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DYNAMIC EFFECTS OF EARTHQUAKES[a]

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SYNOPSIS

The principal factors controlling the dynamic response of structures to earthquakes are summarized, and are related to the lateral force provisions recommended for inclusion in the Uniform Building Code by the Structural Engineers Association of California (SEAOC). These provisions are seen to conform very well with the concepts of dynamic theory.

INTRODUCTION

Experience with recent earthquakes in Tehachapi, Calif. and Mexico City, Mexico has shown that it is possible to build economical, attractive structures which are highly resistant to earthquake affects. But at the same time, these earthquakes demonstrated that where the dynamic effects of earthquakes are not fully understood or properly accounted for, the results can be disastrous.

The purpose of this paper is to summarize the principal factors controlling the dynamic response of structures to earthquakes, and to relate these factors to current trends in the development of the earthquake provisions in building codes. It will be seen that the lateral force requirements recently recommended by the SEAOC take cognizance of the major factors affecting the dynamic response of structures and, thus, provide a rational basis for the design of earthquake-resistant structures.

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Before going into the details of the dynamic response theory, it will be useful to emphasize a few pertinent facts regarding the nature of earthquakes. An earthquake is, of course, simply a ground-vibration phenomenon. Since the earth is elastic in its gross characteristics, and possesses mass, it will vibrate when subjected to a shock loading just as will any other mechanical system. Thus, when a slippage occurs suddenly at a fault zone, shock waves are propagated through the earth in all directions, and when the surface manifestations of these waves pass any given point on the earth, it (and any structure located on it) will be caused to vibrate. Motions induced by the ground vibrations may have both vertical and horizontal components, but since buildings normally have considerable excess strength in the vertical direction it is customary to consider only the effects of the horizontal motions in earthquake-resistant design. It should be emphasized that the forces developed during an earthquake

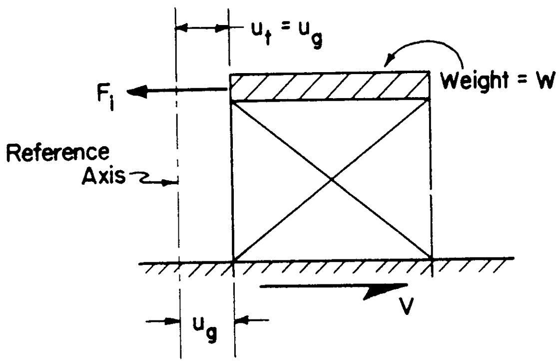


FIG. 1.—EARTHQUAKE FORCES, RIDGID STRUCTURE

are not applied directly to the structure, but rather are inertia forces resulting from the motions of the structure.

EARTHQUAKE EFFECTS ON A RIGID STRUCTURE

In order to provide a suitable background for this study of the dynamic-response problem, it will be useful to consider first the effect of an earthquake on a rigid structure. Such a structure is shown in Fig. 1. It is assumed that both the building and its foundations are rigid so that the earthquake motions of the ground, u_g, are transmitted directly to the building. In this case, it is clear that an effective earthquake force, F_i, will be developed in the structure equal to the product of the ground acceleration and the mass of the structure,

$$F_i = \ddot{u}_g \frac{W}{g} \quad \ldots\ldots\ldots\ldots \quad (1)$$

in which $\ddot{u}_g$ is the ground acceleration, W denotes the weight of the structure, and g is the acceleration of gravity. For convenience, Eq. 1 is usually rearranged so that the force is given as the product of the weight of the structure

and a seismic coefficient, C, which represents the ratio of the ground acceleration to the acceleration of gravity:

$$F_i = \frac{\ddot{u}_g}{g} W = C\,W \quad \ldots\ldots\ldots\ldots \quad (2)$$

in which

$$C = \frac{\ddot{u}_g}{g} \quad \ldots\ldots\ldots\ldots \quad (3)$$

For design purposes, it is common practice to express the earthquake force in terms of the shearing force developed at the base of the structure. In this case, simple statics show that the base shear, V, is equal to the force F_i, and is given by

$$V = C\,W \quad \ldots\ldots\ldots\ldots \quad (4)$$

Eq. 4 demonstrates that the dynamic analysis of a rigid structure is very simple. All that is required is an estimate of the maximum ground acceleration which will occur during the earthquake. This acceleration, expressed as a ratio to the acceleration of gravity is the seismic coefficient C in the formula.

The rigid-structure concept provided the basis for the lateral-force provisions of some of the earliest earthquake codes, which specified that a structure should be designed for a certain percentage of gravity (say 10% or 12%), regardless of the characteristics of the structure. Unfortunately, the dynamic-response characteristics of actual structures are not so simple. Their flexibility and mass impart to them vibration characteristics which directly affect the magnitude of the seismic forces to which they will be subjected during an earthquake.

DYNAMIC RESPONSE OF A FLEXIBLE STRUCTURE

The effect of a structure's flexibility on its response may be discussed most easily by reference to a simple, one-story structure, as shown in Fig. 2. The weight of the structure, W, is assumed to be concentrated at the roof level. Such a structure is said to have a single degree of freedom (considering plane motion only) because only one type of deformation is possible, represented here by the displacement, u. The significant dynamic properties of this structure, in addition to its weight, are the stiffness of the columns, k, which represents the force developed per unit displacement, and the damping, c, which represents the force per unit velocity. In the explanation which follows, damping will be omitted for simplicity, but the effect of damping will be included with the final results.

In the absence of damping, the base shear in this structure may be expressed as the product of the displacement and the column stiffness,

$$V = k\,u \quad \ldots\ldots\ldots\ldots \quad (5)$$

Dynamic equilibrium conditions (using d'Alembert's principle) show that the base shear must balance the inertia force of the mass, that is,

$$\frac{W}{g}\ddot{u}_t + k\,u = 0 \quad \ldots\ldots \quad (6)$$

(noting the sign convention assumed in Fig. 2). It will be noted that the inertia force here depends on the total motion of the mass, rather than the ground motion, as was the case only in Fig. 1. It is convenient to express the total acceleration as the sum of the ground acceleration and the relative acceleration of the mass with respect to the ground, thus

$$\ddot{u}_t = \ddot{u}_g + \ddot{u} \quad \ldots\ldots \quad (7)$$

Then Eq. 6 may be rewritten as

$$\frac{W}{g}\ddot{u} + k\,u = -\frac{W}{g}\ddot{u}_g = F_e \quad \ldots\ldots \quad (8)$$

Eq. 8 is identical with that which would apply to a stationary structure subjected to an effective force, F_e, equal to the product of the mass of the structure and the ground acceleration. Thus the dynamic effects of **earthquakes**

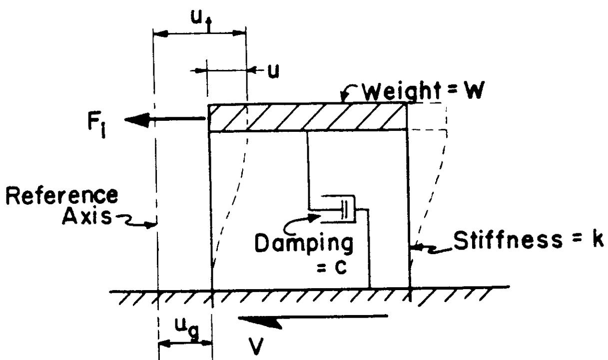


FIG. 2.—EARTHQUAKES FORCES, FLEXIBLE STRUCTURE

may be studied by considering the structure to be stationary and applying to it an effective earthquake force, F_e.

Now it is clear that this effective force is not directly resisted by shears in the columns; the mass must first be accelerated, and, thus, the inertia of the structure modified the dynamic effect of the applied load. The base shear, V, in this case, depends on the nature of the applied force F_e (that is, the time history of the ground acceleration) and also on the vibration characteristics of the structure. If the ground displacements were a simple harmonic motion of period T_p, as shown in Fig. 3, the effective earthquake force could be expressed

$$F_e = F_o \sin p\,t = -\ddot{u}_{go}\frac{W}{g}\sin p\,t \quad \ldots\ldots \quad (9)$$

in which

$$p = \frac{2\pi}{T_p} \quad \ldots\ldots \quad (10)$$

In this case, the maximum (steady state) base-shear force is given by

$$V_{max} = u_{max}\,k = F_o\left[\frac{1}{1-\left(\frac{T}{T_p}\right)^2}\right] = W\frac{\ddot{u}_{go}}{g}\left[\frac{1}{1-\left(\frac{T}{T_p}\right)^2}\right] \quad \ldots \quad (11)$$

in which

$$T = 2\pi\sqrt{\frac{W}{g\,k}} \quad \ldots\ldots \quad (12)$$

is the period of vibration of the structure. From Eq. 11 it is clear that the response of the structure depends, in a very direct fashion, on the natural period

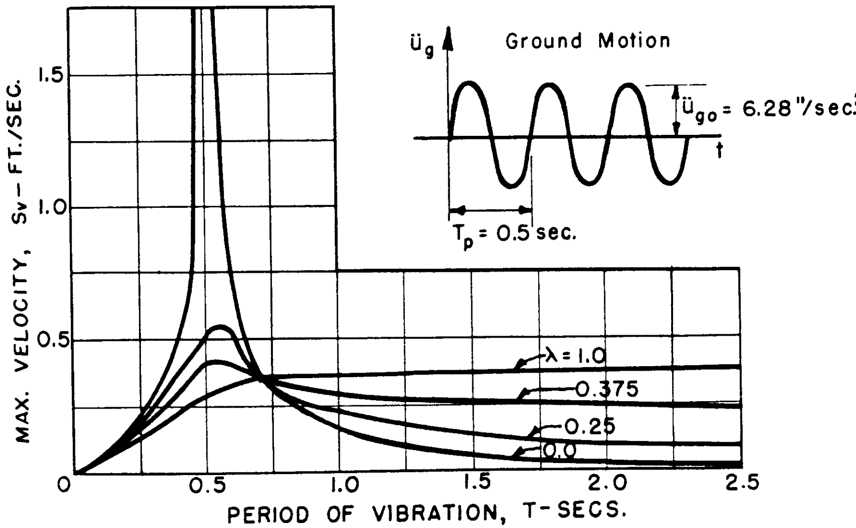


FIG. 3.—RESPONSE TO HARMONIC MOTION

of vibration of the structure, which depends, in turn, on its stiffness and weight. The base shear may be either less than or greater than that of a rigid structure, depending on how close to resonance this system is, that is, how near its natural period of vibration is to the period of the applied ground motion.

It is sometimes convenient to express the response of a structure to a specific ground motion in terms of a velocity coefficient, S_v, as follows:

$$V_{max} = \frac{W}{g}\frac{2\pi}{T}S_v \quad \ldots\ldots \quad (13)$$

where S_v represents the maximum velocity produced in the structure by the particular motion. The velocity coefficient for simple harmonic ground motion, therefore, is given by

$$S_v = \frac{T}{2\pi}\,\ddot{u}_{go}\left[\frac{1}{1-\left(\frac{T}{T_p}\right)^2}\right] \qquad (14)$$

A graph representing the variation of the velocity coefficient, S_v, with the period of vibration of the structure for a harmonic ground motion of a particular amplitude and period, is shown in Fig. 3. Such a graph is called the velocity spectrum of the ground motion because it shows the maximum velocity developed by this motion for a complete spectrum of periods of vibration of the structure.

It will be recalled that the preceding remarks refer to the response of an undamped structure. If the structure is damped, that is, if it has some form of resistance which depends on the velocity of motion, as represented by the viscous damper shown in Fig. 2, the magnitude of this damping force will also affect the response of the structure. The magnitude of the viscous damping of a system, c, is usually expressed as a ratio to a critical or reference damping coefficient, c_c. Thus, the damping ratio λ, is given by $\lambda = c/c_c$. The velocity spectrum of a damped system for a harmonic-ground motion is given by

$$S_v = \frac{T}{2\pi}\,\ddot{u}_{go}\left[\frac{1+\left(2\lambda\frac{T}{T_p}\right)^2}{\left[1-\left(\frac{T}{T_p}\right)^2\right]^2+\left(2\lambda\frac{T}{T_p}\right)^2}\right]^{1/2} \qquad (15)$$

and, also, is shown graphically in Fig. 3, for various values of the damping ratio. The important effect that damping has in limiting the response of the system at frequencies approaching resonance is clearly shown in Fig. 3.

EARTHQUAKE RESPONSE OF A SIMPLE FLEXIBLE STRUCTURE

The preceding material is not intended to imply that earthquake motion of the ground may be represented by simple harmonic motion. The only reason for including this explanation is to emphasize, with a familiar example, the important influence of the period of vibration of the structure on its response to a given ground motion. That an earthquake is far from a simple harmonic motion is clearly shown by Fig. 4, which presents the ground acceleration measured at Taft, Calif., from the Tehachapi earthquake of July 21, 1952. The motion may be characterized best as a series of erratic, almost random, acceleration pulses. Thus, the concept of resonance which was applied to harmonic motions has no place in the treatment of earthquake response.

On the other hand, the response to an earthquake motion can be expressed in terms of a velocity spectrum, just as was described previously, if the velocity spectrum is determined properly; and, again, the response will be found to depend on the period of vibration of the structure. For a completely arbitrary ground motion, such as is represented by an earthquake, the velocity

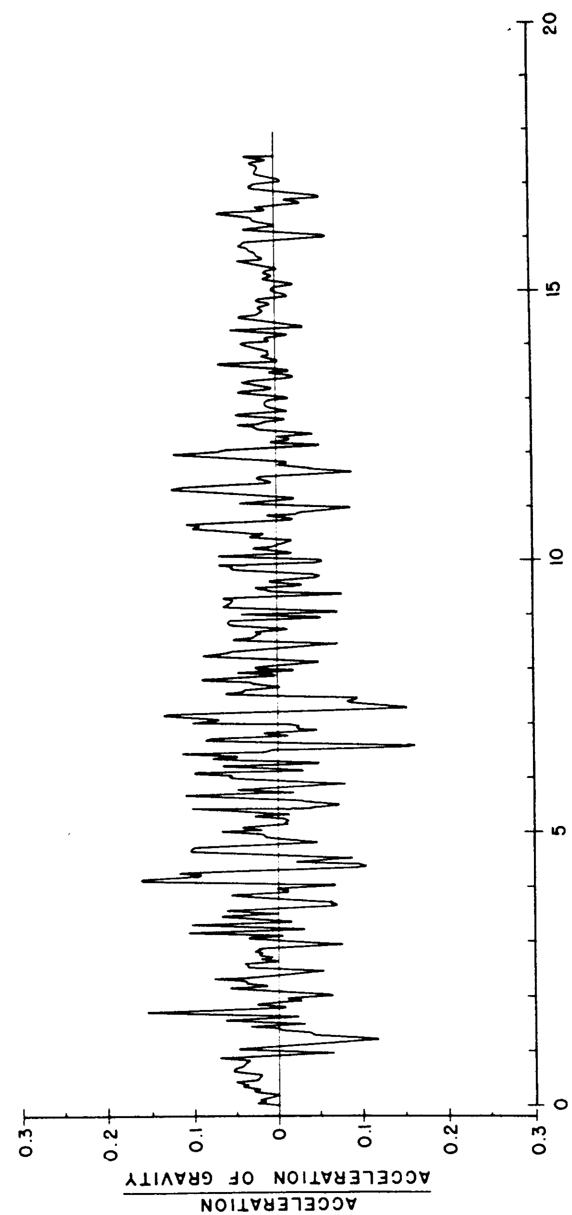


FIG. 4.—ACCELEROGRAM RECORDED AT TAFT, CALIFORNIA JULY 21, 1952 (COMPONENT N21E)

spectrum of an undamped system may be evaluated from

$$S_v = \left[\int_0^t \ddot{u}_g \sin \frac{2\pi}{T} (t - \tau) \, d\tau \right]_{max} \quad \ldots\ldots\ldots\ldots (16)$$

while for a damped system, the velocity spectrum is given by

$$S_v = \left[\int_0^t \ddot{u}_g \, e^{-\lambda\left(\frac{2\pi}{T}\right)(t - \tau)} \sin \frac{2\pi}{T} (t - \tau) d\tau \right]_{max} \quad \ldots\ldots (17)$$

In both Eqs. 16 and 17, the primary dependence of the spectral values on the period of the structure is evident. Damped and undamped velocity spectra calculated for the motion recorded at Taft, during the Tehachapi earthquake, are shown[2] in Fig. 5. The spectral curves are less regular in this case than they were for the simple harmonic-ground motion because of the erratic nature of the earthquake, but they have the same significance and the maximum base shear can be obtained by the use of Eq. 13, as before. Since Eq. 16 (or 17) must be evaluated throughout the entire history of the earthquake, for any given period of vibration, in order to find the maximum velocity developed for that one period, the calculation effort required to obtain a complete velocity spectrum is enormous. Such work is generally done by either analog or automatic digital computers.

The importance of the velocity-spectrum concept in earthquake engineering cannot be over-emphasized. The complete dynamic effect of the earthquake is represented by the spectrum, and to determine the force which would be developed in a given structure, by a given quake, it is necessary only to evaluate the damping and period of vibration of the structure, and then find the appropriate value of S_v from the velocity spectrum. For example, if the structure of Fig. 2 had a period of vibration of 0.7 sec and 10% critical damping, the corresponding spectral velocity, S_v, for the Taft earthquake would be 0.9 ft per sec. Then, if the mass of the structure were 20 kips, the base shear produced in this structure by this earthquake motion would be

$$V = \frac{W}{g} \frac{2\pi}{T} (0.9) = 20 \left(\frac{6.28}{0.7}\right) \left(\frac{1}{32.2}\right) (0.9) = 5.0 \text{ kips}$$

The response of any other single-degree-of-freedom system to this earthquake could be evaluated similarly.

EARTHQUAKE RESPONSE OF MULTI-STORY STRUCTURES

Although the dynamic effect of an earthquake on a simple elastic structure is completely represented by the velocity spectrum, there still remains the important question of how to evaluate the effect of earthquakes on more complex systems such as multi-story buildings. Fortunately, the procedure developed for structures having a single degree of freedom may be applied similarly to multiple-degree-of-freedom systems. It is necessary only to evaluate first the vibration properties of the structure, that is, its vibration periods and mode

2 "Behaviour of Structures During Earthquakes," by G. W. Housner, Proceedings, ASCE, Vol. 85, No. EM 4, Ocotober, 1959, p. 109.

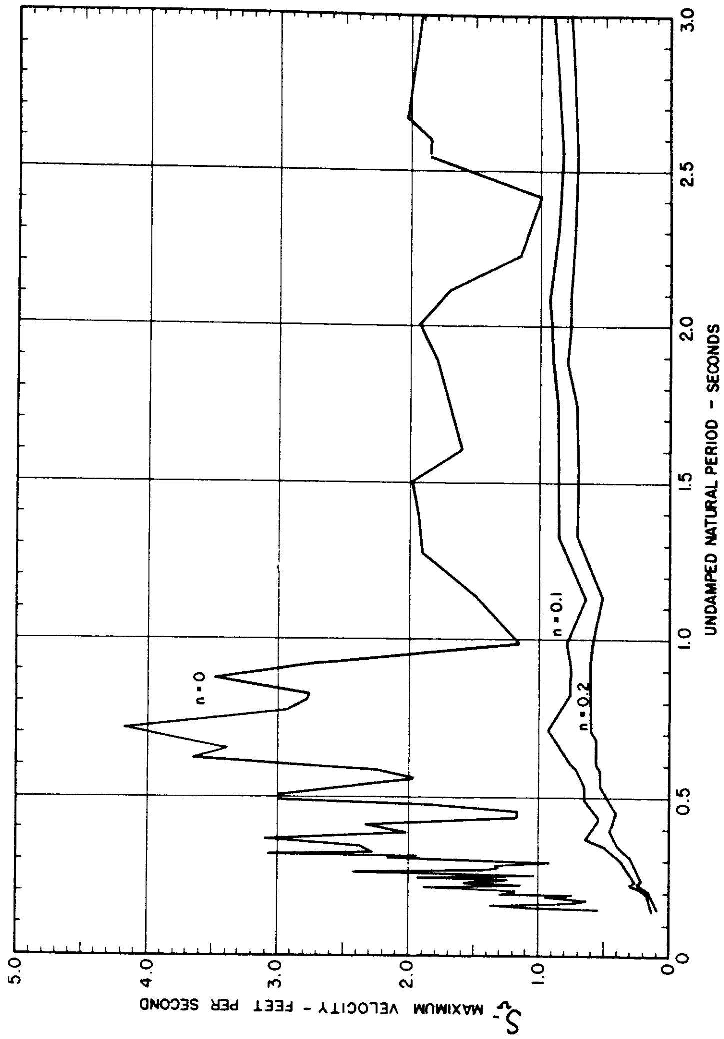
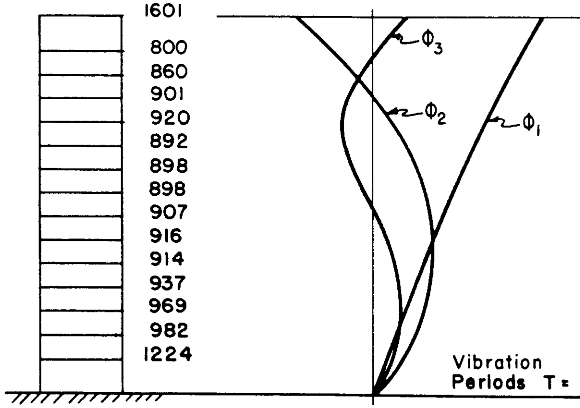


FIG. 5.—VELOCITY SPECTRUM FOR TAFT EARTHQUAKE, JULY 21, 1952

shapes. A structure may vibrate with as many different mode shapes and periods as it has degrees of freedom, and a multi-story building will have one degree of freedom for each story (considering plane motion only) if the weight is assumed to be concentrated at the floor levels. Thus a 10-story building will have ten vibration mode shapes and periods.

Now the important characteristic of these vibration modes is that they are completely independent of each other. Thus, the response to a given ground motion can be calculated independently for each mode, exactly as was described for the one-story system illustrated previously. The total effect of the earthquake may then be obtained by simply adding together the individual mode effects.

To demonstrate the procedure, the structure shown in Fig. 6 will be considered. This is the Alexander Building in San Francisco, Calif. undoubtedly the subject of more technical discussions on earthquake effects than any other



Story Weight - w_x (kips)	Φ_1	Φ_2	Φ_3
1601	1.408	-0.638	0.271
800	1.228	-0.350	0.017
860	1.142	-0.141	-0.163
901	1.052	0.018	-0.243
920	0.964	0.154	-0.267
892	0.866	0.283	-0.238
898	0.776	0.387	-0.161
898	0.673	0.456	-0.069
907	0.581	0.495	0.035
916	0.494	0.502	0.122
914	0.409	0.482	0.192
937	0.332	0.441	0.230
969	0.258	0.377	0.236
982	0.191	0.301	0.210
1224	0.131	0.215	0.162
Vibration Periods T =	1.25	0.35	0.20 secs

Total Weight = $14{,}619^k$; Eff. Weights = $10{,}377^k$ 2431^k 595^k = W_n

where $W_n = \dfrac{\left(\sum_x \Phi_{xn} w_x\right)^2}{\sum_x \Phi_{xn}^2 w_x}$

FIG. 6.—VIBRATION PROPERTIES OF THE ALEXANDER BUILDING

building in the world. The first, second, and third mode shapes for this building, and the corresponding vibration periods are shown. Also indicated is the effective weight, W_n, associated with each mode. This weight is used, together with period of vibration and spectral velocity value for each mode, to calculate the base shear for that mode, using the equation

$$V_n = \frac{W_n}{g} \frac{2\pi}{T_n} S_{v_n} \quad \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

which is the same as Eq. 13 except that the subscript n has been added to indicate that values appropriate to the n^{th} mode of vibration are to be used. The

effective-weight for the n^{th} mode is obtained from the relationship

$$W_n = \frac{\left(\sum_x \phi_{xn} w_x\right)^2}{\Sigma \phi_{xn}^2 w_x} \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

in which ϕ_{xn} represents the displacement at the x^{th} floor level in the n^{th} mode of vibration, and w_x represents the weight of the x^{th} floor. The base shear calculated for each of the three modes of vibration, using the velocity spectrum of the Taft earthquake record and assuming 10% damping, is shown in Fig. 7.

In addition to the base shear for the multi-story structure, of course, the manner in which the forces are distributed through the height of the structure is also required. In general, the force in the n^{th} mode at height x, F_{xn}, is given by the base shear for that mode multiplied by a distribution coefficient, as follows:

$$F_{xn} = V_n \left[\frac{\phi_{xn} w_x}{\sum_x \phi_{xn} w_x} \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

The distribution of forces for the three modes considered in this analysis is also shown in Fig. 6.

As was mentioned previously, the total response of the structure to the earthquake motion may be obtained by superposition of the responses calculated for each mode. Thus, if we had the time-history of the base-shear variation for each mode, the time-history of the total base shear could be determined by merely adding the individual response terms at each instant of time. However, it should be recognized that the total maximum base shear developed by the Taft earthquake cannot be obtained by merely adding the base shears shown in Fig. 6, even though each value represents the maximum force developed in that particular mode. This is because the maximum velocities represented by the velocity-spectrum values for the different periods of vibration would occur at different times during the history of the quake, and thus they do not represent simultaneous affects. Accordingly, the value obtained by direct superposition of the maximum modal forces will always exceed the true maximum forces. For example, a complete analysis of the response of the Alexander Building to the El Centro earthquake of 1940, showed[3] that the superposed modal maxima gave a base shear force which exceeded the true maximum base shear by about 28%.

Since it is possible to obtain only an approximation to the maximum response by direct superposition of the modal maxima, it is equally rational and considerably simpler to calculate only the fundamental mode response, and to increase it by a factor to account for higher mode effects. Referring again to the analysis of the response of the Alexander Building to the El Centro quake, it was found that the true maximum base shear was about 19% greater than that given by the first mode spectral-response value. This increase applies only to this particular building and earthquake, of course, but it may be considered representative of the order of magnitude of higher mode effects in tall buildings. (However, it may be noted here that increasing the fundamental mode

3 "On the Importance of Higher Modes of Vibration on the Earthquake Response of a Tall Building," by R. W. Clough, Bulletin, Seismological Soc. of Amer., Vol. 45, No. 4, October, 1955, p. 289.

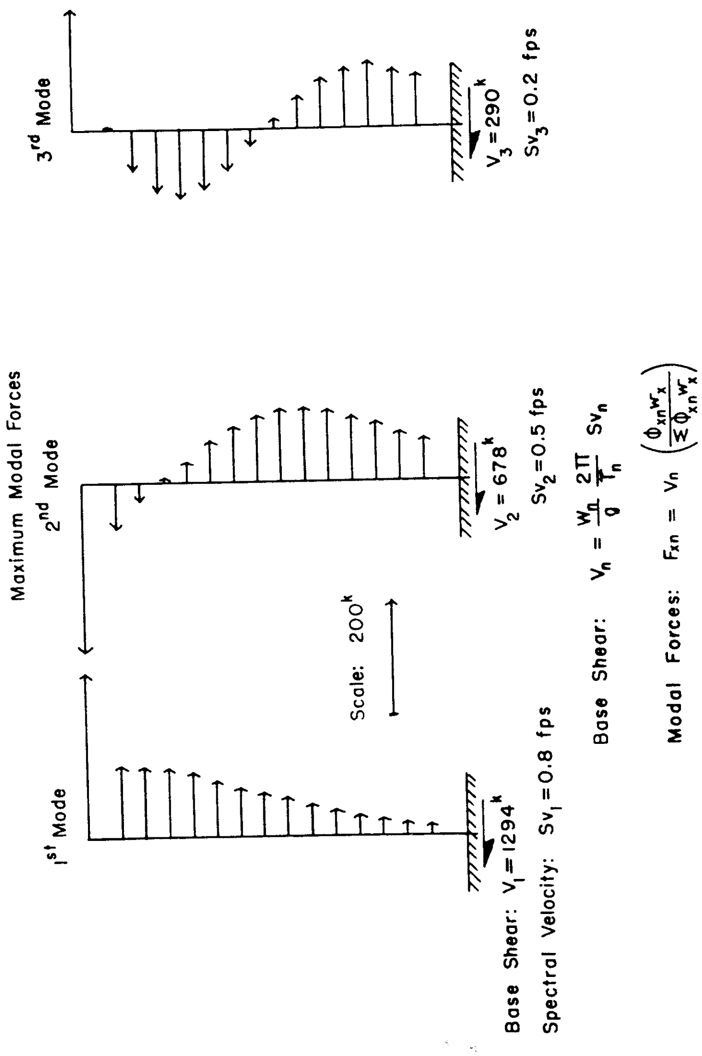


FIG. 7.—RESPONSE OF ALEXANDER BLDG. TO TAFT QUAKE

response by a constant factor to account for higher mode effects is not entirely rational, because the higher modes provide different effects at different heights. A better procedure would be to add a specified fraction, say 50%, of the second and third mode maxima to the first mode maximum to obtain an estimate of the total forces developed throughout the height.)

INELASTIC RESPONSE TO EARTHQUAKES

The procedure described previously makes possible the analysis of earthquake forces in any type of structure, and would apparently provide a complete picture of the dynamic effects of earthquakes. However, when the forces due to a moderately severe quake are calculated by this procedure, it is found that they exceed, by a significant amount, the design forces which would be specified by building codes. For example, in the previously mentioned study of the response of the Alexander Building to the El Centro earthquake of 1940, it was found that the base shear was about 20% of the weight of the building. Even the relatively moderate Taft earthquake would have produced a base shear of about 10% of the buildings' weight. On the other hand, building codes would specify a value of about 3% to 5% for the base-shear coefficient for this structure. This would appear to indicate that the lateral-force provisions of building codes are quite unconservative in providing resistance to a severe quake.

At the same time, however, it must be recognized that buildings having considerably less strength than is required by modern codes have withstood rather severe quakes with only moderate damage. This apparent discrepancy may be attributed, in part, to the fact that buildings possess considerable strength in excess of the design values due to use of conservative design stresses and to the participation of non-structural elements in resisting lateral deformations. Nevertheless, this factor does not fully explain the relatively slight damage exhibited by many ordinary buildings which have gone through heavy quakes. Even more important in many cases, is the fact that, as the response of the building builds up, cracking and yielding begin to take place, and these inelastic deformations absorb a large part of the vibrational energy of the structure. As a result, the continued build-up of energy which is required to develop the maximum velocities indicated by the spectral response curves is prevented.

On this basis, it is clear that inelastic deformations of the structure are a predominate factor in limiting the forces developed in a structure by a strong earthquake. Moreover, it is evident that earthquake codes have empirically taken account of this effect, since the code provisions provide strengths which are not sufficient to resist the earthquake forces elastically. To account for this effect rationally requires that inelastic action be incorporated into the analysis, and this may be accomplished effectively only through the use of automatic digital or analog computers. A study of this type was performed[4] by J. Penzien in which he evaluated the inelastic response of a single-story system to the El Centro earthquake of 1940. A part of the results of this study is presented in Fig. 8, in a form somewhat similar to the velocity-spectrum curves discussed previously (except that maximum displacement rather than velocity is the quantity presented.) The dashed curve in Fig. 8 represents the

4 "Elasto-Plastic Response of a Single Mass System Subjected to a Strong-Motion Earthquake," by J. Penzien, presented at a meeting of the ASCE, Los Angeles, Calif., February, 1959.

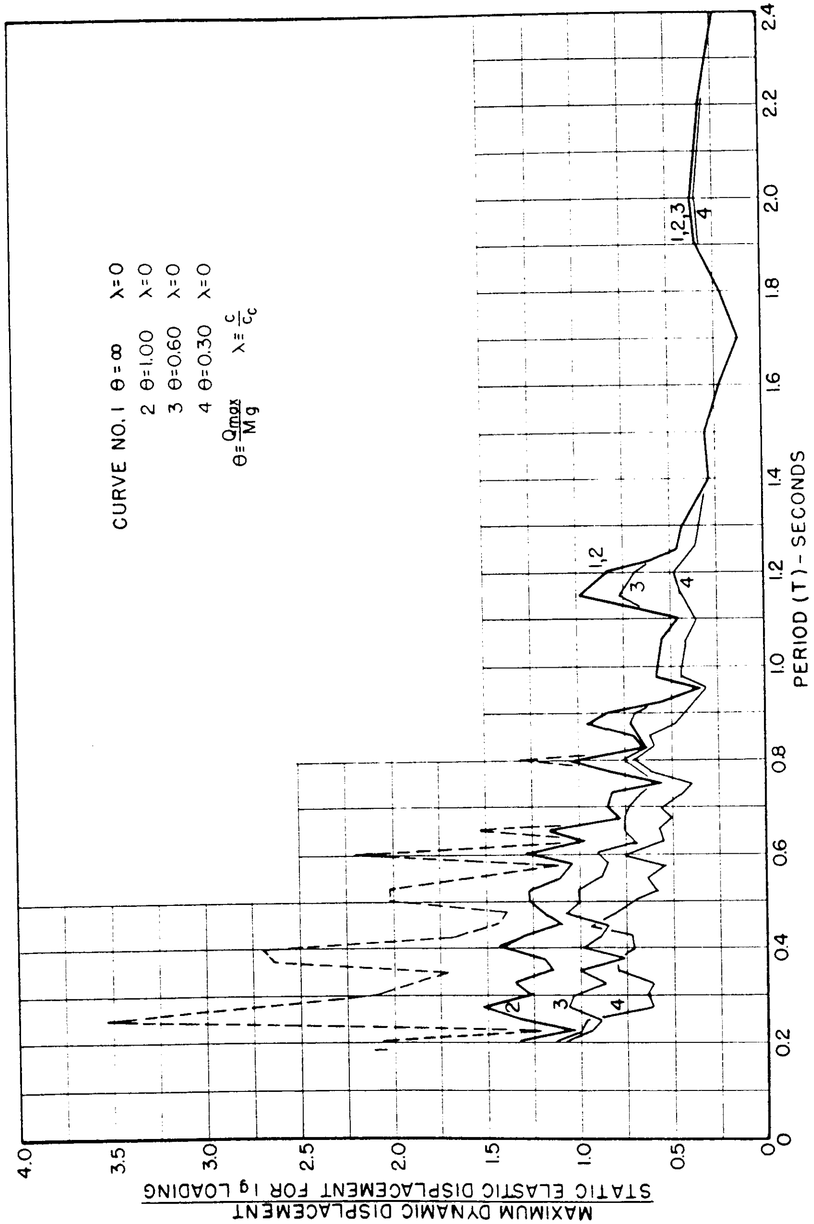


FIG. 8.—MAXIMUM DYNAMIC RESPONSE FOR EL CENTRO, CALIFORNIA EARTHQUAKE OF MAY 18, 1940 E-W COMPONENT (FROM REFERENCE 3)

maximum elastic response of the structure for varying values of period of vibration, while the solid lines indicate the inelastic response for several values of plastic limit. This limit is represented by the parameter θ, which is the ratio of the lateral force which would initiate yielding to the weight of the structure. Thus, decreasing values of θ indicate decreasing elastic strength. The important effect that inelastic deformations have in limiting displacements is evident in this figure, since the weaker structures are seen to undergo smaller displacements.

DYNAMIC CHARACTERISTICS OF THE EARTHQUAKE

One other aspect of the earthquake-response problem should be considered herein—the dynamic characteristics of the earthquake itself. As was noted

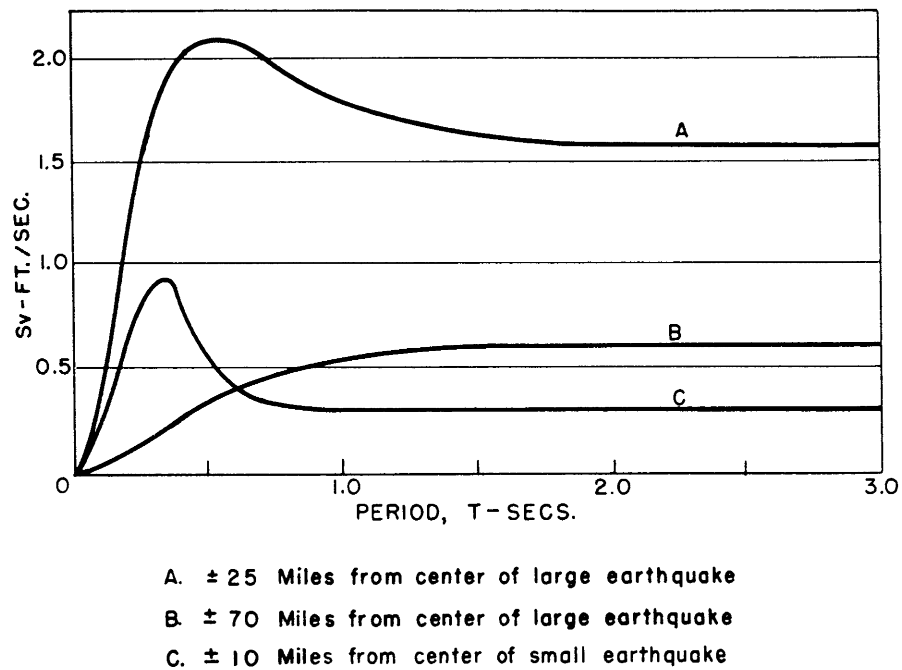


FIG. 9.—UNDAMPED VELOCITY SPECTRUM CURVES (FROM REFERENCE 1)

previously, the velocity spectrum depends on the nature of the earthquake motion and may be quite different for different quakes. Thus, the difficulties of establishing a standard spectrum for use in earthquake codes is obvious. However, enough earthquake records have now been obtained to establish certain general characteristics of the velocity spectra. These average characteristics were evaluated by G. W. Housner and are presented[2] in Fig. 9. Two basic points may be recognized from the curves of Fig. 9. First, comparison of curves A and B shows that propagation of the quake through the ground for a long distance not only reduces the general intensity of the motion (as might be expected), but of equal importance, it tends to filter out the short-period components of the motion more effectively than the long-period components. Thus, while a nearby earthquake will tend to cause the most severe damage to stiff, short-period structures, a quake at a greater distance will not affect such structures appreciably and will concentrate its effects instead on the flexible, long-period buildings.

The second basic principle is demonstrated by curve C, which shows that small, nearby quakes still further emphasize the short-period components of the motion, and thus may be expected even more exclusively to limit their damaging effects to the short-period, stiff structures. Flexible, tall buildings will show very slight effects from such local quakes.

SUMMARY

The preceding brief presentation of the principal dynamic effects of earthquakes will be summarized by comparing the results of theory with some of the lateral-force requirements recently proposed by the SEAOC.[5] The comparison is presented, in brief, in Table 1 and will be discussed subsequently.

Considering first the dynamic theory, the base-shear force developed in the n^{th} mode of vibration of a structure is given by Eq. Ia of Table 1, which shows that the force depends on the effective weight, the period of vibration and the spectral-velocity value. Eq. Ib indicates that the spectral velocity depends on the period of vibration, and that the effective weight varies with the mode shape and weight distribution. Eq. Ic shows that the base shear is distributed through the height of the building in proportion to the weight distribution and modal displacements. Eq. Id is simply a reminder that dynamic forces may be greatly reduced by inelastic action.

Compared with these basic facts of dynamic theory, in the right-hand column of Table 1 are presented some of the principal provisions of the proposed SEAOC. Eq. IIa shows that the total base shear is to be given by the product of the weight, a seismic coefficient C, and a factor k. This latter factor will be discussed later. An empirical expression for the seismic coefficient is given By Eq. IIb. It is clear that the selection of this coefficient must be carefully considered because of the many factors for which it is intended to account. A primary factor, of course, is the quantity $\frac{2\pi}{T} S_v$ (frequency times spectral velocity) and the variation of this quantity is represented in Eq. IIb by the negative cube root of the period. This, however, is only a part of the task assigned to the expression of Eq. IIb. Other factors which must be represented by the equation are the difference between the effective first mode weight, W_1 and the total weight and the influence of higher modes of vibration (because only a single mode is considered in Eq. Ia). Finally, but still of great importance, the factor C, in Eq. IIa, must take account of the energy-absorption effects of inelastic action which greatly alter the maximum response values. Accounting for all of these factors places a heavy burden on the expression of Eq. IIb, but it would appear to do the job as well as any possible choice on the basis of current knowledge.

Eq. IIc provides for distribution of the calculated base shear through the height of the building. Comparison with Eq. Ic shows that the two are identical

5 "Recommended Lateral Force Requirements," Seismology Committee, SEAOC, presented at a meeting of the SEAOC, Yosemite, Calif., October 2, 1958.

if the vibration displacements increase linearly with height. It will be noted in Fig. 6 that the first mode for the Alexander Building is essentially of this shape. Other buildings may tend to emphasize either the shear or the flexural distortion to a greater extent but this appears to be a reasonable assumption for a typical building of tall, slender proportions.

Finally as shown in Eq. IId, the factor k is assigned a value between limits of 2/3 and 4/3. The purpose of this factor is to account for the varying plastic-deformation capacities of different types of construction. It is evident that considerable amounts of energy must be absorbed in plastic deformations if the response of a structure is to be reduced materially below the amplitude of motion which would be developed elastically. Consequently, it is important that the structure possess adequate capacity for plastic deformation. The "k" value of 2/3 is intended to be applied to structural types which may undergo

TABLE 1.—COMPARISON OF DYNAMIC THEORY WITH PROPOSED SEAOC CODE

I.—Dynamic Theory	II.—Proposed SEAOC Code
a. $V_n = \frac{W_n}{g} \frac{2\pi}{T} S_v$	a. $V = k\ C\ W$
b. $S_v = f(T)$ $W_n = \left(\frac{\Sigma\phi_{xn}\ w_x}{\Sigma\phi_{xn}^2\ w_x}\right)^2$	b. $C = \frac{0.05}{3\sqrt{T}}$ accounts for: velocity spectrum effective weight higher modes inelastic action
c. $F_{xn} = V_n \frac{\Sigma\phi_{xn}\ w_x}{\Sigma\phi_{xn}\ w_x}$	c. $F_x = V \frac{w_x\ h_x}{w_x\ h_x}$
d. Inelastic action greatly affects magnitudes of dynamic forces	d. $k = 0.67$ to 1.33 accounts for varying yield capacity of different types of construction

sizeable amounts of plastic deformation without suffering major damage, while the factor of 4/3 would be applied to structures which can undergo only minor amounts of plastic deformation.

It is apparent from the preceding remarks that current concepts regarding the lateral-force provisions of building codes have advanced considerably beyond the original rigid-structure treatment. The seismology committee which promulgated the lateral-force provisions proposed by the SEAOC has done a remarkable job of relating the practical requirements of a building code to the essential features of dynamic theory. There is still a need for extensive research on the inelastic response of structures, and further studies of the characteristics of earthquakes will be needed to define a standard earthquake. However, it is encouraging to find a proposed building code which so nearly represents the current state of the art in this rapidly developing field.