REINFORCED CONCRETE COLUMN INVESTIGATION*

*Tentative Final Report of Committee 105***

F. E. RICHART, CHAIRMAN

In the following report, the Committee reviews the principal findings from its recent column investigation and, by way of interpretation, presents recommended design formulas for tied and spirally reinforced concrete columns.

The report has the approval of five of the seven members. Mr. R. L. Bertin and Prof. Inge Lyse approve the summary of test data but disapprove the design formulas presented.† The committee membership is as follows:

P. H. Bates	*W. S. Thomson*
R. L. Bertin	*W. F. Zabriskie*
Inge Lyse	*F. E. Richart*
F. R. McMillan	

The committee wishes to record its indebtedness to Prof. W. A. Slater, deceased October 1931, who was the original chairman of the committee, and to R. D. Snodgrass, member of the committee, 1929-31. Acknowledgment is also made of the generous assistance of many members of the Institute, who contributed to the Column Test Fund and thereby made the work of this committee possible.

1. RESUME OF PRINCIPAL TEST RESULTS

A few of the outstanding results of the recent column investigation conducted at Lehigh University and the University of Illinois under the auspices of Committee 105 will be summarized here. For the sake of brevity and simplicity, minor limitations and variations mentioned in previous test reports will not be discussed.

1. The following notation is used throughout the report:

*For presentation and discussion at the 29th Annual Convention, Chicago, February 21-24, 1933. For first, second, third and fourth progress reports on tests in this investigation at Lehigh University and University of Illinois, see this JOURNAL for February, March (Vol. 27), and November 1931, and January 1932 (Vol. 28).

**The committee has under consideration several minor additions upon which it hopes to report finally at the time of the convention.

†A minority report by Messrs. Bertin and Lyse, here follows the majority report.

P = ultimate load on column, in pounds, applied axially at an ordinary rate of loading.
A = cross-sectional area of core (area within outer circumference of spiral) of spirally reinforced columns without shells or with light shells, or overall area of tied column (see Section 7 for spiral columns with heavy shells).
A_c = cross-sectional area of core of spirally reinforced column.
A_g = overall or gross area of column.
R = ratio, A_g/A_c.
C = a constant, for which the value from these tests was about 0.85.
f_c' = compressive strength of 6 by 12 in. concrete control cylinders.
f_y = yield point stress of vertical reinforcement.
f_s' = useful limit stress of spiral reinforcement (assumed as the stress at a unit deformation of 0.005).
p = ratio of cross-sectional area of vertical reinforcement to the area A.
p_g = ratio of cross-sectional area of vertical reinforcement to the gross area A_g.
p' = ratio of volume of spiral reinforcement to volume of concrete core.
k = a constant, for which 80 per cent of the test values for columns with flat ends ranged from 1.5 to 2.5 with a general average of about 2.).

2. The tests showed that the ultimate strength of reinforced concrete columns could be expressed by a single formula of the following form:

$$\frac{P}{A} = Cf_c' (1-p) + f_y p + k f_s' p' \quad \text{......................} (1)$$

3. Equation 1 applies to columns made with a large range in amount and quality of materials, as follows:

(a) Concrete, of strengths varying from 2000 to 8000 p. s. i.
(b) Vertical reinforcing steel, of 1.5, 4.0, and 6.0 per cent, and yield point stresses ranging from 39,000 to 68,000 p. s. i.*
(c) Spiral reinforcement, of 1.2 and 2.0 per cent, and useful limit stresses ranging from 41,000 to 83,000 p. s. i.
(d) Scope of tests, included columns of varying size, age, and storage conditions.

*Supplementing the committee's tests, later tests have been made by members of the Committee using in one case vertical steel with 96,000 p. s. i. yield point and in another case using intermediate grade vertical steel in percentages up to 17½. The results of these tests also confirm Equation 1.

4. The yield point of all columns (and the ultimate strength of tied columns) is given by the equation:

$$\frac{P}{A_g} = Cf_c' (1-p) + f_y p_g \quad \ldots \ldots (2)$$

5. When a column is subjected to working loads, the initial distribution of stress between vertical reinforcement and concrete follows the usual theory based upon the moduli of elasticity and the cross-sectional areas of the two materials. However, if the load is sustained for some time, the stress distribution changes very rapidly due to plastic yielding and volume change of the concrete. The effect is to increase the steel stress and to decrease the concrete stress. In test columns in dry air storage and subject to ordinary design or working loads (as determined by A. C. I. or New York City Code formulas) for 5 months, the steel stresses reached values of 30,000 to 42,000 p. s. i. in extreme cases, wherein only 1½ per cent of vertical reinforcement was present. The average increase in steel stresses during the 5-month period was about 12,000 p. s. i. in the Illinois tests and about 20,000 p. s. i. in the Lehigh tests. The average increase in steel stress in these columns from the age of 5 months to one year was only about 2000 p. s. i. more, and for 16 columns which were held another year under sustained load the increase of the 2-year stress over the one-year value averaged less than 2000 p. s. i. The increase varied inversely with percentage of vertical steel and was greater for columns in dry storage than for those in moist storage. The effect of shrinkage was much less than that of time yield. The shortening of these columns (proportional in amount to the stress in the vertical steel) due to elastic deformation and time yield of the concrete has not been shown to be of sufficient magnitude to prove harmful, with the working loads used.

6. The redistribution of stress due to time yield, and the resulting development of high steel stresses, has no effect upon the ultimate strength of a column when tested to failure. Equation 1 applies to this case.

7. While in the usual case Equation 1 gives the ultimate strength of a spirally reinforced concrete column, there are cases where the strength of the column may be even greater than this, if the shell (neglected in Equation 1) used outside the spiral is very thick or of high strength concrete. From the nature of spiral column action, the strength of the shell must be exceeded (with accompanying cracking and spalling of shell, lateral bulging and vertical shortening of column) before the spiral reinforcement can come into play. For small columns, with relatively thick shells and made with strong concrete, the shell may add more strength than the portion (represented by the last term of Equation 1) added by the spiral reinforcement. In this case the ultimate strength is the same as the yield point of the column, given by Equation 2. This does not mean that the spiral reinforcement in such a column is of no value, since, due to the large amount of deformation required to produce failure and the initial spalling which is observable, warning will be given when a load approaches the ultimate strength. This is particularly true when the ultimate load involves bending, which will produce indications of distress in localized parts of the column. This property of "toughness" of spirally reinforced columns (similar to ductility in mild steel) deserves consideration, since a column which is capable of deforming greatly before failure will in all probability be relieved of some of its load through the stiffness of connecting members, thus avoiding complete collapse.

8. Columns of different size but made with the same percentages of reinforcement and the same quality of all materials had the same strength per unit area, aside from the effect of the shells noted above.

9. Tests of columns subject to sustained loads approaching the ultimate indicate that failure will generally occur in a few hours when the load is held at 95 per cent of the ordinary "fast-loading" ultimate. Loads of 80 to 90 per cent of the ultimate, causing longitudinal strains several times the yield point strain of the vertical steel, have been held for periods of one to two years. Such columns show extensive spalling and a gradual shortening and bending, but are still carrying the load.

2. CHOICE OF TYPE OF DESIGN FORMULA

Several factors demand consideration when the type of design formula is to be selected. Equations 1 and 2 are evidently reliable expressions for the ultimate strength and yield point of a column. Among the items that have been given careful study by the Committee are:

(a) Factor of safety with regard to ultimate strength of column.

(b) Factor of safety with regard to yield point of column.

(c) Tendency to excessive steel stresses and column shortening due to time yield and shrinkage, especially with small percentages of vertical reinforcement.

(d) The structural action of the concrete shell outside the spiral.

(e) The effect of the large deformations required to bring the spiral reinforcement into action.

(f) The adaptability of the design formula to include bending stresses.

(g) Simplicity and convenience of the design formula.

In approaching the question of new formulas for columns with spiral reinforcement and for those with lateral ties, an attempt was made to discover any difference between such columns in their behavior under load, and to determine whether different factors of safety should be used. Evidently in columns in which the strength added by the spiral is less than that of the shell, the only advantage of the spiral is in the quality of toughness it may give the column. To provide certain and unquestioned spiral action, the strength due to the spiral should exceed that of the shell. With care taken to avoid a close spacing of spirals, it seems safe to take the strength of shell concrete as about nine-tenths that of the core concrete, or as $0.75\, f_c'$.

If a spirally reinforced column be now defined as a column in which the strength added by the spiral is definitely greater, by about 15 per cent, than the strength of the shell, the ratio of spiral reinforcement required is given by the equation

$$2\, p'\, f_s'\, A_c = 1.15 \times 0.75\, f_c'\, (A_g - A_c) \text{ or}$$

$$p' = \frac{0.43\, f_c'\, (R - 1)}{f_s'} \qquad (3)$$

The amount of spiral given by Equation 3 will now be taken as a minimum requisite for spiral columns, except as noted later. It is seen that when this requirement is just fulfilled, the ultimate strength of the column is given by either Equation 1 or Equation 2 with a factor C of 0.85 and the yield point of such a column is very slightly less, due to the use of $C = 0.75$ on the shell. This makes it possible to base formulas for both spiral and tied columns on the gross or overall area. Where the amount of spiral (when used) is that determined by Formula 3, the ultimate strength may be expressed by the formula

$$P = 0.85\, f_c'\, A_c\, (1 - p) + f_y p\, A_c + 0.86\, f_c'\, (A_g - A_c)$$

or for simplicity we may say,

$$\frac{P}{A_g} = 0.85\, f_c'\, (1 - p_g) + f_y p_g \qquad (4)$$

The use of formulas based on gross areas not only produces consistency in design between tied and spirally reinforced columns, but is also of decided advantage when the effect of combined axial and bending stresses is to be considered. While the subject of eccentric loading is outside the scope of the present investigation and is not included in this report, the subject has been given some analytical study.

A design formula for axially loaded spirally reinforced columns is now derived by combining Equation 4 with a suitable factor of safety. In choosing the latter, some recognition is given to the tendency for high steel stresses due to time yield and shrinkage when the steel percentage is low, by the use of a slightly higher factor with low percentages. Choosing limiting values of the steel percentage (on gross area) at 0.01 and 0.08, corresponding factors of safety at these limits were taken at 3 and 2.5, with intervening values given by the expression, $F = 3.07 - 7\, p_g$. Values from Equation 4, divided by the corresponding values of F, gave design curves which were nearly linear, and which were very closely represented by the equation

$$\frac{P}{A_g} = 0.25\, f_c' + 0.45\, f_y\, p_g \qquad (5)$$

Equation 5 fits the calculated values very well, even for ranges of 2000 to 5000 p. s. i. for f_c', and 40,000 to 50,000 and higher for f_y.

The values of f_y contemplated for use in Equation 5 are minimum specification values, as, for example, 40,000 p. s. i. for intermediate grade and 50,000 p. s. i. for hard grade steel.

The values of f_s' contemplated for use in Equation 3 are 40,000 p. s. i. for intermediate grade hot rolled spirals and 60,000 p. s. i. for cold drawn wire spirals.

It is evident that Equation 5 recognizes the value of spiral reinforcement in producing toughness in a column, but does not allow any increase in load because of spiral percentages in excess of those given by Equation 3. To supplement Equation 3 in fixing a minimum percentage of spiral reinforcement, certain flat minimum percentages will also be specified to insure that proper restraint is given the vertical steel in all cases.

A design formula for axially loaded columns with lateral ties has been developed by a procedure similar to that used with Equation 5, except that a factor of safety 25 per cent greater has been used in all cases. This is equivalent to reducing the constants in Equation 5 by 20 per cent, giving the following equation for tied columns:

$$\frac{P}{A_g} = 0.2\, f_c' + 0.36\, f_y\, p_g \qquad (6)$$

3. RECOMMENDED DESIGN FORMULAS

The design formulas which the committee recommends, after a thorough analysis of the test data collected and consideration of many practical features of design and construction, are given herewith. In each case certain limitations as to design and details of reinforcement, with which the formulas are intended to be used, are given.

1. Spirally Reinforced Columns.—The maximum permissible axial load on columns reinforced with longitudinal bars and closely spaced spirals enclosing a circular core shall be that given by formula 7, using the notation given in Part 1 of this report.

$$P = A_g\,(0.25\,f_c' + 0.45\,f_y\,p_g) \qquad (7)$$

The ratio, p_g, of the effective cross-sectional area of the vertical reinforcement to the gross area, A_g, of the column, shall not be less than 0.01 nor more than 0.08. The minimum number of bars shall be four. The center to center spacing of bars within the periphery of the column core shall not be less than 2½ times the diameter for round bars or 3 times the side dimension for square bars. The clear spacing between bars shall not be less than 1 in. or 1⅓ times the maximum size of the coarse aggregate used. These spacing rules apply to the bars at a lapped splice.

The spiral reinforcement shall be of such amount and quality that the load-carrying capacity of the spiral shall be 15 per cent greater than that of the concrete shell outside the core. The spiral ratio, p', to satisfy this requirement is given by the equation (see notation in Part 1)

$$p' = \frac{0.43\,f_c'\,(R - 1)}{f_s'} \qquad (8)$$

In applying Equation 8, the useful limit stress of the spiral steel, f_s', shall be taken at 40,000 p. s. i. for hot rolled rod of intermediate grade (A. S. T. M. Designation A15-30) and 60,000 p. s. i. for cold drawn wire (A. S. T. M. Designation A82-27).

The spiral ratio, p', shall not be less than the value given by Equation 8, nor shall it be less in any case than 1⅛ percent for hot rolled spirals of intermediate grade or ¾ percent for cold drawn wire spirals.

The spiral reinforcement shall consist of evenly spaced continuous spirals held firmly in place and true to line by at least three vertical spacer bars. Where splices must be made in spiral rod or wire, butt-welds are recommended. The center to center spacing of the spirals shall not exceed 3 in. nor one-sixth of the core diameter. The clear spacing between spirals shall not be less than 1½ in., nor 1⅓ times the maximum size of coarse aggregate used.

2. Tied Columns.—The permissible axial load on columns reinforced with longitudinal bars and separate lateral ties is given by formula 9

$$P = A_g\,(0.2\,f_c' + 0.36\,f_y\,p_g) \qquad (9)$$

The ratio, p_g, of the effective cross-sectional area of the vertical reinforcement to the gross area, A_g, of the column, shall not be less than 0.005 nor more than 0.03. The reinforcement shall consist of not less than four bars, placed with a clear distance from the face of the column of not less than 2 in.

3. Splices in Vertical Bars.—Where lapped splices in the column verticals are used, the minimum amount of lap shall be as follows: (a) *For deformed bars*—with concrete having a strength of 3000 p. s. i. or above, 24 diameters of bar of intermediate grade steel and 30 diameters of bar of hard grade steel. For bars of higher yield point, the amount of lap shall be increased in proportion to the yield point stress. When the concrete strengths are less than 3000 p. s. i., the amount of lap shall be one-third greater than the values just given. (b) *For plain bars*—the minimum amount of lap shall be 25 percent greater than that specified for deformed bars.

For cases in which the requirements for bar spacing do not permit the use of lapped splices, welded splices or other positive connection shall be used.

Readers are referred to the JOURNAL *for June 1933, for discussion of the foregoing majority report of the committee, and of the following minority report. Such discussion should reach the Secretary by April 1, 1933.*

Reinforced Concrete Column Investigation

Committee 105—Minority Recommendation for Design Formula of Reinforced Concrete Columns

We, members of Committee 105, cannot accept the design formulas recommended in the majority report of the Committee, for the following reasons:

(a) Design formulas do not reflect the results of tests.

(b) The formulas do not adhere to fundamental principles established by tests or theory.

(c) The formulas are entirely empirical.

(d) They infer different factors of safety for the concrete and the steel which is brought about through an attempt to control time yield.

We therefore submit the following recommendations, relative to design formulas for consideration by the Institute.

We recognize all factors involved in the following basic formula:

$$\frac{P}{A_g} = \frac{1}{F}\left(.85f_c'(1-p) + f_y p + \frac{kf_s'}{R}\left(p_a - \frac{.85f_c'(R-1)}{kf_s'}\right)\right) \quad \ldots(1)$$

in which $p_a = p' + p''$ = total ratio of spiral to core area of the column

p' = ratio of effective spiral to core area

p'' = ratio of ineffective spiral to core area

the ineffective spiral being that amount required to produce a resistance equal to that of the protective shell

or $$p'' = \frac{.85f_c'(R-1)}{kf_s'} \quad \ldots\ldots(2)$$

substituting for $p_a - \dfrac{.85f_c'(R-1)}{kf_s'}$ its equivalent p'

and making $k = 2$

$F = 3.5$

equation 1 becomes

$$\frac{P}{A_g} = \frac{1}{3.5}\left(.85f_c'(1-p) + f_y p + \frac{2f_s'}{R}p'\right) \quad \ldots\ldots(3)$$

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We therefore submit the following recommendations, relative to design formulas for consideration by the Institute.

We recognize all factors involved in the following basic formula:

$$\frac{P}{A_g} = \frac{1}{F}\left(.85f_c'(1-p) + f_yp + \frac{kf_s'}{R}\left(p_a - \frac{.85f_c'(R-1)}{kf_s'}\right)\right) \quad \text{..(1)}$$

in which $p_a = p' + p''$ = total ratio of spiral to core area of the column

p' = ratio of effective spiral to core area

p'' = ratio of ineffective spiral to core area

the ineffective spiral being that amount required to produce a resistance equal to that of the protective shell

$$\text{or} \quad p'' = \frac{.85f_c'(R-1)}{kf_s'} \quad \text{......(2)}$$

substituting for $p_a - \dfrac{.85f_c'(R-1)}{kf_s'}$ its equivalent p'

and making $k = 2$

$F = 3.5$

equation 1 becomes

$$\frac{P}{A_g} = \frac{1}{3.5}\left(.85f_c'(1-p) + f_yp + \frac{2f_s'}{R}p'\right) \quad \text{......(3)}$$

in which the amount of spiral "p_a" used in the column shall in no case be less than .005 nor more than .02, or as governed by a minimum clear spacing between wires of 1½ in.

The total unit stress shall be limited by the following:

$$\frac{2f_s'p'}{R} \leqq .4\ (.85f_c'\ (1-p) + f_y p) \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(4)}$$

Equation 3 is recommended as a design formula for tied and spiral columns for the reason that it is derived directly from the test results and therefore adheres to fundamental principles.

The test results and investigation of structures in service have failed to reveal any ill effects from time yield and therefore we do not recognize the necessity of modifying equation 3 because of such effects.

In view of the simplicity of equation 3, we do not recognize any need for further simplification with its incidental departure from basic principles.

The basic factor of safety $F = 3.5$ applies to columns, the yield point of which coincides with their ultimate strength.

For columns in which the ultimate strength is raised above the yield point strength by the introduction of effective spiral, the yield point factor of safety is reduced in proportion to the added strength to a minimum of 2.5, as given by equation 4.

We recommend the following limitations of longitudinal reinforcement.

	Minimum	*Maximum*
Tied Columns	.005	.04
Spiral Columns	.010	.08 for reinforcing bars
Spiral Columns		.20 for structural shapes

Signed: R. L. BERTIN
INGE LYSE

Readers please note page 282 as to discussion of this report.