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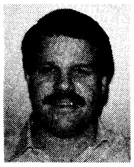
Shrinkage Cracking in Fully Restrained Concrete Members



by R. Ian Gilbert

This paper considers the problem of cracking in fully restrained members subjected to direct tension caused by dry shrinkage. The mechanism of direct tension cracking is discussed, and some popular misconceptions concerning the behavior of restrained members are exposed. Paper presents a rational approach for the determination of the number and spacing of cracks and the average crack width in a member, which is fully restrained and subjected only to an axial restraining force caused by shrinkage. The approach is based on the principles of mechanics and is illustrated by worked examples. Predictions agree well with observed cracking in restrained members. The procedure is used to calculate the quantities of steel required for crack control in a number of practical situations. Finally, the results of the investigation are compared with the provisions for shrinkage and temperature reinforcement in the ACI Building Code (ACI 318-89) and AS 3600-1988.

Keywords: cracking (fracturing); crack width and spacing; creep; **deformation; reinforced concrete;** serviceability; **shrinkage;** slabs; **structural members.**

In reinforced and partially prestressed concrete structures, cracking is to be expected at service loads. Cracks may be caused by the external loads, or by restraint to shrinkage and temperature variations, or by a variety of other causes. If uncontrolled, cracking may spoil the appearance of a structure or otherwise adversely affect its performance.

Excessively wide cracks in floor systems and walls may often be avoided by the inclusion of strategically placed contraction (or control) joints, thereby removing some of the restraint to shrinkage and temperature movements and reducing the internal tension. When cracking does occur, to insure that crack widths remain acceptably small, adequate quantities of well-anchored reinforcement must be included at every location in the structure where significant tension is expected.

The maximum crack width, which may be considered to be acceptable in a given situation, depends on the type of structure, the environment, and the consequences of excessive cracking. In corrosive and aggressive environments, some building codes and specifications recommend that crack widths do not exceed about 0.3 mm (0.012 in.). For members with one or more exposed surfaces, a maximum crack width of 0.3 mm should also provide visual acceptability. For the sheltered interior of most buildings where the concrete is not exposed and esthetic requirements are of secondary importance, a larger crack width may be acceptable (say 0.3 to 0.5 mm).

Most existing techniques for predicting crack widths involve empirical models that have been calibrated from results obtained in laboratory tests and, as such, often fail to accurately predict crack widths in actual structures. It is difficult, therefore, for a structural designer to be confident that the cracks in a structure will in fact satisfy the maximum crack width requirements of the local building code.

In beams and slabs, which have been proportioned to avoid excessive deflection at service loads and which contain sufficient quantities of reinforcement to provide adequate ultimate strength and ductility, flexural crack widths are rarely a problem under normal in-service conditions, provided of course that the reinforcement bars are not spaced too widely apart. Sensible reinforcement detailing is the key to flexural crack control. It is, therefore, not usually critical if flexural crack widths are not specifically checked in design.

When flexural members are also restrained at the supports, shrinkage causes a buildup of tension in the member, in addition to the bending caused by the external loads. Crack control is still not usually a problem, since shrinkage is accommodated by small increases in the widths of the numerous flexural cracks. However, for members not subjected to significant bending in which restraint is provided to the longitudinal movement caused by shrinkage and temperature changes, cracks tend to propagate over the full depth of the section. Exessively wide cracks are not uncommon. Such cracks are commonly called direct tension cracks, since they are caused by direct tension rather than by flexural tension. In fully restrained direct tension members, relatively large amounts of reinforcement are required to control load-independent cracking. The

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minimum quantities of reinforcement specified in the Australian Code AS 3600-1988[1] for crack control in restrained members are much greater than that required in the ACI Building Code (metric version) ACI 318M-89[2] and are based on Australian research by Base and Murray[3,4] and others.

In this paper, the mechanism of direct tension cracking is discussed, and a rational approach is presented for the determination of the average crack width in a member that is fully restrained and subjected only to an axial restraining force caused by shrinkage. The predicted number and width of shrinkage cracks agree well with observed cracking in restrained members. The approach is used to calculate the quantities of steel required for crack control in restrained members for a variety of situations. Finally, the results of the investigation are compared with the provisions for crack control in AS 3600-1988[1] and ACI 318M-89.[2]

CRACKING IN DIRECT TENSION MEMBERS

Consider a reinforced concrete direct tension member that is prevented from shortening by its supports or by adjacent parts of the structure. As the concrete shrinks, an axial tensile restraining force $N(t)$ develops with time. When the concrete stress caused by $N(t)$ at a

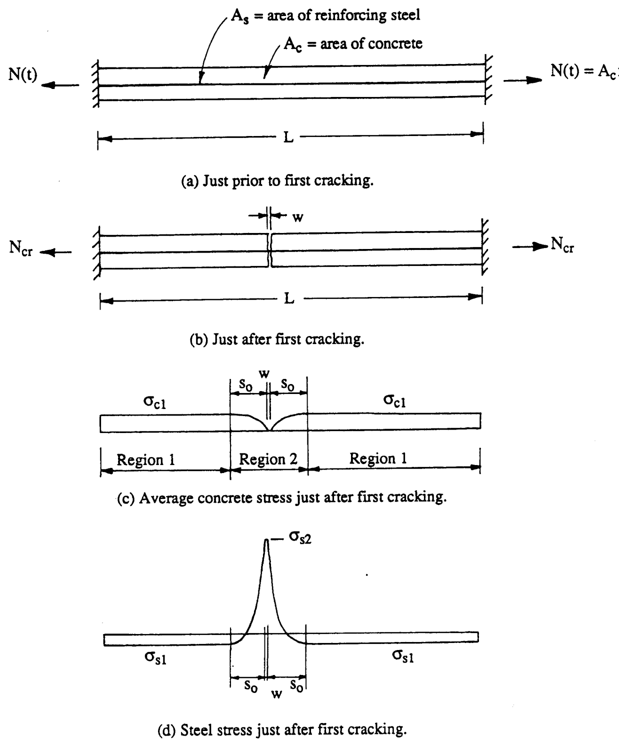


Fig. 1 — First cracking in a restrained direct tension member

particular cross section first reaches the direct (uniaxial) tensile strength of concrete f_t, full-depth direct tension cracking occurs. First cracking may occur less than 1 week after the commencement of drying. The magnitude of $N(t)$ after cracking and the crack width depend primarily on the amount of bonded reinforcement crossing the crack. If the member contains no longitudinal steel, cracking causes the restraining force $N(t)$ to drop to zero and a wide, unsightly crack results. If the member contains only small quantities of reinforcement ($\rho = A_s/bh$ is less than about 0.003 for $f_y = 400$ MPa), the steel at the crack yields (either immediately or after additional shrinkage), the crack opens widely, and the restraining force drops to a value of $A_s f_y$, which may be only a small fraction of its value prior to cracking. If the member contains relatively large quantities of reinforcement (ρ greater than about 0.01 for $f_y = 400$ MPa), the steel at each crack does not yield, the crack width remains small, and because the loss of member stiffness at cracking is not great, the restraining force remains high. Members containing large quanities of steel are therefore likely eventually to suffer many cracks, but each will be fine and well controlled. For intermediate steel quanities ($0.003 < \rho < 0.01$ when $f_y = 400$ MPa), cracking causes a loss of stiffness, a reduction of $N(t)$, and a crack width that may or may not be acceptable.

In addition to the quantity of steel, the width of a crack in a restrained member depends on the quality of bond between the concrete and the steel, the size and distribution of the individual reinforcement bars, the concrete quality, and whether or not the axial restraining force is accompanied by bending. Direct tension cracks are more parallel-sided than flexural cracks, and hence the observed crack width is less dependent on the amount of concrete cover.

FIRST CRACKING

Description and notation

Consider the fully restrained member shown in Fig. 1(a). As the concrete shrinks, the restraining force $N(t)$ gradually increases until the first crack occurs when $N(t) = A_c f_t$. Immediately after first cracking, the restraining force reduces to N_{cr}, and the concrete stress away from the crack is less than the tensile strength of the concrete f_t. The concrete on either side of the crack shortens elastically and the crack opens to a width w, as shown in Fig. 1(b). At the crack, the steel carries the entire force N_{cr}, and the stress in the concrete is obviously zero. In the region immediately adjacent to the crack, the concrete and steel stresses vary considerably, and there exists a region of partial bond breakdown. At some distance s_o on each side of the crack, the concrete and steel stresses are no longer influenced directly by the presence of the crack, as shown in Fig. 1(c) and (d).

In Region 1, where the distance x from the crack is greater than or equal to s_o, the concrete and steel stresses are σ_{c1} and σ_{s1}, respectively. Since the steel stress (and hence strain) at the crack is tensile as shown and the overall elongation of the steel is zero (full restraint), σ_{s1} must be compressive. Equilibrium requires that the sum of the forces carried by the concrete and the steel on any cross section is equal to the restraining force. Therefore, with the force in the steel in Region 1 being compressive, the force carried by the concrete ($\sigma_{c1} A_c$) must be tensile and somewhat greater than the restraining force (N_{cr}). In Region 2, where the distance x from the nearest crack is less than s_o, the concrete stress varies from zero at the crack to σ_{c1} at $x = s_o$. The steel stress varies from σ_{s2} (tensile) at the crack to σ_{s1} (compressive) at $x = s_o$, as shown.

To determine the crack width w and the concrete and steel stresses in Fig. 1, the distance s_o, over which the concrete and steel stresses vary, needs to be known and the restraining force N_{cr} needs to be calculated. An approximation for s_o may be obtained using the following equation, which was proposed by Favre et al.[5] for a member containing deformed bars or welded wire mesh

$$s_o = \frac{d_b}{10\,\rho} \tag{1}$$

where d_b is the bar diameter, and ρ is the reinforcement ratio A_s/A_c. Base and Murray[4] used a similar expression.

Calculation of restraining force and internal stresses

The following procedure may be used to determine the restraining force N_{cr} immediately after first cracking and the corresponding steel and concrete stresses.

The overall elongation of the steel is zero, since the member is fully restrained and therefore prevented from shortening. Integrating the steel strain over the length of the member [and assuming a parabolic variation of stress and, hence, strain in Region 2, as shown in Fig. 1(d)] gives

$$\frac{\sigma_{s1}}{E_s} L + \frac{\sigma_{s2} - \sigma_{s1}}{E_s}\left(\frac{2}{3} s_o + w\right) = 0 \tag{2}$$

Realizing that w is very much less than s_o, Eq. (2) may be rearranged to give

$$\sigma_{s1} = \frac{-2\,s_o}{3\,L - 2\,s_o}\,\sigma_{s2} \tag{3}$$

At the crack, the restraining force N_{cr} is carried entirely by the steel. That is

$$\sigma_{s2} = \frac{N_{cr}}{A_s} \tag{4}$$

By substituting Eq. (4) into Eq. (3), the steel stress away from the crack is expressed in terms of the unknown restraining force

$$\sigma_{s1} = \frac{2\,s_o}{3\,L - 2\,s_o}\,\frac{N_{cr}}{A_s} = -\,C_1\,\frac{N_{cr}}{A_s} \tag{5}$$

where

$$C_1 = \frac{2\,s_o}{3\,L - 2\,s_o} \tag{6}$$

Prior to cracking, the total concrete strain at any point is zero, since shortening is prevented. Although the total concrete strain is zero, the individual strain components of creep, shrinkage, and elastic strain are not. The creep and elastic strains are tensile (positive) and the shrinkage strain is compressive (negative). The sum of the time-dependent creep and shrinkage strain components must be equal and opposite to the elastic strain component. Immediately before the first crack occurs when the concrete stress just reaches f_t, the sum of the creep and shrinkage strain components is therefore

$$\epsilon_c + \epsilon_{sh} = -\frac{f_t}{E_c} \tag{7}$$

where E_c is the elastic modulus of the concrete at the time of first cracking. Immediately after first cracking, the magnitude of the elastic component of strain in the uncracked concrete decreases (as the concrete stress decreases), but the creep and shrinkage strain components are unaltered. Creep and shrinkage strains only change gradually with time.

In Region 1, at any distance greater than s_o from the crack, equilibrium requires that the sum of the forces in the concrete and the steel immediately after first cracking is equal to N_{cr}. That is

$$\sigma_{c1} A_c + \sigma_{s1} A_s = N_{cr} \tag{8}$$

and substituting Eq. (5) into Eq. (8) and rearranging gives

$$\sigma_{c1} = \frac{N_{cr} - \sigma_{s1}\,A_s}{A_c} = \frac{N_{cr}\,(1 + C_1)}{A_c} \tag{9}$$

The compatibility requirement is that the concrete and steel strains in Region 1 are identical, i.e.

$$\epsilon_{s1} = \epsilon_1 \tag{10}$$

With the concrete strain ϵ_1 equal to the sum of the elastic, creep, and shrinkage strain components, Eq. (10) can be reexpressed as

$$\frac{\sigma_{s1}}{E_s} = \frac{\sigma_{c1}}{E_c} + \epsilon_c + \epsilon_{sh} \tag{11}$$

Substituting Eq. (5), (7), and (9) into Eq. (11) and solving for N_{cr} gives

$$N_{cr} = \frac{n\,\rho\,f_t\,A_c}{C_1 + n\,\rho\,(1 + C_1)} \tag{12}$$

where $\rho = A_s/A_c$ and $n = E_s/E_c$.

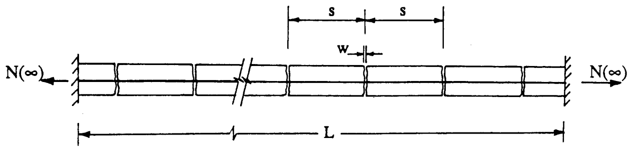
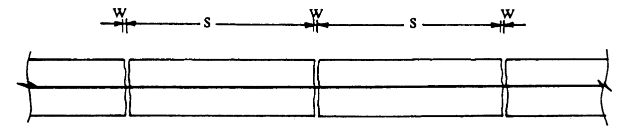
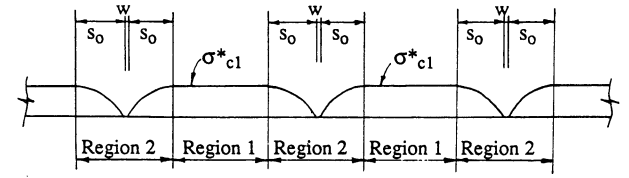


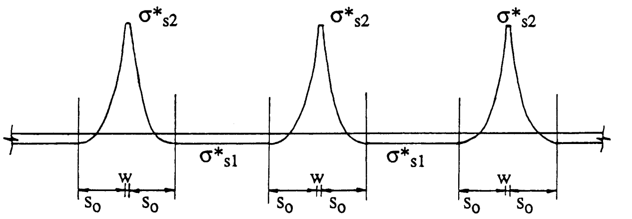
Fig. 2 — Final crack pattern in a restrained direct tension member



(a)Portion of a restrained member after all cracking.



(b) Average concrete stress after all shrinkage.



(c) Steel stress after all shrinkage cracking.

Fig. 3 — Final concrete and steel stresses after direct tension cracking

When N_{cr} is calculated from Eq. (12), the concrete and steel stresses immediately after cracking may be obtained from Eq. (4), (5), and (9).

DETERMINATION OF FINAL CRACK SPACING AND CRACK WIDTH

Discussion and notation

With the stresses and deformations determined immediately after first cracking, the subsequent long-term behavior as shrinkage continues must next be determined. Consider again the fully restrained member shown in Fig. 1(a) and (b). After first cracking, the concrete is no longer fully restrained since the crack width can increase with time as shrinkage continues. A state of parital restraint therefore exists after first cracking. Subsequent shrinkage will cause further gradual increases in the restraining force $N(t)$ and in the concrete stress away from the crack, and a second crack may develop. Additional cracks may occur as the shrinkage strain continues to increase with time. However, as each new crack forms, the member becomes less stiff and the amount of shrinkage required to produce each new crack increases. The process continues until the crack pattern is established, usually in the first few months after the commencement of drying. The number of cracks and the final average crack width depend on the length of the member, the quantity and distribution of reinforcement, the quality of bond between the concrete and steel, the amount of shrinkage, and the concrete strength. A typical final shrinkage crack pattern is shown in Fig. 2. Let the number of cracks be m and the final shrinkage-induced restraining force be $N(\infty)$.

In Fig. 3(a), a portion of a fully restrained direct tension member is shown after all shrinkage has taken place and the final crack pattern is established. The average concrete and steel stresses caused by shrinkage are illustrated in Fig. 3(b) and (c). It is assumed that the distance s_o in which the concrete and steel stresses vary on either side of each crack is the same as the distance s_o given by Eq. (1) and previously used in the analysis at first cracking. In Region 1, where the distance x from the nearest crack is greater than or equal to s_o, the final concrete and steel stresses are σ_{c1}^* and σ_{s1}^*, respectively.

Calculation of final stresses and deformation

The following analysis may be used to determine the number of cracks m, the final restraining force $N(\infty)$, and the final average crack width w.

For the member containing m cracks, the following expressions [similar to Eq. (2) and (3)] are obtained by equating the overall elongation of the steel to zero

$$\frac{\sigma_{s1}^*}{E_s} L + m \frac{\sigma_{s2}^* - \sigma_{s1}^*}{E_s} \left(\frac{2}{3} s_o + w \right) = 0 \tag{13}$$

and rearranging gives

$$\sigma_{s1}^* = \frac{-2 s_o m}{3L - 2 s_o m} \sigma_{s2}^* \tag{14}$$

since w is very much less than s_o. Dividing both the numerator and the denominator on the right-hand side of Eq. (14) by m and letting the crack spacing $s = L/m$, gives

$$\sigma_{s1}^* = \frac{-2 s_o}{3s - 2 s_o} \sigma_{s2}^* = -C_2 \sigma_{s2}^* \tag{15}$$

where

$$C_2 = \frac{2 s_o}{3s - 2 s_o} \tag{16}$$

At each crack

$$\sigma_{s2}^* = \frac{N(\infty)}{A_s} \tag{17}$$

In Region 1, away from each crack, a typical concrete stress history is shown diagrammatically in Fig. 4. The concrete tensile stress increases gradually with time and approaches the direct tensile strength of the concrete f_t. When cracking occurs elsewhere in the member, the tensile stress in the uncracked regions drops suddenly as shown. Although the concrete stress history is continuously changing, for the estimation of creep strain it is not unreasonable to assume that the average concrete stress at any time after the commencement of drying σ_{av} is somewhere between σ_{c1} and f_t, as shown in Fig. 4, and the final creep strain in Region 1 may be approximated by

$$\epsilon_c^* = \frac{\sigma_{av}}{E_c} \phi^* \tag{18}$$

where ϕ^* is the final creep coefficient (defined as the ratio of the final creep strain to elastic strain under the average sustained stress σ_{av}).

In this study, it is assumed that

$$\sigma_{av} = \frac{\sigma_{c1} + f_t}{2} \tag{19}$$

The final concrete strain in Region 1 is the sum of the elastic, creep, and shrinkage components and may be approximated by

$$\begin{aligned} \epsilon_1^* &= \epsilon_e + \epsilon_c^* + \epsilon_{sh}^* \\ &= \frac{\sigma_{av}}{E_c} + \frac{\sigma_{av}}{E_c} \phi^* + \epsilon_{sh}^* \end{aligned} \tag{20}$$

The magnitude of the final creep coefficient ϕ^* is usually between 2 and 4, depending on the age at the commencement of drying and the quality of the concrete. ϵ_{sh}^* is the final shrinkage strain and depends on the relative humidity, the size and shape of the member, and the characteristics of the concrete mix. Numerical estimates of ϕ^* and ϵ_{sh}^* can be obtained from ACI 209[6] and elsewhere. A number of the more well-known methods for predicting both ϕ^* and ϵ_{sh}^* have been presented and compared previously by the author.

Eq. (20) may be expressed as

$$\epsilon_1^* = \frac{\sigma_{av}}{E_e^*} + \epsilon_{sh}^* \tag{21}$$

where E_e^* is final effective modulus for concrete and is given by

$$E_e^* = \frac{E_c}{1 + \phi^*} \tag{22}$$

In Region 1, at any distance from a crack greater than s_o, equilibrium requires that the sum of the force in the concrete and the force in the steel is equal to $N(\infty)$. That is

$$\sigma_{c1}^* A_c + \sigma_{s1}^* A_s = N(\infty)$$

$$\text{or} \quad \sigma_{c1}^* = \frac{N(\infty) - \sigma_{s1}^* A_s}{A_c} \tag{23}$$

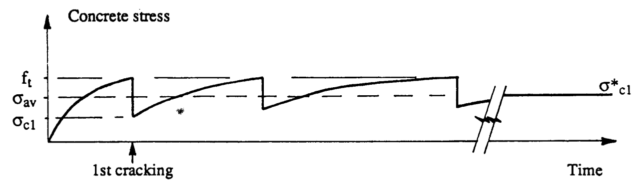


Fig. 4 — Concrete stress history in uncracked Region 1

The compatibility requirement is that the concrete and steel strains are identical

$$\epsilon_{s1}^* = \epsilon_1^*$$

and using Eq. (21)

$$\frac{\sigma_{s1}^*}{E_s} = \frac{\sigma_{av}}{E_c^*} + \epsilon_{sh}^* \tag{24}$$

Substituting Eq. (15) and (17) into Eq. (24) and rearranging gives

$$N(\infty) = -\frac{n^* A_s}{C_2} (\sigma_{av} + \epsilon_{sh}^* E_e^*) \tag{25}$$

where $n^* = E_s/E_e^*$.

The crack spacing s $(= L/m)$ must be first determined to calculate C_2 [as defined in Eq. (16)]. Knowing that σ_{c1}^* must be less than the tensile strength of concrete f_t and making use of Eq. (15) and (17), Eq. (23) becomes

$$\sigma_{c1}^* = \frac{N(\infty)\,(1 + C_2)}{A_c} \leqslant f_t \tag{26}$$

Substituting Eq. (16) and (25) into Eq. (26) gives

$$s \leqslant \frac{2\, s_o\, (1 + \xi)}{3\xi} \tag{27}$$

where

$$\xi = \frac{-\, n^* \rho\, (\sigma_{av} + \epsilon_{sh}^* E_e^*)}{n^* \rho\, (\sigma_{av} + \epsilon_{sh}^* E_e^*) + f_t} \tag{28}$$

The number of cracks m $(= L/s)$ may be taken as the smallest integer that causes Eq. (27) to be satisfied. With m thus determined, the restraining force $N(\infty)$ can be calculated using Eq. (25) and the steel and concrete stresses in the various regions of the member may be determined from Eq. (15), 17, and (23).

The overall shortening of the concrete is an estimate of the sum of the crack widths. The final concrete strain at any point in Region 1 of Fig. 3 is given by Eq. (21), and in Region 2, the final concrete strain is

$$\epsilon_2^* = \frac{fn\, \sigma_{c1}^*}{E_e^*} + \epsilon_{sh}^* \tag{29}$$

where fn varies beween zero at a crack and unity at s_o from a crack. If a parabolic variation of stress is assumed in Region 2, the following expression for the average crack width w is obtained by integrating the concrete strain over the length of the member

$$w = -\left[\frac{\sigma_{c1}^*}{E_e^*}\left(s - \frac{2}{3}s_o\right) + \epsilon_{sh}^* s\right] \tag{30}$$

The preceding analysis may be used to determine the number and width of shrinkage cracks, provided the assumption of linear-elastic behavior in the steel is valid. However, if the area of steel A_s is small, yielding may occur at each crack and the value of $N(\infty)$, calculated from Eq. (12), will not be correct. In such a case, ∞_2^* is equal to the yield stress of the reinforcement f_y and $N(\infty)$ is equal to $f_y A_s$. From Eq. (23), the stress in the concrete away from the crack in Region 1 is now

$$\sigma_{c1}^* = \frac{f_y A_s - \sigma_{s1}^* A_s}{A_c} \tag{31}$$

After the steel at the first crack yields, the tensile concrete stress σ_{c1} increases only slightly as the compressive steel stress σ_{s1} increases with time and the first crack opens. Since the restraining force is constant at all times after yielding of the steel at the first crack (and equal to $f_y A_s$), the tensile stress in the concrete never again approaches the tensile strength of concrete, and subsequent cracking does not occur. The width of the initial crack is usually unacceptably large as the steel at the crack deforms plastically. The crack width w may be found by insuring that the overall elongation of the steel is zero. That is

$$\frac{\sigma_{s1}^*}{E_s}(L - w) + \frac{f_y - \sigma_{s1}^*}{E_s}\frac{2}{3}s_o + w = 0 \tag{32}$$

and since w is very much smaller than L, Eq. (32) can be arranged to give

$$w = -\frac{\sigma_{s1}^*(3L - 2s_o) + 2s_o f_y}{3E_s} \tag{33}$$

Since the tensile stress in the uncracked concrete does not change significantly with time, it is reasonable to assume that the average concrete stress σ_{av} is given by Eq. (31) and the final steel stress in Region 1 may be obtained by substituting Eq. (31) into Eq. (21) and simplifying

$$\frac{\sigma_{s1}^*}{E_s} = \frac{f_{sy} A_s - \sigma_{s1}^* A_s}{A_c E_e^*} + \epsilon_{sh}^*$$

$$\therefore \quad \sigma_{s1}^* = \frac{n^* \rho f_{sy} + \epsilon_{sh}^* E_s}{1 + n^* \rho} \tag{34}$$

NUMERICAL EXAMPLES

Case (a)

Consider a 5-m long and 150-mm thick reinforced concrete slab which is fully restrained at each end. The slab contains 12-mm diameter deformed longitudinal bars at 300-mm centers in both the top and bottom of the slab (A_s = 750 mm²/m). The concrete cover to the reinforcement is 30 mm. Estimate the spacing s and final average width w of the restrained shrinkage cracks. Take

$$\phi^* = 2.5;\ \epsilon_{sh}^* = -600 \times 10^{-6};\ f_t = 2.0 \text{ MPa};$$

$$E_c = 25{,}000 \text{ MPa};\ E_s = 200{,}000 \text{ MPa};$$
$$n = 8; \text{ and } f_y = 400 \text{ MPa}.$$

The concrete area and reinforcement ratio are

$$A_c \doteq A_{gross} = 150{,}000 \text{ mm}^2/\text{m};\ \rho = \frac{A_s}{A_c} = 0.005$$

and from Eq. (1)

$$s_o = \frac{12}{10 \times 0.005} = 240 \text{ mm}$$

The final effective modulus is obtained from Eq. (22)

$$E_e^* = \frac{25{,}000}{1 + 2.5} = 7143 \text{ MPa}$$

and the corresponding effective modular ratio is $n^* = E_s/E_e^* = 28$. Eq. (6) gives

$$C_1 = \frac{2 \times 240}{3 \times 5000 - 2 \times 240} = 0.0331$$

and from Eq. (12), the restraining force immediately after first cracking is

$$N_{cr} = \frac{8 \times 0.005 \times 2.0 \times 150{,}000}{0.0331 + 8 \times 0.005\,(1 + 0.0331)}$$
$$= 161{,}300 \text{ N/m}$$

The concrete stress σ_{c1} is obtained from Eq. (9)

$$\sigma_{c1} = \frac{161{,}300\,(1 + 0.0331)}{150{,}000} = 1.11 \text{ MPa}$$

and from Eq. (19), the average concrete stress may be approximated by

$$\sigma_{av} = \frac{1.11 + 2.0}{2} = 1.56 \text{ MPa}$$

Eq. (28) gives

$$\xi = \frac{-28 \times 0.005\,(1.56 - 0.0006 \times 7143)}{28 \times 0.005\,(1.56 - 0.0006 \times 7143) + 2.0}$$
$$= 0.236$$

and from Eq. (27), the crack spacing must satisfy

$$s \leqslant \frac{2 \times 240\,(1 + 0.236)}{3 \times 0.236} = 839 \text{ mm}$$

The minimum number of cracks m is obtained from

$$m = \frac{L}{s} \geqslant \frac{5000}{839} = 5.96$$

Therefore $m = 6$ and $s = L/m = 833$ mm.

The constant C_2 is obtained from Eq. (16)

$$C_2 = \frac{2 \times 240}{3 \times 833 - 2 \times 240} = 0.238$$

and the final restraining force is calculated using Eq. (25)

$$N(\infty) = -\frac{28 \times 750}{0.238}(1.56 - 0.0006 \times 7143)$$
$$= 240{,}900 \text{ N/m}$$

From Eq. (15), (17), and (23)

$$\sigma^*_{s2} = 321 \text{ MPa};$$
$$\sigma^*_{s1} = -7.643 \text{ MPa; and } \sigma^*_{c1} = 1.99 \text{ MPa}$$

The final crack width is determined using Eq. (30)

$$w = -\left[\frac{1.99}{7143}\left(833 - \frac{2}{3} \times 240\right) - 0.0006 \times 833\right]$$
$$= 0.31 \text{ mm}$$

Case (b)

Consider the slab of Case (a), with one-half the quantity of reinforcement, i.e., $A_s = 375$ mm²/m. The number and final average width of the restrained shrinkage cracks are to be calcualted. All material properties are as for Case (a).

For this slab

$$A_c = 150{,}000 \text{ mm}^2\text{/m and } \rho = 0.0025.$$

If 12-mm diameter bars are used, Eq. (1) gives $s_o = 480$ mm. As for Case (a), $E^*_e = 7143$ MPa and $n^* = 28$.

From Eq. (6), $C_1 = 0.0684$ and Eq. (12) gives $N_{cr} = 66{,}840$ N/m. The concrete stress in Region 1 after first cracking is calculated from Eq. (9) to be $\sigma_{c1} = 0.476$ MPa and, using Eq. (19), $\sigma_{av} = 1.24$. Eq. (28) gives $\xi = 0.119$ and, from Eq. (27), $s \leqslant 3002$ mm. The number of cracks is therefore $m = 2$ since $L/s \geqslant 1.67$. Therefore $s = L/m = 2500$ mm. From Eq. (16), $C_2 = 0.147$ and Eq. (25) gives $N(\infty) = 217{,}900$ N. The steel stress at each crack calculated using Eq. (17) is $\sigma^*_2 = 581$ MPa, which is greater than the yield stress and clearly incorrect. The steel at each crack has yielded and the previous analysis is not valid.

If the steel at the crack has yielded

$$N(\infty) = A_s f_y = 150{,}000 \text{ N/m}$$

and only a single crack occurs. From Eq. (34)

$$\sigma^*_{s1} = \frac{28 \times 0.0025 \times 400 - 600 \times 10^{-6} \times 200{,}000}{1 + 28 \times 0.0025}$$
$$= -86.0 \text{ MPa}$$

and the tensil stress in the concrete away from the crack is given by Eq. (31)

$$\sigma^*_{c1} = \frac{400 \times 375 + 86.0 \times 375}{150{,}000} = 1.22 \text{ MPa}$$

The crack width may be calculated from Eq. (33)

$$w = -\frac{-86.0(3 \times 5000 - 2 \times 480) + 2 \times 480 \times 400}{3 \times 200{,}000}$$
$$= 1.37 \text{ mm}$$

As expected, when the steel is at yield across the crack, the crack width is excessive.

CRACK CONTROL IN RESTRAINED SLABS

As mentioned in the introduction, flexural cracks are rarely a problem in reinforced concrete slabs, provided of course that bonded reinforcement at reasonable spacing crosses the crack and that the member does not deflect excessively. In contrast, direct tension cracks due to restrained shrinkage and temperature changes frequently lead to serviceability problems, particularly in regions of low moment. Such cracks usually extend completely through the slab and are more parallel-sided than flexural cracks. If uncontrolled, these cracks can become very wide and lead to waterproofing and corrosion problems. They can also disrupt the integrity and the structural action of the slab.

Evidence of direct tension-type cracks are common in reinforced concrete slab systems. For example, in a typical one-way beam-slab floor system, the load is usually carried by the slab across the span to the supporting beams, while in the orthogonal direction the bending moment is small. Shrinkage is the same in both directions and restraint to shrinkage usually exists in both directions.

In the span direction, shrinkage will cause small increases in the widths of the many fine flexural cracks and may cause additional flexure-type cracks in the previously uncracked regions. However, in the orthogonal direction, which is virtually a direct tension situation, shrinkage generally causes a few widely spaced cracks that penetrate completely through the slab. Frequently, more reinforcement is required in the orthogonal direction to control these direct tension cracks than is required for bending in the span direction. As far as cracking is concerned, it is not unreasonable to say that shrinkage is a greater problem when it is not accompanied by flexure.

If the amount of reinforcement crossing a direct tension crack is too small, yielding of the steel will occur and a wide unserviceable crack will result. To avoid this eventuality, Campbell-Allen and Hughes[7] proposed the following expression for the minimum steel ratio ρ_{min}

$$\rho_{min} = \left(\frac{A_{st}}{bd}\right)_{min} = \frac{1.2 f_t}{f_{sy}} \tag{35}$$

where f_t is the tensile strength of immature concrete (usually about 3 days old) and may be taken as 0.25 $\sqrt{f'_c}$. For 25-MPa concrete and 400-MPa steel, Eq. (35) gives a minimum reinforcement ratio of $\rho_{min} = 0.0038$ for the control of shrinkage cracking. This is considerably less steel than contained in the slab of the example ($\rho = 0.005$), for which the calculated final crack widths were just over 0.3 mm, and might therefore be considered inadequate for some applications.

ACI 318M-89[2] suggests minimum ratios of reinforcement area to gross concrete area in the direction at right angles to the principal reinforcement direction in a one-way slab. For Grade 400 deformed bars, the minimum ratio is $\rho_{min} = 0.00018$. This is not to say that this is what is required in all situations—it is the absolute mimimun requirement. In many restrained members, significantly more shrinkage steel is required to provide crack control.

The Australian code AS 3600-1988[1] contains the following provisions for crack control in reinforced concrete slabs.

Where the ends of a slab are restrained and the salb is not free to expand or contract, the minimum ratio of reinforcement to gross concrete area in the restrained direction, when the slab is located in a severe exposure condition, is

$$\rho_{min} = \frac{2.5}{f_y} \tag{36}$$

which equals 0.0063 for a slab containing 400-MPa steel.

For a 5-m long and 150-mm thick nonprestressed concrete slab with similar material properties to that analyzed earlier in the numberical examples and with $\rho = 0.0063$ ($d_b = 12$ mm), the analysis described in this paper gives the following results immediately after first cracking

$$N_{cr} = 194 \text{ kN/m}, \sigma_{s1} = -5.36 \text{ MPa}, \sigma_{c1} = 1.33 \text{ MPa, and } \sigma_{s2} = 206 \text{ MPa}$$

and after all shrinkage has taken place

$$s = 500 \text{ mm}, N(\infty) = 203 \text{ kN/m}, \sigma^*_{s2} = 215 \text{ MPA} \quad \sigma^*_{s1} = -73.2 \text{ MPa}, \sigma^*_{c1} = 1.82 \text{ MPa}$$

and the final crack width is $w = 0.21$ mm.

This would normally be considered to be an acceptable maximum crack width in a severe exposure condition, and the analysis presented herein therefore endorses this provision of AS 3600.[1]

For a sheltered environment, in which visible cracks could cause esthetic problems, AS 3600 suggests that the minimum area of steel should be taken from Eq. (36). Where cracks are not likely to cause esthetic problems, in a sheltered environment, AS 3600 specifies

$$\rho_{min} = \frac{1.4}{f_y} \tag{37}$$

This would apply, for example, in the case of an interior slab in which visible cracking could be tolerated or in the case of an interior slab that was later to be covered by a floor covering and/or a false ceiling. Eq. (37) corresponds to a reinforcement ratio of 0.0035 for a reinforced slab with 400-MPa steel.

Analyzing the same slab just considered, except that $\rho = 0.0035$, gives

$$N_{cr} = 109 \text{ kN/m}, \sigma_{c1} = 0.76 \text{ MPa}, \sigma_{s2} = 207 \text{ MPa}$$

and after all shrinkage

$$s = 1250 \text{ mm}, N(\infty) = 191 \text{ kN/m}, \sigma^*_{s2} = 363 \text{ MPa} \quad \sigma^*_{c1} = 1.56 \text{ MPa and } w = 0.53 \text{ mm}$$

In a sheltered environment, this size crack may be reasonable provided visible cracking can be tolerated.

Where the ends of a slab are unrestrained and the slab is free to expand or contract, the minimum reinforcement ratio is

$$\rho_{min} = \frac{0.7}{f_y} \tag{38}$$

This steel area is recommended, for example, in slabs-on-ground with control joints at regular centers and is similar to the minimum provisions in ACI 318M-89.

CONCLUDING REMARKS

A simple procedure is presented for the determination of the stresses and deformation after shrinkage cracking in a fully restrained direct tension member. The predicted number and width of shrinkage cracks are in accordance with observed cracking in restrained members, and the results of the analysis agree with and provide endorsement for the direct tension crack control provisions of AS 3600-1988.[1]

CONVERSION FACTORS

1 mm = 0.039 in.
1 MPa = 145 psi
1 kN/m = 5.71 lbf/in.

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