research report 20

Analysis and design of reinforced concrete columns

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Cement and Concrete Association

Foreword

When the drafting work began on the new Code of Practice, *The structural use of concrete,* due to be published later this year a study of the behaviour of reinforced concrete columns was already in hand and this was extended in scope to provide the necessary understanding and background data to enable an appropriate design procedure to be formulated. Subsequently the relevant design clauses were submitted to the drafting committee for consideration.

This report covers the entire study of the column problem and gives a comprehensive comparison between the predicted or design ultimate loads and those obtained experimentally by various research workers. It thus establishes the validity of certain simplified design formulae and prescribes their limitations, at the same time indicating a rigorous approach which may be relevant in certain applications.

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Contents

Synopsis

The aim of the work described in this report was to provide a practical method for the design of reinforced concrete columns. To this end, extensive studies of the behaviour of such columns have been made, using a computer model. These studies confirmed the suitability of the design method presented by the European Committee for Concrete. This method has been developed and refined to achieve both simplicity in use and, as far as possible, a realistic representation of actual behaviour. Extensive comparisons have been made with the results of tests on actual columns, providing final confirmation of the validity of the method. The method is recommended for general use.

Notation

a = deflection
a_u = deflection at ultimate load
A_c = area of concrete
A_s = area of steel reinforcement
b = width of cross-section
d = effective depth, i.e. depth from compression face to centroid of tension steel
e = eccentricity of load = M/N
e_1 = eccentricity of load at one end of column
e_2 = eccentricity of load at other end of column
e_i = initial eccentricity = M_i/N
E = modulus of elasticity
f_u = ultimate stress
f_{cu} = characteristic cube strength
f_y = characteristic yield strength of reinforcement
h = cross-section depth
I = second moment of area
k = stiffness factor = I/l
k_b = beam stiffness
k_c = column stiffness
k = factor dependent on intensity of axial load
l = length of member
l_0 = clear length (or height) of column between end restraints
M = bending moment
M_a = additional moment
M_i = initial moment
M_t = total moment
M_u = ultimate moment
M_1 = smaller initial end moment due to ultimate loads
M_2 = larger initial end moment due to ultimate loads
N = axial load
N_{crit} = critical or Euler load
N_u = ultimate load *or* design ultimate load
N_{uz} = ultimate axial load in absence of any moment on cross-section
N_{bal} = axial load corresponding to balanced conditions in the cross-section
r = radius of curvature due to bending
r_u = radius of curvature due to M_u
α_{c1} = ratio of column stiffnesses to beam stiffnesses at upper end of column
α_{c2} = ratio of column stiffnesses to beam stiffnesses at lower end of column
$\alpha_{c\,min}$ = lesser of α_{c1} and α_{c2}
ρ = ratio of area of longitudinal steel to area of concrete

Introduction

The behaviour of columns has been the subject of research for many years. Despite this the problem is still not fully understood and design methods are, for the most part, based on empirical formulae. The recent development of the limit state approach to design[1,2] has focussed particular attention on two requirements: accurate information regarding the behaviour of structures throughout the entire range of loading up to ultimate load, and simple procedures to enable designers to assess this behaviour. The work described in this report attempts to satisfy these requirements in the case of reinforced concrete columns.

The first part of the report discusses the column problem to highlight the particular difficulties associated with reinforced concrete columns. A brief description follows of a rigorous analysis[3] capable of dealing with columns in framed structures. Studies of reinforced concrete columns made using this analysis are presented.

A design method, based on proposals put forward by the European Committee for Concrete[1], is then developed. The method is used to predict ultimate loads for those columns where behaviour has been accurately established from the analysis. It is concluded that the method is satisfactory.

The design method is developed further for general application, and some practical upper limits are suggested for its use. Comparisons are then made with an extensive range of tests on columns. It is concluded that the method is suitable for adoption in practice.

The column problem

GENERAL

The problem may be approached from the standpoint of deflections. All structures deflect under loading, but in general the effect of this upon the over-all geometry can be ignored. In the particular case of columns, however, the deflections may be such as to add a significant additional moment.

Any rigorous analysis which attempts to cover this behaviour is destined to be complex, since even for a purely elastic material the use of formal mathematics leads to second-order differential equations. In the plastic or non-linear range, these second-order differential equations are often intractable and recourse must be had to numerical methods to obtain solutions.

In this report, a considerable volume of data is presented which has been obtained by using a particular numerical method. Before the actual presentation and discussion of this information, however, it will be useful to discuss the behaviour to be expected of columns on the basis of certain simple assumptions.

THE PINNED LONG COLUMN UNDER AXIAL LOAD

This is the classical problem treated by Euler, who demonstrated that such a column would remain stable until a critical load was reached at which lateral deflection would develop. Where the column is of length l, the critical load, N_{crit}, is given by

$$N_{crit} = \frac{\pi^2 EI}{l^2}$$

In the more general case of a column with restraints to end rotation provided by beams or slabs, the critical load depends upon the ratio of beam stiffness to column stiffness. It is convenient for such systems to seek the so-called 'effective length' which, when used in the simple formula above, will yield the critical load. Where the column is braced against sidesway, the effective length, l_e, varies between $0{\cdot}5l$ for very stiff beams and $1{\cdot}0l$ for very flexible beams. For columns the ends of which are not braced against sidesway, the effective length l_e varies from $1{\cdot}0l$ for very stiff beams up to a theoretical value of infinity, as the beams become very flexible.

THE SHORT COLUMN UNDER AXIAL LOAD

On the assumption that the material has a strength f_u, the maximum axial load N_{uz} which can be resisted by a short column of area A is given by

$$N_{uz} = Af_u$$

TYPICAL VALUES OF CRITICAL LOAD AND MAXIMUM AXIAL LOAD

Values of N_{crit} and N_{uz} have been calculated for the cross-sections shown in Figure 1. The sections have been assumed to be uncracked, and the moduli of elasticity of the steel (E_s) and concrete (E_c) have been taken as 200 and 20 kN/mm² respectively. The stresses in the steel and concrete at ultimate have been taken as 400 and 20 N/mm² respectively. Figure 2 gives values of N_{crit}/N_{uz} over a range of slenderness ratios l/h. It will be seen that, for $l/h < 12$, the value of N_{crit}/N_{uz} is $> 5{\cdot}0$. It follows that in this range the

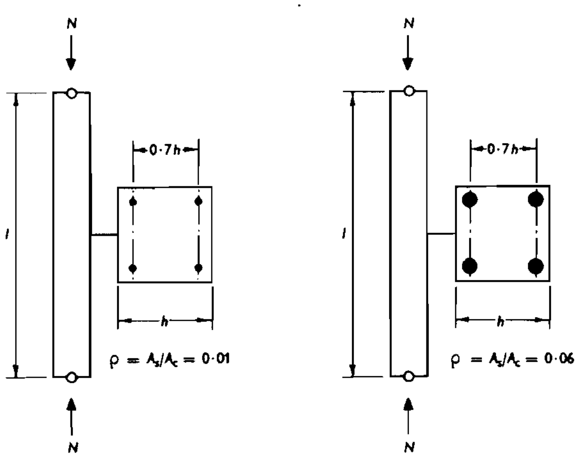


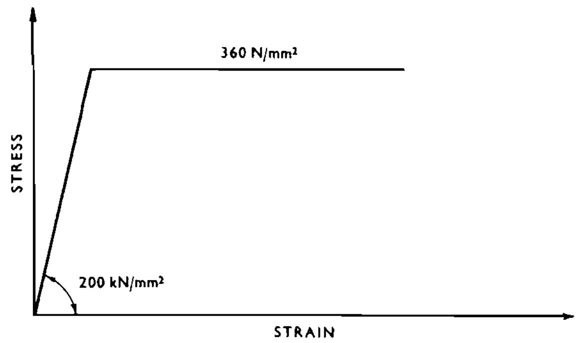
Figure 1: Pinned columns.

effects of stability will be small in controlling the collapse load. As l/h increases up to 60, however, N_{crit}/N_{uz} reduces to 0·2 and it is plain that here the stability or deflection effects will dominate the situation. Also drawn on Figure 2 for interest is a plot of the reduction coefficient from CP 114:1957[13]; it will be seen that this follows the same trend as the line for N_{crit}/N_{uz}. It is plain, therefore, that stability or deflection effects can be significant in practical reinforced concrete construction.

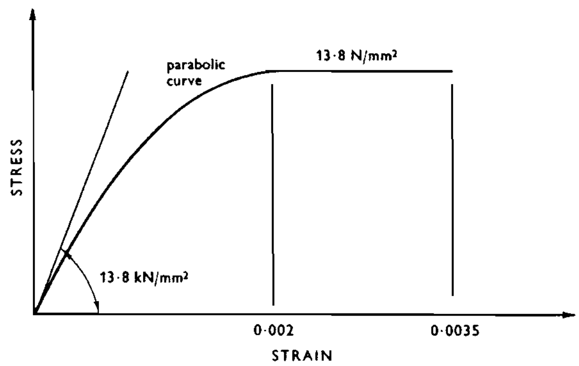
ASSESSMENT OF *EI* AND MOMENT-CURVATURE RELATION

The basic quantity required in any calculations for N_{crit} is the value of *EI*, which here can be considered as the slope of the relation between moment, *M*, and curvature, $1/r$. For an elastic material this presents no difficulties but in the case of reinforced concrete there are problems, because the moment-curvature response is non-linear.

Moment-curvature relations for a cross-section with a small amount of steel are shown in Figure 4, which has been computed from the stress-strain curves given in Figure 3. It will be seen that the intensity of axial load is of major influence. Zero axial load gives the minimum initial stiffness. When, however, the axial load is equal to 0·4 times N_{uz}, the initial response is nearly three times as stiff (essentially because tensile strain does not develop). As the axial load increases beyond this, however, the initial stiffness decreases. Figure 5 shows similar relations for a cross-section with a large amount of steel. Here, apart from a small



(*a*) *For steel in tension and compression*



(*b*) *For concrete in compression*

Figure 3: Stress-strain curves.

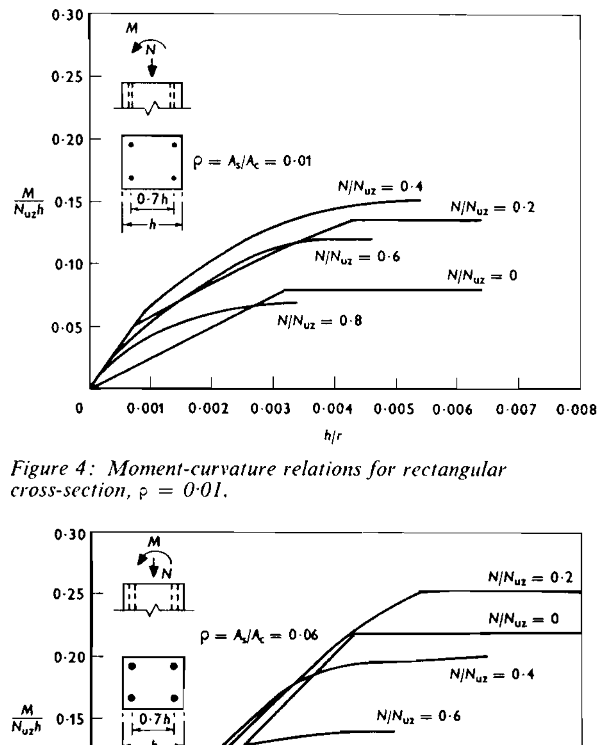


Figure 4: Moment-curvature relations for rectangular cross-section, ρ = 0·01.

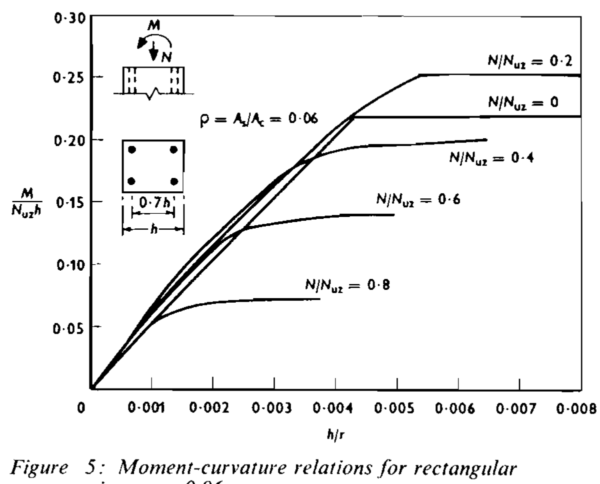


Figure 5: Moment-curvature relations for rectangular cross-section, ρ = 0·06.

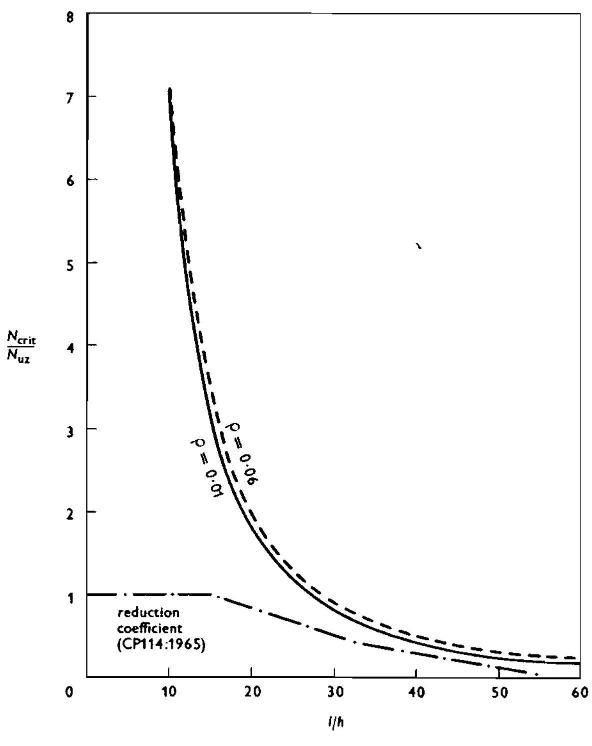


Figure 2: Influence of slenderness.

region under very low moments, the stiffnesses (i.e. slopes of the moment-curvature relations) do not vary with axial load nearly so much as in the previous case; the variation is, however, still significant.

COLUMNS WITH LINEAR MOMENT-CURVATURE RESPONSE UP TO MAXIMUM MOMENT CAPACITY

Here the moment-curvature relation is simplified as shown in Figure 6, and is assumed not to be influenced by axial load. This is probably a reasonable assumption for columns with large amounts of steel, but appreciable errors must be expected for columns with small amounts of steel. Nevertheless the approach is worth pursuing.

Consider the simple pin-ended column under end moments shown in Figure 7. Plainly the moments will be a maximum at the centre of the column and will be given by

$$M_t = M_i + Na$$

The column will be capable of resisting axial load until M_t is equal to M_u, so that the maximum axial capacity N_u will obey the following:

$$M_u = M_i + N_u a_u$$

or

$$N_u = \frac{M_u - M_i}{a_u}$$

At this stage a_u is, of course, not known.

Turning now to the curvature diagram, also sketched in Figure 7, it will be seen that it must be a maximum at the centre with the value $1/r_u$. Formal mathematical treatment of this problem will yield an exact equation for the bent shape which will have the general form shown in Figure 7. Taking the conservative and unconservative curvature diagrams and integrating them to give the deflections gives

$$a_u = \frac{l^2}{8r_u} \quad \text{(conservative)}$$

$$a_u = \frac{l^2}{12r_u} \quad \text{(unconservative)}$$

It is plain, therefore, that a reasonable estimate of a_u, certainly suitable for design purposes, would be to take it as an average value of $l^2/10r_u$.

COLUMNS WITH A BI-LINEAR MOMENT-CURVATURE RELATION (FIGURE 8)

Here the behaviour can be studied by considering first the situation when the maximum moment in the column length reaches M_1, and assuming that the initial moment M_i is less than M_1. If, in addition, the deflection a_1 at this stage is assumed to be given by $l^2/10r_1$, the following equation must hold for N_1:

$$N_1 = \frac{(M_1 - M_i)10r_1}{l^2}$$

Consider now the situation where the maximum moment equals M_u. The curvature diagram will be as sketched in Figure 9. As argued in the previous section, it is reasonable here also to assume that

$$a_u = \frac{l^2}{10r_u}$$

and thus it follows that

$$N_u = \frac{(M_u - M_i)10r_u}{l^2}$$

If, now, the assumption that $a = l^2/10r$ is applied throughout the loading range, it is possible to derive a load-deflection curve from the moment-curvature diagram. This has been done in Figure 11 for the three moment-curvature diagrams in Figure 10.

For cases I and II, it will be seen that the maximum load corresponds to the attainment of ultimate moment at the centre of the column. In case III, however, the maximum load occurs when $M = M_1$, i.e. long before the ultimate strength of the section is reached. This situation is often called a stability failure, to distinguish it from the cases in which the ultimate moment is attained. It is plain that the more pronounced the bi-linearity of the moment-curvature relation, the more likely is a stability failure.

The above is, of course, an extremely simplified approach to the problem; nevertheless the general behaviour illustrated in the load-deflection diagrams of Figure 11 is valid. In order to follow this behaviour and define the maximum or collapse loads of columns accurately, it is therefore necessary to have a method which will trace out the load-deflection response from zero load upwards, taking accurate account of the moment-curvature relation. The method of analysis described briefly in the next section was developed specifically to fulfil this requirement.

It should be noted that where a linear moment-curvature relation applies, the load-deflection response of the column will not show a maximum until ultimate strength is attained at a critical section. Thus, if this simplified relation is accepted, design can be based on a consideration of the deformed shape corresponding to ultimate conditions being present in the critical cross-section. But a rigorous analysis is still necessary to assess the consequences of such a simplification.

Method of analysis

Only a brief description of the method of analysis is given here. A full description has been given in reference 3.

SYSTEM ANALYSED

The system analysed is shown in Figure 12. The column AB is held by systems which provide rigid

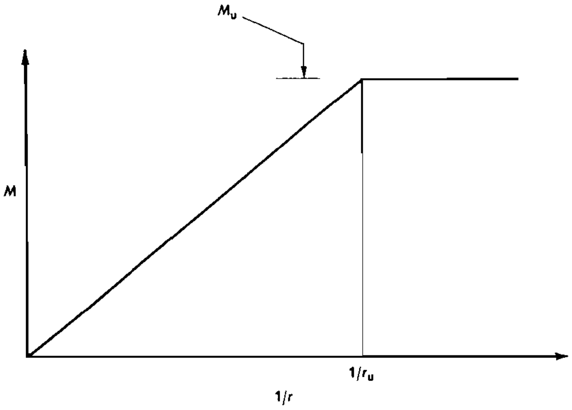


Figure 6: Linear moment-curvature relation.

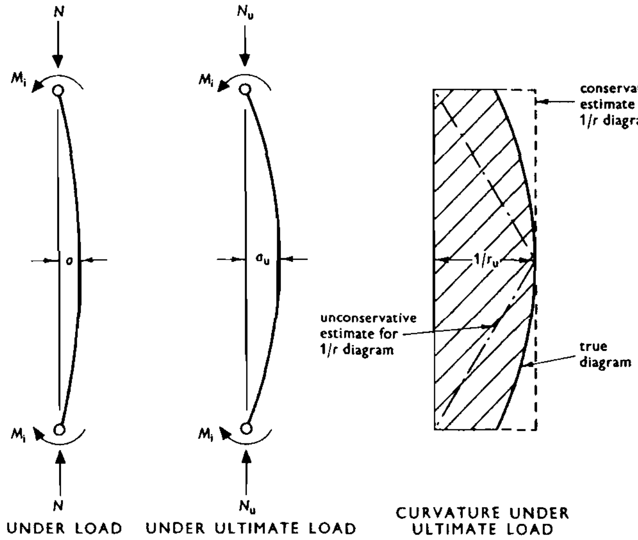


Figure 7: Pinned column with linear moment-curvature relation.

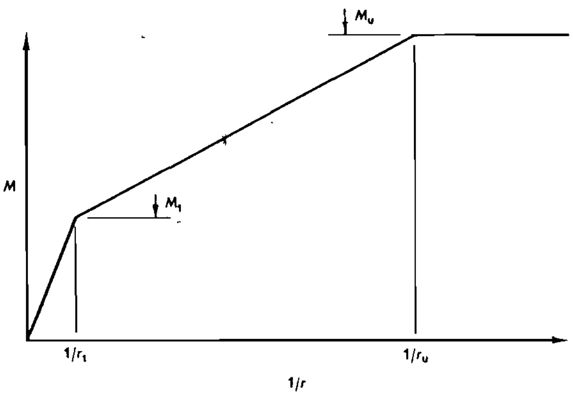


Figure 8: Bi-linear moment-curvature relation.

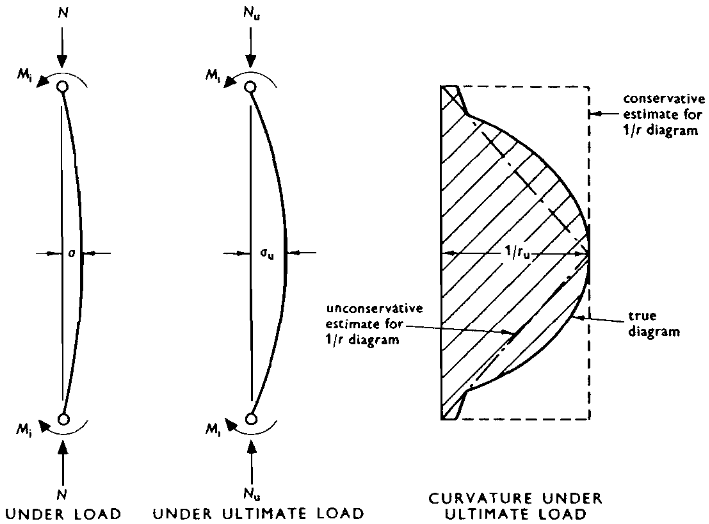


Figure 9: Pinned column with bi-linear moment-curvature relation.

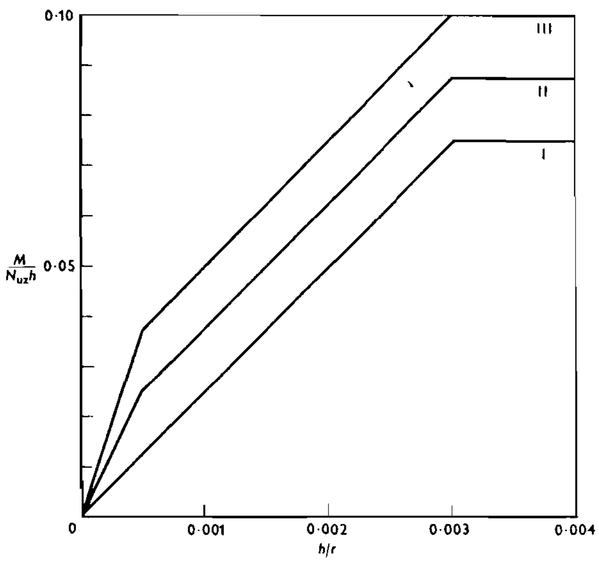


Figure 10: Typical moment-curvature relations.

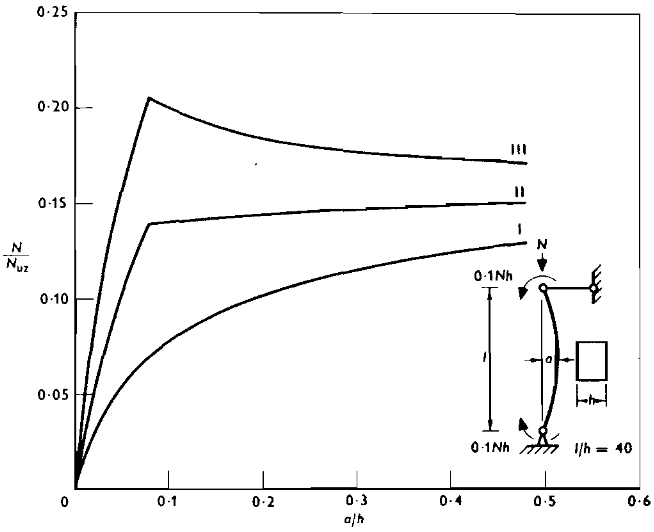
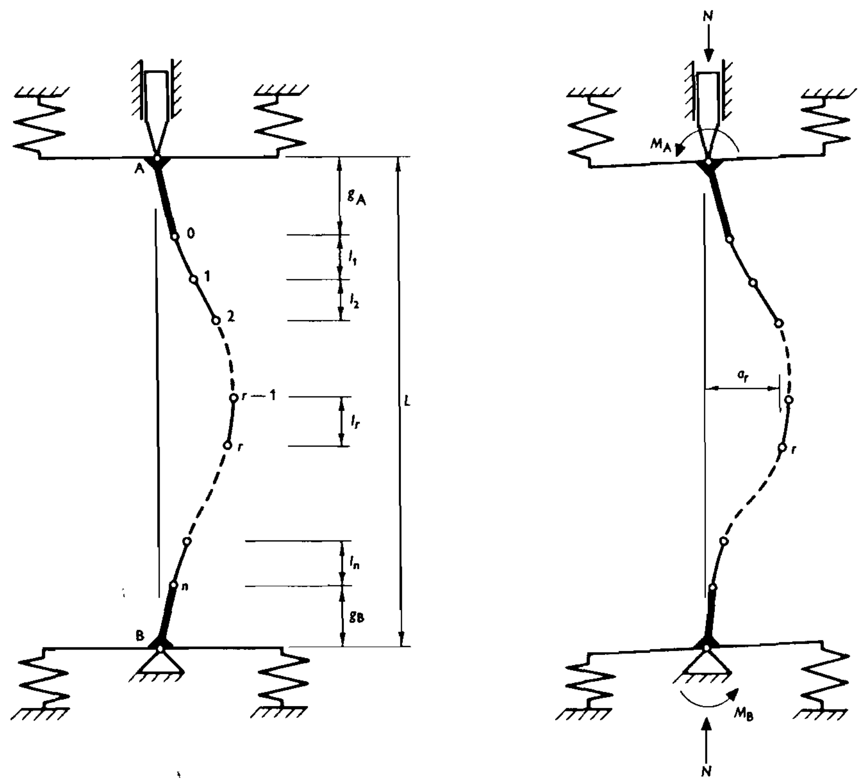


Figure 11: Load-deflection relations.



(*a*) *Unloaded* (*b*) *Loaded*

Figure 12: Systems analysed.

bracing against sway movement but which are capable of rotation. Such a column is defined in this report as a braced* column. The loading consists of an axial load, N, and end moments, M_A and M_B.

The column is considered to have two rigid end lengths, the remainder being divided into a number of straight segments. The analysis is based on the cross-section behaviour at the division points between the segments.

ASSUMPTIONS

The analysis is based on the following assumptions:

plane sections remain plane after bending;

lateral deflections of the column are small in comparison with its length;

the longitudinal stress at any point in the column is dependent only upon the longitudinal strain;

material strained into the inelastic range and subsequently unloaded follows a linear unloading line;

under loading, the curvature varies linearly along segments.

GENERAL DESCRIPTION

The behaviour of the column is studied as loading is applied from zero, the loading being applied initially in specified increments. The general load-deflection response to be expected from reinforced concrete columns may be seen in Figure 11. For curves of the type illustrated as case III, there are two equilibrium positions corresponding to the same loading in regions close to the peak of the curve. To avoid difficulties in such cases, it is convenient to find solutions corresponding to a specified deflection. In this way, the behaviour of the column can be traced up to and beyond maximum load if desired.

The analysis thus consists of finding successive solutions as the load on (or deflection of) the column is increased in steps. The finding of each separate solution is said to comprise a stage in the analysis.

The method of solution is iterative, in that initial proposals are made for the deflected shape of the column and bending moments are computed for each division point. This part of the procedure ensures that equilibrium conditions are satisfied.

The curvature at each division point is then computed by using a subsidiary iterative procedure, in which the cross-sections are idealized into a number of elements. These elements are made small enough for the stresses in them to be assumed uniform. A strain profile across the section is proposed from which calculated values of axial load and bending moment are obtained and, if these agree closely with the loading applied to the section, the curvature corresponding

*In reference 3, the terms restrained and unrestrained are used to mean 'braced' and 'unbraced'. The new terms have been adopted to avoid confusion which has arisen in interpreting reference 3.

to the proposed strain profile is taken as correct. Otherwise the strain profile is modified and the procedure repeated. In this way, the influence of the axial load upon the moment-curvature relation is automatically accounted for.

When the curvatures at all the division points have been computed, the deflected shape is calculated and compared with that initially proposed. If close agreement is obtained, it is plain that compatibility conditions are satisfied and the initial proposals therefore comprise a valid solution. Otherwise the initial proposals are modified and the procedure is repeated.

PINNED AND UNBRACED COLUMNS

The analysis can be used directly to deal with columns with pinned ends by reducing the rotational end restraints to zero. In some structures the ends of the columns are able to sway sideways with respect to one another. The stability in such cases is assured by ensuring that the beams and columns are capable of resisting the moments which will be induced by lateral forces. Columns of this type are defined in this report as unbraced columns. The analysis as described above is not directly applicable to unbraced columns, but a simple idealization described in reference 3 enables such cases to be tackled.

VALIDITY

The validity of the analysis has been demonstrated in reference 4, where comparisons are made with the results of other methods of analysis and also with test results. It is shown that, where the column length is divided into ten segments and the cross-sections into ten elements, the ultimate loads obtained are generally within at least 1 % of formal mathematical solutions of the governing differential equations. Furthermore, the deviations found in the comparisons with actual tests can, in most cases, be attributed to inaccurate knowledge of the stress-strain curves for the materials, rather than to any inherent defect in the analysis.

Design principles

GENERAL

The analyses described in the next section were carried out to help establish a design method. A brief discussion of the principles of limit state design is therefore appropriate at this point.

The basic aim of limit state design is to provide a reasonable margin of safety against the structure's becoming unfit for use, i.e. entering a limit state. The main limit states to be considered are:

the ultimate or collapse limit state, when the strength of the structure is exhausted;

the serviceability limit state of deflection, e.g. when deflections lead to damage to finishes;

the serviceability limit state of cracking, e.g. when wide cracks develop in the concrete.

For columns, the main limit state will be the ultimate limit state. Under service conditions the loading on most columns is axial or nearly axial and thus significant deflection does not normally develop. Cracking, also, will rarely be critical since, even where bending moments are dominant in columns, the tensile stresses in the reinforcement will be less than are usual in beams. In this study, therefore, the main interest has been the behaviour of columns under ultimate conditions.

SAFETY FACTORS

It has been suggested[1] that, in designing for a particular limit state, two safety factors should be used, one applied to the loads, and the other to the strengths of the materials. The safety factors are applied to so-called 'characteristic' loads and strengths, which are derived on a statistical basis. The characteristic strength of a material is defined as that strength below which 5 % of test results may be expected to fall. The characteristic loads should, ideally, be loads which have a defined chance of being exceeded once in the lifetime of the structure. Statistical evidence to define loads in this way is not as yet available and so, for the moment, they are taken as equal to the working loads as laid down in current regulations.

The safety factors currently proposed[5] for the ultimate limit state are 1·4 and 1·6 on dead and imposed loads respectively and 1·15 and 1·5 on steel and concrete respectively. Thus the design load, N, for a column is given by

$$N = 1{\cdot}4N_g + 1{\cdot}6N_q$$

where N_g is the charaeteristic dead load and N_q is the characteristic imposed load.

The cube strengths to be assumed for the concrete in design are $0{\cdot}67f_{cu}$, where f_{cu} is the characteristic cube strength, and the yield strength to be assumed for the steel is $0{\cdot}87f_y$, where f_y is the characteristic yield strength.

In this approach, therefore, an ultimate section capacity just greater than the design load is provided, the material strengths being assumed to be at their design values. At this stage, it would appear correct to analyse a range of columns containing material of design strength, and establish the ultimate loads. However, with long columns, the deformations will affect the moments and hence the ultimate loads, and so not only do the strengths of the materials enter into the matter but also the stiffnesses.

Before specifically tackling this point, it is useful to look further into the thinking that lies behind the use of a safety factor on materials. Its purpose is essentially to allow for the effects of bad workmanship. It is rather more likely that bad workmanship will lead to pockets of poor material being present than that the whole column will be uniformly affected. If this is accepted, and it has been in the present study, it is more realistic to assume in design that the average strength of the concrete is at characteristic strength, and that design strength material is in evidence only at critical points. Thus the analyses have been carried out on the basis of characteristic strength material being present throughout, and the so-called design ultimate load has then been assessed from this analysis by considering the effects of the presence of design strength materials at the most critically loaded points.

LOADING PATTERN

On the question of loading patterns, the usual approach is to seek the most critical condition and design for it. The assessment of the most critical loading condition depends upon the type of column, and it is convenient to consider the three main types of column, pinned, braced and unbraced, separately.

The pinned column is uncommon in practice at this time, although the development of precast concrete construction could lead to more frequent usage. The possible types of loading condition are summarized in Figure 13. Load condition (a) is plainly the most critical because the deflexions caused by the moments will be the largest. Load condition (b) will be less critical, and (c) will be much less critical because the central deflexion caused by the moments will be zero. In practice, pinned columns will be designed for axial or near axial load and the most likely hazard to consider in design will be misalignment at one end. Studies in this report have therefore been limited to case (b) loading.

Braced columns, i.e. those braced against sidesway, are the most common form of structural column since, in most structures, bracing against lateral load is provided by walls, lift shafts and the like. Three cases of beam loading are sketched in Figure 14, and it will be seen that the magnitude of moment transferred to the end of the column for a given beam load intensity increases as the loading becomes asymmetrical. The moments due to the axial load times the deflection (*Na* moments) will, however, become less serious as asymmetry develops, because the deflections will be less. For slender columns, therefore, where the *Na* moments are likely to dominate, load condition (a) with symmetrical bending is likely to be the most critical. For short columns, load condition (c) is going to be the most critical. Thus the most likely candidates for study would appear to be load conditions (a) and (c). But load condition (b) is the condition laid down for design in the existing Codes of Practice[6]. In fact, the major part of the studies carried out have been under load conditions (a) and (b). The work done on load condition (b) indicates that, for short columns, a redistribution of moments will generally take place, making the results of a simple moment distribution rather conservative. Such redistribution will be even more marked for load condition (c) and this, combined with its being rather less likely to arise in practice, led to the decision to exclude it from study.

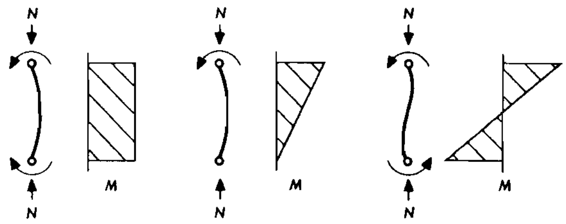


Figure 13: Loading patterns: pinned columns.

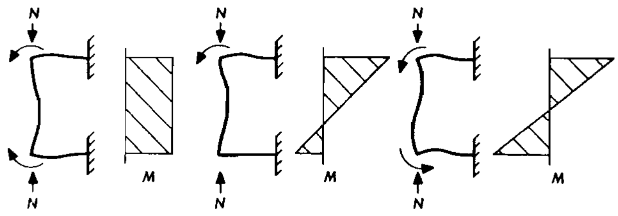


Figure 14: Loading patterns: braced columns.

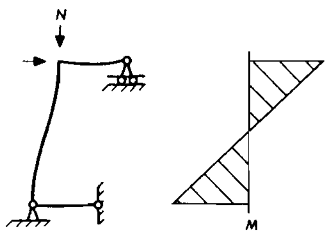


Figure 15: Loading patterns: unbraced columns.

Coming finally to the unbraced column, one of its functions is to carry lateral forces arising from wind loading or other sources, as shown in Figure 15. Unbraced columns in a structure may, of course, be subjected to moments arising from beam loadings in the same way as braced columns. The response in this case is complex and normally will require a study of the whole storey height. The only examples considered in this study are those subjected to lateral loading, as in Figure 15.

ROTATIONAL RESTRAINTS FROM BEAMS

The relation between restraining moment and end rotation is complex, because the restraining beams or slabs can first of all be uncracked or cracked. Here it is not difficult to arrive at some reasonable assumptions for design.

A more difficult point is that, in extreme cases, yielding of the tension steel could occur in the restraining beams or slabs. This would introduce a sudden reduction of stiffness. In considering this problem, it has been assumed that the strengths of the materials in the adjoining beams or slabs are at least at the characteristic values. This is a reasonable application of the limit state philosophy, since the probability that serious defects in workmanship (of the kind associated with the material strength safety factors) will occur in both beams and columns is very low.

If the restraint systems are at characteristic strength, yielding is unlikely to occur even under the full design load envisaged for the columns. Therefore it has been assumed in this study that the restraining systems have an elastic response.

Analyses carried out

VARIABLES

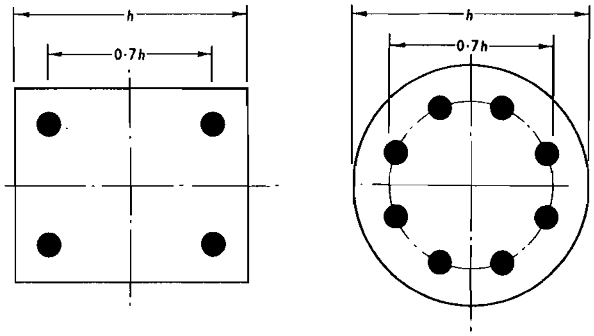
The range of variables which could be considered is very large, and some restriction was necessary to keep the total number of separate analyses to a practical level. At the same time, the range of design possibilities had to be reasonably well covered.

Both circular and rectangular cross-sections as shown in Figure 16, were considered. The positioning of the reinforcement was chosen as being close to a practical average. Two percentages of reinforcement, 1 % and 6 %, were used.

Only one concrete was considered and this was assumed to have a characteristic strength of 31 N/mm². Only one type of steel was considered, having a characteristic strength of 414 N/mm². The characteristic stress-strain curves assumed for these materials are shown in Figures 17 and 18. These curves are slightly different from those that would be obtained from current design proposals[5]. The differences are, however, so small as to have a negligible effect upon the results presented here.

Analyses were carried out for the three types of column, pinned, braced and unbraced. Figure 19 shows the pinned columns, which were subjected to two levels of end moment, 0·1 *Nh* and 0·5 *Nh*. Figure 20 shows the braced columns analysed and also indicates the columns in a frame building of which they are effectively the equivalent. The idealizations were necessary because the method of analysis[3] considers only one column length, with specified end-moment/rotation relations. It will be seen that, where moments are applied at the ends of columns, they are assumed to split equally between the upper and lower column, and half of the beam restraint system available is allowed to assist the column being considered. At column ends where no moments are applied, the column below the one being considered is assumed to assist in resisting end rotation in proportion to its nominal stiffness. These various assumptions are considered to be reasonable. Figure 21 shows the unbraced columns which were analysed and also indicates the equivalent column in a framed structure.

For each loading condition, five different slenderness ratios, *l/h*, were considered, 10, 15, 25, 40 and 60; the slenderness ratio is defined as the clear height of the column between restraining beams. The depth of the restraining beams was assumed to be negligible so that *l* becomes, in this instance, the centre-to-centre distance between beam-column joints.



(*a*) *Rectangular* (*b*) *Circular*

Figure 16: Cross-sections considered.

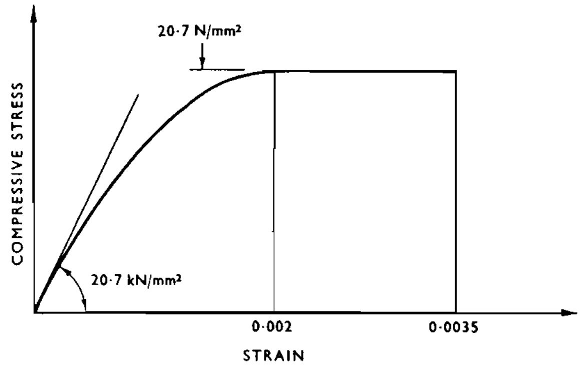


Figure 17: Stress-strain curve assumed for concrete having a characteristic strength of 31 N/mm².

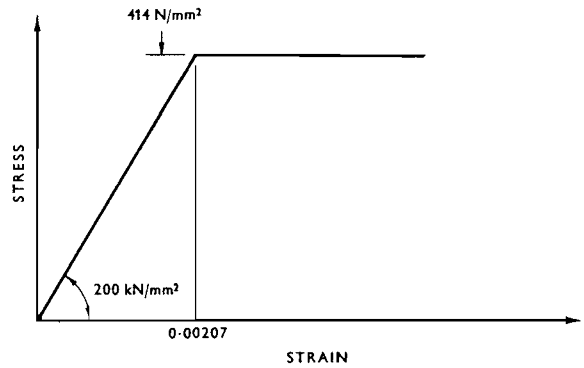


Figure 18: Stress-strain curve assumed for steel having a characteristic strength of 414 N/mm².

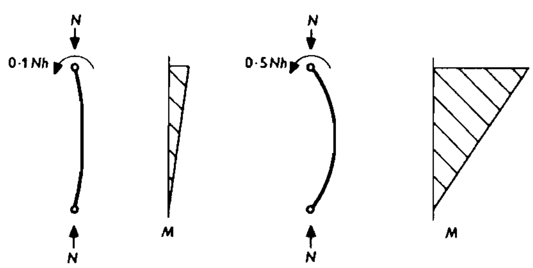


Figure 19: Pinned columns analysed.

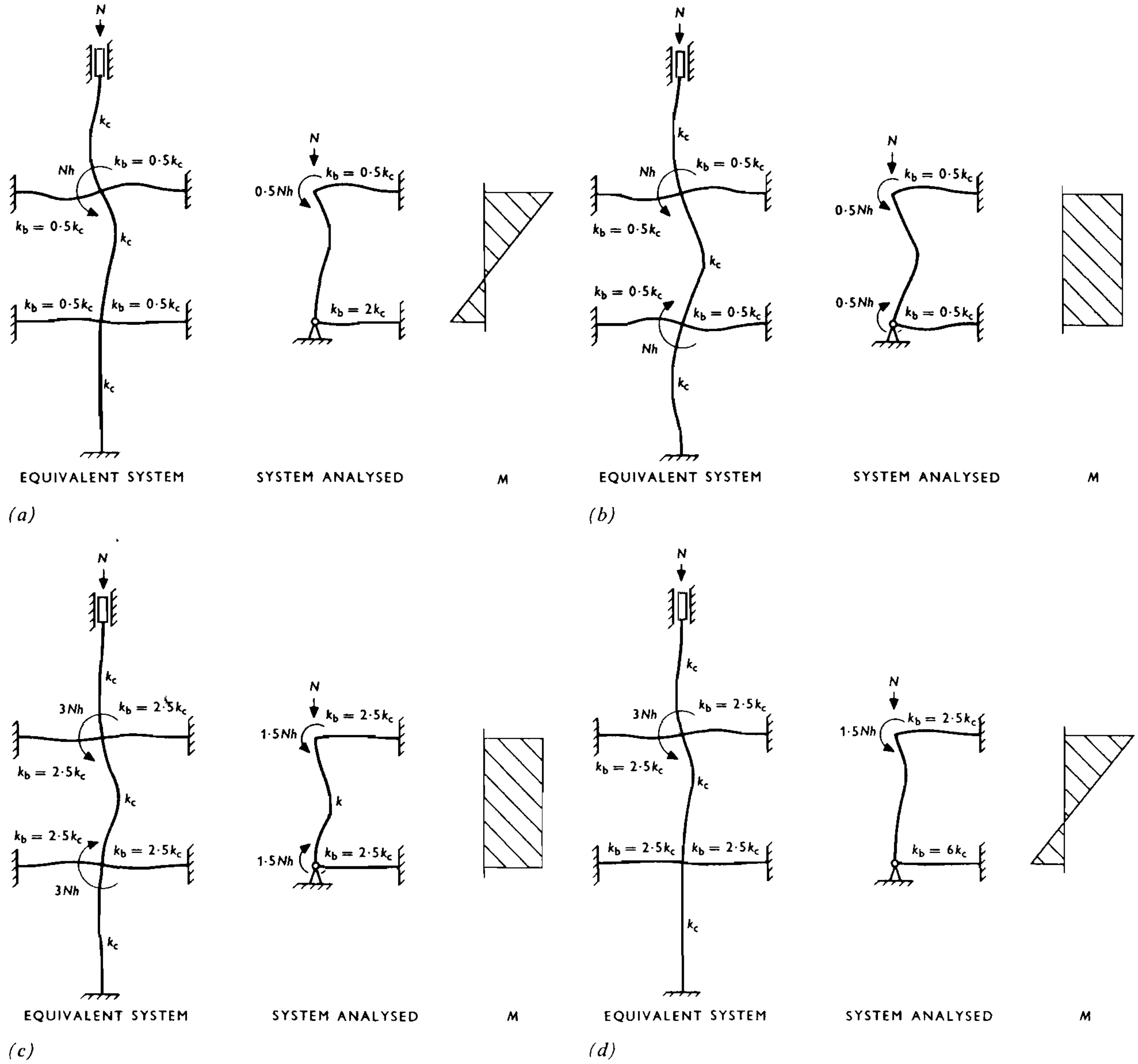


Figure 20: Braced columns analysed (k signifies conventional stiffness = I/l).

Considering the limited range of variables just outlined leads to a total of 200 analyses, which took an average of $1\frac{1}{2}$ hours' computing time each.

IDEALIZATIONS

The column lengths were divided into between 4 and 10 segments for the pinned and restrained columns, and between 8 and 16 segments for the unrestrained columns. The concrete in the cross-section was idealized into ten elements of equal width, allowance being made for the area displaced by the reinforcement. The reinforcement was taken as two elements for the rectangular section and as four for the circular section (Figure 16). These degrees of sub-division of column length and cross-section were known from previous checks of the analysis to lead to errors of less than 1 % in the results, when compared with a rigorous mathematical solution.

For the restrained and unrestrained columns, the moment-rotation relations for the beam restraint systems had to be calculated. The starting point was to calculate the flexural rigidity $(EI)_c$ of the column section, the modulus of elasticity of the steel being assumed to be 200 kN/mm^2 and that of the concrete 20·7 kN/mm^2, and the section to be uncracked. From the stress-strain curves for concrete in Figure 17, it can be seen that 20·7 kN/mm^2 is the tangent modulus at the origin of the curve. Thus the stiffness calculated is the stiffness under a small initial increment of load.

The ratios between the column stiffness and beam stiffness to be considered in analysis have already been discussed and are shown in Figures 20 and 21. To take the braced column in Figure 10a as an example, the relation between restraining moment, M_R, and end rotation, θ, used in the analysis for the upper end of the column was derived from

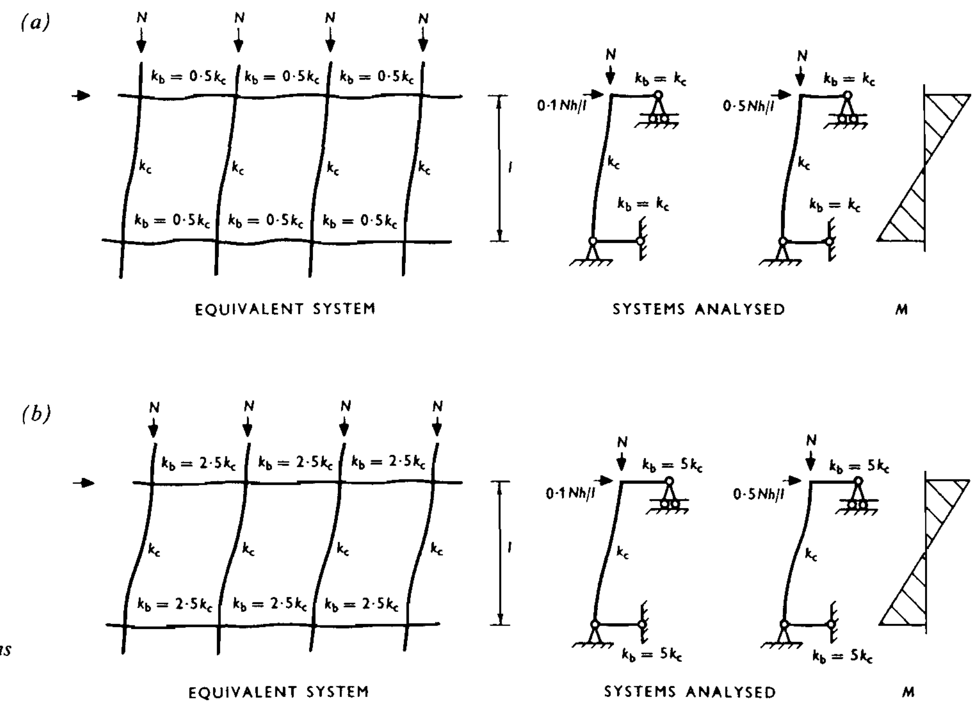


Figure 21: Unbraced columns analysed (k signifies conventional stiffness = I/l).

$$M_R = \theta \times \frac{4(EI)_c}{l} \times 0{\cdot}5$$

For the lower end of the column, the corresponding relation is

$$M_R = \theta \times \frac{4(EI)_c}{l} \times 2{\cdot}0$$

For the unbraced case in Figure 11a, the beams are bent into a symmetrical double curvature and this leads to the relation

$$M_R = \theta \times \frac{6(EI)_c}{l} \times 0{\cdot}5$$

The net result of these assumptions about beam and column stiffness is that, in the initial stages of an analysis, provided that the *Na* moments are not large, and provided that the axial load, *N*, is such as to produce compressive strain throughout the column, the bending moments and deflections in the column will differ only slightly from those obtained from a simple elastic analysis of the systems sketched in Figures 20 and 21.

GENERAL COMMENTS ON BEHAVIOUR

The results obtained from each of the 200 analyses carried out are extensive and cannot all be presented in this report. In this section, 12 columns, four of each type, are considered in detail to bring out points of general behaviour. In the next section, the design ultimate loads for all of the analyses are presented.

The results for these 12 columns are presented in Figures 22 to 33, in which are given load-deflection curves, deflection, moment and curvature diagrams for several load stages and also diagrams indicating strain conditions in the concrete along the length of the column; the load stages chosen generally correspond to about half the maximum load and then close to the maximum load.

The results are presented non-dimensionally. The axial load, *N*, has been divided by N_{uz}, which is defined as the ultimate axial capacity of the section in the absence of moment, account being taken of the design safety factors. In this case, N_{uz} is given by

$$N_{uz} = \frac{A_c \times 20{\cdot}7}{1{\cdot}5} + \frac{A_s \times 414}{1{\cdot}15}$$

where A_c = area of concrete in the section and A_s = area of steel in the section.

The moment *M* has been reduced to non-dimensional terms by dividing by $N_{uz}h$, where *h* is the height of the column section in the direction of bending. The deflection *a* has been divided by *h* and the curvature $1/r$ multiplied by *h*. The quantity h/r at a division point is thus the change in strain from the compression face to the tension face of the section.

To help in interpreting the results, interaction diagrams giving the combinations of axial load and moment corresponding to maximum or ultimate load conditions are also included in the Figures. Plotted on each interaction diagram are values of the most critical combinations of load from which the imminence or otherwise of a material failure may be judged. Interaction diagrams have been drawn first on the basis of characteristic stress-strain curves as shown in Figures 17 and 18. Interaction diagrams based on design strengths for the materials are also given; these are described later.

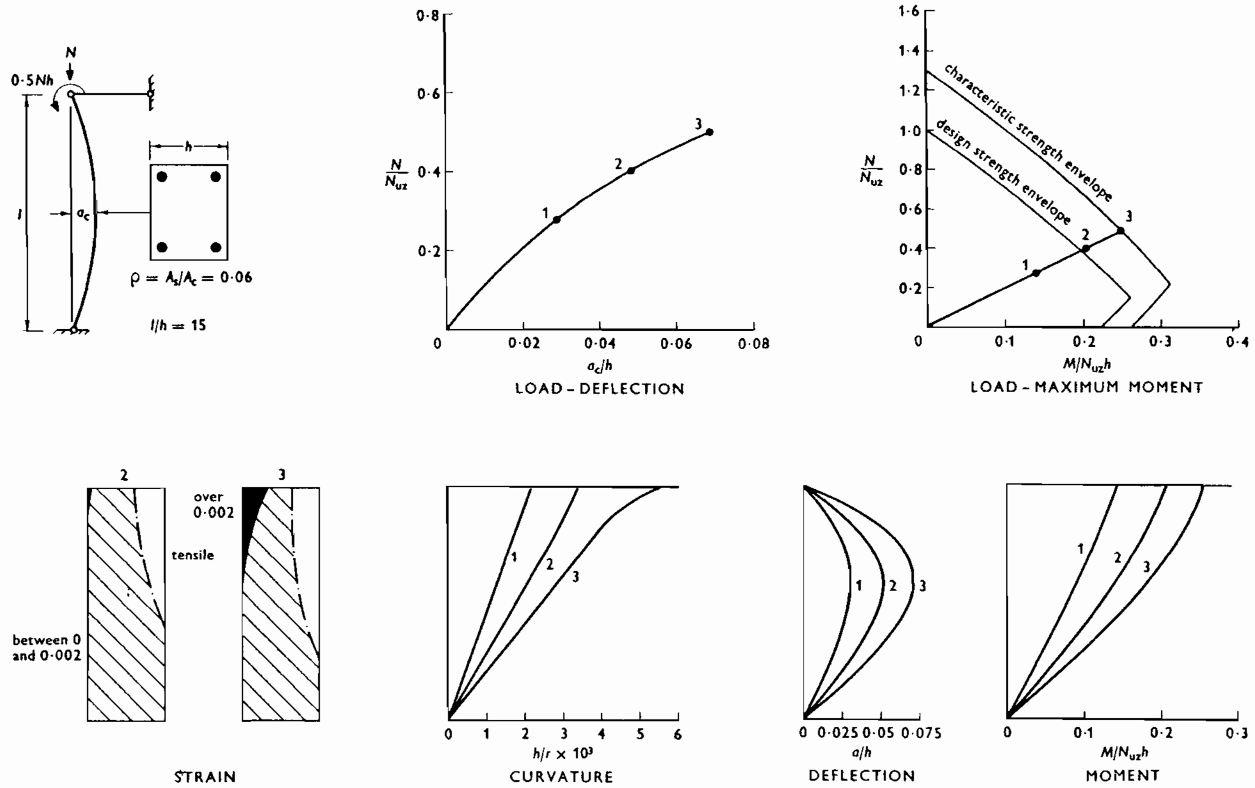


Figure 22: Behaviour of pinned column, l/h = 15, ρ = 0·06.

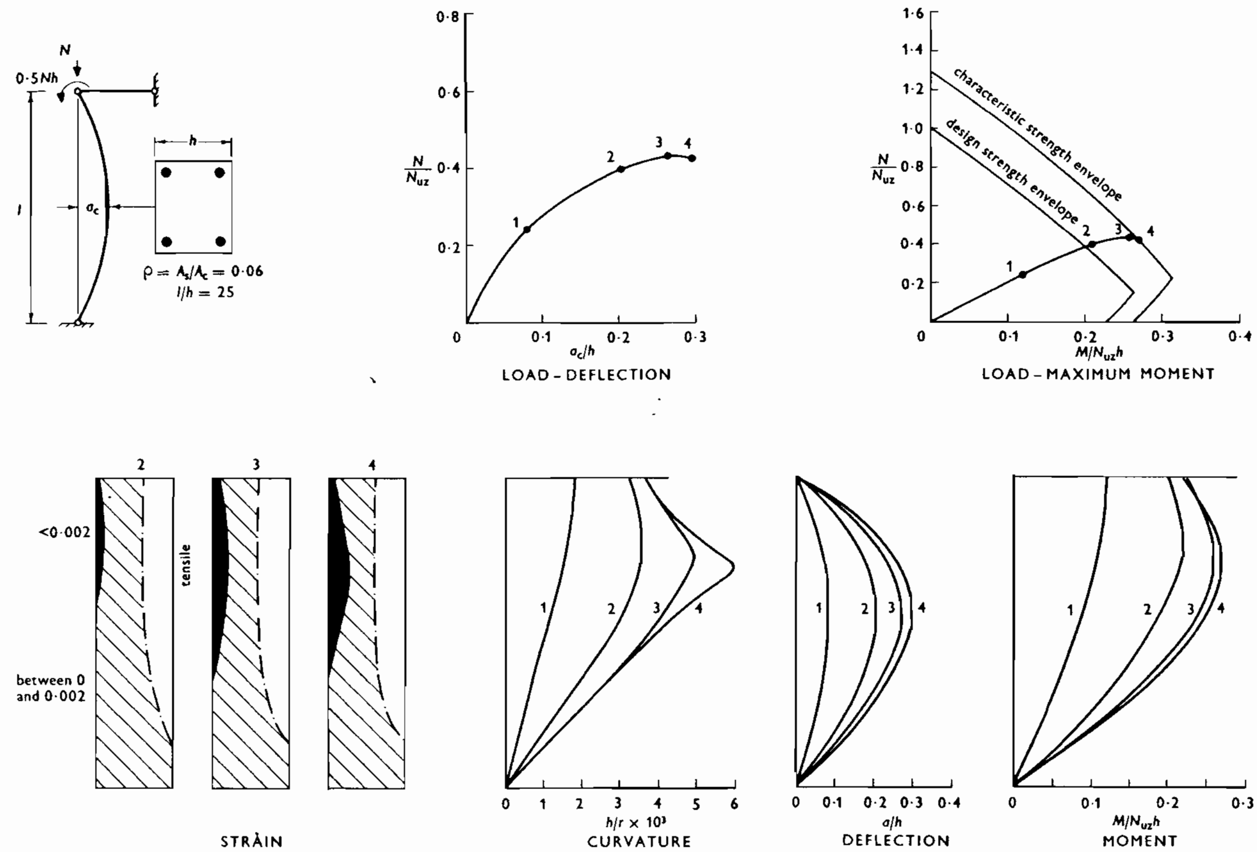


Figure 23: Behaviour of pinned column, l/h = 25, ρ = 0·06.

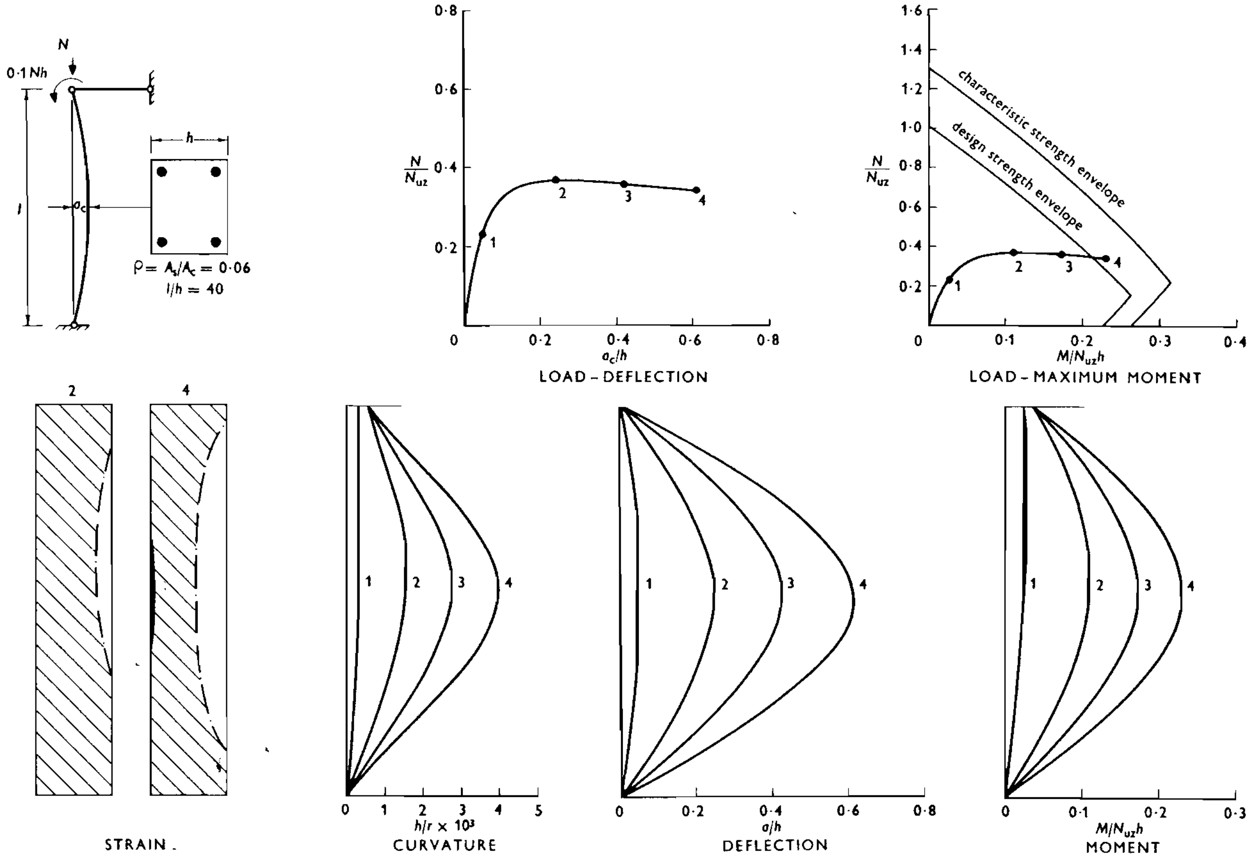


Figure 24: Behaviour of pinned column, l/h = 40, small end moment.

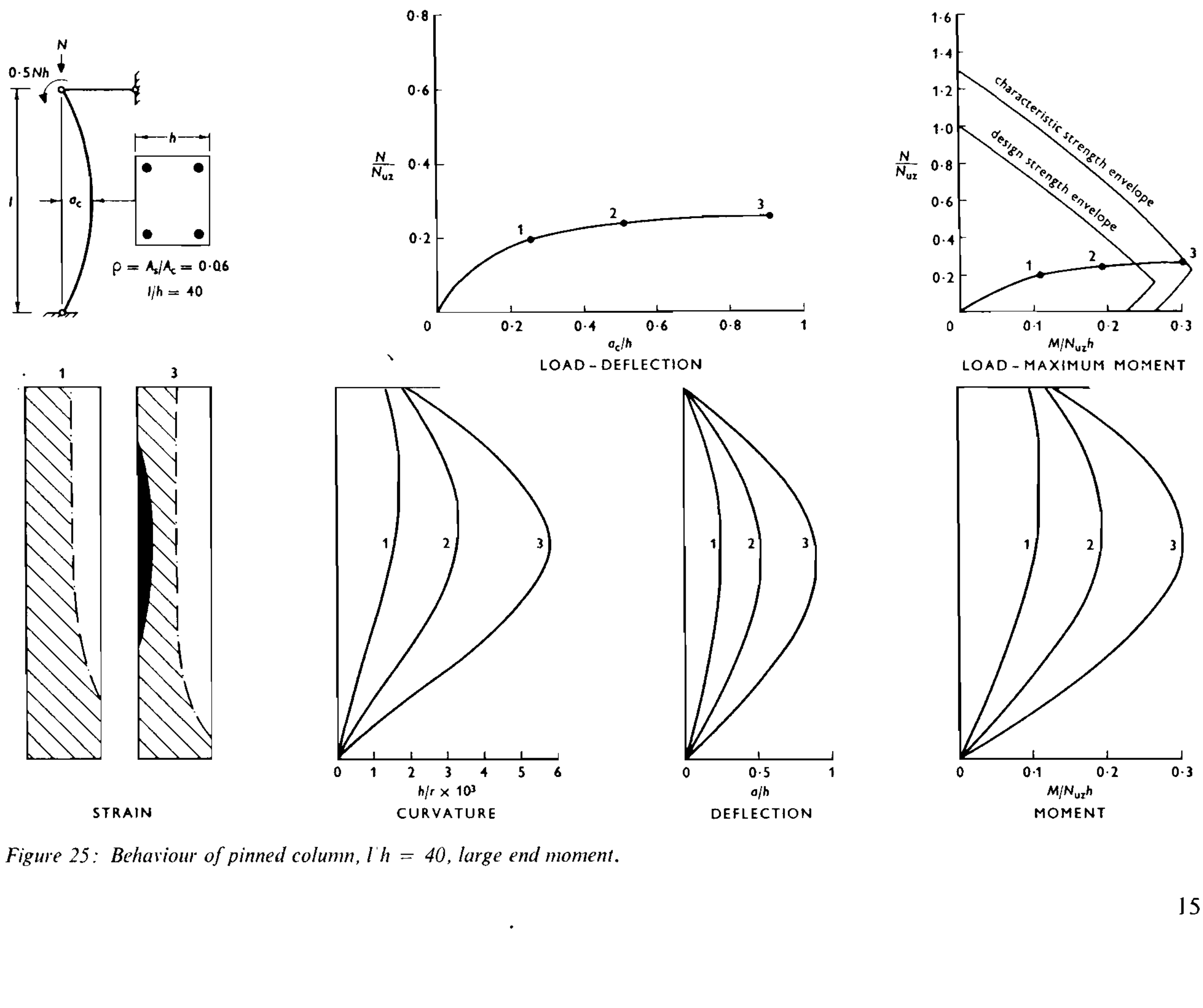


Figure 25: Behaviour of pinned column, l/h = 40, large end moment.

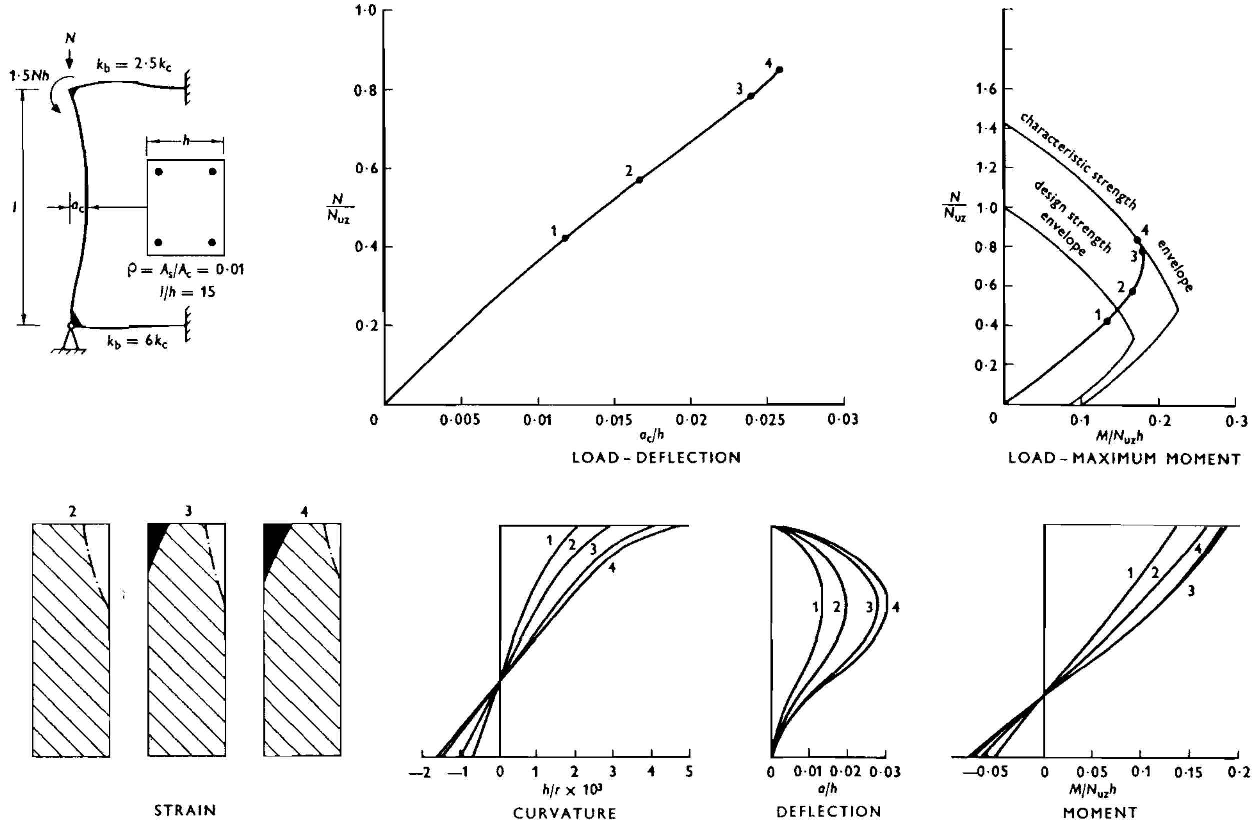


Figure 26: Behaviour of braced column, l/h = 15, moment at one end.

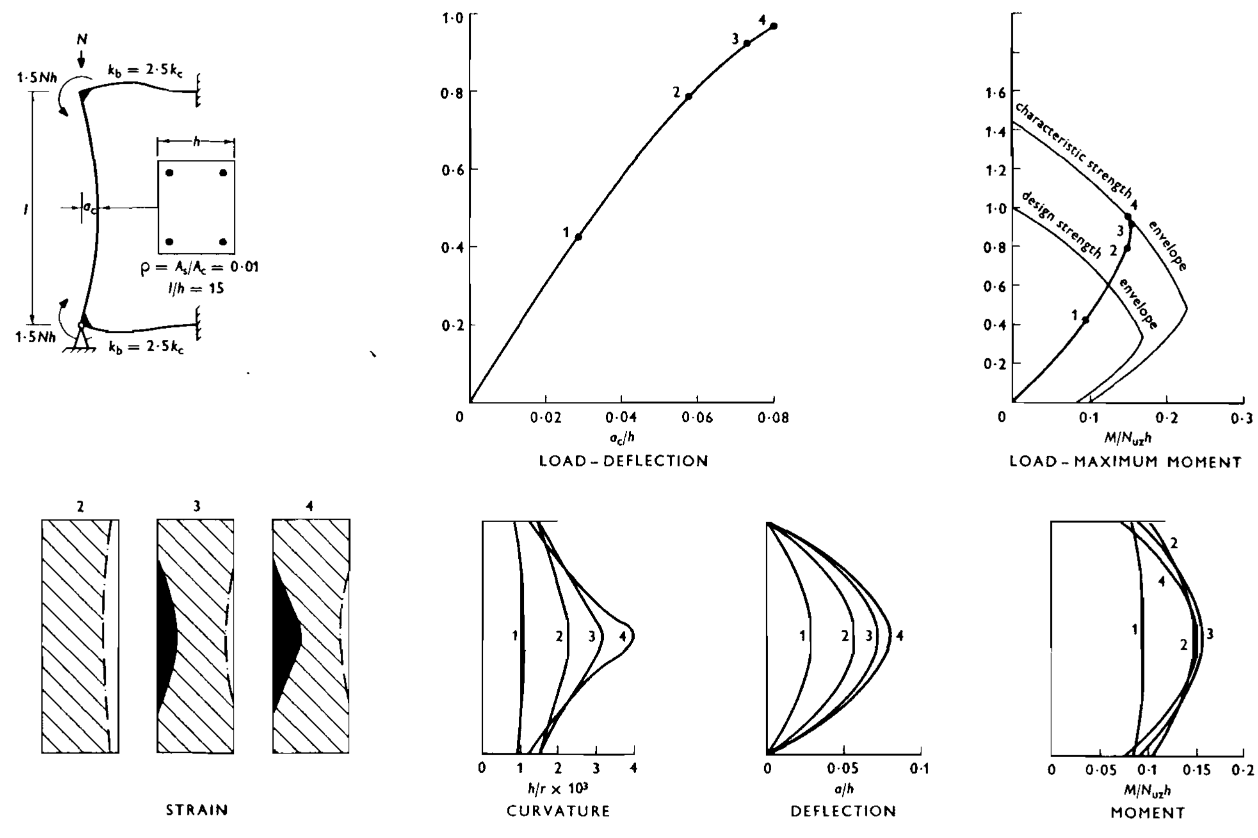


Figure 27: Behaviour of braced column, l/h = 15, moment at both ends.

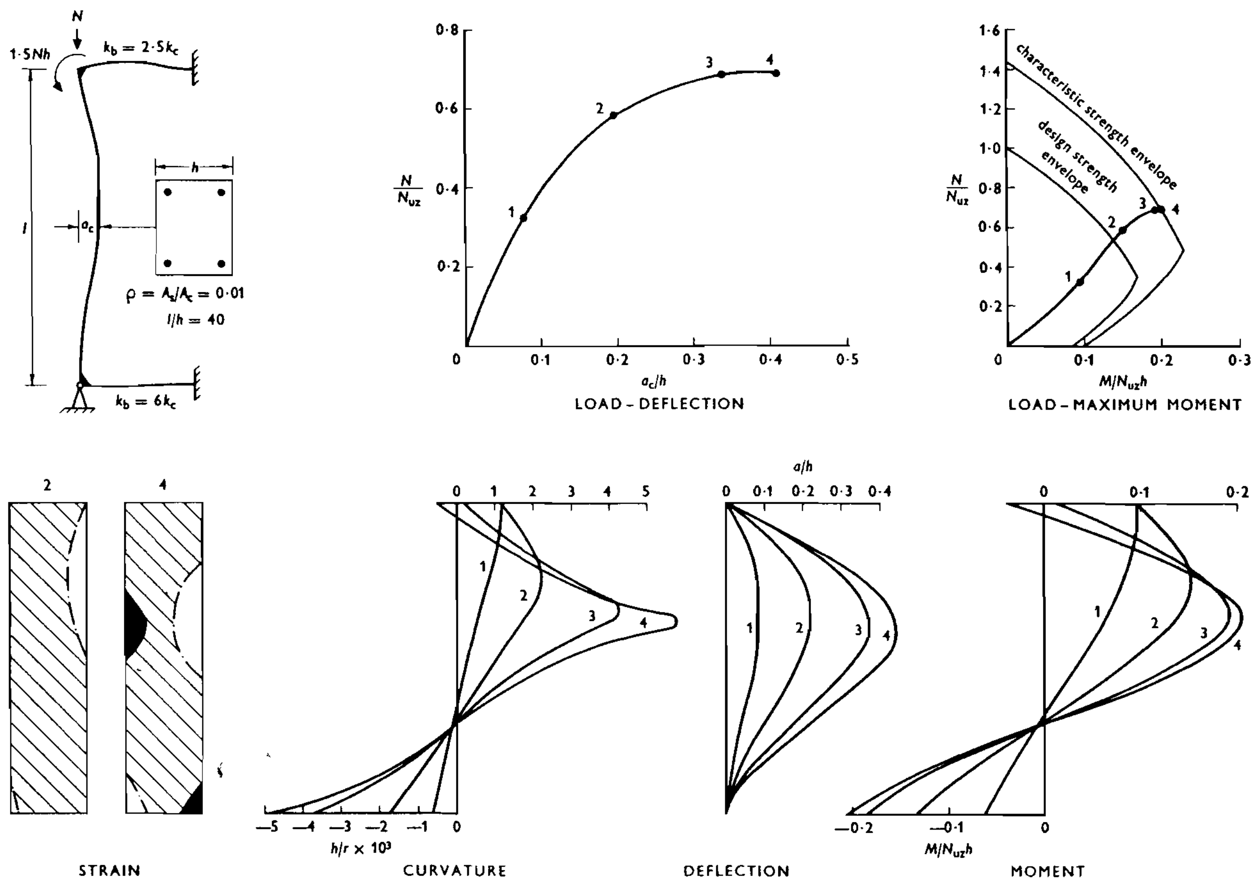


Figure 28: Behaviour of braced column, l/h = 40, moment at one end.

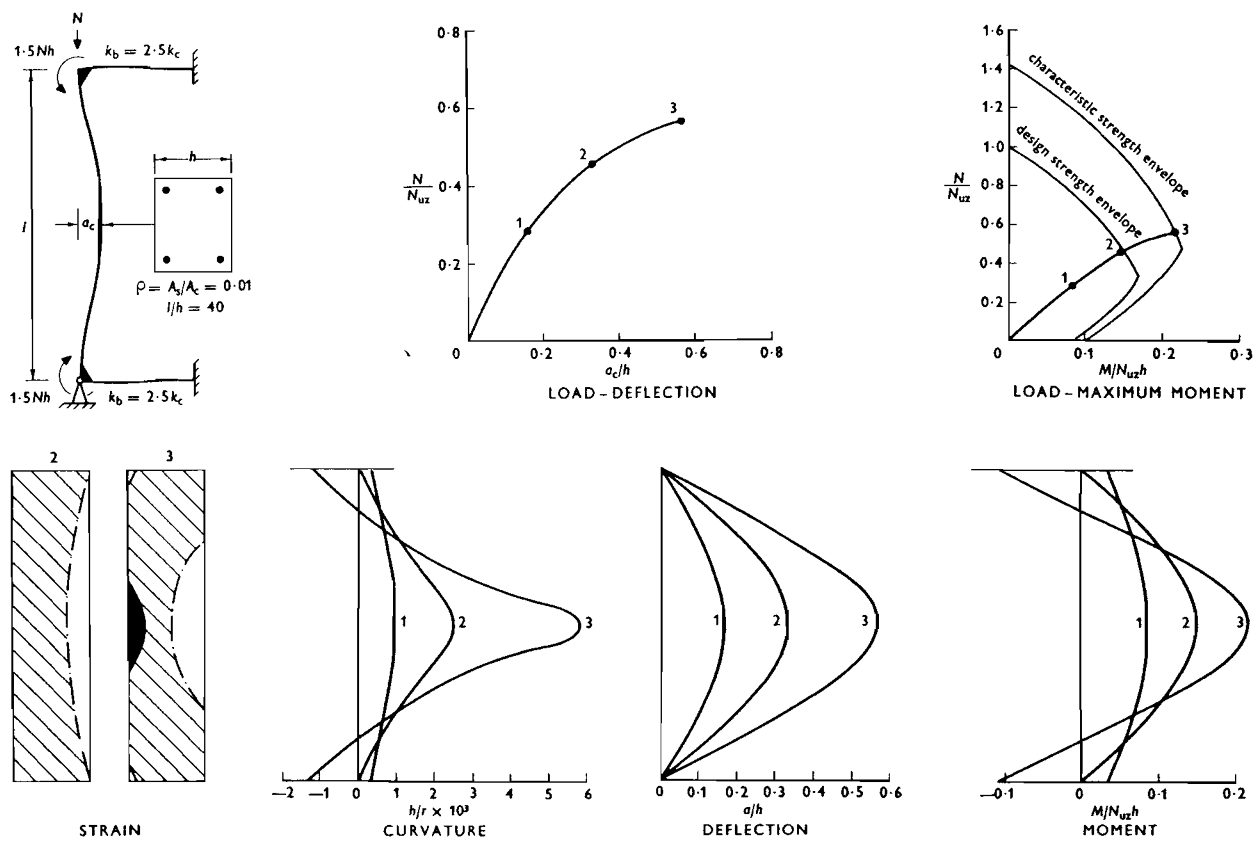


Figure 29: Behaviour of braced column, l/h = 40, moment at both ends.

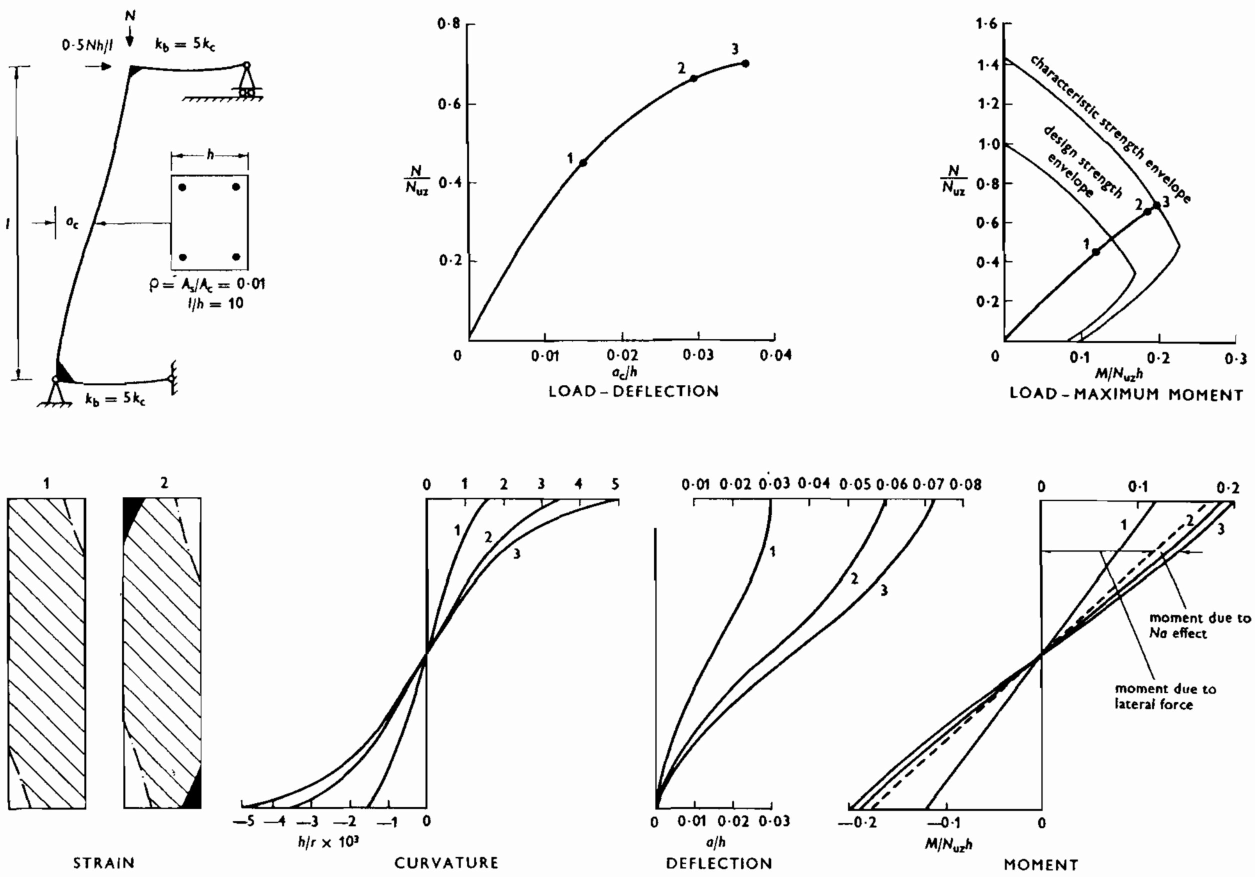


Figure 30: Behaviour of unbraced column, l/h = 10, stiff beams.

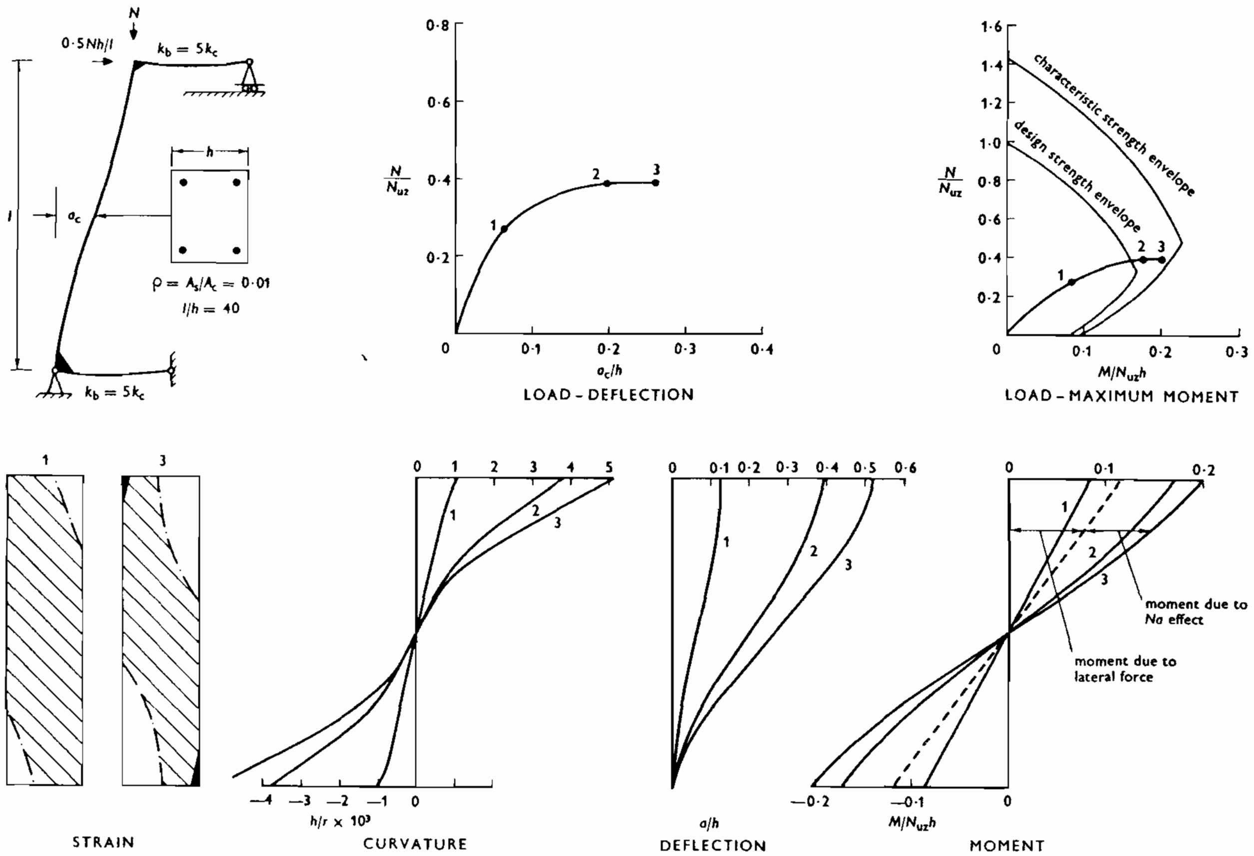


Figure 31: Behaviour of unbraced column, l/h = 40, stiff beams.

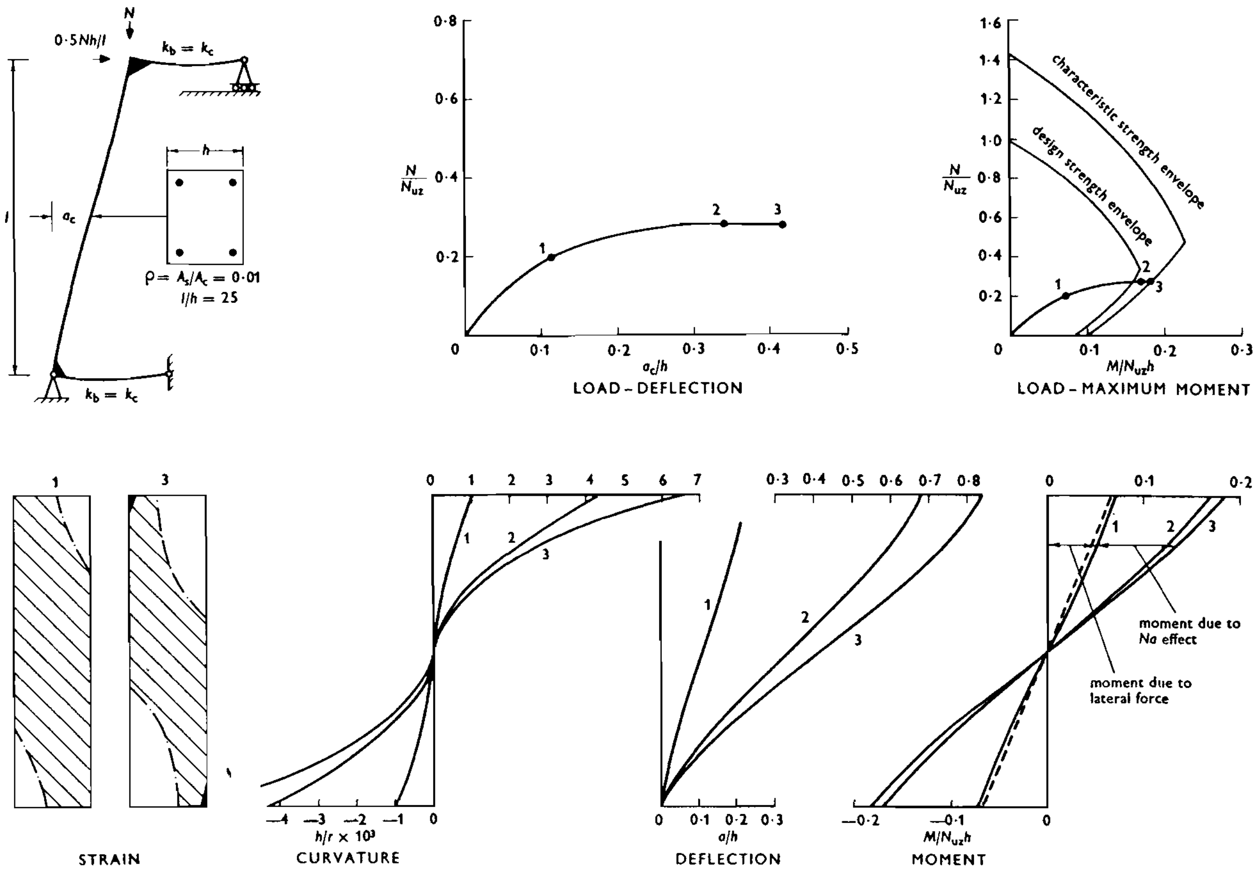


Figure 32: Behaviour of unbraced column, l/h = 25, large lateral load.

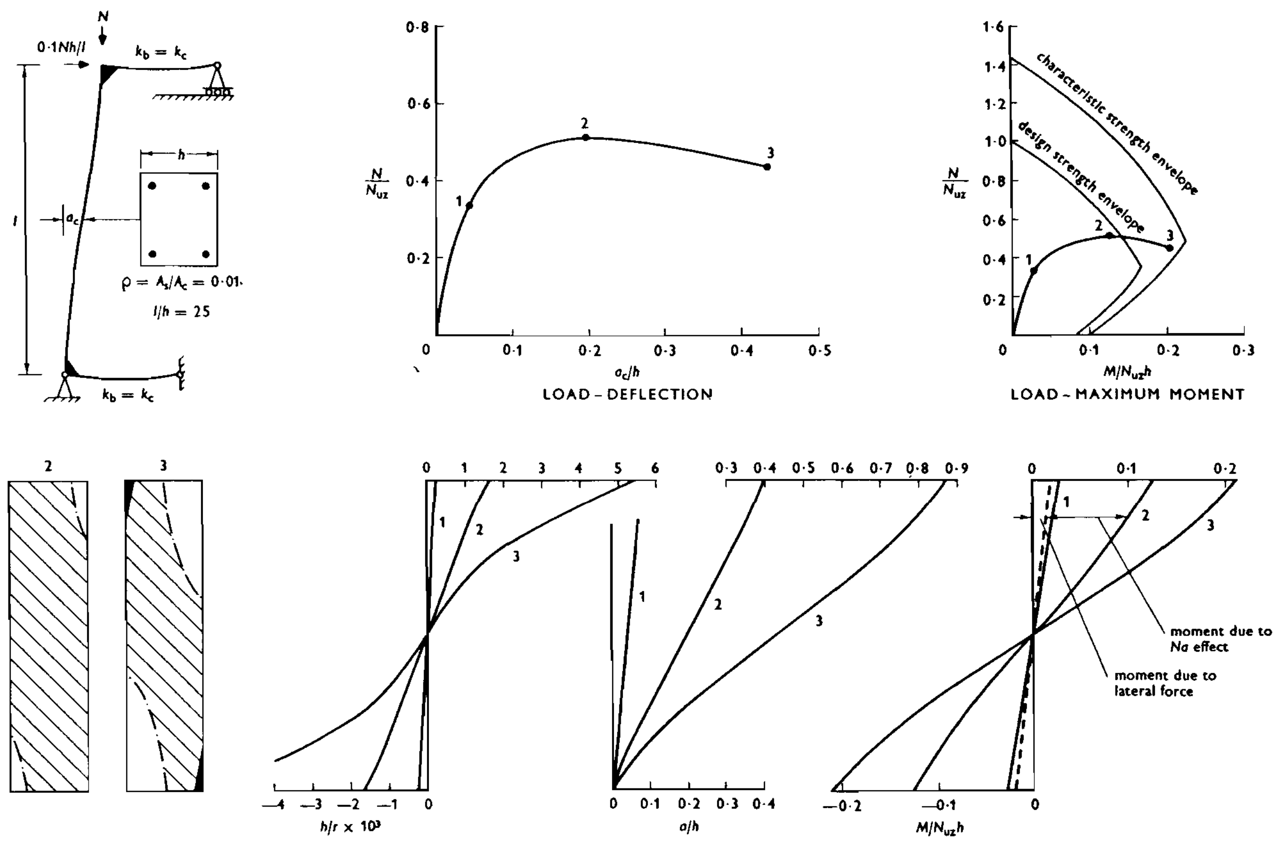


Figure 33: Behaviour of unbraced column, l/h = 25, small lateral load.

The results for each type of column are now discussed. Only results for square columns are discussed here, since essentially similar results were obtained for the circular columns.

Pinned columns (Figures 22 to 25)

Figure 22 deals with a short column with a large end moment. The moments in the column are hardly affected by the *Na* moments, and the maximum load is governed by the attainment of a strain of 0·0035 in the concrete at the end of the column, where the *Na* moment is, of course, zero.

Figure 23 deals with a longer column which is otherwise similar to that treated in Figure 22. Up to load stage 1 the maximum moment occurs at the end of the column, but as the load increases the point of maximum moment moves down the column. At load stage 3 the loads are at a maximum and the point of maximum moment is just beyond the quarter-point of the column length, and the moment there is 20% greater than the end moment. Thus, in this case, the *Na* moments do have some influence upon the maximum load.

Figure 24 deals with a slender column, approaching the upper limit of practical design. At maximum load (load stage 2) the moments at the centre of the column have become several times the moments induced by the end moment loading. The strains in the concrete at this point are still well below 0·002, i.e. the concrete is nowhere close to material failure. The reason for the attainment of maximum load is that the development of tension has reduced the stiffness, i.e. the second stage of bi-linear behaviour has been entered. This is essentially an instability failure. It is of interest to note, however, that in this case the axial load remains almost constant as the deflections increase up to load stage 4. At this stage the concrete strain is 0·0023, i.e. material failure is approaching.

Figure 25 deals again with a slender column but this time subjected to a substantial end moment. In this case the analysis has been terminated at a deflection exceeding 0·8*h*, i.e. more than 1/50 of the length. This is considered to constitute an effective collapse, since deflections of this order in a practical column would almost certainly require a subsequent replacement of the structure. It will be seen that failure by attainment of a strain of 0·0035 in the concrete is not far off at this stage. Despite the fact that the initial end moment is large, the additional *Na* moment becomes dominant at later stages.

A detailed study of the results outlined above leads to the following conclusions. For the short columns (defined as having $l/h \leqslant 15$), the *Na* moments have either no or only a very slight effect upon the maximum load, which is governed by the exhaustion of the strength of the most critically loaded section. In the intermediate range of slenderness (defined as $15 < l/h \leqslant 30$), the *Na* or instability effects can add substantially to the moments on the column and thus reduce the maximum loads; attainment of maximum load is, however, still governed by the exhaustion of the strength of the most critically loaded section. For slender columns ($l/h > 30$), the *Na* moments become dominant, and maximum load can be attained without the strength of the most critically loaded section being exhausted, i.e. there is an instability failure. For slender columns with large end moments, the attainment of a deflection greater than 1/50 of the length is possible without exhaustion of the strength of the most critically loaded section.

Braced columns (Figures 26 to 29)

Figure 26 deals with a short column. The attainment of the maximum load corresponds to the concrete at the end of the column reaching a strain of 0·0035. Between load stages 1 and 3 the axial load doubles but the bending moment diagrams show an increase in bending moment of only about one-third. This markedly non-linear behaviour is because a substantial length of the column is severely deformed and the over-all stiffness has dropped well below the initial value. This is the well-known 'redistribution of moments' effect and is not related to the *Na* moments which, for this column, can be seen to be negligible. Figure 27 shows the same column as in Figure 26 but with moments applied at both ends producing single-curvature loading. The deflections go up to 0·08*h*, as opposed to 0·03*h* in the previous example, but paradoxically the moments in the column at maximum load are lower. This is because the redistribution effect is more in evidence. In fact between load stages 3 and 4 the moments reduce throughout the column. The net result is to increase the maximum load beyond that obtained in the previous example.

Figure 28 shows a much more slender column and, as in the case of the pinned columns, the point of maximum moments moves from the end of the column almost to the centre. The end moment reduces also, practically from the beginning of loading, and is almost zero when the maximum load is reached. Again this is essentially a moment-redistribution effect, but in this instance it is brought about mainly by the influence of the *Na* moments and not so much by a reduction in stiffness of the column cross-section.

Figure 29 deals with the same column as Figure 28 but this time equal end moment loadings are applied. The deflections are somewhat larger at maximum load than in the previous case, but the reduction and reversal of moments at the end of the column are such that the central moment at maximum load is only slightly greater. The axial load at maximum load would thus be expected to be lower and it is, at $0{\cdot}57N_{uz}$ instead of $0{\cdot}69N_{uz}$. This rather large reduction is because these axial loads are close to the load corresponding to balanced conditions in the cross-section, and a small increase in applied moment has a proportionally greater effect upon the axial load capacity of the section. The maximum load of the column is governed by the attainment of a strain of 0·0035 in the concrete at the centre of the column; the load-deflection curve is, however, almost horizontal at this stage, indicating that *Na* effects equally govern.

Reviewing the results above leads to the following general conclusions for braced columns. For short columns, the *Na* effect is small and in any event enhances the maximum load by assisting the redistribution effect. The computed maximum loads of such columns are thus higher than would be obtained from a simple elastic analysis. For the intermediate and slender columns, the *Na* effects become more important, although they do not appear to reduce the carrying capacity significantly until the slenderness ratio approaches 40. This indicates the presence of a considerable reserve of strength compared with the reduction coefficient of around 0·5 which would apply to columns of this slenderness with current regulations[6].

Unbraced columns (Figures 30 to 33)

Figure 30 deals with a short column with a severe sidesway loading. The deflections and moments arising therefrom are small and do not significantly affect the maximum load, which is governed by the attainment of a strain of 0·0035 in the concrete.

Figure 31 shows a more slender column and here the *Na* moments are quite large at maximum load, amounting to about half of the total end moments. The maximum load is reached at load stage 2, when the steel and concrete strains are not too large, but at load stage 3 the tensile steel yields and the strength of the end cross-section is virtually exhausted.

Figure 32 shows the same column as Figure 31, but with the stiffness of the beams reduced. The column is thus more flexible in face of sidesway force. The additional moment is a greater proportion of the total moment than in the previous case and the maximum load is substantially lower. Attainment of maximum load is essentially governed by instability, closely followed by yielding of the steel at load stage 3.

Figure 33 is the same column as in Figure 32 but with a much reduced lateral force, approaching perhaps a more realistic value if wind loading is the source of the load. Here the *Na* moment contributes practically all of the column moment at maximum load. At maximum load, i.e. at load stage 3, the sidesway deflection is about half of that in the previous example and the curvatures and strains are all much lower. Material failure is not reached until a much larger deflection, by which time the load has reduced substantially. This is a clear example of the case III type of instability failure shown in Figure 11. The reason for the clarity of the example is that the maximum load is very close to giving the maximum amount of bilinearity possible in the moment-curvature relation.

The general behaviour of unbraced columns, as exemplified above, is much the same as for pinned columns, as would be expected. An additional point to note is that the moments arising at the ends of the slender columns must in this case be transferred into and be carried by the restraining beams. In the last example, this moment is between four and five times the simple moment due to lateral force. It follows that the additional *Na* moments arising in slender columns should be considered when the beams are being designed.

ASSESSING DESIGN ULTIMATE LOADS FROM ANALYSES

The behaviour outlined in the previous section was for columns of uniform material at characteristic strength. As discussed in the section on design assumptions, it is necessary to consider the presence locally of pockets of understrength material. The degree of understrength considered in this study corresponds to applying safety factors of 1·5 and 1·15 to the concrete and steel respectively. This gives design stress-strain curves as shown in Figures 34 and 35, which in turn lead to interaction diagrams giving combinations of ultimate axial load and moment which the various sections will sustain. These are expressed in non-dimensional terms and have, as previously mentioned, been added to the interaction diagrams based on characteristic strength shown in Figures 22 to 33 for

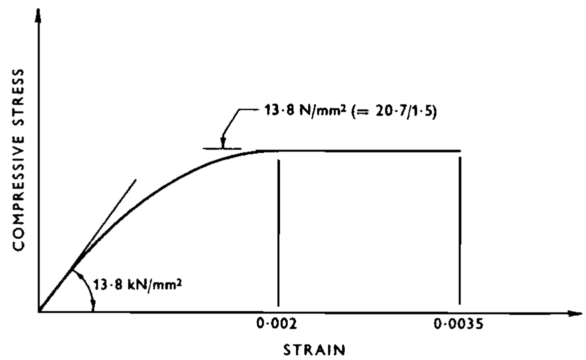


Figure 34: Design stress-strain curve for concrete with characteristic strength of 31 N/mm².

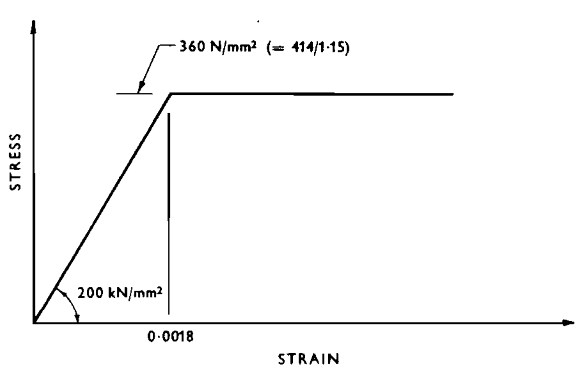


Figure 35: Design stress-strain curve for reinforcing steel with a characteristic strength of 414 N/mm².

comparison. They are also drawn to a larger scale in Figures 36 to 39 with eccentricities added.

The method of obtaining the maximum load which would have occurred in the presence of understrength material is best illustrated by an example. In Figure 22 it will be seen that the curve plotted on the interaction diagram giving the most critical combinations of axial load and moment cuts the design strength interaction diagram at $N/N_{uz} = 0{\cdot}4$; this plainly is the maximum load the column could carry if a pocket of design strength material were present at that critical section. Thus the design ultimate load is equal to $0{\cdot}4N_{uz}$. The maximum load produced by the analysis based on the characteristic strengths corresponds to $N/N_{uz} = 0{\cdot}5$, giving an over-all strength factor of $0{\cdot}5/0{\cdot}4 = 1{\cdot}25$. This is midway between 1·15 and 1·5 as would be expected.

For more slender columns, the instability effects begin to be important and the influence of the reduction in strength is not so marked. For instance, for the column shown in Figure 23, the design ultimate load is $0{\cdot}38N_{uz}$ compared with $0{\cdot}43N_{uz}$ if understrength material is not considered. The strength factor for the column as a whole is thus $0{\cdot}43/0{\cdot}39 = 1{\cdot}10$.

Where the column collapse load is governed by an instability failure, i.e. for very slender columns, the presence of understrength material, while weakening the column locally, does not influence the maximum load since the section strength is not a controlling factor. In fact, of course, a small pocket of understrength concrete will make the column deflect a little more, but this can be ignored. Figure 24 shows an example of this type.

Applying the approach just described to all 200 analyses gives the design ultimate loads which are presented in Figures 42 to 47. Also shown in these Figures are curves relating to the design proposals developed later in the report. The curves are presented non-dimensionally in terms of N_u/N_{uz} against the slenderness ratio l/h.

The results will be discussed in detail when the comparison with the proposed design method is made. It is, however, of interest at this stage to note the remarkably high loads which can be carried by the slender braced column; for instance, Figure 44 shows a capacity of $0{\cdot}4N_{uz}$ corresponding to a slenderness ratio of 50. This is much higher than present design rules would suggest as reasonable. The other major point to note is the drastic reduction in capacity of the unbraced column with respect to slenderness ratios, particularly where the beam stiffnesses are equal to half the column stiffness.

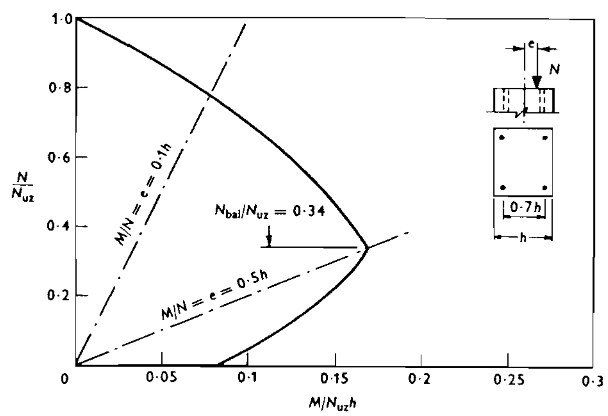


Figure 36: Interaction diagram based on design strength. Rectangular column, $\rho = 0{\cdot}01$.

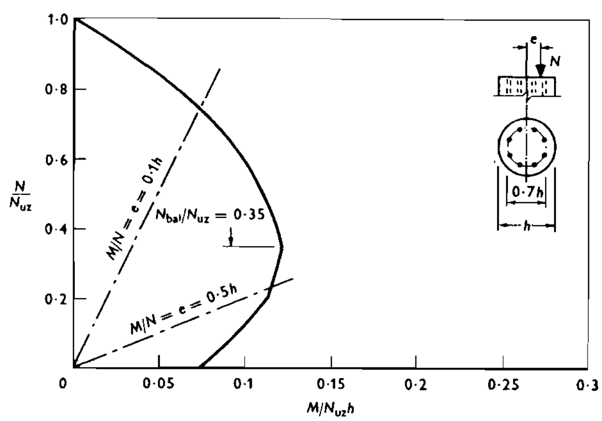


Figure 38: Interaction diagram based on design strength. Circular column, $\rho = 0{\cdot}01$.

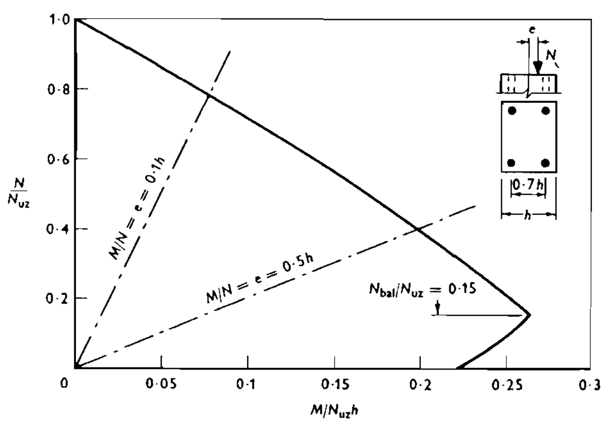


Figure 37: Interaction diagram based on design strength. Rectangular column, $\rho = 0{\cdot}06$.

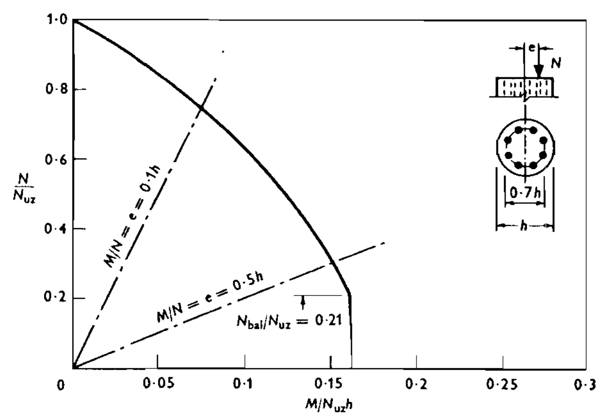


Figure 39: Interaction diagram based on design strength. Circular column, $\rho = 0{\cdot}06$.

The design method

GENERAL

As indicated in the introduction, design methods should be simple to use and should also reflect the actual behaviour reasonably accurately. In this section, the behaviour reviewed in the previous section is summarized, and then some general observations are made regarding the choice of design method. The method itself is then briefly outlined.

The behaviour reviewed in the previous section of this report can be classified into four different types. It is helpful for the time being to continue using the terms short columns for cases where $l/h \leqslant 15$, intermediate columns for those where $15 < l/h \leqslant 30$ and slender columns for those where $l/h > 30$. The types of failure are discussed below.

(1) *Material failure.* This is defined by the attainment of the maximum capacity at a section while the *Na* moments at that section are not significant. This behaviour is typical of short columns but it can also apply to intermediate columns where large end moments are applied in conjunction with the axial load.

(2) *Material failure influenced by Na moments.* This is also defined by the attainment of maximum capacity at a section, but in this case the *Na* moments at that section contribute significantly to the total moment. This behaviour is typical of intermediate columns, and for unbraced and pinned intermediate columns the *Na* effects can considerably reduce the load capacity in comparison with short columns. However, for a braced intermediate column, the *Na* moments can have a beneficial effect in that the effective stiffness of the column is reduced; consequently the end moments reduce and may even reverse from values calculated from a simple elastic analysis.

(3) *Instability failure.* This is defined as the attainment of maximum load before material failure develops at any cross-section. As shown in the introductory section on the column problem, this type of failure can only occur with sections exhibiting a markedly non-linear moment-curvature relation. The column itself also has to be long.

(4) *Deflection failure.* This has been arbitrarily taken to be when the deflection reaches 1/50 of the clear length of the column. Whilst a column reaching this state will generally have a small reserve of strength, it is not really usable, because the owner of the structure would almost certainly insist on complete replacement if deflections of this order actually occurred. This kind of failure only applies to long columns.

To cover these four types of behaviour accurately is plainly a matter of some difficulty. The first type, i.e. material failure, can be dealt with to a large extent by having a classification of short columns for which the design is carried out without reference to the *Na* moments. For the other types of failure, the *Na* moments have to be considered. The question of accuracy, however, decreases in importance as the slenderness of the columns increases. This is because the efficiency of the long column in carrying axial load is low and it will therefore not be used much in design. So not many practical columns should display instability or deflection failure. Therefore it is reasonable to derive a method which will deal accurately with material failure influenced by *Na* moments, i.e. the intermediate columns. Whilst such a method will not deal realistically with instability and deflection failure, it should be possible to ensure that the method is conservative for such cases.

The first method for consideration is an approach which uses the rigorous analysis[3] as used in this report. It is possible that with modern developments in computers this may become feasible, but for the moment the approach cannot be considered because of the time taken to prepare input data and the cost of computer time.

Another approach is the reduction factor method as used at present in this country[6]. It is open to the philosophical objection that it effectively suggests magnifying both the axial load and moments, whereas simple logic suggests that the moment alone should be magnified. A more serious objection is that it does not convey any clear idea of the mode of collapse of the column. Despite these objections, a number of studies were made in an endeavour to develop the method; these were, however, abandoned at an early stage.

The approach finally chosen for detailed study is that suggested by the European Committee for Concrete's Commission on Buckling. Their approach has been developed and refined in various bulletins[7-13] published over the last decade. At stages throughout the development of the method, comparisons have been made with test evidence. For instance, comparisons with 269 individual test results are given in reference 13. A more extensive comparison is given in a later section of this report.

The essential point in the method is the provision of a relatively simple expression for a so-called 'additional moment'. This moment is added to the moments calculated from a first-order theory, i.e. simple elastic analysis. The column section is designed to withstand the axial load and total moment. A more detailed presentation and development of this principle is given in the sub-sections below.

BASIC EXPRESSION FOR ADDITIONAL MOMENT

The expression proposed in the most recent CEB *Recommendations*[14] is

$$M_a = N \frac{l_e^2}{10}\left(\frac{1}{r_u}\right) \quad \ldots\ldots\ldots\ldots (1)$$

where M_a = additional moment;
N = design axial load;

l_e = effective (or buckling) length of the column determined from elastic theory;
$1/r_u$ = curvature due to loading at the centre of the effective length.

To assess the curvature ($1/r_u$), the following expression applies:

$$\frac{1}{r_u} = \frac{\left(0{\cdot}003 + \frac{f_y^*}{E_s} - \frac{l_e}{50{,}000h}\right)}{h} K_1 \ldots\ldots(2)$$

where f_y^* = design strength of the reinforcement;
E_s = modulus of elasticity of the reinforcement;
h = over-all depth of the section;
K_1 = a factor depending upon the intensity of axial load.

K_1 is given by the following expression*:

$$K_1 = \frac{N_{uz} - N}{N_{uz} - N_{bal}} \leqslant 1{\cdot}0 \ldots\ldots\ldots\ldots(3)$$

where N_{uz} is the axial load capacity of the section and N_{bal} is the axial load corresponding to 'balanced' conditions, i.e. when the tension steel has just reached its design strength simultaneously with the attainment of maximum or ultimate strain in the outermost concrete compression fibre.

Equation 1 has already been developed in the introductory section on the column problem and needs no further explanation. Equation 2 is an estimate of the curvature under maximum or ultimate conditions when the section is in the 'balanced' condition, provided K_1 is taken as equal to 1·0. The term $l_e/50{,}000h$ introduces the slenderness ratio l_e/h as a variable here, but its effect is small in the practical range of ratios up to 50. Equation 3 varies K_1 from 1·0 at the point corresponding to the balanced condition to zero as the axial load is increased up to the maximum possible capacity. These equations deal with short-term loading and make no allowance for any long-term deflections which can take place under service load conditions. Suggestions for dealing with this have been made but will be considered later.

CALCULATION OF INITIAL MOMENT

The calculation of initial moments is carried out by simple elastic theory. This, of itself, presents no difficulties but for most braced columns the design loading condition produces double-curvature bending with the maximum moment occurring at one end. It is plain that the additional moment has a major influence at the centre of a braced column and here the initial moments are less than at the ends. MacGregor[12] has suggested the following relation for the effective initial moment to which the additional moment should be added.

$$M_i = 0{\cdot}4M_1 + 0{\cdot}6M_2 \geqslant 0{\cdot}4\, M_2$$

where M_1 is the smaller end moment, taken as negative where the column is bent in double curvature, and M_2 is the larger end moment, taken as positive.

In design, of course, the column section will still have to be designed to carry M_2 at the end, and in many cases this will be found to be the most critical loading.

For unbraced columns, the additional moment must be added at the end of the column and thus the initial moment in this case will be the actual calculated end moment.

APPLICATION TO COLUMNS PREVIOUSLY ANALYSED

For these columns the design strength f_y^* of the steel is 360 N/mm² with a modulus of elasticity E_s of 200 kN/mm². The expression for curvature $1/r_u$ thus becomes

$$\frac{1}{r_u} = \frac{0{\cdot}003 + 0{\cdot}0018 - \frac{l_e}{50{,}000h}}{h} K_1$$

$$= \frac{1}{208h}\left(1 - 0{\cdot}00415\frac{l_e}{h}\right) K_1$$

Substituting now into equation 1 gives

$$M_a = \frac{Nh}{2080}\left(\frac{l_e}{h}\right)^2\left(1 - 0{\cdot}00415\frac{l_e}{h}\right) K_1$$

The interaction diagrams for strength, based on the design stress-strain curves as given in Figures 34 and 35, are presented in Figures 36 to 39. The axial load corresponding to N_{bal} is also marked on Figures 36 to 39. It will be seen that, for the rectangular section, $N_{bal}/N_{uz} = 0{\cdot}34$ for the case with $\rho = 0{\cdot}01$, whilst for the higher steel areas corresponding to $\rho = 0{\cdot}06$, $N_{bal}/N_{uz} = 0{\cdot}15$.

Since K_1 is not known beforehand, it is plain that a trial-and-error procedure is required to reach a solution, in cases where the design ultimate load N_u is greater than N_{bal}. To illustrate the procedures, typical examples will be worked out.

Pinned columns

Consider the case of $l_e/h = 10$, $\rho = 0{\cdot}01$, with end moments $M_2 = 0{\cdot}1Nh$, $M_1 = 0$.

$$M_i = 0{\cdot}06Nh$$

Assume that $N_u/N_{uz} = 0{\cdot}78$, giving

$$K_1 = \frac{1{\cdot}00 - 0{\cdot}78}{1{\cdot}00 - 0{\cdot}34} = 0{\cdot}33$$

*The expression for K_1 given in the *Recommendations*[14] is simpler but less accurate; it was adopted by the editors in response to suggestions about the difficulty of using equation 3 above in design. The more accurate expression has been retained here, but the design difficulty is met by making the use of K_1 optional in the proposals developed later in this report.

Hence

$$M_a = \frac{Nh}{2080} \times 10^2 (1 - 0{\cdot}00415 \times 10)\, 0{\cdot}33$$
$$= 0{\cdot}015Nh$$

Therefore the total moment $M_t = (0{\cdot}06 + 0{\cdot}015)Nh = 0{\cdot}075$ at the centre of the column. In this case M_2 governs and so the design ultimate load is found on the interaction diagram corresponding to an eccentricity of load of $0{\cdot}1h$. Consulting Figure 36 gives a value of 0·78 for N_u/N_{uz}. This agrees with the assumption made when assessing K_1 and is therefore the correct solution.

Consider now a column similar but with $l_e/h = 25$. Assume that $N_u/N_{uz} = 0{\cdot}55$, giving

$$K_1 = \frac{1{\cdot}00 - 0{\cdot}55}{1{\cdot}00 - 0{\cdot}34} = 0{\cdot}68$$

Hence

$$M_a = \frac{Nh}{2080} \times 25^2 (1 - 0{\cdot}00415 \times 25) \times 0{\cdot}68$$
$$= 0{\cdot}175Nh$$

Therefore

$$M_t = (0{\cdot}06 + 0{\cdot}175)Nh = 0{\cdot}235Nh > 0{\cdot}1Nh,$$

and therefore M_t governs. Thus the design ultimate load can be found from the interaction diagram in Figure 36 corresponding to an eccentricity of $0{\cdot}235h$. This gives $N_u/N_{uz} = 0{\cdot}55$; this checks with the value of N_u/N_{uz} assumed to give K_1 and thus no further trials are necessary.

Braced columns

Consider the example illustrated in Figure 40.

$$M_i = 0{\cdot}32Nh\left[0{\cdot}6 - 0{\cdot}4\left(\frac{0{\cdot}12}{0{\cdot}32}\right)\right]$$
$$= 0{\cdot}144Nh$$

From elastic buckling theory, l_e in this case equals $0{\cdot}69l$. Therefore

$$\frac{l_e}{h} = 34{\cdot}5$$

Assume that $N_u/N_{uz} = 0{\cdot}40$, giving

$$K_1 = \frac{1{\cdot}00 - 0{\cdot}40}{1{\cdot}00 - 0{\cdot}15} = 0{\cdot}70$$

Hence

$$M_a = \frac{Nh}{2080} \times 34{\cdot}5^2 (1 - 0{\cdot}00415 \times 34{\cdot}5) \times 0{\cdot}70$$
$$= 0{\cdot}344Nh$$

Therefore

$$M_t = 0{\cdot}344 + 0{\cdot}144 = 0{\cdot}49Nh > 0{\cdot}32Nh,$$

and therefore M_t governs. This gives $N_u/N_{uz} = 0{\cdot}40$, from Figure 37; this agrees with the initial assumption and is thus correct.

Unbraced columns

Consider the example shown in Figure 41.

$$M_i = 0{\cdot}05Nh$$

From buckling theory, $l_e = 1{\cdot}58l$. Therefore

$$l_e/h = 23{\cdot}7$$

Assume that $N_u/N_{uz} = 0{\cdot}61$, giving

$$K_1 = \frac{1{\cdot}00 - 0{\cdot}61}{1{\cdot}00 - 0{\cdot}34} = 0{\cdot}59$$

Hence

$$M_a = \frac{Nh}{2080} \times 23{\cdot}7^2 (1 - 0{\cdot}00415 \times 23{\cdot}7)\, 0{\cdot}59$$
$$= 0{\cdot}145Nh$$

Therefore

$$M_t = (0{\cdot}145 + 0{\cdot}05)Nh = 0{\cdot}195h > 0{\cdot}05Nh$$

and therefore M_t governs. From Figure 36, $N_u/N_{uz} > 0{\cdot}61$, which agrees with the initial assumption and is therefore correct.

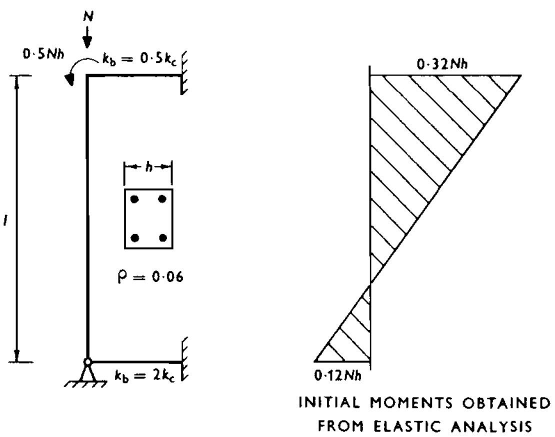


Figure 40: Braced column example, l/h = 50.

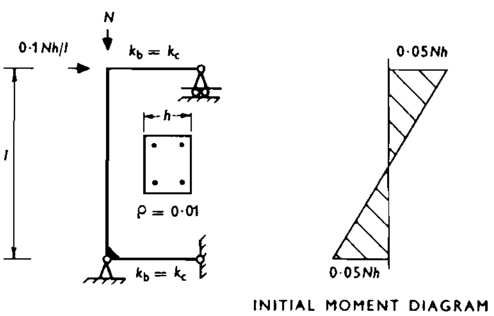


Figure 41: Unbraced column example, l/h = 15.

Comparison between design method and analysis

The results for rectangular columns are presented in Figures 42 to 44, covering the various categories studied. The results for each category are discussed in turn below. Circular columns are covered in Figures 45 to 47.

It should be noted that in Figures 42 to 47:

'Analysis' denotes ultimate loads obtained from computer studies;

'Design I' denotes ultimate loads assessed from the design procedure taking

$$M_a = \frac{Nh}{2080}\left(\frac{l_e}{h}\right)^2\left(1 - 0{\cdot}00415\,\frac{l_e}{h}\right)K_1$$

and using elastic theory to determine the effective length l_e;

'Design II' denotes ultimate loads assessed from the design procedure taking

$$M_a = \frac{Nh}{1750}\left(\frac{l_e}{h}\right)^2\left(1 - 0{\cdot}0035\,\frac{l_e}{h}\right)$$

and using the simple Code procedures (p. 36) to determine the effective length.

PINNED RECTANGULAR COLUMNS

These can be seen in Figure 42, in which the solid lines give the analytical results and the broken lines the results from the design method. It will be seen that agreement is excellent, apart from the case with $M = 0{\cdot}1Nh$ and $\rho = 0{\cdot}01$. For the higher slenderness ratios, the design method underestimates the capacity by a considerable amount. This is a region where instability rather than material failures occur and the discrepancy is to be expected.

BRACED RECTANGULAR COLUMNS

The results for $\rho = 0{\cdot}01$ are given in Figures 43a to d, and those for $\rho = 0{\cdot}06$ in Figures 43e to h. Where $\rho = 0{\cdot}01$, the design method gives an underestimate in all cases, of the same order throughout the slenderness range. The discrepancy in the lower slenderness range is due to redistribution of moments which takes place in the analysis, which takes rigorous account of the non-linear behaviour of the column. For the cases with $\rho = 0{\cdot}06$, the agreement is excellent.

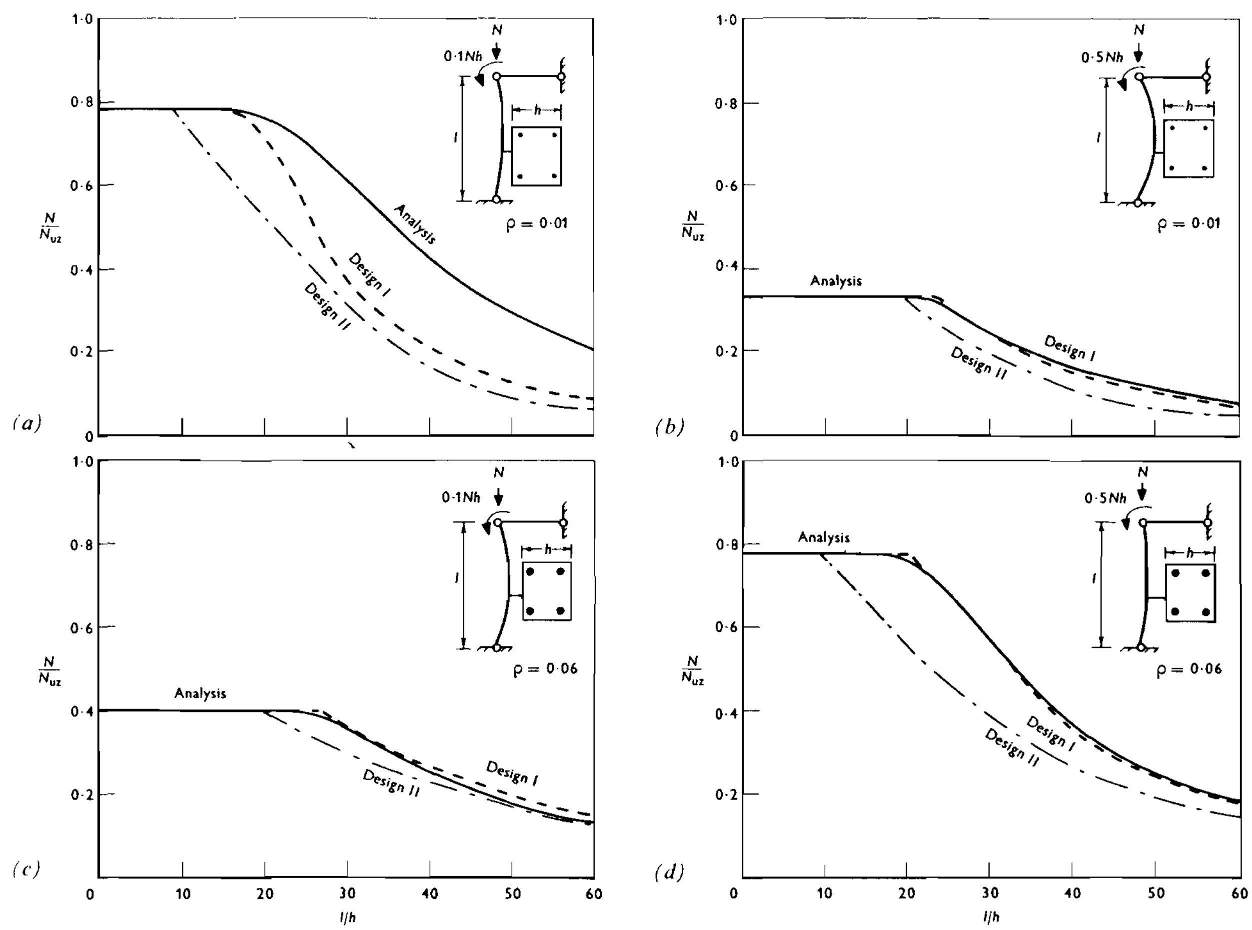
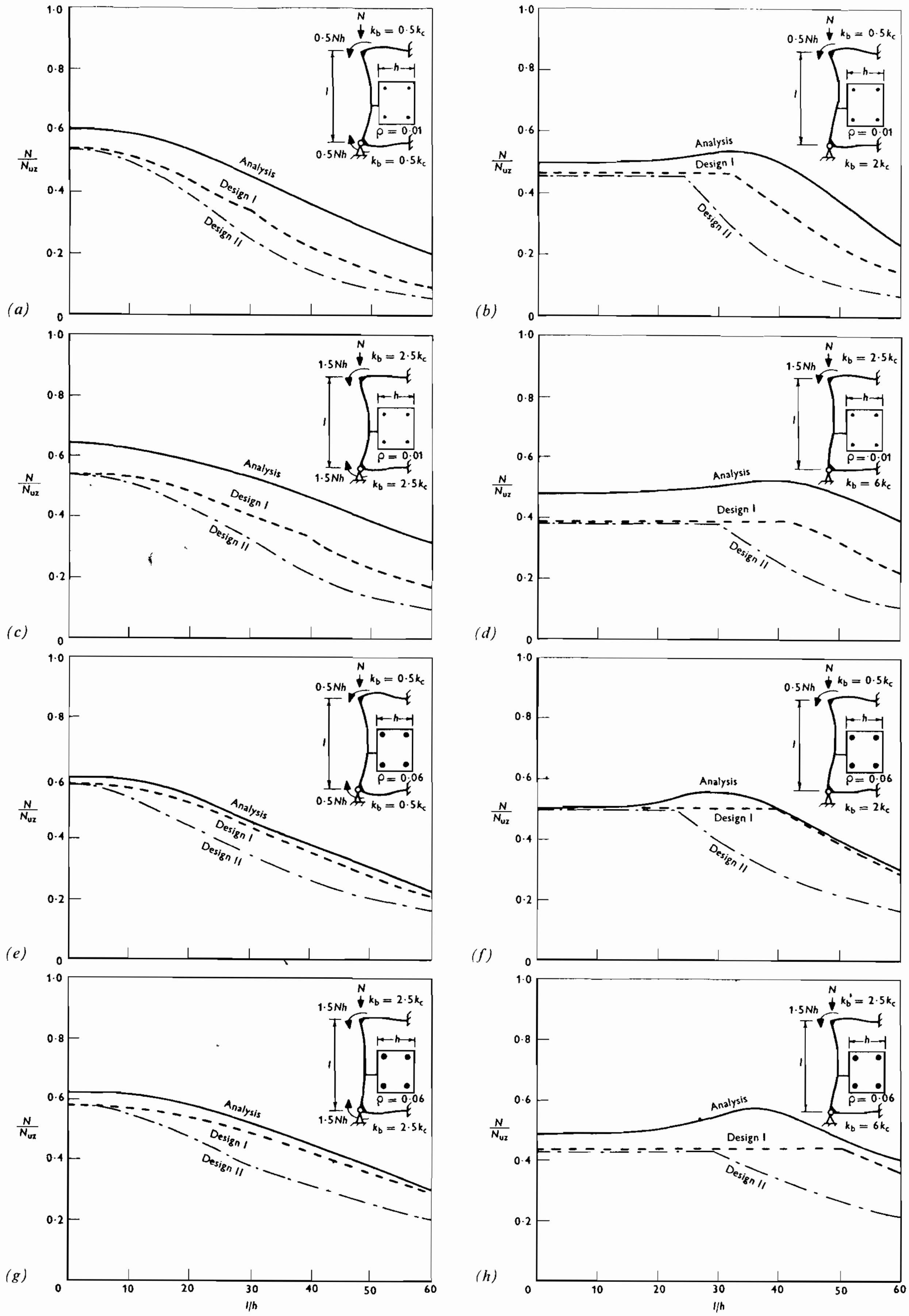


Figure 42: Ultimate axial loads for pinned rectangular columns.


(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
N/N_uz
l/h
0
0·2
0·4
0·6
0·8
1·0
10
20
30
40
50
60
Analysis
Design I
Design II
N
0·5Nh
1·5Nh
k_b = 0·5k_c
k_b = 2k_c
k_b = 2·5k_c
k_b = 6k_c
h
l
ρ = 0·01
ρ = 0·06

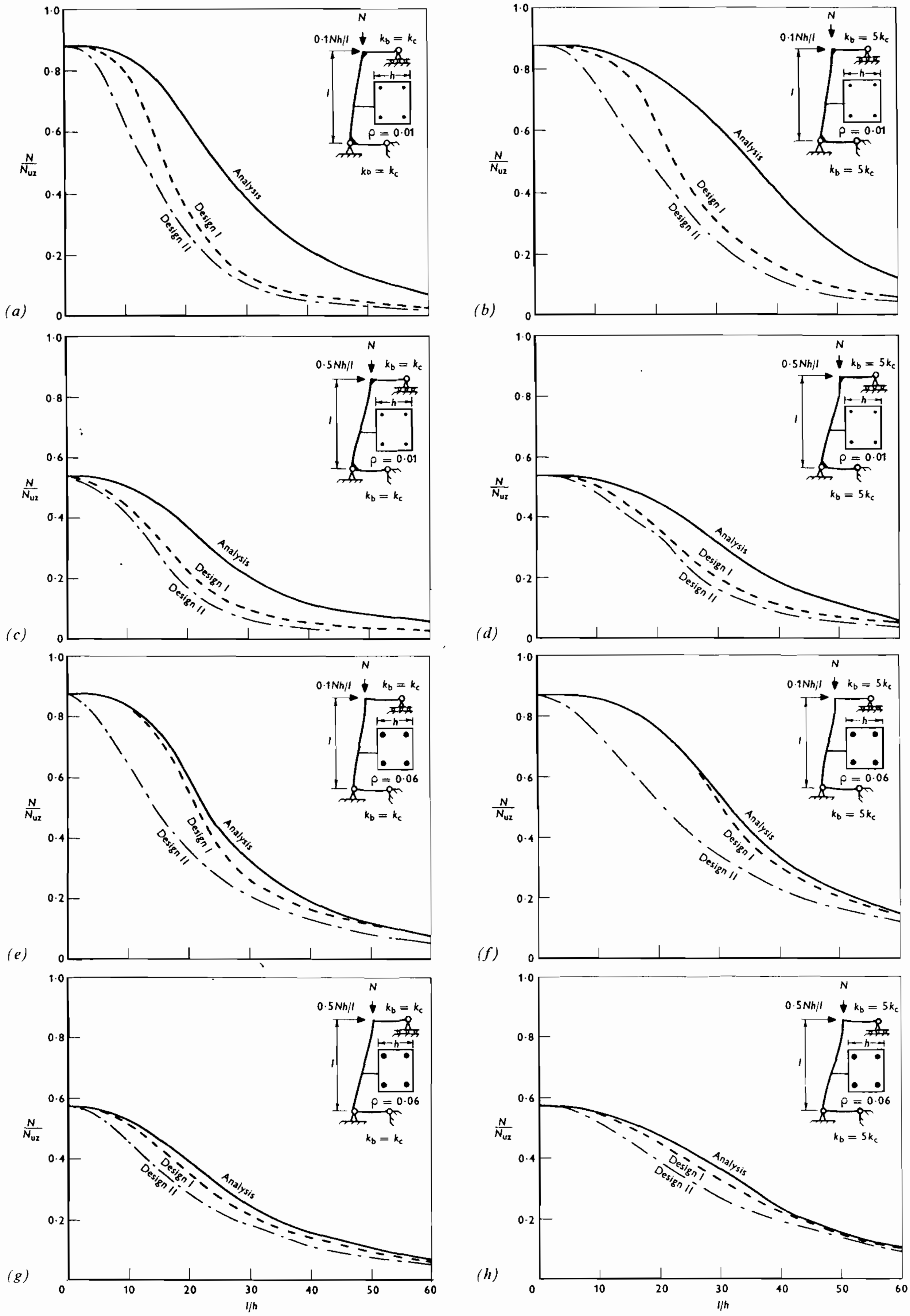


Figure 44: Ultimate axial loads for unbraced rectangular columns.

UNBRACED COLUMNS

The cases for $\rho = 0{\cdot}01$ are covered by Figures 44a to d, whilst those for $\rho = 0{\cdot}06$ are given in Figures 44e to h. The results for $\rho = 0{\cdot}01$ show considerable discrepancies, the design method being conservative particularly in the case with low initial moments and low beam stiffness (Figure 44). This represents an extreme practical case. For $\rho = 0{\cdot}06$, the agreement is excellent in all cases.

CIRCULAR COLUMNS

The results for circular columns are presented in Figures 45 to 47. A comparison between Figures 42 to 44 and 45 to 47 shows only very slight differences between the rectangular and circular columns. The design method tends to be slightly more optimistic for the circular columns but the difference is very small. This is at first sight surprising since the design method is based on the actual depth of the member, *h*, whereas the radius of gyration for a circular section is 0·25/0·289 = 0·86 times that for a rectangle of depth *h*. However, study of the design interaction diagrams (Figures 36 to 39) shows the circular section to be much weaker in bending in proportion and this is reflected in the ultimate loads obtained from the design method.

DISCUSSION

The over-all result is very satisfactory in that the design approach can be seen to be accurate over a wide range. Particularly satisfying are the analyses of the braced columns for which very few test data are available and for which, in consequence, few comparisons have been made to date.

A degree of conservatism does enter the picture for cases where the applied initial moments are low and the steel percentages are low. This is not too important since, in practice, accidental moments arise which will reduce the possibility of very low initial moments being present; and with the lower percentages of steel, concrete creep may have a greater effect upon deflexions and some degree of conservatism could well be desirable.

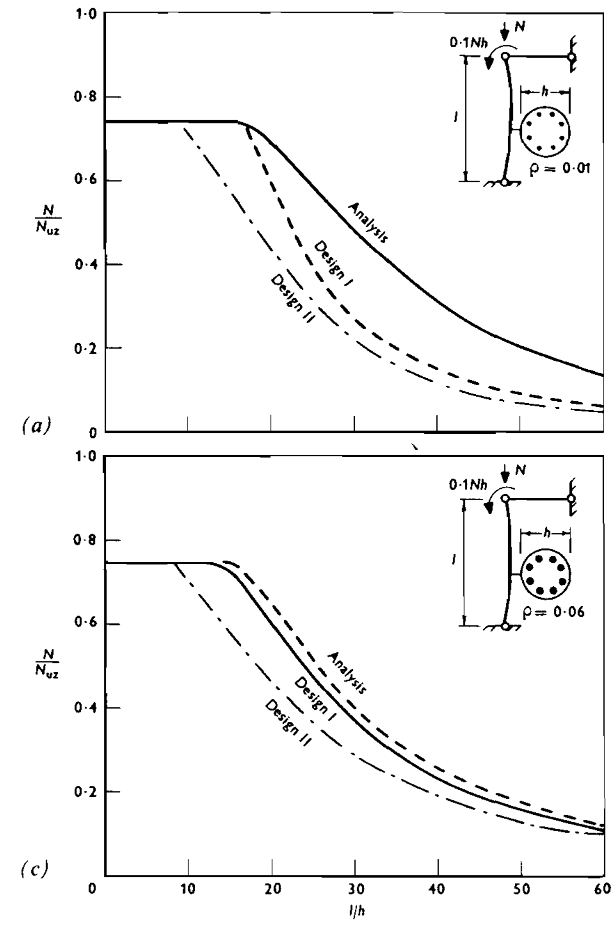


Figure 45: Ultimate axial loads for pinned circular columns.

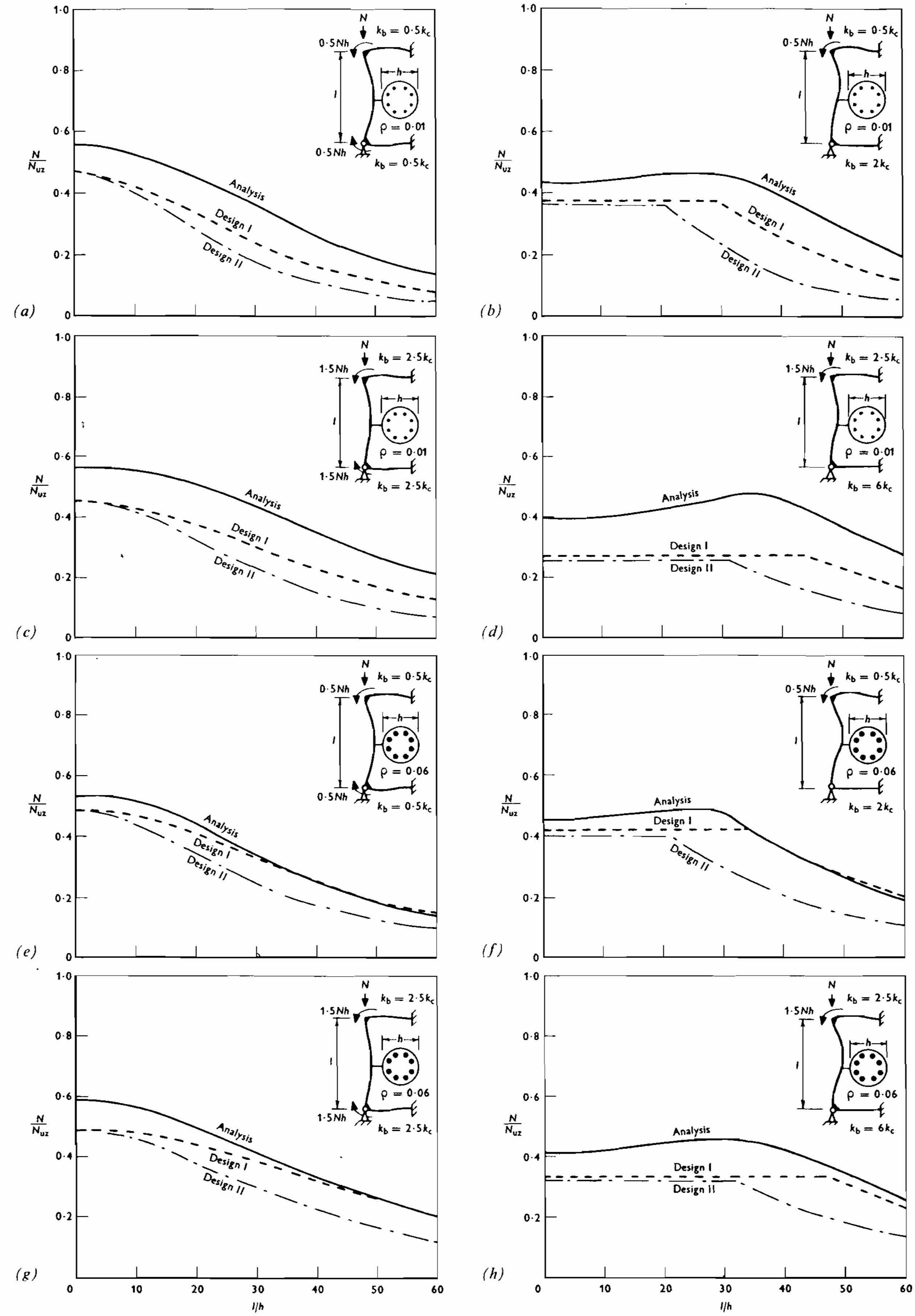


Figure 46: Ultimate axial loads for braced circular columns.

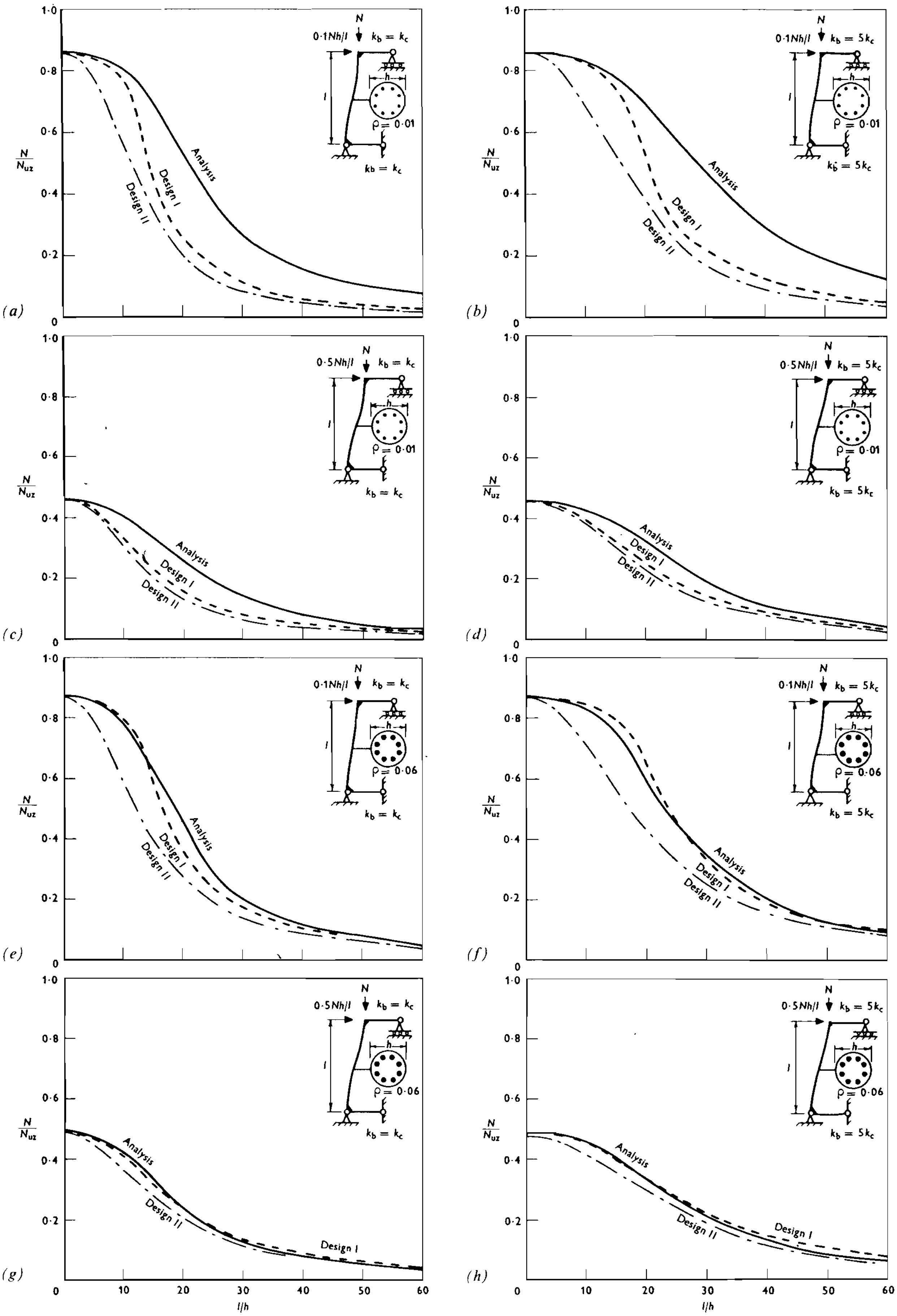


Figure 47: Ultimate axial loads for unbraced circular columns.

Further development of design method

GENERAL

The comparison presented in the previous section has confirmed the accuracy of the basic approach, when applied to a given range of columns. It is necessary to develop the approach further for general application.

The expression for additional moment requires to be generalized to deal with all types of reinforcement and to take account of possible long-term deformations under service loads. Simplified approaches to the calculation of effective lengths and of initial moments also appear desirable. The possibility that column sections may be stronger about one axis than another should be considered. And finally the approach must be developed to deal with slender beams. These points are covered in some detail below, after which a summary of the approach, written in the style of a Code of Practice, is given.

GENERAL EXPRESSION FOR ADDITIONAL MOMENT

In the basic equation for curvature (equation 2) the concrete strain is taken as 0·0030 and, as stated on page 24, no allowance has been made for long-term effects under service conditions. In the most recent CEB *Recommendations*[14], it is suggested that this concrete strain be multiplied by a factor dependent upon the age of loading, atmospheric conditions, and the ratio of the moment under the long-term or permanent load to that under full characteristic load. It is only under exceptional circumstances that this factor will exceed 1·25 and for this reason it is suggested that the concrete strain be taken as $0{\cdot}003 \times 1{\cdot}25 = 0{\cdot}00375$.

Turning now to the steel strain, given by f_y^*/E_s in equation 2, the value appropriate to the comparisons was 0·0018, corresponding to steel with the fairly high characteristic strength of 410 N/m². A strain of 0·002 will be sufficient to cover the range of steels likely to be used as compression reinforcement.

Substituting these two values into equation 2 gives:

$$\frac{1}{r_u} = \frac{1}{175h}\left(1 - 0{\cdot}0035\frac{l_e}{h}\right) K_1$$

Substituting now into equation 1 (page 23) to obtain the additional moment gives:

$$M_a = \frac{Nh}{1750}\left(\frac{l_e}{h}\right)^2 \left(1 - 0{\cdot}0035\frac{l_e}{h}\right) K_1$$

It remains to give some guidance in the assessment of N_{bal}, the axial load corresponding to 'balanced' conditions. This is required to calculate K_1, which is given by:

$$K_1 = \frac{N_{uz} - N}{N_{uz} - N_{bal}} \leqslant 1{\cdot}0$$

TABLE 1: **Expression for additional moment when K_1 is taken to be equal to 1·0.**

$\frac{l_e}{h}$	$\frac{M_a}{Nh}$
5	0·014
10	0·055
15	0·122
20	0·212
25	0·325
30	0·46
35	0·62
40	0·79
45	0·98
50	1·18
55	1·40
60	1·63
80	2·64
100	3·71

A constant value of 0·002 having been chosen for the steel strain in the expression for curvature, it is appropriate to define N_{bal} as the axial load corresponding to the attainment of a tensile strain of 0·002 in the outermost layer of tension steel along with the appropriate ultimate strain in the concrete. N_{bal} defined in this way can easily be calculated and inserted in design charts or tables.

The retention of K_1 in the expression for additional moment will require a 'cut and try' approach in design, because a specific section will have to be chosen in order to give values for N_{uz} and N_{bal}. This can be avoided, of course, by taking K_1 to be equal to 1·0 in all cases, at the expense of some conservatism. The expression for additional moment becomes dependent in this case upon l_e/h above, and a simple tabular presentation may be adopted (Table 1).

CALCULATION OF INITIAL MOMENT IN BRACED COLUMNS

When the moments at each end of a column are different, it is necessary to work out an 'equivalent' initial moment. It will be appropriate to use the expression given in the comparisons, which was:

$$M_i = 0{\cdot}4M_1 + 0{\cdot}6M_2 \geqslant 0{\cdot}4M_2$$

where M_1 is the smaller of the end moments, taken as negative if the column is bent in double curvature and M_2 is the larger end moment, taken as positive. In the comparisons, the values of M_1 and M_2 were worked out by doing a complete moment distribution for the simple systems under consideration. It would appear reasonable to simplify this for design purposes by recommending that the moments arising from loadings involving moment applied at only one end of the column should be calculated on the assumption that the other end of the column remains fixed.

CALCULATION OF EFFECTIVE LENGTH

In physical terms the effective length is the length of the pin-ended column which embodies the column being considered, as shown in Figure 48. It will be seen that for the braced column the effective length must always be less than the actual storey height but that for the unbraced column it will always be greater. Where the relative stiffnesses of the beams and columns are known, the effective length can be computed directly by using formal mathematics.[15]

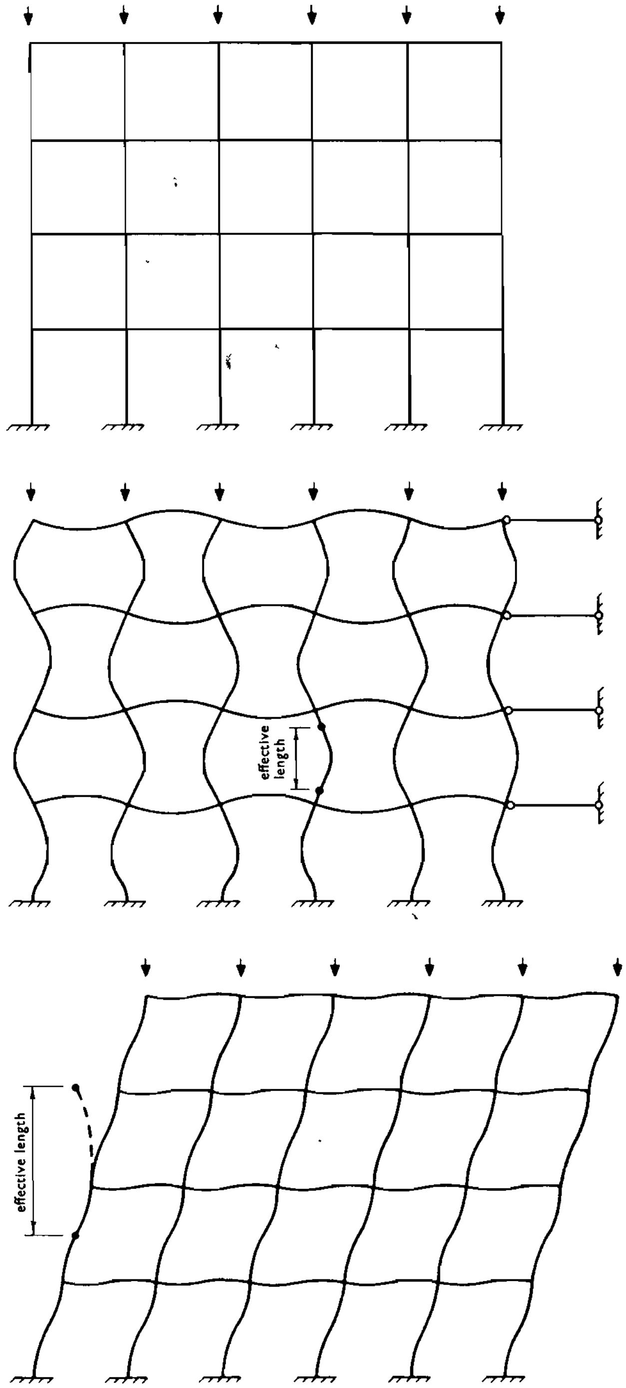


Figure 48: Buckling modes for rectangular frames.

Since the computation of relative stiffness is always approximate in reinforced concrete, it is reasonable to take a simple lower bound to the effective lengths. In the case of braced columns, the effective length can be taken as the lesser of

$$l[0{\cdot}7 + 0{\cdot}05(\alpha_{c1} + \alpha_{c2})]$$

and

$$l(0{\cdot}85 + 0{\cdot}05\alpha_{c\,min})$$

where α_{c1} and α_{c2} are the ratios of the column stiffness to the beam stiffnesses at the upper and lower ends respectively, and $\alpha_{c\,min}$ is the lesser of α_{c1} and α_{c2}.

In the particular case of a regular framed building where the beam stiffness is equal to the column stiffness, α_{c1} and α_{c2} become equal to 1 and the effective length is 0·8. The over-all results from the expressions above are compared in Figures 49 to 51 with the results of a formal mathematical analysis. It will be noted that the beams providing restraint are assumed to be bent in symmetrical single curvature, i.e. their actual stiffnesses are half the nominal value. This, of course, only applies in the general case if all columns are loaded critically together; this is a safe assumption, since loading surveys indicate that severe overload is likely to be a local phenomenon.

For unbraced columns, the effective length is given by the lesser of

$$l[1 + 0{\cdot}15(\alpha_{c1} + \alpha_{c2})]$$

and

$$l(2 + 0{\cdot}3\alpha_{c\,min})$$

where α_{c1}, α_{c2} and $\alpha_{c\,min}$ have the same meaning as above.

Figures 52 to 54 give appropriate comparisons. It will be seen that here it is assumed that the beams are bent in symmetrical single curvature, which must be the case since the columns must all sway sideways the same amount.

BENDING ABOUT A MAJOR AXIS

The method as developed so far deals with bending about one axis. This is the normal design situation and, provided the column is square or circular, the method does not require further development. However, it is often the case that the column is made deeper about the axis where bending moment is applied. In these cases the slenderness ratio about the minor axis can be much greater than about the major axis. In extreme circumstances the column could fail in the minor axis direction, before the full strength about the major axis can be mobilized.

The obvious solution is to consider additional moments in the minor axis direction, and this leads to the following design conditions:

$$M_{tx} = M_i + \frac{Nh}{1750}\left(\frac{l_{ex}}{h}\right)^2\left(1 - 0{\cdot}0035\frac{l_{ex}}{h}\right)K_{1x}$$

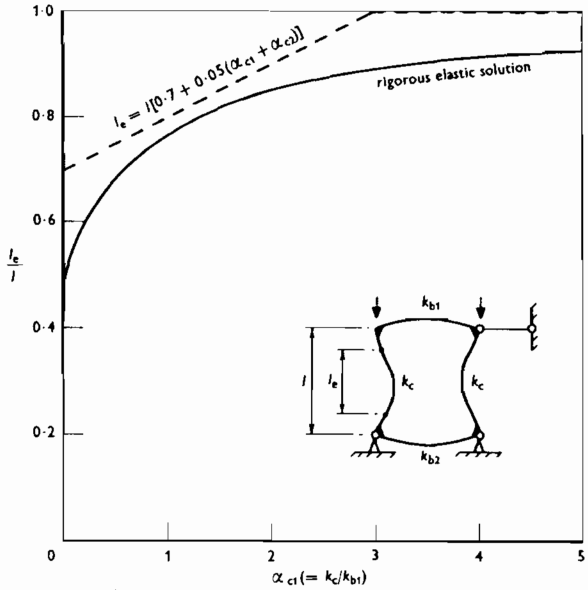


Figure 49: Effective lengths for braced columns, $\alpha_{c2} = \alpha_{c1}$.

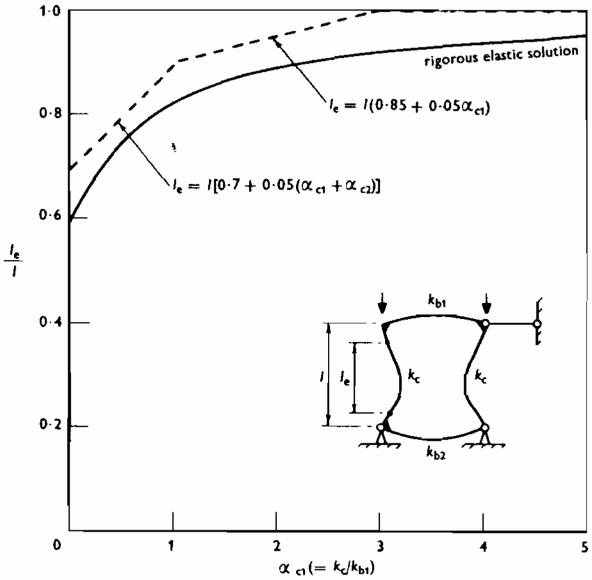


Figure 50: Effective lengths for braced columns, $\alpha_{c2} = 3\alpha_{c1}$.

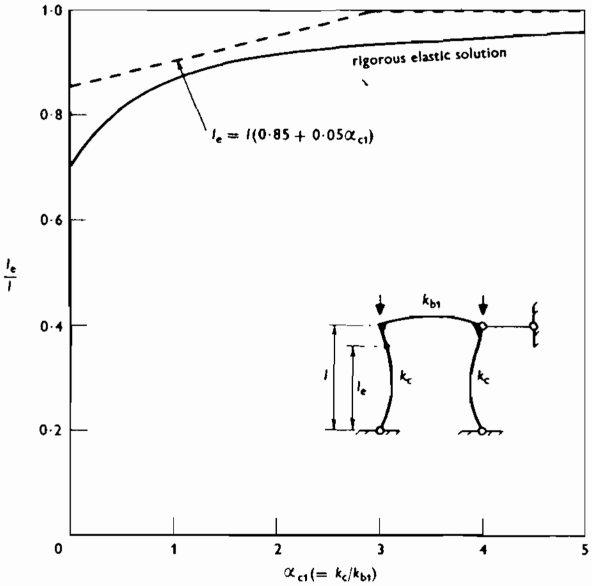


Figure 51: Effective lengths for braced columns, $\alpha_{c2} = \infty$

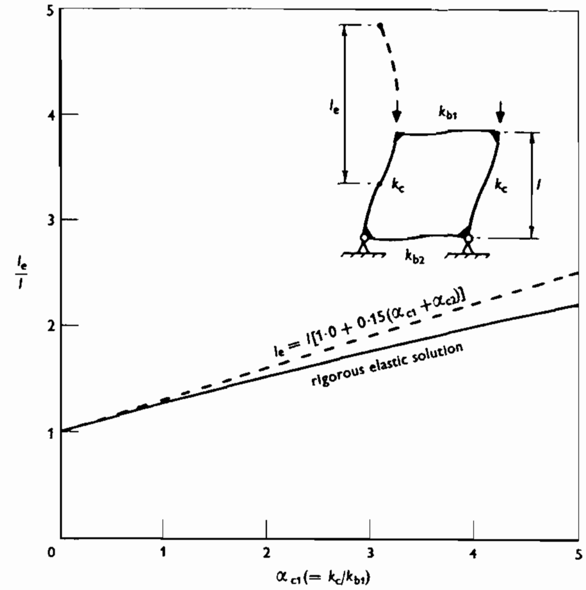


Figure 52: Effective lengths for unbraced columns, $\alpha_{c2} = \alpha_{c1}$.

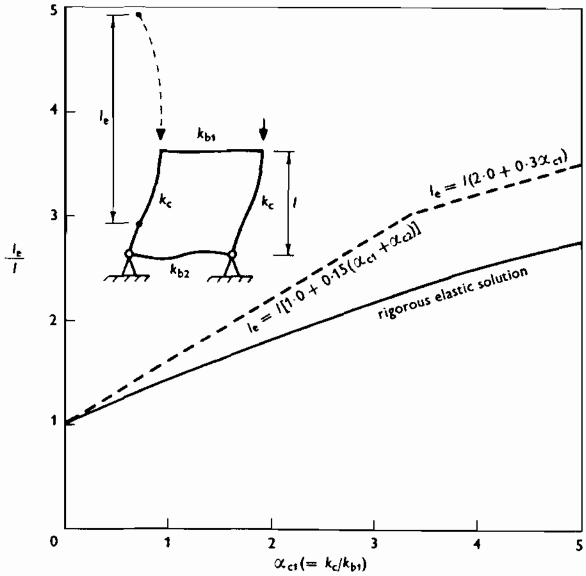


Figure 53: Effective lengths for unbraced columns, $\alpha_{c2} = 3\alpha_{c1}$.

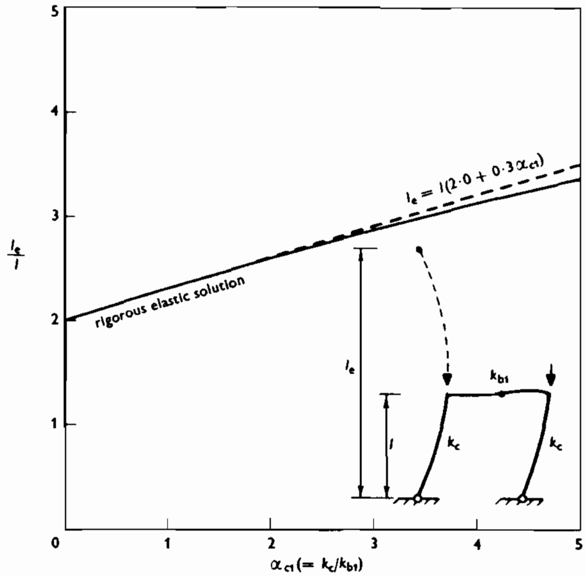


Figure 54: Effective lengths for unbraced columns, $\alpha_{c2} = \infty$.

$$M_{ty} = \frac{Nb}{1750}\left(\frac{l_{ey}}{b}\right)^2\left(1 - 0{\cdot}0035\,\frac{l_{ey}}{b}\right)K_{1y}$$

where M_{tx} and M_{ty} = design bending moments about the major and minor axis respectively;

b = width of the section;

l_{ex}, l_{ey} = effective lengths computed in the plane of bending about the major and minor axis respectively;

K_{1x}, K_{1y} = values of $K_1 = \dfrac{N_{uz} - N}{N_{uz} - N_{bal}}$

where N_{bal} is assessed for bending about the major and minor axis respectively.

In cases where M_{ty} becomes significant, a biaxial type of failure can be expected, with bending taking place about a diagonal axis. The actual curvatures about the major and minor axis in such cases will be somewhat less than about the diagonal axis and it follows that the expressions above will be conservative.

A simplification is desirable where the designer does not wish to tackle the complexities of designing the cross-section for a biaxial system of moments. It is demonstrated in Appendix 1 that the design can be based on bending about the major axis above, provided the total design moment is taken as

$$M_t = M_i + \frac{Nh}{1750}\left(\frac{l_e}{b}\right)^2\left(1 - 0{\cdot}0035\,\frac{l_e}{b}\right)K_1$$

where l_e is the effective length calculated in the plane of bending about the major or minor axis, whichever is the greater, and b is the width of the section.

The term l_e/b means that a larger additional moment is considered in design, providing a theoretical excess of strength about the major axis. It is shown in Appendix 1 that this should provide sufficient strength to deal with possible minor axis loading.

SLENDER BEAMS

The provisions so far deal with the possibility of buckling by flexure about the major and minor axes. Slender beams are also subject to buckling, in this case by a combination of lateral bending and twisting. The existing Code[6] contains certain restrictions on the span/breadth ratios which may be used and suggests that, where the effective length/breadth (l_e/b) ratio for a column exceeds the given limit of 30, interpolation be carried out to obtain a reduction factor for design.

Work by Marshall[16], based on experimental work such as that by Hansell and Winter[17], suggests that the limitations imposed on slender beams by present practice are conservative and, in addition, are not expressed in terms of the major governing parameter. This he shows to be

$$\frac{ld}{b^2}$$

where l = span between points of simple support where restraint is provided against twisting, but not against warping of the cross-section;

d = effective depth;

b = breadth of the compression face.

Marshall concludes from a review of all the test data available that lateral torsional buckling will not influence collapse, provided that

$$\frac{ld}{b^2} \leqslant 500$$

For design purposes, therefore, it is reasonable to set a limit for columns of

$$\frac{lh}{b^2} \leqslant 250$$

below which no provisions need be made for lateral torsional buckling. The end restraints provided for columns will generally be more effective than the supports at the ends of simply supported beams.

In the special case of precast columns for single-storey construction, it is possible for the members connecting with the top of the column not to provide any restraint against twisting deformation. Here it will be prudent to insist that

$$\frac{lh}{b^2} = 100$$

Provided the above conditions are met, the design procedures previously established can be used without modification.

SUMMARY OF DESIGN METHOD

The method is now summarized in the form and style of a Code of Practice. Explanatory notes are added. The actual Clauses are set in *italic type* with the explanatory notes immediately following.

1. Columns

1.1 General. *A column should be considered as braced in a given plane if lateral stability is provided by walls or other suitable bracing designed to resist all lateral forces in that plane. It should be considered as unbraced if lateral stability is provided by the columns alone.*

A column should be considered as short where the ratio of its effective length, l_e, to the lateral dimension h, of its cross-section in the plane of bending does not exceed 12. The effective length should be assessed for buckling about both the major and the minor axes of bending.

The ratio of the clear length, l_0, of the column between end restraints to the lateral dimension h should not exceed 60.

Application of the design method to columns with $l_e/h < 12$ shows that only very rarely is the additional moment large enough to affect the design. Even where it does, its effect is not more than 5 to 10%. For many practical columns l_e/h is less than 12, and this provision thus avoids superfluous design calculations.

The introduction of the limitation on l_0/h arises from the fact that in this study comparisons are made for cases where $l_0/h \leqslant 60$. This value was chosen as representing an extreme practical upper limit.

1.2 Forces in columns. *The cross-sections of columns should resist all possible combinations of axial load and bending moment, corresponding to the various possible load patterns. In general, the most critical loading to consider will be bending with maximum axial load.*

Moments in columns arise from loading on members framing into the ends of the column, and from the loads (if any) applied at points along it; moments from this source are conveniently called initial moments. These moments should be evaluated from simple elastic analysis.

Moments also arise from accidental eccentricities due to construction tolerances; these should be allowed for by taking the initial moments to be not less than 0·05Nh where N is the design axial load and h the total depth of the section at right-angles to the axis of bending. This accidental eccentricity need be considered about only one axis.

Finally moments may arise from the lateral deflections of the column under load; moments from this source are conveniently called additional moments and are significant only for long columns. Appropriate allowance for additional moments is made in the equations for total moment given for long columns in clauses 1.5 to 1.7.

It should be noted that, for unbraced long columns, the additional moments act at the column ends, and thus the beams (and bases if appropriate) into which the columns frame must resist these total moments. The beams (and bases) need, however, no specific check except where the average value of l_e/h for all columns at a particular level or storey is greater than 20.

Information to assist in a more accurate treatment of additional moments is given in clauses 1.8 and 1.9.

If the maximum design axial load for a column cross-section is at or below the balance point, the axial load is assisting the cross-section to carry bending moment. In such cases the minimum design axial load with maximum bending may be more critical. But to reach a minimum axial load means reducing the imposed load which in turn reduces the bending moment. Only in exceptional circumstances, therefore, is it necessary to consider other than the maximum design axial load to act.

In assessing the initial moments in regular framed buildings it is usual to introduce simplifications into the elastic analysis. It will be adequate for a braced column to consider the ends of members away from the joint at which out-of-balance moment is applied to be fixed. For unbraced columns, analysis may be based on dividing the lateral shear force between the columns in proportion to their stiffness, but care should be taken if the beam stiffnesses are much less than the column stiffnesses.

A reasonable allowance for accidental eccentricity is, of course, built into the partial factors of safety. Columns are, however, particularly susceptible to this type of constructional inaccuracy—hence the specific provision for 0·05*Nh*.

The warning given for unbraced long columns about designing the beams and bases is an obvious requirement. Its effect upon design will be minimal since in most frames columns are neither unbraced nor long and the warning applies only to structures where both misfortunes apply !

1.3 Short columns subjected to moments. *The column cross-section should be designed to resist the critical combination of axial load and initial moment.*

For normal cases, only single-axis bending need be considered. Even where it is possible for significant bending moments to arise simultaneously about both axes, it will generally be adequate to design for the maximum possible moment about the major axis.

For most internal and edge columns in framed structures, the load patterns necessary to produce biaxial bending produce a major axis moment which is less than if major axis bending alone is considered. For corner columns, biaxial bending will occur but in this case about a diagonal axis. The axial load is, however, much less and, in general, it will be adequate to use the same steel as in an edge column.

1.4 Effective length. *The effective length l_e of a braced column may be taken as equal to the clear length l_0 between end restraints. A more accurate estimate may be made by using the smaller of the following two expressions:*

$$l_e = l_0[0{\cdot}7 + 0{\cdot}05(\alpha_{c1} + \alpha_{c2})] \leqslant l_0$$

$$l_e = l_0(0{\cdot}85 + 0{\cdot}05\alpha_{c\,min} \geqslant) \leqslant l_0$$

where α_{c1} is the ratio of the sum of the column stiffnesses to the sum of the beam stiffnesses at one end of the column;

α_{c2} is the ratio of the sum of the column stiffnesses to the sum of the beam stiffnesses at the other end of the column.

$\alpha_{c\,min}$ is the minimum of α_{c1} and α_{c2}.

For an unbraced column, the effective length should be taken as the smaller of the following:

$$l_e = l_0[1{\cdot}0 + 0{\cdot}15(\alpha_{c1} + \alpha_{c2})]$$

$$l_e = l_0(2{\cdot}0 + 0{\cdot}30\alpha_{c\,min})$$

where l_0, α_{c1}, α_{c2} and $\alpha_{c\,min}$ are as above.

When calculating α, only the members properly framed into the end of the column in the appropriate *plane of bending should be considered. The stiffnesses should be obtained by dividing the second moments of area of the concrete sections by the appropriate lengths.*

In the case of flat-slab construction, the beam cross-section should be taken as a width of slab extending over the column strip.

Where a column is cast into a base which is not designed to resist moment (or supporting beams which are designed as simply supported), α at that end should be taken as 10. Where a base is designed to resist the applied moments, α may be taken as 1·0.

The expressions for effective length are simplifications of the rigorous buckling formulae for columns in regular framed buildings. For braced frames, the results are conservative—if overload is clearly confined to just one column and the beams framing in are fixed at their far ends, the values of α_{c1} and α_{c2} can be halved.

For columns which are not in regular frames, the formulae do not apply since the deformation of such columns at collapse may involve a significant amount of sway deflection, but it need not totally dominate collapse. The behaviour in such cases will be intermediate between that for a braced and an unbraced column. The designer must have recourse to first principles here.

1.5 Long columns bent about a minor axis. *The cross-sections should be designed to resist the ultimate axial load, N, and a total moment, M_t, given by:*

$$M_t = M_i + \frac{Nh}{1750}\left(\frac{l_e}{h}\right)^2\left(1 - 0{\cdot}0035\frac{l_e}{h}\right)$$

where M_i is the maximum initial moment in the length of the column, but not less than 0·05Nh, calculated by using simple elastic analysis;

h is the total depth of the cross-section in the plane of bending;

l_e is the effective length either in the plane of bending or in the plane at right-angles, whichever is the greater.

In the case of braced columns where no transverse loads are applied in the length of the column, a reduced value of M_i may be taken as follows:

$$M_i = 0{\cdot}4M_1 + 0{\cdot}6M_2$$

where M_1 is the smaller initial end moment, taken as negative, when the column is bent in double curvature, and M_2 is the larger initial end moment, taken as positive. In no case, however, should M_i be taken less than $0{\cdot}4M_2$ or such that M_t is less than M_2.

It will be noted that the factor K_1 has been omitted from the equation, i.e. K_1 has been set to 1·0 in all cases. It has been omitted to achieve simplicity in the general run of designs. Section 1.8 below allows it to be included in design as an option. It could be argued that the term $(1 - 0{\cdot}0035l_e/h)$ should also be omitted, but since designers will generally represent the equation above graphically it is no real disadvantage to leave the factor in.

1.6 Long columns bent about a major axis. *Provided the cross-section is such that $h/b \leqslant 3{\cdot}00$, it should be designed to resist the ultimate axial load, N, and a total moment, M_t, given by:*

$$M_t = M_i + \frac{Nh}{1750}\left(\frac{l_e}{b}\right)^2\left(1 - 0{\cdot}0035\frac{l_e}{b}\right)$$

where M_i, l_e and h are as in clause 1.5 and b is the width of the column cross-section at right-angles to the plane of bending.

Alternatively, columns bent about the major axis may be treated as biaxially loaded columns with the initial moment about the minor axis taken as zero.

As described in the derivation, the use of l_e/b as the effective slenderness ratio will lead to a reasonable solution for columns bent about a major axis. A significant economy may result, however, from adopting the biaxial approach.

1.7 Long columns bent about both axes. *The cross-section should be designed to resist the ultimate axial load, N, and total moments, M_{tx} about the major axis and M_{ty} about the minor axis, given by:*

$$M_{tx} = M_{ix} + \frac{Nh}{1750}\left(\frac{l_{ex}}{h}\right)^2\left(1 - 0{\cdot}0035\frac{l_{ex}}{h}\right)$$

$$M_{ty} = M_{iy} + \frac{Nb}{1750}\left(\frac{l_{ey}}{b}\right)^2\left(1 - 0{\cdot}0035\frac{l_{ey}}{b}\right)$$

where M_{ix} = design initial moment in the plane of bending about the major axis;

M_{iy} = design initial moment in the plane of bending about the minor axis;

l_{ex} = effective length computed in the plane of bending about the major axis;

l_{ey} = effective length computed in the plane of bending about the minor axis;

h = depth of column cross-section in the plane of bending about the major axis;

b = width of the column cross-section in the plane of bending about the minor axis.

1.8 Adjustments to additional moment. *In each of the equations for total moment in 1.5, 1.6 and 1.7, the second term comprises the additional moment. These terms may, if desired, be reduced by multiplying by the factor, K_1, given by*

$$K_1 = \frac{N_{uz} - N}{N_{uz} - N_{bal}} \leqslant 1{\cdot}0$$

where N = ultimate axial load;

N_{uz} = axial capacity of the cross-section in the absence of bending moments;

N_{bal} = axial load corresponding to 'balanced' conditions in the cross-section, which

should be taken when the tensile strain in the outermost layer of tension steel is 0·002.

Unbraced columns at a given level or storey subject to a lateral load are usually constrained to deflect sideways by the same amount. In such cases, l_e/h (or l_e/b, l_{ex}/h, l_{ey}/b as appropriate) may be taken for all columns as the average of the values computed for each of the individual columns.

The provisions for reducing the additional moment can only be applied in cases where a reasonably accurate estimate of the cross-section is already available, since the value of N_{uz} and, to a lesser extent, of N_{bal} depends upon the amount of reinforcement provided. The provision for average l_e/h ensures that an individual slender column at a particular storey level is not grossly over-designed.

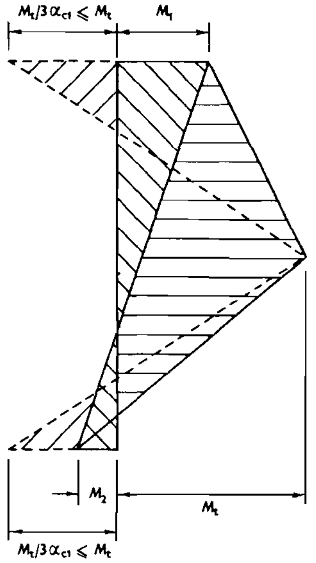
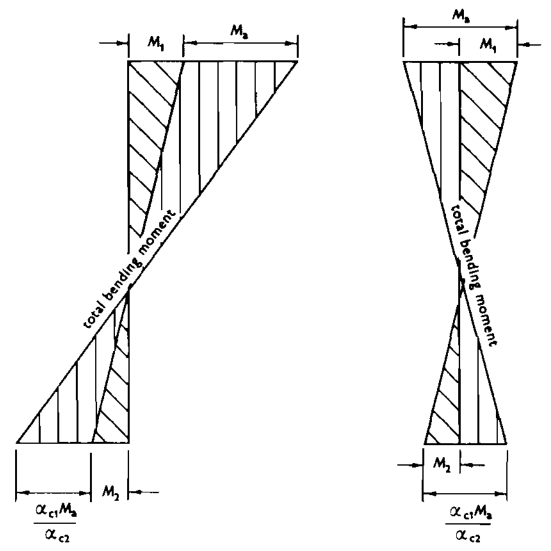


Figure 55: Bending moment diagram for braced long column.

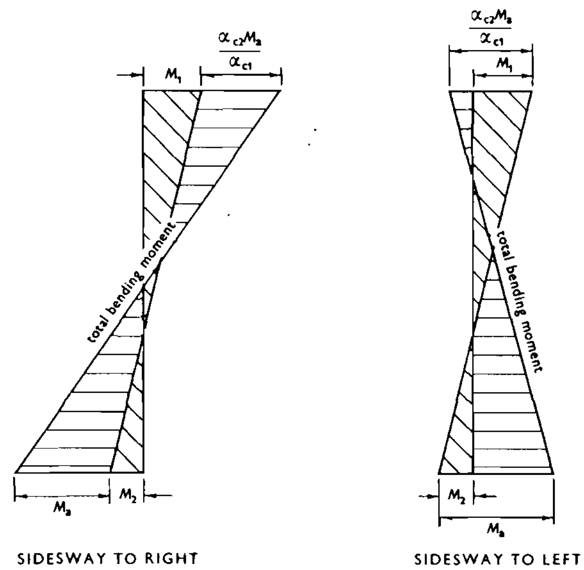
1.9 Diagrams for additional moment. *The procedures given above for obtaining total moments do not enable the designer to establish the bending moments throughout the column length; this is not necessary in general, since it is usual to provide a uniform cross-section with symmetrically arranged steel. If desired, however, bending moments as indicated in Figures 55 and 56 may be used.*

In the case of unbraced columns as indicated in Figure 56, the additional moment M_a is introduced and should be taken as equal to the second term in the expression for total moment given in 1.5, 1.6 and 1.7 as appropriate.

The diagrams shown in Figures 55 and 56 have been derived from a detailed study of the bending moments developing in the column under ultimate conditions. It should be noted that, in Figure 55, allowance is made for restraint moment effect where the beams are stiff (i.e. when α_c is small) via the broken lines of the bending moment diagram. The development of such moments can be seen in Figures 28 and 29. In Figure 56, M_a is added to the column end with the smaller value of α_e, i.e. the end with the stiffer beams. The moments added at the other end is reduced in proportion to the ratio of the beam stiffness, which allows for the fact that, where these are different, the point of inflection must move towards the end with the more flexible beams.



(*a*) *Cases where* $\alpha_{c1} \leqslant \alpha_{c2}$



(*b*) *Cases where* $\alpha_{c2} \leqslant \alpha_{c1}$

Figure 56: Bending moment diagram for unbraced columns.

COMPARISON BETWEEN ANALYSES AND THE DEVELOPED DESIGN METHOD

The ultimate loads of the columns analysed in the computer studies have been assessed by using the developed design method. The effective lengths used in the calculations were assessed not from buckling theory, but on the basis of clause 1.4 above, the values of α_{c1} and α_{c2} being assessed from the equivalent systems as sketched in Figures 20 and 21. The design total moments were assessed by using the provisions of clause 1.5, the reduced value of $M_i = 0{\cdot}4M_1 +$

$0{\cdot}6M_2$ being used where appropriate. In analysis of the braced columns where moments are applied at only one end, the initial moments in the column length have been calculated by simple moment distribution assuming the other end of the column to remain fixed. The adjustment to the additional moment as given in clause 1.8 above has *not* been applied. The results (labelled 'Design II') are presented in Figures 42 to 47, where they can be compared with the results of the computer analyses (labelled 'Analysis') and with the results (labelled 'Design I') obtained from the design method as developed to apply strictly to the computer analyses.

As expected, the developed method yields more conservative answers throughout, arising from the various simplifications introduced, and the adjustment introduced to cater for long-term effects. For the pinned columns an appreciable degree of conservatism is introduced in the intermediate range of slenderness where low moments are applied. This is due to the neglect of the adjustment factor, K_1. In the case of braced columns conservatism is not apparent until the slender range of columns is entered, and here the influence at work is the conservative approach used to determine the effective length. For the unbraced columns, conservatism emerges for the cases with low end moment, and this again can be traced to the neglect of the adjustment factor.

The areas where results from the developed method differ greatly from the results obtained from the method developed to apply strictly to the computer analyses are not large. This indicates that the simplifications introduced have not brought any great sacrifice in over-all accuracy. Application of the adjustment factor K_1 to the additional moments (which has been included in the design method as an option) would reduce the areas of difference considerably.

COMPARISON WITH TESTS

Reference has already been made to comparisons which have been made with test data. These comparisons have demonstrated the general validity of the approach over a wide range.

Nevertheless, it is of value to prepare a further comparison for several reasons. Firstly, the method contains recommendations with regard to the calculation of effective length and of initial moment which are developments from the original approach. Secondly, there are now available test data not hitherto considered. Thirdly, the computer studies against which the method has been compared ignore the influence of the tensile strength of the concrete, which should increase the ultimate load in an actual test. Fourthly, the influence of long-term loading, which would tend to decrease the ultimate load by inducing creep deflection, has also been ignored in the computer studies.

The various test series are described in references 18 to 38 and are summarized in Table 2. The end conditions are classified as pinned, framed, or biaxial as shown in Figure 57 and the loading is further classified into short- or long-term. 381 tests have been considered.

The classification of long-term tests presented some difficulty (Table 2). Two types of results have been obtained, the first where the load is maintained constant until failure, and the second where a load is applied and maintained for a considerable time but the result is merely an increase in deflections; a short-term load to failure is then applied. A long-term test for the purpose of this comparison has been taken to be either one which reaches ultimate conditions under a constantly maintained load, or a column for which the long-term maintained load comes out to be 85% or more of the short-term ultimate achieved at the termination of the test. The remarks given in Table 2 give details of this classification.

A full description of how the comparisons were made is given in Appendix 2, where detailed results are given in Tables 3 to 6. This Appendix is provided for detailed reference only, since it is difficult to gain any reliable impression from scanning the 381 results. In the following, the results are looked at in various other ways.

The key factor is $N_{u\,test}/N_{u\,calc}$, which ideally should be 1. The major variable involved in the design method is the slenderness ratio. In Figure 58 can be seen the relationship between $N_{u\,test}/N_{u\,calc}$ and slenderness ratio; separate histograms have been prepared for results separated out into various ranges of slenderness.

It will be seen from Figure 58 that in the slenderness range 7·5 to 17·5 the mean result is 1·16 with a standard deviation of 0·22. In this region the effects of slenderness are small and the scatter is thus due to inevitable experimental variations in concrete strength and workmanship.

It is of interest to compare with this the work of Massonnet and Moenart[39], who produced evidence on the validity of the CEB's proposals for stress-strain curves for concrete and steel when used to assess strengths of sections. They found a mean value and standard deviation for the ratio of test to calculated value of $1{\cdot}02 \pm 0{\cdot}14$. These are lower than the figures of $1{\cdot}16 \pm 0{\cdot}22$. It is considered that, if the effects of slenderness could be separated from the data in this range, the mean and standard deviation would then be very close to the figures of Massonnet and Moenart.

At higher slenderness ratios, the results become rather more spread out and more conservative. Some rather large values occur, but these are mainly in cases with steel percentages $< 1\%$ and/or eccentricity $\leqslant 0{\cdot}1h$. In practice it is, of course, essential to allow for the arbitrary eccentricity of $0{\cdot}05h$ and, had this been included in *both* test and design, the discrepancies would be somewhat reduced. In practice also, it is usual to insist on a minimum steel percentage of

TABLE 2: **Column test series considered in comparisons.**

Series	Date	Number of tests	End conditions*						Loading†			Remarks
			P1	P2	F1	F2	F3	B	S	L	L-S	
Baumann[18] (Series 1)	1934	27	14		13				27			
Baumann[18] (Series 2)	1934	16	16						16			
Thomas[19]	1939	14	14						14			
Rambøll[20]	1951	38	38						38			
Ernst, Hromadik and Riveland[21]	1952	8	8						8			Tests with l_e/h omitted
Gehler and Hutter[22]	1954	50	50						50			
Gaede[23]	1958	16	16						8	8		
Kordina[24]	1960?	4	4						4			Not published, details taken from reference 11
Aas-Jakobsen[25]	1960?	20	20						20			
Saenz and Martín[26]	1963	52			52				52			
Chang and Ferguson[27]	1963	6	6						6			
Breen and Ferguson[28]	1964	6				6			5		1	L-S test taken as short-term (see text)
Martín and Olivieri[29]	1966	8	2	6					8			
MacGregor and Barter[30]	1966	8		4		4			8			
Furlong and Ferguson[31]	1966	7				7			6		1	L-S test taken as short-term (see text)
Ferguson and Breen[32]	1966	8					8		7	1		
Green[33]	1966	6	6						1	5		
Pannell and Robinson[34]	1968	16	9					7	16			
Mehmel, Schwarz, Kasparek and Makovi[35]	1969	16	14	2					16			
Breen and Ferguson[36]	1969	10	10						10			Cantilever columns
Ramu, Grenacher, Baumann and Thürlimann[37]	1969	37	37						6	19	12	Seven of the 12 L-S columns taken as long-term (see text)
Sturrock and Cranston[38]	1971	8	3					5	8			
Totals		381	267	12	65	17	8	12	334	33	14	

* For classification of end conditions, see Figure 57.

† The load classifications are as follows:

S = Short-term loading up to ultimate

L-S = Long-term load followed by short-term loading up to ultimate load

L = Long-term load maintained till failure.

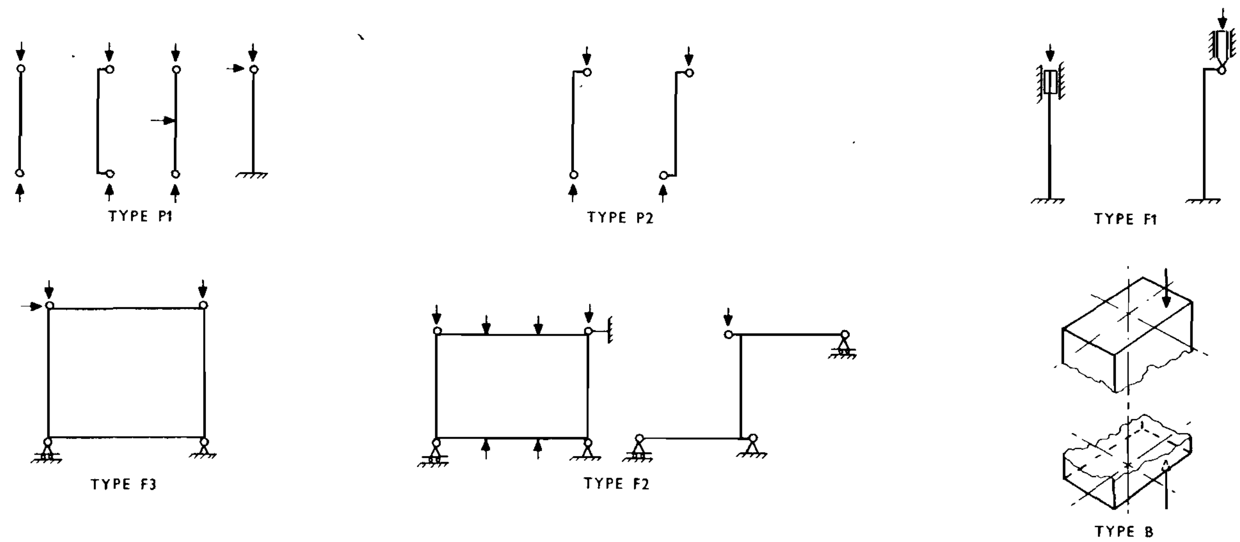


Figure 57: Categories of loading arrangements.

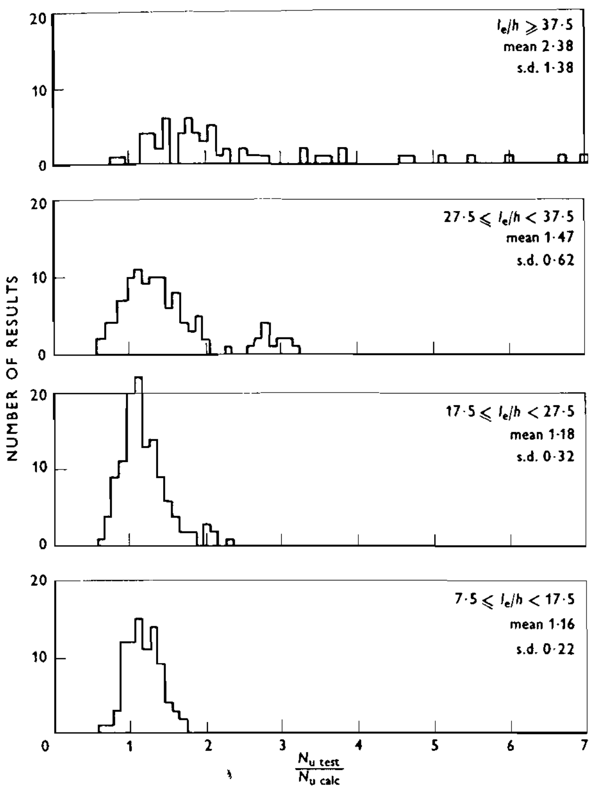


Figure 58: $N_{u\ test}/N_{u\ calc}$ *for various ranges of slenderness ratio.*

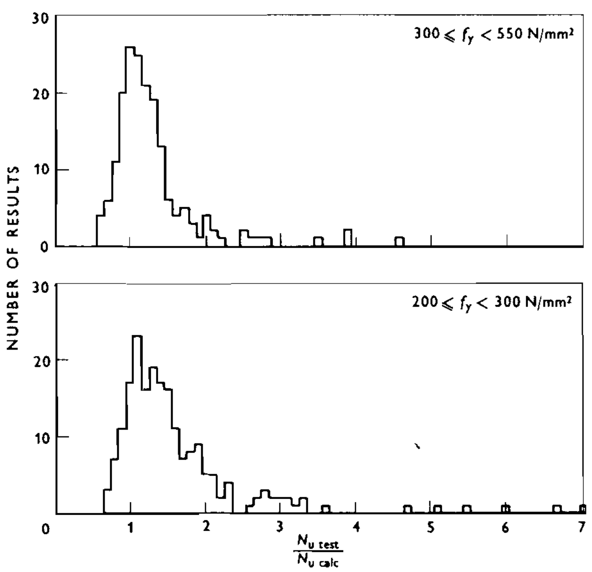


Figure: 59 $N_{u\ test}/N_{u\ calc}$ *for various ranges of steel strength.*

around 1 % and this would rule out some of the worst discrepancies observed.

Figure 59 and 60 give comparisons which investigate the influence of steel strength and concrete strength respectively. It will be seen that no significant trend or variation in $N_{u\ test}/N_{u\ calc}$ can be traced; this indicates that these variables are adequately covered by the design method which considers them only when designing the section.

Figure 61 gives histograms for the tests subdivided

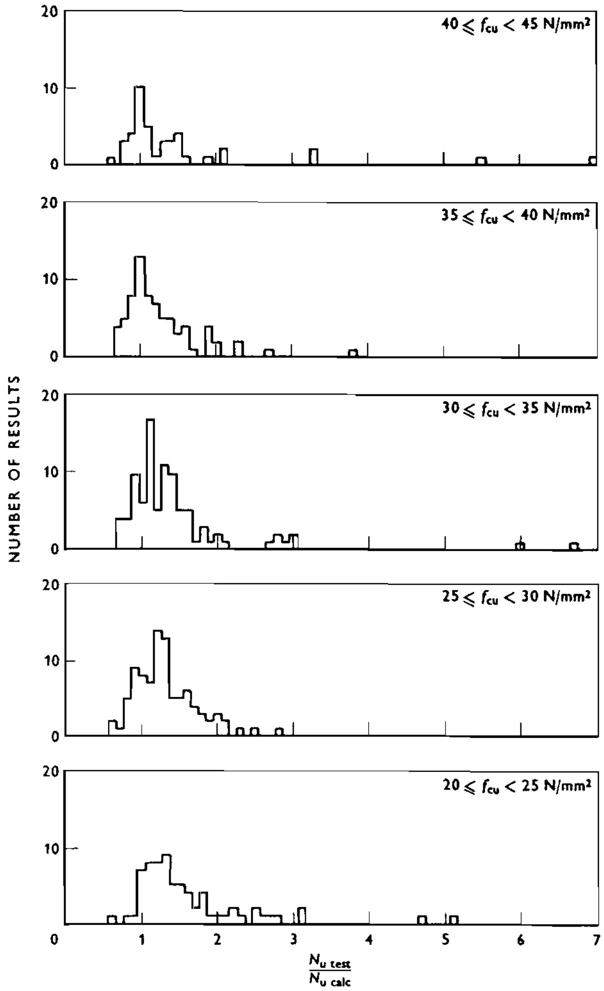


Figure 60: $N_{u\ test}/N_{u\ calc}$ *for various ranges of concrete strength.*

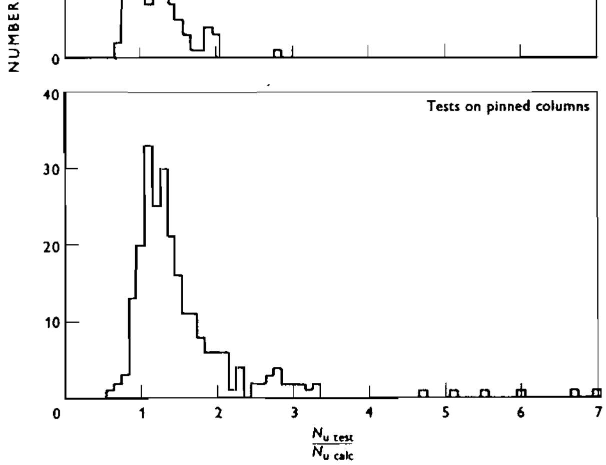


Figure 61: $N_{u\ test}/N_{u\ calc}$ *for various types of test.*

into the three categories of pinned, framed and long-term. No noticeable difference emerges between the frame and pinned tests, indicating that the methods for assessing effective lengths are reasonable. The long-term results show a tendency to be on the low side, as expected. However, only a small fraction give test loads of less than two-thirds of the short-term load as calculated, and this is considered an adequate safety margin to provide against this drastic condition of loading. The biaxial tests have not been presented as a separate histogram since there are so few of them. The values of $N_{u\,test}/N_{u\,calc}$ are conservative for these tests, ranging from 1·5 to 4·6. This is to be expected because the full additional moment is being considered to act about both axes simultaneously. It should be noted, however, that these biaxial tests are in the main for high slenderness ratios, and the results are thus influenced by the basic conservatism revealed in Figure 58.

Figure 62 gives a histogram for all 381 results. A number of very high results are present but, as will be seen from the notes, they are all for cases with high slenderness ratios, small steel percentages and zero initial eccentricities. Conservative results are to be expected for such cases. At the lower end of the histogram, there are 62 results below 0·95. A cursory impression might be that in some respects the approach is inadequate. Results with $N_{u\,test}/N_{u\,calc}$ in the range 0·85 to 0·95 can reasonably be assumed to be due to normal scatter of test evidence. Values below 0·85, of which there are 31, give cause for concern, and it seems prudent to examine them in some detail, to establish whether any common factor might be present which should be taken into account.

Fifteen of the 31 are long-term tests, already classified as a rather drastic loading, and extremely unlikely to arise in practice. If this type of loading was foreseen in practice, the additional moments would be increased from those proposed in any case. Eight of the remainder are from tests under pure axial load by Saenz and Martin[26] and are all for slenderness ratios

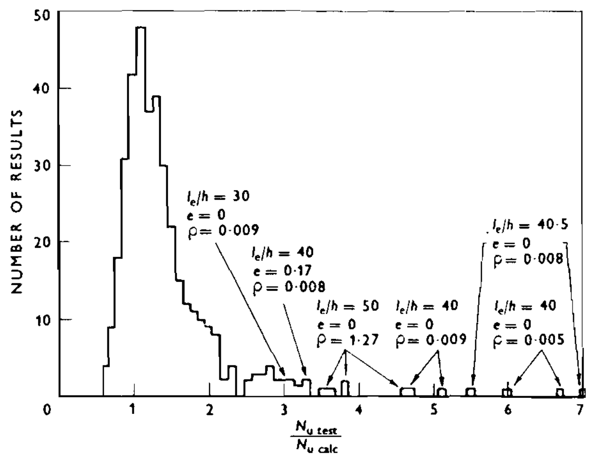


Figure 62: $N_{u\,test}/N_{u\,calc}$ for all tests.

of 17·5 or 21·0. These tests were described by the authors as being of a practical nature and considerable out-of-straightness was reported for some specimens. In the design assessment for this series, it would probably have been more appropriate to include the 0·05h minimum eccentricity. Columns at this slenderness are very sensitive to the presence of the minimum eccentricity and the discrepancies become much less serious if it is included. Three of the tests are from the lateral load series of Ferguson and Breen[32]. This series is discussed in Appendix 2, where it is concluded that the joints in the frames were probably much more flexible than assumed. Two of the tests are from the series by Thürlimann[37], classified as short-term. They were, however, subjected to considerable long-term loads prior to a short-term load to ultimate. The remaining three tests are as follows:

No. 15 from Ernst, Hromadik and Riveland's series
No. 28 from Rambøll's series
No. 7 from Baumann's series

These three are isolated low values, plainly well away from the trends being shown by the remainder of the respective test series. They can be accepted as the extreme edge of experimental scatter.

It may be concluded that the above comparison provides further confirmation of the design method. Indeed, it could be argued that at the higher slenderness ratios the approach is rather too conservative. However, since it is possible that errors in workmanship will have a rather greater influence upon the ultimate loads of slender columns than upon the ultimate loads of elements which are not slender, it is considered prudent at this time to leave the approach as it is.

Taking these experimental comparisons along with the rather wider range of analytical comparisons previously described, the reliability of the design method for general use can be taken as established.

Conclusions

A considerable amount of data has been obtained and presented concerning the behaviour of reinforced concrete columns. Against the background of this information a design method, based essentially on proposals put forward by the European Committee for Concrete, is derived and developed. The general validity of the approach is demonstrated by comparisons with computed collapse loads. Finally, some essential practical upper limits and extensions are made to the method, widening its scope considerably.

Throughout this development it has been recognized that a balance must be struck between the requirement for realistic representation of column behaviour and the requirement for simplicity in use. It is considered that in both these respects the method is satisfactory. Taking this point in conjunction with the demonstrated validity of the approach, it is recommended for general use.

ACKNOWLEDGEMENTS

The preparation of this report had its origin in a suggestion made by Dr D. D. Matthews, Chairman of the Committee drafting the new Code for the structural use of concrete.

The author gratefully acknowledges the considerable effort by his colleague Mr R. D. Sturrock, who wrote the computer program to work out the interaction diagrams for the 381 columns tested by other authors. The help of Mr Liaqat Hayat in preparing the comparisons with test results is also acknowledged.

APPENDIX 1

The design of slender columns bent about the major axis

A rigorous approach to the problem is plainly not possible at this time, because no precise data are available either from experiments or from analyses. Therefore an approximate approach must be derived, based on the technique developed for single-axis bending.

For the purpose of developing the approach, a braced column will be considered with the effective lengths about the major and minor axes being equal. In addition, the initial moment about the major axis equals $0{\cdot}1Nh$ and is constant throughout the column length; this means that the centre moments must govern the design. The proper design approach will give the following moments for a biaxial design:

$$M_{tx} = 0{\cdot}1Nh + \frac{Nh}{1750}\left(\frac{l_e}{h}\right)^2$$

$$M_{ty} = \frac{Nb}{1750}\left(\frac{l_e}{b}\right)^2$$

A very crude and certainly conservative method of design for biaxial bending would be to add an area of steel over and above that required for M_{tx} in each face capable of carrying a force equal to $M_{ty}/0{\cdot}8b$, assuming the steel to be located as shown in Figure 63. This gives in this instance an area,

$$A_sf_y = \frac{Nb}{1750 \times 0{\cdot}8b}\left(\frac{l_e}{b}\right)^2$$

$$= \frac{N}{1400}\left(\frac{l_e}{b}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

If, now, a design is carried out ignoring M_{tx} but instead using $(l_e/b)^2$ in the expression for M_{ty}, an extra moment will be catered for about the strong axis, equal to

$$\frac{Nh}{1750}\left[\left(\frac{l_e}{b}\right)^2 - \left(\frac{l_e}{h}\right)^2\right]$$

If it is assumed that this is resisted by extra steel at a lever arm of $0{\cdot}8h$, we get:

$$A_sf_y = \frac{N}{1400}\left[\left(\frac{l_e}{b}\right)^2 - \left(\frac{l_e}{h}\right)^2\right] \ldots\ldots\ldots(5)$$

Where h/b is in the range 1 to 1·5, it is plain that the area of steel provided by expression 4 is considerably less than by expression 5. For such cases, the differences in slenderness ratio are not large, and with bending only about the major axis, it is probable that deflection will develop only in the major axis direction; thus there is no cause for concern about the possible slight lack of conservatism here. As h/b moves up towards 2·0, i.e. towards a situation where deflection about the minor axis can be envisaged, the results become much closer and the basic soundness of the approach is demonstrated.

A basic assumption is that the steel provided in the column will be detailed in the four corners. This will normally be the case.

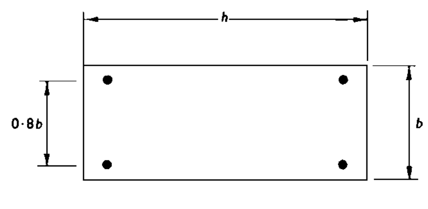


Figure 63: Location of steel.

APPENDIX 2

Comparison of design method with tests

The various test series considered are given in Table 2. Over such a large number of individual series there is considerable variation in the units employed, and the types of concrete and steel used. The results have all been reduced to a common basis in SI units. In the assessment of the strength of sections, the stress-strain curves given in Figures 64 and 65 have been used;

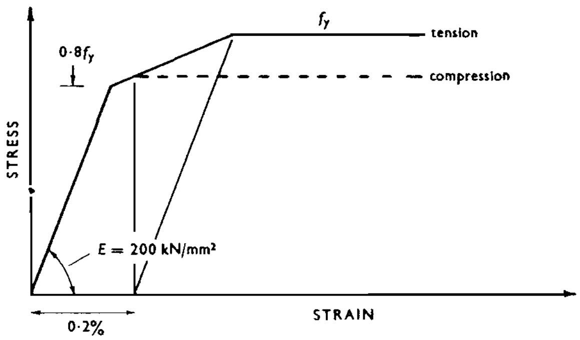


Figure 64: Stress-strain curves assumed for reinforcement.

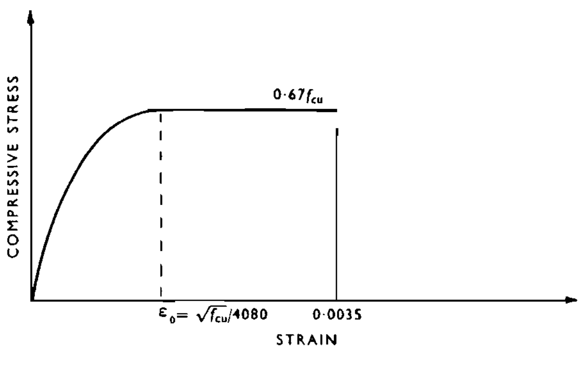


Figure 65: Stress-strain curve for concrete in compression, f_{cu} in N/mm².

these curves have been taken from the Draft British Code[5].

It will be noted that in Figure 65 the curve is related to the cube strength f_{cu}. Where individual authors have reported prism or cylinder strengths, the cube strength has been assumed to be 1·25 times these strengths. Figure 64 for the reinforcing steel shows a deviation from linear behaviour at $0{\cdot}8f_y$, strictly applicable to cold-worked steels. This has no effect if the tension steel is yielding but the compressive strength is limited since it is taken as the value appropriate to 0·2% strain. For steel with a yield stress f_y of 250 N/mm^2, the effective compressive yield is 220 N/mm^2. For steels with a tensile yield stress greater than or equal to 500 N/mm^2, the effective compressive yield is limited to 400 N/mm^2. Steels of this high strength tend to be of the cold-worked type, and so the general use of this curve is reasonable.

The interaction diagrams giving the section strengths for each individual column cross-section were produced by a specially written computer program. To avoid conversion errors, the program was designed to accept data in a variety of different systems of units and to convert automatically to the SI system.

In finding $N_{u\ calc}$ the fully developed design method was used, i.e. the provisions of clauses 1.5, 1.6, 1.7 and 1.8 were employed in their entirety. The process of calculation has already been illustrated in the main text of the report. With regard to the assessment of effective length, this has for pinned columns been taken simply as the centre-to-centre distance between the end bearings. For columns tested with fixed or flat ends, the effective length has been taken as $0{\cdot}7l_0$ where both ends are fixed and $0{\cdot}85l_0$ where just one end is fixed. Where beams are present, i.e. for framed columns, the simplified formula in clause 1.4, based on the clear length between restraints, has been used. For the cantilever columns of Breen and Ferguson[36], the effective length has been taken as $2l_0$. A strict reading of clause 1.4 would lead to $2{\cdot}3l_0$ but in the experiments a rigid base was provided, and thus $2l_0$ is more realistic. With regard to initial eccentricity, the arbitrary $0{\cdot}05h$ was *not* included, because this is essentially for construction tolerances. Whilst initial eccentricity was undoubtedly present for some tests, it is not present generally in laboratory tests. Including it would thus have given an unfair advantage to the design method.

The results are presented in Tables 3 to 6, separated into the categories of pinned, biaxial, framed and long-term tests. Sufficient information has been given in the Tables to allow other workers to operate on the data if desired.

Nearly all the pinned tests are either axially loaded, or with equal end eccentricities, although a few have different eccentricities at each end. In some of these, the end eccentricity was found to govern the design, and allowance then had to be made for the strengthened zones at the ends of the columns, which cause failure some distance down the column where the eccentricity is reduced.

For the biaxial tests, it was necessary to prepare strength data for biaxial bending. This was done by preparing interaction diagrams between M_{ux} and M_{uy} for various levels of ultimate load. A guess was made at the design ultimate load and the appropriate interaction diagram selected. Calculation of additional moment about each axis gave a relation between M_x and M_y from which the ultimate values could be assessed. Dividing the M_{ux} (or M_{uy}) moment by the appropriate eccentricity of load gave a value for ultimate load to compare with the initial guess.

In the frame tests, the moments have been calculated from straightforward moment distribution based on stiffnesses calculated from the concrete sections alone. The effective lengths were calculated on the basis of the clear height between end restraints. It should be noted that, in the tests with framing beams, the axial load was always applied through a roller seating at some distance above the beam centre-line. Where the end slope of the beam is significant, an additional Na moment is introduced here. This moment has not been taken into account in design. The figures given for e_1 and e_2 in Table 3 are those calculated at the face of the column-beam joints. Where e_1 governs the design assessment, this point is noted in the Table.

No modification to the design method was made when assessing the ultimate loads appropriate to the tests classified as long-term. The concrete strength used was that appropriate to the time the column actually collapsed. These long-term tests either had a constant load applied until eventual collapse, or a constant load greater than 85% of the final short-term load at failure. The design method includes an increase in ultimate strain from 0·003 to 0·0375 to allow for long-term effects but this is purely under service or working load conditions. The short-term elastic strains under such conditions are unlikely to exceed 0·00050 and the effective allowance of 0·00075 for creep strain is reasonable when considered in this light. Thus the comparisons in this report for long-term tests should give an indication of the maximum possible reductions for long-term loading.

Whilst it is difficult to obtain any reliable over-all impression from the Tables, some individual points are worthy of attention. It can be seen that, for the lower slenderness ratios of 15 or less, the values of $N_{u\ test}/N_{u\ calc}$ are generally, as expected, close to unity. Some test series show deviations from this.

For instance, the series by Gehler and Hutter[22] have values ranging from 1·20 to 1·40. Almost certainly, in this case, the method of testing the control specimens is in doubt. It has been shown[40] that different testing machines can introduce markedly different cube results. Another possibility is that the basic load-measuring equipment in the column testing machine was at fault. Such points can arise in all research work, but raise particular difficulty in conducting comparisons of this kind. It is appropriate at this point to mention that the work of Hanson and Rosenstrom[41], included in comparisons made by others, has been rejected here because even wider discrepancies are in evidence than with Gehler and Hutter. In particular, prism tests reported by them give higher strengths than the corresponding cubes, indicating a serious experimental error in the control tests.

Results giving concern because of an opposite trend are those from the lateral load series by Ferguson and Breen[32]. All values of $N_{u\ test}/N_{u\ calc}$ except one are below unity and the single long-term result at 0·56 is the lowest value of all 381 results. The three values corresponding to an l_e/h of 13 are 0·82, 0·94 and 0·80, indicating something amiss, because the effects of slenderness should be minimal for such a slenderness ratio. In the report on the tests, the occurrence of "wide flexural cracks" is reported at the joints, at a stage where the stresses in the reinforcement should be well below yield. Tests of corner joints by Swann[42] with similar reinforcement details show deformations considerably greater than can be calculated on the usual assumptions that the members extend to the centre of the joint. More recent analytical studies of a variety of framed tests by Drysdale, Mirza and McCutcheon[43] indicate that joints contribute greatly to over-all deformation. In view of this evidence, it is considered that the effect of joint flexibility, markedly increasing the effective length, is a major contribution to the over-all poor performance of these tests. The individual low long-term value of 0·56 merits some further attention. In this case, very high compressive strains were recorded. It should be noted, however, that the compression steel, which might have been expected to reduce these strains somewhat, continued into the beam only some 75 mm. Most of the stress in this bar, which would certainly have been yielding, would be taken out in simple end bearing. Some severe local deformation of the concrete at this point would be inevitable in these circumstances, leading to further significant joint deformation.

TABLE 3: **Data from tests on pinned columns.**

Test No.	b (mm)	h (mm)	d/h	100ρ	f_{cu} (N/mm^2)	f_y (N/mm^2)	e_i/h	l_e/h	$N_{u\ test}$ (kN)	$N_{uz\ calc}$ (kN)	$N_{bal\ calc}$ (kN)	$N_{u\ calc}$	$\frac{N_{u\ test}}{N_{u\ calc}}$	Remarks
THOMAS[19]														
LC1					30·3	309	0	14·7	590	597	191	597	0·99	
LC2					33·1	279	0	20·8	540	627	206	590	0·92	
LC3					29·7	334	0·01	23·8	480	597	185	430	1·12	
LC4					29·7	309	0·01	26·8	470	587	189	280	1·68	
LC5					31·4	305	0·04	26·8	460	611	195	230	2·00	
LC6	152	152	0·75	2·18	34·3	327	0·04	23·8	450	664	212	340	1·33	
LC7					32·9	282	0·03	20·8	460	626	206	440	1·04	
LC8					28·7	312	0·02	14·7	470	573	178	520	0·90	
LC9					25·0	308	0·02	26·8	360	515	159	240	1·50	
LC10					23·5	283	0·04	23·8	370	483	149	265	1·40	
LC11					23·3	271	0·04	20·8	420	475	147	320	1·31	
LC12					26·1	282	0·03	14·7	440	522	167	470	0·94	
PLC1	76	76	0·73	4·91	23·5	310	0·06	33·2	81	163	33	49	1·65	
PLC2					24·1	310	0·06	33·2	82	166	32	50	1·64	
GEHLER AND HÜTTER[22] (FIRST SERIES)														
IA1					24·1	282	0	40	240	408	171	51	4·70	
IA2					24·1	282	0	40	260	408	171	51	5·10	
IB1					24·1	282	0	30	390	408	171	127	3·07	
IB2					24·1	282	0	30	400	408	171	127	3·15	
IC1					25·9	282	0	25	500	435	182	240	2·11	
IC2	160	140	0·84	0·89	25·9	282	0	25	540	435	182	240	2·25	
ID1					25·6	282	0	20	490	430	180	430	1·14	
ID2					25·6	282	0	20	550	430	180	430	1·28	
IE1					24·7	282	0	15	600	417	175	417	1·44	
IE2					24·7	282	0	15	570	417	175	417	1·37	
IF1					23·8	282	0	10	480	400	168	400	1·20	
IF2					23·8	282	0	10	500	400	168	400	1·25	
IIA1	160	140	0·82	2·75	24·3	337	0	40	330	533	160	128	2·57	
IIA2					24·3	337	0	40	350	533	160	128	2·73	
GEHLER and HÜTTER[22] (THIRD SERIES)														
1/1	160	140	0·85	0·50	31·2	235	0	40	174	489	220	29	6·00	
1/2					31·2	235	0	40	195	489	220	29	6·70	
2/1	160	140	0·83	2·02	32·5	289	0	40	218	592	219	103	2·12	
2/2					32·5	289	0	40	289	592	219	103	2·76	
3/1	160	140	0·80	5·61	30·0	275	0	40	325	729	204	180	1·81	
3/2					30·0	275	0	40	286	729	204	180	1·54	
4/1					16·7	206	0	30	272	285	111	88	3·10	
4/2					16·7	206	0	30	252	285	111	88	2·86	
5/1	160	140	0·85	0·89	26·3	206	0	30	310	428	175	118	2·63	
5/2					26·3	206	0	30	326	428	175	118	2·76	
6/1					32·7	206	0	30	405	523	219	136	3·00	
6/2					32·7	206	0	30	405	523	219	136	3·00	
7/1					23·3	289	0·04	15	472	457	155	380	1·24	
7/2					23·3	289	0·04	15	450	459	155	390	1·18	
8/1					29·8	289	0·05	20	480	552	198	390	1·23	
8/2					29·8	289	0·05	20	445	552	198	390	1·14	
9/1					27·2	289	0·08	30	320	515	185	165	1·94	
9/2					27·2	289	0·08	30	298	515	185	165	1·80	
10/1					27·2	289	0·01	40	152	515	185	82	1·85	
10/2	160	140	0·83	2·02	27·2	289	0·01	40	165	515	185	82	2·01	Bending moment applied by lateral load at mid-height of column
11/1					23·3	289	0·08	15	430	457	155	340	1·26	
11/2					23·3	289	0·08	15	470	457	155	340	1·38	
12/1					29·8	289	0·10	20	396	552	198	320	1·24	
12/2					29·8	289	0·10	20	405	552	198	320	1·27	
13/1					30·4	289	0·15	30	236	561	201	146	1·62	
13/2					30·4	289	0·15	30	240	561	201	146	1·64	
14/1					26·1	289	0·20	40	126	498	174	75	1·68	
14/2					26·1	289	0·20	40	113	498	174	75	1·50	
15/1	160	140	0·85	2·02	23·3	289	0·08	15	400	457	155	340	1·33	
15/2					23·3	289	0·08	15	440	457	155	340	1·46	

TABLE 3 *continued*

Test No.	b (mm)	h (mm)	d/h	100ρ	f_{cu} (N/mm^2)	f_y (N/mm^2)	e_i/h	l_e/h	$N_{u\,test}$ (kN)	$N_{uz\,calc}$ (kN)	$N_{bal\,calc}$ (kN)	$N_{u\,calc}$ (kN)	$N_{u\,test}/N_{u\,calc}$	Remarks
16/1	160	140	0·83	2·02	23·3	289	0·15	20	263	457	155	230	1·14	Bending moment applied by lateral load at mid-height of column
16/2					23·3	289	0·15	20	306	457	155	230	1·33	
17/1					23·3	289	0·23	30	196	457	155	120	1·63	
17/2					23·3	289	0·23	30	174	457	155	120	1·45	
18/1					25·1	289	0·30	40	105	483	169	60	1·75	
18/2					25·1	289	0·30	40	100	483	169	60	1·67	
GAEDE[23]														
I1	154	100	0·87	1·00	24·7	335	0·20	29·4	75	297	125	65	1·15	
I5					32·1	288	0·20	29·4	97	366	161	66	1·47	
II4					30·1	273	0·50	29·4	35	345	152	35	1·00	
II5					31·6	272	0·50	29·4	38	359	299	36	1·06	
III1					32·9	326	0·50	35·4	33	379	164	28	1·18	
III2					28·6	326	0·50	35·4	33	336	146	27	1·22	
III3					28·4	327	0·50	35·4	34	334	143	27	1·26	
III4					38·8	314	0·50	35·4	37	438	188	30	1·23	
PANNELL AND ROBINSON[34]														
1A	95	63	0·80	3·25	24·8	352	0	41·6	61	156	42	35	1·74	Bending moment applied by lateral load at mid-height
2A					23·7	365	0	41·6	75	154	40	35	2·14	
3A					22·1	365	0	27·2	100	148	39	96	1·04	
4A					27·4	365	0	15·2	174	169	48	169	1·03	
5A					27·4	365	0	32·0	99	169	48	69	1·43	
6B					29·5	352	1·42	41·6	15	175	51	12	1·25	
8B					20·7	352	0·17	41·6	55	140	36	28	1·96	
7B	63	95	0·87	3·25	31·7	352	1·77	27·7	20	184	61	15	1·33	
9B					20·7	352	0·72	27·7	40	140	41	29	1·38	
STURROCK AND CRANSTON[38]														
8	400	100	0·81	1·27	55·0	420	0·15	50	160	1630	619	90	1·78	
9					55·0	420	0·15	50	170	1630	619	90	1·89	
10					55·0	420	0·15	50	200	1630	619	90	2·20	
MARTÍN AND OLIVIERI[29]														
402.1	127	90	0·80	2·50	37·5	276	0	40	147	349	116	63	2·23	
402.2					30·4	276	0	40	125	296	99	55	2·27	
412.1					42·1	276	0·08	40	118	383	134	55	2·14	$e_1/h = 0{\cdot}211$;
412.2					31·3	276	0·08	40	90	303	101	48	1·87	$e_2/h = -0{\cdot}106$
422.1					43·6	276	0·16	40	94	394	139	49	1·92	$e_1/h = 0{\cdot}388$;
422.2					32·1	276	0·16	40	76	309	106	43	1·77	$e_2/h = -0{\cdot}194$
432·1					46·6	276	0·11	40	96	417	147	52	1·85	$e_1/h = 0{\cdot}282$;
432·2					33·0	276	0·11	40	94	315	107	47	2·00	$e_2/h = -0{\cdot}141$
MACGREGOR AND BARTER[30]														
A1	112	63	0·82	4·02	42·1	308	0·08	27·3	170	268	82	113	1·50	$e_1/h = 0{\cdot}2$
A2					40·8	308	0·08	27·3	170	262	81	110	1·55	$= -e_2/h$
B1					26·3	308	0·60	27·3	33	242	73	31	1·06	$e_1/h = 1{\cdot}5$
B2					40·8	308	0·60	27·3	31	262	81	33	0·94	$= -e_2/h$
CHANG AND FERGUSON[27]														
1	156	103	0·84	1·78	29·5	338	0·07	31	168	394	146	118	1·42	
2					43·7	338	0·39	31	69	544	212	68	1·01	
3					36·1	338	0·06	31	189	464	176	135	1·40	
4					37·6	338	0·38	31	73	479	185	72	1·01	
5					41·0	338	0·21	31	123	515	200	98	1·26	
6					42·0	400	0·06	31	198	539	204	150	1·32	
ERNST, HROMADIK AND RIVELAND[21]														
3	152	152	0·83	1·23	25·16	357	0	15	490	473	184	473	1·03	
4					25·16	357	0	25	450	473	184	280	1·60	
7					25·16	357	0·125	15	360	473	184	310	1·16	
8					25·16	357	0·125	25	290	473	184	170	1·70	
11					25·16	357	0·250	15	260	473	184	210	1·24	
12					25·16	357	0·250	25	170	473	184	130	1·31	
15					25·16	357	0·375	15	90	473	184	150	0·60	
16					25·16	357	0·375	25	110	473	184	99	1·11	

TABLE 3 *continued*

Test No.	b (*mm*)	h (*mm*)	d/h	100ρ	f_{cu} (*N/mm²*)	f_y	e_i/h	l_e/h	$N_{u\,test}$ (*kN*)	$N_{uz\,calc}$ (*kN*)	$N_{bal\,calc}$ (*kN*))	N_{ucalc} (*kN*)	$N_{u\,test}/N_{u\,calc}$	Remarks
MEHMEL, SCHWARZ, KASPAREK AND MAKOVI[35]														
0·1	253	159	0·88	1·11	37·4	500	0·08	8·8	940	1174	481	940	1·00	
0·2	254	156	0·85	1·12	40·5	500	1·00	9·0	137	1241	483	124	1·10	
1·1	253	203	0·84	1·22	39·2	480	0·18	16·8	855	1576	614	760	1·12	
1·2	253	202	0·84	1·22	37·8	480	0·48	16·8	320	1518	592	350	0·92	
2·1	252	202	0·84	1·23	37·2	480	0·18	22·3	590	1497	583	540	1·09	
2·2	252	203	0·84	1·22	40·7	480	0·48	22·2	258	1620	632	292	0·88	
3·1	252	152	0·84	1·23	38·2	500	0·16	22·4	470	1158	451	430	1·09	
3·2	252	151	0·83	1·25	41·1	500	0·50	22·5	176	1225	457	196	0·90	
3·3	254	159	0·84	1·10	35·3	500	0·08	21·5	780	1123	442	575	1·36	
3·4	253	158	0·84	1·12	42·8	500	1·00	21·5	102	1311	524	101	1·01	
4·1	253	150	0·83	1·25	40·5	500	0·16	30·0	368	1207	476	265	1·39	
4·2	253	148	0·83	1·27	41·5	500	0·49	30·4	145	1218	467	142	1·02	
5·1	253	158	0·81	3·19	40·6	412	0·16	21·5	735	1488	729	670	1·10	
5·2	252	159	0·84	3·18	37·0	412	0·50	21·4	370	1396	446	363	1·02	
6·1	254	159	0·84	1·11	42·5	500	0·10	14·5	940	1315	526	890	1·06	$e_1/h = 0{\cdot}17$; $e_2/h = 0$
6·2	253	157	0·85	1·12	44·1	500	0·30	21·6	343	1339	536	350	0·98	$e_1/h = 0{\cdot}50$; $e_2/h = 0$
RAMBØLL[20]														
1	182	144	0·79	0·97	35·6	294	0	9·1	860	686	268	686	1·26	
2	181	141	0·79	1·00	31·8	294	0	9·1	640	603	235	603	1·06	
3	182	143	0·81	0·98	33·0	294	0·08	9·1	690	636	254	500	1·38	
4	181	141	0·82	1·00	28·4	294	0·08	9·1	590	513	230	400	1·47	
5	181	143	0·77	0·98	34·7	294	0·17	9·1	510	661	253	390	1·31	
6	181	143	0·79	0·98	31·4	294	0·17	9·1	530	604	235	360	1·47	
7	180	145	0·79	0·97	29·6	294	0·33	9·1	340	578	225	215	1·58	
8	181	144	0·79	0·97	31·6	294	0·33	9·1	305	613	239	225	1·35	
9	181	142	0·79	0·99	29·2	294	0·67	9·1	118	563	219	73	1·62	
10	181	144	0·80	0·97	30·6	294	0·67	9·1	106	594	237	80	1·32	
11	181	141	0·80	1·00	32·2	294	0·83	9·1	78	611	244	55	1·42	
12	181	141	0·80	1·00	26·9	294	0·83	9·1	78	522	203	55	1·42	
13	181	142	0·79	0·99	35·6	294	0	13·2	580	672	262	672	0·86	
14	181	142	0·83	0·99	32·0	294	0	13·2	690	612	251	612	1·13	
15	181	147	0·78	0·95	30·9	294	0	13·2	650	610	238	610	1·06	
16	183	146	0·78	0·95	30·9	294	0	13·2	650	610	238	610	1·06	
17	180	142	0·79	0·99	31·4	294	0·08	13·2	580	597	233	450	1·29	
18	181	144	0·80	0·97	29·5	294	0·08	13·2	530	576	230	430	1·23	
19	180	142	0·79	0·99	30·2	294	0·17	13·2	470	577	225	310	1·52	
20	182	143	0·79	0·98	30·4	294	0·17	13·2	510	590	230	320	1·59	
21	183	145	0·80	0·96	28·8	294	0·33	13·2	305	573	189	180	1·70	
22	182	144	0·79	0·97	29·1	294	0·33	13·2	305	572	223	183	1·67	
23	181	144	0·78	0·97	29·3	294	0·67	13·2	94	572	223	69	1·36	
24	181	144	0·79	0·97	27·4	294	0·67	13·2	94	540	216	70	1·34	
25	182	144	0·79	0·97	35·2	294	0·83	13·2	69	677	264	51	1·35	
26	181	141	0·80	1·00	33·4	294	0·83	13·2	67	631	252	51	1·31	
27	182	141	0·77	0·99	36·6	294	0	13·2	580	689	262	620	0·93	
28	183	146	0·79	0·95	35·7	294	0	13·2	490	700	273	660	0·74	
29	182	144	0·79	0·97	36·9	294	0·17	13·2	335	708	276	240	1·40	
30	182	143	0·76	0·98	33·9	294	0·33	13·2	195	651	247	124	1·57	
31	183	144	0·79	0·96	36·4	294	0·67	13·2	73	703	274	56	1·30	
32	183	142	0·79	0·98	36·8	294	0·83	13·2	57	701	273	45	1·27	
33	183	143	0·81	0·97	34·4	294	0	30·1	495	663	74	185	2·88	
34	182	145	0·80	0·96	36·9	294	0·08	29·6	410	713	60	150	2·73	
35	183	144	0·70	1·71	33·2	294	0·17	29·9	235	692	52	145	1·62	
36	182	145	0·80	0·96	34·0	294	0·33	29·6	118	661	27	69	1·71	
37	183	143	0·79	0·97	33·6	294	0·67	30·1	56	649	15	39	1·44	
38	182	145	0·79	0·96	40·5	294	0·83	29·6	44	776	15	39	1·13	
BAUMANN[18]														
1	200	100	0·87	1·57	18·4	293	0	32·1	264	323	122	116	2·28	
1A	200	100	0·87	1·57	19·7	293	0·08	32·1	152	340	130	102	1·49	
3	140	140	0·87	1·60	20·1	293	0	22·9	343	340	130	340	1·01	
3A	140	140	0·87	1·60	20·3	293	0·08	22·9	235	344	130	193	1·22	
5	177	139	0·87	2·50	33·0	281	0	23·3	645	681	258	610	1·06	

TABLE 3 *continued*

Test No.	b (mm)	h (mm)	d/h	100ρ	f_{cu} (N/mm^2)	f_y	e_i/h	l_e/h	$N_{u\,test}$ (kN)	$N_{uz\,calc}$ (kN)	$N_{bal\,calc}$ (kN)	$N_{u\,calc}$ (kN)	$\frac{N_{u\,test}}{N_{u\,calc}}$	Remarks
5A	178	140	0·87	2·47	33·0	281	0	23·1	685	688	261	620	1·10	
6	198	98	0·87	1·62	29·9	293	0	32·8	390	463	189	139	2·80	
6A	200	100	0·87	1·57	29·9	293	0	32·1	400	475	195	147	2·72	
7	182	178	0·88	1·90	35·3	281	0	18·0	685	904	361	904	0·76	} local end failures
7A	180	180	0·87	1·90	35·3	281	0	17·8	820	904	361	904	0·91	} in test
8	182	178	0·87	1·90	36·0	281	0	17·5	1070	918	367	918	1·17	
8A	180	180	0·87	1·90	36·0	281	0	15·6	1215	918	367	918	1·32	
2/1	250	250	0·87	1·29	41·9	272	0	11·9	2040	1923	826	1923	1·06	
2/2	250	125	0·88	0·64	41·9	304	0	25·8	695	924	415	350	1·98	
2/3	250	160	0·87	0·78	41·9	294	0	40·6	665	1194	525	95	7·00	
2/4	250	250	0·87	1·29	40·2	272	0·17	12·0	960	1854	797	1110	0·87	
2/5	250	125	0·88	0·65	40·1	304	0·17	25·8	345	888	399	177	1·95	
2/6	250	160	0·87	0·78	40·2	294	0·17	40·6	225	1150	506	69	3·26	
2/7	250	250	0·87	1·29	25·5	272	0·17	11·7	840	1246	834	750	1·12	
2/8	250	126	0·88	0·64	25·5	304	0·17	25·6	235	588	258	147	1·60	
2/9	250	160	0·87	0·78	30·6	294	0·17	40·6	205	894	393	62	3·30	
2/10	253	251	0·88	1·27	37·4	272	0·33	11·7	690	1761	757	635	1·09	
2/11	252	126	0·88	0·64	37·4	304	0·33	25·6	195	843	379	93	2·10	
2/12	250	162	0·87	0·78	37·2	295	0·33	40·1	113	1081	475	54	2·09	
2/13	247	251	0·88	1·30	41·1	272	0·33	11·8	700	1877	807	675	1·04	
2/14	248	126	0·88	0·65	41·1	304	0·33	25·6	163	908	408	91	1·79	
2/15	247	161	0·87	0·79	41·3	294	0	40·5	550	1172	515	100	5·50	
3/19	250	130	0·87	0·97	29·8	293	0·1	24·7	385	723	318	252	1·53	
3/26	252	250	0·87	0·99	37·2	281	0·1	12·6	1320	1705	750	1140	1·16	
3/32	250	250	0·87	2·00	34·3	282	0·1	12·6	1350	1718	687	1160	1·16	
AAS-JAKOBSEN[25]														
1·01					26·4	498	2·42	21·9	9·8	12·5	45·2	9·5	1·03	
1·02					26·4	498	1·23	21·9	19·6	12·5	45·2	18·5	1·06	
1·03					26·4	471	0·60	21·9	39·2	12·3	45·2	32·5	1·20	
1·04					26·4	471	0·32	21·9	59·0	12·3	45·2	46·5	1·27	
1·05					36·5	530	2·60	21·9	9·8	159	60·4	9·2	1·07	
1·06			0·93	2·04	36·5	530	1·38	21·9	19·6	159	60·4	18·0	1·08	
1·07					36·5	530	0·78	21·9	40·0	159	60·4	31·8	1·26	
1·08					36·5	530	0·52	21·9	56·0	159	60·4	43·0	1·30	
1·09					27·6	431	2·30	21·9	9·8	124	45·8	8·6	1·14	
1.10	70	70			27·6	412	1·07	21·9	19·6	123	45·7	17·7	1·11	
1·11					27·6	412	0·55	21·9	39·2	123	45·7	32·5	1·21	
1·12					27·6	500	1·30	21·9	19·6	129	46·4	17·6	1·11	
1·13			0·93	3·20	27·6	461	3·20	21·9	9·8	146	45·2	10·0	0·98	
1·14			0·93	1·63	27·6	461	3·20	21·9	10·3	146	45·2	10·0	1·03	
1·15			0·93	0·92	27·6	461	1·63	21·9	20·3	146	45·2	19·5	1·04	
1·16			0·93	3·20	27·6	461	0·92	21·9	37·0	146	45·4	32·2	1·15	
2·01			0·94	2·33	23·5	206	1·10	42·8	9·8	97	41·0	6·6	1·48	
2·02			0·94	2·33	24·5	206	0·47	42·8	19·6	100	41·6	11·2	1·75	
2·03			0·93	2·04	29·4	471	0·80	42·8	19·6	133	49·0	15·2	1·29	
2·04			0·91	3·20	31·9	480	0·72	42·8	31·4	162	52·0	23·8	1·32	
BREEN AND FERGUSON[36]														
G1	156	101	0·82	1·82	32·1	410	0·30	20	151	427	149	139	1·09	
G2	154	102	0·82	1·82	31·6	405	0·60	40	47·8	421	147	39	1·22	
G3	153	102	0·82	1·82	31·9	410	0·76	50	30·0	425	148	25	1·20	
G4	154	101	0·82	1·83	31·9	403	0·30	50	53·3	423	148	35·5	1·50	
G5	153	102	0·83	1·84	35·9	465	0·91	60	29·4	474	166	21·2	1·39	
G6	153	102	0·82	1·83	37·7	450	0·38	50	49·0	491	171	38	1·29	
G7	155	102	0·79	1·80	41·8	441	0·30	40	66·7	538	188	55	1·21	
G8	152	102	0·82	1·84	35·0	428	0·25	60	48·0	457	159	28	1·71	
G9	153	101	0·81	1·84	34·3	420	0·33	20	147	448	157	135	1·09	
G10	152	102	0·79	1·84	34·7	412	0·30	10	209	450	153	194	1·08	
KORDINA[24]														
A1				1·01	35·7	294	0·20	29	117	404	173	70	1·67	
A2	154	100	0·87	1·01	42·6	294	0·50	29	51·5	474	203	35	1·47	
A3				1·01	32·6	294	0·20	29	118	373	160	67	1·76	
A4				1·34	42·8	305	0·17	29	139	490	206	100	1·39	

TABLE 3 *continued*

Test No.	b (*mm*)	h (*mm*)	d/h	100ρ	f_{cu} (*N/mm²*)	f_y	e_i/h	l_e/h	$N_{u\ test}$ (*kN*)	$N_{uz\ calc}$ (*kN*)	$N_{bal\ calc}$ (*kN*)	$N_{u\ calc}$ (*kN*)	$\frac{N_{u\ test}}{N_{u\ calc}}$	Remarks
RAMU, GRENACHER, BAUMANN AND THÜRLIMANN[37]														
41			0·82	1·67	30·1	459	0·033	28·8	515	978	342	365	1·41	
11			0·83	1·67	38·2	459	0·10	28·8	380	1177	423	350	1·09	
12			0·82	1·67	41·6	459	0·10	28·8	290	1262	454	365	0·80	
14	25	15	0·81	1·67	32·1	459	0·10	28·8	381	1027	359	310	1·23	
53			0·82	1·67	46·5	459	0·10	28·8	468	1383	497	390	1·20	
23			0·82	1·67	37·3	459	0·25	28·9	185	1138	409	265	0·70	
24			0·82	1·67	31·5	459	0·25	28·9	236	1012	354	245	0·96	
31			0·82	1·67	28·8	459	1·00	28·9	78·5	895	304	90	0·87	
82	25	10	0·82	1·71	45·0	459	0·05	43·2	174	901	324	113	1·54	
73	25	15	0·81	1·67	35·0	459	0·033	14·4	880	1099	384	950	0·93	
74	25	15	0·81	1·67	41·2	459	0·033	14·4	900	1251	450	1090	0·83	
GREEN[33]														
S9	152	102	0·81	1·83	35·3	43·8	0·42	18·8	138	461	161	120	1·15	

TABLE 4: **Data from tests on biaxially loaded columns.**

Test No.	b (*mm*)	h (*mm*)	d/h	100ρ	f_{cu} (*N/mm²*)	f_y	e_i/h	l_e/h	$N_{u\ test}$ (*kN*)	$N_{uz\ calc}$ (*kN*)	$N_{bal\ calc}$ (*kN*)	$N_{u\ calc}$ (*kN*)	$\frac{N_{u\ test}}{N_{u\ calc}}$	Remarks
PANNELL AND ROBINSON[34]														
11			0·80 0·87		22·3	352	0·38 0·43	41·6	40	147	51	15·8	2·53	
12			0·80 0·87		23·2	285	0·29 0·09	41·6	40	140	42	18·1	2·21	
13			0·80 0·87		20·0	285	0·79 0·49	41·6	20	127	39·3	11·0	1·81	
14	95	64	0·80 0·87	3·25	21·0	285	0·67 1·12	41·6	20	131	40·6	9·5	2·12	Slenderness ratio about strong axis is 27·8
15			0·80 0·87		20·0	366	0·17 0·21	41·6	50	139	36·6	19·7	2·54	
16			0·80 0·87		20·1	366	0·30 0·37	41·6	30	139	39	16·7	1·80	
17			0·80 0·87		36·5	352	0·77 0·86	41·6	20	202	68·6	13·7	1·46	
STURROCK AND CRANSTON[38]														
3			0·81 0·87		49·0	290	0 0·38	50	273	1430	600	75	3·64	
4			0·81 0·92		50·0	420	0 0·25	50	463	1500	645	100	4·63	
5	400	100	0·81 0·92	1·27	50·0	420	0 0·30	50	349	1500	645	100	3·49	Slenderness ratio about strong axis is 12·5
6			0·81 0·92		38·0	420	0 0·38	50	321	1234	530	85	3·78	
7			0·92 0·92		50·0	420	0 0·34	50	378	1500	645	100	3·78	

NOTES: 1. Two values are given for d/h and e_i/h. These refer to the minor and major axis, the value for the minor axis being given first.

2. The slenderness ratios given are about the minor axes since, in both test and design, deformations at the ultimate stage dominate about this axis.

TABLE 5: **Data from tests on framed columns.**

Test No.	b (*mm*)	h (*mm*)	d/h	100ρ	f_{cu} (*N/mm²*)	f_y	e_i/h	l_e/h	$N_{u\,test}$ (*kN*)	$N_{uz\,calc}$ (*kN*)	$N_{bal\,calc}$ (*kN*)	$N_{u\,calc}$ (*kN*)	$\frac{N_{u\,test}}{N_{u\,calc}}$	Remarks
SAENZ AND MARTÍN[26]														
1E-3			0·84	1·10	42·5	263	0	15·1	396	350	147	350	1·13	
10E-1			0·83	2·48	39·9	248	0	15·1	381	360	148	360	1·05	
26D-1						263	0	17·5	261	250	105	250	0·86	
2			0·84	1·10	28·9	263	0	17·5	272	250	105	250	1·10	
3						263	0	17·5	236	250	105	250	0·94	
23D-1						248	0	17·5	277	246	84	245	1·13	
2			0·83	2·48	24·5	248	0	17·5	233	246	84	245	0·95	
3						248	0	17·5	251	246	84	245	1·02	
3E-1			0·84	1·10	39·3	263	0	17·5	333	329	138	330	1·01	
2						263	0	17·5	236	329	138	330	0·72	
31D-1						248	0	17·5	395	389	144	390	1·02	
2			0·83	2·48	43·5	248	0	17·5	375	389	144	390	0·96	
3						248	0	17·5	343	389	144	390	0·88	
27D-1						263	0	21	215	263	136	265	0·82	
2			0·84	1·10	30·7	263	0	21	216	263	136	265	0·82	
3						263	0	21	192	263	136	265	0·73	
24D-1						248	0	21	211	258	87	260	0·82	
2			0·83	2·48	26·0	248	0	21	198	258	87	260	0·77	
3						248	0	21	215	258	87	260	0·83	
1E-1			0·84	1·10	42·5	263	0	21	297	253	148	355	0·84	
2						263	0	21	343	253	148	355	0·97	
10E-2			0·83	2·48	39·9	248	0	21	365	362	130	360	1·01	
3						248	0	21	370	362	130	360	1·02	
29D-1						263	0	24·5	177	239	100	130	1·36	
2			0·84	1·10	27·5	263	0	24·5	159	239	100	130	1·22	
3						263	0	24·5	189	239	100	130	1·46	
30D-1	127	90·4				248	0	24·5	171	260	91	195	0·88	
2			0·83	2·48	26·3	248	0	24·5	195	260	91	195	1·00	
3						248	0	24·5	192	260	91	195	1·00	
2E-1						263	0	24·5	248	381	160	180	1·38	
2			0·84	1·10	46·2	263	0	24·5	246	381	160	180	1·37	
3						263	0	24·5	250	381	160	180	1·39	
20D-1						248	0	24·5	230	332	120	230	1·00	
2			0·83	1·10	35·9	248	0	24·5	229	332	120	230	1·00	
3						248	0	24·5	242	332	120	230	1·05	
6E-1						263	0	28	148	212	86	81	1·83	
2			0·84	1·10	24·0	263	0	28	151	212	86	81	1·86	
3						263	0	28	141	212	86	81	1·74	
15E-1						247	0	28	190	251	85	125	1·52	
2			0·83	2·48	25·1	247	0	28	161	251	85	125	1·29	
3						247	0	28	168	251	85	125	1·34	
5E-1			0·84	1·10	48·2	263	0	28	238	396	166	125	1·90	
2						263	0	28	244	396	166	125	1·95	
14E-1						248	0	28	227	337	121	150	1·51	
2			0·83	2·48	36·6	248	0	28	243	337	121	150	1·62	
3						248	0	28	223	337	121	150	1·48	
21E-1						248	0	30·1	169	234	79	100	1·57	
2			0·83	2·48	22·8	248	0	30·1	152	234	79	100	1·42	
3						248	0	30·1	137	234	79	100	1·29	
28F-1						248	0	30·1	196	345	125	120	1·63	
2			0·83	2·48	37·7	248	0	30·1	225	345	125	120	1·87	
3						248	0	30·1	226	345	125	120	1·88	
BARTER AND MACGREGOR[30]														
C1					33·1	308	0·05	21	170	227	68	159	1·07	$e_1/h = 0{\cdot}11$
C2	112	63·5	0·82	4·02	38·0	308	0·05	21	175	250	77	175	1·00	$= -e_2/h$
D1					31·3	308	0·39	21	42	219	73	48	0·88	$e_1/h = 0{\cdot}82$
D2					39·2	308	0·39	21	54	255	76	54	1·00	$= -e_2/h$

TABLE 5 *continued*

Test No.	b (*mm*)	h (*mm*)	d/h	100ρ	f_{cu} (*N/mm²*)	f_y	e_i/h	l_e/h	$N_{u\,test}$ (*kN*)	$N_{uz\,calc}$ (*kN*)	$N_{bal\,calc}$ (*kN*)	$N_{u\,calc}$ (*kN*)	$\frac{N_{u\,test}}{N_{u\,calc}}$	Remarks
BREEN AND FERGUSON[28]														
F1					34·9	365	0·13	24·0	260	443	164	181	1·44	$e_1/h = 0{\cdot}30$; $e_2/h = -0{\cdot}12$
F2					26·2	359	0·04	24·0	260	354	127	220	1·18	$e_1/h = 0{\cdot}10$; $e_2/h = -0{\cdot}04$
F3	152	102	0·84	1·84	33·4	364	0·13	11·2	270	428	158	214	1·26	$e_1/h = 0{\cdot}30$; $e_2/h = -0{\cdot}12$
F4					28·1	361	0·04	11·2	370	373	134	300	1·24	$e_1/h = 0{\cdot}10$;
F5					32·4	372	0·04	23·0	320	419	155	280	1·14	$e_2/h = -0{\cdot}04$
F6					33·1	354	0·13	24·0	260	423	158	182	1·43*	$e_1/h = 0{\cdot}30$; $e_2/h = -0{\cdot}12$
FURLONG AND FERGUSON[31]														
1				3·28	29·8	350	0·1	15·6	267	450	130	300	0·89	
2				1·84	37·0	379	0·1	17·0	274	468	160	290	0·94	
3				1·84	28·8	394	0·3	17·0	177	387	133	140	1·26	
4	152	102	0·80	1·84	27·9	372	0·2	15·6	234	374	125	185	1·26	
5				1·84	27·9	364	0·1	14·0	247	372	124	260	0·95	
6				1·84	30·6	349	0·3	14·0	200	396	132	160	1·25	
7				1·84	41·3	350	0·1	14·0	349	506	178	350	1·00*	
FERGUSON AND BREEN[32] (Lateral load series)†														
L1			0·84	1·95	24·3	383	0·1	26	167	451	162	175	0·95	
L2			0·84	1·93	35·9	407	0·3	26	111	475	171	119	0·93	
L3			0·84	1·93	27·6	389	0·1	32	138	485	136	112	1·23	
L4	153	103	0·85	1·94	32·8	393	0·1	13	245	438	159	300	0·82	
L5			0·84	1·93	35·0	398	0·3	13	189	463	166	200	0·94	
L6			0·84	1·94	31·9	384	0·1	13	245	426	153	305	0·80	
L7			0·84	1·92	25·8	393	0·1	21	178	368	130	200	0·80	
BAUMANN[18]														
2/17	200	90	0·87	1·12	24·6	304	0	22·7	375	347	145	310	1·21	
2/18	201	91	0·87	1·10	24·6	304	0	29·1	360	352	147	130	2·77	
2/20	250	130	0·87	0·97	29·8	293	0	19·6	845	723	319	723	1·17	
2/21	200	89	0·88	1·13	42·6	304	0	23·0	550	555	241	420	1·31	
2/22	200	89	0·88	1·13	42·6	304	0	23·0	620	555	241	420	1·48	
2/23	248	129	0·88	0·98	46·8	293	0	15·0	1075	1073	472	1073	1·00	
2/24	248	129	0·88	0·98	46·8	293	0	19·8	945	1073	472	1073	0·88	
2/25	250	248	0·87	0·99	37·2	281	0·07	7·1	1300	1680	743	1130	1·15	
2/27	201	92	0·88	1·70	38·2	272	0·07	7·1	340	541	222	168	2·02	
2/28	200	89	0·88	1·76	35·9	272	0·07	29·8	290	496	205	144	2·01	
2/29	250	130	0·87	1·89	39·8	281	0·07	19·6	735	1002	410	600	1·22	
2/30	250	132	0·83	1·86	39·8	281	0·07	19·3	770	1015	414	640	1·20	
2/31	250	250	0·87	2·00	34·2	282	0·07	9·8	1430	1718	687	1160	1·23	

* Long-term test but deflections insignificant.
† See note in text on this series.

TABLE 6: **Data from long-term tests on columns.**

Test No.	b (mm)	h (mm)	d/h	100ρ	f_{cu} (N/mm^2)	f_y	e_i/h	l_e/h	$N_{u\,test}$ (kN)	$N_{uz\,calc}$ (kN)	$N_{bal\,calc}$ (kN)	$N_{u\,calc}$ (kN)	$\frac{N_{u\,test}}{N_{u\,calc}}$	Remarks
FERGUSON AND BREEN[32]														
L8	154	103	0·84	1·93	40·3	390	0·3	26	67	517	191	119	0·56	
GAEDE[23]														
I2					25·0	334	0·2	29·1	62	300	125	63	0·99	
I3					27·3	342	0·2	29·1	62	323	137	68	0·91	
I4					31·2	280	0·2	29·4	63	356	153	60	1·05	
I6					38·6	292	0·2	29·1	80	434	186	74	1·08	
I7	154	100	0·87	1·00	37·5	286	0·2	29·4	80	421	181	71	1·13	
II1					25·9	327	0·5	29·4	21·5	308	129	37	0·58	
II2					27·7	324	0·5	29·4	23·5	325	136	35	0·67	
II13					24·9	333	0·5	29·4	23·5	298	125	37	0·63	
RAMU, GRENACHER, BAUMANN AND THÜRLIMANN[37]														
61			0·81	1·67	42·7	459	0	28·8	645	1288	463	465	1·39	
81			0·81	1·67	38·2	459	0	28·8	600	1179	424	450	1·33	
42			0·81	1·67	31·2	459	0·033	28·8	407	1005	344	370	1·10	
43			0·82	1·67	32·1	459	0·033	28·8	426	1026	359	380	1·12	
44			0·81	1·67	26·8	459	0·033	28·8	427	896	295	330	1·29	
51			0·83	1·67	54·4	459	0·033	28·8	490(430)*	1576	583	510	0·96	
13			0·82	1·67	34·2	459	0·10	28·8	305	1080	378	335	0·91	
15			0·82	1·67	36·2	459	0·10	28·8	342	1128	359	340	1·00	
16			0·82	1·67	27·4	459	0·10	28·8	323	910	309	295	1·09	
21			0·82	1·67	34·8	459	0·10	28·8	300(260)	1094	382	330	0·91	
22	25	15	0·82	1·67	37·3	459	0·25	28·9	185	1155	415	265	0·71	
25			0·82	1·67	36·6	459	0·25	28·9	190(161)	959	398	235	0·81	
52			0·82	1·67	31·5	459	0·25	28·9	219(185)	1579	354	300	0·73	
32			0·81	1·67	35·8	459	1·0	28·9	76·5(68·5)	1119	391	95	0·81	
33			0·82	1·67	34·6	459	1·0	28·9	81·5(71·5)	1090	381	93	0·88	
55			0·81	0.99†	39·5	459	0·033	28·9	426	1121	336	370	1·15	
56			0·82	0·99†	45·5	459	0·25	28·9	183	1270	406	254	0·72	
83			0·82	4·28	39·2	550	0	28·9	760	1635	425	900	0·84	
64			0·81	4·28	33·0	550	0·033	28·9	630	1484	356	710	0·89	
63			0·81	4·28	49·4	550	0·25	28·9	355(344)	1881	507	375	0·95	
54			0·82	1·67	42·2	459	0·033	28·9	430	1276	459	420	1·02	
62			0·85	1·67	46·1	459	0·033	28·9	485	1373	521	40	1·05	
65			0·83	1·71	30·6	459	0·05	43·2	156	663	232	110	1·42	
71	25	10	0·82	1·71	39·2	459	0·05	43·2	137	806	290	114	1·20	
66			0·82	1·71	31·9	459	0·375	43·2	66	684	239	72	0·92	
72			0·83	1·71	40·2	459	0·375	43·2	61	822	254	75	0·81	
GREEN[33]														
51			0·81	1·86	34·5	452	0·035	18·8	236	450	153	360	0·65	
54			0·82	1·79	34·5	386	0·18	18·3	187	452	162	212	0·88	
55	152	104	0·82	1·79	34·1	403	0·105	18·3	185	452	162	270	0·69	
56			0·82	1·79	28·6	443	0·155	18·3	162	402	136	205	0·79	
58			0·82	1·79	35·3	412	0·27	18·3	133	466	167	168	0·79	

* Where a second value for $N_{u\,test}$ is given in parentheses, this signifies the long-term load in an L-S test (see footnote to Table 2).

†In these cases 0·15 is in compression and 0·84 in tension.

REFERENCES

1. AMERICAN CONCRETE INSTITUTE AND CEMENT AND CONCRETE ASSOCIATION. *Recommendations for an international code of practice for reinforced concrete.* London, 1964. pp. 156.
2. ROWE, R. E., CRANSTON, W. B. and BEST, B. C. New concepts in the design of structural concrete. *The Structural Engineer.* Vol. 43, No. 12. December 1965. pp. 399–403.
3. CRANSTON, W. B. *A computer method for the analysis of restrained columns.* London, Cement and Concrete Association, April 1967. pp. 20. Technical Report TRA 402.
4. CRANSTON, W. B. *Restrained metal columns.* Thesis submitted to the University of Glasgow for the degree of PhD. 1963. pp. 98.
5. BRITISH STANDARDS INSTITUTION. Draft British Standard Code of Practice for the Structural use of concrete. London, 1969. pp. 241. Document No. 69/23007.
6. BRITISH STANDARDS INSTITUTION. CP 114:1957. *The structural use of reinforced concrete in buildings.* London. pp. 83.
7. COMITE EUROPEEN DU BETON. Commission No. 8. Flambage. *Bulletin d'Information.* No. 5. Paris, June 1958. pp. 23–40.
8. COMITE EUROPEEN DU BETON. Kommission Nr. 8. Knickung. *Bulletin d'Information.* No. 17. Paris, April 1959. pp. 1–81.
9. COMITE EUROPEEN DU BETON. 5e session de travail. Flambement. *Bulletin d'Information.* No. 25. Paris, July 1960. pp. 8.00–8.35.
10. COMITE EUROPEEN DU BETON. Commission VIII. Flambement. *Bulletin d'Information.* No. 31. Paris, February 1961. pp. 1–90.
11. COMITE EUROPEEN DU BETON. XIe session plénière. Flambement. 1e partie. *Bulletin d'Information.* No. 62. Paris, September 1967. pp. 173.
12. COMITE EUROPEEN DU BETON. XIe session plénière. Flambement. 2e partie. *Bulletin d'Information.* No. 63. Paris, October 1967. pp. 153.
13. COMITE EUROPEEN DU BETON. Conclusions techniques XIIe session plénière. 2e partie. Flambement. *Bulletin d'Information.* No. 69. Paris, October 1968. pp. 117–270.
14. COMITE EUROPEEN DU BETON and FEDERATION INTERNATIONALE DE LA PRECONTRAINTE. *International Recommendations for the design and construction of concrete structures.* English edition. London, Cement and Concrete Association, 1970. pp. 80.
15. JOHNSTON, B. C. (Ed.) *Column Research Council guide to design criteria for metal compression members.* Second edition. New York, John Wiley & Sons Inc., 1966. pp. 217.
16. MARSHALL, W. T. A survey of the problem of lateral instability in reinforced concrete beams. *Proceedings of the Institution of Civil Engineers.* Vol. 43. July 1969. pp. 397–406.
17. HANSELL, W. and WINTER, G. Lateral stability of reinforced concrete beams. *Journal of the American Concrete Institute. Proceedings* Vol. 56, No. 3. September 1959. pp. 193–214.
18. BAUMANN, O. *Die Knickung der Eisenbeton-Säulen.* (Buckling of reinforced concrete columns.) Zürich, Eidgenossische Materialsprüfungsanstalt an der Eidgenossische Technischen Hochschule, Zürich, 1934. Bericht No. 89.
19. THOMAS, F. G. *The strength of long reinforced concrete columns in short period tests to destruction.* London, HMSO, 1939. pp. 29. Building Research Station Technical Paper No. 24.
20. RAMBØLL, B. J. *Reinforced concrete columns concentrically and eccentrically loaded.* Copenhagen, Teknisk Førlag, 1951.
21. ERNST, G. C., HROMADIK, J. J. and RIVELAND, A. R. *Inelastic buckling of plain and reinforced concrete columns, plates and shells.* Lincoln, University of Nebraska, 1963. pp. 66. Engineering Experiment Station Bulletin No. 3.
22. GEHLER, W. and HÜTTER, A. *Knickversuche mit Stahlbetonsäulen.* (Buckling tests on reinforced concrete columns.) Berlin, Wilhelm Ernst und Sohn, 1954. Deutscher Ausschuss für Stahlbeton. Heft 113. pp. 1–56.
23. GAEDE, K. *Knicken von Stahlbetonstäben unter Kurz- und Langzeitbelastung.* (Buckling tests on reinforced concrete columns under short- and long-term loading.) Berlin, Wilhelm Ernst und Sohn, 1958. pp. 80. Deutscher Ausschuss für Stahlbeton. Heft 129.
24. KORDINA, K. Unpublished work, details of which are given in reference 11.
25. AAS-JAKOBSEN, A. Unpublished work, details of which are given in reference 11.
26. SAENZ, L. P. and MARTÍN, I. Test of reinforced concrete columns with high slenderness ratio. *Journal of the American Concrete Institute. Proceedings* Vol. 60, No. 5. May 1963. pp. 589–616.
27. CHANG, W. F. and FERGUSON, P. M. Long hinged reinforced concrete columns. *Journal of the American Concrete Institute. Proceedings* Vol. 60, No. 1. January 1963. pp. 1–26.
28. BREEN, J. E. and FERGUSON, P. M. The restrained long concrete column as a part of a rectangular frame. *Journal of the American Concrete Institute. Proceedings* Vol. 61, No. 5. May 1964. pp. 563–588.
29. MARTÍN, I. and OLIVIERI, E. Test of reinforced concrete columns bent in double curvature. *Symposium on reinforced concrete columns.* Detroit, American Concrete Institute, 1966. Special Publication SP-13. pp. 121–138.
30. MACGREGOR, J. G. and BARTER, S. L. Long eccentrically loaded concrete columns bent in double curvature. *Symposium on reinforced concrete columns.* Detroit, American Concrete Institute, 1966. Special Publication SP-13. pp. 139–156.
31. FURLONG, R. W. and FERGUSON, P. M. Tests of frames with columns in single curvature. *Symposium on reinforced concrete columns.* Detroit, American Concrete Institute, 1966. Special Publication SP-13. pp. 55–73.
32. FERGUSON, P. M. and BREEN, J. E. Investigation of the long concrete column in a frame subject to lateral loads. *Symposium on reinforced concrete columns.* Detroit, American Concrete Institute, 1966. Special Publication SP-13. pp. 75–114.
33. GREEN, R. *Behavior of unrestrained reinforced concrete columns under sustained load.* Dissertation submitted to the University of Texas, Austin, for the degree of PhD. 1966.
34. PANNELL, F. N. and ROBINSON, J. L. Slender reinforced concrete columns with biaxial eccentricity of loading. *Magazine of Concrete Research.* Vol. 20, No. 65. December 1968. pp. 195–204.
35. MEHMEL, A., SCHWARZ, H., KASPAREK, K. H. and MAKOVI, J. *Tragverhalten ausmittig beanspruchter Stahlbetondruckglieder.* (Load-carrying capacity of eccentrically loaded reinforced concrete compression members.) Berlin, Wilhelm Ernst und Sohn, 1969. pp. 54. Deutscher Ausschuss für Stahlbeton. Heft 204.
36. BREEN, J. E. and FERGUSON, P. M. Long cantilever columns subject to lateral forces. *Journal of the American Concrete Institute. Proceedings* Vol. 66, No. 11. November 1969. pp. 884–893.
37. RAMU, P., GRENACHER, M., BAUMANN, M. and THÜRLIMANN, B, *Versuche an gelenkig gelagerten Stahlbetonstützen unter Dauerlast.* (Long-term tests on pin-ended reinforced concrete columns.) Zürich, Institut für Baustatik Eidgenossische Technischen Hochschule, 1969. pp. 86. Bericht Nr. 6418-1.
38. CRANSTON, W. B. and STURROCK, R. D. Lateral instability of slender reinforced concrete columns. *Experimental analysis of instability problems on reduced and full-scale models.* RILEM International Symposium, Buenos Aires, 1971. Theme 1. pp. 117–141.

39. MASSONNET, C. and MOENAERT, P. Calcul du béton armé à la rupture en flexion simple ou composée. Comparison statistique de diverses théories avec l'ensemble des résultats des recherches expérimentales. *International Association for Bridge and Structural Engineering, Sixth Congress, Stockholm* 1960. Preliminary publication. pp. 105–127.
40. SPOONER, D. C. Discrepancies in concrete cube testing—current problems and future developments in cube testing techniques. *Symposium: The Evolution of Mechanical Equipment for Testing Building Materials, Stuttgart, 25–30 March 1968*. RILEM. Final Report. Vol. 2. pp. B1–B24.
41. HANSON, R. and ROSENSTRØM, S. Trykkforsøk med slanka betonpelare. (Some tests of slender concrete columns subjected to compressive forces.) *Betong*. Vol. 32, No. 3. 1947. pp. 247–262.
42. SWANN, R. A. *Flexural strength of corners of reinforced concrete portal frames*. London, Cement and Concrete Association, 1969. pp. 14. Technical Report TRA 434.
43. DRYSDALE, R. G., MIRZA, M. S. and MCCUTCHEON, J. O. Recent research in deflection of concrete structures. *Deflection of structural concrete: Proceedings of the Symposium, Montreal*, 1971. Montreal, Canadian Capital Chapter of ACI, 1971. pp. 161–182.