Shear transfer in reinforced concrete with moment or tension acting across the shear plane

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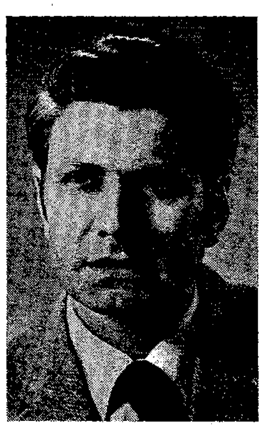
Reports results from a comprehensive investigation to study the shear transfer strength of reinforced concrete. Specific design recommendations are proposed.



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In the design of precast concrete connections it is frequently necessary to consider the transfer of moment and normal force across a shear plane, as well as shear. Such a situation occurs at the interface between a corbel (or bracket) and its supporting column.

The ACI Building Code, ACI 318-71,[1] permits the use of the shear-friction provisions of Section 11.15 for the design of corbels in which the shear span to depth ratio a/d is one-half or less, providing the limitations on the quantity and spacing of reinforcement in corbels specified in Section 11.14 are observed.

In using shear-friction according to Section 11.15 of ACI 318-71 to design the reinforcement crossing the interface between the corbel and the column, the following assumptions are made:

1. The concurrent action of moment across the shear plane will not reduce the effectiveness of the reinforcement crossing the shear plane in resisting shear, i.e., no interaction between moment and shear transfer.

2. The shear transfer reinforcement need not be uniformly distributed over the shear plane but may be distributed so as to be more effective in resisting moment.

3. If a normal tension force acts across the shear plane, it may be provided for by providing reinforcement additional to that required for shear transfer and having a yield strength equal to the tension force, i.e., linear interaction between shear and normal force.

Before this study was undertaken, no systematic experimental study had been made to validate these assumptions. However, their use within the limits imposed by Section 11.14 of ACI 318-71 is justified by the fact that they lead to generally conservative estimates of the yield strength (or ultimate strength if

A comprehensive study of the shear transfer strength of reinforced concrete, subject to both single direction and cyclically reversing loading (the latter simulating earthquake conditions), is currently in progress at the University of Washington. This paper reports that part of the study concerned with the effect of normal force and moment in the shear plane on single direction shear transfer strength. Tests are reported of corbel type push-off specimens and of push-off specimens with tension acting across the shear plane. It was found that

1. Moments in the shear plane less than or equal to the flexural ultimate moment of the shear plane do not reduce the shear transfer strength.
2. Tension across the shear plane results in a reduction in shear transfer strength equal to that which would result from a reduction in the reinforcement parameter ρf_y by an amount equal to the tension stress.

A future paper will extend the results of this investigation to lightweight concrete and provide a firm basis for corbel design.

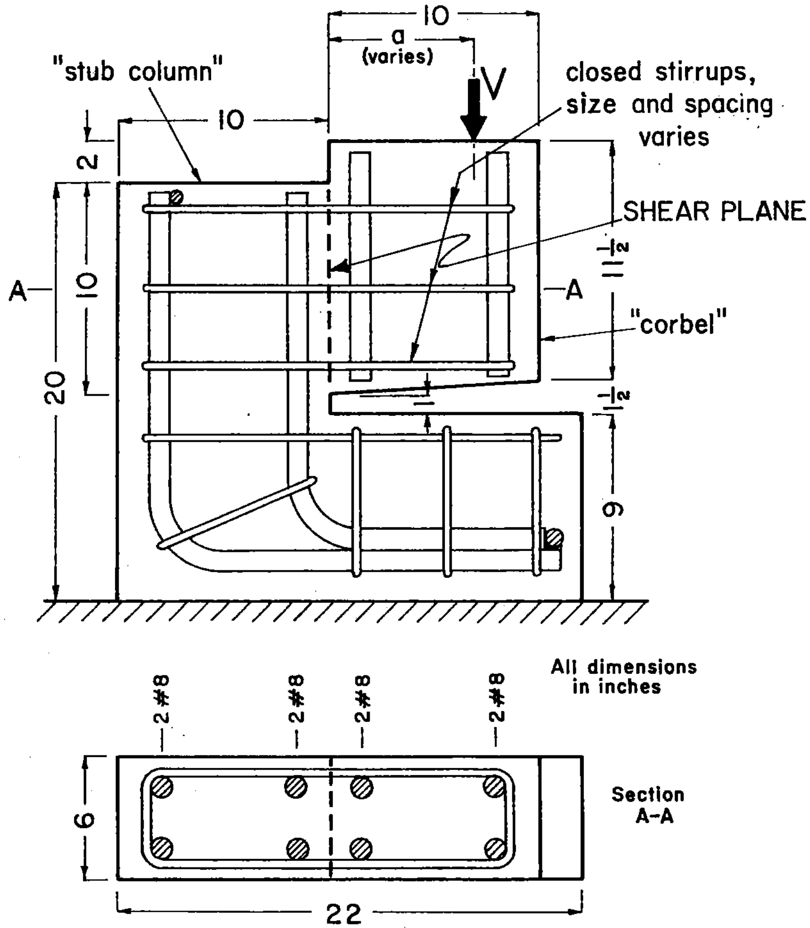

Fig. 1. Corbel type push-off specimen (Series A, B, C, and D).

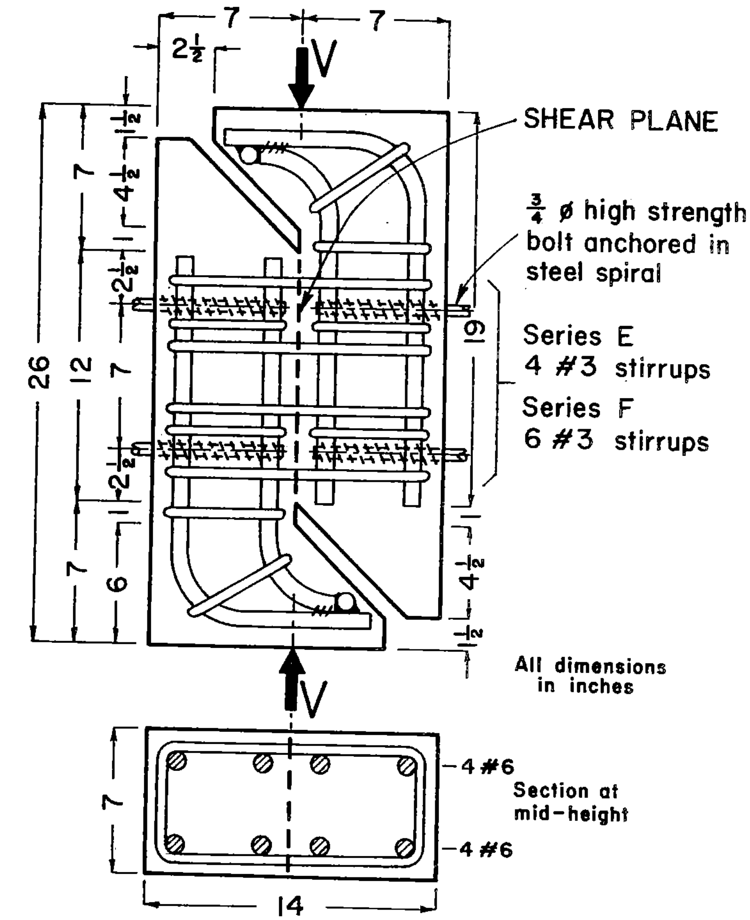

Fig. 2. Push-off specimen for test with tension across shear plane (Series E and F).

the main tension reinforcement did not yield) of those corbels tested by Kriz and Raths[2] which satisfy the requirements of Section 11.14.

This was demonstrated in the Report[3] of ACI-ASCE Committee 426, Shear and Diagonal Tension. However, the limitations contained in Section 11.14 of ACI 318-71 are quite arbitrary. They simply reflect the range of various parameters included in Kriz and Raths' tests for which satisfactory behavior was obtained, and the ranges of these parameters were chosen arbitrarily when their test program was planned.

It appeared that if the assumptions listed above could be validated in a more general manner, then the use of shear-friction concepts in corbel design could be extended beyond the arbitrary limits set out in Section 11.14, with consequent simplification of design procedures. This study was therefore undertaken with the following objectives:

1. To determine the effect of moment acting on the shear plane, on the shear which can be transferred across a shear plane by a given quantity and arrangement of reinforcement.

2. To study the influence of the arrangement of the reinforcement crossing the shear plane on the shear and moment which can be transferred across the shear plane.

3. To determine the effect of a tension force acting normal to the shear plane, on the shear which can be transferred across the shear plane.

Experimental Study

Six series of specimens were tested. Series A through D were corbel type push-off specimens, as shown in Fig. 1. Series E and F were standard push-off specimens, except that anchorages were provided on opposite sides of the shear plane through which a tensile force could be applied across the shear plane (see Fig. 2).

Table 1. Data concerning corbel type push-off specimens.

Specimen No.	Arrangement of Shear[3] Transfer Reinforcement	Concrete[1] Compressive Strength f'_c (psi)	Concrete[2] Tensile Strength f_{ct} (psi)	Eccentricity of Applied Load a (in.)	Ultimate Load V_u (kips)	Observed Failure Mode
A1	Uniformly distributed over shear plane	3975	395	0	60.00	Shear
A2		4130	410	2.50	60.00	Shear
A3		4200	415	5.00	41.25	Flexure
A4		3895	385	7.50	31.50	Flexure
B1	Distributed in upper part of shear plane	3890	380	0	52.00	Shear
B2		4125	410	5.00	51.50	Flexure
C1	Concentrated at top of shear plane (d - 8.5 in.)	3980	395	0	50.50	Shear
C2		3915	385	5.00	53.00	Shear
C3		3805	375	6.00	51.00	Shear
C4		4210	420	6.66	40.50	Flexure
D1		3920	385	0	39.00	Shear
D2		4090	400	5.70	36.00	Flexure

(1) Measured on 6 x 12 in. cylinders
(2) Split cylinder tensile strength measured on 6 x 12 in. cylinders
(3) Reinforcement in Series A, B & C, 3 #3 bar closed stirrups, A_{vf} = 0.66 in.^{2}, f_y = 53.0 ksi
Reinforcement in Series D, 1 #4 bar closed stirrup, A_{vf} = 0.40 in.^{2}, f_y = 53.0 ksi

Data concerning the specimens of Series A through D are shown in Table 1 and data concerning the specimens of Series E and F are shown in Tables 2 and 3, respectively. All specimens were made from sand and gravel concrete with ¾ in. maximum size aggregate and a design strength of 4000 psi at the time of test.

Corbel type push-off tests

The specimens were tested using a Baldwin hydraulic testing machine to apply a load V at distance a from the shear plane, as shown in Fig. 1. This resulted in a shear V and a moment Va acting in the shear plane simultaneously. The "corbel" part of the specimen was made to project above the top of the "stub column" in order that the load V could be applied in line with the shear plane in certain of the tests, i.e., $a = 0$.

Typical arrangements for test are shown in Fig. 3. The specimen was stood on the lower platten of the testing machine and was loaded through the system of rollers shown. The fixed upper roller served to define the point of application of the load and the lower free rollers prevented the testing machine from restraining horizontal movement of the "corbel" relative to the remainder of the specimen.

The 2-in. thick steel bearing plate resting on the top face of the "corbel" was anchored to the corbel by an embedded screw anchor at the side of the corbel remote from the point of application of the load, when zero eccentricity or a very large eccentricity was used. Measurements were made of slip along the shear plane and separation across it using 0.0001-in. dial gages, mounted on the specimen as shown in Fig. 3.

Mast[4] pointed out the need to consider the case of a crack existing in the shear plane before shear acts. Therefore, prior to test, the specimens were cracked along the shear plane by applying line loads to their front and rear faces. These loads were applied

Table 2. Data concerning Series E push-off specimens with tension across shear plane.

Specimen No.	Concrete[2] Compressive Strength f'_c (psi)	Concrete[3] Tensile Strength f_{ct} (psi)	Stirrup Yield Point f_y (ksi)	Reinforcement Parameter ρf_y (psi)	Normal[4] Stress σ_{Nx} (psi)	$(\rho f_y + \sigma_{Nx})$ (psi)	$v_u = \frac{V_u}{A_{cr}}$ (psi)
E1C[1]	3855	359	51.8	543	0	543	881
E2C	4220	397	52.1	546	-100	446	929
E3C	3960	343	52.7	552	-163	389	714
E4C	3820	362	50.5	529	-200	329	673
E5C	4020	383	52.3	548	-300	248	527
E6C	3985	373	50.9	533	-400	133	369
E1U	4060	372	52.7	552	0	552	1089
E4U	3860	392	49.1	514	-200	314	946
E6U	4120	335	50.8	532	-400	132	607

Shear transfer reinforcement area A_{vf} = 0.88 in.^{2} in all Series E specimens. (4 #3 bar closed stirrups).

(1) C denotes specimens initially cracked; U denotes specimens initially uncracked.
(2) Measured on 6 x 12 - in. cylinders.
(3) Split cylinder tensile strength measured on 6 x 12 - in. cylinders.
(4) Tension negative, Compression positive.

Table 3. Data concerning Series F push-off specimens with tension across shear plane.

Specimen	Concrete[2] Compressive Strength f'_c (psi)	Concrete[3] Tensile Strength f_{ct} (psi)	Stirrup Yield Point f_y (ksi)	Reinforcement Parameter ρf_y (psi)	Normal[4] Stress σ_{Nx} (psi)	$(\rho f_y + \sigma_{Nx})$ (psi)	$v_u = \frac{V_u}{A_{cr}}$ (psi)
F1C[1]	4220	350	50.1	787	0	787	988
F4C	3890	336	51.3	806	-200	606	839
F6C	4150	380	51.7	812	-400	412	804
F1U	4035	420	52.2	820	0	820	1369
F4U	4175	354	53.2	836	-200	636	1143
F6U	4245	389	51.0	801	-400	401	1066

Shear transfer reinforcement area A_{vf} = 1.32 in.^{2} in all Series F specimens. (6 #3 bar closed stirrups).

(1) C denotes specimens initially cracked; U denotes specimens initially uncracked.
(2) Measured on 6 x 12 - in. cylinders.
(3) Split cylinder tensile strength measured on 6 x 12 - in. cylinders.
(4) Tension negative, Compression positive.

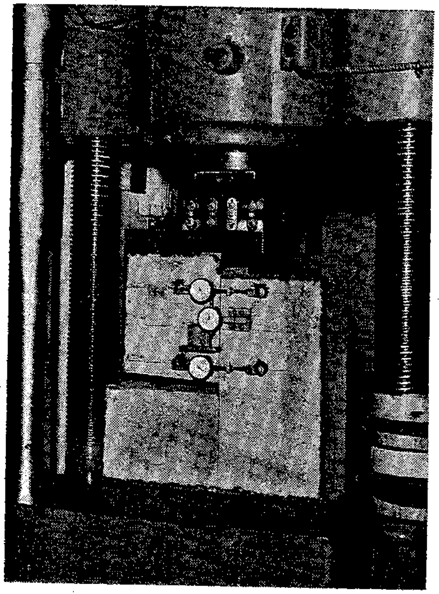

Fig. 3. Arrangements for test of corbel type push-off specimen.

through steel wedges with the specimen in a horizontal position. The dilation of the specimen across the shear plane was measured during the cracking operation, using dial gages mounted in a reference frame. The width of crack produced was about 0.01 in. in the region of the reinforcement crossing the crack.

The specimens were subject to an incrementally increasing load to failure. Failure was regarded as having occurred when the load could not be increased further and slip increased rapidly. At each increment of load readings were taken of slip and separation, and the growth of any cracks was marked on the specimen. Due to the nature of the instrumentation used, it was not possible to obtain data relating to the falling branch of the load-slip and load-separation curves.

Behavior—No additional cracking occurred during the testing of the specimens loaded along the shear plane (i.e., $a = 0$). In the case of the specimens loaded with an intermediate eccentricity (i.e. insufficient to cause a flexural failure), diagonal tension cracks occurred in the "stub column" during test, but they did not link up with the crack in the shear plane. At failure, minor compression spalling occurred adjacent to the crack in the shear plane, in the specimens loaded with zero or intermediate eccentricity.

Both flexural cracks and independent diagonal tension cracks formed in the "stub column" of those specimens loaded at an eccentricity sufficiently large to cause a flexural failure. The flexural cracks started in the top face of the stub column and propagated downward and towards the shear plane. In some cases a flexural crack linked up with the crack in the shear plane.

In flexural failures extensive compression spalling occurred in the concrete adjacent to the lower end of the shear plane and the shear plane crack opened wide at the top of the shear plane. Failure became less sudden as the eccentricity of the applied load increased and the failure mode changed from shear to flexure.

Both slip and separation occurred at all levels of loading. These have been reported in detail elsewhere.[5] It was found that for zero eccentricity of load (i.e., shear only in the shear plane), the ultimate slip increased as the distribution of the reinforcement changed from uniform distribution over the shear plane to concentration at the upper end of the shear plane.

It appears that when a crack in a shear plane is subject to shear only, then only that part of the crack crossed by the reinforcement is fully effective in resisting slip. By concentrating the reinforcement, the average shear stress is increased in that part of the shear plane fully effective in resisting slip, as compared with the case of uniformly distributed reinforcement. This may be the reason for the increase in slip.

In the case of specimens with uniformly distributed reinforcement the ultimate slip increased as the eccentricity of the load increased, except for A4 in which a flexural failure occurred at a load only about half that sustained in shear. This trend in behavior may also be due to an increase in the effective average shear stress. In this case the moment acting in the shear plane results in only part of the shear plane being subject to compression and the average shear stress in that part of the crack across which compression acts will therefore be greater than when the whole length of the crack is active in resisting shear.

In the specimens having the reinforcement concentrated near the top of the shear plane there is little variation in ultimate slip with eccentricity of load. This is probably due to the fact that the length of the crack subject to compression and therefore active in resisting shear will not change appreciably going from small to large eccentricity, although its location will change. For zero or small eccentricity, that part of the crack crossed by the reinforcement will be subject to compression, while for large eccentricities that part of the crack lying in the flexural compression zone of the "corbel" will be subject to compression, but the length of crack involved is probably similar in both these cases.

Ultimate strength—In Figs. 4 and 5 the measured ultimate strengths are compared with the calculated strengths which correspond to shear failure and flexure failure. The calculated shear strength V_u (calc.), is obtained using the shear-friction provision of Section 11.15 of ACI 318-71, with $\mu = 1.4$ for

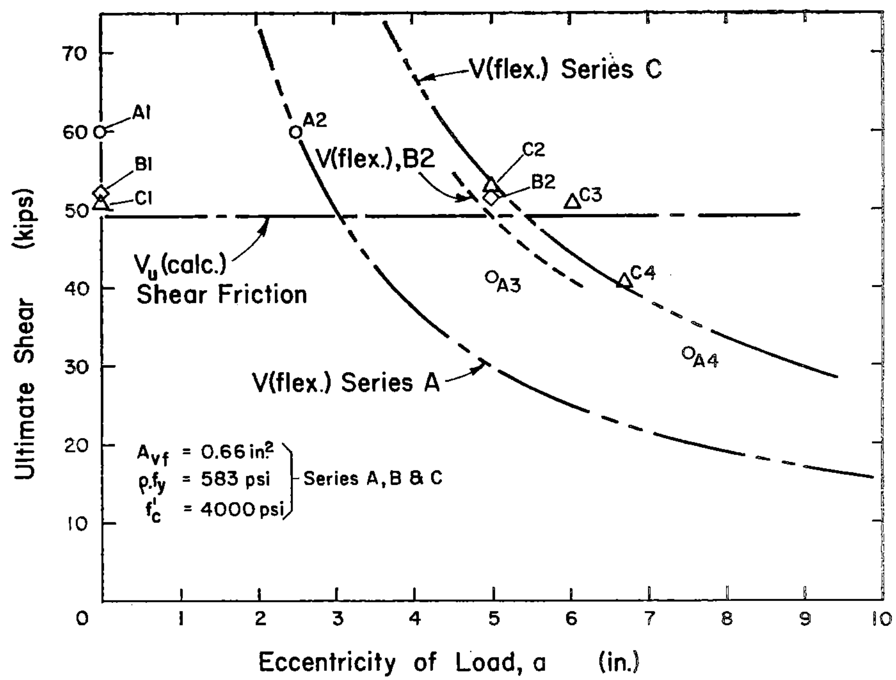

Fig. 4. Variation of ultimate shear with eccentricity of applied load (Series A, B, and C).

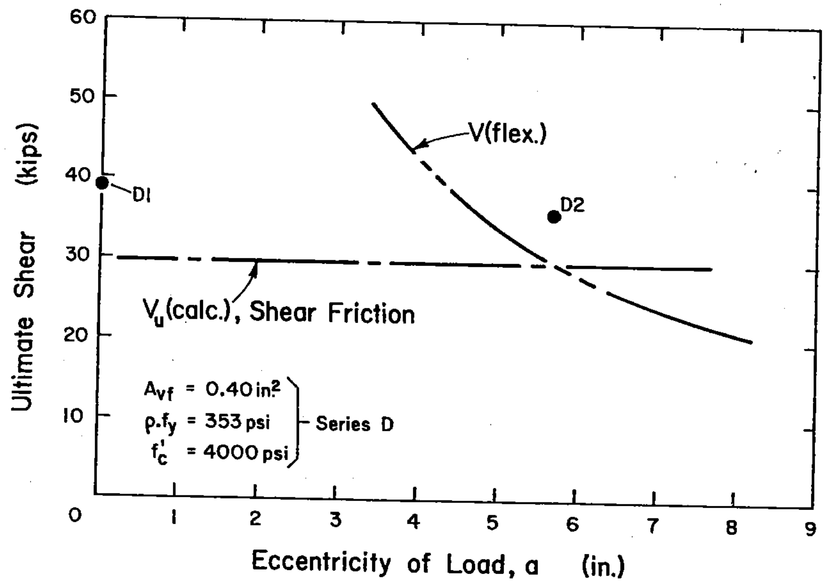

Fig. 5. Variation of ultimate shear with eccentricity of applied load (Series D).

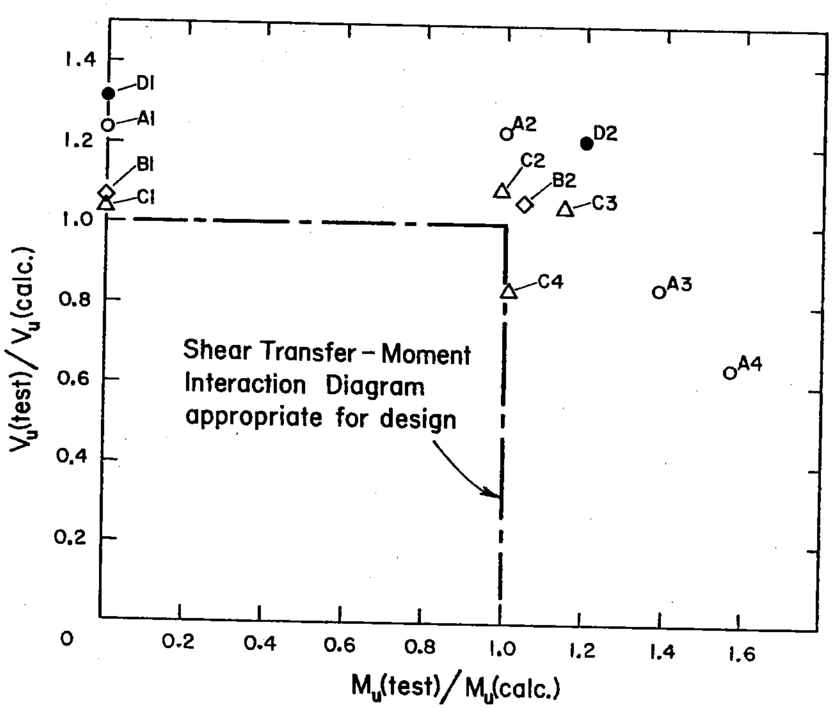

Fig. 6. Interaction of shear and moment transferred across a crack.

a crack in monolithic concrete, and setting $\phi = 1.0$ since the material strengths and specimen dimensions are known accurately.

The calculated shear corresponding to flexural failure V (flex.), is obtained by dividing the calculated ultimate moment capacity by the eccentricity of loading. The ultimate moment capacity was calculated using the assumptions set out in Section 10.2 of ACI 318-71, ϕ being taken as 1.0 in this case also.

It can be seen that if the calculated strength is taken to be the lesser of V_u (calc.) and V (flex.), then in all cases the actual strength exceeds the calculated strength. It appears that the ultimate shear which can be transferred across a crack is not significantly affected by the presence of moment in the crack, providing the applied moment is less than or equal to the flexural capacity of the cracked section.

A comparison of the strengths of Specimens A1, B1, and C1 which were all reinforced with three #3 bar stirrups, indicates that a small decrease in shear strength resulted when the distribution of the reinforcement was changed from uniformly distributed to concentrated at one end of the shear plane. This is probably due to the local increase in shear stress in the latter case due to the reinforced part only of the shear plane effectively resisting shear. It should be noted however, that the shear capacity was still greater than the calculated capacity for all three specimens.

The greater conservatism of Specimen D1 than C1, although both have their reinforcement concentrated near the top of the shear plane, is due to the lower reinforcement parameter of Specimen D1 (353 psi), as compared to that of Specimen C1 (583 psi). The shear-friction equation is most conservative for low values of ρf_y and becomes progressively less conservative as ρf_y increases.

The flexural capacity of Specimens A3 and A4 is considerably greater than the calculated capacity because the calculation assumes that the maximum stress that can be developed in the reinforcement is the yield stress. In this case the effective flexural reinforcement ratio is very low and consequently the upper reinforcement was strained into the strain hardening range, resulting in a reinforcement stress considerably greater than the yield stress. This occurred to a lesser extent in Specimen D2 where the flexural reinforcement ratio was 0.8 percent, but did not occur in Specimen C4 where the flexural reinforcement ratio was 1.3 percent. This is the trend to be expected.

The interaction of shear and moment in all the specimens tested is shown in Fig. 6. The test eccentricities for Specimens A2, B2, C2, C3, and D2 were deliberately chosen to check whether the shear transfer strength according to shear-friction theory could be developed simultaneously with the calculated flexural ultimate strength, i.e., the most severe interaction situation. The results obtained indicate that the shear transfer strength according to shear-friction theory and the flexural ultimate strength can be developed simultaneously across a crack in monolithic concrete.

Section 11.14 of ACI 318-71 allows the shear-friction provisions of Section 11.15 to be used for the design of corbels providing a/d is less than 0.5. The results obtained in this study indicate that at least for the case of vertical load only, this arbitrary limit on a/d is unnecessary, provided the corbel is designed for shear according to the shear friction provisions of Section 11.15 and for flexure using the assumptions of Section 10.2. The value of a/d at which flexure will begin to control and reduce the shear transfer strength below that calculated according to the shear-friction theory, will depend on

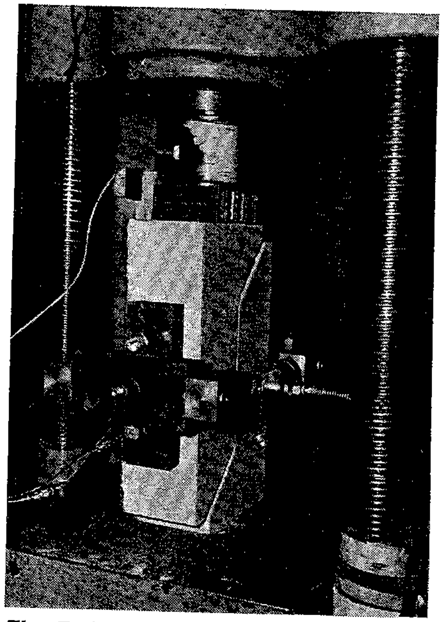

Fig. 7. Arrangement for test of push-off specimen with tension acting across shear plane.

the distribution of the reinforcement over the shear plane.

Push-off tests with tension across the shear plane

The specimens were tested using a Baldwin hydraulic testing machine to apply a load V concentric with the shear plane as shown in Fig. 2. This produced shear without moment in the shear plane. Simultaneously, tension across the shear plane was produced by pulling on the pairs of high strength bolts anchored in the specimen on opposite sides of the shear plane. The tension force was provided by a pair of 20-kip capacity rams, acting through short distribution beams as may be seen in Fig. 7.

Both the shear V and the tensile force were measured using SR4 gage load cells. The slip and separation were measured by linear differential transformers attached to reference points embedded in the concrete on opposite sides of the shear plane. Both the linear differential transformers and the load cells were monitored continuously during the tests using a Sanborn strip chart recorder.

Prior to test, those specimens indicated in Tables 2 and 3 were cracked along the shear plane by applying line loads to their front and rear faces. The loads were applied through steel wedges with the specimen in a horizontal position. The dilation of the specimen across the shear plane was measured during the cracking operation, using dial gages mounted in a reference frame. A crack width of approximately 0.01 in. was produced.

The tensile force across the shear plane was applied before the specimen was subjected to shear loading, and was maintained constant during the test. The shear load was then increased continuously until failure occurred. Failure was regarded as having occurred when the shear load could not be increased further and slip increased rapidly.

Behavior of initially cracked specimens—When the tension force across the shear plane was applied, additional separations of up to 0.0012 in. were measured, the magnitude of the separation increasing with increase in the applied stress. Both slip and separation occurred from the commencement of shear loading.

These measurements have been reported in detail elsewhere.[5] No diagonal tension cracks occurred in the Series E initially cracked specimens, but a few occurred in the more heavily reinforced initially cracked specimens of Series F. Failure of this type of specimen was characterized by a rapid increase in slip and separation, and by compression spalling of the concrete adjacent to the shear plane.

(Separation is caused by the over-riding of roughness on the crack faces. This will be accompanied by very high local compression stresses at the points of contact of the crack faces, which must be the cause of the compression spalling observed adjacent to the crack at failure.)

Behavior of initially uncracked specimens—When the largest tensile stress (400 psi) was applied to the initially uncracked specimens E6U and F6U fine cracks occurred near the shear plane and roughly parallel to it. An additional "separation" of about 0.001 in. was recorded in these cases.

When shear was applied to the initially uncracked specimens no slip or separation was recorded until short diagonal tension cracks commenced to form across the shear plane. These cracks were first observed at shear stresses of from 330 to 700 psi, the shear stress at cracking decreasing as the tensile stress across the shear plane is increased. The inclination of the diagonal tension cracks to the shear plane varied from about 10 to 40 deg, the inclination decreasing as the tension stress across the shear plane increased.

Failure was quite brittle and was characterized by the extension of one

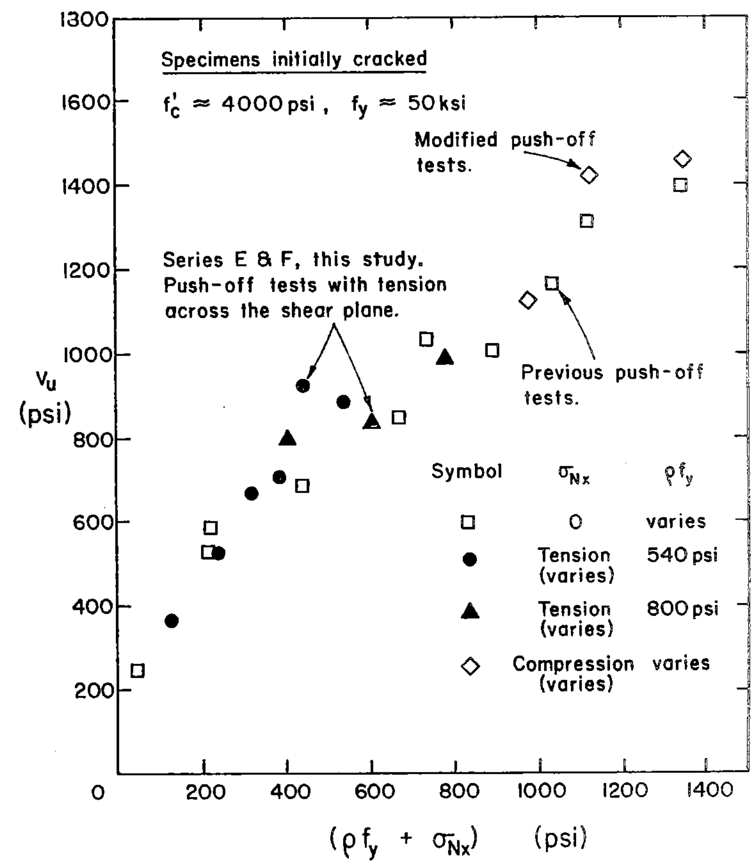

Fig. 8. Comparison of shear transfer strength of initially cracked specimens tested in this program with that of "push-off" and "modified push-off" specimens tested previously.

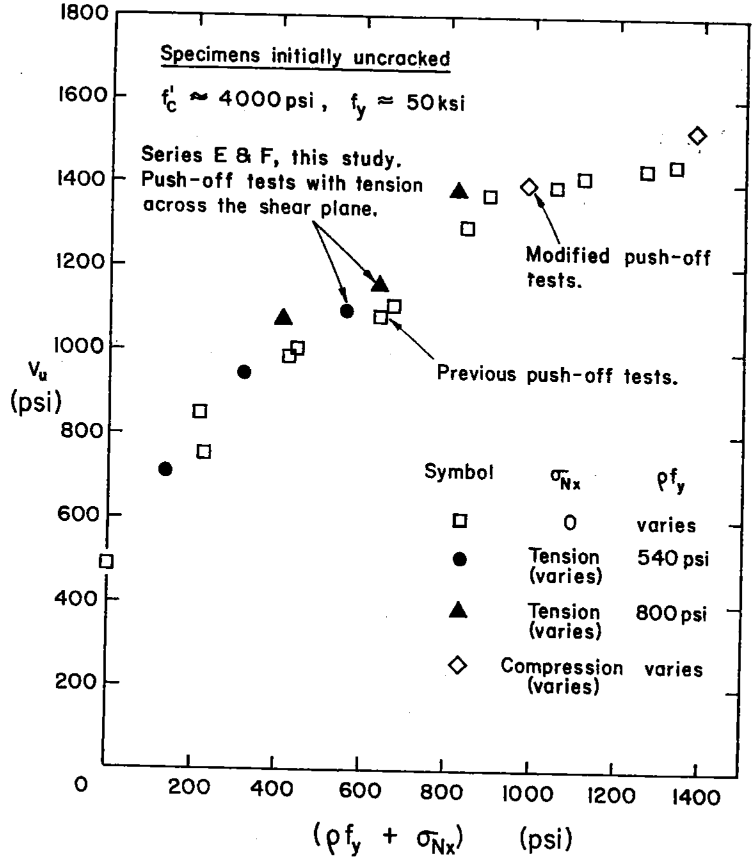

Fig. 9. Comparison of shear transfer strength of initially uncracked specimens tested in this program with that of "push-off" and "modified push-off" specimens tested previously.

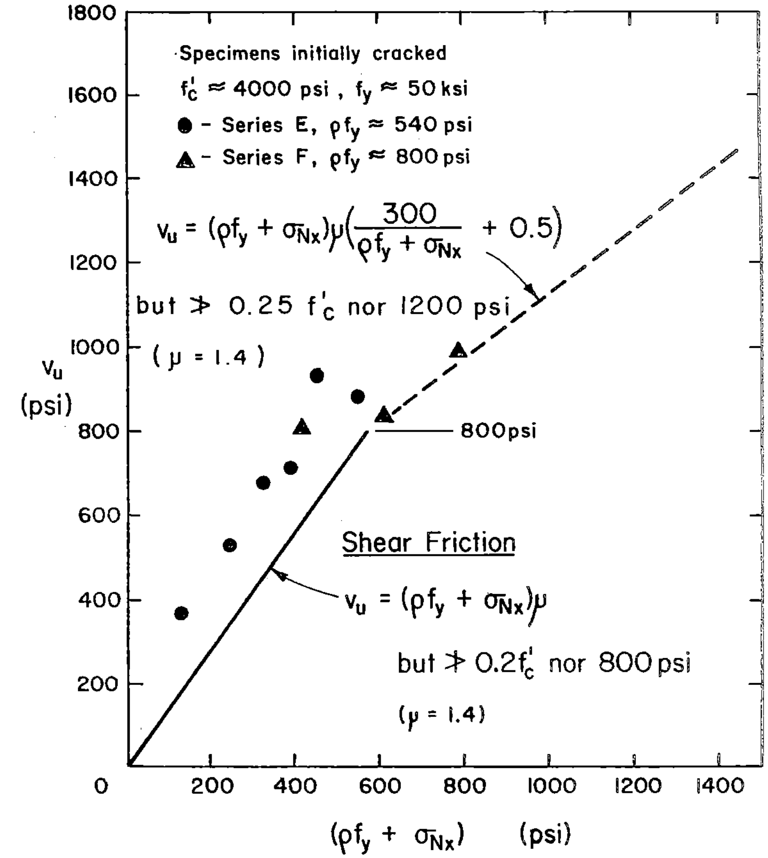

Fig. 10. Comparison of shear transfer strength of initially cracked concrete having a tension stress across shear plane, with the strength predicted by current ACI Code and PCI Handbook equations.

of the larger diagonal tension cracks roughly parallel to the shear plane, linking up with other diagonal tension cracks, and by compression spalling of the concrete, particularly near the ends of the shear plane.

The slip which occurred at all levels of load in the initially cracked specimens was greater than that which occurred in companion initially uncracked specimens. The slip at ultimate load tended to decrease as the tensile stress acting across the shear plane increased.

In the case of the initially uncracked specimen, the "slip" is not a true slip but rather the component parallel to the shear plane of the relative motion of the two halves of the specimen, due to rotation and compression of the inclined concrete struts formed by the diagonal tension cracking.

The reduction in "slip" at ultimate in an initially uncracked specimen as the tensile stress across the shear plane increases, is consistent with the reduction in the angle between the diagonal tension cracks and the shear plane, (and hence between the inclined concrete struts and the shear plane), as the tension across the shear plane increases.

In the case of the initially cracked specimens, it may be that because of the increasing initial separation of the crack faces as the tension across the shear plane increased, less slip became necessary to over-ride the minor roughness of the crack faces and cause failure.

Ultimate strength—The ultimate shear transfer strength of the specimens is shown in Tables 2 and 3, in the form of nominal shear stresses at failure of the specimens.

In Figs. 8 and 9, the shear transfer strengths obtained in the tests of initially cracked and initially uncracked push-off specimens, respectively, are plotted against $(\rho f_y + \sigma_{Nx})$. Also plotted in these figures are shear transfer strength data obtained previously in simple push-off tests[6] and in modified push-off tests[7] in which compression acted across the shear plane simultaneously with the shear.

In making these plots, a positive value of σ_{Nx} corresponds to a compressive stress across the shear plane and a negative value of σ_{Nx} corresponds to a tensile stress across the shear plane.

In both the initially cracked concrete

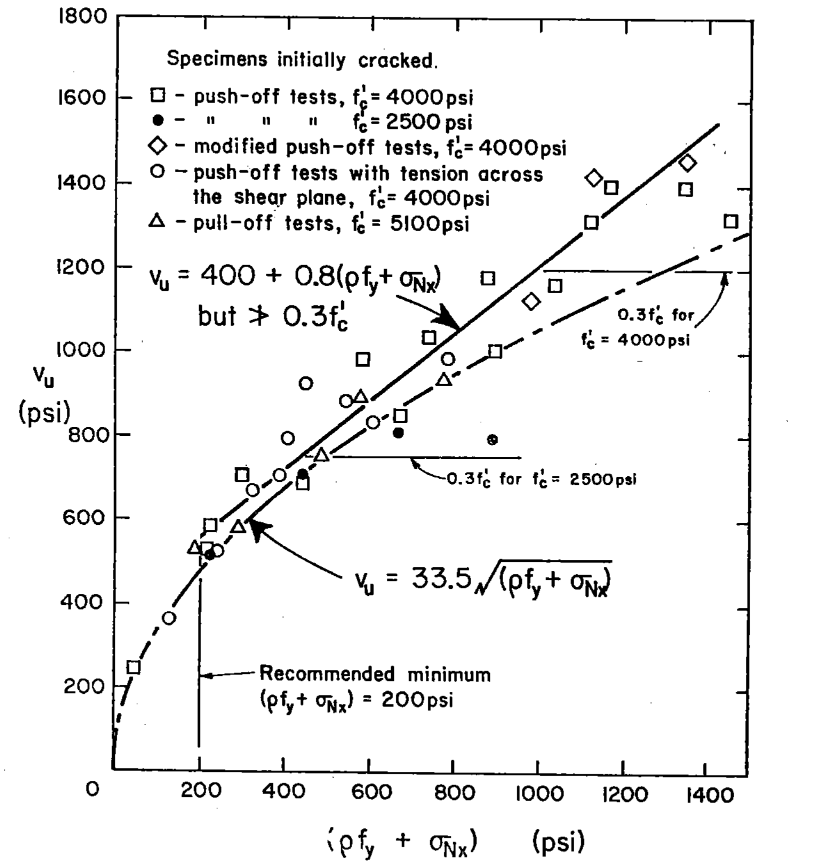

Fig. 11. Comparison of shear transfer strength of initially cracked concrete with and without direct stress across shear plane with strength predicted by equations proposed by Mattock[9,10] and Birkeland.[11]

and the initially uncracked concrete, the grouping of the data points in these figures indicates that the change in shear transfer strength which occurs when $(\rho f_y + \sigma_{Nx})$ is varied is the same whether this parameter is changed by varying ρf_y or σ_{Nx}. This indicates that it is appropriate to combine the normal stress σ_{Nx} with the reinforcement parameter when calculating shear transfer strength.

In Fig. 10, the shear transfer strength of the initially cracked push-off specimens with tension across the shear plane are plotted against the parameter $(\rho f_y + \sigma_{Nx})$. Also plotted in this figure are lines corresponding to the shear-friction provisions of Section 11.15 of ACI 318-71 (solid line) and the design equation contained in Section 6.1.9 of the *PCI Design Handbook*[8] (dash line). (Note that the capacity reduction factor ϕ was taken as 1.0 in both cases, since the material properties were accurately known for these specimens.)

It can be seen that for this sand and gravel concrete, both equations yield conservative estimates of shear transfer strength.

For the situation in which a tension force N_u acts across a shear plane area A_{cr}, the shear-friction equation for design:

$$v_u = (\phi \rho f_y + \sigma_{Nx})\mu \tag{1}$$

becomes:

$$v_u = (\phi A_s f_y / A_{cr} - N_u / A_{cr})\mu \tag{2}$$

or

$$V_u/\mu = \phi A_s f_y - N_u \tag{3}$$

where A_s is the total area of steel crossing the shear plane and ϕ is the capacity reduction factor (0.85 for shear). Hence

$$\phi A_s f_y = V_u/\mu + N_u$$

that is

$$A_s = \frac{V_u}{\phi \mu f_y} + \frac{N_u}{\phi f_y} \tag{4}$$

or

$$A_s = A_{vf} + A_t \tag{5}$$

where A_{vf} is the area of reinforcement required to carry the shear V_u according to shear-friction, and A_t is the area of reinforcement required to carry the tension force N_u acting across the shear plane.

The test results shown in Fig. 10 therefore validate the assumption made in Section 11.15 of ACI 318-71 that the total amount of reinforcement needed to carry a shear V_u and a tension N_u across a crack may be obtained by simply adding together the area of reinforcement required to resist the shear [according to Eq. (11-30)], and the area of reinforcement required to resist the tension force N_u.

While the shear-friction equation has the advantage of simplicity, it also has the disadvantages of being rather conservative for small values of ρf_y and of artificially limiting the maximum ultimate shear transfer stress to 800 psi. The PCI equation is an attempt to remedy the second disadvantage, but has the disadvantage of complexity.

Alternate simple expressions for shear transfer strength, aimed at eliminating these disadvantages, have been proposed previously by Birkeland:[11]

$$v_u = 33.5\sqrt{\rho f_y} \tag{6}$$

and by Mattock:[9,10]

$$v_u = 400 + 0.8\,\rho f_y \tag{7}$$

but not less than $0.3 f_c'$.

These alternate design equations have previously been validated only for the case of shear alone acting in the shear plane. In Fig. 11 they are extended to the case of shear and direct stress acting across the shear-plane, and are compared with measured shear transfer strengths of initially cracked sand and gravel concrete, both with and without direct stress acting across the shear plane.

It can be seen that both expressions are applicable to this general combination of stress, Birkeland's parabola being slightly more conservative than Mattock's straight line. Use of either of these relationships (suitably modified by the inclusion of the capacity reduction factor ϕ) would lead to more economical shear transfer designs than are currently yielded by the shear-friction provisions of ACI 318-71.

In design, either of these relationships could be used to design the reinforcement required for shear transfer, and then the reinforcement required to carry the tension across the shear plane should simply be added to the shear reinforcement.

Conclusions for Design

On the basis of the study reported here, the following conclusions are drawn concerning shear transfer in sand and gravel concrete.

1. The simultaneous action of a moment less than or equal to the flexural ultimate strength of the cracked section will not reduce the shear which can be transferred across the crack.

2. The arbitrary limitation of a/d to less than 0.5 when shear-friction is used in corbel design, contained in Section 11.14 of ACI 318-71, should be replaced by a requirement that both the flexural and shear capacity of corbels shall be checked according to Sections 10.2 and 11.15 of ACI 318-71, respectively.

3. If both moment and shear are to be transferred across a crack, then in order to be fully effective the shear transfer reinforcement should be located in the flexural tension zone.

4. It is appropriate to add the normal stress σ_{Nx} to the reinforcement parameter ρf_y when calculating shear transfer strength in both initially cracked and initially uncracked reinforced concrete. (σ_{Nx} is positive when compression and negative when tension.)

5. The shear friction provisions of Section 11.15 of ACI 318-71 yield a conservative estimate of the shear transfer strength of reinforced concrete, both with and without a tension stress acting across the shear plane. The results obtained validate the assumption made in these provisions that the total amount of reinforcement needed to carry a shear V_u and a tension N_u across a crack may be obtained by simply adding together the area of reinforcement required to resist the shear [calculated using Eq. (11-30)], and the area of reinforcement required to resist the tension force N_u.

6. The equation proposed in Section 6.1.9 of the *PCI Design Handbook* for use when ρf_y exceeds 600 psi, yields conservative results for the case when a normal stress σ_{Nx} acts across the plane, providing $(\rho f_y + \sigma_{Nx})$ is substituted for ρf_y in the equation. (σ_{Nx} is positive when compression and negative when tension.)

7. The alternate shear transfer design Eq. (6) and (7) proposed by Birkeland[11] and Mattock[9,10] are equally applicable to the general case of both shear and direct stress acting across a shear plane. Use of these equations in design (suitably modified by the inclusion of the capacity reduction factor ϕ), instead of the shear friction equation, would lead to economies.

Appendix—Notation

A_{cr} = area of shear plane, sq in.

A_s = total area of reinforcement crossing shear plane when both shear and tension act, sq in.

A_t = area of reinforcement necessary to resist tension force N_u, sq in.

A_{vf} = area of shear-friction reinforcement, sq in.

a = eccentricity of applied load with respect to shear plane, in.

d = distance from extreme compression fiber to centroid of tension reinforcement, in.

f_c' = compressive strength of concrete measured on 6 x 12-in. cylinders, psi

f_{ct} = splitting tensile strength of concrete, measured on 6 x 12 in. cylinders, psi

f_y = yield point stress of reinforcement, psi

M_u = ultimate moment of resistance, in.-kips

N_u = ultimate tensile force acting across shear plane simultaneously with V_u, kips

$V(\text{flex})$ = M_u/a

V_u = ultimate shear strength, kips

v_u = nominal ultimate shear stress, psi

μ = coefficient of friction used in shear-friction calculations

ρ = A_s/A_{cr} when both shear and tension act

ρ = A_{vf}/A_{cr} when shear only acts

σ_{Nx} = externally applied normal stress acting across shear plane, psi (compression positive, tension negative)

ϕ = capacity reduction factor, as per Section 9.2 of ACI 318-71

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References

1. "Building Code Requirements for Reinforced Concrete (ACI 318-71)," American Concrete Institute, Detroit, 1971.
2. Kriz, L. B., and Raths, C. H., "Connections in Precast Concrete Structures —Strength of Corbels," PCI JOURNAL, V. 10, No. 1, February 1965, pp. 16-61.
3. ACI-ASCE Committee 426, "The Shear Strength of Reinforced Concrete Members," (Chapters 1 to 4), *Journal of the Structural Division, ASCE,* V. 99, No. ST6, June 1973, pp. 1091-1187.
4. Mast, R. F., "Auxiliary Reinforcement in Concrete Connections," *Journal of the Structural Division, ASCE,* V. 94, ST6, June 1968, pp. 1485-1504.
5. Mattock, A. H., "Effect of Moment and Tension Across the Shear Plane on Single Direction Shear Transfer Strength in Monolithic Concrete," Report SM 74-3, Department of Civil Engineering, University of Washington, Seattle, Washington, October 1974.
6. Hofbeck, J. A., Ibrahim, I. O., and Mattock, A. H., "Shear Transfer in Reinforced Concrete," *ACI Journal,* V. 66, No. 2, February 1969, pp. 119-128.
7. Mattock, A. H., and Hawkins, N. M., "Research on Shear Transfer in Reinforced Concrete," PCI JOURNAL, V. 17, No. 2, March-April 1972, pp. 55-75.
8. *PCI Design Handbook,* Prestressed Concrete Institute, Chicago, 1971.
9. Mattock, A. H., "Shear Transfer in Concrete Having Reinforcement at an Angle to the Shear Plane," American Concrete Institute Publication SP-42, *Shear in Reinforced Concrete,* 1974, pp. 17-42.
10. Mattock, A. H., Discussion of the paper, "Modified Shear-Friction Theory for Bracket Design," by B. R. Hermansen and J. Cowan, *ACI Journal,* V. 71, No. 8, August 1974, pp. 421-423.
11. Birkeland, H. W., Class Notes for Course, "Precast and Prestressed Concrete," University of British Columbia, Spring 1968.

Discussion of this paper is invited. Please forward your discussion to PCI Headquarters by December 1, 1975.