Report UMCEE 01-04

Reliability-Based Calibration for Structural Concrete, Phase 2

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Executive Summary

The report documents Phase 2 of research carried out in conjunction with calibration of the Building Requirements for Structural Concrete (ACI 318-99). Calibration was performed to determine the resistance factors corresponding to load factors specified by ASCE 7 Standard on Minimum Design Loads for Buildings and Other Structures (1998). The presented research is a continuation of Phase 1 and covers the selection of representative structural types and materials, resistance models, reliability analysis, and selection of resistance factors. The development of load models, the reliability analysis procedures and selection of target the reliability indices were presented in the previous report covering Phase 1. The structural elements considered in Phase 2 include eccentrically loaded reinforced concrete columns and slabs and foundation beams subjected to shear. The load components include dead load and live load. The statistical parameters for loads are based on the information available in literature.

Resistance parameters are determined on the basis of statistical data on materials (material factors) and other factors (fabrication and professional factors), established using statistical information on dimensions of cross section, fabrication and methods of structural analysis. The statistical parameters of resistance are calculated using Monte Carlo simulations.

Reliability indices are calculated for structural components designed using the load factors specified by ASCE 7 Standard (1998) for several possible values of resistance factor. For comparison, reliability analysis is also performed for the components designed according to old ACI 318-99 Code. The calculations are performed for the new statistical models for load and resistance, and for the statistical parameters of resistance used in previous studies. The target reliability index for columns and shear was assumed to be the same as in Phase 1. The acceptance criterion for selection of resistance factors is closeness of the calculated reliability index to the target reliability index.

The recommended resistance factors for columns are $\varphi = 0.70$ for tied columns, $\varphi = 0.75$ for spiral columns, $\varphi = 0.80$ for shear in foundation beams, and $\varphi = 0.75$ for shear in slabs.

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1. Introduction

The report documents the results of research work on calibration of the Building Code Requirements for Structural Concrete (ACI 318). The objective of this study is to determine the resistance factors that are consistent with load and load combination factors specified by ASCE 7 Standard on Minimum Design Loads for Buildings and Other Structures (1998).

Report on Phase 1 of this project, revised in October 2001, presented limit state design criteria covering possible cases for design of reinforced concrete and prestressed concrete structural elements, in particular: reinforced and prestressed concrete beams in flexure and shear, reinforced concrete slabs in flexure, and reinforced and prestressed concrete columns in axial loading. Phase 2 covers the remaining cases of design of reinforced concrete structural elements, in particular, eccentrically loaded columns, slabs and beams in shear (without shear reinforcement).

The calibration procedure is based on the reliability analysis, with load and resistance parameters treated as random variables. It is assumed that the available load and load combination models are adequate for reinforced concrete structural elements (Ellingwood et al. 1980 and Nowak and Collins 2000). The load factors and load combinations follow ASCE 7 Standard (1998) and new ACI 318 Building Code Requirements for Structural Concrete (2002). The resistance factors, strongly affected by statistical parameters of material properties, dimensions and fabrication are the main focus of this study.

Statistical parameters of materials, fabrication and professional factors are presented in the Phase 1 of this report. The same statistical parameters are used for resistance models considered in Phase 2 of the project.

The discussion on general calibration procedure, calculation of statistical parameters of resistance (Monte Carlo simulations), reliability analysis procedures, and selection of the target reliability indices are included in the first part of the report (Phase 1) and are not repeated in this report.

Phase 2 of the report presents the resistance models for eccentrically loaded columns, reliability indices calculated for the considered loading cases, and proposed resistance factors. As a continuation of the shear analysis from Phase 1 of the report, the ultimate limit state of shear capacity for structural elements without shear reinforcement is also considered in this report.

Design cases considered in Phase 2 of the report include eccentrically loaded columns and flexural components (slabs and beams) subjected to shear. Both topics are a continuation of the work presented in Phase 1 of the report. Analysis of eccentrically loaded columns is a continuation of the axially loaded columns, and additional shear analysis is focused on the design cases with the shear force being transferred by the concrete section (structural elements without shear reinforcement).

2. Eccentrically Loaded Columns

This Chapter deals with the design of eccentrically loaded reinforced concrete columns. Four reinforcement ratios are considered: 1%, 2%, 3% and 4%. The calculations are performed for ordinary concrete and high strength concrete, and for various eccentricities (e/h ratios) according to the interaction diagram curve shown in Figure 2-1.

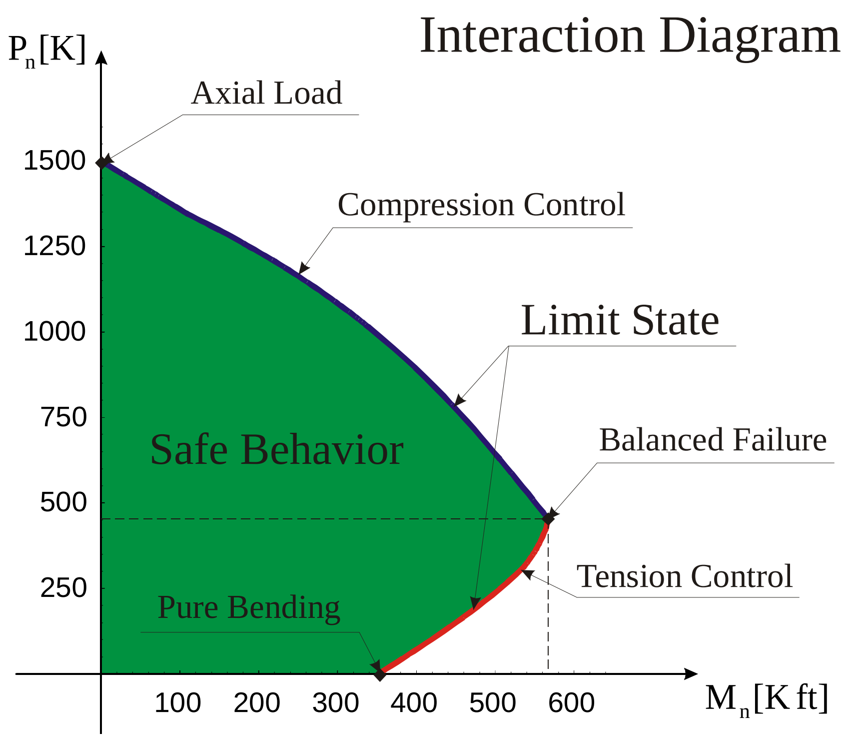


Figure 2-1. Interaction diagram for eccentrically loaded columns.

2.1. Resistance of Eccentrically Loaded Columns

The statistical parameters of resistance, R, were calculated using Monte Carlo simulations and formulas for resistance (load carrying capacity of eccentrically loaded columns). The statistical parameters of material fctor, M, fabrication factor, F, and professional factor, P, presented in the first part of the report (Phase 1) were used in the calculations. Parameters considered as random variables include:

- Strength of concrete, f'_c, for ordinary and high strength concrete
- Yield strength of reinforcing steel, f_y

- Dimensions of the cross section and area of reinforcing steel
- Construction type (cast-in-place and plant-cast)

Resistance formulas for eccentrically loaded columns were developed based on the equilibrium of the external and internal forces in the section.

2.1.1. Basic Assumptions

The notation for stresses and strains as well as geometry for a rectangular cross section of an eccentrically loaded column is shown in Figure 2-2.

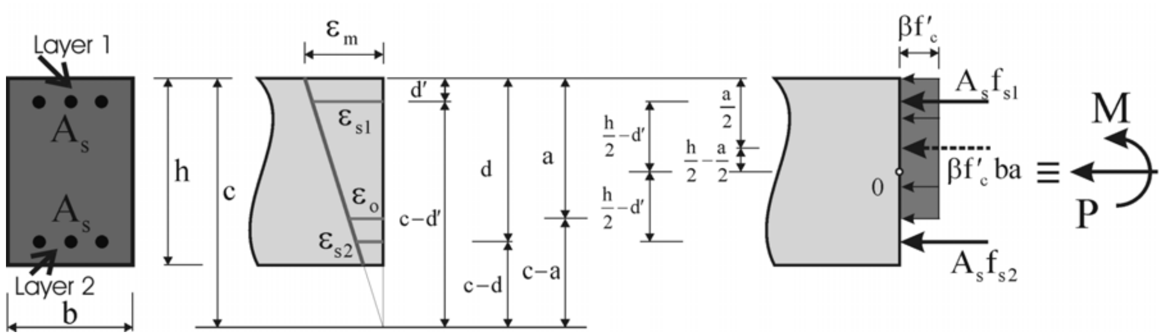


Figure 2-2. Strain and Stress Distribution in Cross Section of an Eccentrically Loaded Column.

Notation:

Layer 1 of reinforcement is the closest to the extreme compression fibers

Layer 2 of the reinforcement is the furthest from the extreme compression fibers

a	=	depth of equivalent rectangular stress block
b	=	width of the section
c	=	distance from the fiber of maximum strain to the neutral axis
d	=	distance from extreme compression fiber to centroid of layer 2 of reinforcement
d’	=	cover of reinforcement
h	=	depth of the cross section
f_c’	=	specified compressive strength of concrete
f_y	=	yield strength of reinforcing steel

f_{s1} = calculated stress in reinforcement close to the extreme compression fiber in the section

f_{s2} = calculated stress in the layer 2 of reinforcement

A_s = area of reinforcement in each layer (layer 1 and layer 2)

E_s = modulus of elasticity of reinforcement

M = moment acting in the section

P = axial force acting in the section

β = reduction factor for compressive strength of concrete

β_1 = reduction factor of compressive zone in concrete

ε_c = strain in concrete

ε_m = extreme compressive strain in concrete

ε_o = strain in concrete at the end of rectangular stress block

ε_{s1} = strain in layer 1 of

ε_{s2} = strain in layer 2 of reinforcement

ε_s = strain in reinforcing steel

ε_y = yield strain in reinforcing steel

- **Assumption of plane cross section**

Strain compatibility relations can be obtained from Figure 2-2 using a linear distribution of strains over the height of the cross section of the column:

$$\boxed{\frac{\varepsilon_m}{c} = \frac{\varepsilon_{s2}}{c-d}} \quad \Rightarrow \quad \varepsilon_{s2} = \varepsilon_m\left(1-\frac{d}{c}\right) \tag{2.1}$$

$$\boxed{\frac{\varepsilon_m}{c} = \frac{\varepsilon_{s1}}{c-d'}} \quad \Rightarrow \quad \varepsilon_{s1} = \varepsilon_m\left(1-\frac{d'}{c}\right) \tag{2.2}$$

- **Reduction factor of compression zone in concrete**

Relationship for reduction of a compression zone in concrete is as follows,

$$\boxed{a = \begin{cases} \beta_1 c & \text{for} \quad c \le \dfrac{h}{\beta_1} \\ h & \text{for} \quad c > \dfrac{h}{\beta_1}. \end{cases}} \tag{2.3}$$

Reduction factor β_1 dependents on strength of concrete in uniaxial compression test, f'_c, and is given by the relationship:

$$\beta_1 = \begin{cases} 0.85 & \text{for} \quad f'_c \le 4.0 \text{ ksi} \\ 1.05 - 0.05 f'_c & \text{for} \quad 4.0 \text{ ksi} \le f'_c \le 8.0 \text{ ksi} \\ 0.65 & \text{for} \quad f'_c \ge 8.0 \text{ psi}. \end{cases} \tag{2.4}$$

- **Mechanical behavior of reinforcing steel**

Material behavior of reinforcing steel is shown in Figure 2-3, and it is described by the following formula,

$$\boxed{f_s = \begin{cases} -f_y & \text{for} \quad \varepsilon_s < -\varepsilon_y \\ E_s \varepsilon_s & \text{for} \quad -\varepsilon_y \le \varepsilon_s \le \varepsilon_y \\ f_y & \text{for} \quad \varepsilon_y < \varepsilon_s < \varepsilon_m \\ 0 & \text{for} \quad \varepsilon_m < \varepsilon_s, \end{cases}} \tag{2.5}$$

where

$$\boxed{E_s = \frac{f_y}{\varepsilon_y}}. \tag{2.6}$$

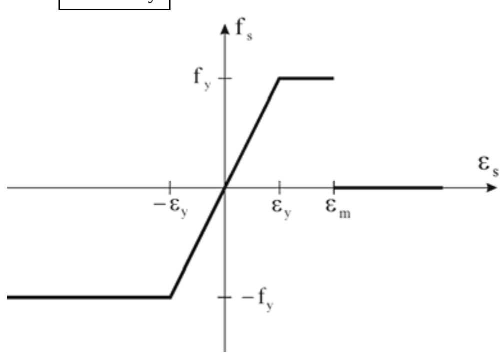


Figure 2-3. Stress vs. Strain Relationship for Reinforcing Steel.

- **Mechanical behavior of concrete**

Material behavior of concrete is shown in Figure 2-4, and it is described by the following formula,

$$\boxed{f_c = \begin{cases} 0 & \text{for} \quad \varepsilon_c < \varepsilon_o \\ \beta f'_c & \text{for} \quad \varepsilon_o \le \varepsilon_c \le \varepsilon_m \\ 0 & \text{for} \quad \varepsilon_m < \varepsilon_c \ . \end{cases}} \tag{2.7}$$

Based on the strain diagram from Figure 2-2, and linear distribution of strains over the height of the cross section of a column, the relationship for strains in the section is as follows:

$$\frac{\varepsilon_m}{c} = \frac{\varepsilon_o}{c-a} \tag{2.8}$$

and applying Eq.$(2.3)_1$ gives,

$$\boxed{\varepsilon_o = \varepsilon_m (1-\beta_1)}. \tag{2.9}$$

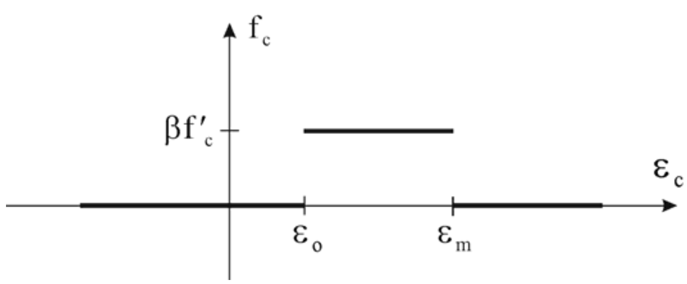


Figure 2-4. Stress vs. Strain Relationship for Concrete.

- **Force and moment caring capacity**

From the equilibrium of axial forces and moment acting in the section, the following equations are obtained,

$$\begin{gathered} \boxed{P = A_s f_{s1} + A_s f_{s2} + \beta f'_c \, ba}, \\ \boxed{M = A_s f_{s1}\left(\frac{h}{2} - d'\right) - A_s f_{s2}\left(\frac{h}{2} - d'\right) + \beta f'_c \, ba\left(\frac{h}{2} - \frac{a}{2}\right)}. \end{gathered} \tag{2.10}$$

- **Eccentricity of force**

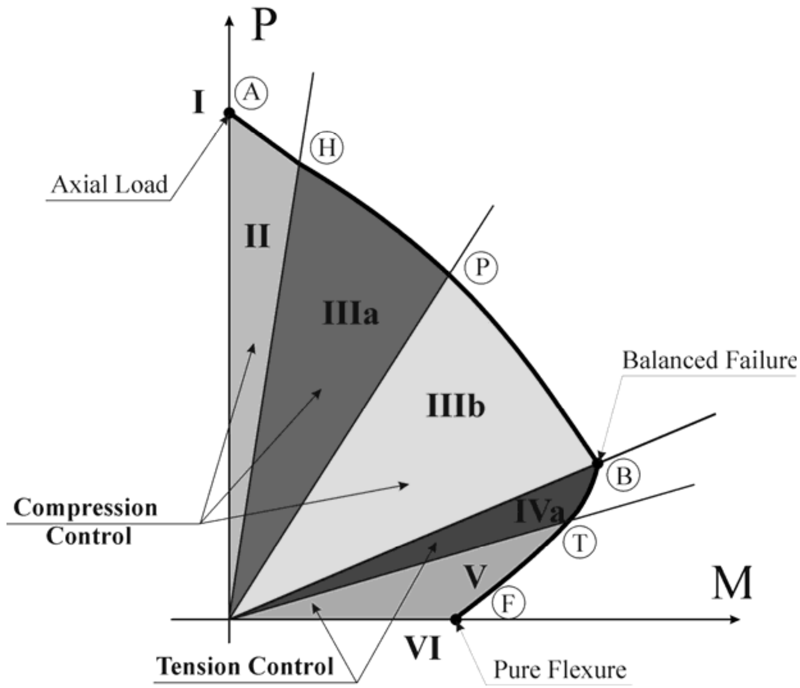
The eccentricity of the axial force is defined as follows:

$$\boxed{e = \frac{M}{P}} \tag{2.11}$$

2.1.2. Analysis of Possible Cases of Cross Section Behavior

In the analysis of possible design cases, it is assumed that in the upper fibers of the section, the strain is equal to ε_m, as shown in Figure 2-2. Formulas are developed for calculation of P and M, for various cases of (P,M) combinations , grouped as shown in Figure 2-5.

a)



b)

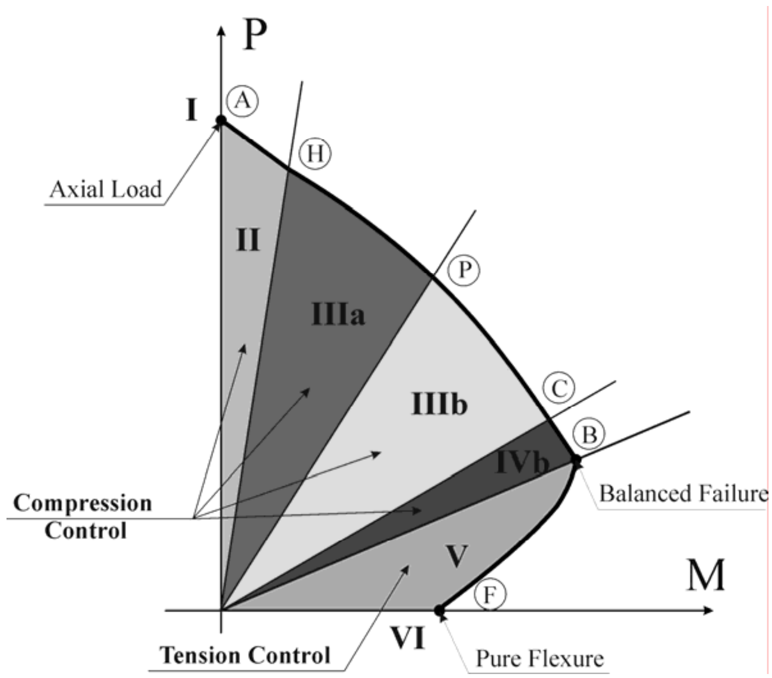


Figure 2-5. Interaction Diagram for Eccentrically Compressed Columns;
a) Cross Sections Type I ($49d' < 9d$), b) Cross Sections Type II ($49d' > 9d$).

2.1.3. Types of Cross Section

Two types of cross section can be distinguished as shown in Figure 2-5. Type I represents large cross sections, with case IVa in the tension control zone, as shown in Figure 2-5a. Type II is representative for small cross sections or those with larger cover of reinforcement, with case IVb in the compression control zone, as shown in Figure 2-5b.

The basic relationship for a balanced failure case is as follows:

$$\varepsilon_{s2} = -\varepsilon_y . \tag{2.12}$$

From Eq. (2.1) , and Eq. (2.12),

$$c = \frac{\varepsilon_m d}{\varepsilon_m + \varepsilon_y} . \tag{2.13}$$

Assuming

$$\varepsilon_{s1} = \varepsilon_y , \tag{2.14}$$

from Eq.(2.2), Eq.(2.13) and Eq.(2.14), the ratio of reinforcement cover to effective depth of the section is,

$$\frac{d'}{d} = \frac{\varepsilon_m - \varepsilon_y}{\varepsilon_m + \varepsilon_y} . \tag{2.15}$$

Assuming strain values for concrete and steel are,

$$\varepsilon_m = \frac{3}{1000} = 0.003, \qquad \varepsilon_y = \frac{f_y}{E_s} = \frac{3}{1450} \cong 0.002069 , \tag{2.16}$$

the ratio is equal to,

$$\frac{d'}{d} = \frac{\varepsilon_m - \varepsilon_y}{\varepsilon_m + \varepsilon_y} = \frac{9}{49} . \tag{2.17}$$

For cross section Type I (see Figure 2-5a, and Case IVa),

$$\varepsilon_{s1} \geq \varepsilon_y , \tag{2.18}$$

and

$$\frac{d'}{d} \leq \frac{9}{49} . \tag{2.19}$$

For cross sections Type II (see Figure 2-5b, and Case IVb),

$$\varepsilon_{s1} < \varepsilon_y, \tag{2.20}$$

and

$$\frac{d'}{d} > \frac{9}{49}. \tag{2.21}$$

For example, if $d' = 2.5\,\text{in}$, then from Eq.(2.17), $d = 13.61\,\text{in}$ and $h = d + d' = 16.11\,\text{in}$. Therefore, a cross section with $d' = 2.5\,\text{in}$ and $h \geq 16.11\,\text{in}$ is of Type I, and if $h < 16.11\,\text{in}$, the cross section is of Type II.

2.1.4. Cross Section Type I

For the cross section Type I, the axial force and moment (Eq. 2.10) are calculated as follows.

- **Case I (Axial Load)**

In this case, the following failure scenario is assumed: the whole concrete cross section is in compression and both layers of reinforcements yield in compression,

$$a = h, \quad f_{s1} = f_y, \quad f_{s2} = f_y. \tag{2.22}$$

Applying Eq.(2.22) to Eq. (2.10) and Eq. (2.11) yields

$$\boxed{P = 2A_s f_y + \beta f'_c\, bh},$$

$$\boxed{M = 0}, \tag{2.23}$$

$$\boxed{e = 0}.$$

The limit state is reached when the axial strain in concrete is ε_m (point *A* in Figure 2-5a) while layer 2 of reinforcement yields (also point *A* in Figure 2-5a).

From the overall axial strain, strains in layer 1 and 2 of the reinforcement are,

$$\varepsilon_{s1} = \varepsilon_m, \quad \varepsilon_{s2} = \varepsilon_m, \quad \Rightarrow \quad c = \infty. \tag{2.24}$$

From yielding in layer 2 of reinforcement,

$$\varepsilon_{s2} = \varepsilon_y \, . \tag{2.25}$$

From Eq.(2.25) and Eq.(2.1), the distance from the maximum strain fiber to the neutral axis is equal to:

$$c = \frac{\varepsilon_m d}{\varepsilon_m - \varepsilon_y} \, , \tag{2.26}$$

and from Eq.(2.26) and Eq.(2.2),

$$\varepsilon_{s1} = \varepsilon_m - (\varepsilon_m - \varepsilon_y)\frac{d'}{d} \, . \tag{2.27}$$

Finally, the following constraints are obtained for the basic parameters $(e, a, c, \varepsilon_{s1}, \varepsilon_{s2}, f_{s1}, f_{s2})$,

$$\begin{gathered} e = 0 \, , \\ a = h \, , \\ \frac{\varepsilon_m d}{\varepsilon_m - \varepsilon_y} \le c \le \infty \, , \\ \varepsilon_m - (\varepsilon_m - \varepsilon_y)\frac{d'}{d} \le \varepsilon_{s1} \le \varepsilon_m \, , \\ \varepsilon_y \le \varepsilon_{s2} \le \varepsilon_m \, , \\ f_{s1} = f_y \, , \\ f_{s2} = f_y \, . \end{gathered} \tag{2.28}$$

- **Case II (Whole Concrete Cross Section in Compression)**

In this case, the following failure scenario is assumed: the whole concrete cross section is in compression and layer 1 of reinforcing steel yields in compression but layer 2 of reinforcing steel is in an elastic range. Then,

$$a = h, \quad f_{s1} = f_y \, , \quad -f_y \le f_{s2} \le f_y \, . \tag{2.29}$$

Combining Eq.(2.5)$_2$ and Eq.(2.1) gives the following stress in layer 2 of reinforcement,

$$f_{s2} = E_s \varepsilon_m \left(1 - \frac{d}{c}\right). \tag{2.30}$$

From Eq.(2.30) and Eq.(2.10),

$$\boxed{P = A_s f_y + A_s E_s \varepsilon_m \left(1 - \frac{d}{c}\right) + \beta f'_c\, bh},$$
$$\boxed{M = A_s f_y \left(\frac{h}{2} - d'\right) - A_s E_s \varepsilon_m \left(1 - \frac{d}{c}\right)\left(\frac{h}{2} - d'\right)}, \tag{2.31}$$

and from Eq.(2.11) the eccentricity is,

$$e = \frac{A_s f_{s1}\left(\frac{h}{2} - d'\right) - A_s E_s \varepsilon_m \left(1 - \frac{d}{c}\right)\left(\frac{h}{2} - d'\right)}{A_s f_y + A_s E_s \varepsilon_m \left(1 - \frac{d}{c}\right) + \beta f'_c\, bh}. \tag{2.32}$$

The obtained relationship Eq.(2.32), is used to determine parameter "c", for a given value of eccentricity. After some rearrangements,

$$\boxed{c = \frac{A_s E_s \varepsilon_m d(2e + h - 2d')}{A_s f_y (2e - h + 2d') + A_s E_s \varepsilon_m (2e + h - 2d') + 2e\beta f'_c\, bh}}. \tag{2.33}$$

In Case II, the cross section behavior is limited by two conditions: the end of yielding in layer 2 of reinforcement (point *A* in Figure 2-5a) and end of compression of the whole concrete cross section (point *H* in Figure 2-5a).

The end of yielding in layer 2 of reinforcement was analyzed in the previous section.

The of end of compression of the whole concrete cross section corresponds to,

$$a = h. \tag{2.34}$$

From Eq.(2.34) and Eq.(2.3),

$$c = \frac{h}{\beta_1}, \tag{2.35}$$

and from Eq.(2.35), Eq.(2.1) and Eq.(2.2), strains in both layers of reinforcement are,

$$\varepsilon_{s2} = \varepsilon_m \left(1 - \frac{\beta_1 d}{h}\right), \tag{2.36}$$

$$\varepsilon_{s1} = \varepsilon_m \left(1 - \frac{\beta_1 d'}{h}\right). \tag{2.37}$$

From Eq.(2.35), Eq.(2.31) and Eq.(2.32),

$$P_H = A_s f_y + A_s E_s \varepsilon_m \left(1 - \frac{\beta_1 d}{h}\right) + \beta f'_c\, bh\,,$$

$$M_H = A_s f_y \left(\frac{h}{2} - d'\right) - A_s E_s \varepsilon_m \left(1 - \frac{\beta_1 d}{h}\right)\left(\frac{h}{2} - d'\right), \tag{2.38}$$

$$\boxed{e_H = \frac{A_s f_{s1}\left(\frac{h}{2} - d'\right) - A_s E_s \varepsilon_m \left(1 - \frac{\beta_1 d}{h}\right)\left(\frac{h}{2} - d'\right)}{A_s f_y + A_s E_s \varepsilon_m \left(1 - \frac{\beta_1 d}{h}\right) + \beta f'_c\, bh}}.$$

Finally, the following constraints are obtained for the basic parameters $(e, a, c, \varepsilon_{s1}, \varepsilon_{s2}, f_{s1}, f_{s2})$,

$$\begin{gathered}
0 \le e \le e_H\,, \\
a = h\,, \\
\frac{h}{\beta_1} \le c \le \frac{\varepsilon_m d}{\varepsilon_m - \varepsilon_y}\,, \\
\varepsilon_m \left(1 - \frac{\beta_1 d'}{h}\right) \le \varepsilon_{s1} \le \varepsilon_m - \left(\varepsilon_m - \varepsilon_y\right)\frac{d'}{d}\,, \\
\varepsilon_m \left(1 - \frac{\beta_1 d}{h}\right) \le \varepsilon_{s2} \le \varepsilon_y\,, \\
f_{s1} = f_y\,, \\
E_s \varepsilon_m \left(1 - \frac{\beta_1 d}{h}\right) \le f_{s2} \le f_y\,.
\end{gathered} \tag{2.39}$$

- **Case III (a Part of Concrete Cross Section is in Compression – Compression Control)**

In this case, the following failure scenario is assumed: a part of concrete cross section is in compression and layer 1 of reinforcing steel yields in compression but layer 2 of reinforcing steel is in an elastic range,

$$f_{s1} = f_y, \quad -f_y \le f_{s2} \le f_y. \tag{2.40}$$

Combining Eq.(2.3)$_1$, Eq.(2.5)$_2$ and Eq.(2.1), stress in layer 2 of reinforcement is equal to:

$$f_{s2} = E_s \varepsilon_m \left(1 - \frac{\beta_1 d}{a}\right). \tag{2.41}$$

Applying Eq.(2.41) in Eq.(2.10) yields,

$$\boxed{P = A_s f_y + A_s E_s \varepsilon_m \left(1 - \frac{\beta_1 d}{a}\right) + \beta f'_c\, ba},$$
$$\boxed{M = A_s f_y \left(\frac{h}{2} - d'\right) - A_s E_s \varepsilon_m \left(1 - \frac{\beta_1 d}{a}\right)\left(\frac{h}{2} - d'\right) + \frac{1}{2}\beta f'_c\, ba(h - a)}, \tag{2.42}$$

and Eq.(2.11) gives the eccentricity,

$$e = \frac{A_s f_y \left(\frac{h}{2} - d'\right) - A_s E_s \varepsilon_m \left(1 - \frac{\beta_1 d}{a}\right)\left(\frac{h}{2} - d'\right) + \frac{1}{2}\beta f'_c\, ba(h - a)}{A_s f_y + A_s E_s \varepsilon_m \left(1 - \frac{\beta_1 d}{a}\right) + \beta f'_c\, ba}. \tag{2.43}$$

Eq.(2.43) is used to evaluate parameter "a", knowing the value of eccentricity. After some rearrangement, the following equation has to be solved:

$$A_1 a^3 + A_2 a^2 + A_3 a + A_4 = 0, \tag{2.44}$$

with the following notation,

$$\boxed{A_1 = \beta f'_c\, b},$$
$$\boxed{A_2 = \beta f'_c\, b(2e - h)},$$
$$\boxed{A_3 = A_s f_y (2e - h + 2d') + A_s E_s \varepsilon_m (2e + h - 2d')}, \tag{2.45}$$
$$\boxed{A_4 = -A_s E_s \varepsilon_m \beta_1 d(2e + h - 2d')}.$$

In order to solve Eq.(2.44), the following notation is introduced:

$$\boxed{t = \frac{A_2}{3A_1}},$$

$$\boxed{p = \left(\frac{A_2}{3A_1}\right)^2 - \frac{A_3}{3A_1}}, \tag{2.46}$$
$$\boxed{q = \frac{A_2 A_3}{6A_1^2} - \left(\frac{A_2}{3A_1}\right)^3 - \frac{A_4}{2A_1}}.$$

The solution depends on sign of parameter p. Therefore,

$$\boxed{a = 2\sqrt{p}\cosh\left[\frac{1}{3}\operatorname{arccosh}\left(\frac{q}{\sqrt{p^3}}\right)\right] - t \qquad \text{for} \qquad p > 0}, \tag{2.47}$$

$$\boxed{a = \sqrt[3]{2q} - t \qquad \text{for} \qquad p = 0}, \tag{2.48}$$

$$\boxed{a = 2\sqrt{-p}\sinh\left[\frac{1}{3}\operatorname{arcsinh}\left(\frac{q}{\sqrt{-p^3}}\right)\right] - t \qquad \text{for} \qquad p < 0}, \tag{2.49}$$

For $p = 0$, the eccentricity that is used to distinguish solution Eq.(2.47) from Eq.(2.49) can be evaluated. After some rearrangements, the following value of eccentricity separates **Case IIIa** (Eq.(2.47)) from **Case IIIb** (Eq.(2.49)), see point *P* in Figure 2-5,

$$\boxed{e_P = \frac{h}{2} + \frac{3A_s\left(f_y + E_s\varepsilon_m\right) - \sqrt{\left[3A_s\left(f_y + E_s\varepsilon_m\right)\right]^2 + 24\beta f'_c b A_s\left[E_s\varepsilon_m h + \left(f_y - E_s\varepsilon_m\right)d'\right]}}{4\beta f'_c b}} \tag{2.50}$$

Accordingly, the cross section behavior is limited by two conditions: the end of compression of the whole concrete cross section (point *H* in Figure 2-5a) and the end of elastic behavior of layer 2 of reinforcement (point *B* in Figure 2-5a).

The end of compression of the whole concrete cross section was analyzed in the previous section.

For the end of elastic behavior of layer 2 of reinforcement (balanced failure) the strains are,

$$\varepsilon_{s2} = -\varepsilon_y \quad \Rightarrow \quad f_{s2} = -f_y. \tag{2.51}$$

From Eq.(2.51), Eq.(2.1) and Eq.(2.3)$_1$,

$$c = \frac{\varepsilon_m d}{\varepsilon_m + \varepsilon_y}, \tag{2.52}$$

$$a = \frac{\varepsilon_m \beta_1 d}{\varepsilon_m + \varepsilon_y}. \tag{2.53}$$

From Eq.(2.52) and Eq.(2.2),

$$\varepsilon_{s1} = \varepsilon_m - \left(\varepsilon_m + \varepsilon_y\right)\frac{d'}{d}. \tag{2.54}$$

Applying Eq.(2.53) in Eq.(2.42) and Eq.(2.43) yields,

$$P_B = \beta f'_c\, b \frac{\varepsilon_m \beta_1 d}{\varepsilon_m + \varepsilon_y},$$

$$M_B = A_s f_y \left(h - 2d'\right) + \frac{1}{2}\beta f'_c\, b \frac{\varepsilon_m \beta_1 d}{\varepsilon_m + \varepsilon_y}\left(h - \frac{\varepsilon_m \beta_1 d}{\varepsilon_m + \varepsilon_y}\right), \tag{2.55}$$

$$\boxed{e_B = \frac{A_s f_y \left(\varepsilon_m + \varepsilon_y\right)\left(h - 2d'\right)}{\beta f'_c\, b \varepsilon_m \beta_1 d} + \frac{1}{2}\left(h - \frac{\varepsilon_m \beta_1 d}{\varepsilon_m + \varepsilon_y}\right)}.$$

Finally, the following constraints are obtained for the basic parameters $(e, a, c, \varepsilon_{s1}, \varepsilon_{s2}, f_{s1}, f_{s2})$,

$$\begin{gathered}
e_H \le e \le e_B, \\
\frac{\varepsilon_m \beta_1 d}{\varepsilon_m + \varepsilon_y} \le a \le h, \\
\frac{\varepsilon_m d}{\varepsilon_m + \varepsilon_y} \le c \le \frac{h}{\beta_1}, \\
\varepsilon_m - \left(\varepsilon_m + \varepsilon_y\right)\frac{d'}{d} \le \varepsilon_{s1} \le \varepsilon_m\left(1 - \frac{\beta_1 d'}{h}\right), \\
-\varepsilon_y \le \varepsilon_{s2} \le \varepsilon_m\left(1 - \frac{\beta_1 d}{h}\right), \\
f_{s1} = f_y, \\
-f_y \le f_{s2} \le E_s \varepsilon_m\left(1 - \frac{\beta_1 d}{h}\right).
\end{gathered} \tag{2.56}$$

Note: Case IIIa is for $e_H \le e < e_P$, and **Case IIIb** is for $e_P < e \le e_B$.

- **Case IVa (Part of Concrete Cross Section is in Compression – Tension Control)**

In this case, the following failure scenario is assumed: a part of concrete cross section is in compression and both layers of reinforcement yield, layer 1 in compression but layer 2 in tension,

$$f_{s1} = f_y\,, \quad f_{s2} = -f_y\,. \tag{2.57}$$

Applying Eq.(2.57) in Eq.(2.10) yields

$$\boxed{P = \beta f'_c\, ba}\,,$$
$$\boxed{M = A_s f_y\left(h - 2d'\right) + \frac{1}{2}\beta f'_c\, ba\left(h - a\right)}\,, \tag{2.58}$$

from Eq.(2.11) the eccentricity is calculated as,

$$e = \frac{A_s f_y\left(h - 2d'\right) + \frac{1}{2}\beta f'_c\, ba\left(h - a\right)}{\beta f'_c\, ba}\,. \tag{2.59}$$

Relationship Eq.(2.59) is used to evaluate parameter “a”, for a known value of eccentricity. After rearrangements, following equation has to be solved,

$$A_1 a^2 + A_2 a + A_3 = 0\,, \tag{2.60}$$

where the following notation is used,

$$\boxed{A_1 = \beta f'_c\, b}\,,$$
$$\boxed{A_2 = \beta f'_c\, b\left(2e - h\right)}\,, \tag{2.61}$$
$$\boxed{A_3 = -2A_s f_y\left(h - 2d'\right)}\,.$$

The solution of Eq.(2.60) is,

$$\boxed{a = \frac{\sqrt{A_2^2 - 4A_1A_3} - A_2}{2A_1}}. \tag{2.62}$$

Accordingly, the cross section behavior is limited by two conditions: the end of elastic behavior of layer 2 of reinforcement (point *B* in Figure 2-5a) and the end of yielding of layer 1 of reinforcement (point *T* in Figure 2-5a).

The end of elastic behavior of layer 2 of reinforcement was analyzed in the previous section.

For the end of yielding of layer 1 reinforcement, the strain is

$$\varepsilon_{s1} = \varepsilon_y . \tag{2.63}$$

From Eq.(2.63), Eq.(2.2) and Eq.(2.3)$_1$,

$$c = \frac{\varepsilon_m d'}{\varepsilon_m - \varepsilon_y}, \tag{2.64}$$

$$a = \frac{\varepsilon_m \beta_1 d'}{\varepsilon_m - \varepsilon_y}. \tag{2.65}$$

From Eq.(2.64) and Eq.(2.1),

$$\varepsilon_{s2} = \varepsilon_m - (\varepsilon_m - \varepsilon_y)\frac{d}{d'}, \tag{2.66}$$

Applying Eq.(2.65) in Eq.(2.58) and Eq.(2.59) yields,

$$P_T = \beta f'_c\, b \frac{\varepsilon_m \beta_1 d'}{\varepsilon_m - \varepsilon_y},$$

$$M_T = A_s f_y (h - 2d') + \frac{1}{2}\beta f'_c\, b \frac{\varepsilon_m \beta_1 d'}{\varepsilon_m - \varepsilon_y}\left(h - \frac{\varepsilon_m \beta_1 d'}{\varepsilon_m - \varepsilon_y} \right), \tag{2.67}$$

$$\boxed{e_T = \frac{A_s f_y (\varepsilon_m - \varepsilon_y)(h - 2d')}{\beta f'_c\, b \varepsilon_m \beta_1 d'} + \frac{1}{2}\left(h - \frac{\varepsilon_m \beta_1 d'}{\varepsilon_m - \varepsilon_y} \right)}.$$

Finally, the following constraints are obtained for the basic parameters $(e, a, c, \varepsilon_{s1}, \varepsilon_{s2}, f_{s1}, f_{s2})$,

$$e_B \le e \le e_T ,$$

$$\frac{\varepsilon_m\beta_1 d'}{\varepsilon_m-\varepsilon_y}\le a\le\frac{\varepsilon_m\beta_1 d}{\varepsilon_m+\varepsilon_y},$$
$$\frac{\varepsilon_m d'}{\varepsilon_m-\varepsilon_y}\le c\le\frac{\varepsilon_m d}{\varepsilon_m+\varepsilon_y},$$
$$\varepsilon_y\le\varepsilon_{s1}\le\varepsilon_m-\left(\varepsilon_m+\varepsilon_y\right)\frac{d'}{d},\tag{2.68}$$
$$\varepsilon_m-\left(\varepsilon_m-\varepsilon_y\right)\frac{d}{d'}\le\varepsilon_{s2}\le-\varepsilon_y,$$
$$f_{s1}=f_y,$$
$$f_{s2}=-f_y.$$

- **Case V (Part of Concrete Cross Section is in Compression – Tension Control)**

In this case, the following failure scenario is assumed: a part of concrete cross section is in compression and layer 1 of reinforcing steel is in elastic range in compression but layer 2 of reinforcing steel is yielding,

$$f_{s2}=-f_y,\quad -f_y\le f_{s1}\le f_y.\tag{2.69}$$

Combining Eq.(2.3)$_1$, Eq.(2.5)$_2$ and Eq.(2.2), stress in layer 1 of reinforcement is,

$$f_{s1}=E_s\varepsilon_m\left(1-\frac{\beta_1 d'}{a}\right).\tag{2.70}$$

Applying Eq.(2.70) in Eq.(2.10) yields,

$$\boxed{P=A_sE_s\varepsilon_m\left(1-\frac{\beta_1 d'}{a}\right)-A_sf_y+\beta f'_c\,ba},$$
$$\boxed{M=A_sf_y\left(\frac{h}{2}-d'\right)+A_sE_s\varepsilon_m\left(1-\frac{\beta_1 d'}{a}\right)\left(\frac{h}{2}-d'\right)+\frac{1}{2}\beta f'_c\,ba\left(h-a\right)},\tag{2.71}$$

and using Eq.(2.11) the eccentricity is,

$$e=\frac{A_sf_y\left(\frac{h}{2}-d'\right)+A_sE_s\varepsilon_m\left(1-\frac{\beta_1 d'}{a}\right)\left(\frac{h}{2}-d'\right)+\frac{1}{2}\beta f'_c\,ba\left(h-a\right)}{A_sE_s\varepsilon_m\left(1-\frac{\beta_1 d'}{a}\right)-A_sf_y+\beta f'_c\,ba}.\tag{2.72}$$

Eq.(2.72) is used to evaluate parameter "a" for a known value of eccentricity. After rearrangements, the following equation has to be solved:

$$A_1 a^3 + A_2 a^2 + A_3 a + A_4 = 0\,, \tag{2.73}$$

where the following notation is used,

$$\begin{gathered}\boxed{A_1 = \beta f'_c\, b}\,,\\ \boxed{A_2 = \beta f'_c\, b(2e - h)}\,,\\ \boxed{A_3 = A_s E_s \varepsilon_m (2e - h + 2d') - A_s f_y (2e + h - 2d')}\,,\\ \boxed{A_4 = -A_s E_s \varepsilon_m \beta_1 d' (2e - h + 2d')}\,.\end{gathered} \tag{2.74}$$

In order to solve Eq.(2.73), the following notation is introduced,

$$\begin{gathered}\boxed{t = \frac{A_2}{3A_1}}\,,\\ \boxed{p = \left(\frac{A_2}{3A_1}\right)^2 - \frac{A_3}{3A_1}}\,,\\ \boxed{q = \frac{A_2 A_3}{6A_1^2} - \left(\frac{A_2}{3A_1}\right)^3 - \frac{A_4}{2A_1}}\,.\end{gathered} \tag{2.75}$$

The solution is,

$$\boxed{a = 2\sqrt{p}\,\cos\left[\frac{1}{3}\arccos\left(\frac{q}{\sqrt{p^3}}\right)\right] - t}\,, \tag{2.76}$$

Accordingly, the cross section behavior is limited by two conditions: the end of elastic behavior of layer 2 of reinforcement (point *T* in Figure 2-5a) and a pure flexure (point *F* in Figure 2-5a).

The end of elastic behavior of layer 2 of reinforcement was analyzed in the previous section.

For the **pure flexure (Case VI)**, the axial force is,

$$P = 0\,. \tag{2.77}$$

Applying Eq.(2.77) in Eq.(2.71)$_1$ and after some rearrangments,

$$\alpha_1 a_M^2 + \alpha_2 a_M + \alpha_3 = 0\,, \tag{2.78}$$

where the following notation is used

$$\begin{aligned} \boxed{\alpha_1 = \beta f'_c\, b}, \\ \boxed{\alpha_2 = A_s\left(E_s\varepsilon_m - f_y\right)}, \\ \boxed{\alpha_3 = -A_s E_s \varepsilon_m \beta_1 d'}. \end{aligned} \tag{2.79}$$

The solution of Eq.(2.78) is,

$$\boxed{a_M = \frac{\sqrt{\alpha_2^2 - 4\alpha_1\alpha_3} - \alpha_2}{2\alpha_1}}. \tag{2.80}$$

From Eq.(2.80) and Eq.(2.3)$_1$,

$$c = \frac{a_M}{\beta_1}\,, \tag{2.81}$$

From Eq.(2.80), Eq.(2.1) and Eq.(2.2),

$$\begin{aligned} \varepsilon_{s2} &= \varepsilon_m\left(1 - \frac{\beta_1 d}{a_M}\right), \\ \varepsilon_{s1} &= \varepsilon_m\left(1 - \frac{\beta_1 d'}{a_M}\right). \end{aligned} \tag{2.82}$$

Applying Eq.(2.80) in Eq.(2.42) yields

$$\boxed{P = 0}, \qquad \boxed{e = \infty},$$

$$\boxed{M = A_s f_y\left(\frac{h}{2} - d'\right) + A_s E_s \varepsilon_m\left(1 - \frac{\beta_1 d'}{a_M}\right)\left(\frac{h}{2} - d'\right) + \frac{1}{2}\beta f'_c\, b a_M\left(h - a_M\right)} \tag{2.83}$$

Finally, the following constraints are obtained for the basic parameters $(e, a, c, \varepsilon_{s1}, \varepsilon_{s2}, f_{s1}, f_{s2})$,

$$\begin{aligned} e_T \le e \le \infty\,, \\ a_M \le a \le \frac{\varepsilon_m \beta_1 d'}{\varepsilon_m - \varepsilon_y}\,, \end{aligned}$$

$$\frac{a_M}{\beta_1} \le c \le \frac{\varepsilon_m d'}{\varepsilon_m - \varepsilon_y},$$

$$\varepsilon_m\left(1 - \frac{\beta_1 d'}{a_M}\right) \le \varepsilon_{s1} \le \varepsilon_y, \tag{2.84}$$

$$\varepsilon_m\left(1 - \frac{\beta_1 d}{a_M}\right) \le \varepsilon_{s2} \le \varepsilon_m - (\varepsilon_m - \varepsilon_y)\frac{d}{d'},$$

$$E_s\varepsilon_m\left(1 - \frac{\beta_1 d'}{a_M}\right) \le f_{s1} \le f_y,$$

$$f_{s2} = -f_y.$$

2.1.5. Cross Section Type II

For the cross section Type II, the axial force and moment (Eq.2.10) are calculated as follows. Cases I, II, III, and V, VI remain unchanged, but in Cases III and V, there are changes in the boundaries of application for the developed formulas.

- **Case III (Part of Concrete Cross Section is in Compression – Compression Control)**

The only change, compared to the previously considered Case III, is the range of plastic behavior of layer 1 of reinforcement (point *C* in Figure 2-5b),

$$\varepsilon_{s1} = \varepsilon_y \quad \Rightarrow \quad f_{s1} = f_y. \tag{2.85}$$

From Eq.(2.85), Eq.(2.2) and Eq.$(2.3)_1$,

$$c = \frac{\varepsilon_m d'}{\varepsilon_m - \varepsilon_y}, \tag{2.86}$$

$$a = \frac{\varepsilon_m \beta_1 d'}{\varepsilon_m - \varepsilon_y}. \tag{2.87}$$

From Eq.(2.86) and Eq.(2.1),

$$\varepsilon_{s2} = \varepsilon_m - (\varepsilon_m - \varepsilon_y)\frac{d}{d'}, \tag{2.88}$$

Applying Eq.(2.87) in Eq.(2.42) and Eq.(2.43) yields,

$$P_C = A_s f_y + A_s E_s \left[\varepsilon_m - (\varepsilon_m - \varepsilon_y) \frac{d}{d'} \right] + \beta f'_c \, b \frac{\varepsilon_m \beta_1 d'}{\varepsilon_m - \varepsilon_y},$$

$$M_C = A_s \left\{ f_y - E_s \left[\varepsilon_m - (\varepsilon_m - \varepsilon_y) \frac{d}{d'} \right] \right\} \left(\frac{h}{2} - d' \right) + \frac{1}{2} \beta f'_c \, b \frac{\varepsilon_m \beta_1 d'}{\varepsilon_m - \varepsilon_y} \left(h - \frac{\varepsilon_m \beta_1 d'}{\varepsilon_m - \varepsilon_y} \right), \qquad (2.89)$$

$$\boxed{e_C = \frac{A_s \left\{ f_y - E_s \left[\varepsilon_m - (\varepsilon_m - \varepsilon_y) \frac{d}{d'} \right] \right\} \left(\frac{h}{2} - d' \right) + \frac{1}{2} \beta f'_c \, b \frac{\varepsilon_m \beta_1 d'}{\varepsilon_m - \varepsilon_y} \left(h - \frac{\varepsilon_m \beta_1 d'}{\varepsilon_m - \varepsilon_y} \right)}{A_s \left\{ f_y + E_s \left[\varepsilon_m - (\varepsilon_m - \varepsilon_y) \frac{d}{d'} \right] \right\} + \beta f'_c \, b \frac{\varepsilon_m \beta_1 d'}{\varepsilon_m - \varepsilon_y}}}$$

Finally, the following constraints are obtained for the basic parameters $(e, a, c, \varepsilon_{s1}, \varepsilon_{s2}, f_{s1}, f_{s2})$,

$$\begin{gathered}
e_H \le e \le e_C, \\
\frac{\varepsilon_m \beta_1 d'}{\varepsilon_m - \varepsilon_y} \le a \le h, \\
\frac{\varepsilon_m d'}{\varepsilon_m - \varepsilon_y} \le c \le \frac{h}{\beta_1}, \\
\varepsilon_y \le \varepsilon_{s1} \le \varepsilon_m \left(1 - \frac{\beta_1 d'}{h} \right), \\
\varepsilon_m - (\varepsilon_m - \varepsilon_y) \frac{d}{d'} \le \varepsilon_{s2} \le \varepsilon_m \left(1 - \frac{\beta_1 d}{h} \right), \\
f_{s1} = f_y, \\
E_s \left[\varepsilon_m - (\varepsilon_m - \varepsilon_y) \frac{d}{d'} \right] \le f_{s2} \le E_s \varepsilon_m \left(1 - \frac{\beta_1 d}{h} \right).
\end{gathered} \qquad (2.90)$$

- **Case IVb (Part of Concrete Cross Section is in Compression – Compression Control)**

In this case, the following failure scenario is assumed: a part of concrete cross section is in compression and both layers of reinforcing steel are in elastic range,

$$-f_y \le f_{s1} \le f_y, \quad -f_y \le f_{s2} \le f_y. \qquad (2.91)$$

Combining Eq.(2.3)$_1$, Eq.(2.5)$_2$ and Eq.(2.2), stress in layer 1 of reinforcement is,

$$f_{s1} = E_s \varepsilon_m \left(1 - \frac{\beta_1 d'}{a}\right). \tag{2.92}$$

Similarly, combining Eq.(2.3)$_1$, Eq.(2.5)$_2$ and Eq.(2.1), stress in layer 2 of reinforcement is,

$$f_{s2} = E_s \varepsilon_m \left(1 - \frac{\beta_1 d}{a}\right). \tag{2.93}$$

Applying Eq.(2.92) and Eq.(2.93) in Eq.(2.10) yields,

$$\boxed{P = A_s E_s \varepsilon_m \left(2 - \frac{\beta_1 h}{a}\right) + \beta f'_c\, ba},$$

$$\boxed{M = A_s E_s \varepsilon_m \frac{(h - 2d')^2 \beta_1}{2a} + \frac{1}{2}\beta f'_c\, ba(h - a)}, \tag{2.94}$$

and from Eq.(2.11) the eccentricity is,

$$e = \frac{A_s E_s \varepsilon_m \frac{(h - 2d')^2 \beta_1}{2a} + \frac{1}{2}\beta f'_c\, ba(h - a)}{A_s E_s \varepsilon_m \left(2 - \frac{\beta_1 h}{a}\right) + \beta f'_c\, ba}. \tag{2.95}$$

Eq.(2.95) is used to evaluate the parameter "a", for a known value of eccentricity. After some rearrangments, the following equation has to be solved,

$$A_1 a^3 + A_2 a^2 + A_3 a + A_4 = 0, \tag{2.96}$$

where the following notation is used,

$$\boxed{A_1 = \beta f'_c\, b},$$
$$\boxed{A_2 = \beta f'_c\, b(2e - h)},$$
$$\boxed{A_3 = 4 A_s E_s \varepsilon_m e}, \tag{2.97}$$
$$\boxed{A_4 = -A_s E_s \varepsilon_m \beta_1 \left[2he + (h - 2d')^2\right]}.$$

In order to solve Eq.(2.96), the following notation is introduced,

$$\boxed{t = \frac{A_2}{3A_1}},$$

$$\boxed{p = \left(\frac{A_2}{3A_1}\right)^2 - \frac{A_3}{3A_1}}, \tag{2.98}$$
$$\boxed{q = \frac{A_2 A_3}{6A_1^2} - \left(\frac{A_2}{3A_1}\right)^3 - \frac{A_4}{2A_1}}.$$

Therefore, the solution is,

$$\boxed{a = 2\sqrt{-p}\,\sinh\left[\frac{1}{3}\operatorname{arcsinh}\left(\frac{q}{\sqrt{-p^3}}\right)\right] - t}. \tag{2.99}$$

Accordingly, the cross section behavior is limited by two conditions: the end of elastic behavior of layer 1 of reinforcement (point *C* in Figure 2-5b) and beginning of yielding of layer 2 of reinforcement (point *B* in Figure 2-5b).

The end point of elastic behavior of layer 1 of reinforcement was analyzed in the previous section.

For the beginning of yielding of layer 2 of reinforcement in tension (balanced failure) the strain is,

$$\varepsilon_{s2} = -\varepsilon_y . \tag{2.100}$$

From Eq.(2.100), Eq.(2.1) and Eq.(2.3)$_1$,

$$c = \frac{\varepsilon_m d}{\varepsilon_m + \varepsilon_y}, \tag{2.101}$$

$$a = \frac{\varepsilon_m \beta_1 d}{\varepsilon_m + \varepsilon_y}. \tag{2.102}$$

From Eq.(2.101) and Eq.(2.2),

$$\varepsilon_{s1} = \varepsilon_m - (\varepsilon_m + \varepsilon_y)\frac{d'}{d}. \tag{2.103}$$

Applying Eq.(2.102) in Eq.(2.94) and Eq.(2.95) yields,

$$P_B = A_s E_s\left(2\varepsilon_m - (\varepsilon_m + \varepsilon_y)\frac{h}{d}\right) + \beta f'_c\, b\frac{\varepsilon_m \beta_1 d}{\varepsilon_m + \varepsilon_y},$$

$$M_B = A_s E_s(\varepsilon_m + \varepsilon_y)\frac{(h-2d')^2}{2d} + \frac{1}{2}\beta f'_c\, b\frac{\varepsilon_m \beta_1 d}{\varepsilon_m + \varepsilon_y}\left(h - \frac{\varepsilon_m \beta_1 d}{\varepsilon_m + \varepsilon_y}\right), \tag{2.104}$$

$$\boxed{e_B = \frac{A_s E_s(\varepsilon_m + \varepsilon_y)\frac{(h-2d')^2}{2d} + \frac{1}{2}\beta f'_c\, b\frac{\varepsilon_m \beta_1 d}{\varepsilon_m + \varepsilon_y}\left(h - \frac{\varepsilon_m \beta_1 d}{\varepsilon_m + \varepsilon_y}\right)}{A_s E_s\left(2\varepsilon_m - (\varepsilon_m + \varepsilon_y)\frac{h}{d}\right) + \beta f'_c\, b\frac{\varepsilon_m \beta_1 d}{\varepsilon_m + \varepsilon_y}}}$$

Finally, the following constraints are obtained for the basic parameters $(e, a, c, \varepsilon_{s1}, \varepsilon_{s2}, f_{s1}, f_{s2})$,

$$\begin{gathered}
e_C \le e \le e_B, \\
\frac{\varepsilon_m \beta_1 d}{\varepsilon_m + \varepsilon_y} \le a \le \frac{\varepsilon_m \beta_1 d'}{\varepsilon_m - \varepsilon_y}, \\
\frac{\varepsilon_m d}{\varepsilon_m + \varepsilon_y} \le c \le \frac{\varepsilon_m d'}{\varepsilon_m - \varepsilon_y}, \\
\varepsilon_m - (\varepsilon_m + \varepsilon_y)\frac{d'}{d} \le \varepsilon_{s1} \le \varepsilon_y, \\
-\varepsilon_y \le \varepsilon_{s2} \le \varepsilon_m - (\varepsilon_m - \varepsilon_y)\frac{d}{d'}, \\
E_s\left[\varepsilon_m - (\varepsilon_m + \varepsilon_y)\frac{d'}{d}\right] \le f_{s1} \le f_y, \\
-f_y \le f_{s2} \le E_s\left[\varepsilon_m - (\varepsilon_m - \varepsilon_y)\frac{d}{d'}\right].
\end{gathered} \tag{2.105}$$

- **Case V (Part of Concrete Cross Section is in Compression – Tension Control)**

In this case, there is only one change in boundary constraints. The following constraints for the basic parameters $(e, a, c, \varepsilon_{s1}, \varepsilon_{s2}, f_{s1}, f_{s2})$ can be formulated in this case,

$$\begin{gathered}
e_B \le e \le \infty, \\
a_M \le a \le \frac{\varepsilon_m \beta_1 d}{\varepsilon_m + \varepsilon_y}, \\
\frac{a_M}{\beta_1} \le c \le \frac{\varepsilon_m d}{\varepsilon_m + \varepsilon_y},
\end{gathered}$$

$$\varepsilon_m\left(1-\frac{\beta_1 d'}{a_M}\right) \le \varepsilon_{s1} \le \varepsilon_m - \left(\varepsilon_m + \varepsilon_y\right)\frac{d'}{d}, \tag{2.106}$$

$$\varepsilon_m\left(1-\frac{\beta_1 d}{a_M}\right) \le \varepsilon_{s2} \le -\varepsilon_y,$$

$$E_s\varepsilon_m\left(1-\frac{\beta_1 d'}{a_M}\right) \le f_{s1} \le E_s\left[\varepsilon_m - \left(\varepsilon_m + \varepsilon_y\right)\frac{d'}{d}\right],$$

$$f_{s2} = -f_y.$$

2.2. Statistical Parameters of Resistance

The statistical parameters of resistance were determined by the Monte Carlo simulations using design equations specified for each design case. The design parameters: b, h, d, f_c', f_y, A_s, E_s, were treated as random variables, and constants were treated as deterministic values. The cumulative distribution function (CDF) of resistance was obtained by generating 200,000 values of resistance, R, for each design case. An example of simulated interaction diagram is shown in Figure 2-6. Based on this information, it was possible to calculate the mean of R, m_R, bias factor, λ_R, and coefficient of variation, V_R. The resistance simulations were performed for concrete strength, f_c' = 3 ksi, 5ksi, 8 ksi and 12 ksi. For comparison, the simulations were also performed for material statistical parameters used in the previous code calibration (Elingwood et al. 1980), denoted here as "old concrete".

For example, the resulting mean and nominal interaction diagrams are shown in Figures 2-7 through 2-10, for cast-in-place tied columns with f_c' = 3 ksi, 5 ksi, 8 ksi and 12 ksi respectively. Also shown are the interaction diagrams corresponding to one standard deviation above and below the mean.

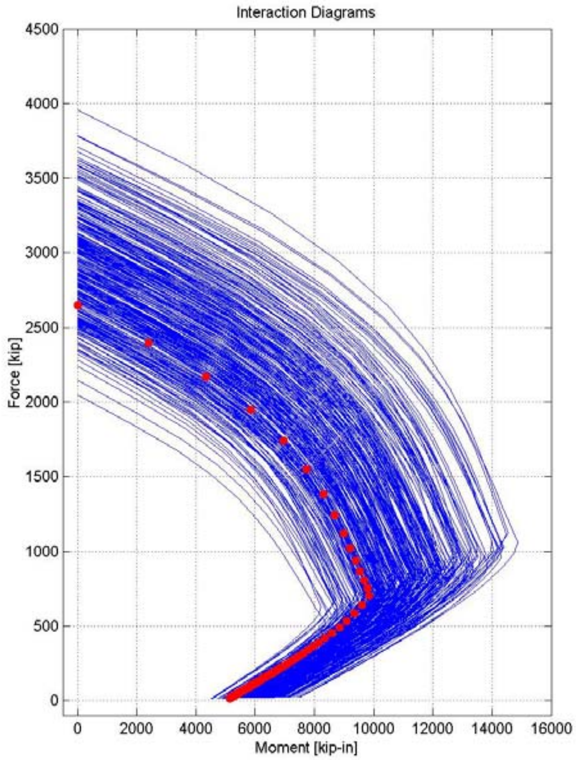


Figure 2-6. Simulated Interaction Diagrams (for concrete strength of 8 ksi, tied columns, cast-in-place), Red Dots are Nominal Values.

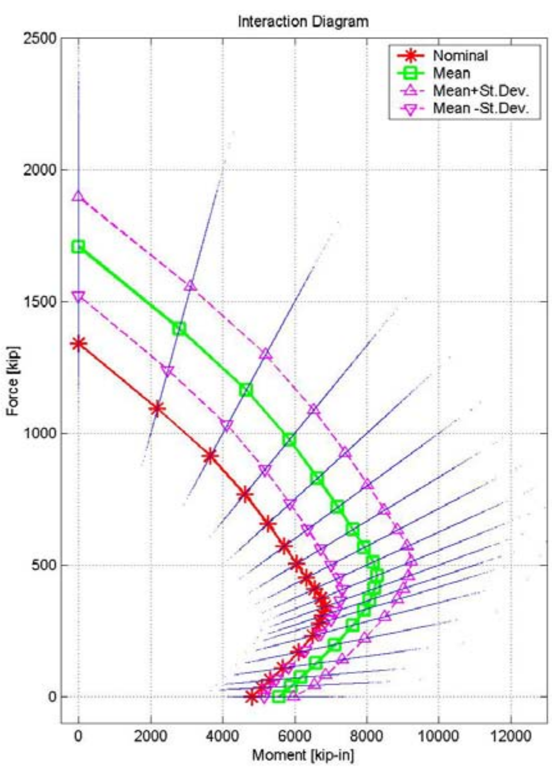


Figure 2-7. Mean and Nominal Interaction Diagrams (for concrete strength of 3 ksi, tied columns, cast-in-place).

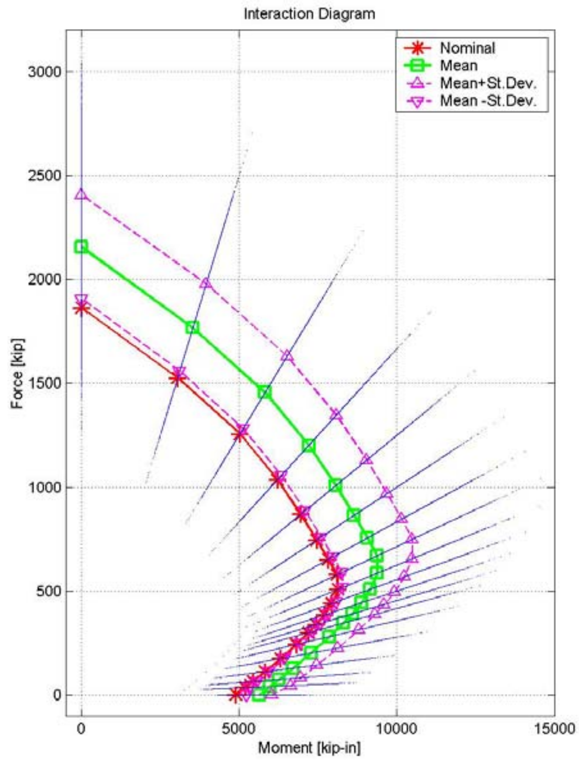


Figure 2-8. Mean and Nominal Interaction Diagrams (for concrete strength of 5 ksi, tied columns, cast-in-place).

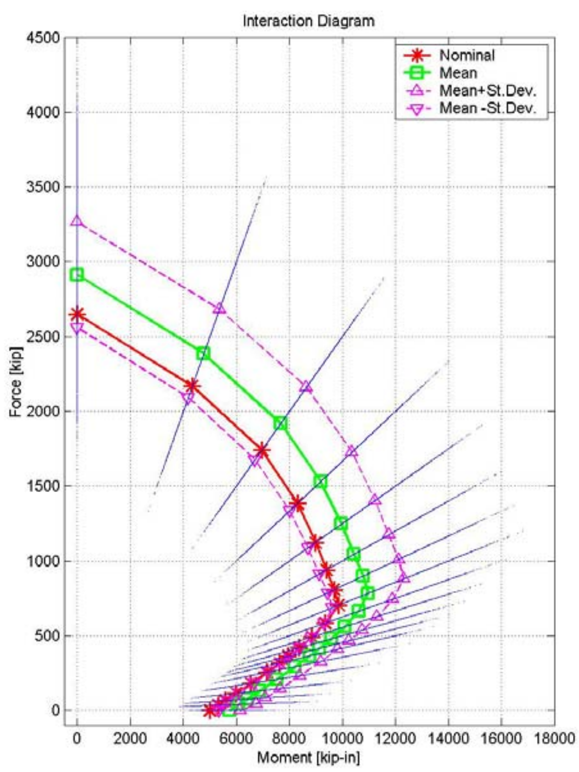


Figure 2-9. Mean and Nominal Interaction Diagrams (for concrete Strength of 8 ksi, tied columns, cast-in-place).

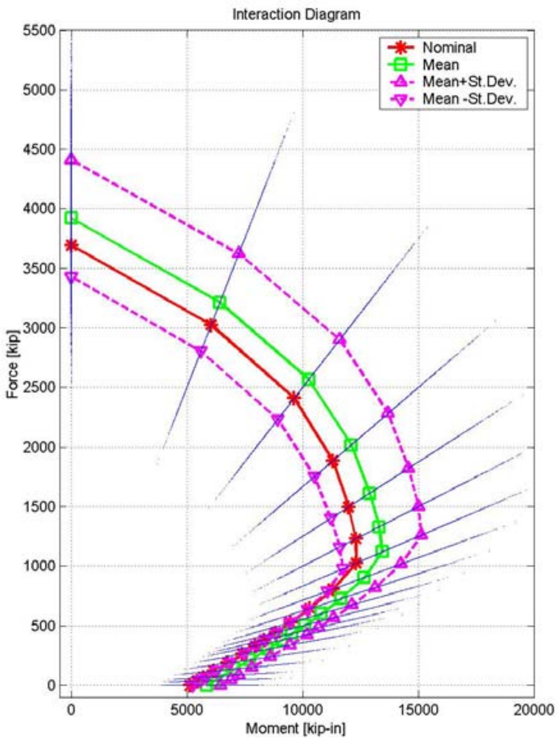


Figure 2-10. Mean and Nominal Interaction Diagrams (for concrete strength of 12 ksi, tied columns, cast-in-place).

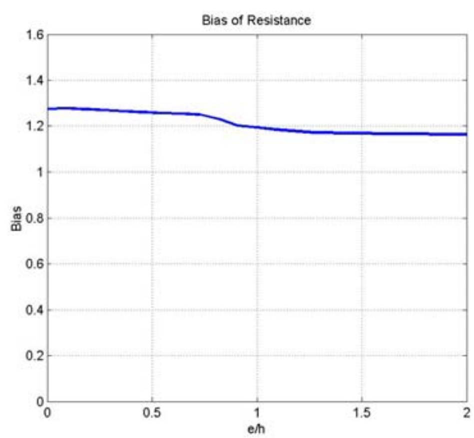


Figure 2-11. Bias Factor of Resistance (calculated for tied columns cast-in-place made of concrete with strength of 3 ksi).

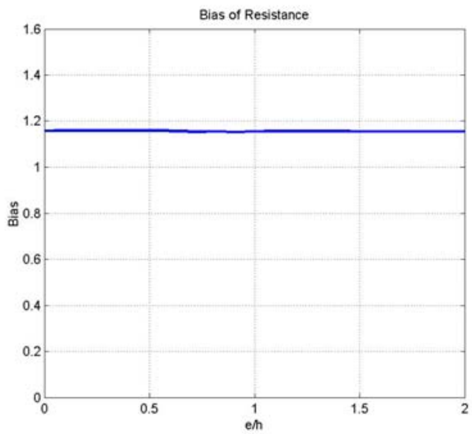


Figure 2-12. Bias Factor of Resistance (calculated for tied columns cast-in-place made of concrete with strength of 5 ksi).

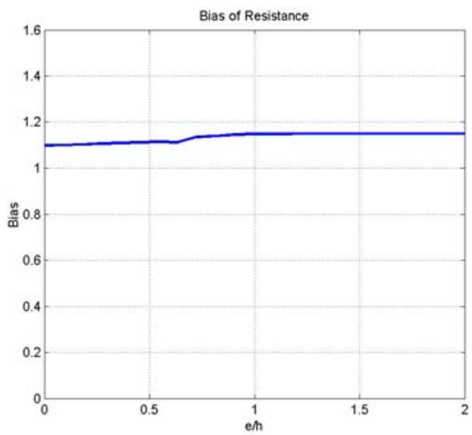


Figure 2-13. Bias Factor of Resistance (calculated for tied columns cast-in-place made of concrete with strength of 8 ksi).

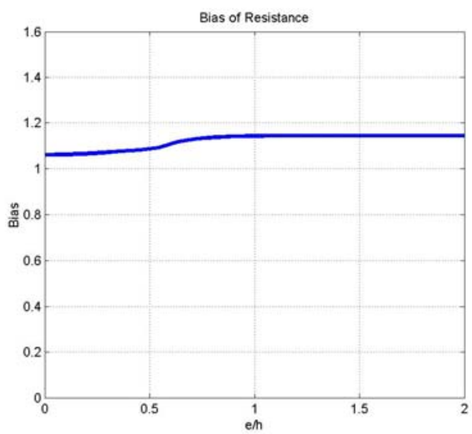


Figure 2-14. Bias Factor of Resistance (calculated for tied columns cast-in-place made of concrete with strength of 12 ksi).

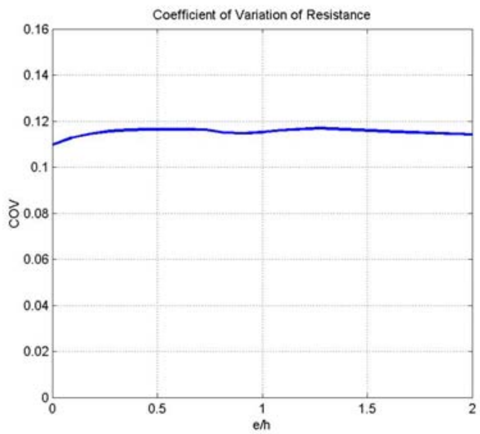


Figure 2-15. Coefficient of Variation of Resistance (calculated for tied columns cast-in-place made of concrete with strength of 3 ksi).

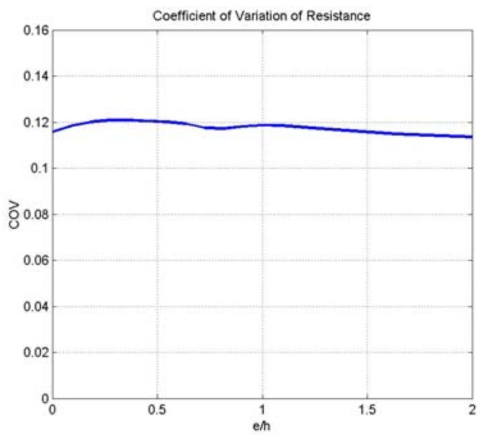


Figure 2-16. Coefficient of Variation of Resistance (calculated for tied columns cast-in-place made of concrete with strength of 5 ksi).

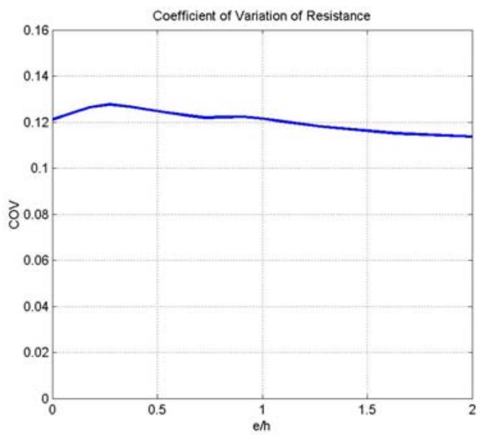


Figure 2-17. Coefficient of Variation of Resistance (calculated for tied columns cast-in-place made of concrete with strength of 8 ksi).

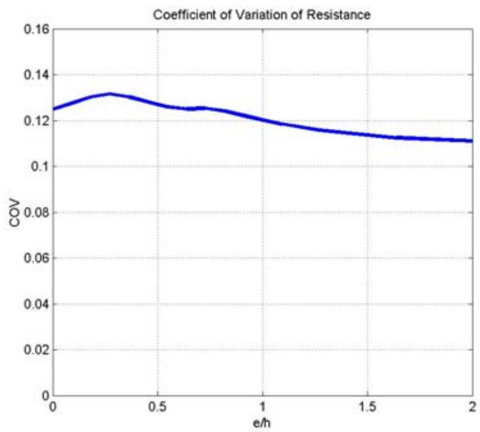


Figure 2-18. Coefficient of Variation of Resistance (calculated for tied columns cast-in-place made of concrete with strength of 12 ksi).

The bias factors of resistance for cast-in-place tied columns are plotted vs. e/h (ratio of eccentricity and total depth of the cross section) in Figures 2-11 through 2-14, for f_c' = 3 ksi, 5 ksi, 8 ksi and 12 ksi, respectively.

The coefficients of variation of resistance for cast-in-place tied columns are plotted vs. e/h (ratio of eccentricity and total depth of the cross section) in Figures 2-15 through 2-18, for f_c' = 3 ksi, 5 ksi, 8 ksi and 12 ksi respectively.

Monte Carlo simulations were performed for three column cross sections: square (16 in x 16 in), rectangular (14 in x 22 in, with h/b = 1.57), and larger rectangular section (18 in x 36 in, with h/b = 2). No significant difference in statistical parameters for resistance was found for square and smaller rectangular (h/b = 1.57) cross sections. For larger rectangular cross sections (h/b = 2), the analysis was performed separately.

Tables 2-1a and b, 2-3a and b, contain bias factors of resistance for tied columns, cast-in-place and plant cast, calculated for reinforcement ratios of 1%, 2%, 3% and 4%, and concrete strengths of 3 ksi, 5 ksi, 8 ksi and 12 ksi (for h/b = 1.57). Tables 2-2a and b, 2-4a and b, contain bias factors of resistance for tied columns, cast-in-place and plant cast, calculated for reinforcement ratios of 1%, 2%, 3% and 4%, and concrete strengths of 3 ksi, 5 ksi, 8 ksi and 12 ksi (for h/b = 2). Tables 2-9a and b, and 2-10a and b, contain bias factors of resistance for spiral columns, cast-in-place and plant cast, calculated for reinforcement ratios of 1%, 2%, 3% and 4%, and concrete strengths of 3 ksi, 5 ksi, 8 ksi and 12 ksi.

Tables 2-5a and b, 2-7a and b, contain coefficient of variation of resistance for tied columns, cast-in-place and plant cast, calculated for reinforcement ratios of 1%, 2%, 3% and 4%, and concrete strengths of 3 ksi, 5 ksi, 8 ksi and 12 ksi (for h/b = 1.57). Tables 2-6a and b, 2-8a and b, contain coefficient of variation of resistance for tied columns, cast-in-place and plant cast, calculated for reinforcement ratios of 1%, 2%, 3% and 4%, and concrete strengths of 3 ksi, 5 ksi, 8 ksi and 12 ksi (for h/b = 2). Tables 2-11a and b, 2-12a and b, contain coefficient of variation of resistance for spiral columns, cast-in-place and plant cast, calculated for reinforcement ratios of 1%, 2%, 3% and 4%, and concrete strengths of 3 ksi, 5 ksi, 8 ksi and 12 ksi.

Table 2-1a. Bias Factor of Resistance for Eccentrically Loaded Columns (tied, cast-in-place, for h/b = 1.57)

Design case			Tied				
% of reinf.	e (in)	e/h	Cast-in-place bias				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
1%	0	0.00	0.982	1.320	1.159	1.092	1.050
	4	0.18	0.986	1.320	1.161	1.095	1.056
	8	0.36	1.001	1.302	1.162	1.103	1.069
	10	0.45	1.024	1.282	1.161	1.116	1.095
	12	0.55	1.055	1.246	1.156	1.133	1.124
	14	0.64	1.078	1.216	1.160	1.142	1.136
	16	0.73	1.095	1.202	1.161	1.147	1.141
	18	0.82	1.101	1.194	1.160	1.147	1.144
	20	0.91	1.110	1.187	1.159	1.150	1.143
	24	1.09	1.112	1.177	1.157	1.148	1.140
	40	1.82	1.116	1.169	1.156	1.147	1.141
	72	3.27	1.115	1.167	1.154	1.145	1.140
2%	0	0.00	0.997	1.292	1.160	1.096	1.055
	4	0.18	1.002	1.292	1.160	1.099	1.060
	8	0.36	1.012	1.279	1.161	1.105	1.072
	10	0.45	1.022	1.274	1.159	1.111	1.077
	12	0.55	1.021	1.269	1.151	1.115	1.110
	14	0.64	1.055	1.250	1.154	1.131	1.130
	16	0.73	1.074	1.219	1.152	1.141	1.141
	18	0.82	1.095	1.200	1.156	1.147	1.139
	20	0.91	1.102	1.187	1.157	1.146	1.143
	24	1.09	1.112	1.175	1.155	1.148	1.143
	40	1.82	1.121	1.165	1.154	1.149	1.143
	72	3.27	1.121	1.163	1.152	1.147	1.144

Note: Highlighted values are for the balanced failure

Table 2-1b. Bias Factor of Resistance for Eccentrically Loaded Columns (tied, cast-in-place, h/b = 1.57)

Design case			Tied				
% of reinf.	e (in)	e/h	Cast-in-place bias				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
3%	0	0.00	1.007	1.274	1.159	1.096	1.061
	4	0.18	1.016	1.273	1.158	1.104	1.064
	8	0.36	1.025	1.261	1.159	1.110	1.076
	10	0.45	1.028	1.262	1.159	1.110	1.084
	12	0.55	1.032	1.257	1.157	1.113	1.094
	14	0.64	1.036	1.254	1.156	1.114	1.120
	16	0.73	1.049	1.248	1.155	1.133	1.132
	18	0.82	1.074	1.228	1.154	1.138	1.136
	20	0.91	1.085	1.198	1.151	1.143	1.141
	24	1.09	1.109	1.184	1.154	1.148	1.144
	40	1.82	1.119	1.165	1.153	1.147	1.147
	72	3.27	1.125	1.158	1.154	1.150	1.143
4%	0	0.00	1.017	1.260	1.157	1.099	1.067
	4	0.18	1.022	1.256	1.160	1.104	1.068
	8	0.36	1.032	1.249	1.159	1.110	1.078
	10	0.45	1.039	1.248	1.158	1.116	1.085
	12	0.55	1.037	1.243	1.156	1.112	1.086
	14	0.64	1.041	1.241	1.155	1.115	1.106
	16	0.73	1.044	1.240	1.155	1.110	1.125
	18	0.82	1.042	1.239	1.148	1.132	1.135
	20	0.91	1.071	1.234	1.153	1.141	1.140
	24	1.09	1.095	1.194	1.149	1.148	1.147
	40	1.82	1.124	1.162	1.154	1.151	1.146
	72	3.27	1.125	1.158	1.153	1.149	1.145

Note: Highlighted values are for the balanced failure

Table 2-2a. Bias Factor of Resistance for Eccentrically Loaded Columns (tied, cast-in-place, for h/b = 2)

Design case			Tied				
% of reinf.	e (in)	e/h	Cast-in-place bias				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
1%	0	0.00	0.982	1.320	1.164	1.088	1.049
	6	0.17	0.984	1.315	1.161	1.090	1.053
	12	0.33	1.001	1.301	1.162	1.107	1.067
	15	0.42	1.004	1.291	1.157	1.105	1.087
	18	0.50	1.043	1.256	1.160	1.126	1.111
	21	0.58	1.062	1.239	1.160	1.140	1.152
	24	0.67	1.079	1.222	1.159	1.142	1.140
	27	0.75	1.091	1.207	1.156	1.147	1.148
	30	0.83	1.104	1.192	1.157	1.157	1.149
	39	1.08	1.119	1.169	1.160	1.150	1.150
	60	1.67	1.123	1.164	1.154	1.152	1.150
	108	3.00	1.125	1.159	1.151	1.149	1.146
2%	0	0.00	0.996	1.292	1.158	1.094	1.154
	6	0.17	0.967	1.299	1.157	1.094	1.062
	12	0.33	1.012	1.278	1.156	1.106	1.070
	15	0.42	1.019	1.274	1.159	1.105	1.078
	18	0.50	1.017	1.267	1.160	1.106	1.099
	21	0.58	1.034	1.265	1.156	1.125	1.116
	24	0.67	1.062	1.236	1.155	1.137	1.124
	27	0.75	1.079	1.212	1.155	1.137	1.136
	30	0.83	1.087	1.204	1.158	1.141	1.141
	39	1.08	1.108	1.184	1.152	1.148	1.149
	60	1.67	1.127	1.155	1.154	1.153	1.149
	108	3.00	1.127	1.155	1.150	1.152	1.148

Note: Highlighted values are for the balanced failure

Table 2-2b. Bias Factor of Resistance for Eccentrically Loaded Columns (tied, cast-in-place, h/b = 2)

Design case			Tied				
% of reinf.	e (in)	e/h	Cast-in-place bias				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
3%	0	0.00	1.011	1.274	1.157	1.095	1.058
	6	0.17	1.013	1.272	1.159	1.100	1.063
	12	0.33	1.023	1.264	1.158	1.108	1.076
	15	0.42	1.030	1.259	1.161	1.109	1.078
	18	0.50	1.033	1.256	1.156	1.110	1.084
	21	0.58	1.034	1.255	1.156	1.113	1.106
	24	0.67	1.035	1.252	1.155	1.125	1.123
	27	0.75	1.051	1.247	1.154	1.136	1.127
	30	0.83	1.076	1.229	1.156	1.135	1.135
	39	1.08	1.102	1.189	1.153	1.145	1.145
	60	1.67	1.121	1.166	1.151	1.152	1.149
	108	3.00	1.129	1.158	1.152	1.149	1.149
4%	0	0.00	1.018	1.257	1.155	1.102	1.067
	6	0.17	1.018	1.256	1.155	1.103	1.071
	12	0.33	1.031	1.249	1.156	1.107	1.076
	15	0.42	1.035	1.250	1.161	1.110	1.082
	18	0.50	1.039	1.246	1.155	1.117	1.081
	21	0.58	1.042	1.240	1.156	1.115	1.090
	24	0.67	1.043	1.242	1.158	1.116	1.112
	27	0.75	1.042	1.240	1.154	1.111	1.123
	30	0.83	1.041	1.237	1.145	1.135	1.130
	39	1.08	1.093	1.205	1.156	1.145	1.143
	60	1.67	1.114	1.170	1.150	1.150	1.151
	108	3.00	1.130	1.152	1.153	1.152	1.150

Note: Highlighted values are for the balanced failure

Table 2-3a. Bias Factor of Resistance for Eccentrically Loaded Columns (tied, plant cast, for h/b = 1.57)

Design case			Tied				
% of reinf.	e (in)	e/h	Plant cast bias				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
1%	0	0.00	1.021	1.375	1.207	1.136	1.095
	4	0.18	1.023	1.369	1.205	1.133	1.096
	8	0.36	1.035	1.351	1.205	1.140	1.104
	10	0.45	1.063	1.335	1.207	1.155	1.131
	12	0.55	1.093	1.293	1.199	1.173	1.152
	14	0.64	1.122	1.260	1.203	1.183	1.176
	16	0.73	1.136	1.249	1.204	1.189	1.179
	18	0.82	1.145	1.240	1.202	1.190	1.182
	20	0.91	1.152	1.233	1.203	1.193	1.184
	24	1.09	1.155	1.227	1.205	1.195	1.187
	40	1.82	1.162	1.218	1.202	1.194	1.189
	72	3.27	1.162	1.214	1.203	1.195	1.190
2%	0	0.00	1.039	1.352	1.204	1.138	1.102
	4	0.18	1.042	1.344	1.204	1.139	1.100
	8	0.36	1.053	1.330	1.209	1.148	1.113
	10	0.45	1.062	1.326	1.206	1.152	1.116
	12	0.55	1.059	1.319	1.196	1.160	1.149
	14	0.64	1.121	1.298	1.205	1.174	1.165
	16	0.73	1.098	1.264	1.198	1.185	1.182
	18	0.82	1.139	1.249	1.201	1.190	1.183
	20	0.91	1.149	1.234	1.206	1.192	1.187
	24	1.09	1.156	1.227	1.201	1.196	1.191
	40	1.82	1.165	1.214	1.204	1.198	1.193
	72	3.27	1.169	1.211	1.202	1.197	1.193

Note: Highlighted values are for the balanced failure

Table 2-3b. Bias Factor of Resistance for Eccentrically Loaded Columns (tied, plant cast, for h/b = 1.57)

Design case			Tied				
% of reinf.	e (in)	e/h	Plant cast bias				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
3%	0	0.00	1.051	1.329	1.206	1.143	1.105
	4	0.18	1.054	1.325	1.202	1.147	1.103
	8	0.36	1.065	1.311	1.206	1.151	1.116
	10	0.45	1.073	1.309	1.203	1.156	1.120
	12	0.55	1.071	1.307	1.201	1.156	1.132
	14	0.64	1.075	1.306	1.201	1.156	1.161
	16	0.73	1.094	1.298	1.202	1.177	1.176
	18	0.82	1.118	1.279	1.199	1.184	1.182
	20	0.91	1.132	1.252	1.198	1.189	1.186
	24	1.09	1.156	1.232	1.201	1.195	1.192
	40	1.82	1.169	1.213	1.205	1.198	1.194
	72	3.27	1.172	1.208	1.204	1.198	1.192
4%	0	0.00	1.062	1.314	1.203	1.146	1.112
	4	0.18	1.065	1.313	1.204	1.148	1.112
	8	0.36	1.077	1.302	1.204	1.156	1.120
	10	0.45	1.080	1.301	1.204	1.157	1.124
	12	0.55	1.082	1.294	1.204	1.159	1.128
	14	0.64	1.087	1.292	1.204	1.159	1.150
	16	0.73	1.085	1.293	1.207	1.157	1.169
	18	0.82	1.087	1.291	1.194	1.178	1.179
	20	0.91	1.117	1.286	1.197	1.185	1.185
	24	1.09	1.139	1.241	1.197	1.193	1.189
	40	1.82	1.173	1.210	1.203	1.198	1.194
	72	3.27	1.173	1.208	1.202	1.199	1.196

Note: Highlighted values are for the balanced failure

Table 2-4a. Bias Factor of Resistance for Eccentrically Loaded Columns (tied, plant cast, for h/b = 2)

Design case			Tied				
% of reinf.	e (in)	e/h	Plant cast bias				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
1%	0	0.00	1.022	1.377	1.204	1.133	1.097
	6	0.17	1.025	1.372	1.205	1.135	1.096
	12	0.33	1.036	1.353	1.206	1.144	1.104
	15	0.42	1.043	1.348	1.198	1.145	1.121
	18	0.50	1.082	1.302	1.202	1.165	1.142
	21	0.58	0.104	1.282	1.203	1.174	1.163
	24	0.67	1.121	1.267	1.206	1.185	1.180
	27	0.75	1.134	1.252	1.201	1.189	1.187
	30	0.83	1.146	1.240	1.199	1.194	1.190
	39	1.08	1.165	1.217	1.203	1.196	1.194
	60	1.67	1.170	1.209	1.203	1.199	1.195
	108	3.00	1.176	1.208	1.202	1.199	1.197
2%	0	0.00	1.037	1.348	1.208	1.142	1.096
	6	0.17	1.039	1.343	1.204	1.143	1.104
	12	0.33	1.052	1.330	1.203	1.145	1.111
	15	0.42	1.059	1.330	1.203	1.150	1.117
	18	0.50	1.063	1.319	1.207	1.149	1.140
	21	0.58	1.076	1.316	1.199	1.173	1.157
	24	0.67	1.107	1.289	1.203	1.177	1.170
	27	0.75	1.122	1.262	1.201	1.182	1.180
	30	0.83	1.132	1.253	1.203	1.187	1.188
	39	1.08	1.153	1.234	1.202	1.197	1.193
	60	1.67	1.173	1.209	1.202	1.201	1.198
	108	3.00	1.174	1.207	1.203	1.201	1.196

Note: Highlighted values are for the balanced failure

Table 2-4b. Bias Factor of Resistance for Eccentrically Loaded Columns (tied, plant cast, for h/b = 2)

Design case			Tied				
% of reinf.	e (in)	e/h	Plant cast bias				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
3%	0	0.00	1.055	1.327	1.204	1.147	1.105
	6	0.17	1.055	1.324	1.204	1.145	1.107
	12	0.33	1.064	1.319	1.206	1.152	1.115
	15	0.42	1.070	1.312	1.205	1.153	1.121
	18	0.50	1.072	1.308	1.206	1.156	1.125
	21	0.58	1.074	1.307	1.203	1.155	1.152
	24	0.67	1.076	1.301	1.200	1.168	1.164
	27	0.75	1.094	1.298	1.199	1.180	1.173
	30	0.83	1.118	1.285	1.202	1.186	1.177
	39	1.08	1.147	1.237	1.203	1.194	1.193
	60	1.67	1.171	1.218	1.204	1.201	1.197
	108	3.00	1.176	1.205	1.202	1.202	1.200
4%	0	0.00	1.064	1.313	1.205	1.149	1.108
	6	0.17	1.066	1.308	1.207	1.147	1.109
	12	0.33	1.071	1.304	1.204	1.154	1.119
	15	0.42	1.077	1.297	1.206	1.155	1.126
	18	0.50	1.081	1.293	1.205	1.156	1.128
	21	0.58	1.086	1.293	1.206	1.160	1.129
	24	0.67	1.085	1.294	1.203	1.163	1.159
	27	0.75	1.086	1.287	1.203	1.158	1.170
	30	0.83	1.087	1.289	1.190	1.179	1.176
	39	1.08	1.135	1.253	1.202	1.193	1.189
	60	1.67	1.163	1.223	1.202	1.200	1.197
	108	3.00	1.176	1.206	1.203	1.198	1.197

Note: Highlighted values are for the balanced failure

Table 2-5a. Coefficient of Variation of Resistance for Eccentrically Loaded Columns (tied, cast-in-place, for h/b = 1.57)

Design case			Tied				
% of reinf.	e (in)	e/h	Cast-in-place COV				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
1%	0	0.00	0.160	0.126	0.129	0.132	0.136
	4	0.18	0.162	0.130	0.135	0.138	0.140
	8	0.36	0.157	0.128	0.135	0.140	0.143
	10	0.45	0.154	0.128	0.134	0.143	0.151
	12	0.55	0.147	0.128	0.134	0.143	0.156
	14	0.64	0.148	0.129	0.136	0.140	0.147
	16	0.73	0.148	0.127	0.122	0.136	0.139
	18	0.82	0.147	0.127	0.127	0.136	0.131
	20	0.91	0.145	0.121	0.124	0.126	0.124
	24	1.09	0.143	0.118	0.120	0.120	0.117
	40	1.82	0.135	0.113	0.111	0.110	0.111
	72	3.27	0.135	0.109	0.109	0.108	0.106
2%	0	0.00	0.149	0.117	0.125	0.128	0.131
	4	0.18	0.149	0.120	0.126	0.131	0.135
	8	0.36	0.145	0.121	0.126	0.130	0.136
	10	0.45	0.147	0.121	0.126	0.132	0.135
	12	0.55	0.143	0.122	0.123	0.127	0.136
	14	0.64	0.141	0.120	0.124	0.129	0.136
	16	0.73	0.139	0.118	0.122	0.128	0.133
	18	0.82	0.141	0.119	0.123	0.126	0.129
	20	0.91	0.142	0.118	0.122	0.125	0.124
	24	1.09	0.140	0.118	0.119	0.119	0.119
	40	1.82	0.140	0.114	0.114	0.114	0.112
	72	3.27	0.137	0.110	0.109	0.109	0.108

Note: Highlighted values are for the balanced failure

Table 2-5b. Coefficient of Variation of Resistance for Eccentrically Loaded Columns (tied, cast-in-place, for h/b = 1.57)

Design case			Tied				
% of reinf.	e (in)	e/h	Cast-in-place COV				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
3%	0	0.00	0.138	0.110	0.116	0.121	0.126
	4	0.18	0.140	0.114	0.120	0.126	0.132
	8	0.36	0.139	0.116	0.120	0.126	0.132
	10	0.45	0.141	0.116	0.119	0.127	0.130
	12	0.55	0.137	0.117	0.121	0.123	0.128
	14	0.64	0.139	0.115	0.118	0.124	0.126
	16	0.73	0.137	0.117	0.117	0.122	0.129
	18	0.82	0.136	0.116	0.116	0.121	0.126
	20	0.91	0.136	0.114	0.119	0.121	0.124
	24	1.09	0.137	0.116	0.119	0.118	0.121
	40	1.82	0.140	0.114	0.113	0.114	0.112
	72	3.27	0.137	0.112	0.110	0.110	0.109
4%	0	0.00	0.131	0.105	0.110	0.117	0.122
	4	0.18	0.135	0.109	0.115	0.122	0.128
	8	0.36	0.134	0.112	0.116	0.122	0.128
	10	0.45	0.134	0.113	0.116	0.120	0.127
	12	0.55	0.135	0.114	0.115	0.121	0.126
	14	0.64	0.133	0.112	0.116	0.119	0.122
	16	0.73	0.134	0.112	0.115	0.119	0.121
	18	0.82	0.133	0.112	0.115	0.118	0.123
	20	0.91	0.134	0.113	0.114	0.118	0.122
	24	1.09	0.137	0.113	0.115	0.120	0.120
	40	1.82	0.139	0.115	0.113	0.113	0.114
	72	3.27	0.139	0.111	0.111	0.111	0.110

Note: Highlighted values are for the balanced failure

Table 2-6a. Coefficient of Variation of Resistance for Eccentrically Loaded Columns (tied, cast-in-place, for h/b = 2)

Design case			Tied				
% of reinf.	e (in)	e/h	Cast-in-place COV				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
1%	0	0.00	0.160	0.123	0.128	0.132	0.135
	6	0.17	0.159	0.128	0.133	0.138	0.141
	12	0.33	0.157	0.126	0.130	0.138	0.144
	15	0.42	0.152	0.127	0.131	0.135	0.145
	18	0.50	0.145	0.124	0.128	0.137	0.147
	21	0.58	0.142	0.120	0.126	0.136	0.143
	24	0.67	0.143	0.121	0.128	0.138	0.141
	27	0.75	0.140	0.122	0.128	0.132	0.135
	30	0.83	0.145	0.122	0.124	0.128	0.129
	39	1.08	0.143	0.119	0.119	0.119	0.121
	60	1.67	0.138	0.113	0.111	0.112	0.113
	108	3.00	0.135	0.108	0.108	0.110	0.107
2%	0	0.00	0.148	0.116	0.121	0.126	0.129
	6	0.17	0.149	0.121	0.125	0.131	0.134
	12	0.33	0.144	0.119	0.124	0.128	0.136
	15	0.42	0.145	0.121	0.124	0.129	0.134
	18	0.50	0.145	0.117	0.123	0.127	0.130
	21	0.58	0.141	0.118	0.118	0.124	0.126
	24	0.67	0.138	0.115	0.119	0.119	0.127
	27	0.75	0.134	0.114	0.118	0.121	0.125
	30	0.83	0.135	0.111	0.115	0.120	0.126
	39	1.08	0.138	0.112	0.116	0.120	0.120
	60	1.67	0.137	0.113	0.114	0.112	0.113
	108	3.00	0.136	0.109	0.109	0.110	0.108

Note: Highlighted values are for the balanced failure

Table 2-6b. Coefficient of Variation of Resistance for Eccentrically Loaded Columns (tied, cast-in-place, for h/b = 2)

Design case			Tied				
% of reinf.	e (in)	e/h	Cast-in-place COV				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
3%	0	0.00	0.138	0.109	0.117	0.122	0.127
	6	0.17	0.139	0.112	0.120	0.124	0.132
	12	0.33	0.138	0.113	0.117	0.127	0.130
	15	0.42	0.140	0.115	0.116	0.121	0.126
	18	0.50	0.139	0.114	0.117	0.122	0.126
	21	0.58	0.133	0.115	0.115	0.120	0.123
	24	0.67	0.134	0.112	0.116	0.119	0.123
	27	0.75	0.132	0.112	0.117	0.116	0.121
	30	0.83	0.131	0.111	0.112	0.116	0.121
	39	1.08	0.137	0.111	0.112	0.115	0.118
	60	1.67	0.138	0.113	0.113	0.112	0.114
	108	3.00	0.135	0.111	0.111	0.109	0.109
4%	0	0.00	0.134	0.105	0.111	0.118	0.122
	6	0.17	0.134	0.110	0.114	0.120	0.125
	12	0.33	0.135	0.109	0.115	0.120	0.125
	15	0.42	0.133	0.110	0.113	0.119	0.126
	18	0.50	0.132	0.110	0.114	0.117	0.121
	21	0.58	0.132	0.110	0.115	0.118	0.123
	24	0.67	0.130	0.111	0.113	0.118	0.118
	27	0.75	0.135	0.109	0.113	0.118	0.117
	30	0.83	0.131	0.107	0.112	0.114	0.118
	39	1.08	0.131	0.108	0.113	0.114	0.117
	60	1.67	0.135	0.109	0.109	0.112	0.114
	108	3.00	0.138	0.109	0.112	0.110	0.108

Note: Highlighted values are for the balanced failure

Table 2-7a. Coefficient of Variation of Resistance for Eccentrically Loaded Columns (tied, plant cast, for h/b = 1.57)

Design case			Tied				
% of reinf.	e (in)	e/h	Plant cast COV				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
1%	0	0.00	0.146	0.108	0.111	0.115	0.118
	4	0.18	0.147	0.110	0.112	0.116	0.118
	8	0.36	0.138	0.104	0.108	0.113	0.116
	10	0.45	0.130	0.100	0.101	0.106	0.114
	12	0.55	0.120	0.095	0.099	0.104	0.108
	14	0.64	0.117	0.095	0.099	0.102	0.105
	16	0.73	0.120	0.094	0.095	0.098	0.099
	18	0.82	0.119	0.091	0.093	0.097	0.094
	20	0.91	0.117	0.090	0.092	0.092	0.092
	24	1.09	0.116	0.089	0.091	0.089	0.090
	40	1.82	0.117	0.086	0.084	0.085	0.085
	72	3.27	0.116	0.085	0.086	0.085	0.082
2%	0	0.00	0.134	0.098	0.103	0.108	0.112
	4	0.18	0.134	0.099	0.105	0.110	0.114
	8	0.36	0.127	0.096	0.101	0.106	0.111
	10	0.45	0.124	0.096	0.099	0.103	0.105
	12	0.55	0.121	0.095	0.095	0.097	0.100
	14	0.64	0.114	0.090	0.091	0.092	0.098
	16	0.73	0.114	0.089	0.093	0.093	0.096
	18	0.82	0.117	0.090	0.092	0.093	0.094
	20	0.91	0.115	0.089	0.090	0.091	0.093
	24	1.09	0.117	0.089	0.089	0.091	0.089
	40	1.82	0.117	0.086	0.087	0.088	0.086
	72	3.27	0.119	0.086	0.087	0.85	0.085

Note: Highlighted values are for the balanced failure

Table 2-7b. Coefficient of Variation of Resistance for Eccentrically Loaded Columns (tied, plant cast, for h/b = 1.57)

Design case			Tied				
% of reinf.	e (in)	e/h	Plant cast COV				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
3%	0	0.00	0.124	0.091	0.097	0.103	0.108
	4	0.18	0.124	0.094	0.099	0.104	0.110
	8	0.36	0.120	0.093	0.095	0.102	0.107
	10	0.45	0.118	0.093	0.096	0.100	0.105
	12	0.55	0.117	0.092	0.095	0.098	0.098
	14	0.64	0.117	0.090	0.093	0.092	0.094
	16	0.73	0.115	0.091	0.087	0.089	0.095
	18	0.82	0.112	0.089	0.088	0.090	0.093
	20	0.91	0.114	0.086	0.088	0.091	0.091
	24	1.09	0.118	0.087	0.090	0.090	0.090
	40	1.82	0.118	0.088	0.090	0.087	0.087
	72	3.27	0.121	0.087	0.086	0.085	0.086
4%	0	0.00	0.117	0.088	0.093	0.100	0.105
	4	0.18	0.119	0.090	0.094	0.102	0.105
	8	0.36	0.116	0.088	0.095	0.097	0.102
	10	0.45	0.114	0.087	0.091	0.095	0.100
	12	0.55	0.114	0.088	0.092	0.095	0.099
	14	0.64	0.114	0.087	0.090	0.094	0.093
	16	0.73	0.115	0.089	0.091	0.093	0.092
	18	0.82	0.114	0.090	0.087	0.089	0.092
	20	0.91	0.111	0.087	0.087	0.089	0.094
	24	1.09	0.112	0.085	0.088	0.089	0.089
	40	1.82	0.119	0.088	0.087	0.089	0.086
	72	3.27	0.119	0.088	0.086	0.086	0.085

Note: Highlighted values are for the balanced failure

Table 2-8a. Coefficient of Variation of Resistance for Eccentrically Loaded Columns (tied, plant cast, for h/b = 2)

Design case			Tied				
% of reinf.	e (in)	e/h	Plant cast COV				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
1%	0	0.00	0.147	0.108	0.112	0.115	0.117
	6	0.17	0.143	0.109	0.111	0.118	0.117
	12	0.33	0.137	0.104	0.111	0.112	0.116
	15	0.42	0.135	0.101	0.102	0.108	0.111
	18	0.50	0.121	0.094	0.098	0.102	0.108
	21	0.58	0.118	0.091	0.095	0.100	0.104
	24	0.67	0.115	0.092	0.094	0.097	0.103
	27	0.75	0.113	0.092	0.092	0.095	0.100
	30	0.83	0.116	0.887	0.092	0.095	0.098
	39	1.08	0.120	0.889	0.090	0.090	0.090
	60	1.67	0.121	0.087	0.086	0.089	0.086
	108	3.00	0.120	0.087	0.084	0.085	0.084
2%	0	0.00	0.133	0.098	0.104	0.109	0.112
	6	0.17	0.134	0.099	0.103	0.108	0.113
	12	0.33	0.126	0.096	0.100	0.105	0.111
	15	0.42	0.124	0.095	0.098	0.104	0.105
	18	0.50	0.122	0.094	0.097	0.100	0.099
	21	0.58	0.118	0.094	0.092	0.094	0.096
	24	0.67	0.114	0.089	0.090	0.091	0.095
	27	0.75	0.110	0.086	0.090	0.089	0.093
	30	0.83	0.110	0.087	0.087	0.090	0.093
	39	1.08	0.114	0.088	0.089	0.090	0.091
	60	1.67	0.119	0.086	0.087	0.088	0.086
	108	3.00	0.118	0.088	0.087	0.084	0.085

Note: Highlighted values are for the balanced failure

Table 2-8b. Coefficient of Variation of Resistance for Eccentrically Loaded Columns (tied, plant cast, for h/b = 2)

Design case			Tied				
% of reinf.	e (in)	e/h	Plant cast COV				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
3%	0	0.00	0.123	0.091	0.097	0.103	0.108
	6	0.17	0.122	0.093	0.098	0.104	0.107
	12	0.33	0.118	0.091	0.096	0.102	0.105
	15	0.42	0.118	0.090	0.094	0.098	0.105
	18	0.50	0.117	0.090	0.092	0.098	0.101
	21	0.58	0.115	0.090	0.091	0.097	0.095
	24	0.67	0.116	0.092	0.091	0.092	0.091
	27	0.75	0.113	0.090	0.088	0.089	0.089
	30	0.83	0.111	0.086	0.086	0.089	0.089
	39	1.08	0.112	0.084	0.087	0.087	0.090
	60	1.67	0.118	0.086	0.088	0.087	0.089
	108	3.00	0.119	0.087	0.086	0.086	0.087
4%	0	0.00	0.116	0.088	0.092	0.099	0.104
	6	0.17	0.119	0.089	0.093	0.100	0.106
	12	0.33	0.116	0.087	0.093	0.098	0.101
	15	0.42	0.116	0.088	0.092	0.097	0.099
	18	0.50	0.114	0.088	0.091	0.094	0.097
	21	0.58	0.112	0.088	0.089	0.094	0.095
	24	0.67	0.114	0.085	0.089	0.094	0.091
	27	0.75	0.114	0.088	0.088	0.090	0.087
	30	0.83	0.111	0.087	0.088	0.087	0.086
	39	1.08	0.110	0.085	0.086	0.087	0.087
	60	1.67	0.112	0.084	0.086	0.087	0.087
	108	3.00	0.120	0.087	0.086	0.087	0.085

Note: Highlighted values are for the balanced failure

Table 2-9a. Bias Factor of Resistance for Eccentrically Loaded Columns (spiral, cast-in-place)

Design case			Spiral				
% of reinf.	e (in)	e/h	Cast-in-place bias				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
1%	0	0.00	1.027	1.387	1.217	1.147	1.100
	4	0.18	1.025	1.386	1.223	1.149	1.111
	8	0.36	1.039	1.364	1.221	1.158	1.123
	10	0.45	1.065	1.347	1.223	1.190	1.150
	12	0.55	1.092	1.303	1.222	1.200	1.175
	14	0.64	1.119	1.272	1.217	1.208	1.194
	16	0.73	1.136	1.260	1.217	1.204	1.201
	18	0.82	1.139	1.253	1.213	1.206	1.201
	20	0.91	1.150	1.245	1.220	1.202	1.203
	24	1.09	1.154	1.237	1.213	1.208	1.198
	40	1.82	1.160	1.227	1.210	1.203	1.194
	72	3.27	1.164	1.225	1.209	1.202	1.198
2%	0	0.00	1.045	1.362	1.214	1.151	1.111
	4	0.18	1.040	1.357	1.219	1.153	1.114
	8	0.36	1.056	1.343	1.219	1.160	1.126
	10	0.45	1.059	1.334	1.219	1.165	1.127
	12	0.55	1.060	1.330	1.210	1.170	1.163
	14	0.64	1.102	1.311	1.215	1.190	1.185
	16	0.73	1.118	1.275	1.210	1.195	1.189
	18	0.82	1.136	1.264	1.211	1.204	1.197
	20	0.91	1.149	1.243	1.210	1.203	1.200
	24	1.09	1.155	1.233	1.217	1.205	1.196
	40	1.82	1.163	1.222	1.210	1.207	1.200
	72	3.27	1.170	1.218	1.211	1.204	1.199

Note: Highlighted values are for the balanced failure

Table 2-9b. Bias Factor of Resistance for Eccentrically Loaded Columns (spiral, cast-in-place)

Design case			Spiral				
% of reinf.	e (in)	e/h	Cast-in-place bias				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
3%	0	0.00	1.053	1.339	1.216	1.153	1.112
	4	0.18	1.059	1.337	1.216	1.156	1.117
	8	0.36	1.070	1.323	1.216	1.163	1.132
	10	0.45	1.072	1.318	1.217	1.164	1.129
	12	0.55	1.071	1.319	1.213	1.170	1.147
	14	0.64	1.076	1.314	1.205	1.165	1.171
	16	0.73	1.092	1.311	1.209	1.187	1.184
	18	0.82	1.114	1.290	1.206	1.192	1.197
	20	0.91	1.129	1.259	1.209	1.204	1.196
	24	1.09	1.155	1.237	1.213	1.207	1.201
	40	1.82	1.162	1.220	1.210	1.203	1.204
	72	3.27	1.169	1.214	1.209	1.206	1.200
4%	0	0.00	1.065	1.320	1.217	1.158	1.116
	4	0.18	1.072	1.321	1.215	1.159	1.121
	8	0.36	1.077	1.314	1.212	1.167	1.134
	10	0.45	1.078	1.306	1.218	1.168	1.135
	12	0.55	1.082	1.304	1.216	1.166	1.136
	14	0.64	1.084	1.304	1.213	1.174	1.161
	16	0.73	1.086	1.299	1.208	1.165	1.178
	18	0.82	1.088	1.300	1.207	1.189	1.185
	20	0.91	1.112	1.293	1.210	1.196	1.195
	24	1.09	1.142	1.251	1.209	1.201	1.201
	40	1.82	1.168	1.216	1.210	1.204	1.203
	72	3.27	1.167	1.213	1.209	1.204	1.201

Note: Highlighted values are for the balanced failure

Table 2-10a. Bias Factor of Resistance for Eccentrically Loaded Columns (spiral, plant cast)

Design case			Spiral				
% of reinf.	e (in)	e/h	Plant cast bias				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
1%	0	0.00	1.022	1.378	1.208	1.132	1.092
	4	0.18	1.023	1.371	1.207	1.134	1.093
	8	0.36	1.039	1.350	1.204	1.142	1.107
	10	0.45	1.064	1.331	1.205	1.154	1.128
	12	0.55	1.090	1.290	1.197	1.167	1.155
	14	0.64	1.118	1.261	1.198	1.179	1.170
	16	0.73	1.132	1.245	1.202	1.190	1.180
	18	0.82	1.145	1.236	1.201	1.189	1.180
	20	0.91	1.146	1.227	1.202	1.187	1.183
	24	1.09	1.155	1.222	1.200	1.192	1.185
	40	1.82	1.162	1.216	1.199	1.194	1.188
	72	3.27	1.164	1.217	1.201	1.193	1.189
2%	0	0.00	1.038	1.348	1.206	1.141	1.101
	4	0.18	1.043	1.341	1.204	1.139	1.102
	8	0.36	1.050	1.328	1.203	1.144	1.109
	10	0.45	1.061	1.325	1.207	1.147	1.116
	12	0.55	1.059	1.319	1.191	1.158	1.144
	14	0.64	1.097	1.296	1.200	1.173	1.169
	16	0.73	1.117	1.263	1.197	1.183	1.174
	18	0.82	1.134	1.249	1.198	1.186	1.179
	20	0.91	1.142	1.231	1.199	1.190	1.183
	24	1.09	1.154	1.222	1.201	1.193	1.187
	40	1.82	1.161	1.211	1.199	1.194	1.191
	72	3.27	1.166	1.207	1.201	1.196	1.192

Note: Highlighted values are for the balanced failure

Table 2-10b. Bias Factor of Resistance for Eccentrically Loaded Columns (spiral, plant cast)

Design case			Spiral				
% of reinf.	e (in)	e/h	Plant cast bias				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
3%	0	0.00	1.053	1.332	1.204	1.142	1.107
	4	0.18	1.052	1.325	1.203	1.143	1.105
	8	0.36	1.061	1.313	1.207	1.151	1.116
	10	0.45	1.069	1.309	1.204	1.153	1.120
	12	0.55	1.071	1.303	1.203	1.156	1.133
	14	0.64	1.070	1.301	1.203	1.152	1.156
	16	0.73	1.091	1.297	1.196	1.177	1.176
	18	0.82	1.117	1.276	1.197	1.181	1.179
	20	0.91	1.130	1.246	1.198	1.188	1.184
	24	1.09	1.152	1.232	1.197	1.193	1.189
	40	1.82	1.166	1.209	1.201	1.196	1.192
	72	3.27	1.170	1.206	1.200	1.195	1.194
4%	0	0.00	1.064	1.313	1.205	1.147	1.110
	4	0.18	1.066	1.309	1.201	1.150	1.108
	8	0.36	1.073	1.298	1.204	1.153	1.119
	10	0.45	1.079	1.296	1.201	1.157	1.122
	12	0.55	1.082	1.294	1.199	1.157	1.124
	14	0.64	1.083	1.293	1.202	1.159	1.148
	16	0.73	1.087	1.287	1.204	1.152	1.167
	18	0.82	1.083	1.285	1.188	1.177	1.176
	20	0.91	1.110	1.284	1.197	1.181	1.182
	24	1.09	1.139	1.237	1.194	1.191	1.189
	40	1.82	1.167	1.208	1.201	1.195	1.192
	72	3.27	1.171	1.203	1.201	1.196	1.190

Note: Highlighted values are for the balanced failure

Table 2-11a. Coefficient of Variation of Resistance for Eccentrically Loaded Columns (spiral, cast-in-place)

Design case			Spiral				
% of reinf.	e (in)	e/h	Cast-in-place COV				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
1%	0	0.00	0.149	0.114	0.120	0.122	0.125
	4	0.18	0.151	0.119	0.125	0.127	0.132
	8	0.36	0.140	0.117	0.126	0.130	0.138
	10	0.45	0.129	0.117	0.121	0.136	0.142
	12	0.55	0.121	0.116	0.125	0.130	0.145
	14	0.64	0.119	0.117	0.123	0.126	0.139
	16	0.73	0.119	0.117	0.121	0.118	0.127
	18	0.82	0.118	0.114	0.113	0.111	0.118
	20	0.91	0.119	0.112	0.112	0.106	0.112
	24	1.09	0.117	0.107	0.106	0.109	0.106
	40	1.82	0.115	0.101	0.097	0.098	0.096
	72	3.27	0.116	0.094	0.094	0.095	0.093
2%	0	0.00	0.136	0.104	0.108	0.115	0.120
	4	0.18	0.133	0.108	0.112	0.119	0.125
	8	0.36	0.130	0.109	0.113	0.120	0.126
	10	0.45	0.126	0.109	0.113	0.121	0.126
	12	0.55	0.124	0.109	0.110	0.117	0.126
	14	0.64	0.116	0.106	0.111	0.114	0.123
	16	0.73	0.114	0.106	0.112	0.115	0.122
	18	0.82	0.114	0.105	0.111	0.113	0.118
	20	0.91	0.118	0.105	0.110	0.113	0.113
	24	1.09	0.119	0.105	0.106	0.107	0.107
	40	1.82	0.119	0.100	0.100	0.101	0.100
	72	3.27	0.117	0.096	0.097	0.095	0.094

Note: Highlighted values are for the balanced failure

Table 2-11b. Coefficient of Variation of Resistance for Eccentrically Loaded Columns (spiral, cast-in-place)

Design case			Spiral				
% of reinf.	e (in)	e/h	Cast-in-place COV				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
3%	0	0.00	0.125	0.097	0.103	0.108	0.114
	4	0.18	0.127	0.101	0.106	0.115	0.121
	8	0.36	0.122	0.102	0.108	0.116	0.121
	10	0.45	0.121	0.102	0.108	0.115	0.115
	12	0.55	0.119	0.103	0.109	0.115	0.117
	14	0.64	0.117	0.104	0.107	0.111	0.114
	16	0.73	0.116	0.103	0.105	0.110	0.116
	18	0.82	0.113	0.100	0.104	0.109	0.113
	20	0.91	0.115	0.098	0.107	0.110	0.110
	24	1.09	0.114	0.101	0.107	0.107	0.107
	40	1.82	0.119	0.101	0.101	0.101	0.100
	72	3.27	0.119	0.098	0.097	0.096	0.098
4%	0	0.00	0.118	0.092	0.096	0.102	0.110
	4	0.18	0.120	0.095	0.103	0.107	0.115
	8	0.36	0.118	0.098	0.105	0.110	0.115
	10	0.45	0.116	0.100	0.104	0.108	0.114
	12	0.55	0.115	0.100	0.103	0.108	0.113
	14	0.64	0.117	0.100	0.103	0.106	0.111
	16	0.73	0.112	0.098	0.103	0.107	0.111
	18	0.82	0.113	0.099	0.102	0.104	0.111
	20	0.91	0.113	0.100	0.101	0.106	0.111
	24	1.09	0.115	0.099	0.105	0.105	0.109
	40	1.82	0.119	0.100	0.103	0.099	0.100
	72	3.27	0.118	0.098	0.100	0.097	0.097

Note: Highlighted values are for the balanced failure

Table 2-12a. Coefficient of Variation of Resistance for Eccentrically Loaded Columns (spiral, plant cast)

Design case			Spiral				
% of reinf.	e (in)	e/h	Plant cast COV				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
1%	0	0.00	0.146	0.108	0.112	0.115	0.118
	4	0.18	0.147	0.107	0.114	0.117	0.121
	8	0.36	0.138	0.105	0.108	0.116	0.116
	10	0.45	0.126	0.102	0.101	0.105	0.111
	12	0.55	0.120	0.096	0.099	0.104	0.110
	14	0.64	0.121	0.096	0.097	0.101	0.105
	16	0.73	0.119	0.094	0.097	0.098	0.099
	18	0.82	0.118	0.091	0.096	0.094	0.095
	20	0.91	0.120	0.091	0.091	0.092	0.093
	24	1.09	0.118	0.090	0.089	0.090	0.089
	40	1.82	0.115	0.088	0.086	0.084	0.087
	72	3.27	0.113	0.086	0.086	0.083	0.083
2%	0	0.00	0.132	0.098	0.103	0.109	0.113
	4	0.18	0.131	0.101	0.104	0.109	0.112
	8	0.36	0.127	0.097	0.101	0.106	0.110
	10	0.45	0.124	0.096	0.099	0.103	0.108
	12	0.55	0.121	0.095	0.095	0.098	0.098
	14	0.64	0.115	0.092	0.093	0.095	0.099
	16	0.73	0.114	0.089	0.091	0.095	0.096
	18	0.82	0.115	0.088	0.091	0.095	0.093
	20	0.91	0.115	0.088	0.091	0.091	0.092
	24	1.09	0.118	0.089	0.091	0.090	0.091
	40	1.82	0.118	0.088	0.086	0.086	0.086
	72	3.27	0.117	0.086	0.088	0.086	0.083

Note: Highlighted values are for the balanced failure

Table 2-12b. Coefficient of Variation of Resistance for Eccentrically Loaded Columns (spiral, plant cast)

Design case			Spiral				
% of reinf.	e (in)	e/h	Plant cast COV				
			old	f_c'=3.0 ksi	f_c'=5.0 ksi	f_c'=8.0 ksi	f_c'=12.0 ksi
3%	0	0.00	0.123	0.091	0.097	0.103	0.108
	4	0.18	0.124	0.093	0.099	0.105	0.110
	8	0.36	0.121	0.091	0.096	0.102	0.107
	10	0.45	0.119	0.092	0.095	0.099	0.103
	12	0.55	0.119	0.091	0.094	0.096	0.099
	14	0.64	0.118	0.090	0.093	0.093	0.095
	16	0.73	0.116	0.091	0.092	0.091	0.094
	18	0.82	0.114	0.089	0.089	0.091	0.093
	20	0.91	0.112	0.086	0.089	0.090	0.093
	24	1.09	0.119	0.088	0.089	0.091	0.090
	40	1.82	0.119	0.087	0.088	0.089	0.086
	72	3.27	0.118	0.087	0.088	0.086	0.085
4%	0	0.00	0.118	0.087	0.093	0.098	0.105
	4	0.18	0.117	0.090	0.094	0.100	0.107
	8	0.36	0.115	0.090	0.092	0.099	0.104
	10	0.45	0.116	0.090	0.093	0.097	0.102
	12	0.55	0.116	0.089	0.092	0.095	0.098
	14	0.64	0.114	0.088	0.092	0.092	0.094
	16	0.73	0.112	0.086	0.089	0.092	0.092
	18	0.82	0.112	0.089	0.089	0.088	0.091
	20	0.91	0.111	0.087	0.087	0.090	0.091
	24	1.09	0.114	0.086	0.089	0.090	0.091
	40	1.82	0.121	0.089	0.087	0.088	0.087
	72	3.27	0.117	0.087	0.088	0.086	0.087

Note: Highlighted values are for the balanced failure

2.3. Reliability Indices for Eccentrically Loaded Columns

Based on the limit state functions established for each design case, the reliability analysis was performed to determine the reliability indices for new material statistics and new load models (according to ASCE 7 -1998 and ACI 318 -2000), and for comparison, for old material statistics and old load models (ACI 318-99). The “old” reliability indices served as a basis for the selection of the target reliability index for eccentrically loaded columns. The reliability analysis procedure used in this study was the same as presented in the first part of the report (Phase I). The reliability index calculated for each design case is based on the mean value of resistance and load effect, and standard deviations of these two variables. The reliability indices calculated for the considered design cases, four types of concrete, and three selected resistance factors, are shown in Tables 2-13 through 2-18.

Table 2-13. Reliability indices and possible resistance factors for eccentrically loaded columns (tied, cast-in-place, for h/b = 1.57)

Design case			Tied												
% of reinforc..	e (in)	e/h	Cast-in-place beta												
			old	f_c'=3.0 ksi			f_c'=5.0 ksi			f_c'=8.0 ksi			f_c'=12.0 ksi		
		φ	0.70	0.85	0.70	0.75	0.85	0.70	0.75	0.85	0.70	0.75	0.85	0.70	0.75
1%	4	0.18	3.04	3.64	4.36	4.12	3.0	3.77	3.51	2.69	3.48	3.21	2.50	3.30	3.03
	10	0.45	3.31	3.57	4.32	4.07	3.02	3.79	3.53	2.69	3.44	3.18	2.49	3.21	2.97
	18	0.82	3.68	3.28	4.09	3.82	3.15	3.97	3.70	3.04	3.85	3.58	3.01	3.81	3.54
	72	3.27	4.03	3.61	4.57	4.25	3.55	4.52	4.19	3.53	4.52	4.19	3.56	4.57	4.23
2%	8	0.36	3.45	3.74	4.53	4.27	3.18	4.01	3.73	2.87	3.70	3.42	2.63	3.44	3.17
	12	0.55	3.53	3.68	4.47	4.20	3.20	4.06	3.77	2.97	3.82	3.53	2.78	3.58	3.31
	18	0.82	3.81	3.49	4.35	4.06	3.22	4.08	3.79	3.12	3.96	3.68	3.03	3.85	3.57
	72	3.27	3.99	3.56	4.52	4.20	3.54	4.51	4.18	3.52	4.49	4.16	3.53	4.51	4.18
3%	8	0.36	3.63	3.81	4.65	4.37	3.30	4.18	3.88	2.97	3.83	3.54	2.71	3.55	3.27
	14	0.64	3.67	3.81	4.66	4.38	3.34	4.23	3.93	3.02	3.90	3.60	3.01	3.86	3.58
	20	0.91	3.91	3.61	4.51	4.21	3.29	4.18	3.88	3.21	4.09	3.79	3.14	3.99	3.71
	72	3.27	4.00	3.49	4.43	4.11	3.52	4.48	4.16	3.51	4.47	4.14	3.50	4.47	4.14
4%	8	0.36	3.78	3.87	4.76	4.46	3.40	4.30	4.00	3.05	3.94	3.64	2.79	3.66	3.36
	14	0.64	3.84	3.84	4.73	4.43	3.38	4.29	3.98	3.13	4.04	3.74	3.03	3.92	3.62
	24	1.09	3.91	3.37	4.20	3.92	3.15	3.99	3.71	3.26	4.14	3.84	3.15	4.00	3.73
	72	3.27	3.95	3.52	4.46	4.14	3.49	4.45	4.12	3.48	4.43	4.11	3.48	4.45	4.12

Table 2-14. Reliability indices and possible resistance factors for eccentrically loaded columns (tied, plant cast, for h/b = 1.57)

Design case			Tied												
% of reinforc.	e (in)	e/h	Plant cast beta												
			old	f_c'=3.0 ksi			f_c'=5.0 ksi			f_c'=8.0 ksi			f_c'=12.0 ksi		
		φ	0.70	0.85	0.70	0.75	0.85	0.70	0.75	0.85	0.70	0.75	0.85	0.70	0.75
1%	4	0.18	3.45	4.38	5.21	4.93	3.70	4.61	4.30	3.28	4.20	3.89	3.07	4.00	3.68
	10	0.45	4.00	4.61	5.56	5.24	4.03	5.04	4.70	3.63	4.63	4.29	3.32	4.26	3.94
	18	0.82	4.62	4.53	5.64	5.27	4.27	5.38	5.00	4.07	5.14	4.78	4.13	5.24	4.87
	40	1.82	4.74	4.61	5.81	5.40	4.60	5.84	5.42	4.52	5.75	5.33	4.49	5.72	5.30
2%	4	0.18	3.82	4.68	5.64	5.32	3.89	4.87	4.54	3.45	4.60	4.10	3.17	4.13	3.81
	12	0.55	4.24	4.74	5.75	5.41	4.17	5.26	4.89	3.92	5.01	4.64	3.77	4.84	4.48
	20	0.91	4.77	4.58	5.72	5.33	4.39	5.55	5.15	4.29	5.43	5.04	4.19	5.31	4.93
	40	1.82	4.75	4.59	5.79	5.38	4.50	5.69	5.28	4.43	5.61	5.21	4.47	5.69	5.28
3%	8	0.36	4.30	4.78	5.83	5.47	4.22	5.30	4.94	3.73	4.77	4.41	3.42	4.43	4.09
	16	0.73	4.58	4.81	5.88	5.52	4.49	5.68	5.27	4.27	5.46	5.06	4.07	5.17	4.80
	20	0.91	4.75	4.79	5.97	5.57	4.43	5.61	5.21	4.27	5.42	5.03	4.25	5.40	5.01
	40	1.82	4.73	4.51	5.68	5.28	4.39	5.54	5.15	4.46	5.66	5.25	4.44	5.64	5.23
4%	4	0.18	4.33	4.92	6.00	5.63	4.24	5.34	4.97	3.71	4.75	4.40	3.45	4.48	4.13
	12	0.55	4.56	4.91	6.03	5.66	4.31	5.44	5.06	3.98	5.09	4.71	3.69	4.78	4.41
	18	0.82	4.58	4.82	5.91	5.54	4.44	5.64	5.23	4.28	5.46	5.06	4.18	5.32	4.94
	40	1.82	4.71	4.49	5.67	5.27	4.49	5.69	5.28	4.39	5.56	5.16	4.48	5.69	5.28

Table 2-15. Reliability indices and possible resistance factors for eccentrically loaded columns (tied, cast-in-place, for h/b = 2)

Design case			Tied												
% of reinforc.	e (in)	e/h	Cast-in-place beta												
			old	f_c'=3.0 ksi			f_c'=5.0 ksi			f_c'=8.0 ksi			f_c'=12.0 ksi		
		φ	0.70	0.85	0.70	0.75	0.85	0.70	0.75	0.85	0.70	0.75	0.85	0.70	0.75
1%	6	0.17	3.08	3.67	4.40	4.16	3.04	3.82	3.55	2.67	3.46	3.19	2.47	3.27	3.00
	15	0.42	3.28	3.62	4.37	4.12	3.06	3.86	3.59	2.78	3.58	3.31	2.55	3.30	3.05
	27	0.75	3.82	3.45	4.28	4.00	3.12	3.93	3.66	3.00	3.80	3.53	2.95	3.73	3.46
	60	1.67	3.97	3.49	4.42	4.11	3.50	4.45	4.13	3.46	4.41	4.09	3.43	4.37	4.05
2%	12	0.33	3.47	3.79	4.60	4.33	3.20	4.05	3.76	2.91	3.76	3.47	2.62	3.44	3.16
	21	0.58	3.62	3.77	4.59	4.32	3.34	4.23	3.93	3.07	3.94	3.64	2.99	3.85	3.56
	27	0.75	3.94	3.67	4.56	4.26	3.33	4.22	3.92	3.19	4.07	3.77	3.10	3.95	3.66
	60	1.67	4.01	3.45	4.39	4.07	3.42	4.35	4.04	3.47	4.41	4.09	3.43	4.36	4.05
3%	12	0.33	3.65	3.90	4.77	4.48	3.37	4.27	3.96	2.94	3.79	3.50	2.75	3.60	3.31
	24	0.67	3.79	3.84	4.77	4.47	3.38	4.29	3.98	3.18	4.08	3.78	3.08	3.96	3.66
	30	0.83	4.01	3.82	4.73	4.42	3.48	4.42	4.10	3.29	4.21	3.90	3.18	4.06	3.76
	60	1.67	3.96	3.50	4.43	4.11	3.43	4.37	4.05	3.46	4.41	4.09	3.40	4.33	4.02
4%	12	0.33	3.75	3.96	4.87	4.57	3.41	4.32	4.01	3.08	3.98	3.68	2.84	3.72	3.42
	21	0.58	3.87	3.90	4.80	4.50	3.41	4.32	4.01	3.16	4.07	3.76	2.94	3.83	3.53
	30	0.83	3.89	3.97	4.91	4.59	3.43	4.38	4.06	3.34	4.27	3.96	3.22	4.13	3.82
	60	1.67	4.02	3.62	4.58	4.26	3.53	4.50	4.17	3.45	4.40	4.08	3.41	4.34	4.02

Table 2-16. Reliability indices and possible resistance factors for eccentrically loaded columns (tied, plant cast, for h/b = 2)

Design case			Tied												
			Plant cast beta												
% of reinforc.	e (in)	e/h	old	f_c'=3.0 ksi			f_c'=5.0 ksi			f_c'=8.0 ksi			f_c'=12.0 ksi		
		φ	0.70	0.85	0.70	0.75	0.85	0.70	0.75	0.85	0.70	0.75	0.85	0.70	0.75
1%	6	0.17	3.54	4.42	5.26	4.98	3.72	4.64	4.33	3.24	4.15	3.84	3.09	4.03	3.71
	15	0.42	3.79	4.62	5.55	5.24	3.95	4.96	4.62	3.53	4.52	4.19	3.34	4.32	3.99
	27	0.75	4.80	4.55	5.64	5.27	4.30	5.42	5.04	4.13	5.23	4.86	3.96	5.00	4.65
	60	1.67	4.63	4.52	5.71	5.31	4.53	5.74	5.33	4.39	5.56	5.17	4.48	5.70	5.29
2%	12	0.33	4.07	4.74	5.74	5.40	4.04	5.07	4.72	3.61	4.63	4.28	3.30	4.27	3.94
	21	0.58	4.40	4.77	5.79	5.45	4.29	5.41	5.03	4.08	5.20	4.82	3.93	5.04	4.67
	30	0.83	4.90	4.75	5.92	5.52	4.49	5.69	5.28	4.29	5.46	5.06	4.19	5.32	4.94
	60	1.67	4.71	4.56	5.77	5.36	4.49	5.68	5.27	4.44	5.62	5.22	4.50	5.71	5.30
3%	15	0.42	4.38	4.91	5.99	5.63	4.25	5.35	4.97	3.85	4.94	4.57	3.49	4.52	4.17
	24	0.67	4.47	4.78	5.84	5.48	4.33	5.47	5.08	4.12	5.27	4.88	4.13	5.30	4.90
	39	1.08	4.88	4.79	6.01	5.60	4.49	5.69	5.28	4.44	5.64	5.23	4.33	5.49	5.09
	60	1.67	4.74	4.61	5.81	5.40	4.46	5.64	5.24	4.48	5.68	5.27	4.38	5.55	5.16
4%	12	0.33	4.45	5.01	6.13	5.75	4.28	5.39	5.01	3.86	4.94	4.57	3.59	4.66	4.30
	21	0.58	4.65	4.91	6.03	5.65	4.43	5.60	5.20	4.01	5.14	4.76	3.81	4.95	4.57
	30	0.83	4.69	4.93	6.07	5.69	4.38	5.57	5.17	4.36	5.57	5.16	4.38	5.61	5.19
	60	1.67	4.94	4.72	5.94	5.53	4.52	5.73	5.32	4.47	5.67	5.26	4.46	5.66	5.25

Table 2-17. Reliability indices and possible resistance factors for eccentrically loaded columns (spiral, cast-in-place)

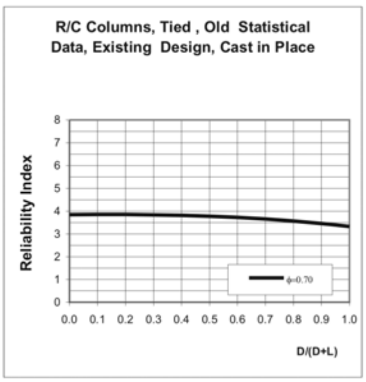
Design case			Spiral												
			Cast-in-place beta												
% of reinforc.	e (in)	e/h	old	f_c'=3.0 ksi			f_c'=5.0 ksi			f_c'=8.0 ksi			f_c'=12.0 ksi		
		φ	0.70	0.85	0.70	0.75	0.85	0.70	0.75	0.85	0.70	0.75	0.85	0.70	0.75
1%	4	0.18	3.37	4.14	4.90	4.64	3.44	4.24	3.97	3.11	3.94	3.66	2.86	3.68	3.40
	10	0.45	4.03	4.08	4.87	4.60	3.53	4.36	4.08	3.08	3.83	3.58	2.83	3.56	3.32
	18	0.82	4.63	3.83	4.70	4.41	3.70	4.60	4.30	3.73	4.65	4.34	3.52	4.39	4.09
	72	3.27	4.79	4.35	5.43	5.06	4.27	5.36	4.99	4.20	5.29	4.92	4.25	5.36	4.98
2%	8	0.36	3.97	4.32	5.18	4.89	3.73	4.62	4.32	3.31	4.18	3.89	3.04	3.88	3.60
	12	0.55	4.16	4.27	5.14	4.85	3.77	4.70	4.38	3.42	4.31	4.01	3.19	4.01	3.73
	24	1.09	4.65	4.02	4.98	4.65	3.92	4.88	4.55	3.84	4.79	4.47	3.80	4.76	4.43
3%	10	0.45	4.29	4.47	5.41	5.09	3.86	4.80	4.48	3.44	4.35	4.05	3.29	4.22	3.90
	14	0.64	4.44	4.38	5.30	4.99	3.84	4.79	4.47	3.55	4.49	4.17	3.50	4.41	4.10
	40	1.82	4.67	4.09	5.09	4.75	4.04	5.05	4.71	4.01	5.03	4.68	4.04	5.07	4.72
4%	10	0.45	4.48	4.49	5.46	5.13	3.98	4.96	4.63	3.64	4.61	4.28	3.34	4.27	3.96
	16	0.73	4.65	4.54	5.53	5.19	3.97	4.96	4.63	3.65	4.64	4.30	3.61	4.54	4.23
	40	1.82	4.69	4.10	5.12	4.78	3.98	4.97	4.63	4.08	5.12	4.76	4.04	5.07	4.72

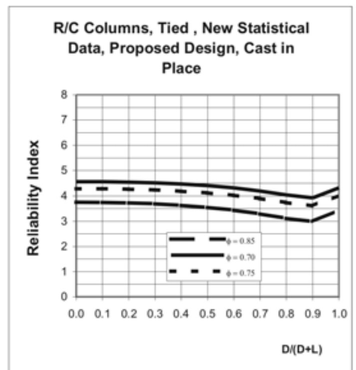
Table 2-18. Reliability indices and possible resistance factors for eccentrically loaded columns (spiral, plant cast).

Design case			Spiral												
% of reinforc..	e (in)	e/h	Plant cast beta												
			old	f_c’=3.0 ksi			f_c’=5.0 ksi			f_c’=8.0 ksi			f_c’=12.0 ksi		
		φ	0.70	0.85	0.70	0.75	0.85	0.70	0.75	0.85	0.70	0.75	0.85	0.70	0.75
1%	4	0.18	3.45	4.49	5.35	5.06	3.65	4.54	4.24	3.26	4.18	3.87	2.99	3.90	3.59
	10	0.45	4.11	4.52	5.45	5.14	4.02	5.03	4.69	3.66	4.67	4.32	3.38	4.35	4.02
	24	1.09	4.68	4.48	5.62	5.23	4.40	5.57	5.17	4.32	5.48	5.09	4.32	5.50	5.10
2%	8	0.36	4.03	4.70	5.68	5.35	4.01	5.03	4.68	3.58	4.59	4.25	3.31	4.30	3.96
	12	0.55	4.24	4.74	5.75	5.41	4.14	5.24	4.87	3.88	4.96	4.59	3.81	4.90	4.53
	24	1.09	4.68	4.52	5.67	5.28	4.33	5.47	5.09	4.33	5.49	5.09	4.26	5.41	5.02
3%	8	0.36	4.25	4.88	5.94	5.58	4.19	5.26	4.90	3.73	4.77	4.41	3.42	4.43	4.09
	14	0.64	4.38	4.86	5.95	5.58	4.27	5.38	5.01	4.00	5.15	4.76	3.96	5.08	4.70
	40	1.82	4.69	4.52	5.71	5.31	4.44	5.62	5.22	4.38	5.55	5.15	4.47	5.68	5.27
4%	10	0.45	4.48	4.84	5.93	5.56	4.26	5.38	5.00	3.90	5.00	4.63	3.58	4.64	4.28
	18	0.82	4.64	4.83	5.94	5.56	4.33	5.51	5.11	4.31	5.51	5.10	4.20	5.36	4.96
	72	3.27	4.77	4.49	5.69	5.28	4.44	5.62	5.22	4.49	5.70	5.29	4.42	5.62	5.21

Examples of calculated reliability indices for eccentrically loaded columns are shown in Figures 2-19 through 2- 28 .

R/C Columns, Cast-in-place, Tied							D + L,		e/h = 0.64		balance failure					
3% of reinforcement				fc =5.0 ksi									beta			beta
				new	old	old	mR new data/design						new			old
D/D+L	D	L	S	Q	Q	mR	0.75	0.70	0.85	mQ	sQ	VQ	0.85	0.70	0.75	0.70
0.00	0.00	1.00	0.00	1.60	1.70	2.516	2.466	2.642	2.176	1.000	0.180	0.18	3.75	4.56	4.28	3.85
0.10	0.10	0.90	0.00	1.56	1.67	2.472	2.404	2.576	2.122	1.005	0.162	0.16	3.74	4.56	4.28	3.86
0.20	0.20	0.80	0.00	1.52	1.64	2.427	2.343	2.510	2.067	1.010	0.146	0.14	3.72	4.55	4.27	3.86
0.30	0.30	0.70	0.00	1.48	1.61	2.383	2.281	2.444	2.013	1.015	0.130	0.13	3.69	4.52	4.24	3.84
0.40	0.40	0.60	0.00	1.44	1.58	2.338	2.220	2.378	1.958	1.020	0.116	0.11	3.63	4.47	4.19	3.82
0.50	0.50	0.50	0.00	1.40	1.55	2.294	2.158	2.312	1.904	1.025	0.104	0.10	3.55	4.41	4.12	3.78
0.60	0.60	0.40	0.00	1.36	1.52	2.250	2.096	2.246	1.850	1.030	0.096	0.09	3.44	4.32	4.02	3.73
0.70	0.70	0.30	0.00	1.32	1.49	2.205	2.035	2.180	1.795	1.035	0.091	0.09	3.30	4.19	3.89	3.66
0.80	0.80	0.20	0.00	1.28	1.46	2.161	1.973	2.114	1.741	1.040	0.091	0.09	3.12	4.04	3.73	3.57
0.90	0.90	0.10	0.00	1.26	1.43	2.116	1.942	2.081	1.714	1.045	0.096	0.09	2.99	3.93	3.61	3.46
1.00	1.00	0.00	0.00	1.40	1.40	2.072	2.158	2.312	1.904	1.050	0.105	0.10	3.44	4.32	4.02	3.33
										average beta			3.34	4.23	3.93	3.67

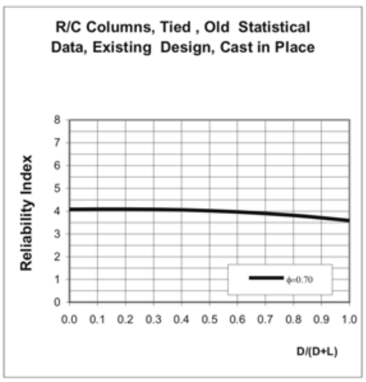


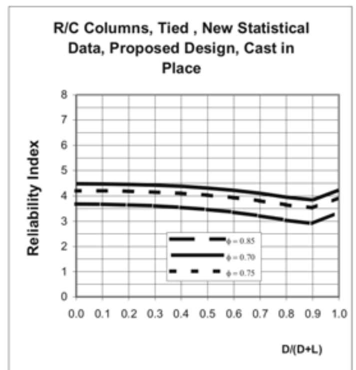


Reliability Indices Calculated for R/C Column Made of Ordinary Concrete for D+L Load Combination

Figure 2-19. Reliability Indices Calculated for Eccentrically Loaded R/C Tied Columns for D+L Load Combination (balance failure).

R/C Columns, Cast-in-place, Tied							D + L,		e/h = 1.09		tension control					
4% of reinforcement				fc =5.0 ksi									beta			beta
				new	old	old	mR new data/design						new			old
D/D+L	D	L	S	Q	Q	mR	0.75	0.70	0.85	mQ	sQ	VQ	0.85	0.70	0.75	0.70
0.00	0.00	1.00	0.00	1.60	1.70	2.659	2.449	2.624	2.161	1.000	0.180	0.18	3.68	4.48	4.20	4.08
0.10	0.10	0.90	0.00	1.56	1.67	2.612	2.388	2.558	2.107	1.005	0.162	0.16	3.67	4.47	4.20	4.09
0.20	0.20	0.80	0.00	1.52	1.64	2.565	2.327	2.493	2.053	1.010	0.146	0.14	3.64	4.46	4.18	4.09
0.30	0.30	0.70	0.00	1.48	1.61	2.519	2.265	2.427	1.999	1.015	0.130	0.13	3.61	4.43	4.15	4.08
0.40	0.40	0.60	0.00	1.44	1.58	2.472	2.204	2.362	1.945	1.020	0.116	0.11	3.55	4.38	4.10	4.06
0.50	0.50	0.50	0.00	1.40	1.55	2.425	2.143	2.296	1.891	1.025	0.104	0.10	3.47	4.31	4.03	4.02
0.60	0.60	0.40	0.00	1.36	1.52	2.378	2.082	2.230	1.837	1.030	0.096	0.09	3.36	4.22	3.93	3.97
0.70	0.70	0.30	0.00	1.32	1.49	2.331	2.020	2.165	1.783	1.035	0.091	0.09	3.22	4.10	3.80	3.90
0.80	0.80	0.20	0.00	1.28	1.46	2.284	1.959	2.099	1.729	1.040	0.091	0.09	3.04	3.95	3.64	3.82
0.90	0.90	0.10	0.00	1.26	1.43	2.237	1.929	2.066	1.702	1.045	0.096	0.09	2.91	3.84	3.53	3.71
1.00	1.00	0.00	0.00	1.40	1.40	2.190	2.143	2.296	1.891	1.050	0.105	0.10	3.36	4.23	3.93	3.59
										average beta			3.26	4.14	3.84	3.91

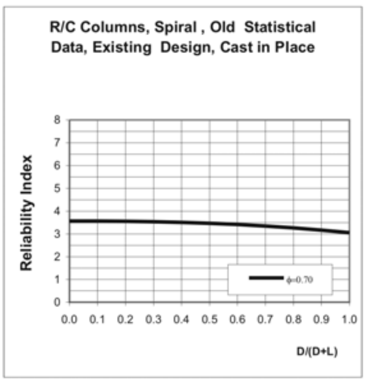


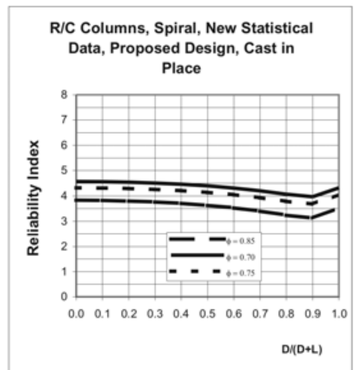


Reliability Indices Calculated for R/C Column Made of Ordinary Concrete for D+L Load Combination

Figure 2-20. Reliability Indices Calculated for Eccentrically Loaded R/C Tied Columns for D+L Load Combination (tension control).

R/C Columns, Cast-in-place, Spiral							D + L,		e/h = 0.18		compression control					
1% of reinforcement				fc =5.0 ksi									beta			beta
				new	old	old	mR new data/design						new			old
D/D+L	D	L	S	Q	Q	mR	0.75	0.70	0.85	mQ	sQ	VQ	0.85	0.70	0.75	0.70
0.00	0.00	1.00	0.00	1.60	1.70	2.489	2.609	2.795	2.302	1.000	0.180	0.18	3.84	4.57	4.32	3.57
0.10	0.10	0.90	0.00	1.56	1.67	2.445	2.544	2.726	2.245	1.005	0.162	0.16	3.82	4.56	4.31	3.57
0.20	0.20	0.80	0.00	1.52	1.64	2.401	2.479	2.656	2.187	1.010	0.146	0.14	3.80	4.54	4.29	3.56
0.30	0.30	0.70	0.00	1.48	1.61	2.358	2.413	2.586	2.129	1.015	0.130	0.13	3.76	4.51	4.26	3.54
0.40	0.40	0.60	0.00	1.44	1.58	2.314	2.348	2.516	2.072	1.020	0.116	0.11	3.71	4.46	4.21	3.51
0.50	0.50	0.50	0.00	1.40	1.55	2.270	2.283	2.446	2.014	1.025	0.104	0.10	3.63	4.40	4.14	3.47
0.60	0.60	0.40	0.00	1.36	1.52	2.226	2.218	2.376	1.957	1.030	0.096	0.09	3.53	4.31	4.05	3.42
0.70	0.70	0.30	0.00	1.32	1.49	2.182	2.152	2.306	1.899	1.035	0.091	0.09	3.40	4.20	3.93	3.35
0.80	0.80	0.20	0.00	1.28	1.46	2.138	2.087	2.236	1.842	1.040	0.091	0.09	3.24	4.07	3.79	3.27
0.90	0.90	0.10	0.00	1.26	1.43	2.094	2.055	2.201	1.813	1.045	0.096	0.09	3.12	3.97	3.68	3.17
1.00	1.00	0.00	0.00	1.40	1.40	2.050	2.283	2.446	2.014	1.050	0.105	0.10	3.53	4.32	4.05	3.06
										average beta			3.44	4.24	3.97	3.37

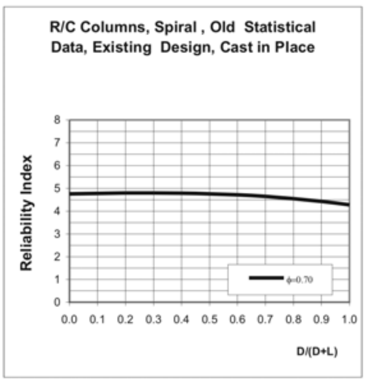


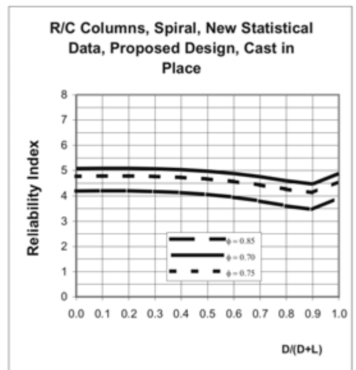


Reliability Indices Calculated for R/C Column Made of Ordinary Concrete for D+L Load Combination

Figure 2-21. Reliability Indices Calculated for Eccentrically Loaded R/C Spiral Columns for D+L Load Combination (compression control).

R/C Columns, Cast-in-place, Spiral							D + L,		e/h = 1.09		tension control					
2% of reinforcement				fc =8.0 ksi									beta			beta
				new	old	old	mR new data/design						new			old
D/D+L	D	L	S	Q	Q	mR	0.75	0.70	0.85	mQ	sQ	VQ	0.85	0.70	0.75	0.70
0.00	0.00	1.00	0.00	1.60	1.70	2.805	2.571	2.754	2.268	1.000	0.180	0.18	4.20	5.08	4.78	4.76
0.10	0.10	0.90	0.00	1.56	1.67	2.756	2.506	2.685	2.212	1.005	0.162	0.16	4.20	5.09	4.79	4.78
0.20	0.20	0.80	0.00	1.52	1.64	2.706	2.442	2.617	2.155	1.010	0.146	0.14	4.20	5.09	4.79	4.80
0.30	0.30	0.70	0.00	1.48	1.61	2.657	2.378	2.548	2.098	1.015	0.130	0.13	4.18	5.08	4.77	4.80
0.40	0.40	0.60	0.00	1.44	1.58	2.607	2.314	2.479	2.041	1.020	0.116	0.11	4.13	5.04	4.73	4.79
0.50	0.50	0.50	0.00	1.40	1.55	2.558	2.249	2.410	1.985	1.025	0.104	0.10	4.06	4.98	4.67	4.76
0.60	0.60	0.40	0.00	1.36	1.52	2.508	2.185	2.341	1.928	1.030	0.096	0.09	3.95	4.89	4.57	4.72
0.70	0.70	0.30	0.00	1.32	1.49	2.459	2.121	2.272	1.871	1.035	0.091	0.09	3.80	4.76	4.44	4.65
0.80	0.80	0.20	0.00	1.28	1.46	2.409	2.057	2.203	1.815	1.040	0.091	0.09	3.61	4.60	4.27	4.55
0.90	0.90	0.10	0.00	1.26	1.43	2.360	2.024	2.169	1.786	1.045	0.096	0.09	3.46	4.47	4.13	4.43
1.00	1.00	0.00	0.00	1.40	1.40	2.310	2.249	2.410	1.985	1.050	0.105	0.10	3.95	4.88	4.57	4.28
										average beta			3.84	4.79	4.47	4.65

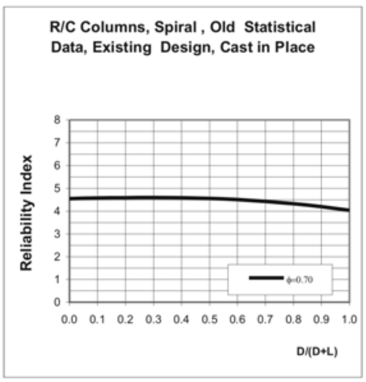


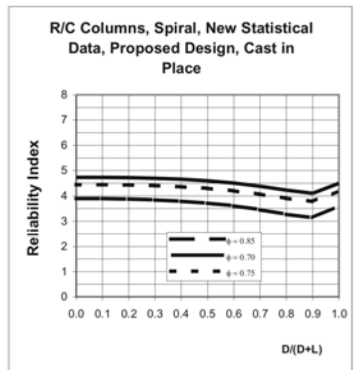


Reliability Indices Calculated for R/C Column Made of Ordinary Concrete for D+L Load Combination

Figure 2-22. Reliability Indices Calculated for Eccentrically Loaded R/C Spiral Columns for D+L Load Combination (tension control).

R/C Columns, Cast-in-place, Spiral							**D + L,**		**e/h = 0.64**		tension control					
3% of reinforcement				fc =12.0 ksi									beta			beta
				new	old	old	mR new data/design						new			old
D/D+L	D	L	S	Q	Q	mR	0.75	0.70	0.85	mQ	sQ	VQ	0.85	0.70	0.75	0.70
0.00	0.00	1.00	0.00	1.60	1.70	2.613	2.498	2.677	2.204	1.000	0.180	0.18	3.90	4.73	4.45	4.55
0.10	0.10	0.90	0.00	1.56	1.67	2.567	2.436	2.610	2.149	1.005	0.162	0.16	3.89	4.73	4.45	4.58
0.20	0.20	0.80	0.00	1.52	1.64	2.521	2.373	2.543	2.094	1.010	0.146	0.14	3.88	4.73	4.44	4.59
0.30	0.30	0.70	0.00	1.48	1.61	2.475	2.311	2.476	2.039	1.015	0.130	0.13	3.85	4.70	4.41	4.60
0.40	0.40	0.60	0.00	1.44	1.58	2.429	2.248	2.409	1.984	1.020	0.116	0.11	3.79	4.66	4.37	4.59
0.50	0.50	0.50	0.00	1.40	1.55	2.383	2.186	2.342	1.929	1.025	0.104	0.10	3.71	4.60	4.30	4.56
0.60	0.60	0.40	0.00	1.36	1.52	2.336	2.123	2.275	1.874	1.030	0.096	0.09	3.60	4.50	4.20	4.51
0.70	0.70	0.30	0.00	1.32	1.49	2.290	2.061	2.208	1.818	1.035	0.091	0.09	3.46	4.38	4.07	4.43
0.80	0.80	0.20	0.00	1.28	1.46	2.244	1.999	2.141	1.763	1.040	0.091	0.09	3.28	4.23	3.90	4.33
0.90	0.90	0.10	0.00	1.26	1.43	2.198	1.967	2.108	1.736	1.045	0.096	0.09	3.14	4.11	3.78	4.20
1.00	1.00	0.00	0.00	1.40	1.40	2.152	2.186	2.342	1.929	1.050	0.105	0.10	3.61	4.50	4.20	4.04
										average beta			3.50	4.41	4.10	4.44

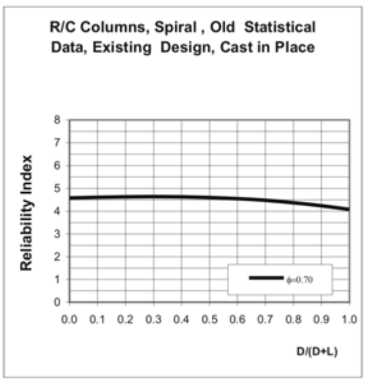


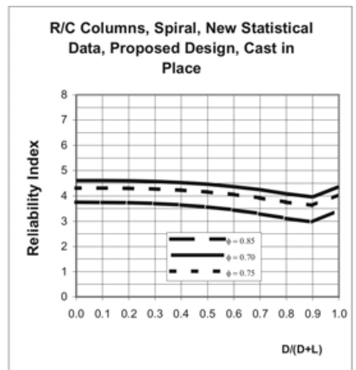


Reliability Indices Calculated for R/C Column Made of Ordinary Concrete for D+L Load Combination

Figure 2-23. Reliability Indices Calculated for Eccentrically Loaded R/C Spiral Columns for D+L Load Combination (tension control).

R/C Columns, Cast-in-place, Spiral							D + L,		e/h = 0.45		compression control					
4% of reinforcement				fc =12.0 ksi									beta			beta
				new	old	old	mR new data/design						new			old
D/D+L	D	L	S	Q	Q	mR	0.75	0.70	0.85	mQ	sQ	VQ	0.85	0.70	0.75	0.70
0.00	0.00	1.00	0.00	1.60	1.70	2.618	2.421	2.594	2.136	1.000	0.180	0.18	3.75	4.60	4.31	4.58
0.10	0.10	0.90	0.00	1.56	1.67	2.572	2.361	2.529	2.083	1.005	0.162	0.16	3.75	4.61	4.31	4.61
0.20	0.20	0.80	0.00	1.52	1.64	2.526	2.300	2.465	2.030	1.010	0.146	0.14	3.73	4.60	4.30	4.63
0.30	0.30	0.70	0.00	1.48	1.61	2.479	2.240	2.400	1.976	1.015	0.130	0.13	3.70	4.57	4.28	4.64
0.40	0.40	0.60	0.00	1.44	1.58	2.433	2.179	2.335	1.923	1.020	0.116	0.11	3.64	4.53	4.23	4.63
0.50	0.50	0.50	0.00	1.40	1.55	2.387	2.119	2.270	1.869	1.025	0.104	0.10	3.56	4.46	4.16	4.60
0.60	0.60	0.40	0.00	1.36	1.52	2.341	2.058	2.205	1.816	1.030	0.096	0.09	3.45	4.37	4.06	4.55
0.70	0.70	0.30	0.00	1.32	1.49	2.295	1.998	2.140	1.763	1.035	0.091	0.09	3.30	4.24	3.92	4.48
0.80	0.80	0.20	0.00	1.28	1.46	2.248	1.937	2.075	1.709	1.040	0.091	0.09	3.11	4.08	3.75	4.37
0.90	0.90	0.10	0.00	1.26	1.43	2.202	1.907	2.043	1.682	1.045	0.096	0.09	2.97	3.96	3.63	4.24
1.00	1.00	0.00	0.00	1.40	1.40	2.156	2.119	2.270	1.869	1.050	0.105	0.10	3.45	4.37	4.06	4.08
										average beta			3.34	4.27	3.96	4.48

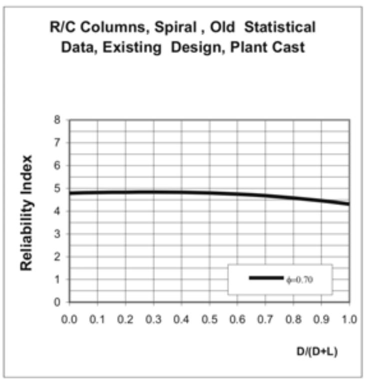


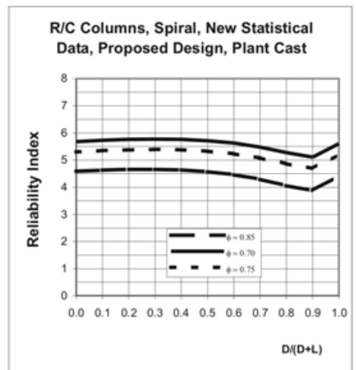


Reliability Indices Calculated for R/C Column Made of Ordinary Concrete for D+L Load Combination

Figure 2-24. Reliability Indices Calculated for Eccentrically Loaded R/C Spiral Columns for D+L Load Combination (compression control).

R/C Columns, Plant cast, Spiral							**D + L,**		**e/h = 1.09**		tension control					
1% of reinforcement				fc =12.0 ksi									beta			beta
				new	old	old	mR new data/design						new			old
D/D+L	D	L	S	Q	Q	mR	0.75	0.70	0.85	mQ	sQ	VQ	0.85	0.70	0.75	0.70
0.00	0.00	1.00	0.00	1.60	1.70	2.805	2.528	2.709	2.231	1.000	0.180	0.18	4.59	5.68	5.30	4.79
0.10	0.10	0.90	0.00	1.56	1.67	2.756	2.465	2.641	2.175	1.005	0.162	0.16	4.63	5.73	5.35	4.82
0.20	0.20	0.80	0.00	1.52	1.64	2.706	2.402	2.573	2.119	1.010	0.146	0.14	4.66	5.76	5.38	4.83
0.30	0.30	0.70	0.00	1.48	1.61	2.657	2.338	2.505	2.063	1.015	0.130	0.13	4.66	5.78	5.39	4.84
0.40	0.40	0.60	0.00	1.44	1.58	2.607	2.275	2.438	2.008	1.020	0.116	0.11	4.64	5.76	5.38	4.83
0.50	0.50	0.50	0.00	1.40	1.55	2.558	2.212	2.370	1.952	1.025	0.104	0.10	4.58	5.72	5.33	4.80
0.60	0.60	0.40	0.00	1.36	1.52	2.508	2.149	2.302	1.896	1.030	0.096	0.09	4.46	5.63	5.23	4.75
0.70	0.70	0.30	0.00	1.32	1.49	2.459	2.086	2.235	1.840	1.035	0.091	0.09	4.30	5.48	5.08	4.68
0.80	0.80	0.20	0.00	1.28	1.46	2.409	2.022	2.167	1.784	1.040	0.091	0.09	4.06	5.28	4.87	4.58
0.90	0.90	0.10	0.00	1.26	1.43	2.360	1.991	2.133	1.757	1.045	0.096	0.09	3.88	5.11	4.69	4.46
1.00	1.00	0.00	0.00	1.40	1.40	2.310	2.212	2.370	1.952	1.050	0.105	0.10	4.44	5.60	5.21	4.31
										average beta			4.32	5.50	5.10	4.68

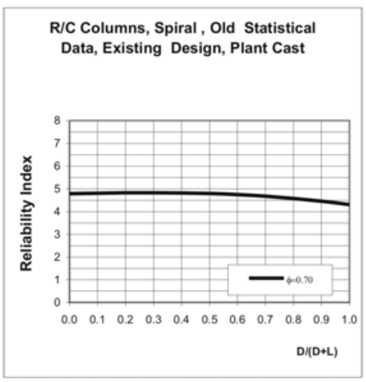


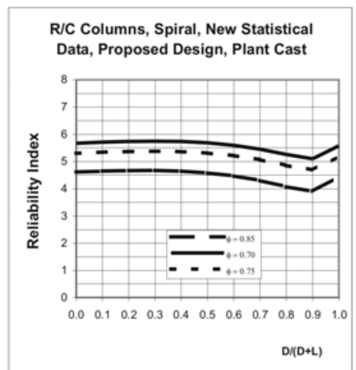


Reliability Indices Calculated for R/C Column Made of Ordinary Concrete for D+L Load Combination

Figure 2-25. Reliability Indices Calculated for Eccentrically Loaded R/C Spiral Columns for D+L Load Combination (tension control).

R/C Columns, Plant cast, Spiral							**D + L,**		**e/h = 1.09**		tension control					
2% of reinforcement				fc =5.0 ksi									beta			beta
				new	old	old	mR new data/design						new			old
D/D+L	D	L	S	Q	Q	mR	0.75	0.70	0.85	mQ	sQ	VQ	0.85	0.70	0.75	0.70
0.00	0.00	1.00	0.00	1.60	1.70	2.803	2.562	2.745	2.261	1.000	0.180	0.18	4.61	5.67	5.30	4.79
0.10	0.10	0.90	0.00	1.56	1.67	2.753	2.498	2.677	2.204	1.005	0.162	0.16	4.65	5.71	5.35	4.81
0.20	0.20	0.80	0.00	1.52	1.64	2.704	2.434	2.608	2.148	1.010	0.146	0.14	4.67	5.74	5.37	4.83
0.30	0.30	0.70	0.00	1.48	1.61	2.654	2.370	2.539	2.091	1.015	0.130	0.13	4.67	5.75	5.38	4.83
0.40	0.40	0.60	0.00	1.44	1.58	2.605	2.306	2.471	2.035	1.020	0.116	0.11	4.65	5.74	5.36	4.82
0.50	0.50	0.50	0.00	1.40	1.55	2.555	2.242	2.402	1.978	1.025	0.104	0.10	4.58	5.69	5.31	4.80
0.60	0.60	0.40	0.00	1.36	1.52	2.506	2.178	2.333	1.922	1.030	0.096	0.09	4.47	5.60	5.22	4.75
0.70	0.70	0.30	0.00	1.32	1.49	2.456	2.114	2.265	1.865	1.035	0.091	0.09	4.31	5.46	5.07	4.68
0.80	0.80	0.20	0.00	1.28	1.46	2.407	2.050	2.196	1.809	1.040	0.091	0.09	4.08	5.26	4.86	4.58
0.90	0.90	0.10	0.00	1.26	1.43	2.357	2.018	2.162	1.780	1.045	0.096	0.09	3.90	5.10	4.69	4.46
1.00	1.00	0.00	0.00	1.40	1.40	2.308	2.242	2.402	1.978	1.050	0.105	0.10	4.45	5.58	5.19	4.31
										average beta			4.33	5.47	5.09	4.68

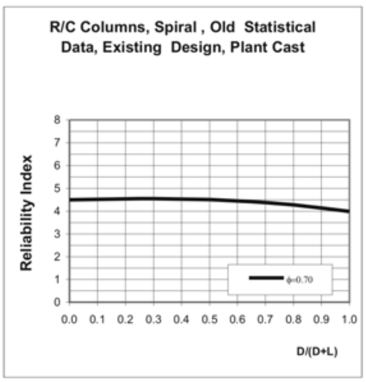


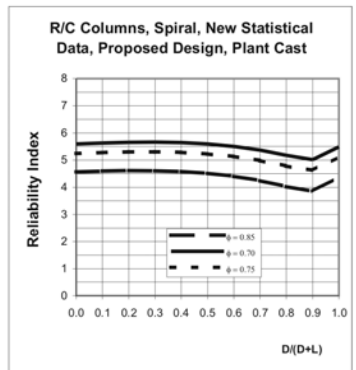


Reliability Indices Calculated for R/C Column Made of Ordinary Concrete for D+L Load Combination

Figure 2-26. Reliability Indices Calculated for Eccentrically Loaded R/C Spiral Columns for D+L Load Combination (tension control).

R/C Columns, Plant cast, Spiral							D + L,		e/h = 0.64		balance failure					
3% of reinforcement				fc =5.0 ksi									beta			beta
				new	old	old	mR new data/design						new			old
D/D+L	D	L	S	Q	Q	mR	0.75	0.70	0.85	mQ	sQ	VQ	0.85	0.70	0.75	0.70
0.00	0.00	1.00	0.00	1.60	1.70	2.599	2.566	2.750	2.264	1.000	0.180	0.18	4.56	5.60	5.24	4.50
0.10	0.10	0.90	0.00	1.56	1.67	2.553	2.502	2.681	2.208	1.005	0.162	0.16	4.60	5.63	5.28	4.52
0.20	0.20	0.80	0.00	1.52	1.64	2.507	2.438	2.612	2.151	1.010	0.146	0.14	4.61	5.66	5.30	4.54
0.30	0.30	0.70	0.00	1.48	1.61	2.461	2.374	2.543	2.095	1.015	0.130	0.13	4.61	5.66	5.31	4.55
0.40	0.40	0.60	0.00	1.44	1.58	2.415	2.310	2.475	2.038	1.020	0.116	0.11	4.58	5.65	5.28	4.53
0.50	0.50	0.50	0.00	1.40	1.55	2.369	2.246	2.406	1.981	1.025	0.104	0.10	4.52	5.59	5.23	4.51
0.60	0.60	0.40	0.00	1.36	1.52	2.323	2.181	2.337	1.925	1.030	0.096	0.09	4.41	5.50	5.13	4.45
0.70	0.70	0.30	0.00	1.32	1.49	2.278	2.117	2.269	1.868	1.035	0.091	0.09	4.25	5.37	4.99	4.38
0.80	0.80	0.20	0.00	1.28	1.46	2.232	2.053	2.200	1.812	1.040	0.091	0.09	4.03	5.18	4.79	4.28
0.90	0.90	0.10	0.00	1.26	1.43	2.186	2.021	2.165	1.783	1.045	0.096	0.09	3.85	5.02	4.62	4.14
1.00	1.00	0.00	0.00	1.40	1.40	2.140	2.246	2.406	1.981	1.050	0.105	0.10	4.39	5.49	5.11	3.99
										average beta			4.27	5.38	5.01	4.38

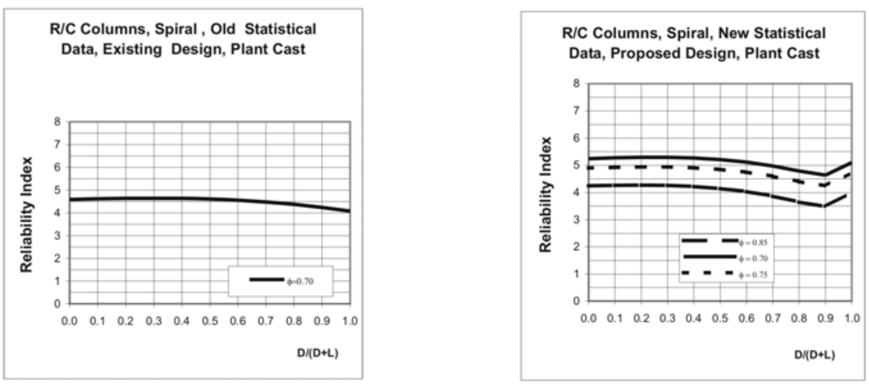




Reliability Indices Calculated for R/C Column Made of Ordinary Concrete for D+L Load Combination

Figure 2-27. Reliability Indices Calculated for Eccentrically Loaded R/C Spiral Columns for D+L Load Combination (balance failure).

R/C Columns, Plant cast, Spiral							D + L,		e/h = 0.45		compression control					
4% of reinforcement				fc =8.0 ksi									beta			beta
				new	old	old	mR new data/design						new			old
D/D+L	D	L	S	Q	Q	mR	0.75	0.70	0.85	mQ	sQ	VQ	0.85	0.70	0.75	0.70
0.00	0.00	1.00	0.00	1.60	1.70	2.620	2.468	2.645	2.178	1.000	0.180	0.18	4.24	5.25	4.90	4.59
0.10	0.10	0.90	0.00	1.56	1.67	2.574	2.407	2.578	2.123	1.005	0.162	0.16	4.26	5.28	4.93	4.62
0.20	0.20	0.80	0.00	1.52	1.64	2.528	2.345	2.512	2.069	1.010	0.146	0.14	4.27	5.29	4.94	4.64
0.30	0.30	0.70	0.00	1.48	1.61	2.482	2.283	2.446	2.015	1.015	0.130	0.13	4.26	5.29	4.94	4.64
0.40	0.40	0.60	0.00	1.44	1.58	2.435	2.221	2.380	1.960	1.020	0.116	0.11	4.22	5.27	4.91	4.64
0.50	0.50	0.50	0.00	1.40	1.55	2.389	2.160	2.314	1.906	1.025	0.104	0.10	4.15	5.21	4.85	4.61
0.60	0.60	0.40	0.00	1.36	1.52	2.343	2.098	2.248	1.851	1.030	0.096	0.09	4.04	5.11	4.75	4.56
0.70	0.70	0.30	0.00	1.32	1.49	2.297	2.036	2.182	1.797	1.035	0.091	0.09	3.87	4.98	4.60	4.48
0.80	0.80	0.20	0.00	1.28	1.46	2.250	1.975	2.116	1.742	1.040	0.091	0.09	3.66	4.79	4.40	4.38
0.90	0.90	0.10	0.00	1.26	1.43	2.204	1.944	2.083	1.715	1.045	0.096	0.09	3.49	4.64	4.25	4.24
1.00	1.00	0.00	0.00	1.40	1.40	2.158	2.160	2.314	1.906	1.050	0.105	0.10	4.02	5.10	4.74	4.08
										average beta			3.90	5.00	4.63	4.48



Reliability Indices Calculated for R/C Column Made of Ordinary Concrete for D+L Load Combination

Figure 2-28. Reliability Indices Calculated for Eccentrically Loaded R/C Spiral Columns for D+L Load Combination (compression control).

Figures 2-29, 2-31, 2-33, 2-35, 2-37 and 2-39 present the relationship between the reliability index and strength of concrete. The analysis was performed with regard to percentage of reinforcement (for ρ = 1%, 2%, 3% and 4%) and eccentricity of load. Particular curves indicate changes of reliability index depending on the mode of failure: compression control, balanced failure, transition or tension control. Figures 2-30, 2-32, 2-34, 2-36, 2-38 and 2-40 present the relationship between reliability index and eccentricity of load. For particular strength of concrete, curves show changes of the reliability index depending on the normalized eccentricity and strength of concrete. In compression control zone up to the balance failure, reliability index differs with regard to strength of concrete. After transition, all curves converge to the same direction in tension control zone. This behavior describes sensitivity of the reliability index to a contribution of concrete or reinforcing steel (with their statistics) to the capacity of the section. All curves prepared for tied columns were calculated using resistance factor equal to 0.70, and curves prepared for spiral columns were calculated using resistance factor equal to 0.75.

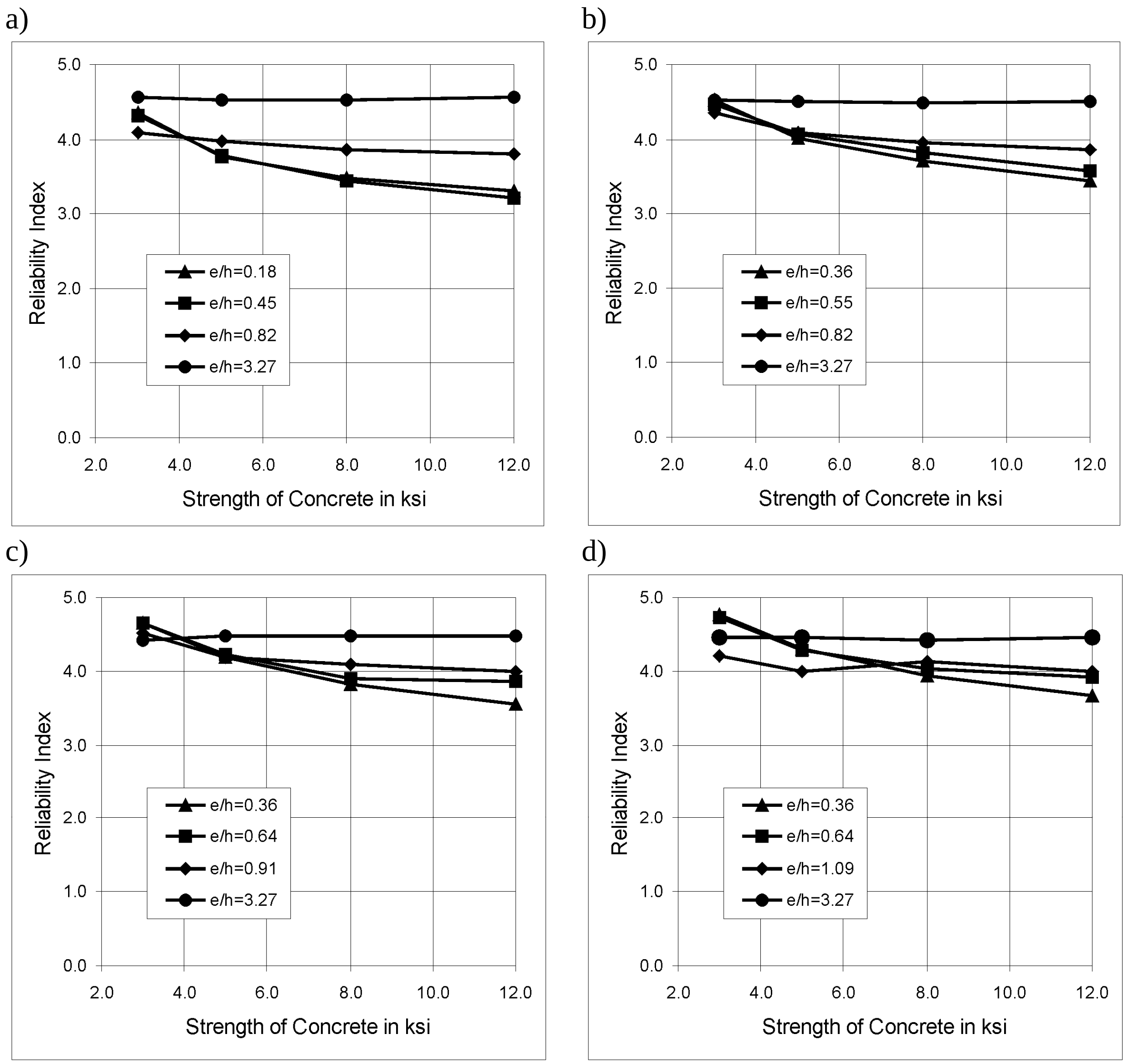


Figure 2-29. Reliability index versus strength of concrete; tied columns, cast-in-place, for h/b = 1.57 and resistance factor, φ = 0.70: a) reinforcement ratio, ρ = 1%, b) reinforcement ratio, ρ = 2%, c) reinforcement ratio, ρ = 3%, d) reinforcement ratio, ρ = 4%,

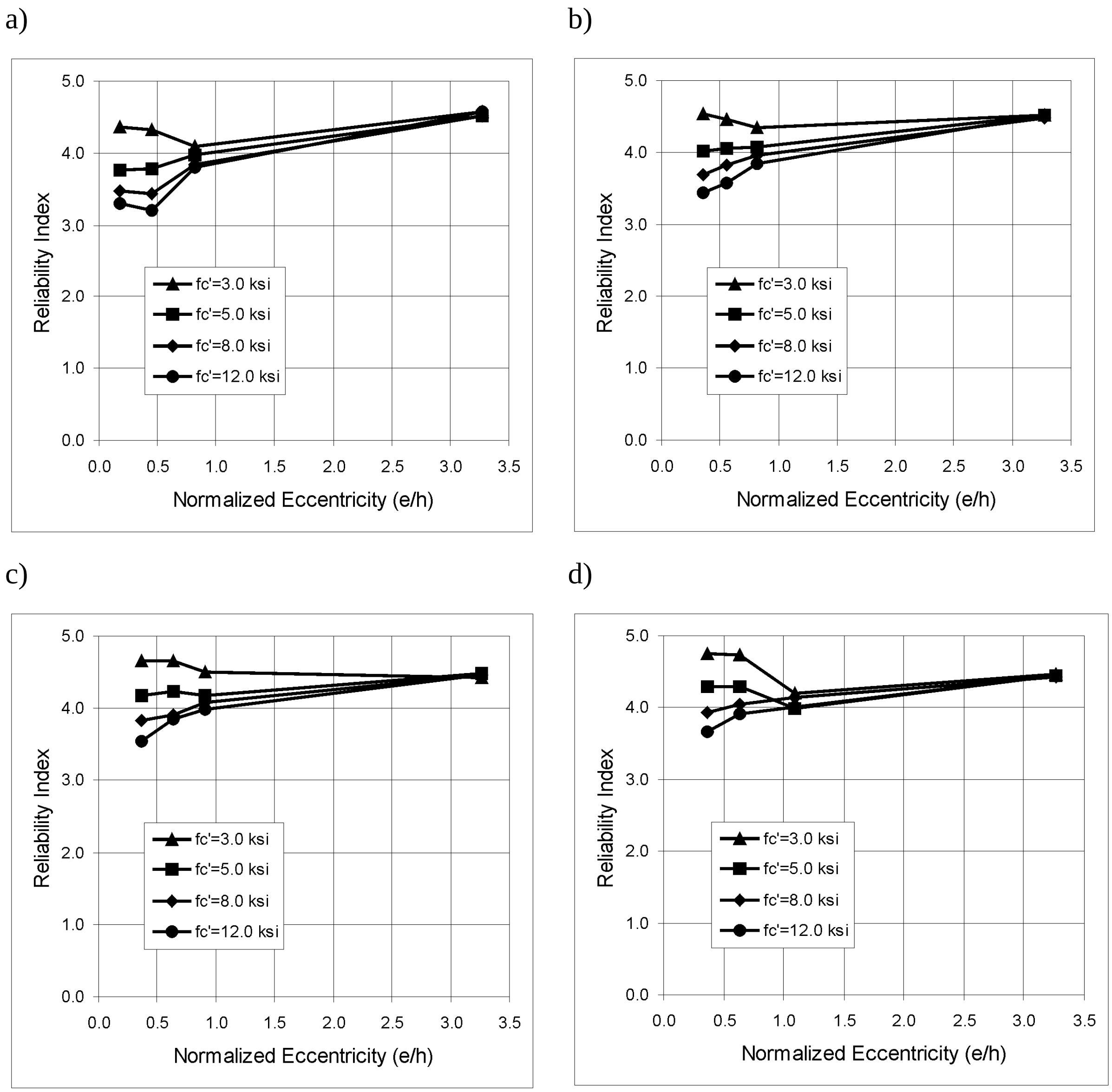


Figure 2-30. Reliability index versus load eccentricity; tied columns, cast-in-place, for h/b = 1.57 and resistance factor, φ = 0.70: a) reinforcement ratio, ρ = 1%, b) reinforcement ratio, ρ = 2%, c) reinforcement ratio, ρ = 3%, d) reinforcement ratio, ρ = 4%,

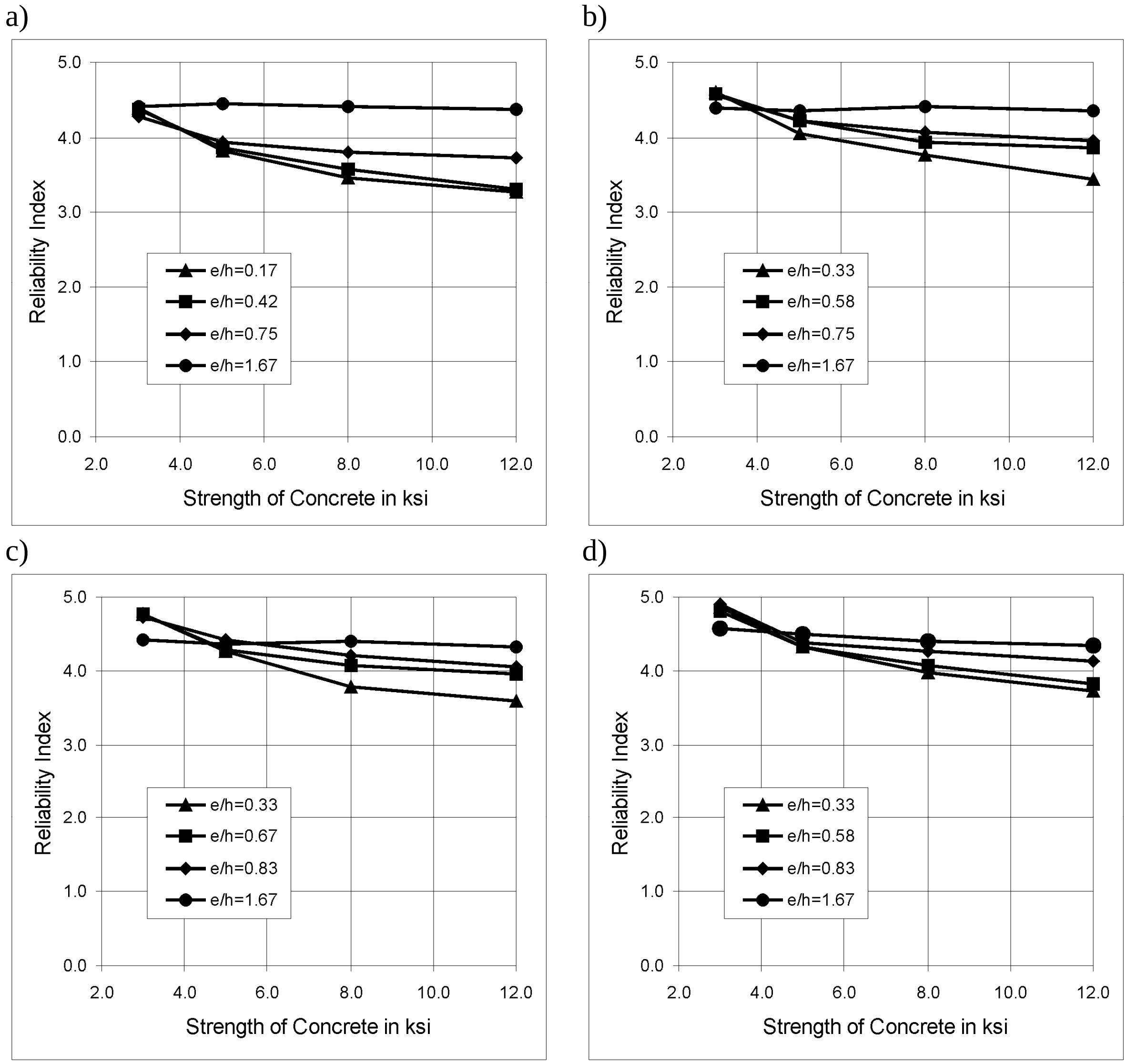


Figure 2-31. Reliability index versus strength of concrete; tied columns, cast-in-place, for h/b = 2.0 and resistance factor, φ = 0.70: a) reinforcement ratio, ρ = 1%, b) reinforcement ratio, ρ = 2%, c) reinforcement ratio, ρ = 3%, d) reinforcement ratio, ρ = 4%,

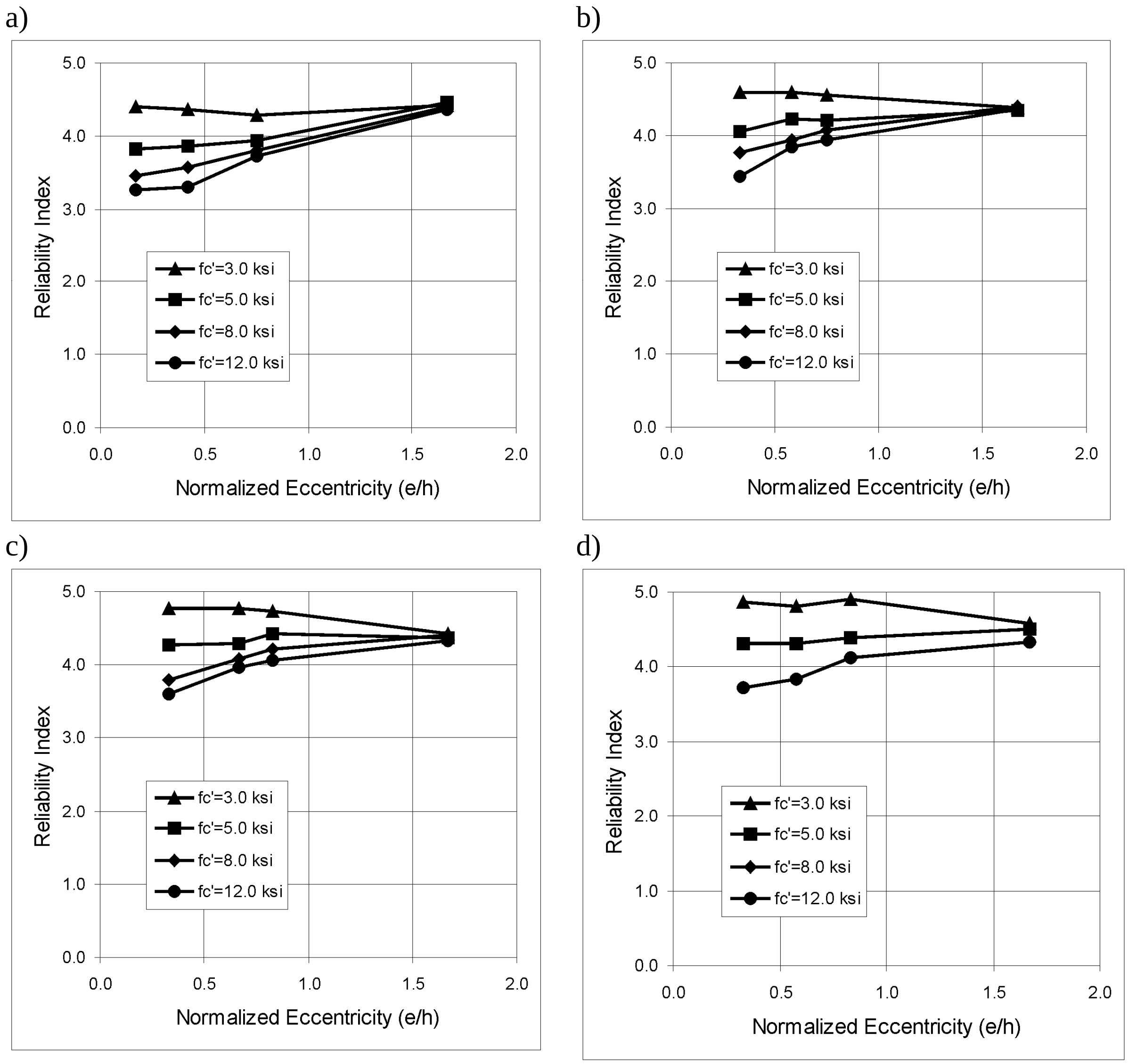


Figure 2-32. Reliability index versus load eccentricity; tied columns, cast-in-place, for h/b = 2.0 and resistance factor, φ = 0.70: a) reinforcement ratio, ρ = 1%, b) reinforcement ratio, ρ = 2%, c) reinforcement ratio, ρ = 3%, d) reinforcement ratio, ρ = 4%,

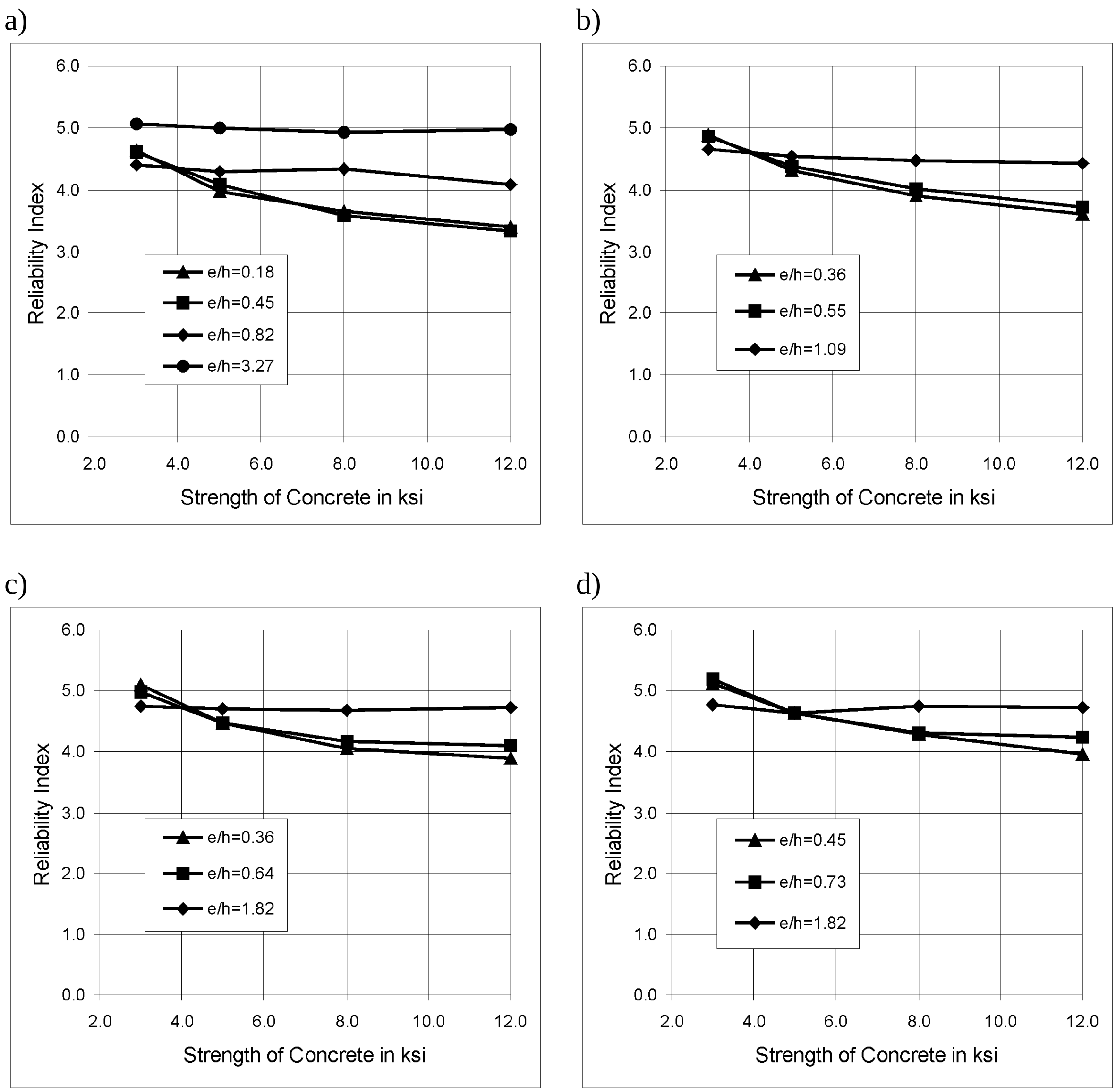


Figure 2-33. Reliability index versus strength of concrete; spiral columns, cast-in-place, for resistance factor, $\varphi = 0.70$: a) reinforcement ratio, $\rho = 1\%$, b) reinforcement ratio, $\rho = 2\%$, c) reinforcement ratio, $\rho = 3\%$, d) reinforcement ratio, $\rho = 4\%$,

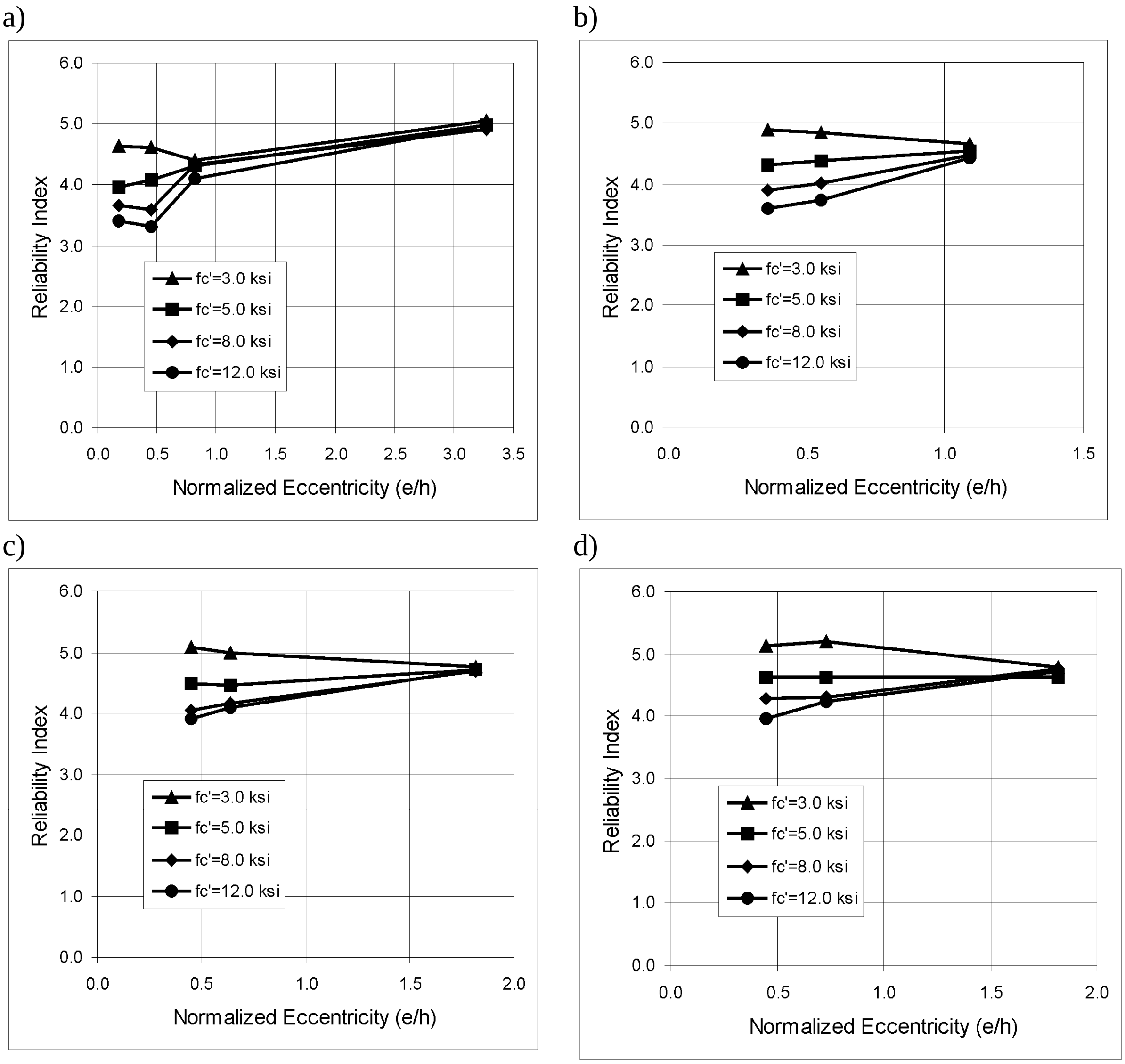


Figure 2-34. Reliability index versus load eccentricity; spiral columns, cast-in-place, resistance factor, $\varphi = 0.70$: a) reinforcement ratio, $\rho = 1\%$, b) reinforcement ratio, $\rho = 2\%$, c) reinforcement ratio, $\rho = 3\%$, d) reinforcement ratio, $\rho = 4\%$,

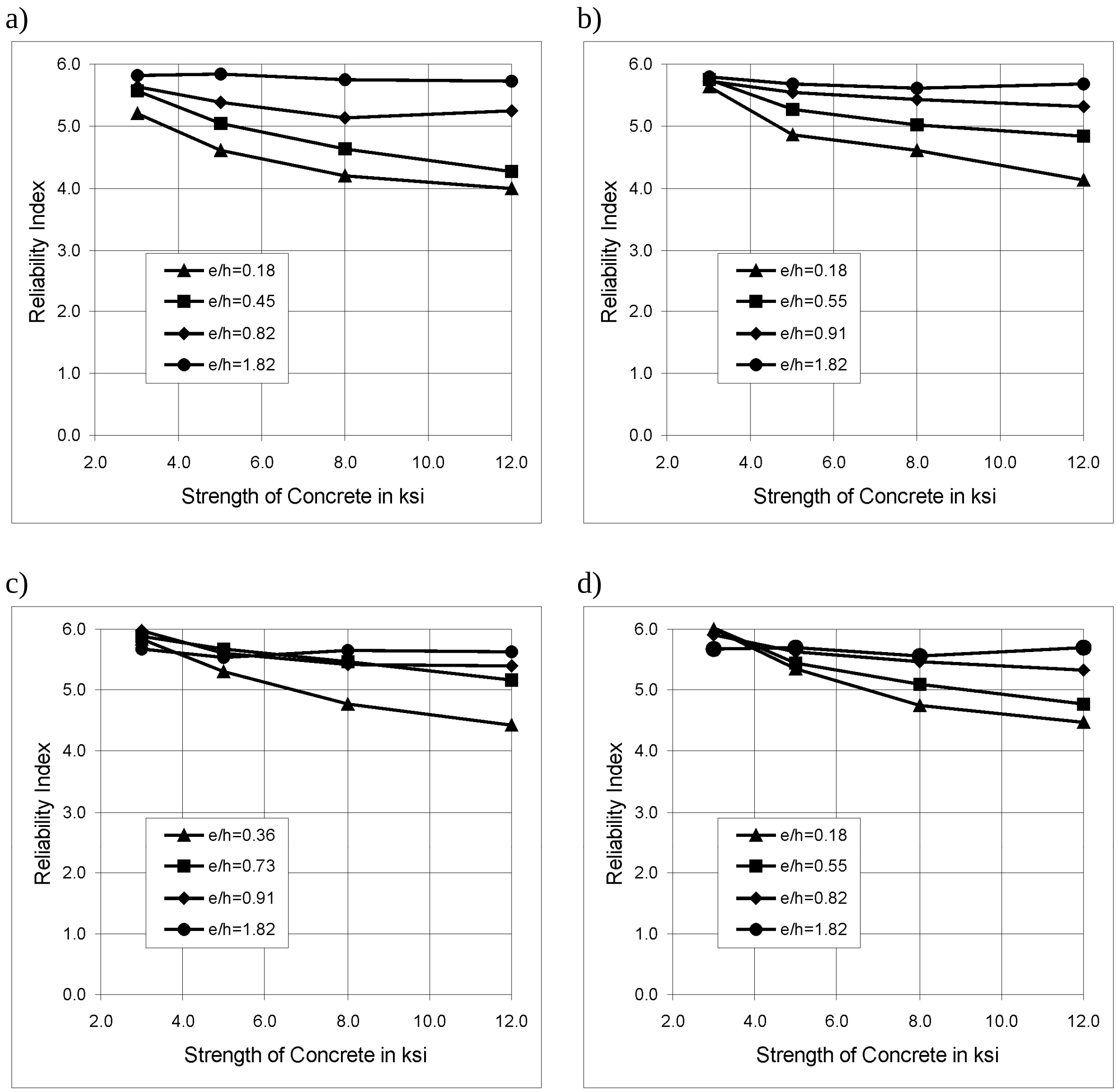


Figure 2-35. Reliability index versus strength of concrete; tied columns, plant cast, for h/b = 1.57 and resistance factor, φ = 0.70: a) reinforcement ratio, ρ = 1%, b) reinforcement ratio, ρ = 2%, c) reinforcement ratio, ρ = 3%, d) reinforcement ratio, ρ = 4%,

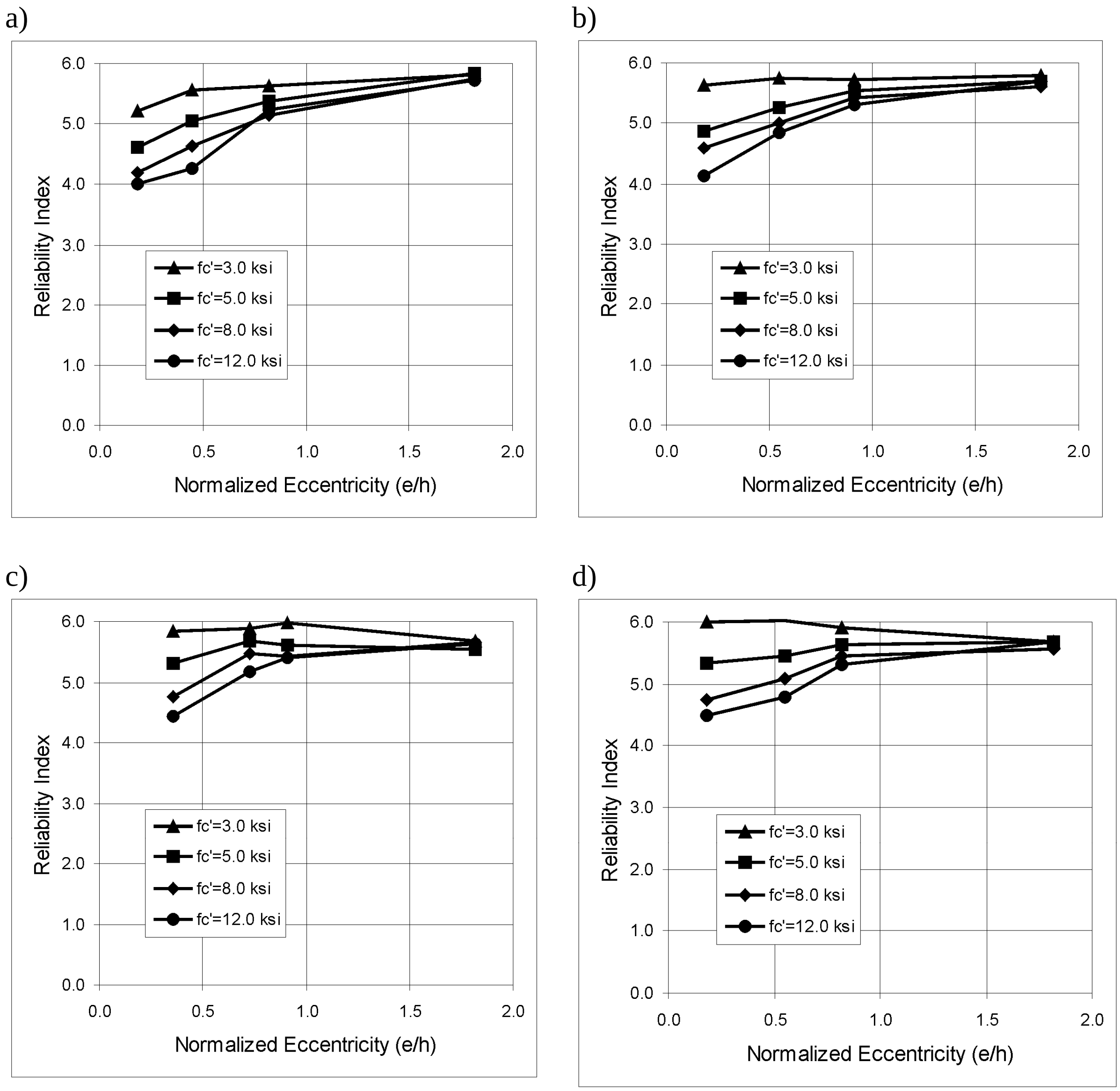


Figure 2-36. Reliability index versus load eccentricity; tied columns, plant cast, for h/b = 1.57 and resistance factor, $\varphi = 0.70$: a) reinforcement ratio, $\rho = 1\%$, b) reinforcement ratio, $\rho = 2\%$, c) reinforcement ratio, $\rho = 3\%$, d) reinforcement ratio, $\rho = 4\%$,

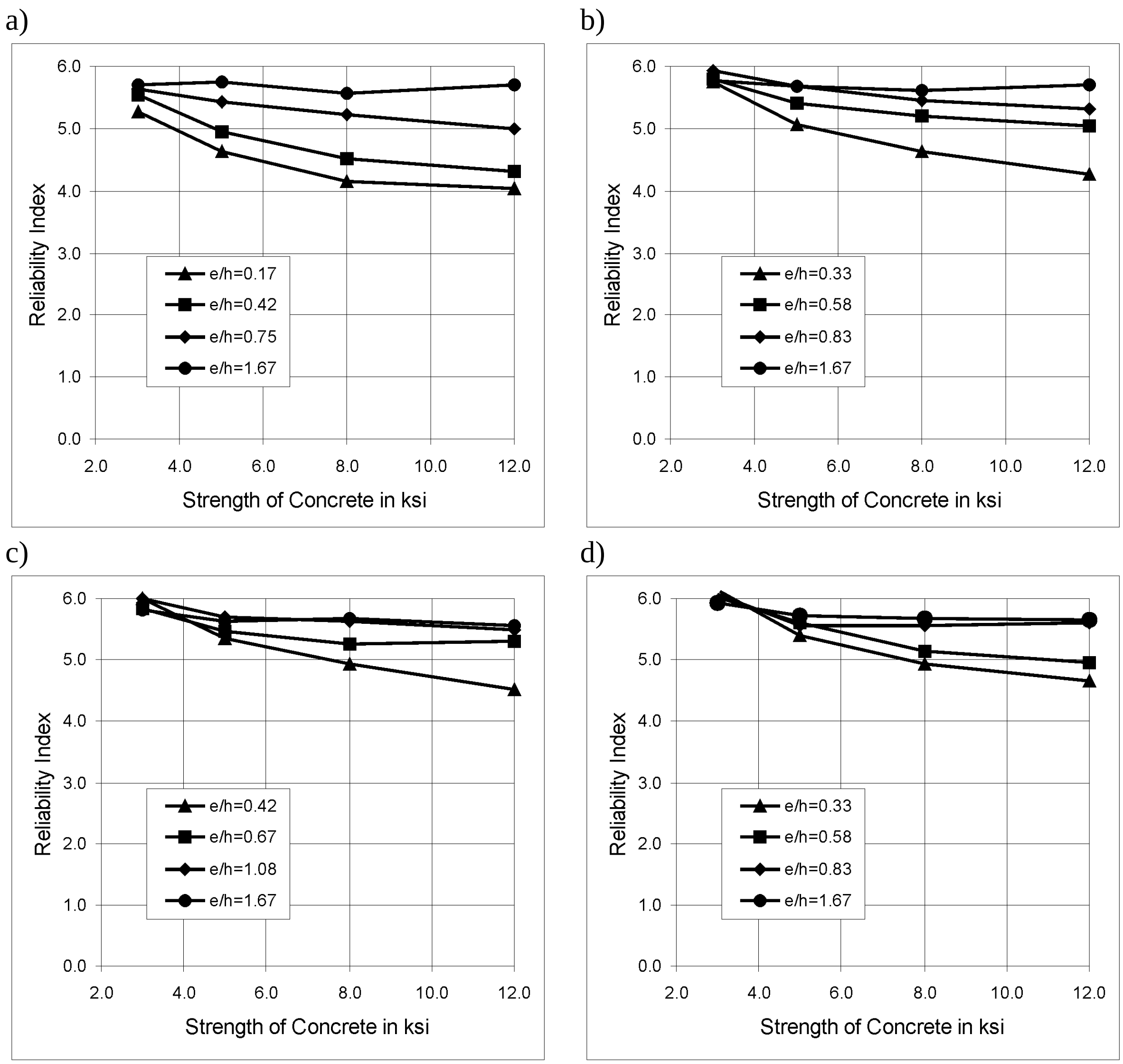


Figure 2-37. Reliability index versus strength of concrete; tied columns, plant cast, for h/b = 2.0 and resistance factor, φ = 0.70: a) reinforcement ratio, ρ = 1%, b) reinforcement ratio, ρ = 2%, c) reinforcement ratio, ρ = 3%, d) reinforcement ratio, ρ = 4%,

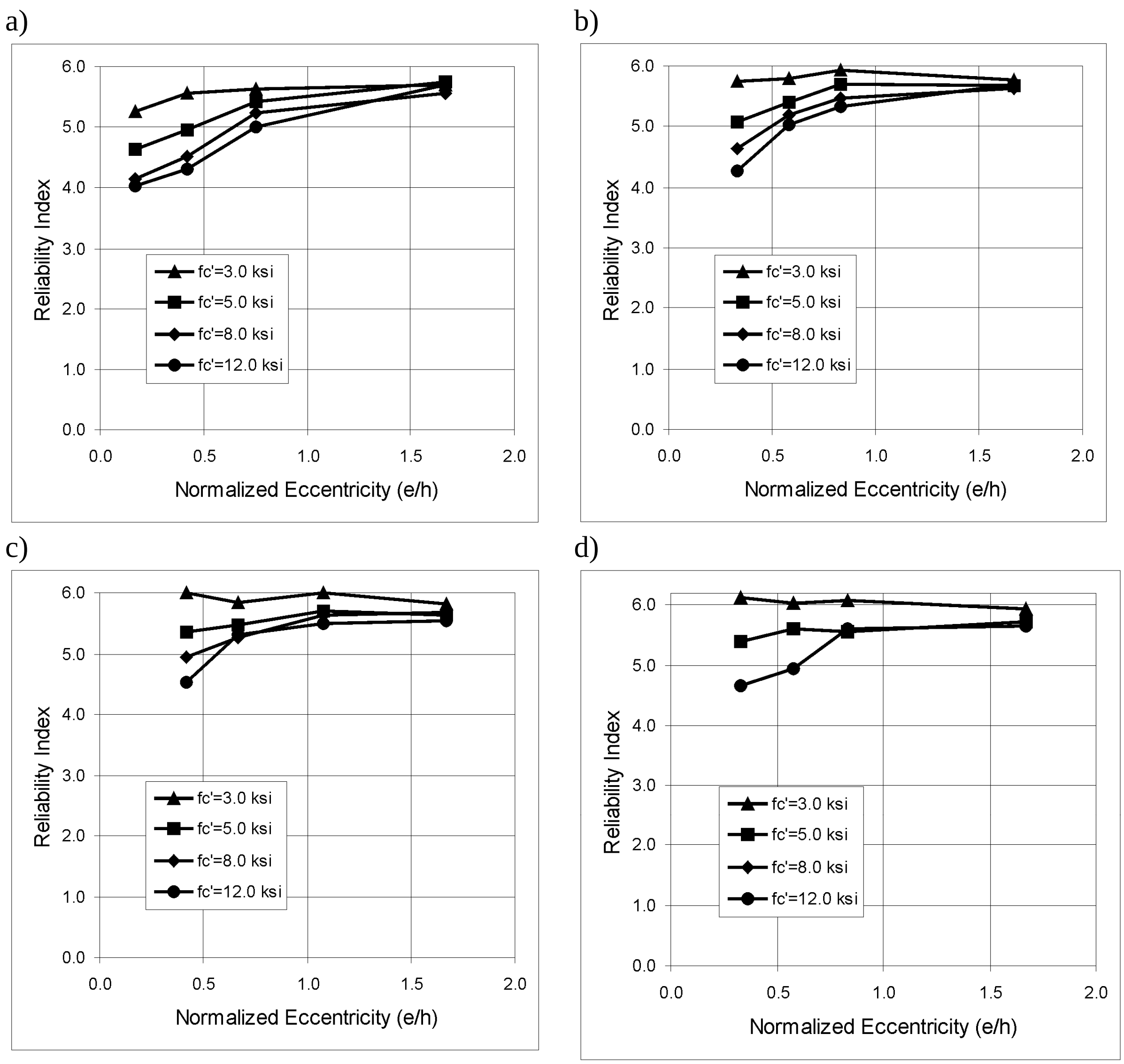


Figure 2-38. Reliability index versus load eccentricity; tied columns, plant cast, for h/b = 2.0 and resistance factor, φ = 0.70: a) reinforcement ratio, ρ = 1%, b) reinforcement ratio, ρ = 2%, c) reinforcement ratio, ρ = 3%, d) reinforcement ratio, ρ = 4%,

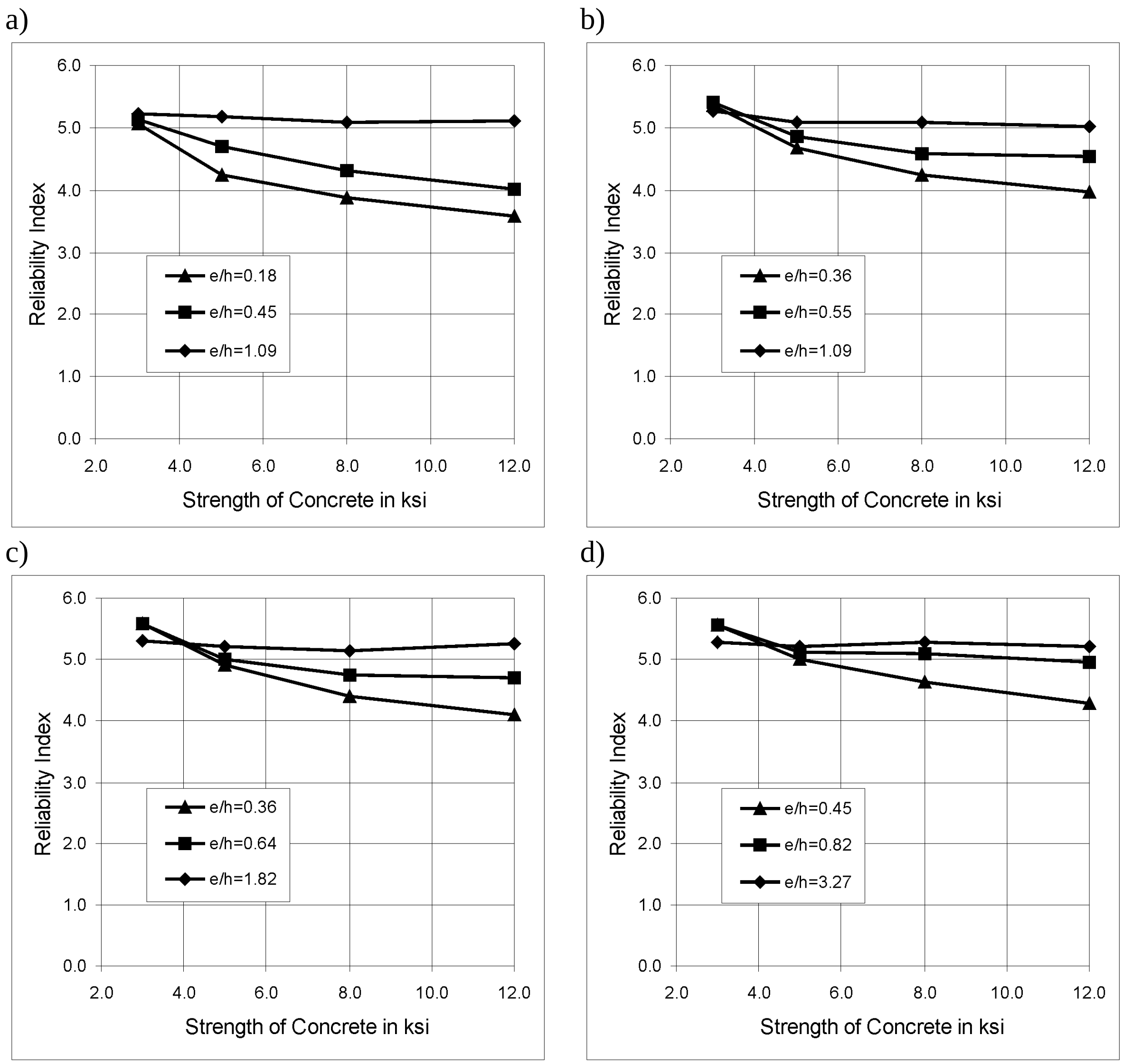


Figure 2-39. Reliability index versus strength of concrete; spiral columns, plant cast, for resistance factor, $\varphi = 0.70$: a) reinforcement ratio, $\rho = 1\%$, b) reinforcement ratio, $\rho = 2\%$, c) reinforcement ratio, $\rho = 3\%$, d) reinforcement ratio, $\rho = 4\%$,

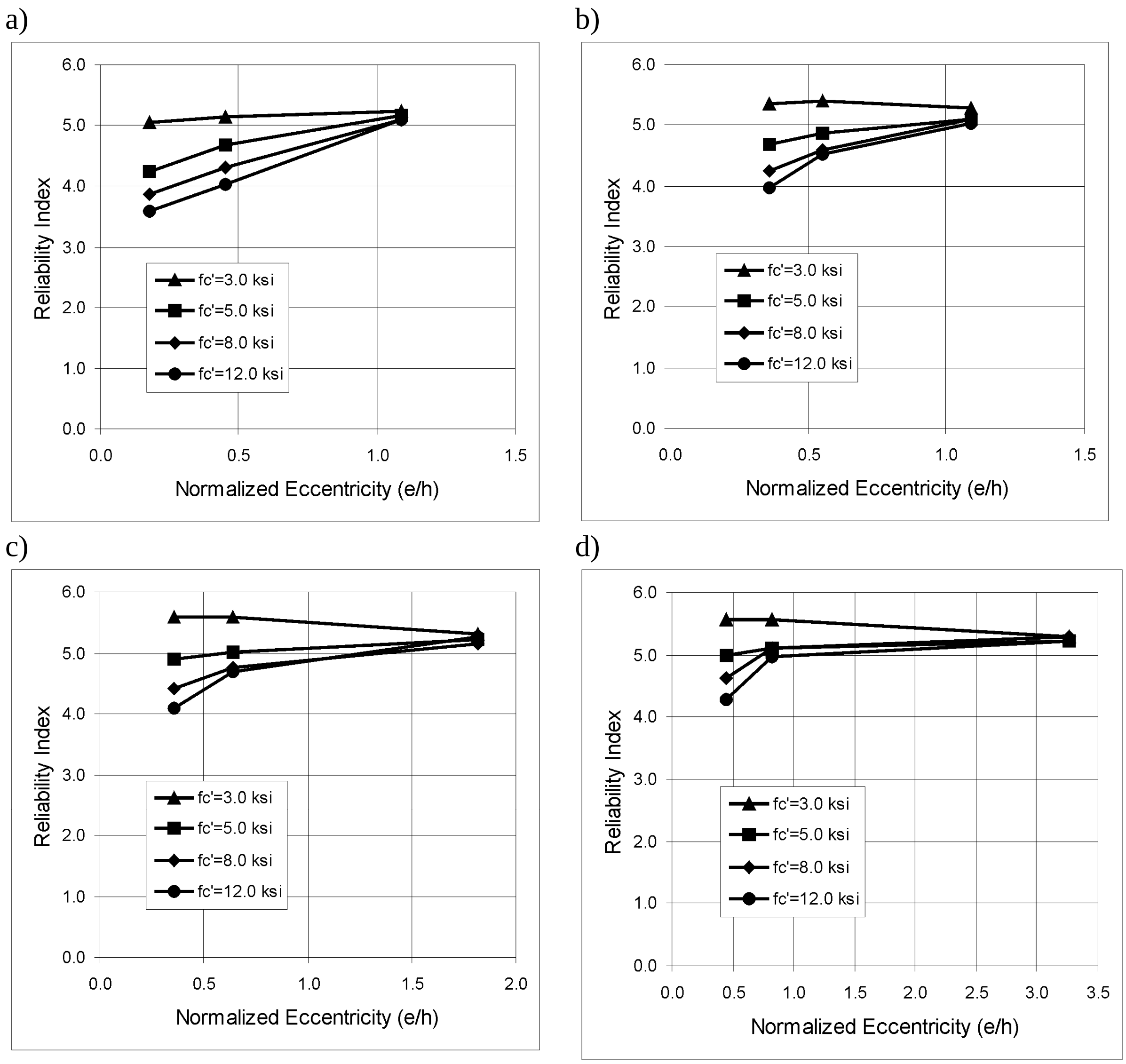


Figure 2-40. Reliability index versus load eccentricity; spiral columns, plant cast, for resistance factor, $\varphi = 0.70$: a) reinforcement ratio, $\rho = 1\%$, b) reinforcement ratio, $\rho = 2\%$, c) reinforcement ratio, $\rho = 3\%$, d) reinforcement ratio, $\rho = 4\%$,

3. Shear in Slabs and Foundation Beams

3.1. Shear Resistance

Two design cases are considered of the shear resistance transferred by concrete only (without shear reinforcemnt). The first case deals with reinforced concrete slabs supported by columns and subjected to shear force close to column (along the column perimeter). In that case, the shear force is carried by concrete only. The other case deals with foundation beams without shear reinforcement.

The minimum shear force transferred by a concrete section of the slab (two-way shear) supported by an interior, edge or corner column shall be smallest of:

$$V_c = \left(2 + \frac{4}{\beta_c}\right)\sqrt{f_c^{'}}\,b_o d \qquad (3.1.a)$$

$$V_c = \left(\frac{\alpha_s \times d}{b_o} + 2\right)\sqrt{f_c^{'}}\,b_o d \qquad (3.1.b)$$

$$V_c = 4\sqrt{f_c'}\,b_o d \qquad (3.1.c)$$

where:

f'_c = compressive strength of the concrete

b_o = critical perimeter

d = effective depth of the slab

β_c= ratio of long side to short side of the column

α_s= coefficient equal to 40 for interior columns, 30 for edge columns, and 20 for corner columns

Three types of columns are considered: interior, edge and corner, with two different shapes of the column cross section: rectangular and square.

The design equation for the shear force (one-way shear) transferred by concrete in foundation beams is:

$$V_c = 2\sqrt{f_c'}\,b_w d \qquad (3.2)$$

where:

f'_c = compressive strength of the concrete

b_w = width of the beam

d = effective depth of the beam

All parameters in Eq.(3.1.a,b,c) and Eq.(3.2) except coefficients α_s and β_c, are treated as random variables. The statistical parameters of material and fabrication factors require some additional comments because they are different than those used in the first part of the analysis (Phase 1). In fact, the shear force transferred by concrete (Eq.3.1.a,b,c and Eq.3.2) depends on the modulus of rupture of concrete, that has a different physical meaning than the compressive strength of concrete. The statistical parameters of concrete, bias factor, $\lambda_{m,}$ and coefficient of variation, V_m, discussed in Phase 1 of the report, were established based on the cylinder compressive strength test data. The modulus of rupture of concrete can be measured in the cylinder splitting test and has different statistical parameters. Based on the published results (Davies and Bose, 1968, Rocco et al., 2001, Tang, 1994 and Raphael, 1984), the coefficient of variation for modulus of rupture was selected as 0.15 (V = 0.10 for concrete strength in compression); the bias factor for concrete strength was assumed to be the same as in previous study (Phase 1). The statistical parameters of fabrication factor were selected as follows: for an area of column cross section, λ = 1.005 and, V = 0.04, for a depth of the slab, λ = 1.03, 1.02 and 1.015, and V = 0.09, 0.06 and 0.04, for the slab thickness equal to 4 in, 6 in and 8 in respectively (based on test data published by Elingwood et al. 1980), for width of the foundation beam, λ = 1.005 and V = 0.04, and for the effective depth of the foundation beam, λ = 0.99 and V = 0.04.

Analysis of minimum shear force, V_c, provided by concrete only (without shear reinforcement) shown that in case of thin slabs (4 in) Eq.(3.1.b) governs; for thicker slabs (6 in or 8 in) Eq. (3.1.c) gives the smallest value of shear capacity, V_c. The same tendency was found for rectangular and square columns, and for interior, edge or corner columns. Following this analysis, Eqs. (3.1.b and 3.1.c) were used in Monte Carlo simulations to obtain statistical parameters for two-way shear resistance. Eq. (3.2) was used in Monte Carlo simulations to obtain statistical parameters for one-way shear resistance. The bias factors and coefficients of variation for shear resistance of slabs and foundation beams are presented in Table 3-1.

Table 3-1. Statistical Parameters of Shear Resistance

Structural Element		Ordinary Concrete (OC)		High Strength Concrete (HSC)		Ordinary Concrete (Old Statistical Data)	
	Slab thickness	Bias	COV	Bias	COV	Bias	COV
Two-way shear	For interior columns						
	4 in	1.29	0.25	1.19	0.25	1.07	0.24
	6 in	1.37	0.19	1.25	0.19	1.13	0.19
	8 in	1.40	0.18	1.29	0.18	1.16	0.18
	For edge columns						
	4 in	1.22	0.23	1.12	0.23	1.01	0.24
	6 in	1.19	0.19	1.08	0.19	0.98	0.19
	8 in	1.18	0.18	1.08	0.18	0.97	0.18
	For corner columns						
	4 in	1.22	0.23	1.12	0.23	1.01	0.24
	6 in	1.19	0.19	1.09	0.19	0.98	0.19
	8 in	1.18	0.18	1.08	0.18	0.97	0.18
One-way shear	Foundation beam depth						
	12 in	1.138	0.180	1.045	0.180	0.942	0.184
	36 in	1.140	0.180	1.046	0.180	0.943	0.180

For comparison, the statistical parameters of resistance were also calculated using "old" statistical information on concrete strength (used in the previous code calibration by Elingwood et al. 1980). These values are also presented in Table 3-1.

3.2. Reliability Indices for Shear in Slabs and Foundation Beams

The reliability analysis was performed for the limit states of shear capacity of slabs and foundation beams for cases where shear reinforcement is not provided, and the shear force is transferred by concrete section only. Load combinations and load factors with their statistical parameters were assumed to be the same as in the first part of the reliability analysis (Phase 1). The statistical parameters of resistance were calculated using Monte Carlo simulations and are presented in Section 3.1 of this report. Calculated reliability indices for three different resistance factors are shown in Table 3-2.

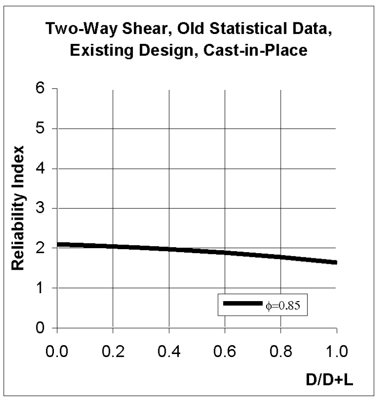
Table 3-2. Reliability Indices and Resistance Factors for Shear in Slabs and Foundation Beams

Structural Element		Reliability Indices (OC)			Reliability Indices (HSC)			Reliability Indices (Old Statistical Data)
	Slab thickness	φ =0.85	φ =0.80	φ =0.75	φ =0.85	φ =0.80	φ =0.75	φ =0.85
Two-way shear	For interior columns							
	4 in	1.93	2.05	2.18	1.76	1.89	2.02	1.85
	6 in	2.66	2.81	2.97	2.42	2.59	2.76	2.46
	8 in	2.87	3.03	3.19	2.64	2.81	2.98	2.66
	For edge columns							
	4 in	1.96	2.10	2.24	1.75	1.91	2.06	1.71
	6 in	2.28	2.45	2.63	2.01	2.20	2.39	2.04
	8 in	2.37	2.56	2.74	2.08	2.28	2.48	2.10
	For corner columns							
	4 in	1.96	2.10	2.24	1.75	1.91	2.06	1.71
	6 in	2.28	2.45	2.63	2.01	2.20	2.39	2.04
	8 in	2.37	2.56	2.74	2.08	2.28	2.48	2.10
One-way shear	Foundation beam depth							
	12 in	2.24	2.43	2.63	1.97	2.17	2.38	1.97
	36 in	2.26	2.45	2.64	1.97	2.18	2.38	2.01

Based on reliability analysis and results presented in Table 3-2, resistance factor equal to 0.75 is proposed for two-way shear and for one-way shear.

Examples of calculated reliability indices for shear in slabs and foundation beams are shown in Figures 3-1, 3-2, 3-3, 3-4 and 3-5.

Two-way shear							D + L,									
0% shear reinforcement				OC	Slab thickness = 4 in				Interior columns				beta			beta
				new	old	old	mR new data/design						new			old
D/D+L	D	L	S	Q	Q	mR	0.80	0.85	0.75	mQ	sQ	VQ	0.75	0.85	0.80	0.85
0.00	0.00	1.00	0.00	1.60	1.70	2.140	2.580	2.428	2.752	1.000	0.180	0.18	2.46	2.26	2.36	2.09
0.10	0.10	0.90	0.00	1.56	1.67	2.102	2.516	2.368	2.683	1.005	0.162	0.16	2.43	2.22	2.33	2.07
0.20	0.20	0.80	0.00	1.52	1.64	2.064	2.451	2.307	2.614	1.010	0.146	0.14	2.40	2.18	2.29	2.04
0.30	0.30	0.70	0.00	1.48	1.61	2.027	2.387	2.246	2.546	1.015	0.130	0.13	2.36	2.14	2.25	2.01
0.40	0.40	0.60	0.00	1.44	1.58	1.989	2.322	2.185	2.477	1.020	0.116	0.11	2.31	2.09	2.20	1.97
0.50	0.50	0.50	0.00	1.40	1.55	1.951	2.258	2.125	2.408	1.025	0.104	0.10	2.26	2.03	2.15	1.93
0.60	0.60	0.40	0.00	1.36	1.52	1.913	2.193	2.064	2.339	1.030	0.096	0.09	2.21	1.97	2.09	1.88
0.70	0.70	0.30	0.00	1.32	1.49	1.876	2.129	2.003	2.270	1.035	0.091	0.09	2.15	1.90	2.03	1.83
0.80	0.80	0.20	0.00	1.28	1.46	1.838	2.064	1.943	2.202	1.040	0.091	0.09	2.08	1.83	1.95	1.77
0.90	0.90	0.10	0.00	1.26	1.43	1.800	2.032	1.912	2.167	1.045	0.096	0.09	2.04	1.78	1.91	1.71
1.00	1.00	0.00	0.00	1.40	1.40	1.762	2.258	2.125	2.408	1.050	0.105	0.10	2.22	1.98	2.10	1.63
										average beta			2.18	1.93	2.05	1.85



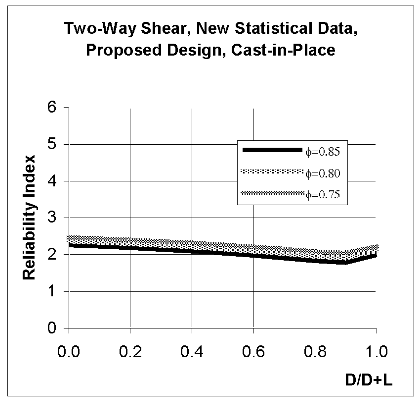
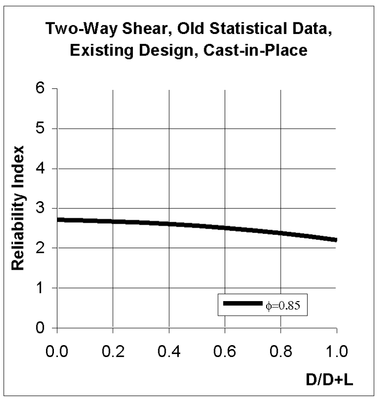


Fig.3-1. Reliability Indices Calculated for R/C Slab Made of Ordinary Concrete for D+L Load Combination (for slab thickness equal to 4 in and without shear reinforcement).

Two-way shear							D + L,									
0% shear reinforcement				OC	Slab thickness = 6 in				Interior columns				beta			beta
				new	old	old	mR new data/design						new			old
D/D+L	D	L	S	Q	Q	mR	0.80	0.85	0.75	mQ	sQ	VQ	0.75	0.85	0.80	0.85
0.00	0.00	1.00	0.00	1.60	1.70	2.260	2.730	2.569	2.912	1.000	0.180	0.18	3.29	3.02	3.15	2.71
0.10	0.10	0.90	0.00	1.56	1.67	2.220	2.662	2.505	2.839	1.005	0.162	0.16	3.26	2.98	3.12	2.69
0.20	0.20	0.80	0.00	1.52	1.64	2.180	2.594	2.441	2.766	1.010	0.146	0.14	3.22	2.94	3.08	2.67
0.30	0.30	0.70	0.00	1.48	1.61	2.140	2.525	2.377	2.694	1.015	0.130	0.13	3.18	2.90	3.04	2.64
0.40	0.40	0.60	0.00	1.44	1.58	2.100	2.457	2.312	2.621	1.020	0.116	0.11	3.13	2.84	2.99	2.60
0.50	0.50	0.50	0.00	1.40	1.55	2.061	2.389	2.248	2.548	1.025	0.104	0.10	3.08	2.78	2.93	2.56
0.60	0.60	0.40	0.00	1.36	1.52	2.021	2.321	2.184	2.475	1.030	0.096	0.09	3.01	2.71	2.86	2.50
0.70	0.70	0.30	0.00	1.32	1.49	1.981	2.252	2.120	2.402	1.035	0.091	0.09	2.94	2.63	2.78	2.44
0.80	0.80	0.20	0.00	1.28	1.46	1.941	2.184	2.056	2.330	1.040	0.091	0.09	2.85	2.53	2.69	2.37
0.90	0.90	0.10	0.00	1.26	1.43	1.901	2.150	2.023	2.293	1.045	0.096	0.09	2.80	2.47	2.63	2.29
1.00	1.00	0.00	0.00	1.40	1.40	1.861	2.389	2.248	2.548	1.050	0.105	0.10	3.02	2.72	2.87	2.20
										average beta			2.97	2.66	2.81	2.46



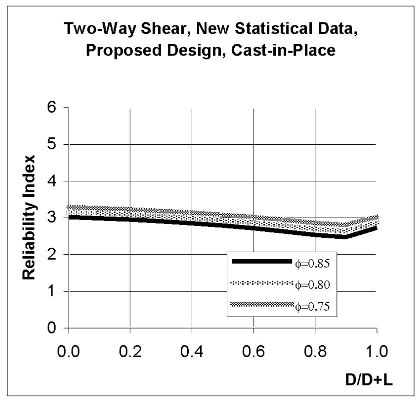


Fig. 3-2. Reliability Indices for R/C Slab Made of Ordinary Concrete for D+L Load Combination (for slab thickness equal to 6 in and without shear reinforcement).

Two-way shear							D + L,									
0% shear reinforcement				OC	Slab thickness = 8 in				Interior columns				beta			beta
				new	old	old	mR new data/design						new			old
D/D+L	D	L	S	Q	Q	mR	0.80	0.85	0.75	mQ	sQ	VQ	0.75	0.85	0.80	0.85
0.00	0.00	1.00	0.00	1.60	1.70	2.320	2.800	2.635	2.987	1.000	0.180	0.18	3.50	3.22	3.36	2.90
0.10	0.10	0.90	0.00	1.56	1.67	2.279	2.730	2.569	2.912	1.005	0.162	0.16	3.48	3.19	3.33	2.89
0.20	0.20	0.80	0.00	1.52	1.64	2.238	2.660	2.504	2.837	1.010	0.146	0.14	3.44	3.15	3.30	2.87
0.30	0.30	0.70	0.00	1.48	1.61	2.197	2.590	2.438	2.763	1.015	0.130	0.13	3.40	3.11	3.25	2.84
0.40	0.40	0.60	0.00	1.44	1.58	2.156	2.520	2.372	2.688	1.020	0.116	0.11	3.35	3.06	3.20	2.81
0.50	0.50	0.50	0.00	1.40	1.55	2.115	2.450	2.306	2.613	1.025	0.104	0.10	3.30	2.99	3.14	2.76
0.60	0.60	0.40	0.00	1.36	1.52	2.074	2.380	2.240	2.539	1.030	0.096	0.09	3.23	2.92	3.08	2.71
0.70	0.70	0.30	0.00	1.32	1.49	2.033	2.310	2.174	2.464	1.035	0.091	0.09	3.16	2.83	3.00	2.65
0.80	0.80	0.20	0.00	1.28	1.46	1.992	2.240	2.108	2.389	1.040	0.091	0.09	3.07	2.74	2.90	2.57
0.90	0.90	0.10	0.00	1.26	1.43	1.952	2.205	2.075	2.352	1.045	0.096	0.09	3.01	2.67	2.84	2.49
1.00	1.00	0.00	0.00	1.40	1.40	1.911	2.450	2.306	2.613	1.050	0.105	0.10	3.24	2.93	3.09	2.39
										average beta			3.19	2.87	3.03	2.66

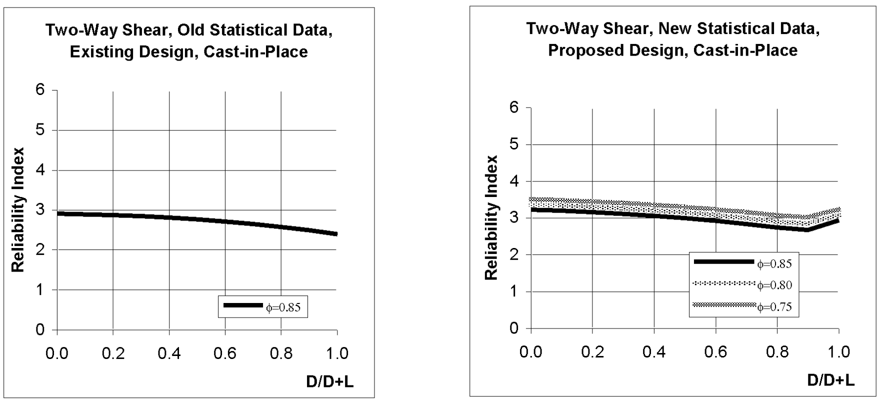
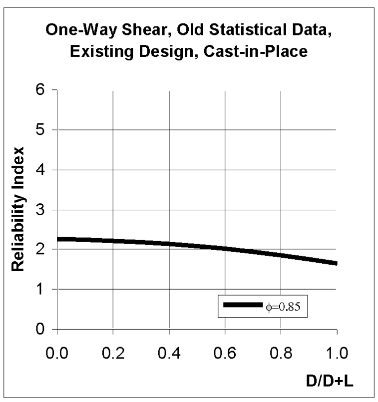


Fig. 3-3. Reliability Indices for R/C Slab Made of Ordinary Concrete for D+L Load Combination (for slab thickness equal to 8 in and without shear reinforcement).

One-way shear							**D + L,**									
0% shear reinforcement				OC	Foundation depth = 12 in								beta			beta
				new	old	old	mR new data/design						new			old
D/D+L	D	L	S	Q	Q	mR	0.80	0.85	0.75	mQ	sQ	VQ	0.75	0.85	0.80	0.85
0.00	0.00	1.00	0.00	1.60	1.70	1.884	2.276	2.142	2.428	1.000	0.180	0.18	3.01	2.67	2.84	2.26
0.10	0.10	0.90	0.00	1.56	1.67	1.851	2.219	2.089	2.367	1.005	0.162	0.16	2.97	2.63	2.80	2.24
0.20	0.20	0.80	0.00	1.52	1.64	1.818	2.162	2.035	2.306	1.010	0.146	0.14	2.93	2.59	2.76	2.21
0.30	0.30	0.70	0.00	1.48	1.61	1.784	2.105	1.981	2.246	1.015	0.130	0.13	2.88	2.53	2.71	2.18
0.40	0.40	0.60	0.00	1.44	1.58	1.751	2.048	1.928	2.185	1.020	0.116	0.11	2.83	2.47	2.65	2.14
0.50	0.50	0.50	0.00	1.40	1.55	1.718	1.992	1.874	2.124	1.025	0.104	0.10	2.76	2.39	2.58	2.08
0.60	0.60	0.40	0.00	1.36	1.52	1.685	1.935	1.821	2.064	1.030	0.096	0.09	2.68	2.30	2.49	2.02
0.70	0.70	0.30	0.00	1.32	1.49	1.651	1.878	1.767	2.003	1.035	0.091	0.09	2.59	2.20	2.39	1.94
0.80	0.80	0.20	0.00	1.28	1.46	1.618	1.821	1.714	1.942	1.040	0.091	0.09	2.48	2.08	2.28	1.86
0.90	0.90	0.10	0.00	1.26	1.43	1.585	1.792	1.687	1.912	1.045	0.096	0.09	2.41	2.01	2.21	1.76
1.00	1.00	0.00	0.00	1.40	1.40	1.552	1.992	1.874	2.124	1.050	0.105	0.10	2.70	2.32	2.51	1.65
										average beta			2.63	2.24	2.43	1.97



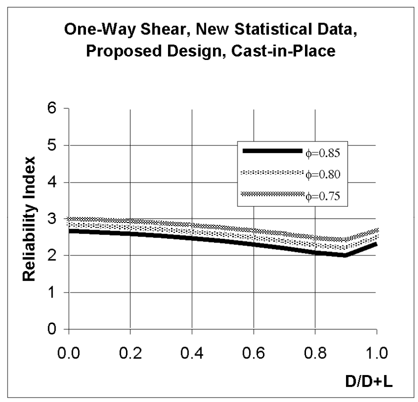
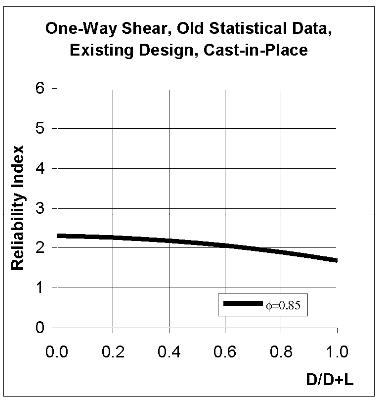


Fig. 3-4. Reliability Indices for R/C Foundation Beam Made of Ordinary Concrete for D+L Load Combination (for foundation depth equal to 12 in and without shear reinforcement).

One-way shear							D + L,									
0% shear reinforcement				OC	Foundation depth = 36 in								beta			beta
				new	old	old	mR new data/design						new			old
D/D+L	D	L	S	Q	Q	mR	0.80	0.85	0.75	mQ	sQ	VQ	0.75	0.85	0.80	0.85
0.00	0.00	1.00	0.00	1.60	1.70	1.886	2.280	2.146	2.432	1.000	0.180	0.18	3.03	2.69	2.86	2.31
0.10	0.10	0.90	0.00	1.56	1.67	1.853	2.223	2.092	2.371	1.005	0.162	0.16	2.99	2.65	2.82	2.29
0.20	0.20	0.80	0.00	1.52	1.64	1.819	2.166	2.039	2.310	1.010	0.146	0.14	2.95	2.61	2.78	2.26
0.30	0.30	0.70	0.00	1.48	1.61	1.786	2.109	1.985	2.250	1.015	0.130	0.13	2.90	2.55	2.73	2.22
0.40	0.40	0.60	0.00	1.44	1.58	1.753	2.052	1.931	2.189	1.020	0.116	0.11	2.85	2.49	2.67	2.18
0.50	0.50	0.50	0.00	1.40	1.55	1.720	1.995	1.878	2.128	1.025	0.104	0.10	2.78	2.41	2.59	2.13
0.60	0.60	0.40	0.00	1.36	1.52	1.686	1.938	1.824	2.067	1.030	0.096	0.09	2.70	2.32	2.51	2.06
0.70	0.70	0.30	0.00	1.32	1.49	1.653	1.881	1.770	2.006	1.035	0.091	0.09	2.61	2.22	2.41	1.99
0.80	0.80	0.20	0.00	1.28	1.46	1.620	1.824	1.717	1.946	1.040	0.091	0.09	2.50	2.10	2.30	1.90
0.90	0.90	0.10	0.00	1.26	1.43	1.586	1.796	1.690	1.915	1.045	0.096	0.09	2.43	2.02	2.23	1.80
1.00	1.00	0.00	0.00	1.40	1.40	1.553	1.995	1.878	2.128	1.050	0.105	0.10	2.71	2.34	2.53	1.68
										average beta			2.64	2.26	2.45	2.01



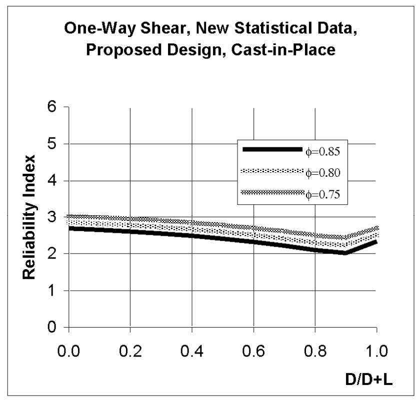


Fig. 3-5. Reliability Indices for R/C Foundation Beam Made of Ordinary Concrete for D+L Load Combination (for foundation depth equal to 36 in and without shear reinforcement).

4. Proposed Resistance Factors

The reliability indices calculated for different design cases of eccentrically loaded columns and shear capacity of slabs and foundation beams were compared to the target values. Based on that analysis, the recommended values of resistance factors are given in Table 4.1 for columns, slabs and foundation beams.

Table 4-1. Recommended resistance factors for eccentrically loaded columns, slabs and foundation beams

Structural Element and Limit State	Recommended Resistance Factor
Eccentrically Loaded Columns, Tied	0.70
Eccentrically Loaded Columns, Spiral	0.75
Slabs, Two-Way Shear	0.75
Foundation Beams, One-Way Shear	0.75

5. Conclusions and Recommendations

The reliability-based calibration was performed for eccentrically loaded columns and shear in slabs and foundation beams. This work is the continuation of a study presented in the Phase 1 of the report. The objective was to calculate resistance factors for the design of concrete structures corresponding to load and load combination factors specified by the ASCE 7-98 Standard.

Statistical parameters for materials are the same as in Phase 1 of the project with the exception of the coefficient of variation for concrete modulus of rupture.

In general, the reliability indices calculated for eccentrically loaded columns are slightly lower than β's for axially loaded columns presented in Phase 1 of the report. Assuming the same target value of the reliability index and resistance factor for eccentrically loaded columns and axially loaded columns, $\beta_T = 4.0$, the recommended resistance factor for columns is 0.75 for spiral columns and 0.70 for tied columns. In the result, the axially loaded columns can have a higher reliability index than eccentrically loaded columns. However, this is reasonable because in most design cases, columns are eccentrically loaded. The same value of resistance factor for axially and eccentrically loaded columns will simplify the code and it will be convenient for the designers.

Based on the reliability analysis performed for design cases with shear force resisted by concrete section only, the recommended resistance factor for shear, is 0.75. This resistance factor is recommended for one-way and two-way shear.

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