SP 55-7

Stress-Strain Characteristics of High-Strength Concrete

By P.H. Kaar, N.W. Hanson, and H.T. Capell

Synopsis: The stress-strain relationship and flexural stress distribution for ultimate strength design has been well established from previous work. Generally, normal-weight concretes with strengths ranging from 1,000 psi to 7,500 psi (6.9 MPa to 51.7 MPa) have been investigated. In the present study, flexural characteristics of high-strength concretes were obtained from a series of specimens tested at the Portland Cement Association laboratories.

The test series included concrete strengths ranging from 6,500 psi to 14,850 psi (44.8 MPa to 102.4 MPa) for normal-weight concretes and from 3,560 psi to 12,490 psi (24.5 MPa to 86.1 MPa) for lightweight concretes. Concretes containing three different normal-weight aggregates and two different lightweight aggregates were included in the study. Stress-strain curves, flexural constants, and moduli of elasticity are reported for the complete range of concrete strengths.

Results of this investigation have been combined with those of other investigators. The data are compared with the latest ACI Building Code revisions pertaining to flexural constants for strength design.

Keywords: beams (supports); flexural strength; high-strength concretes; lightweight aggregate concretes; modulus of elasticity; strains; stress block; stresses; stress-strain relationships; structural analysis

ACI Fellow, Paul H. Kaar, is a Senior Structural Engineer, Portland Cement Association, Skokie, Illinois. Currently he is chairman of ACI-ASCE Committee 512, Precast Structural Concrete Design and Construction, and a member of ACI Committee 439, Steel Reinforcement, and 408, Bond. Mr. Kaar is a Registered Structural Engineer in Illinois, a Fellow of ASCE, and a member of ASTM.

ACI Fellow, Norman W. Hanson, is a Principal Structural Engineer, Portland Cement Association, Skokie, Illinois. His activities have included research on the behavior and strength of reinforced and prestressed concrete. Mr. Hanson is Secretary of ACI-ASCE Committee 352, Joints and Connections in Monolithic Concrete Structures and member of ACI Committee 545, Concrete Railroad Ties.

H. Thomas Capell, formerly Associate Structural Engineer, Portland Cement Association, Skokie, Illinois, is now Supervisor, Technical Services, Ecodyne Cooling Products Division, Santa Rosa, California. After service in the Corps of Engineers, U.S. Army, Mr. Capell worked in the fields of consulting, prestressed concrete production, and research.

HIGHLIGHTS AND DESIGN RECOMMENDATIONS

Strength design of reinforced concrete requires accurate knowledge of material properties for correct proportioning of sections. Previous work at the Portland Cement Association Laboratories by Hognestad, Hanson, and McHenry (1)* established stress-strain relationships for concrete. This included the descending curve beyond the maximum stress. The flexural constants for strength design were also clearly defined. Normal-weight concretes with strengths ranging from 1,000 psi to 7,500 psi (6.9 MPa to 51.7 MPa) were included in the earlier investigation. In recent years, concretes with strengths higher than 7,500 psi (51.7 MPa) have been developed and are now being used in the construction of tall buildings.

The objective of the investigation reported in this paper was to determine the stress-strain curves and the flexural constants for concretes with extremely high compressive strengths. Concretes with strengths of 6,500 psi to 14,850 psi (44.8 MPa to 102.4 MPa) for normal-weight concrete and from 3,560 psi to 12,490 psi (24.5 MPa to 86.1 MPa) for lightweight aggregate concrete were tested.

---

*Numbers in parenthesis designate references at end of this paper.

In the American Concrete Institute Building Code Requirements for Reinforced Concrete (ACI 318-71),(2) rectangular, trapezoidal, parabolic or other stress blocks may be assumed, and assumption is acceptable so long as the relationship between concrete compressive stress distribution and the resulting concrete strain is in agreement with test data. The data developed in this investigation are interpreted into the form of design constants suitable for use with a rectangular distribution of concrete stress and a linear distribution of strain.

The primary constant affecting design is the value of β_1. This defines the fraction of the neutral-axis depth that can be used for the rectangular stress-block depth. The test data confirm ACI Building Code (ACI 318-71) revision 10.2.7, (3) which states: "The fraction β_1 shall be taken as 0.85 for strengths, f'_c, up to and including 4,000 psi (27.6 MPa). For strengths above 4,000 psi (27.6 MPa), β_1 shall be reduced continuously at a rate of 0.05 for each 1,000 psi of strength in excess of 4,000 psi (27.6 MPa), but β_1 shall not be taken less than 0.65." While not in the proposed revision, data from this investigation indicate that the constant β_1 should be taken as 0.65 for all strengths of lightweight aggregate concrete.

INTRODUCTION

In 1955, Hognestad, Hanson, and McHenry (1) presented test data demonstrating the validity of the plasticity concepts involved in strength design. A test method using an eccentrically loaded C-shaped specimen was developed. This permitted the flexural stress distribution to be determined for the complete range of loading including the descending portion of the stress-strain curve.

In the earlier paper, complete information regarding the flexural stress distribution was reported for water-cement ratios from 1.00 to 0.33. Test ages ranged from 7 to 90 days. The corresponding range of concrete strengths was from 1000 psi to 7500 psi (6.9 MPa to 51.7 MPa).

Flexural constants designated by symbols k_1, k_2 and k_3 were defined in the Hognestad, Hanson, and McHenry paper as illustrated in Fig. 1. Based on test data, the authors determined equations relating k_1, k_2 and k_3 to concrete cylinder strength. These equations were then used to develop a table of constants for each 1,000 psi of concrete cylinder strength up to 8000 psi.

In 1973, Nedderman (4) conducted experiments at the University of Texas at Arlington to determine flexural constants for normal-weight concrete with strengths from 11,000 psi to 14,000 psi (75.8 MPa to 96.5 MPa). Nedderman used a C-shaped specimen similar to Hognestad, Hanson, and McHenry's. The data from Hognestad, Hanson, and McHenry; Nedderman; and some other data by J. A. Hanson (5) on lightweight-aggregate concrete are used in this report along with the new test data.

EXPERIMENTAL INVESTIGATION

Test Specimens

The C-shaped specimen shown in Fig. 2 has identical dimensions to those used previously by Hognestad, Hanson, and McHenry. The central 16-in. (406-mm.) long test section of each specimen was not reinforced. Flexural and shear reinforcement was used in the two ends of the specimen. Instrumentation is described in Appendix B. A specimen ready for test is shown in Fig. 3.

Test Variables

Controlled variables were concrete strength and type of coarse aggregate. Design strengths of the concrete ranged from 6,000 psi to 14,000 psi (41.4 MPa to 96.6 MPa) for normal-weight concrete and from 4,000 psi to 12,000 psi (27.6 MPa to 82.7 MPa) for lightweight concrete. Concrete strengths were incremented in steps of 2,000 psi. Three normal-weight aggregates and two structural lightweight aggregates were investigated. Further details of the aggregates and concrete mixes are discussed in Appendix B.

TEST RESULTS

Stress-Strain Curves

Stress-strain curves are shown in Fig. 4. The specimen number indicates the target strength in kips per square inch. Prefix letters indicate the aggregate used and the suffix letter "P" indicates duplicate specimens generally confined to target strengths of 6,000, 10,000, and 14,000 psi (41.4, 68.9, and 96.5 MPa) for normal-weight aggregate concrete and 4,000, 8,000, and 12,000 psi (27.6, 55.1, and 82.7 MPa) for lightweight aggregate concrete. The curves were obtained using the analysis described in Appendix C.

Flexural Stress Distribution

Flexural constants required to define the flexural stress distribution are given in Table 1 for normal-weight and Table 2 for lightweight aggregate concrete. The constants were calculated by the procedure explained in Appendix C.

Ultimate strain was taken as the maximum strain measured before sudden crushing of the concrete. The commonly accepted 0.003 value for ultimate strain appears to be suitable for use in design.

ANALYSIS OF DATA

Coefficients for Strength Design

To determine the necessary flexural design coefficients, data from the present study were combined with data of Hognestad, Hanson, and McHenry; Nedderman; and J. A. Hanson (5). As far as possible test results from all these sources have been included in Figs. 5 through 8. However, in some cases, the information was not available.

Three stress-block properties needed for strength design are k_1k_3, k_2/k_1k_3 and ε_u. Values of these properties and of k_2 are shown in Figs. 5 and 6 as a function of concrete strength. The coefficients were determined by the procedures described in Appendix C.

While values of k_2/k_1k_3 for lightweight vary considerably, the scatter effect is diminished in values of ultimate moment as computed using Equation (C-7). This scatter is comparable with that obtained by Mattock, Kriz, and Hognestad (6).

Values of the maximum strain, ε_u, have considerable scatter. While most code recommendations represent conservative lower bounds of experimental data, continued acceptance of the value 0.003 for the ultimate strain of concrete seems justified in spite of the scatter shown in this report. Tests of confined concrete have shown that even the lightest transverse reinforcement (7)(8) would extend the ultimate strain value beyond 0.003. This transverse reinforcement in flexural members would be provided by stirrups even at maximum allowable spacing.

Plots of the values k_1 and k_3 in relation to concrete cylinder strength are shown in Fig. 7. The value of k_3 was obtained by dividing the maximum flexural concrete stress expressed by Equations (C-1) and (C-2) by the concrete cylinder strength. Having k_3, k_1 was obtained from k_1k_3 by division.

Moduli of elasticity obtained from stress-strain curves produced from a compression cylinder and from a flexural test are shown in Tables 1 and 2. Hognestad, Hanson, and McHenry showed that the modulus determined from compression tests of cylinders was about 10% higher than the values determined from flexural data. This trend is also shown in Fig. 8, where the two moduli are compared.

A polynomial equation was determined by regression analysis of all data presented in the figures of this report. In most cases the equation chosen was of either the first or second degree. Coefficients for the polynomials are listed in Table 3. The curve of each equation is shown as a solid line on the figures discussed previously. In addition, the flexural constants obtained from the polynomials for concrete in the range 1000 psi to 16,000 psi (6.9 MPa to 110.3 MPa) are given in Table 4.

It should be emphasized that these values are from the polynomial equation and are not recommended for design. The values recommended for design are based on a lower bound limit. The polynomial values however should be of interest to researchers and those interested in the origin of design or code expressions.

The authors of Reference 1 also developed equations to describe data up to 8,000 psi (55.2 MPa) concrete. Results of tests reported in this paper indicated that these equations could not be extrapolated to higher concrete strengths with sufficient accuracy. Therefore, new equations were required to give a better fit for the range of strengths from 1,000 to 15,000 psi (6.9 MPa to 103.4 MPa).

Rectangular Stress Distribution Parameters

The ACI 318-71 (2) rectangular stress distribution shown in Fig. 1 is defined as follows:

> *"A concrete stress of $0.85f'_c$ shall be assumed uniformly distributed over an equivalent compression zone bounded by the edges of the cross section and a straight line located parallel to the neutral axis at a distance $a = \beta_1 c$ from the fiber of maximum compressive strain. The distance c from the fiber of maximum strain to the neutral axis is measured in a direction perpendicular to that axis. The fraction β_1 shall be taken as 0.85 for strengths, f'_c, up to 4,000 psi (27.6 MPa) and shall be reduced continuously at a rate of 0.05 for each 1,000 psi (6.9 MPa) of strength in excess of 4,000 psi (27.6 MPa)."*

The relationships between β_1 and the flexural constants k_1k_3 and k_2 are as follows:

$$k_1k_3 = 0.85\beta_1 \tag{1}$$

$$k_2 = \frac{\beta_1}{2} \tag{2}$$

The broken line in Fig. 5 shows the variation of k_1k_3, as given in Equation (1). The reduction of β_1 below 0.65 at 8,000 psi (55.2 MPa) places the rectangular stress-block parameter well below the data points. A new lower limit of 0.65 has been proposed (3). For the lightweight-aggregate concrete shown in Fig. 5, the rectangular stress block parameter defined by ACI Committee 318 would pass through the middle of the data rather than the lower boundary. For this reason a constant value for β_1 of 0.65 is proposed for lightweight aggregate concretes of all strengths.

The relationship of β_1 with k_2 is also shown in Fig. 5. The proposed variation of β_1 is shown, along with the proposed cutoffs. The representation of k_2 by β_1 is on the lower fringe of the data and, therefore, is nonconservative. This is in contrast with the conservative representation of k_1k_3.

The combined effect is shown on Fig. 6 where the rectangular stress-block parameters combine to a constant value of 0.588 as compared to the slightly convex curve of the polynomial representing the k_2/k_1k_3 data.

SUMMARY

The test method previously developed (1) has been used to establish the flexural characteristics and constants for high-strength concretes. Both normal weight and lightweight aggregates were included. Recommendations currently under consideration by ACI Committee 318 are reviewed and the basis for revisions is documented. Specific conclusions are contained in "Highlights and Design Recommendations" at the beginning of this report.

ACKNOWLEDGEMENTS

This investigation was carried out in the Structural Development Section of the Portland Cement Association under the direction of Dr. H. G. Russell, Manager, and Dr. W. G. Corley, Director, Engineering Development Department. Particular credit is due B. W. Fullhart, R. K. Richter, B. J. Doepp, W. Hummerich, O. A. Kurvits, J. P. McCarthy and W. H. Graves for their assistance in the manufacture and testing of the specimens. W. W. Hotaling, Jr., formerly of the Expanded Shale Clay and Slate Institute, and R. C. Jones of the Solite Corporation provided valuable information on material properties and mix proportions.

REFERENCES

1. Hognestad, Eivind, Hanson, N. W., and McHenry, Douglas, "Concrete Stress Distribution in Ultimate Strength Design," Journal of the American Concrete Institute, Proceedings, Vol. 52, December 1955, pages 455-479. Also PCA Development Bulletin D6.

2. "Building Code Requirements for Reinforced Concrete (ACI 318-71)," American Concrete Institute, Detroit, 78 pages.

3. ACI Committee 318, "Proposed Revision to Building Code Requirements for Reinforced Concrete (ACI 318-71)," Journal of the American Concrete Institute, Vol. 73, No. 1, January 1976, pages 15-25.

4. Nedderman, Howard, "Flexural Stress Distribution in Extra High-Strength Concrete," Masters Degree Thesis 1973, Department of Civil Engineering, University of Texas at Arlington.

5. Abeles, P. W., and others, Discussion of a paper by Eivind Hognestad, N. W. Hanson, and Douglas McHenry: "Concrete Stress Distribution in Ultimate Strength Design." Journal of the American Concrete Institute, Vol. 28, No. 6, December 1956, pages 1305-1330. Also PCA Development Bulletin D6A.

6. Mattock, A.H., Kriz, L.B., and Hognestad, Eivind., "Rectangular Concrete Stress Distribution in Ultimate Strength Design," Journal of the American Concrete Institute, Proceedings, Vol. 56, February 1961, pages 875-928. Also PCA Development Bulletin D49.

7. Soliman, M. T. M., and Yu, C. W., "The Flexural Stress Strain Relationship of Concrete Confined by Rectangular Transverse Reinforcement," Magazine of Concrete Research, Vol. 19, No. 61, December 1967, pages 223-238.

8. Kaar, P. H., Fiorato, A. E., Carpenter, J. E., and Corley, W. G., "Confined Concrete in Compression Zones of Structural Walls Designed to Resist Lateral Load Due to Earthquakes," Proceedings of the International Symposium on Earthquake Structural Engineering, St. Louis, Mo., August 19-21, 1976, pages 1207-1218.

9. Hognestad, Eivind, Hanson, N. W., Kriz, L. B., and Kurvits, O. A., "Facilities and Test Methods of PCA Structural Laboratory," papers under various titles in Journal of the PCA Research and Development Laboratories, Vol. 1, No. 1, January 1959, pages 12-20, 40-44; Vol. 1, No. 2, May 1959, pages 30-37; and Vol. 1, No. 3, September 1959, pages 35-41; reprinted jointly as PCA Development Bulletin D33.

APPENDIX A. NOTATION

A_s = area of nonprestressed tension reinforcement

C_c = total internal compressive force in concrete

b = width of compressive face of member

c = distance from extreme compression fiber to neutral axis

d = distance from extreme compression fiber to tension reinforcement

E_{Cc} = modulus of elasticity of concrete as determined by compressive tests of 6x12-in. (152x305-mm) cylinders

E_{Cf} = modulus of elasticity of concrete as determined from stress-strain curves of flexural test specimens

f_c = compressive stress in concrete

f'_c = compressive strength of concrete test cylinders

f_o = average compressive stress in concrete compressive zone

f_y = yield strength of nonprestressed reinforcement

f_{su} = stress in tension reinforcement at ultimate load

M = applied bending moment

$m_o = \frac{M}{bc^2}$

β_1, k_1, k_2, k_3 = coefficients pertaining to amount and position of internal compressive force in compression block

ε_c = strain in concrete

ε_{su} = strain in tension reinforcement at ultimate load

ε_u = ultimate concrete strain in flexure

ρ = $\frac{A_s}{bd}$, reinforcement ratio

APPENDIX B. LABORATORY WORK

Materials and Fabrication

Extensive trial batching was conducted prior to casting of specimens. Three normal-weight concretes were investigated. Elgin sand and gravel, limestone with Elgin sand, and traprock with Elgin sand constituted the aggregates for the normal-weight concretes. Two expanded shale lightweight aggregates manufactured by the rotary kiln method were used. In both cases, full replacement of fines by Elgin sand was used.

The range of water-cement ratios varied from 0.40 to 0.97 so that a broad range of compressive strengths could be attained. Within each type of concrete, the amount of coarse aggregate was held constant and the amount of water held nearly constant. Cement and sand content was varied to attain the desired strength. Water was increased for concrete below a target strength of 12,000 psi (82.7 MPa) until a slump was attained in the range 2-1/2 to 3-1/2 in. (63.5 mm to 88.9 mm). Concretes higher than 12,000-psi (82.7-MPa) target strength were of no slump mixes. A blend made of three brands of Type I portland cement was used. No admixtures were used.

Specimens were cast in a horizontal position. Each specimen was moist-cured under a plastic sheet at 73°F (23°C) for 3 days after casting. Subsequently, the concrete was cured at 73°F (23°C) and 50% relative humidity. Strengths given in Tables 1 and 2 are the average from three 6x12-in. (152x305-mm) cylinders taken from the concrete placed in the central test section. Cylinders were tested at the same age as the specimens.

Instrumentation

During testing, strains were measured at mid height of the test section by two 2.5-in. (63.5 mm) electrical resistance strain gages on the neutral surface; two gages on the compressive surface; one gage at middepth of each of the two side faces. A direct current differential transformer (DCDT) was used to monitor the deflection of the test section relative to the ends. This information was used to adjust the load lever arm distances a_1 and a_2, Fig. 2, for calculation of bending moment. Strain gages and the DCDT are shown in Fig. 3.

Signals from the DCDT and electrical resistance strain gages were monitored at regular intervals during the continuous loading of the specimen. Loads were also monitored by sensors described elsewhere (9). Data were both printed and punched on paper tape. The punched tape was read into a META 4 computer for reduction and analysis.

Test Apparatus

The major load, P_1, shown in Fig. 3 was applied by a testing machine having a 1-million-lb. capacity. Force was applied through a system of bearing plates and rollers that accommodated rotation of the specimen during the test.

The minor load, P_2, shown in Fig. 3, was applied by a hydraulic ram through a system of rods, crossheads and rollers.

Test Procedure

During the test, the major load, P_1, was increased at a constant rate. By manually controlling the value of the minor load, P_2, the strain at the extreme fibers on the left side of the specimen in Fig. 3 was kept at zero. This zero-strain surface represented the neutral axis boundary of the compressive stress block of a flexural member. On the opposite side of the cross section, the right side in Fig. 3, the extreme fibers were subjected to a monotonically increasing compressive strain. This surface represented the extreme compressive surface of a section in flexure.

APPENDIX C. DATA ANALYSIS

With loads and moments on the concrete determined, analysis followed that used by Hognestad, Hanson, and McHenry. Closely spaced readings of data during the test enabled the differentials in the following equations for concrete stress determination to be approximated by finite differences $\Delta f_o/\Delta\varepsilon_c$ and $\Delta m_o/\Delta\varepsilon_c$.

$$f_c = \varepsilon_c \frac{df_o}{d\varepsilon_c} + f_o \qquad \text{(C-1)}$$

$$f_c = \varepsilon_c \frac{dm_o}{d\varepsilon_c} + 2\, m_o \qquad \text{(C-2)}$$

where

f_c = compressive stress in concrete

ε_c = strain in concrete

f_o = average compressive stress in concrete

$m_o = \frac{\text{applied moment}}{bc^2}$

b = width of rectangular member

c = distance from neutral axis to compressive edge of member

The resulting concrete stress on a continuous stress strain curve is f_c calculated by both Equations (C-1) and (C-2). For each set of data, this approximation was accomplished by consideration of consecutive readings. The slope established by the (n+1) and (n-1) reading approximated the differential at the n^{th} reading point. Stress-strain curves given in Fig. 4 are an average of the calculated values obtained using Equations (C-1) and (C-2).

The numerical differentiation process tended to amplify experimental scatter. Accuracy of the test data was indicated by agreement of stresses resulting from independent use of Equations (C-1) and (C-2). Experimental sources of error affect the two equations differently. For instance, Equation (C-2) involves forces while Equation (C-2) involves moments. Therefore, any error in determining a_1 and a_2 will affect only Equation (C-2).

The value of k_1k_3 was determined by dividing the maximum applied load by test area and concrete cylinder strength. The value, less than 1.0, indicates the fraction of the cylinder strength that can be used as an average over the compression block area at ultimate load. The value is expressed as

$$k_1k_3 = \frac{C}{f'_c bc} = \frac{P_1 + P_2}{f'_c bc} \qquad \text{(C-3)}$$

where terms are as defined in Fig. 2. Values of k_1k_3 are shown in Fig. 5 for normal-weight and lightweight aggregate concrete both from this report and from other work.

The value of k_2 was determined by dividing the maximum applied moment by the maximum applied force times the section depth and subtracting from 1. The value defines the depth to the compression force as shown in Fig. 2,

$$k_2 = 1 - \frac{P_1a_1 + P_2a_2}{(P_1 + P_2)\ c} \qquad \text{(C-4)}$$

in which the lever arms a_1 and a_2 include both the initial values and those due to deflection of the specimen. Values for k_2 are shown graphically in Fig. 5. Also shown is a plot of the rectangular stress-block parameter β_1 versus concrete strength.

The value $\frac{k_2}{k_1k_3}$ is of importance in a fundamental equation used in strength design. From Fig. 1 equations summing forces and moments are:

$$A_sf_y = C = k_1k_3bcf'_c \qquad \text{(C-5)}$$

$$M = k_1k_3\ bc\ f'_c\ (d - k_2c) \qquad \text{(C-6)}$$

Solving Equations (C-5) and (C-6) we obtain:

$$\frac{M}{bd^2f'_c} = q\ (1 - \frac{k_2}{k_1\ k_3}\ q) \qquad \text{(C-7)}$$

in which the tension reinforcement index q, is:

$$q = \frac{A_sf_y}{bd\ f'_c} = \rho\ \frac{f_y}{f'_c} \qquad \text{(C-8)}$$

where f_y is the specified yield stress of reinforcement. The above equations are based on a section without compression reinforcement. Plots of the $\frac{k_2}{k_1k_3}$ values considered for normal-weight and lightweight concrete are shown in Fig. 6.

Table 1 - Strength Factors - Normal Weight Concrete

Specimen Number	Cylinder Strength, psi	k_1k_3	k_3	$\frac{k_2}{k_1 k_3}$	ε_u	k_1	k_2	Modulus of Elasticity: Compress-ometer Test, ksi	Modulus of Elasticity: Stress-Strain Curve, ksi
Elgin Sand and Gravel									
A6	6,500	0.76	1.01	0.60	0.0045	0.75	0.45	3,720	4,210
A6P	6,900	0.72	0.98	0.62	0.0038	0.73	0.45	3,820	4,610
A8	8,470	0.65	0.93	0.61	0.0034	0.70	0.39	4,270	4,380
A10	10,410	0.63	0.92	0.60	0.0036	0.68	0.37	4,680	5,100
A10P	9,380	0.64	1.00	0.57	0.0030	0.64	0.36	4,550	4,610
A12	11,340	0.69	1.12	0.53	0.0031	0.62	0.36	5,290	5,770
A14	14,000	0.58	1.03	0.58	0.0033	0.56	0.34	5,820	5,670
A14P	13,280	0.71	1.09	0.53	0.0038	0.65	0.37	5,650	5,900
Limestone and Elgin Sand									
D6	6,570	0.72	0.98	0.58	0.0028	0.75	0.42	5,200	5,830
D6P	7,100	0.66	0.89	0.63	0.0033	0.74	0.42	4,650	5,110
D8	8,420	0.61	0.95	0.65	0.0024	0.64	0.40	5,350	5,570
D10	9,810	0.62	0.94	0.62	0.0031	0.66	0.38	5,250	5,260
D10P	10,100	0.62	0.98	0.67	0.0032	0.63	0.41	4,850	6,200
D12	11,180	0.71	1.20	0.51	0.0032	0.59	0.36	5,930	6,610
D14	12,870	0.59	1.08	0.64	0.0028	0.55	0.38	5,760	6,320
D14P	14,850	0.62	1.03	0.57	0.0035	0.60	0.36	6,220	6,350
Trap Rock and Elgin Sand									
F10	11,290	0.64	0.99	0.58	0.0032	0.65	0.37	4,710	6,770
F12	12,690	0.64	1.12	0.55	0.0029	0.57	0.36	5,460	6,890
F14	13,700	0.59	0.91	0.64	0.0035	0.65	0.38	6,410	6,220

1000 psi = 6.89 MPa

Table 2 - Strength Factors - Lightweight Concrete

Specimen Number	Cylinder Strength, psi	k_1k_3	k_3	$\frac{k_2}{k_1 k_3}$	ε_u	k_1	k_2	Modulus of Elasticity: Compress-ometer Test, ksi	Modulus of Elasticity: Stress-Strain Curve, ksi
Lightweight Aggregate E and Elgin Sand									
E4	4,240	0.76	1.16	0.46	0.0024	0.66	0.35	2,810	3,230
E6	6,440	0.66	0.92	0.71	0.0052	0.61	0.47	3,130	3,140
E8	8,210	0.66	0.99	0.55	0.0038	0.67	0.36	3,330	4,000
E8P	8,450	0.57	0.84	0,81	0.0037	0.68	0.46	3,520	3,480
E10	11,330	0.61	1.06	0.54	0.0037	0.58	0.33	4,270	4,250
E12	12,490	0.47	0.75	0.79	0.0031	0.63	0.37	4,210	4,230
E12P	11.950	0.58	0.97	0.59	0.0036	0.60	0.34	3,930	4,360
Lightweight Aggregate C and Elgin Sand									
C4	3,630	0.77	1.03	0.56	0.0036	0.75	0.43	2,510	2,460
C4P	3,560	0.74	0.95	0.59	0.0051	0.77	0.44	2,270	2,560
C6	6,010	0.61	0.96	0.69	0.0035	0.64	0.42	2,820	2,710
C8	8,210	0.57	0.94	0.60	0.0030	0.61	0.34	3,290	3,460
C8P	8,150	0.56	0.96	0.68	0.0030	0.58	0.38	3,170	3,440
C10	9,570	0.61	1.06	0.56	0.0032	0.57	0.34	3,520	3,900
C12	9,680	0.64	1.20	0.61	0.0029	0.53	0.39	4,170	4,390
C12P	9,990	0.53	1.19	0.81	0.0029	0.45	0.43	4,230	4,890

1,000 psi = 6.89 MPa

Table 3 - Polynomial Coefficients for Strength Factors

$$\text{Factor} = A(f'_c)^4 + B(f'_c)^3 + C(f'_c)^2 + D(f'_c) + E^*$$

Factor	Coefficient A	Coefficient B	Coefficient C	Coefficient D	Coefficient E
Normal-Weight Concrete					
k_1k_3	0.00003	-0.0013	+0.0214	-0.1672	+1.1564
k_3	0	0	+0.0028	-0.042	+1.1178
k_2/k_1k_3	0	0	-0.0016	+0.0302	+0.4824
ε_u	0	0	0	-0.00003	+0.0035
k_1	0	0	+0.0015	-0.0448	+0.9381
k_2	0	0	+0.0005	-0.0175	+0.5073
Lightweight-Aggregate Concrete					
k_1k_3	0	0	+0.0015	-0.0403	+0.8175
k_3	0	0	0	-0.0045	+1.0176
k_2/k_1k_3	0	0	-0.0001	+0.0108	+0.5615
ε_u	0	0	0	0	+0.0035
k_1	0	0	0	-0.0152	+0.7413
k_2	0	0	+0.0002	-0.0087	+0.4383

*f'_c in units of ksi

Table 4 - Strength Factors from Polynomial Regression Analysis

Concrete Cylinder Strength f'_c, psi	k_1k_3	k_3	$\frac{k_2}{k_1k_3}$	ε_u	k_1	k_2
Normal-Weight Concrete						
1,000	1.01	1.07	0.51	0.0035	0.89	0.49
2,000	0.90	1.04	0.54	0.0035	0.85	0.47
3,000	0.81	1.02	0.56	0.0034	0.82	0.46
4,000	0.75	0.99	0.58	0.0034	0.78	0.45
5,000	0.71	0.98	0.59	0.0034	0.75	0.43
6,000	0.68	0.97	0.61	0.0033	0.72	0.42
7,000	0.66	0.96	0.62	0.0033	0.70	0.41
8,000	0.65	0.96	0.62	0.0033	0.69	0.40
9,000	0.63	0.97	0.62	0.0032	0.66	0.39
10,000	0.62	0.98	0.62	0.0032	0.64	0.38
11,000	0.61	0.99	0.62	0.0032	0.63	0.38
12,000	0.61	1.02	0.61	0.0031	0.62	0.37
13,000	0.60	1.04	0.61	0.0031	0.61	0.36
14,000	0.59	1.08	0.59	0.0031	0.60	0.36
15,000	0.59	1.12	0.58	0.0030	0.60	0.36
Lightweight-Aggregate Concrete						
1,000	0.78	1.01	0.57	0.0035	0.73	0.43
2,000	0.74	1.01	0.58	0.0035	0.71	0.42
3,000	0.71	1.00	0.59	0.0035	0.70	0.41
4,000	0.68	1.00	0.60	0.0035	0.68	0.41
5,000	0.65	1.00	0.61	0.0035	0.67	0.40
6,000	0.63	0.99	0.62	0.0035	0.65	0.39
7,000	0.61	0.99	0.63	0.0035	0.63	0.39
8,000	0.59	0.98	0.64	0.0035	0.62	0.38
9,000	0.58	0.98	0.65	0.0035	0.60	0.38
10,000	0.56	0.97	0.66	0.0035	0.59	0.37
11,000	0.56	0.97	0.67	0.0035	0.57	0.37
12,000	0.55	0.96	0.68	0.0035	0.56	0.36
13,000	0.55	0.96	0.68	0.0035	0.54	0.36
14,000	0.55	0.95	0.69	0.0035	0.53	0.36
15,000	0.55	0.95	0.70	0.0035	0.51	0.35

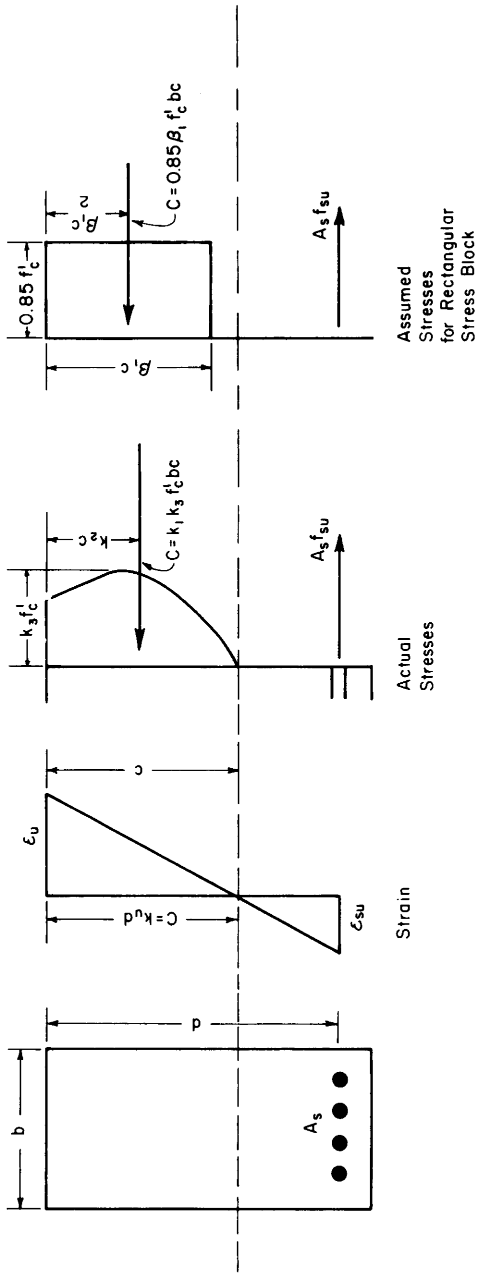


Fig. 1--Conditions at ultimate load

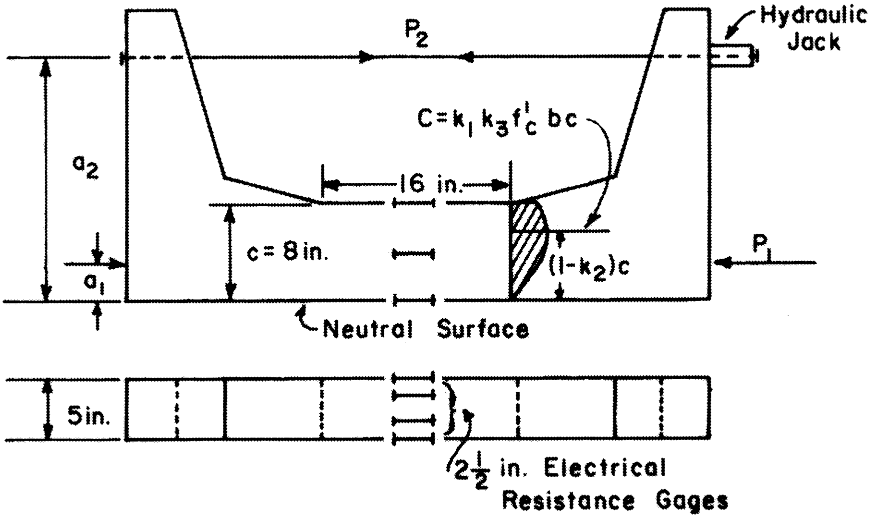


Fig. 2--Test specimen

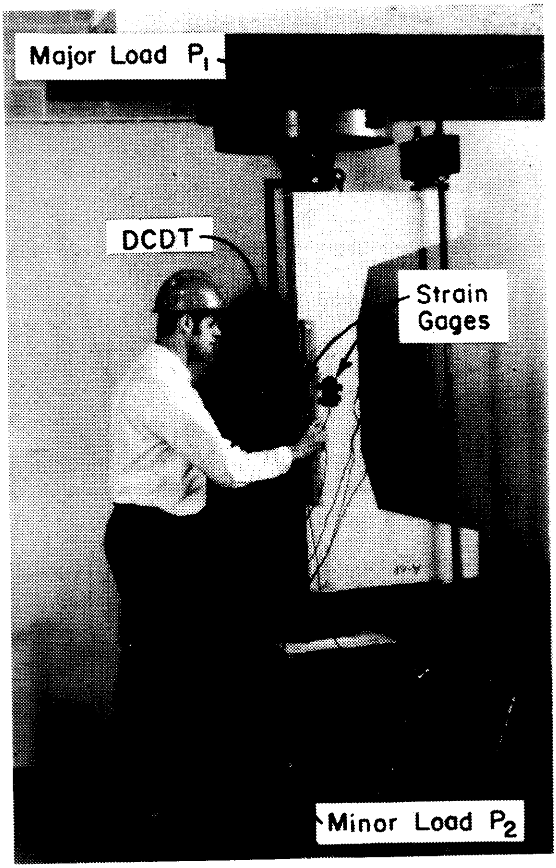


Fig. 3--Specimen ready for test

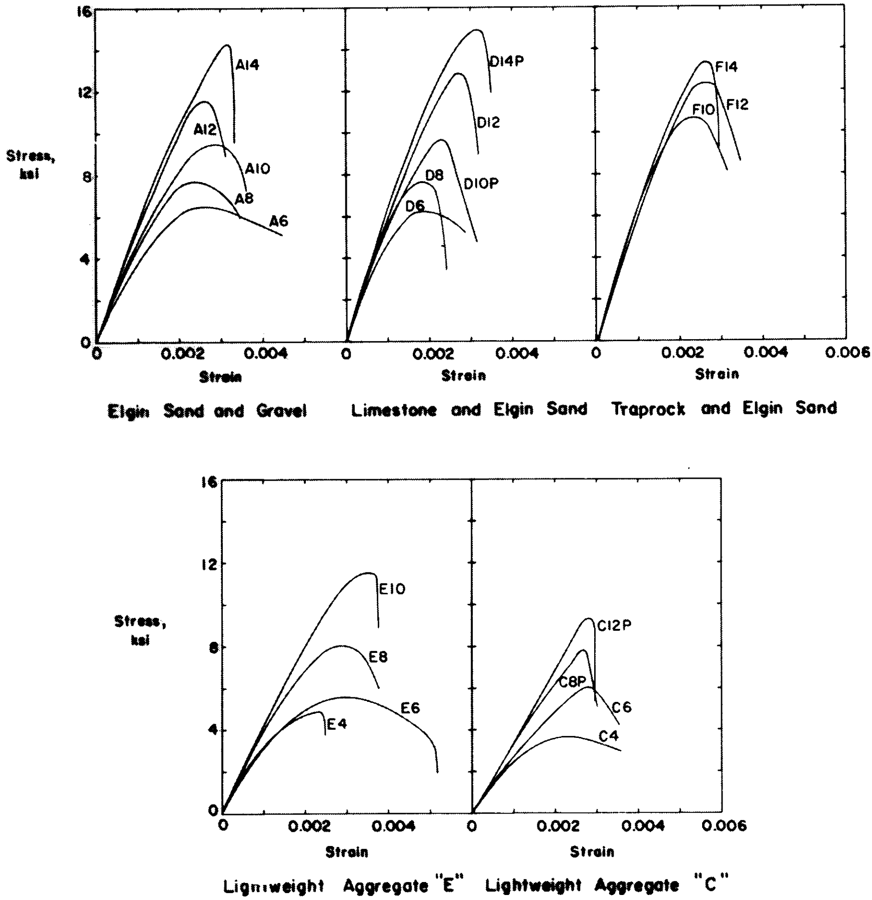


Fig. 4--Stress-strain curves of concrete

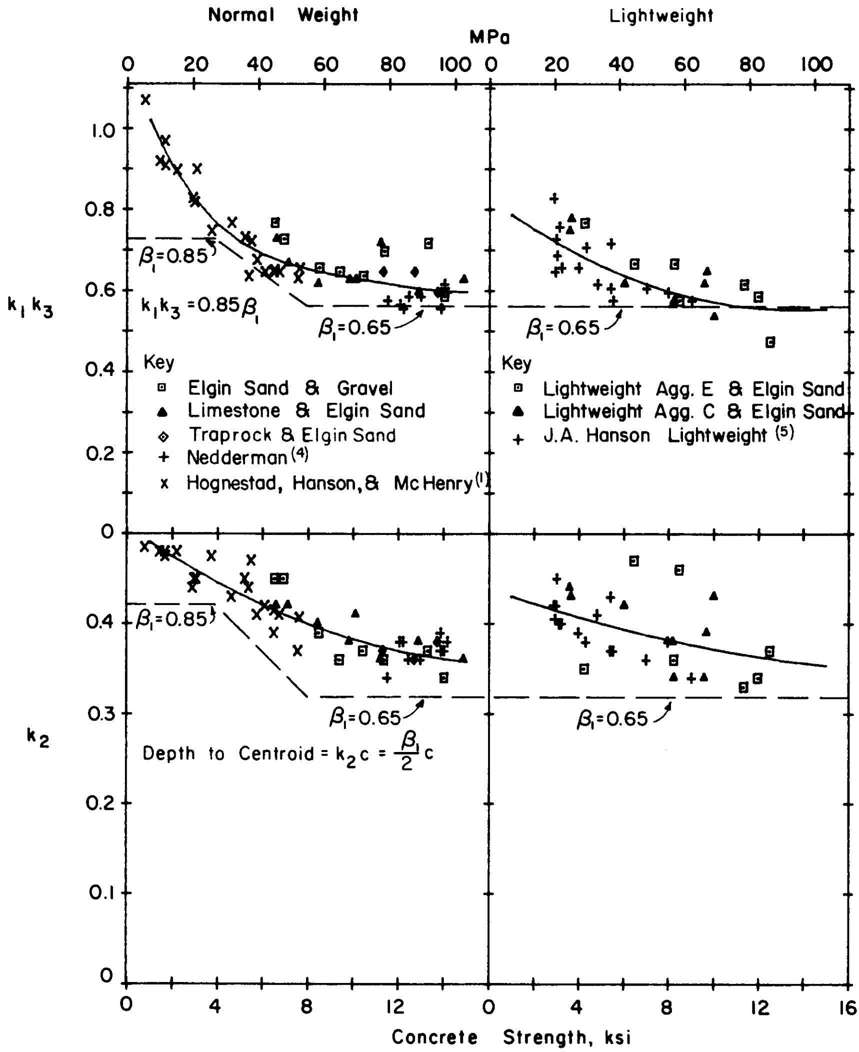
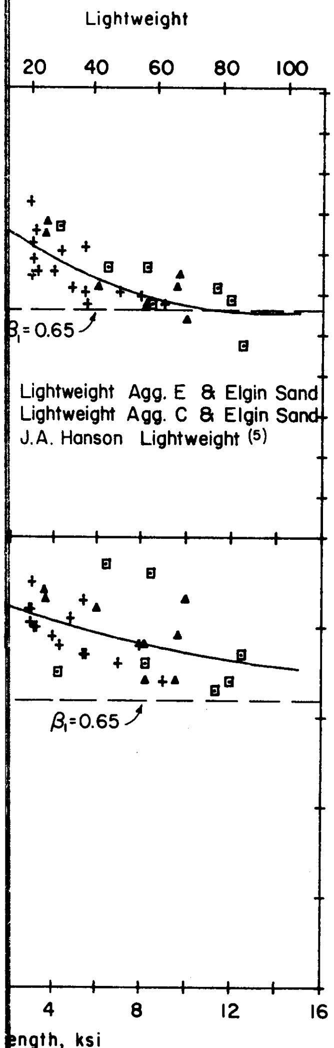


Fig. 5--Values of k_1k_3 and k_2 versus concrete strength



concrete strength

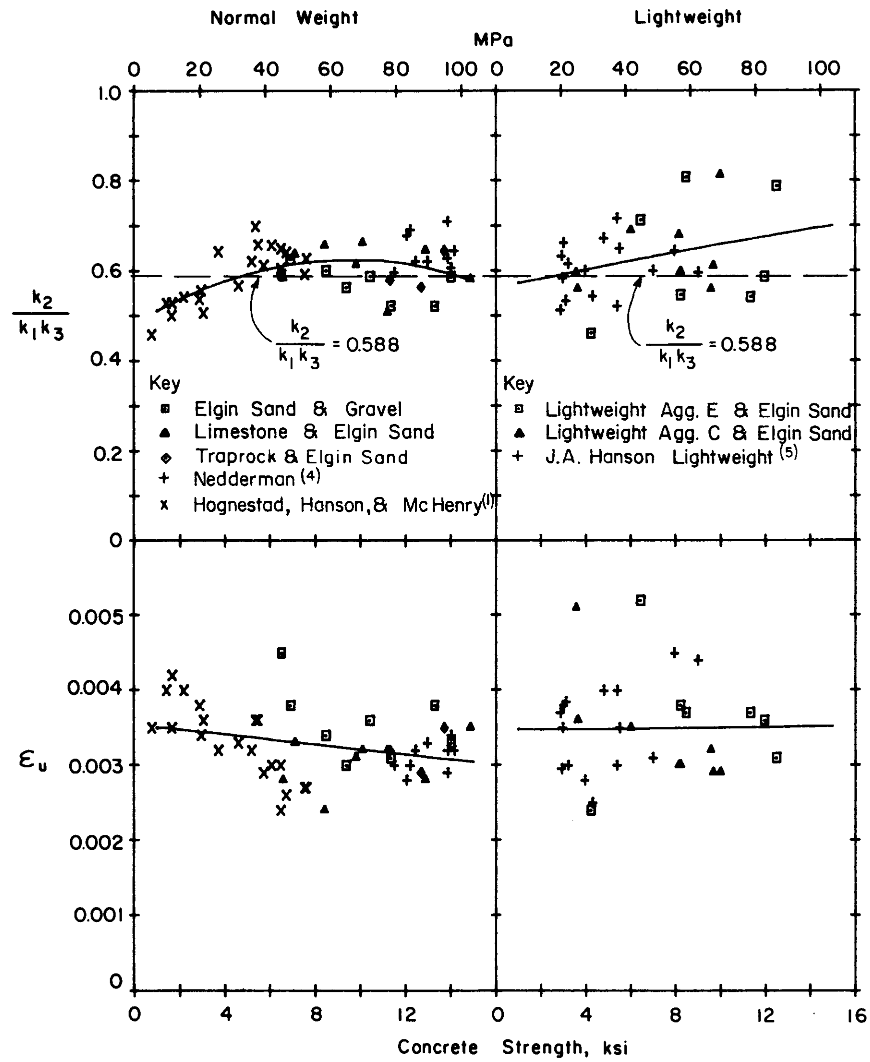


Fig. 6--Values of $\frac{k_2}{k_1 k_3}$ and ε_u versus concrete strength

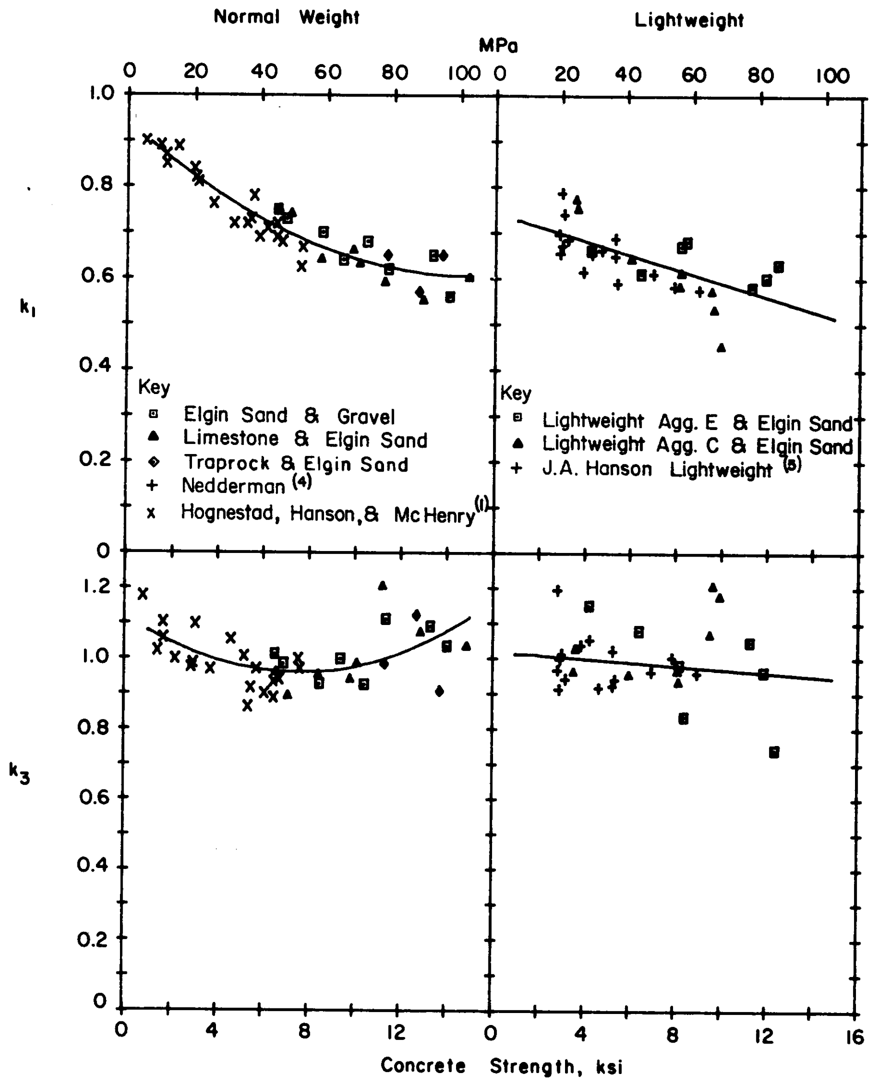


Fig. 7--Values of k_1 and k_3 versus concrete strength

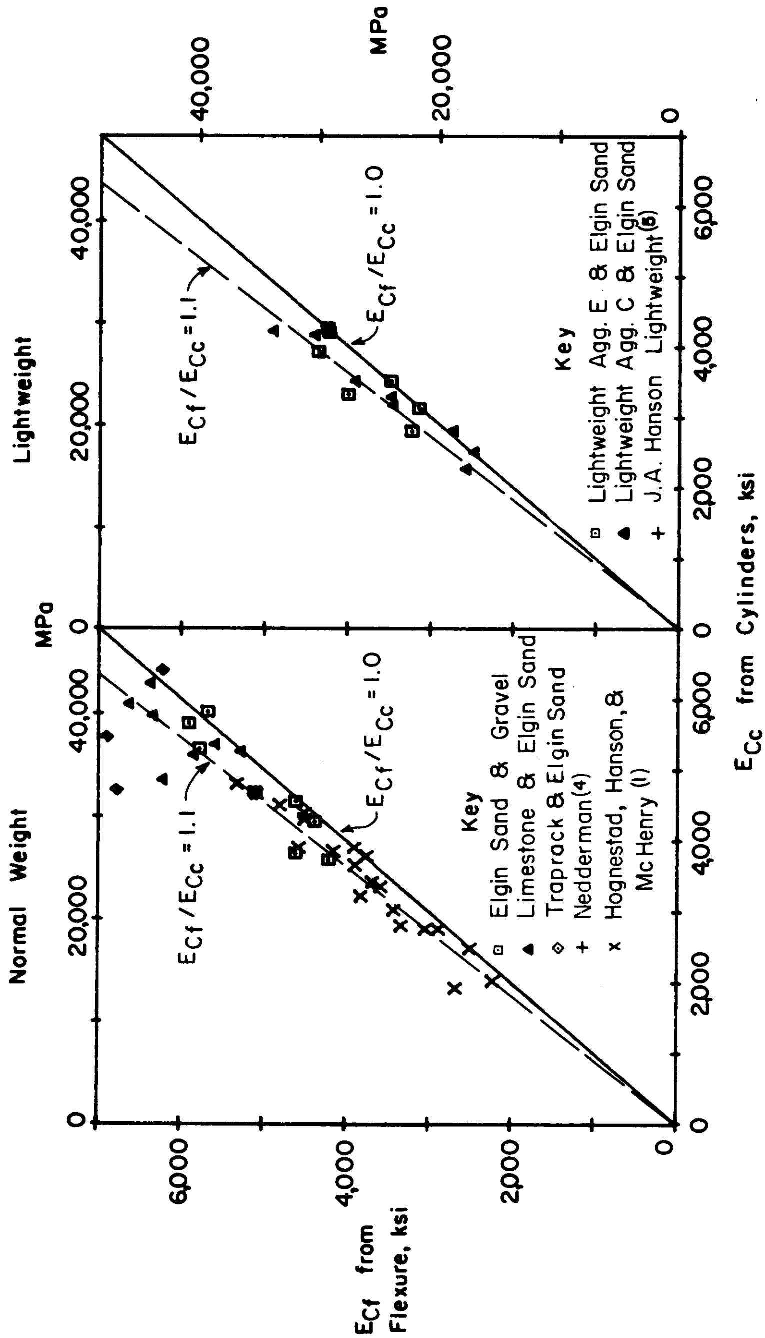


Fig. 8--Flexural versus compressive moduli of elasticity