SP 59-3

SAFETY AND SERVICEABILITY PROVISIONS IN THE ACI BUILDING CODE

By George Winter

Synopsis: The development of the safety provisions of the ACI Building Code is traced through the four most recent editions, since 1956, with special emphasis on the present load and resistance factor (LRF) format and its relation to more rigorous probabilistic approaches to structural safety. Serviceability provisions regarding cracking and deflections at service loads and their background are also discussed.

Keywords: building codes; cracking (fracturing); deflection; load factors; reinforced concrete; safety factor; serviceability; structural design; ultimate strength method.

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Introduction

In the last three editions of the ACI Building Code, those of 1956, 1963, and 1971, the provisions for safety and for serviceability have undergone a radical evolution. During that same time, two other far-reaching changes have occurred, which have in turn affected the safety and serviceability provisions.

The first of these is the tendency toward the use of higher strength materials. Prior to 1956 the usual stipulated cylinder strength of concrete was 2,500 to 3,000 psi (17.2 to 20.7 MPa) with 3,750 psi (25.8 MPa) used in special cases. But, by 1971 the usual concrete strengths ranged from 3,000 to 4,000 psi (20.7 to 27.6 MPa) and concretes up to 6,000 psi (41.4 MPa) are no longer exceptional. Similarly, before 1956 the stipulated minimum yield strength of reinforcing bars was 33,000 or 40,000 psi (227.5 or 275.8 MPa). By 1971 the use of 60,000 psi (413.7 MPa) yield strength steel had become practically universal, with 75,000 psi (510.2 MPa) steel employed under appropriate conditions.

The second relevant change consists in the approach to calculating the strength, or allowable load, on reinforced concrete members. Prior to 1956 the carrying capacity of members was computed on a semi-elastic basis: that is, linear relations between stress and strain were assumed for both steel and concrete, with some exceptions, such as for axially loaded compression members. In contrast, by 1971 the dimensioning of members was based on the recognition of the inelastic behavior of both concrete and steel at loads approaching the ultimate strength of the member.

The development of the safety and serviceability provisions is intimately connected with these two basic changes.

Safety Provisions

In current design codes or specifications, safety provisions are formulated in one of the following three formats:

1. Allowable stress format. (AS-format)

In general terms, in this method some property of the cross-section, S, is multipled by an allowable stress f_{all}. For a member to be safe, this quantity must be equal to or greater than the pertinent load effect or generalized force F (moment, shear axial load, etc.). Here, F is calculated for the stipulated design or service load L_{des}, i.e. $F(L_{des})$. Symbolically, then,

$$Sf_{all} \geq F\,(L_{des}) \tag{1}$$

2. Load factor format. (LF-format)

Here one calculates the expected (nominal) ultimate strength or resistance of the member, R, based on the best available experimental and analytical information. For a member to be safe, this ultimate strength must be equal or greater than the load effect F caused by the various types of design loads, each multiplied by an appropriate load factor γ. Symbolically, then,

$$R \geq F\,(\Sigma \gamma_n L_{des,n}) \tag{2}$$

Here the various load factors $\gamma_n > 1$ provide the required margin of safety. Their magnitudes can be adjusted to reflect the differing character and variability of the various types of loads (self-weight, occupancy loads, wind, etc.), which is not possible in the AS-format.

3. Load and resistance factor format. (LRF-format)

In this format, it is explicitly recognized that both the actual resistance of a member and the load effects acting on it are, in fact, variable quantities which differ in random ways from their nominal values R and $F(L_{des})$. In the LRF-format, the possibility that the actual resistance may be less than its nominal value, and the actual load effect larger than its design value, are recognized separately by understrength or resistance factors $\phi_m < 1$ and by load factors $\gamma_n > 1$. The desired safety margin is then provided by the condition

$$\phi_m R_n \geq F\,(\Sigma \gamma_n L_{des,n}) \tag{3}$$

Here R_n is the particular type of strength or resistance (bending, shear, compression, etc.) and ϕ_m the pertinent resistance factor. This format has the advantage that it lends itself well to the determination of the factors ϕ_m and γ_n by a combination of probabilistic safety theory and calibration against existing designs and structures. The approach known as limit state design, now in use or under development in many countries, is frequently expressed in the LRF-format.

Historically, the safety provisions of the ACI Code have developed as follows:

Prior to 1956: AS-format

1956 edition: AS-format or optionally LF-format

1963 edition: AS-format or alternatively LRF-format

1971 and 1977 editions: LRF-format or optionally AS-format

That is, in the 1971 edition, the basic design provisions are formulated in the LRF-format. Use of the old AS-format is still permitted, but is discouraged, particularly in connection with the new, higher strength materials. This is so because the nominal carrying capacity Sf_{all} utilized in the AS-format and mostly calculated on a semi-elastic basis is increasingly inaccurate when applied to today's higher strength materials.

As indicated, the first radical departure from the traditional AS-format occurred in the 1956 edition of the Code. There, in a separate Appendix, equations were given for the first time for the nominal real strength or carrying capacity of members, R, as distinct from allowable capacities Sf_{all}. The load effects F were calculated for factored loads, namely either

$$F = F(1.2\ DL + 2.4\ LL) \qquad (4)$$

$$F + F(\gamma_{DLL}(DL + LL)) \qquad (5)$$

Here DL is the dead load caused essentially by selfweight, and LL is the live load essentially caused by occupancy loads. The factor γ_{DLL} was to be taken as 2.0 for columns and 1.8 for beams. Additional provisions were made for wind loading in combination with dead and live loads.

This method, presented in an Appendix, did not gain wide acceptance and practical design continued mostly in the traditional

AS-format. However, this approach, which was known as "ultimate strength design" because of the use of real strength expressions R rather than allowable capacities Sf_{all}, represented the stepping stone to the introduction of a fully developed LRF-format of safety provisions in the 1963 Code edition.

It was clear that the traditional design using the AS-format could not be suddenly discontinued by simple fiat. Therefore, the 1963 Code contained two complete alternative sets of provisions, one in the old AS-format and one in the new LRF-format, the use depending on the designer's preference.

At the time when that edition was in preparation, it was already realized that it is illusory to try designing for absolute safety. Considerable work was underway by various individuals and institutions toward the development of a theoretically consistent probabilistic approach to structural safety, but no useable method had yet been devised. Thus, the subcommittee in charge of formulating the safety provisions, of which the writer was chairman, had to develop its own methodology, utilizing whatever useful work by others was available.

It was decided to aim at a probability of failure of the order of 1/100,000. It was realized that the probability of failure represented a combination of the possibility of strength deficiencies on the one hand, and excess loads or load effects on the other. The aim, then, was to keep the probability of strength deficiency to 1/1000 and that for excess load effects to 1/100. The product of these two probabilities would then represent the overall failure probability of the order of 1/100,000. It is easy to aim at such probabilities, but it is next to impossible to achieve them without adequate statistical data on which to base numerical values of load and resistance factors.

Use was made of data available for at least one source of strength deficiency, namely frequency distributions of the actual strengths of the two materials, concrete and steel, as obtained in large-scale test series. Use was also made of such meager statistics on loads as were available. Combining these scanty data with engineering experience and intuition, and with comparisons with accepted design practice, the subcommittee drafted the following formulation: The factored resistance ϕR (see Eq. 3) was to be computed by means of accepted strength equations, but using materials' strength multiplied, for steel, by the resistance factor $\phi_s = 0.8$ and for concrete by $\phi_c = 0.67$. Thus, in Eq. 3.

$$R = R(0.67f_c', \quad 0.8f_y) \qquad (6)$$

It was stated that only part of the resistance factors ϕ_c and ϕ_y accounted for actual possible strength deficiencies of the two materials. The rest was to provide for other possible sources of under-strength, such as dimensional deficiencies, effects of approximations and simplifications in structural analysis, subnornal workmanship, etc. It is worth noting that the application of separate resistance factors ϕ to the specified strengths of concrete and steel is now employed in the codes of several European countries.

Regarding load effects F, the subcommittee recognized the greater certainty of dead loads as compared to live loads by corresponding load factors $\gamma_{DL} = 1.3$ and $\gamma_{LL} = 1.5$. This resulted in the load effects (cf. Eq. 3)

$$F = F(1.3DL + 1.5LL) \qquad (7)$$

with similar expressions for combinations of wind with other loads. Thus, the basic safety requirement, Eq. 2, became

$$R(0.67f'_c,\ 0.8f_y) \geq F(1.3DL + 1.5LL) \qquad (8)$$

(This brief presentation omits a number of secondary details).

When this draft was submitted for discussion to the Code Committee and the entire membership of A.C.I., it met with considerable criticism. The main objections raised were these:
(a) the resistance factors applied to the strengths of concrete and steel could be misinterpreted by builders and others to imply that deficient materials were acceptable in construction provided their actual, tested strength exceeded $0.67f'_c$ or $0.8f_y$, respectively.
(b) The significantly smaller value of ϕ_c for concrete than ϕ_s for steel may be exploited in the competition between steel and concrete construction as indicating that concrete is the less reliable material. (c) The effects of approximations and simplifications in structural analysis represented an uncertainty which affects the magnitude of the load effects F rather than of the member resistance R. Hence, they should be included in the load factors γ rather than the resistance factors ϕ. Considering the latter argument, it was decided to re-distribute the assumed probabilities so that a strength deficiency probability of 1/100 and an overload probability of 1/1000 was assumed, rather than vice versa.

On this basis the load factors were increased from 1.3 to 1.5 for dead load and from 1.5 to 1.8 for live load. In addition, individual resistance factors were applied not to the strengths of the two materials, but rather to the type of member or of internal action in the member. That is, different ϕ-values were developed for flexure, shear, bond, anchorage, and for columns with ties and columns with spiral reinforcement. This way of using ϕ-factors has the advantage that types of member performance and of importance of members for the survival of a structure could be reflected in more detail in the ϕ-values.

Because the safety provisions of the 1971 Code are very similar to those of the 1963 edition, only the presently used values will be presented. There is one substantive difference between these two editions: in 1971 the load factors were lowered slightly, from 1.5 to 1.4 for γ_{DL} and from 1.8 to 1.7 for γ_{LL}. The Commentary states that this was done because of the more comprehensive provisions of the 1971 Code, improved concrete and steel quality control, and satisfactory experience gained with the LRF-format.

In brief, then, the safety provisions of the ACI Code utilize the LRF-criterion given by Eq. 3 with the following values:

Loading	Load factors γ		
	γ_{DL}	γ_{LL}	γ_W
dead plus live	1.4	1.7	
dead, live and wind, additive	1.04	1.275	1.275
dead and wind, counteracting	0.9		1.3

Type of action or member	Resistance factor ϕ
flexure	0.90
compression members with spirals	0.75
compression members with ties	0.70
shear and torsion	0.85
bearing on concrete	0.70
bending in unreinforced concrete	0.65

Except for minor adjustments, no significant changes of these factors are anticipated for the next edition of the Code.

It is evident from this history of these safety provisions, that the present numerical values of the factors ϕ and γ are based only to a minor degree on objective statistical evidence. Such scarce data as existed at the time have been utilized, but the final values were largely determined by engineering judgement and calibration against established design practice and existing structures. In the foreseeable future, however, this is almost certain to change because, during the years when these provisions were in effect, great strides have been made in the development of rigorous, yet practically useable probabilistic approaches to structural safety.

It is fortunate that the method farthest developed in this country, the so-called first order, second moment theory, can be made to result in a criterion of the precise form of Eq. 3, i.e. the LRF-format. When statistical evidence is available on strengths, loads or other relevant quantities, this theory makes use of only two distribution characteristics: the mean value and the coefficient of variation. The safety criterion for the limit state k (e.g. flexure, shear, what have you) due to the load combination j can then be written as

$$\phi_k R_{nk} \geq \gamma_o \left(\sum_{i=1}^{m} c_i \gamma_i L_{ni} \right)_j \qquad (9)$$

where R_{nk} is the nominal resistance for the state k, γ_o is an analysis factor of the order of 1.1, representing the effect of approximations and simplifications in the analysis, and c_i is a deterministic factor by means of which the factored nominal load $\gamma_i L_{ni}$ is transformed into the pertinent internal force (i.e. moment, shear force, etc.) According to the second moment theory, the resistance factor and the load factor are:

$$\phi_k = \frac{R_{mk}}{R_{nk}} e^{-\alpha \beta V_R} \qquad (10)$$

$$\gamma_i = \frac{L_{mi}}{L_{ni}} e^{\alpha \beta V_L} \qquad (11)$$

Here the resistances R and loads L when subscripted m are the mean values of the statistical distributions, and when subscripted n are the nominal design values (e.g. codified loads), α is a numerical quantity of no particular interest in this abbreviated presentation, and V_R and V_L are the coefficients of variation of the particular type of resistance or load, respectively. β is the safety index, which is of the order of 3 to 4 for a probability of failure of about 1/100,000. The same range for β is obtained by calibration against customary design practice.

Consequently, once one has settled on the desired safety index β, it is only necessary to obtain sufficient statistical information on different types of strength (resistance) and of loads, so that the mean values and the coefficients of variation can be determined. Having these, the pertinent ϕ and γ can be calculated rather than being determined mostly by judgement. In regard to resistance, a considerable amount of such work has already been done. The Reinforced Concrete Research Council is in the process of instituting a major research project aimed at establishing resistance factors ϕ chiefly in the above context. Likewise, a Subcommittee on Load Factors of the Committee on Building Code Requirements for Minimum Design Loads in Buildings and Other Structures of the American National Standards Institute, ANSI A58.1, has recently been organized to develop, in the same context, load factors valid for all types of construction, not just for reinforced concrete.

As this much abbreviated and simplified presentation indicates, future safety provisions will be based to a much larger degree than now on objective evidence and on rational criteria, even though engineering experience and judgement will continue to play an important role in this area.

Serviceability Requirements

Once adequate safety against structural distress has been achieved in a design, it is still necessary to ensure that the performance of the structure will be satisfactory at service loads. In most cases this means that the magnitude of deflections and of tension cracking of concrete must be adequately controlled. Control of deflections, important in itself, also furnished some indirect control of undesirable vibrations.

Other things being equal, deflections as well as widths of flexural tension cracks at service loads are approximately proportional to the steel stress. In earlier Code editions through 1956 there was no need for serviceability requirements because fairly low steel stresses at working loads were obtained in low and moderate strength steels in conjunction with the AS-format. However, the general use of higher steel strengths, in conjunction with the LRF-format, resulted in sharply increased steel stresses at service loads so that control of member performance under working conditions became important. This is why the 1963 edition, for the first time, contains provisions for control of deflections. These have been retained and expanded in the 1971 edition together with new provisions for limiting the width of flexural tension cracks.

Control of Deflections

It is well to realize that control of deflections does not mean that a precise prediction of actual deflections is attempted. Deflections, particularly long-time deflections, depend on many factors such as on ambient temperature and humidity, age and moisture content of concrete at time of loading and others. For this reason the usual scatter range of actual vs. predicted deflections is of the order of $\pm$ 25%. Yet for most occupancies, this degree of control is entirely adequate.

The 1971 ACI Code specifies a set of maximum allowable service deflections in terms of fractions of the span ℓ. These range from $\ell/480$ for immediate deflections due to live loads on roofs, to $\ell/480$ for total long-time deflections of roof or floor construction supporting other nonstructural elements, such as partitions, which are likely to be damaged by large deflections.

For non-prestressed construction the deflection requirements can be satisfied in two different ways. For one, the Code stipulates a set of minimum thicknesses of beams and of slabs, in terms of the span length ℓ. For instance, for the outer spans of continuous beams and of continuous one-way slabs, these minimum thicknesses are, respectively, $\ell/18.5$ and $\ell/24$. Separate values are specified for other support conditions. These values are intended for reinforcing steel with 60,000 psi (414 MPa) yield strength and for regular weight concrete. Modifications are given when other steel strengths or light-weight concretes are used. Deflections need not be calculated for members with thicknesses not less than these specified values unless they support or are attached to elements likely to be damaged by large deflections. Similar thickness limits are given for two-way slabs, but not for prestressed construction, for which deflections must always be computed.

When deflections must be calculated, one of the uncertainties is the effective flexural rigidity EI. This rigidity is affected by the extent and distribution of flexural tension cracks along the beam. The Code recognizes this by defining an effective moment of inertia as developed by D. E. Branson, viz.

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_g \qquad (12)$$

where M_a = maximum moment in member, M_{cr} = cracking moment, I_g = moment of inertia of gross concrete section, and I_{cr} = moment of inertia of cracked, transformed section. Immediate deflections are then calculated for the rigidity E_cI_e, where E_c = f (f'_c) is the modulus of elasticity of concrete.

To these instantaneous deflections must be added the long-time or creep deflections caused by the sustained part of the total load. One could do this by appropriate modification of E_c. However, W. W. Yu and G. Winter have developed a simpler way. It consists in multiplying the appropriate part of the immediate deflection by a factor which depends on the ratio of compression reinforcement A_s' to tension reinforcement A_s. This takes account of the fact that the presence of compression reinforcement, which is not subject to creep, reduces the creep deformations of the concrete compression zone. The Code prescribes a simplified version of this factor, namely

$$\left[2 - 1.2\,(A_s'/A_s)\right] \geq 0.6 \qquad (13)$$

The total deflection, then, is the immediate deflection upon application of the load, plus that part of the immediate deflection which is caused by sustained loads multiplied by the quoted factor.

ACI Committee 435 has made a statistical investigation of the reliability of this method, when applied to laboratory beams. It has found that there is a 90% probability that the deflections of a given beam will fall within the range of 20% smaller to 30% larger than the calculated value. This illustrates the previously mentioned uncertainties of deflection calculations.

This simplified presentation omits a number of features, such as special deflection provisions for lightweight concrete, two-way construction, prestressed concrete composite members.

Control of Flexural Cracking

Wide flexural tension cracks, which can develop when high steel stresses are present under working loads, are objectionable from the viewpoint of appearance, i.e., esthetically. In corrosive environments, in addition, cracks of large width can significantly reduce the corrosion protection generally provided by concrete cover. For these reasons, a large number of thin flexural hairline cracks is preferable to the development of a few wide cracks. The sum of the widths of all cracks over a given length of reinforcing bar is closely proportional to the steel stress. However, the maximum width of individual cracks, i.e., the appearance of a few wide cracks vs. the development of many narrow hairline cracks, can significantly be influenced by detailing of the reinforcement. Crack widths, like deflections, are subject to wide scatter, so that great accuracy in crack control computations is not warranted. For this reason, the 1971 Code provides no means of calculating crack widths. Instead, it provides crack control in the form of quantitative and qualitative provisions for detailing of reinforcement.

The quantitative provisions are based on an approximate expression for predicting crack width, obtained in a large experimental investigation by P. Gergely and L. A. Lutz. This expression is

$$w = 0.076\beta \; f_s \; 3\sqrt{d_c A_b} \qquad (14)$$

where w = crack width at tension face a member, β = ratio of distances from the neutral axis to the bottom face and to the centroid of reinforcement, i.e., $\beta = h_2/h_1$ in Fig. 1, f_s = steel stress, d_c = concrete cover as shown on Fig. 1, A_b = effective tension area of concrete divided by number of bars. This effective tension area is centered on the centroid of reinforcement, as shown in Fig. 1.

For a guide to detailing of reinforcing, this expression has been simplified by assuming $\beta = 1.2$. Furthermore, the maximum tolerable crack width has been stipulated as, approximately, 0.013 in (0.34mm) for exterior members and 0.016 in (0.41mm) for interior members, reflecting the importance of crack width for corrosion control. On this basis the equation for w can be transformed into that provided in the Code, viz.

$$z = f_s \; 3\sqrt{d_c A_b} \qquad (15)$$

where it is specified that z shall not exceed 175 kips per inch for interior exposure (corresponding to w = 0.016 in) and 145 kips per inch for exterior exposure (corresponding to w = 0.013 in).

In order to satisfy this requirement, the steel stress f_s could be reduced in design, but this affects economy adversely since it would constitute a poor utilization of the generally available higher strength steels. Hence, the only other way to keep z within the prescribed limits, i.e., to control crack width, is to use a larger number of smaller reinforcing bars. This will reduce A_{b1} the effective concrete tension area per reinforcing bar, and in a minor way will also reduce d_c. In this manner the stipulated limits on z provide guidance for detailing of reinforcement in order to reduce crack width.

Further, in laboratory investigations of T-beams loaded to produce tension in the flange it has been observed that if the tension reinforcement is concentrated over the web, inadmissibly wide tension cracks develop in the outer portions of the flanges. For this reason the Code specified that when flanges are in tension, a portion of the tension reinforcement shall be distributed over the flange width beyond the web, and for unusually wide flanges, special additional longitudinal reinforcement shall be placed in the outer portions.

Just as for deflections, these provisions for crack control were made necessary by the use of higher strength reinforcing bars. For crack control in prestressed concrete, the Code asks for a stipulated amount of bonded reinforcement to be placed in the precompressed tension zone of flexural members with unbonded prestressing steel; it also calls for uniform distribution of this bonded steel near the extreme tension fiber.

This brief review of serviceability requirements has omitted some secondary details, but it does present the essential provisions in the 1971 Code and their background. No substantial changes, except for details, are anticipated in the next Code edition.

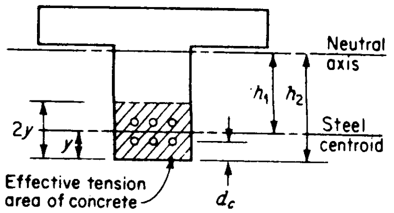

Fig. 1--Effective tension area centered on centroid of reinforcement

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