NON-TENSIONED STEEL IN PRESTRESSED CONCRETE BEAMS

A study of the effects of non-tensioned steel on the behavior of prestressed concrete beams is presented. Effects on camber, loss of prestress force, cracking, and deflections are included. Analytical results are compared with the observed behavior of twelve, simply supported, prestressed concrete beams, ten of which contained non-tensioned steel.

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Non-tensioned steel can be used in prestressed concrete beams to serve various purposes. However, due to a lack of sufficient understanding of the behavior of prestressed concrete beams containing non-tensioned steel, only a very limited application is to be found in practice.

The use of non-tensioned steel in prestressed concrete has been referred to as partial prestressing, which, in general, is taken to mean either or both of the following conditions:

1. Tensile stresses are permitted under working loads
2. Non-tensioned steel is used in addition to tensioned prestressing steel.

Some work on partial prestressing has been done in the last fifteen years[1,2,3,4,5,6,7], but most of it deals with the form of partial prestressing defined under item 1 above. Abeles[1,3] has made brief references to the use of non-tensioned steel and questioned the generally accepted notion of its ineffectiveness in uncracked prestressed concrete beams. Abeles[3] mentions that "with more non-tensioned steel of lower strength, as compared to less non-tensioned steel of higher strength, the loss of prestress will be directly more. However, as better control on camber and cracking is likely, a vital need for research in this direction exists."

The only other report on the use of non-tensioned steel appears to be that of Hutton and Loov[7], which was published in December 1966. This paper contains observed camber and deflection curves of a limited number of beams containing non-tensioned steel.

Practically no analytical work or conclusive experimental work has been reported on the use of non-tensioned steel in prestressed concrete beams with reference to camber, loss of prestress, and deflections of cracked sections.

OBJECTIVES

This paper details the findings of an analytical and experimental study on the effects of both the quality (type) and quantity of non-tensioned steel on the following behavior characteristics of prestressed concrete beams:

1. Camber (short-time and time-dependent)
2. Loss of prestress
3. Crack formation
4. Deflections under working loads and overloads.

DESCRIPTION OF EXPERIMENTAL INVESTIGATION

The experimental program consisted of the testing of four series of pretensioned, prestressed concrete beams. Each series included three simply supported, 6 x 8-in. by 15-ft. beams, for a total of 12 beams. Table 1 shows the details of the test beams.

The three beams in each of the four series were designed for the same prestress force. Series I was designed to study the effect of quantity of non-tensioned steel on the behavior of prestressed concrete beams. Series II and IV were designed to study the effects of quality as well as quantity of non-tensioned steel. The distinction between the two series was the total amount of steel. Series III was designed to study primarily the effect of steel prestress level.

Measurements, methods and instrumentation.

1. All test beams, shrinkage specimens and control cylinders were moist cured for 7 days by keeping them covered with wet burlap. The temperature in the laboratory ranged from 70°F to 80°F, with an average value of 72°F. All test beams were prestressed at 7 days.

2. Steel collars with electrical SR-4 strain gages served as load cells for measuring the individual strand prestressing force. Fully temperature-compensated, four-arm bridge circuitry was employed.

3. Initial and long-time midspan camber values were obtained using two dial gages, one on each side of the beam. The discrepancy between the readings of the two dial gages was found to be insignificant.

4. Initial and long-time concrete strains were obtained using a Whittemore mechanical strain gage with a 10-in. gage length. Each beam had three gages distributed from top to bottom on both sides of the beam.

5. Records of temperature and relative humidity were kept throughout the time-dependent study. A sling psychrometer was used to obtain the relative humidity data.

Table 1. Details of test beams[(1)]

Series No.	I			II			III			IV		
f'_{ci} (psi)	4120 (7 day)	5400 (28 day)		4380 (7 day)	5890 (28 day)		4830 (7 day)	6570 (28 day)		4300 (7 day)	5880 (28 day)	
Beam No.	1	2	3	1	2	3	1	2	3	1	2	3
Non-tensioned steel	#4	#4	#4	5/16	#4	#4			#5	3/8	#5	#4
Prestressing strand dia. (in.)	5/16	5/16	5/16	5/16	5/16	5/16	5/16	3/8	3/8	3/8	3/8	3/8
Design F_o (kips)	30	30	30	20	20	20	30	30	30	26	26	26
Actual F_o (kips)	29.8	29.0	30.1	20.2	20.0	19.7	30.5	29.8	29.8	25.2	25.8	24.4
A_s (sq. in.)	0.173	0.173	0.173	0.116	0.116	0.116	0.173	0.240	0.240	0.160	0.160	0.160
A'_s (sq. in.)	0.200	0.400	0.600	0.058	0.200	0.400	0	0	0.310	0.080	0.310	0.600
$p = A_s/bd$ (%)	0.45	0.45	0.45	0.30	0.30	0.30	0.45	0.60	0.60	0.40	0.40	0.40
$p' = A'_s/bd$ (%)	0.50	1.00	1.50	0.15	0.50	1.00	0	0	0.80	0.20	0.80	1.50
$A'_s/A_s = p'/p$	1.15	2.30	3.46	0.50	1.73	3.46	0	0	1.30	0.50	1.94	3.76
$p\frac{f_{su}}{f'_c} + p'\frac{f_y}{f'_c}$	0.23	0.27	0.31	0.20	0.20	0.21	0.20	0.25	0.31	0.28	0.28	0.29
Design M_u (kip-ft.)	21.3	23.8	26.2	19.0	19.8	19.8	19.5	25.6	29.6	24.8	27.7	26.1

(1) All beams 6 x 8-in.; d = 6.5 in.; span = 15 ft. simply supported.
○ Prestressing steel
Ꝋ Non-tensioned high strength steel
● Non-tensioned 33 ksi minimum yield steel
⊛ Non-tensioned 60 ksi minimum yield steel

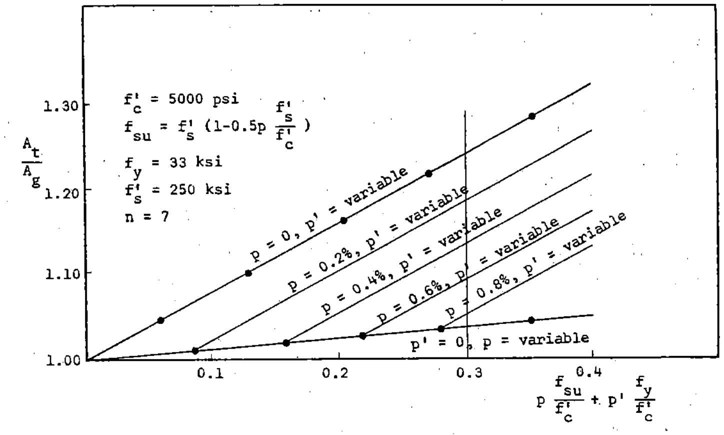


Fig. 1A. Area ratio vs. steel percentage parameter

6. At the end of the time-dependent study period, the beams were loaded to failure under a two-point loading applied with a Universal testing machine using a spreader beam. Mid-span deflections were recorded at one-half kip increments with two long-reach dial gages. As in the case of camber, the discrepancy between the two dial gage readings was found to be negligible. The number and height of visible cracks and the cracked length of each beam were recorded at various stages of loading.

CAMBER

The effect of non-tensioned steel on the factors that influence initial and time-dependent behavior of prestressed concrete beams was

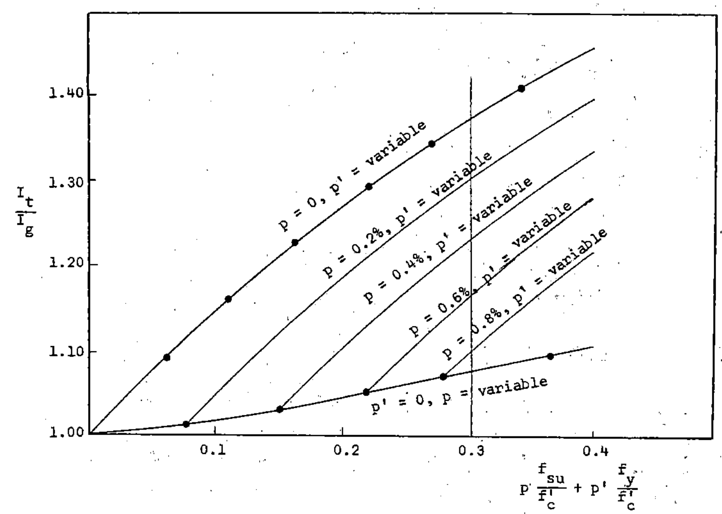


Fig. 1B. Moment of inertia ratio vs. steel percentage parameter studied. A method is presented which, in conjunction with any of the available methods for predicting camber of prestressed concrete beams *without* non-tensioned steel[8,9], will enable the prediction of camber for prestressed concrete beams *with* non-tensioned steel.

Short-time (initial) and long-time (initial plus time-dependent) camber are considered separately.

Short-time camber. For a prestressed concrete beam without non-tensioned steel, it is usually satisfactory to use gross section for computing section properties. However, when non-tensioned steel is used, depending upon the amount and location, the effect on the transformed area and/or the moment of inertia of the transformed section may become quite significant. This is demonstrated in Figs. 1A and 1B, which plot the area ratio (area of transformed section to area of gross section, A_t/A_g) and the moment of inertia ratio (moment of inertia of the uncracked transformed section to moment of inertia of the gross section, I_t/I_g) against the steel percentage parameter, $p\frac{f_{su}}{f_c'} + p'\frac{f_y}{f_c'}$. The steel percentage parameter is indicative of the ductility of the beam cross-section. The plots in Figs. 1A and 1B were obtained for a rectangular cross-section with both the prestressed and non-tensioned steel located at an eccentricity of 0.4 h.

For $p\frac{f_{su}}{f_c'} + p'\frac{f_y}{f_c'} = 0.3$ (which determines the maximum amount of steel permitted for ultimate strength computations per ACI, AASHO and PCI codes), $A_t/A_g = 1.25$ and $I_t/I_g = 1.38$ for a beam which contains no prestressing steel (an ordinary reinforced concrete beam). These ratios are equal to 1.04 and 1.07 respectively for a fully prestressed beam (without non-tensioned steel). The effect of transformed section in the latter case may be ignored, but should be considered in the former case for accurate results.

Between these two extremes, there is a family of curves which pertain to prestressed concrete beams with some non-tensioned steel. Depending upon the amounts of non-tensioned and prestressing steel, and also upon the accuracy desired, a decision regarding the use of the gross section or the transformed section may then be necessary.

For the test beams the maximum area ratio, A_t/A_g, was 1.08 (Beam I B3) and the maximum moment of inertia ratio, I_t/I_g, was 1.05 (Beam IVB2). Thus, for the test beams, the use of the gross section was considered satisfactory. Initial camber values for the test beams were predicted using the gross section and compared with the measured values. These are shown in Fig. 4 and Table 3 to be in good agreement.

Long-time camber. Long-time camber in a prestressed concrete beam consists of initial camber and time-dependent camber. Initial camber occurs at release of prestress; time-dependent camber is caused by strain changes due, primarily, to creep and shrinkage of the concrete.

The strain changes due to shrinkage and creep of concrete bring about a loss of prestress which has a two-fold effect: first, a reduction in initial curvature due to reduction in prestress; and second, a change in creep rate (decrease) due to a reduction in concrete stresses. In other words, the changes in initial deformations are caused by an interaction between creep and shrinkage of the concrete and loss of prestress. Other factors which also influence these changes are steel relaxation and the increase in modulus of elasticity of concrete with time. All of these factors are both time-dependent and inter-dependent.

There are basically two methods for computing deflections of prestressed concrete beams: a detailed method[8,9] that considers the effects of shrinkage and creep separately; and a simplified method that lumps together the effects of shrinkage, creep and the loss of prestress into a combined time-dependent coefficient. This is rather an over-simplified approach, as it does not take into consideration the stress level and distribution, prestress loss, quality of concrete, increase in concrete modulus with age and the presence of non-tensioned steel. The ACI-ASCE Joint Committee[10] method is an example of the simplified method where the ultimate creep coefficient, C_u, of 1.0 to 3.0 is actually the combined time-dependent coefficient. This combined time-dependent coefficient will be referred to as a camber coefficient[8], B_t, as it is directly used with camber.

Some recent studies[11,8,9] have been made to determine the effects of non-uniform stress distribution, shape of specimen, loss of prestress, increase in concrete modulus of elasticity with age and effect of variable stress levels.

In order to use both the detailed method and the simplified method for computing deflections of prestressed concrete beams containing non-tensioned steel, three modification factors are derived. Let α_{sh}, α_c and α be three factors such that, when respectively applied to the shrinkage strain, creep coefficient and camber coefficient values of a prestressed concrete beam without non-tensioned steel, they will give the corresponding quantities to be used in the case of an identical

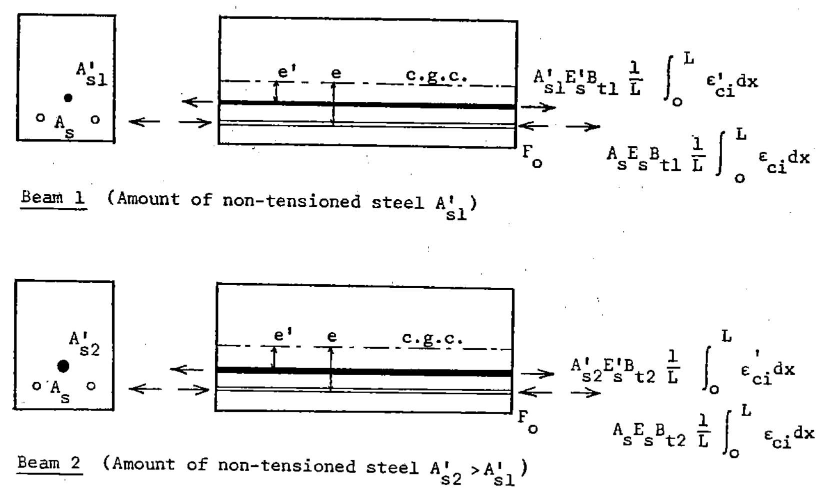


Fig. 2. Comparison of beams with different amounts of non-tensioned steel

beam with non-tensioned steel.

The basis for evaluating these modification factors follows.

During time-dependent deformations, work done by forces in the steel less work done against beam dead load is the change in internal strain energy of the beam, or

$$W_s - W_d = E_b \tag{1}$$

The modification factor, α, for the camber coefficient, B_t, is derived as follows:

Consider two beams which are identical in every respect except the amounts of non-tensioned steel (Fig. 2). Let B_{t1} be the camber coefficient for Beam 1. Let α be a factor (may be time-dependent) such that $\alpha B_{t1} = B_{t2}$, the camber coefficient for Beam 2. Also let $A'_{s1} < A'_{s2}$ where A'_s = area of non-tensioned steel.

Referring to Fig. 2, during an interval of time, dt, Eq. (1) in differential form for Beam 1 is:

$$dW_{s1} - dW_{d1} = dE_{b1} \tag{2}$$

$$dW_{s1} = F_o \frac{dB_{t1}}{dt}\int_0^L \epsilon_{ci}dx - A_sE_sB_{t1}\frac{dB_{t1}}{dt}\int_0^L \epsilon_{ci}dx - A'_{s1}E'_sB_{t1}\frac{dB_{t1}}{dt}\int_0^L \epsilon'_{ci}dx \tag{3}$$

$$dW_{d1} = w\frac{dB_{t1}}{dt}\int_0^L y_i dx \tag{4}$$

where w = the dead load of the beam

y_i = initial camber

Substituting Eqs. (3) and (4) in Eq. (2) and integrating from $t = 0$ to $t = t$:

$$E_{b1} = F_oB_{t1}\int_0^L \epsilon_{ci}dx - \frac{1}{2}A_sE_sB_{t1}^2\int_0^L \epsilon_{ci}^2\,dx - \frac{1}{2}A'_{s1}E'_sB_{t1}^2\int_0^L \epsilon'^2_{ci}\,dx - w\,B_{t1}\int_0^L y_i dx \tag{5}$$

Similarly for Beam 2:

$$E_{b2} = F_oB_{t2}\int_0^L \epsilon_{ci}dx - \frac{1}{2}A_sE_sB_{t2}^2\int_0^L \epsilon_{ci}^2dx - \frac{1}{2}A'_{s2}E'_sB_{t2}^2\int_0^L \epsilon'^2_{ci}\,dx - wB_{t2}\int_0^L y_i dx \tag{6}$$

Now assuming that changes in the strain energies of the two beams during time-dependent deformations are proportional to their respective camber coefficients (intuitively this seems a reasonable assumption as the stress in concrete for the two beams would be approximately the same and the time-dependent strains would be $B_{t1}\epsilon_i$ and $B_{t2}\epsilon_i$):

$$\frac{E_{b1}}{E_{b2}} = \frac{B_{t1}}{B_{t2}} = \frac{1}{\alpha} \tag{7}$$

Using Eqs. (5), (6) and (7):

$$\alpha = \frac{1 + \dfrac{E'_sA'_{s1}}{E_sA_s}\dfrac{\int_0^L \epsilon'^2_{ci}dx}{\int_0^L \epsilon^2_{ci}dx}}{1 + \dfrac{E'_sA'_{s2}}{E_sA_s}\dfrac{\int_0^L \epsilon'^2_{ci}dx}{\int_0^L \epsilon^2_{ci}dx}} \tag{8}$$

where ϵ_{ci} and ϵ'_{ci} are initial strains at the levels of prestressing steel and non-tensioned steel, respectively, and are computed as:

$$\epsilon_{ci} = \frac{1}{E_p}\left[\frac{F_o}{A} + \frac{F_oe_xe_x}{I} - \frac{M_xe_x}{I}\right] \tag{9}$$

$$\epsilon'_{ci} = \frac{1}{E_p}\left[\frac{F_o}{A} + \frac{F_oe_xe'_x}{I} - \frac{M_xe'_x}{I}\right] \tag{10}$$

For constant eccentricities (i.e. $e_x = e$ and $e'_x = e'$):

$$\int_0^L \epsilon^2_{ci}dx = \frac{1}{E_p^2}\left[\left(\frac{F_o}{A} + \frac{F_oe^2}{I}\right) - \frac{wL^2e}{6I}\left(\frac{F_o}{A} + \frac{F_oe^2}{I}\right) + \frac{1}{120}\left(\frac{wL^2e}{I}\right)\right] \tag{11}$$

$$\int_0^L \epsilon'^2_{ci}dx = \frac{1}{E_p^2}\left[\left(\frac{F_o}{A} + \frac{F_oee'}{I}\right) - \frac{wL^2e}{6I}\left(\frac{F_o}{A} + \frac{F_oee'}{I}\right) + \frac{1}{120}\left(\frac{wL^2e'}{I}\right)\right] \tag{12}$$

It is noted here that for the case of constant eccentricities, the x-dependent part in Eqs. (11) and (12) is that due to M_x, and is relatively small. As the quantity of interest is the ratio

$$\frac{\int_0^L \epsilon'^2_{ci}dx}{\int_0^L \epsilon^2_{ci}\,dx}$$

the following approximation may be made:

$$\frac{\int_0^L \epsilon'^2_{ci}dx}{\int_0^L \epsilon^2_{ci}\,dx} = \left(\frac{\epsilon'^2_{ci}}{\epsilon^2_{ci}}\right)_{L/4} \tag{13}$$

where the subscript $L/4$ indicates that the ratio is computed at the quarter-point of the span using Eq. (13). Eq. (8) may be rewritten as:

$$\alpha = \frac{1 + \dfrac{E'_sA'_{s1}}{E_sA_s}\left(\dfrac{\epsilon'^2_{ci}}{\epsilon^2_{ci}}\right)_{L/4}}{1 + \dfrac{E'_sA'_{s2}}{E_sA_s}\left(\dfrac{\epsilon'^2_{ci}}{\epsilon^2_{ci}}\right)_{L/4}} \tag{14}$$

However if $e_x = e'_x$ for all values of A'_s, then Eq. (8) reduces to:

$$\alpha = \frac{1 + \dfrac{E'_sA'_{s1}}{E_sA_s}}{1 + \dfrac{E'_sA'_{s2}}{E_sA_s}} \tag{15}$$

Further, in the case of bonded cables, the effective modulus of elasticity of the prestressing steel is approximately the same as that of non-tensioned steel bars, and Eq. (15) reduces to:

$$\alpha = \frac{1 + \dfrac{A'_{s1}}{A_s}}{1 + \dfrac{A'_{s2}}{A_s}} \tag{16}$$

To establish a comparison of camber coefficients of two beams, one without non-tensioned steel and the other with non-tensioned steel, let $A'_{s1} = 0$ and $A'_{s2} = A'_s$. Eq. (16) is written as:

$$\alpha = \frac{1}{1 + A'_s/A_s} \tag{17}$$

Modification factors for creep and shrinkage may be derived on a similar basis [12,13]. For the condition that Beam 1 contains no non-tensioned steel, the following expressions are obtained:

$$\alpha_c = \frac{C_{t1}}{C_{t2}} = \frac{1}{1 + \dfrac{E'_sA'_s\int_0^L \epsilon'^2_{ci}dx}{E_sA_s\int_0^L \epsilon^2_{ci}dx}} \tag{18}$$

$$\alpha_{sh} = \frac{(\epsilon_{sh})_1}{(\epsilon_{sh})_2} = \frac{1}{1 + \dfrac{E'_s A'_s}{E_s A_s}} \qquad (19)$$

Under the same assumptions as in the case of the modification factor, α, the alternate expressions for α_c are:

$$\alpha_c = \frac{1}{1 + \dfrac{E'_s A'_s}{E_s A_s}\left(\dfrac{\epsilon'^2_{ci}}{\epsilon^2_{ci}}\right)_{L/4}} \qquad (20)$$

$$\alpha_c = \frac{1}{1 + \dfrac{E'_s A'_s}{E_s A_s}} \qquad (21)$$

and for the condition $E'_s = E_s$:

$$\alpha_c = \frac{1}{1 + A'_s/A_s} \qquad (22)$$

$$\alpha_{sh} = \frac{1}{1 + A'_s/A_s} \qquad (23)$$

Table 2. Comparison of modification factor with ACI reduction factor for compression steel

$\frac{A'_s}{A_s}$	α Eq. (17)	Reduction factor for compression steel
1.0	0.50	0.40
0.5	0.67	0.65
0	1.00	1.00

Some comments on the previously derived equations are in order:

1. Even though the modification factors α, α_c and α_{sh} were assumed to be functions of time, the resulting final expressions are independent of time. This was verified by the observed camber behavior of the test beams.
2. It was observed that the reduction of time-dependent camber is not directly proportional to the area of non-tensioned steel. As a matter of fact a law of diminishing returns seems to apply, as shown in Fig. 4, which plots Eqs. (17), (22) and (23).
3. For very large amounts of non-tensioned steel, the modification factors previously derived would no longer be accurate, because of the transformed area effects. However, from design considerations, large percentages of p' are rarely used because of the desirability of achieving ductile (under-reinforced) beams and because of economic considerations.
4. The effect of non-tensioned steel used to control time-dependent camber is similar to the effect of compression steel on deflections of ordinary reinforced concrete beams due to creep and shrinkage. Table 2 compares Eq. (17) to the reduction factor for compression steel in reinforced concrete beams proposed by ACI Committee 435[14], and as found in the ACI Building Code (318-63).

Comparison of theoretical and test results. The time-dependent camber curves for the test beams are shown in Fig. 5. The top curve in each figure is labeled 100 percent, and refers to the beam without non-tensioned steel or with the minimum amount of non-tensioned steel in each of the four series. The other percentages are the average observed time-dependent cambers of the other two beams in each series compared to the beam with minimum non-tensioned steel.

A comparison between test results and theoretical results is made in terms of the modification factor, α, for the camber coefficient, B_t. Table 3 shows the observed and computed values of α along with the observed range of the values of α. The ob-

Table 3. Experimental and theoretical values of the modification factor and of initial and time-dependent camber

Series No.	I			II			III			IV		
Beam No.	1	2	3	1	2	3	1	2	3	1	2	3
A'_s/A_s	1.15	2.30	3.46	0.50	1.73	3.46	0	0	1.30	0.50	1.94	3.76
Experimentally observed range of α, %	100 to 100	70 to 77	60 to 65	100 to 100	62 to 66	32 to 40	100 to 100	70 to 80	43 to 50	100 to 100	78 to 85	39 to 45
Average of the observed values of α, %	100	74	63	100	65	37	100	77	48	100	83	42
Theoretical value of α, Eq. (8), %	100	74	56	100	56	42	100	73	48	100	54	44
Theoretical value of α, Eq. (13), %	100	72	55	100	55	40	100	71	45	100	51	42
Observed initial camber, in.	0.251	*	0.252	*	0.142	0.140	0.225	0.225	0.219	0.204	0.205	0.184
Computed initial camber, in.	0.254	0.254	0.254	0.138	0.138	0.138	0.218	0.218	0.218	0.202	0.202	0.202
Observed time-dependent camber, in.	0.254	0.191	0.157	0.123	0.075	0.042	0.249	0.213	0.106	0.130	0.106	0.041
Time-dependent study period, days	172	172	172	140	140	140	124	124	124	123	123	123

* Readings could not be obtained as beam shifted significantly upon release of prestress force.

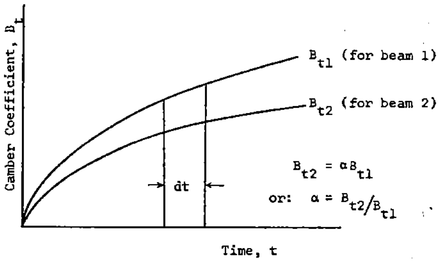


Fig. 3. Camber coefficient vs. time

served range is seen to be fairly narrow, thus verifying the theoretical conclusion that α is independent of time. Table 3 also gives the computed and observed values of initial camber and observed time-dependent camber. A comparison of modification factors is seen to be good, except in the case of Beam IVB2. The discrepancy in this case is attributed to experimental errors.

EFFECT OF NON-TENSIONED STEEL ON LOSS OF PRESTRESS

The initial prestress force applied to a prestressed concrete beam decreases at a decreasing rate with time. The major contribution to the loss of prestress (usually 70 to 80 percent of the total loss) is due to shrinkage and creep of the concrete.

In a prestressed concrete beam without non-tensioned steel any loss of prestress force results in an equivalent reduction of force on the concrete section. However, when non-tensioned steel is included in a prestressed concrete beam, this reduction of force on the concrete is equal to the loss of prestress force plus the force transferred to the non-tensioned steel. Thus, when non-tensioned steel is used, a distinction between the loss of prestress and the reduction of force on the concrete must be made. A determination of the reduction of force on the concrete permits an evaluation of the change in stress level in concrete from which the net stress in concrete can be computed. This net effective stress in concrete is of primary importance from the point of view of creep rate and behavior (deflections and extent of cracking) under service loads.

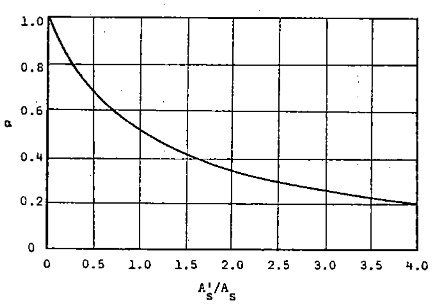


Fig. 4. Relation between modification factor and steel ratios

It has been shown (Fig. 4) that non-tensioned steel reduces creep and shrinkage strains. This reduction in strains results in a reduction in the loss of prestress. To arrive at a relationship between the loss of prestress and the reduction of concrete force, consider two beams: Beam 1 is without non-tensioned steel and Beam 2 contains some non-tensioned steel. Define two parameters β and γ as:

$$\beta = \frac{\text{loss of prestress for Beam 2}}{\text{loss of prestress for Beam 1}} \qquad (24)$$

$$\gamma = \frac{\text{reduction of force in concrete for Beam 2}}{\text{reduction of force in concrete for Beam 1}} \qquad (25)$$

This loss of prestress ratio due to creep effects, β_c, may be expressed as:

$$\beta_c = \frac{A_s E_s C_{t2} \dfrac{1}{L}\displaystyle\int_0^L \epsilon_{ci} dx}{A_s E_s C_{t1} \dfrac{1}{L}\displaystyle\int_0^L \epsilon_{ci} dx}$$

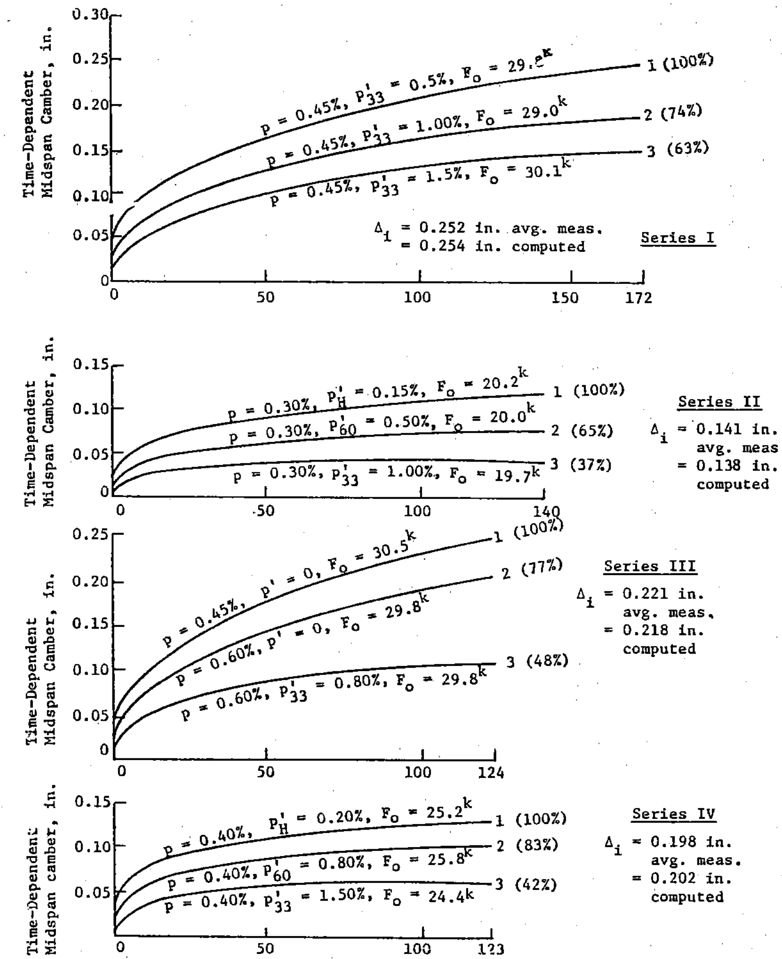


Fig. 5. Time dependent midspan camber (total minus initial) plotted against time in days

$$\beta_c = \frac{C_{t2}}{C_{t1}} = \alpha_c \qquad (26)$$

and the ratio of concrete force reduction due to creep, γ_c, as:

$$\gamma_c = \frac{A_s E_s C_{t2} \dfrac{1}{L}\displaystyle\int_0^L \epsilon_{ci} dx + A_s' E_s' C_{t2} \dfrac{1}{L}\displaystyle\int_0^L \epsilon_{ci}' dx}{A_s E_s C_{t1} \dfrac{1}{L}\displaystyle\int_0^L \epsilon_{ci} dx}$$

$$\gamma_c = \alpha_c \left\{ 1 + \frac{A_s' E_s' \displaystyle\int_0^L \epsilon_{ci}' dx}{A_s E_s \displaystyle\int_0^L \epsilon_{ci} dx} \right\} \qquad (27)$$

If $e_x = e_x'$, then:

$$\int_0^L \epsilon_{ci}' dx = \int_0^L \epsilon_{ci} dx$$

$$\alpha_c = \frac{1}{1 + \dfrac{A_s' E_s'}{A_s E_s}}$$

Table 4. Computed loss of prestress and reduction of concrete force for the test beams

Series No.	I			II			III			IV		
Beam No.	1[1]	2	3	1[1]	2	3	1[1]	2	3	1[1]	2	3
A'_s/A_s	1.15	2.30	3.46	0.50	1.73	3.46	0	0	1.30	0.50	1.94	3.76
β, Eq. (31)[2]	1.00	0.74	0.56	1.00	0.56	0.42	1.00	0.73	0.48	1.00	0.54	0.44
γ, Eq. (32)[2]	1.00	1.05	1.08	1.00	1.00	1.09	1.00	1.00	1.12	1.00	1.00	1.13

[1] The beam in each of the four series without non-tensioned steel or with the minimum amount.

[2] Values of β and γ used to compute the loss of prestress and reduction in the concrete force for the test beams. For example, if the loss of prestress and the reduction of the concrete force in Beam IB1 are $k_{st}F_o$ and $k_{ct}F_o$, respectively, then the loss of prestress and the reduction of the concrete force of Beam IB2 would be $\beta k_{st}F_o$ (where $\beta = 0.74$) and $\gamma k_{ct}F_o$ (where $\gamma = 1.05$), respectively.

Table 5. Number, height and distribution of visible cracks in test beams

Series No.	I			II			II			III			IV			IV		
Load, P	6 kips			5 kips			6 kips			6 kips			6 kips			8 kips		
Beam No.	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
No. of cracks[1]	7	12	9	7	8	7	8	10	11	7	7	7	7	9	9	12	13	12
Max. height of a crack[2] (in.)	4.00	4.55	3.20	5.20	4.55	4.55	5.68	5.20	5.12	4.8	4.64	4.00	4.55	4.00	3.44	5.44	4.48	4.08
Ave. height of cracks (in.)	3.60	3.20	2.04	4.64	4.00	3.84	5.20	4.80	4.48	4.40	3.20	3.20	4.16	3.52	2.56	4.80	4.16	3.76
Length of beam cracked[3] (ft.)	3.2	4.4	5.0	3.4	3.7	5.1	3.7	4.8	5.2	3.8	3.5	4.6	3.7	4.8	5.3	4.4	5.4	5.8

[1] Refers to cracks in one half of each test beam. Observations of the other half indicated approximately the same results.

[2] The beams were all 8 in. deep.

[3] Refers to the distance over which the cracks in one half of each beam were distributed. The beams were all 15 ft. long.

$$\gamma_c = 1 \tag{28}$$

Similarly during shrinkage:

$$\beta_{sh} = \alpha_{sh} \tag{29}$$

$$\gamma_{sh} = 1 \tag{30}$$

For the camber coefficient:

$$\beta = \alpha \tag{31}$$

$$\gamma = \alpha \left\{ 1 + \frac{A'_s E'_s \int_0^L \epsilon'_{ci} dx}{A_s E_s \int_0^L \epsilon_{ci} dx} \right\} \tag{32}$$

If $e_x = e'_x$

$$\gamma = 1 \tag{33}$$

Thus, if a beam without non-tensioned steel has an initial prestress force, F_o, and loss of prestress at a particular time, t, of $(\Delta F_{t_{sh}} + \Delta F_{t_c})$, then the reduction of concrete force will be equal to the loss of prestress, $\Delta F_{t_{sh}} + \Delta F_{t_c}$. If non-tensioned steel is provided in this beam such that modification factors to shrinkage and creep are α_{sh} and α_c, respectively, then the loss of prestress would be $\alpha_{sh}\Delta F_{t_{sh}} + \alpha_c \Delta F_{t_c}$ and the reduction of concrete force would be $\gamma_{sh}\Delta F_{t_{sh}} + \gamma_c \Delta F_{t_c}$, where γ_{sh} and γ_c are given by Eqs. (30) and (27) respectively. If the combined coefficient, B_t, is used and if the loss of prestress in the beam without non-tensioned steel is ΔF_t, then the reduction of concrete force is ΔF_t. However, in the beam with non-tensioned steel the loss of prestress would be $\alpha \Delta F_t$, and the reduction of concrete force would be $\gamma \Delta F_t$, where γ is given by Eq. (32).

The computed loss of prestress and the computed reduction of concrete force are compared in Table 4. For this comparison define a prestress loss coefficient, k_{st}, such that the loss of prestress will be equal to $k_{st}F_o$. Also let k_{ct} be a reduction of concrete force coefficient such that the reduction of concrete force will be equal to $k_{ct}F_o$. The loss of prestress and the reduction of concrete force for the other two beams in each of the four series are expressed relative to the beam with the minimum amount of non-tensioned steel in each series. It is noted that, in the case of a beam without non-tensioned steel, $k_{st} = k_{ct}$.

Table 4 shows that the total concrete force is relatively insensitive to the provision of non-tensioned steel (it is invariant for $e = e'$ since $\gamma_c = 1$, $\gamma_{sh} = 1$ and $\gamma = 1$). In other words, any reduction in the loss of prestress appears as the force in the non-tensioned steel. Even for $e'/e = 0.5$ (Beam IVB3 which contains the maximum amount of non-tensioned steel), where the loss of prestress is 56 percent less than that in Beam IVB1, the reduction of concrete force is only 13 percent more than that in Beam IVB1. In practice the ratio e'/e is usually close to 1 and thus, for all practical purposes, it may be assumed that the provision of non-tensioned steel does not influence the effective force in concrete.

CRACK FORMATION, DEFLECTION AND ULTIMATE STRENGTH BEHAVIOR

The existing philosophy for the design of prestressed concrete members is to allow either no tensile stresses under working loads (fully prestressed concrete) or no cracking under working loads, even though some tensile stresses may exist (a limited form of partially prestressed concrete). Nevertheless, the behavior of cracked prestressed concrete members is of importance from the point of view of overloads. Knowledge regarding ultimate strength is of interest in providing criteria for

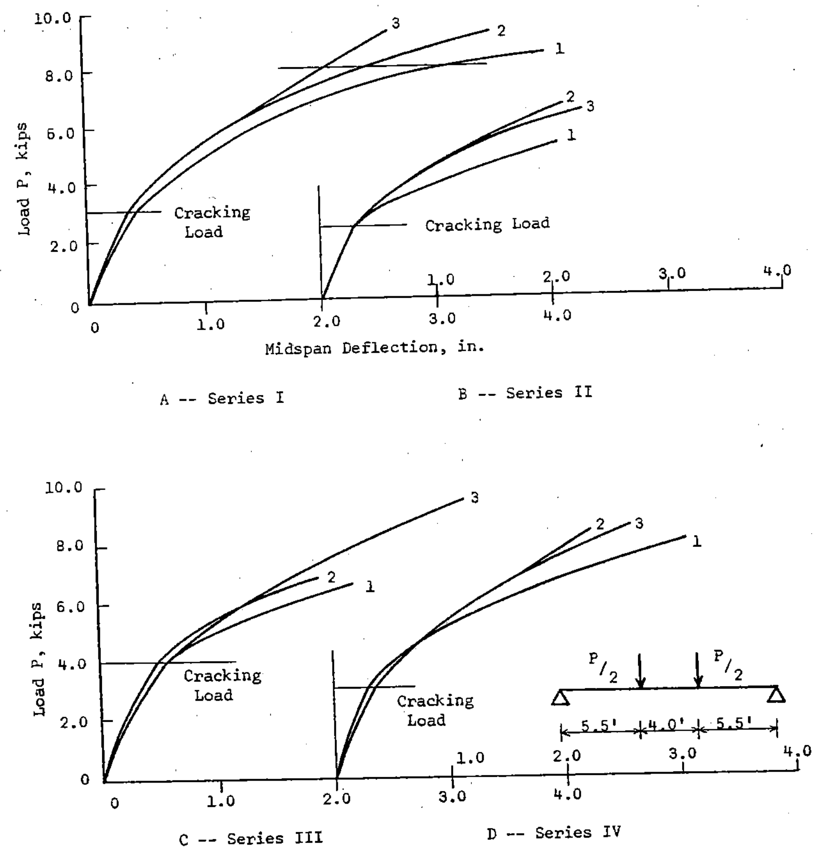
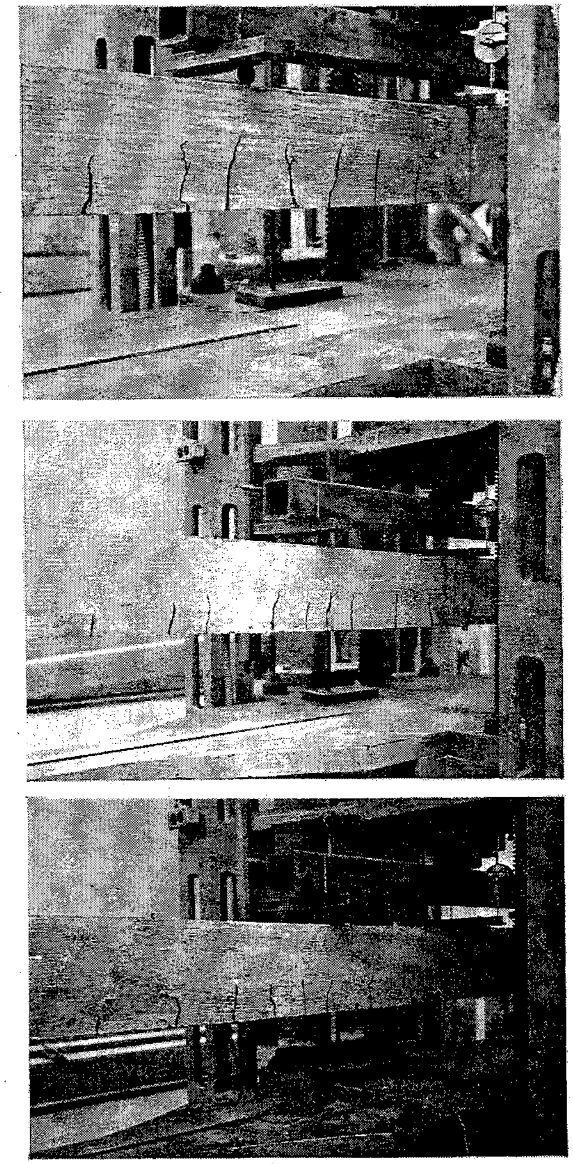


Fig. 6. Observed midspan deflections vs. load

design. Increasing interest is also being shown in the design of prestressed concrete members that would crack under working loads. Since substantial cracking occurs under working loads in ordinary reinforced concrete members, cracking in prestressed concrete members should be acceptable provided that all safety and serviceability requirements are met. The presence of prestress force might then be considered an advantage as compared to the corresponding reinforced concrete member.

The behavior of prestressed concrete members and ordinary reinforced concrete members is similar under cracked conditions. Consequently, the extensive work that has been done on ordinary reinforced concrete members should provide a strong basis for predicting the behavior of cracked prestressed concrete members.

It is reasonable to expect that non-tensioned steel does not influence the cracking moment of prestressed concrete beams. This appears to be verified by the observed load-deflection response of the test beams (Fig. 6). The curves seem to deviate from an initial linear relationship at about the same load for all beams



Beam IV B1

$p = 0.40\%$

$p'_H = 0.20\%$

$F_o = 25.2$ kips

$p\frac{f_{su}}{f'_c} + p'\frac{f_y}{f'_c} = 0.28$

$P_{ult.} = 8.95$ kips

Beam IV B2

$p = 0.40\%$

$p'_{60} = 0.80\%$

$F_o = 25.8$ kips

$p\frac{f_{su}}{f'_c} + p'\frac{f_y}{f'_c} = 0.28$

$P_{ult.} = 10.11$ kips

Beam IV B3

$p = 0.40\%$

$p'_{33} = 1.50\%$

$F_o = 24.4$ kips

$p\frac{f_{su}}{f'_c} + p'\frac{f_y}{f'_c} = 0.29$

$P_{ult.} = 9.23$ kips

Fig. 7. Condition of Beams IVB1, IVB2 and IVB3 under a load of 6 kips

in each of the four series. For example, it is seen that in the case of Series I this load is 3.3 kips. The three beams differ only in the amount of non-tensioned steel.

Whereas non-tensioned steel does not influence first cracking, it has quite a significant cumulative effec on the number, height and distribu tion of cracks. Studies of crack for mation were made on an area o one-half of one side of each of th test beams. Other areas of the tes beams exhibited similar crack for

Table 6. Comparisons of computed and measured values for the test beams

Series No.	I			II			III			IV		
Beam No.	1	2	3	1	2	3	1	2	3	1	2	3
Observed ultimate load, P_u (kips)	8.7	9.4	9.9	6.8	7.2	7.3	7.0	9.3	9.7	8.9	10.1	9.2
Working load, P_w[1] (kips)	3.3	3.3	3.3	2.3	2.3	2.3	4.0	4.0	4.0	3.0	3.0	3.0
Load factor, P_u/P_w	2.62	2.84	3.00	2.96	3.14	3.16	1.75	2.32	2.42	2.96	3.36	3.06
P_{max}[2] (kips)	8.3	9.3	9.5	5.5	6.5	6.5	6.5	7.5	9.5	8.0	8.5	8.5
$P_{max}/P_u \times 100$	95	99	96	81	90	89	93	81	98	90	84	92
P_Δ[3] (kips)	7.0	8.0	8.0	5.5	6.0	6.0	6.0	7.0	8.0	8.0	8.0	8.0
Applied overload ratio, P_Δ/P_w[4]	2.12	2.42	2.42	2.38	2.60	2.60	1.50	1.75	2.00	2.67	2.67	2.67
Worst discrepancy in deflection[5] (percent)	+15	+13	+17	−14	+6	−3	−19	−3	+12	+3	+9	+15
Observed M_u (k-ft.)	23.8	25.9	27.4	18.6	19.9	20.0	19.2	25.6	26.8	24.6	27.8	25.4
Computed M_u (k-ft.)	21.3	23.8	26.2	19.0	19.8	19.8	19.5	25.6	29.6	24.6	27.7	26.1
Ratio of observed to computed M_u	1.12	1.09	1.05	0.98	1.00	1.01	0.98	1.00	0.91	0.99	1.00	0.97

(1) For the test beams, the working load was assumed to represent the condition that cracking would occur as soon as this load is exceeded. P_w were computed values. Note that the load factors, even for this assumption, tend to be on the high side for the test beams.

(2) Represents the maximum load for which deflections were recorded.

(3) Represents the load at which the discrepancy between the observed and computed values of deflection is the greatest.

(4) Gives an indication of the range of overload in which the discrepancy in deflections is the greatest.

(5) Plus or minus indicate that computed deflection is greater than or smaller than the observed deflection.

mation behavior. Table 5 lists the number, maximum and average heights of cracks, and the length of the cracked portion of the beam for each half of the test beams.

As an example, in the case of Beam IVB1 under a load of 6 kips (Table 5 and Fig. 7) there are 7 visible cracks in a length of 3.7 feet with a maximum height of crack of 4.55 in. and an average height of cracks of 4.16 in. The corresponding quantities for Beams IVB2 and IVB3 are: 9 cracks over 4.8 ft., 4.00 in. max. and 3.52 in. ave.; 9 cracks over 5.3 ft., 3.44 in. max. and 2.56 in. ave. The three beams are identical except that Beam IVB2 contains 4 times as much non-tensioned steel and Beam IVB3 contains 8 times as much non-tensioned steel as Beam IVB1. All have roughly the same ultimate load capacity as shown in Fig. 7.

Deflections. The similarity of the behavior of prestressed concrete members and ordinary reinforced concrete members under cracked conditions led to the investigation of the available methods given in the literature[15,16,17,18,19] for computing deflections of reinforced concrete members. Since ordinary reinforced concrete is normally cracked under working loads, most methods for computing these deflections do take into account the effect of flexural cracking.

Branson's method[14, 18] was used to compute the deflections of test beams. Based on a sizable number of tests on rectangular beams (simple span and continuous) and T-beams, Branson[18] presented an empirical expression for the effective moment of inertia at a given section, I_{eff}. The expression is given in a form that includes the effect of cracking as:

$$I_{eff} = \left(\frac{M_{cr}}{M}\right)^4 \times I_g + \left[1 - \left(\frac{M_{cr}}{M}\right)^4\right] I_{cr} \qquad (34)$$

where M = moment at a particular section

I_{cr} = moment of inertia of the cracked transformed section.

An expression for an average effective moment of inertia for the entire length of a simply supported beam under uniformly distributed load is also given:

$$I_{eff} = \left(\frac{M_{cr}}{M_{max}}\right)^3 \times I_g + \left[1 - \left(\frac{M_{cr}}{M_{max}}\right)^3\right] I_{cr} \qquad (35)$$

where M_{max} = maximum moment ir the span. Eqs. (34) and (35) apply only when M or M_{max} is greater thar M_{cr}; otherwise $I_{eff} = I_g$ (or I_t).

For continuous beams, the average of the values for positive and nega tive moment regions is recom mended[14,19]. Although Eq. (35 was originally established for simpl supported beams under uniforml distributed loads, its use is consid ered quite adequate for two-poin test loading as well as for othe loads that are approximately sym metrical about the center line c the beam.

The effect of non-tensioned stee on deflections under cracked cond tions is evident from Fig. 6. Th deflections of the beams with mor non-tensioned steel are considerabl less than the deflections at corr sponding loads of identical bean containing smaller amounts of no tensioned steel. For example, Bea IB1 under a load of 8 kips shows deflection of 2.88 in., whereas Bea

IB3, which contains three times as much non-tensioned steel, shows a deflection of only 1.95 in.

Midspan deflections of the test beams were measured at loads ranging from 81 to 99 percent of the ultimate loads. Eq. (35), along with the following expressions for M_{cr} and I_{cr}, was used to compute these deflections.

$$M_{cr} = F_t e_x + \frac{F_t I_g}{A\, y_t} + \frac{f'_{cb} I_g}{y_t} \tag{36}$$

$$I_{cr} = \frac{b(kd)^3}{3} + n\, A_s\, (d - kd)^2 + n'\, A'_s\, (d' - kd)^2 \tag{37}$$

where $k =$

$$\sqrt{(np + n'p')^2 + \left(2np + 2n'p'\frac{d'}{d}\right)} - (np + n'p') \tag{38}$$

The modulus of rupture, f'_{cb}, was obtained by bending tests on plain concrete specimens of the test beams.

Maximum discrepancies in observed and computed values of deflection are indicated in Table 6. Table 6 also gives the maximum loads for which the deflections were recorded.

The midspan deflections shown in Fig. 6 are relative to the positions of the beams just before the application of the transverse load. If the deflections from the positions of the beams before prestressing are desired, the total camber (initial plus time-dependent, Table 3) just prior to the application of the two-point loading must be subtracted from the deflections in Fig. 6.

Note that, with the use of non-tensioned steel, *greater* not smaller net deflections (as referred to the position of the beam before prestressing) occur under working load. This is because non-tensioned steel reduces time-dependent camber and thus, there is less total camber to be cancelled before the beam deflects downward from the position of the beam before prestressing.

For example, in the case of Beams IB1 and IB3 (having non-tensioned steel percentages of 0.5 and 1.5 percent respectively) the total camber values are $0.251 + 0.254 = 0.505$ in. and $0.252 + 0.157 = 0.409$ in. respectively (Table 3). Under a transverse load of 4 kips the observed deflections (Fig. 6) for the two beams are 0.534 in. and 0.514 in. respectively. Thus the deflections relative to the positions before prestressing are $0.534 - 0.505 = 0.029$ in. and $0.514 - 0.409 = 0.105$ in. respectively. Whereas the deflection of Beam IB3 relative to its position just before application of the transverse load is smaller than the corresponding deflection of Beam IB1, its deflection relative to the position before prestressing is significantly greater.

After first cracking, the increase in deflection of a beam with a smaller amount of non-tensioned steel will be greater than the increase in deflection of an identical beam containing a larger amount of non-tensioned steel. This is due to a better distribution of cracks and a reduction in the extent of crack development with a greater amount of non-tensioned steel. The net deflection (relative to the position of the beam before prestressing) of the beam with a larger amount of non-tensioned steel may be greater, comparable or considerably smaller depending on whether the applied transverse load is approximately equal to, somewhat greater than, or considerably greater than the cracking load.

In the case of most prestressed concrete beams with non-tensioned steel, under increasing load the non-tensioned steel would yield before the ultimate load of the beam is reached. This will certainly be the case if the non-tensioned steel is of lower strength than the prestressing steel, and the beam is under-reinforced. However, for the usual percentages of steel, the reserve strength after yielding of non-tensioned steel is only a small percentage of the ultimate strength of the beam due to the precompression in the non-tensioned steel.

This observation is corroborated by the load-deflection response of the test beams. The only beam in which non-tensioned steel yielded (that is, up to the maximum load for which the deflection was recorded) is Beam IB1. The contribution of its non-tensioned steel to the quantity, $p\frac{f_{su}}{f'_c} + p'\frac{f_y}{f'_c}$, is the least (about 14 percent) of all the test beams. The yielding of non-tensioned steel seems to have originated at a load of about 8 kips. The observed deflection is smaller than the computed deflection at 8 kips, but grows rapidly thereafter. Between 8 kips and 8.34 kips, the increase in observed deflection is about three times the increase in computed deflection. Even in the case of Beam IB1, the load of 8 kips amounts to about 92 percent of the ultimate load.

CONCLUSIONS

1. The use of non-tensioned steel in prestressed concrete beams may necessitate the use of uncracked transformed section properties as opposed to gross section properties for reasonable accuracy (see Fig. 1).

2. The effect of non-tensioned steel on time-dependent camber is primarily due to restraints imposed on creep and shrinkage of the concrete as embodied in the modification factors α_c and α_{sh} (see Eqs. (18) through (23)). The factor α that combines these two effects is given by Eqs. (8) and (14) through (17). The simplified Eqs. (15), (22) and (23), in which $e_x = e'_x$, could be used in practice to estimate the gross effect of non-tensioned steel in reducing prestress loss and camber when the eccentricities of the tensioned and non-tensioned steels are approximately equal and on the same side of the centroidal axis.

3. The effect of non-tensioned steel in reducing time-dependent camber of prestressed concrete beams is similar to the effect of compressive reinforcement in reducing long-time deflections of ordinary reinforced concrete members (see Table 2).

4. A distinction must be made between the loss of prestress force and the reduction of the concrete force in beams containing non-tensioned steel. The loss of prestress is greatly reduced due to the restraining action of the non-tensioned steel on the creep and shrinkage of the concrete. However, the total effective concrete force is quite insensitive to the provision of the non-tensioned steel. (See the discussion of β and γ defined by Eqs. (24) and (25), and the results presented in Table 4.)

5. From a practical point of view, the non-tensioned steel does not influence first cracking (i.e. cracking moment) of prestressed concrete beams.

6. If the net deflection under working loads is downward relative to the position of the beam before prestressing, then, by using non-tensioned steel, this deflection would be larger (assuming no cracking has

occured) because of the substantially reduced time-dependent camber.

7. Whereas non-tensioned steel does not have a substantial effect on first cracking, its effect on subsequent crack formation is quite pronounced. The additional bonded, non-tensioned steel tends to distribute cracks and restrict their progression. Increased flexural rigidity and reduced deflections under cracked conditions are thus realized (see Fig. 6).

8. The total deflection of a prestressed concrete beam containing non-tensioned steel, when compared to the deflection of an identical beam without non-tensioned steel, may be greater if the applied load is equal to the cracking load; comparable if the applied load is slightly larger than the cracking load; or considerably smaller if the applied load is considerably larger than the cracking load.

9. Due to the similarity between the behavior of ordinary reinforced concrete and prestressed concrete under cracked conditions, the methods used for computing deflections of ordinary reinforced concrete members may be applied to the deflections of cracked prestressed concrete members. This is accomplished by properly defining the cracking moment and the effective moment of inertia (see Eqs. (35) through (38) and Table 6). This method predicted deflections up to 80 percent of the ultimate load within 19 percent of the measured values in all cases.

10. For all normal provisions of non-tensioned steel, yielding (even for a 33 ksi yield steel) occurs close to the ultimate load and deflections at loads of about 80 percent of the ultimate load may be computed by assuming that the non-tensioned steel has not yielded.

11. Regarding the contribution of non-tensioned steel to the ultimate strength of an under-reinforced prestressed concrete beam, the usual practice of considering that the non-tensioned steel provides a tension force equal to its area times its stress at ultimate is satisfactory.

12. The selection of type and quantity of non-tensioned steel should be based on the behavior desired under various service conditions: desired reduction in time-dependent camber, acceptable deflections under working loads, desirability of limiting deflections under overloads, and the required factor of safety against failure.

13. The only unfavorable effects appear to be the possibility of greater deflections under working loads (see conclusions 6, 7, 8). In general, non-tensioned steel affords a powerful means which, with proper judgment, can be used to meet even the severest serviceability and safety requirements of prestressed concrete beams.

NOTATION

A_g = area of gross concrete section

A_t = area of uncracked transformed concrete section

A_s = area of prestressing steel

A'_s = area of non-tensioned steel

B_t = camber coefficient for prestressed concrete beam defined as the ratio of time-dependent camber to initial camber

C_t = creep coefficient defined as the ratio of creep strain to initial strain

E_p = modulus of elasticity of concrete at the time of prestressing

E_b = internal strain energy of a beam

e = eccentricity of steel

F_o = prestress force at release

ΔF_t = loss of prestress force at time, t

F'_t = force in non-tensioned steel at time, t

f'_c = 28-day concrete strength

f'_{cb} = modulus of rupture of concrete

f'_s = nominal ultimate strength of prestressing steel

f_{su} = calculated stress in prestressing steel at ultimate load

f_y = nominal yield strength of steel

h = total depth of a beam

I = moment of inertia

I_g = moment of inertia of gross concrete section

I_t = moment of inertia of uncracked transformed concrete section

I_{cr} = moment of inertia of cracked transformed concrete section

I_{eff} = effective moment of inertia of concrete section

k = coefficient determining the depth of neutral axis under cracked conditions

k_{st} = prestress loss coefficient

k_{ct} = reduction of concrete force coefficient

L = beam span (center to center of supports)

M = bending moment

M_{cr} = cracking moment

M_{max} = maximum moment in a beam

n = modular ratio: $n = E_s/E_c$; $n' = E'_s/E_c$

p = ratio of area of steel to area of concrete: $p = A_s/bd$; $p' = A'_s/bd$ (p'_{33} for 33 ksi yield steel, p'_{60} for 60 ksi yield steel and p'_H for high strength steel)

W_d = work done against dead load of a beam

W_s = work done by force in steel

w = uniform distributed beam dead load

y = camber of a beam

y_t = distance of extreme tension fiber from centroid of concrete section

α = modification factor for combined time-dependent camber coefficient

α_c = modification factor for creep coefficient

α_{sh} = modification factor for shrinkage strain

β = ratio of loss of prestress in a beam *with* non-tensioned steel to the loss of prestress in a beam *without* non-tensioned steel

β_c = ratio of loss of prestress due to creep in a beam *with* non-tensioned steel to the loss of prestress in a beam *without* non-tensioned steel

β_{sh} = ratio of loss of prestress due to shrinkage in a beam *with* non-tensioned steel to the loss of prestress in a beam *without* non-tensioned steel

γ = ratio of reduction of the concrete force in a beam *with* non-tensioned steel to reduction of the concrete force in a beam *without* non-tensioned steel

γ_c = ratio of reduction of the concrete force due to creep in a beam *with* non-tensioned steel to reduction of the concrete force in a beam *without* non-tensioned steel

γ_{sh} = ratio of the reduction of the concrete force due to shrinkage in a beam *with* non-tensioned steel to reduction of concrete force in a beam *without* non-tensioned steel

ϵ_{ci} = initial concrete strain at the level of steel

ϕ = curvature or angle change per unit length of the beam

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ALLOWABLE TENSILE STRESSES FOR PRESTRESSED CONCRETE

Prepared by
PCI Committee on Allowable Stresses in Prestressed Concrete Design
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Shortly after the ACI Building Code (ACI 318-63) was revised in 1963, there was concern that some of the Code provisions for prestressed concrete products made it difficult, in many instances, for the products to meet serviceability requirements. This committee was formed to recommend higher allowable tensile stresses in prestressed concrete flexural members while maintaining serviceability requirements. The work of the committee was largely responsible for the changes in allowable tensile stresses for prestressed concrete in the draft of the revised ACI Building Code (see ACI Journal, February 1970). The committee report gives the rationale behind the use of higher tensile stresses in flexural members. Discussion of this report is invited.

—George G. Goble, *Chairman*

NOTATION

A = overload factor

f_c = apparent cracking stress*

f_t = allowable tensile stress

f_o = difference between cracking stress and allowable service load tensile stress

f_s = stress change in the bottom fiber due to superimposed load

I_c = cracked section moment of inertia**

I_g = gross section moment of inertia

$R = \dfrac{I_g}{I_c}$

Δ = allowable deflection at overload

δ = deflection due to superimposed service load computed using I_g

δ_g = deflection at which cracking occurs

δ_c = added deflection after crack-

**For normal percentages and good distribution of bonded steel, f_c can be taken as $10\sqrt{f'_c}$.*

***The cracked section moment of inertia shall be calculated neglecting the effect of the prestress force on the location of the neutral axis. I_c is approximately equal to $nA_sd^2(1-\sqrt{p})$, an empirical relationship that may be used in lieu of a more exact analysis.*