 2. Thus, combining Eq. (24), (25), and (26), a close approximation for Eq. (24), which can be evaluated without the necessity for a summation procedure, is as follows:

$$\phi_{pt} = -\frac{P_i\, e_x}{EI}\left[1 - \frac{P_t'}{P_i} + \left(1 - \frac{P_t'}{2P_i}\right) C_t\right] \qquad (27)$$

404 — Illustrative example

The purpose of the numerical example given in this section is to illustrate the use of the procedure outlined in the preceding section. A simple case is analyzed so that attention may be focused on the basic steps of the procedure rather than on computational details.

Consider an 18-in. double-T-beam with a simple span of 54 ft. Assume a dead load of 275 lb per ft and a live load equal to the dead load.

The longitudinal reinforcement ($A_s = 1.52$ sq in.) has an effective prestress (after release) of 140,000 psi. The effective depth varies linearly from 14.7 in. at the third-points of the span to 10.9 in. at the supports.

The pertinent geometrical properties of the cross section are:

Area = 267 sq in.

Moment of inertia (plain section) = 7550 in.^{4}

Distance from centroidal axis to bottom extreme fiber = 12.5 in.

The material properties are assumed as follows:

Concrete strength = 5000 psi (at release, no further increase in strength)

Modulus of deformation for concrete, $E_c = 4.3 \times 10^6$ psi

Modulus of deformation for embedded strand reinforcement, $E_s = 30 \times 10^6$ psi

Concrete creep coefficient, $C_t = 2.0$

Concrete shrinkage strain = 0.0006 (after release of prestress)

Steel relaxation loss = 5 percent of effective prestress after release

For the simply supported beam considered, Eq. (24) may be written directly in terms of the deflections caused by the load and the prestress:

$$\Delta_t = \Delta_{mt} + \Delta_{pt} \qquad (28)$$

where

Δ_t = total midspan deflection
Δ_{mt} = midspan deflection caused by the transverse load
Δ_{pt} = midspan deflection caused by the prestress

In accordance with Eq. (23):

$$\Delta_{mt} = (1 + C_t)\,\Delta_{mi}$$

where

Δ_{mi} = instantaneous deflection caused by the transverse load

The instantaneous deflection for a simply supported beam under uniform load is

$$\Delta_{mi} = \frac{5}{384}\,\frac{WL^3}{E_c I}$$

when

W = total load on the beam

$= 275 \times 54 = 14{,}850$ lb for the dead *or* the live load

$$\Delta_{mi} = \frac{5 \times 14{,}850 \times (54 \times 12)^3}{384 \times 4.3 \times 10^6 \times 7550}$$

$= 1.6$ in. downward for the dead *or* the live load

$$\Delta_{mt} = (1 + 2)\ 1.6$$

$= 4.8$ in. downward for the dead *or* the live load

This step concludes the calculations to determine the deflection caused by the transverse loads. The calculation of the deflection caused by the prestress follows.

The position of the prestressing force varies linearly in the outer thirds of the span. Therefore:

$$\Delta = \frac{L^2}{216} (19\,\phi_1 + 8\,\phi_2)$$

where

Δ = midspan deflection
ϕ_1 = curvature existing in middle third of span
ϕ_2 = curvature at the support

To obtain the instantaneous deflection Δ_{pi} caused by the prestress, ϕ_1 and ϕ_2 can be expressed as follows:

$$\phi_1 = \frac{A_s\, f_{se}\, e_1}{E_c I}$$

$$\phi_2 = \frac{A_s\, f_{se}\, e_2}{E_c I}$$

where

e_1 = eccentricity of the prestressing force in the middle third of the span
e_2 = eccentricity of the prestressing force at the support

$$\phi_1 = \frac{1.52 \times 140{,}000 \times 9.2}{7550 \times 4.3 \times 10^6} = 0.060 \times 10^{-3} \text{ in.}^{-1}$$

$$\phi_2 = \frac{1.52 \times 140{,}000 \times 5.4}{7550 \times 4.3 \times 10^6} = 0.035 \times 10^{-3} \text{ in.}^{-1}$$

$$\Delta_{pi} = \frac{(54 \times 12)^2}{216} (19 \times 0.060 + 8 \times 0.035) \times 10^{-3} = 2.8 \text{ in. upward}$$

The loss in prestress must be determined before calculating the upward time-dependent deflection. Since the reinforcement is draped, the compressive stress at the center of gravity of the steel varies in the outer thirds of the span. Hence, the loss caused by creep of the concrete will be evaluated at the two extreme points of the draped portion of the reinforcement.

In the middle third of the span, the initial compressive stress at the center of gravity of the prestressed reinforcement is

$$f_{ci} = A_s f_{se}\left(\frac{1}{A} + \frac{e_1^2}{I}\right) = 1.52 \times 140{,}000\left(\frac{1}{267} + \frac{\overline{9.2}^2}{7550}\right) = 3200 \text{ psi}$$

At the support:

$$f_{ci} = 1.52 \times 140{,}000\left(\frac{1}{267} + \frac{\overline{5.4}^2}{7550}\right) = 1600 \text{ psi}$$

The unit loss in prestress is determined in accordance with the discussion preceding Eq. (27).

In the middle third of the span:

$$\frac{P_t'}{A_s} = \text{Relaxation loss} + \text{Shrinkage loss} + \text{Creep loss}$$

$$= 0.05 \times 140{,}000 + 0.0006 \times 30 \times 10^6 + 2 \times 7 \times 3200\left(1 - \frac{2 \times 7 \times 3200}{2 \times 140{,}000}\right)$$

$$= 7{,}000 + 18{,}000 + 38{,}000$$

$$= 63{,}000 \text{ psi}$$

$$\frac{P_t'}{P_i} = \frac{63{,}000}{140{,}000} = 0.45$$

At the support:

$$\frac{P_t'}{A_s} = 0.05 \times 140{,}000 + 0.0006 \times 30 \times 10^6 + 2 \times 7 \times 1600\left(1 - \frac{2 \times 7 \times 1600}{2 \times 140{,}000}\right)$$

$$= 7{,}000 + 18{,}000 + 21{,}000$$

$$= 46{,}000 \text{ psi}$$

$$\frac{P_t'}{P_i} = \frac{46{,}000}{140{,}000} = 0.33$$

It should be noted that the creep losses calculated above are fictitious since the calculations refer to a beam with no transverse loading. The total curvature at the two sections considered is evaluated using Eq. (27).

In the middle third of the span:

$$(\phi_{pt})_1 = \phi_1\left[1 - \frac{P_t'}{P_i} + \left(1 - \frac{P_t'}{2P_i}\right)C_t\right]$$

$$= 0.060 \times 10^{-3}\left[1 - 0.45 + \left(1 - \frac{0.45}{2}\right)2\right]$$

$$0.060 \times 10^{-3} \times 2.1 = 0.13 \times 10^{-3} \text{ in.}^{-1}$$

At the support:

$$(\phi_{pt})_2 = 0.035 \times 10^{-3}\left[1 - 0.33 + \left(1 - \frac{0.33}{2}\right)2\right]$$

$$0.035 \times 10^{-3}\,[2.3] = 0.08 \times 10^{-3} \text{ in.}^{-1}$$

Total deflection at midspan caused by prestress Δ_{pt}:

$$\Delta_{pt} = \frac{L^2}{216}[19\ (\phi_{pt})_1 + 8\ (\phi_{pt})_2]$$
$$= \frac{(54 \times 12)^2}{216}(19 \times 0.13 + 8 \times 0.08)$$
$$= 6.0 \text{ in. upward}$$

Although the deflections Δ_{mt} and Δ_{pt} have been evaluated, Eq. (28) cannot be used without knowledge of the loading sequence. The deflections caused by the transverse load and prestress have to be combined depending on the datum plane for the deflection and the program of loading as illustrated below. Downward deflections are assumed to be positive.

If the dead and live loads are applied immediately after release of prestress, the total deflection referred to the position of the beam before the application of prestress is:

$$\Delta_t = 9.6 - 6.0 = 3.6 \text{ in. downward}$$

If the live load is transient, and therefore not included

$$\Delta_t = 4.8 - 6.0 = -1.2 \text{ in. upward}$$

Often the permanent live load is applied some time after the beam is prestressed. If it is desired to find the deflection referred to the position of the beam just before the application of the permanent live load, this cannot be done simply by adding the time-dependent deflections under the different effects. Part of the time-dependent deflection under prestress and dead load would have already occurred. For an extreme example consider the application of the live load after almost all of the time-dependent deflection has taken place. In that case, the deflection under the live load would be 4.8 in. If the live load is applied at an earlier time, it is necessary to make an assumption about the rate of the time-dependent deflections. Unless pertinent data are available, it is satisfactory to assume that one-quarter of the time-dependent deflection occurs in 2 weeks, one-half in 3 months, and three-quarters in 1 year. Accordingly, if the permanent live load is placed 3 months after release of prestress, the total deflection under the live load would be:

$$\begin{aligned}\Delta_t &= \text{Instantaneous and time-dependent deflection under live load} + \\ &+ \text{one-half of time-dependent deflection under dead load} + \\ &+ \text{one-half of time-dependent deflection under prestress} \\ &= 4.8 + 0.5 \times 3.2 - 0.5 \times 3.2 \\ &= 4.8 \text{ in. downward}\end{aligned}$$

In the foregoing discussion, it was assumed that the basic properties of the concrete did not change after release of the prestress. If the strength of the concrete increases between the time the prestress is released and the time the live load is applied, the deflections corresponding to these effects must be calculated on the basis of the pertinent moduli of elasticity and creep factors. However, the effect of changes in the properties of the concrete on time-dependent deflections caused by the prestress or the load may be ignored. The time-dependent deflections may be based on the properties of the concrete at the time of application of the force producing the deflection.

CHAPTER 5 — SUMMARY

The principal factors affecting the short-time and long-time deflection behavior of prestressed concrete beams have been discussed and methods for calculating these deflections have been presented.

The factors influencing the deflection behavior are external load, amount of prestress, shrinkage, creep, and relaxation of steel stress. The determination of long-time deflections is complex because these factors, which are interrelated, produce a constantly changing strain and thus stress distribution over the depth and span of the beam. The changes in stress with time are primarily due to losses in prestress force caused by creep, shrinkage, and relaxation of steel stress. Changes in strain with time are due to creep and shrinkage as well as changes in stress distribution.

The deflection of a beam from a reference base of zero stress, strain, and deflection can be evaluated at any time if the magnitude and the longitudinal distribution of the curvatures (angle change per unit length) for the beam span are known for that instant. Any suitable integration procedure or equivalent method, such as conjugate beam or moment area, may be used to obtain the deflections from the curvatures.

Two methods have been presented in this report by which the curvatures can be evaluated at any particular time. The first method is a general one, which involves a step by step summation procedure. This summation is performed using any desired number of time intervals from the original state of stress, strain and deflection to the time for which the deflection is desired. This procedure can be used in almost all cases provided the basic material properties and the program of prestressing and loading of the beam are known. The procedure has been verified by checking computed and measured deflections on several groups of experimental beams. Results indicate that long-time deflections may be predicted within 5 to 10 percent by this method. The method unfortunately involves a considerable amount of computation.

The second method is an approximate one in which the curvatures at a particular time are estimated by substituting into a single formula average values for constants related to the creep, shrinkage and prestress loss characteristics of the beams. In most cases this procedure will yield values which are sufficiently accurate for design purposes.

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Sinopsis — Résumés — Zusammenfassung

Deflexiónes en Miembros de Concreto Presforzado

Reporte del Subcomité 5 del Comité ACI 435 (335), Deflexiónes en Estructuras de Concreto. Discute factores que afectan las deflexión de corta larga duración en miembros de concreto presforzado.

Se presentan métodos analíticos para calcular estas deflexiones, tomando en cuenta presfuerzo, carga transversal, flujo plástico, contracción y relajación del esfuerzo en el acero.

Déflexion dans les Poutres en Béton Précontraint

Ce compte-rendu du sous-comité #5 du Comité ACI 435 (335), Déflexions dans les Bâtiments en Béton, discute les facteurs qui influent le comportement de la déflexion pour périodes courtes et longues dans les poutres en béton précontraint. On présente des méthodes analytiques pour calculer ces déflexions, en tenant compte de la précontrainte, les charges transversales, fluage, contraction, et la relaxation de la contrainte dans l'acier.

Verformung von Spannbetonbauteilen

Dieser Bericht des Unterausschusses 5 des ACI Komitee 435 (335), Verformung von Betonkonstruktionen, erörtert die Faktoren, die das kurzfristige und langfristige Verformungsverhalten von Spannbetonbauteilen beeinflussen. Analytische Methoden für die Berechnung dieser Verformungen werden angegeben, in denen Vorspannung, Querbelastung, Kriechen, Schwinden sowie das Nachlassen der Stahlspannung berücksichtigt werden.