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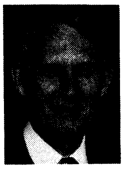
Design of Slender Concrete Columns—Revisited



by J. G. MacGregor

Revisions are proposed to Sections 10.10 and 10.11 of the ACI Building Code to simplify the design of slender columns and to recognize the use of second-order analyses. These changes are undergoing letter ballot in ACI Committee 318 and if accepted will appear in the 1995 ACI Code. Major changes include the listing of a series of EI *values for use in second-order frame analyses, a test for sway and nonsway frames, a flat* φ *value for stability calculations, new slenderness limits, and the method of combining and magnifying the nonsway and sway moments.*

Keywords: columns (supports); frames; reinforced concrete; stability; **structural design.**

The current ACI BuildingCode[1] provisions for the design of slender columns were developed in the late 1960s[2] and were incorporated in the Code in 1971. Additions to the Commentary in 1977 and 1983 amplified the calculation of k factors and the differentiation between braced and unbraced frames. A major change in 1983 distinguished between sway and nonsway moments and magnified these separately.

In the last decade, second-order analysis programs have become widely available. This proposed revision to the slenderness provisions gives guidance for the use of such analyses in column design.

The proposed revision, presented in an appendix to this paper, has the following major subdivisions:

1. *Slenderness effects in compression members*—This section allows either a general method or a moment magnifier method for the design of slender columns.

2. *Magnified moments—General*—This section gives general rules applicable in the moment magnifier method given in Sections 3. and 4. These include values of E and I for use in frame analyses and a test of whether frames are sway or nonsway frames.

3. *Magnified moments—Nonsway frames*—This section gives rules for designing columns in nonsway frames. Nonsway frames have been separated to make the application of the procedures more evident. The major changes are a new slenderness limit equation and a requirement that the moments in the bracing elements be magnified.

4. *Magnified moments—Sway frames*—Major changes in this section include the method of calculation of the magnified sway moments and the method by which these are combined with the nonsway moments. A stability check under gravity loads is also required.

The rest of this paper will discuss the individual changes.

SECTION PROPERTIES FOR FRAME ANALYSIS

Traditionally engineers have used the gross moments of inertia of the columns and beams in frame analyses. With the advent of second-order analyses as design tools, however, it is important that the computed lateral deflections closely resemble the anticipated deflections so that realistic $P\Delta$ moments are obtained. This requires realistic EI values.

Member stiffness EI values are used for three things in the proposed code sections on slenderness: (a) in frame analysis, (b) when calculating the effective length factor k, and (c) in the design of individual columns. Two different sets of EI values are given. Since the lateral deflections from the frame analysis are affected by the stiffnesses of all the members in the structure, the EI values used in frame analysis should approach the mean values for the individual members. On the other hand, when dealing with the stability of a single isolated member the value used should be a safe lower bound estimate to the EI value for a single column. As a result, the EI values for columns given in Section 2.1 for frame analysis are larger than those for member design given in Section 3.3.

The EI values for second-order frame analysis should be representative of the member stiffnesses immediately before the ultimate condition. At this stage, parts of the beams, slabs, and walls will be cracked in flexure. It is too conservative to base the EI on the cracked moment of inertia because the beam will not be completely cracked at all sections. Instead the EI should be back-calculated from the member stiffness, $K = 4EI/l$, taking into account the distribution of cracking along the member. When dealing with a 20-story building with more than 1000 members and more than 2000 critical sections, it is not economically feasible for designers to go

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through such calculations, and simplified methods must be used to compute *EI*.

Kordina[3] and Hage[4] have studied the variation of stiffness for various types of frame members subjected to gravity load moments, lateral load moments, and combinations of the two. Based on these studies, MacGregor and Hage[5] concluded that a reasonable estimate of *EI* for second-order analysis would be based on the ACI value of E_c and I = 0.4 I_g for beams and 0.8 I_g for columns.

Fig. 1, taken from Hage[4] shows the variation in the effective *EI* for a T-beam as the load level is increased.

Fig. 1(a) considers gravity load moments. The term η is the ratio of the fixed end moment to the nominal moment capacity of the end of the beam

$$\mu = \frac{wl^2}{12}/M_n \qquad (1)$$

For small loads (small μ), the effective *EI* slightly exceeds E_cI_g due to the presence of the reinforcement. As η increases, parts of the beam crack and the effective *EI* approaches $0.4E_cI_g$. Fig. 1 is plotted for one particular cross section. Similar trends were obtained for other sections including rectangular sections.

Fig. 1(b) gives the effective *EI* for beam moments due to lateral loads. The term μ in Fig. 1(b) is the ratio of the end moment due to lateral loads to the nominal moment capacity. Again *EI* approaches 0.4 E_cI_g as μ approaches 1.0. Fig. 1(c) considers combinations of μ and η. Similar graphs are obtained for rectangular cross sections. Hage proposed *EI* for beams equal to $0.4E_cI_g$.

Once the effective *EI* of beams had been obtained, Hage obtained the value of the effective *EI* for columns by back calculating from the lateral deflection of laboratory tests of reinforced concrete frames. This gave $EI = 0.8E_cI_g$.

Furlong[6] proposed that the *EI* of T-beams be taken as the gross *EI* of the stem but not less than 0.5 E_cI_g where I_g is for the T-shaped cross section. For lower floor columns he suggested $EI = 0.6\ E_cI_g$, for upper floor columns 0.3 E_cI_g.

Dixon[7] back-calculated *EI* for columns in 13 frame tests using a second-order analysis program. Based on Hage's work, he assumed the *EI* of the beams as $0.5E_cI_g$. Using this beam stiffness, the column stiffness which gave the best conservative estimate of the measured lateral deflections was 0.5 E_cI_g.

McDonald[8] generated moment-end rotation relationships for T-beams, one-way slabs, and columns. For T-beams with 1.2 percent steel, he found *EI* ranged from 0.37 to 0.44 E_cI_g. For one-way slabs with 0.5 percent steel, *EI* varied from 0.16 to 0.22 E_cI_g. For columns *EI* varied from 0.66 to 0.89 E_cI_g. MacDonald proposed *EI* values of 0.42 E_cI_g, 0.20 E_cI_g, and 0.7 E_cI_g for T-beams, one-way slabs, and columns, respectively.

A strength reduction factor φ should be included in the second-order analysis to account for the variability in the

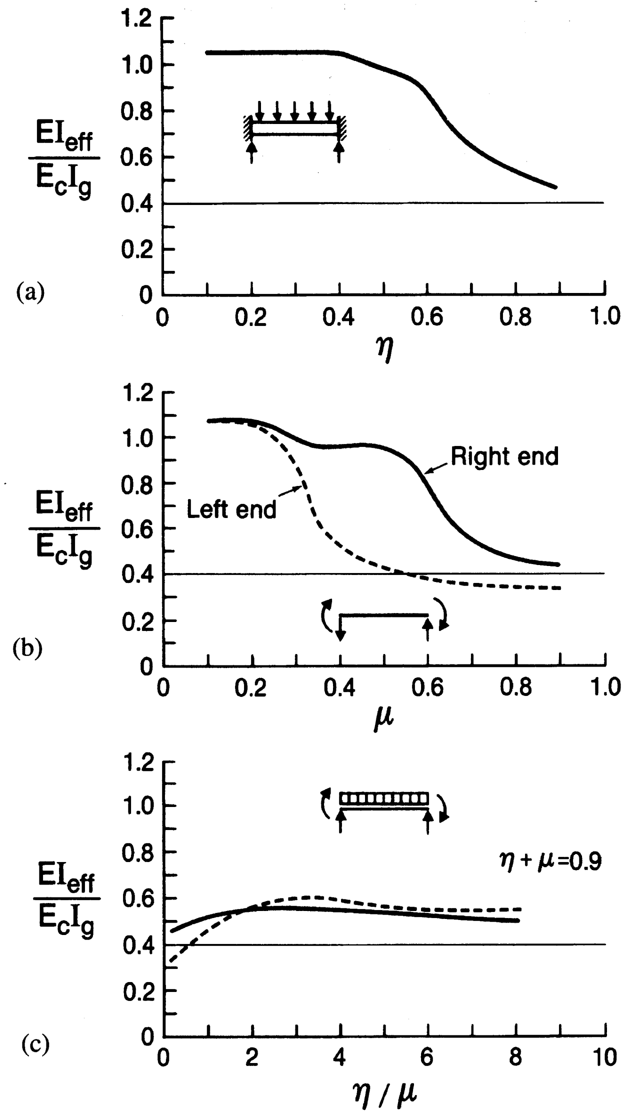

Fig. 1—Variation of beam stiffness with load

predicted lateral deflections resulting from simplifications in modeling the structure and the assumed values of *E* and *I*. Later in this paper a single value of $\varphi = 0.75$ is proposed for use in the moment magnifier equations. This is related to the probability that an individual column will be understrength. The variability of the lateral deflections of a frame are related to the variability of the mean E and I values of all the members of the frame. Since this is considerably less than the variability of an individual member in the frame, the φ factor applied to the second order analysis should be closer to 1.0 than that for an individual member. A value of 0.875 is proposed.

The *EI* values proposed by MacGregor and Hage[5] are recommended for use in frame analysis and are incorporated in the proposed revisions. When these are multiplied by φ = 0.875 they become:

a. Modulus of elasticity = E_c from Section 8.5.1.

b. Moment of inertia

Beams: 0.35 I_g

Columns: 0.70 I_g

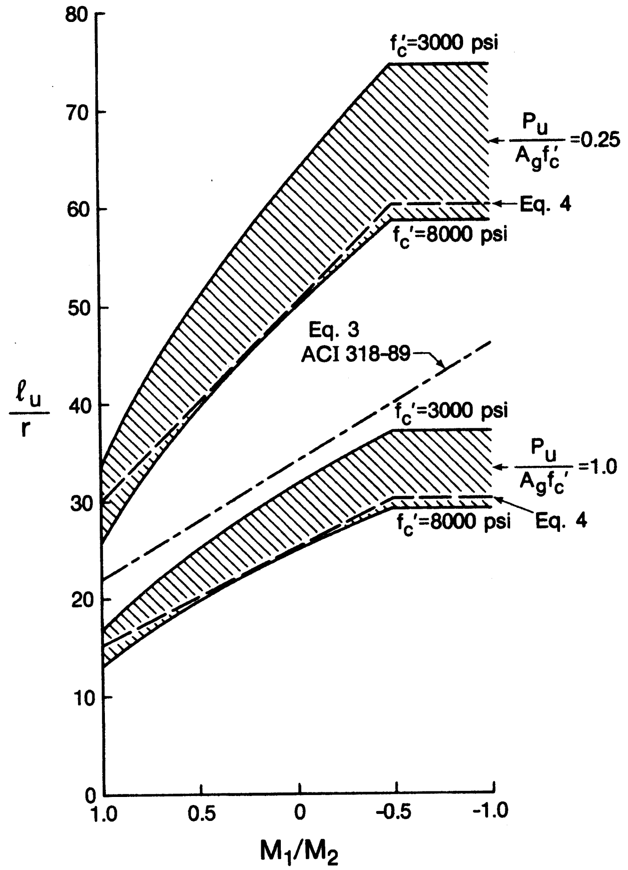

Fig. 2—Comparison of Eq. (4) with columns having a magnifier of 1.05

Walls—Uncracked: 0.70 I_g

Cracked: 0.35 I_g

Flat plates and flat slabs: 0.25 I_g

c. Area: 1.0 A_g

The moments of inertia used in second-order frame analyses must be divided by $(1 + \beta_d)$ in (a) the rare situation when sustained lateral loads act, or (b) in gravity load stability checks.

The first value given for walls assumes the walls are uncracked. If the moments from an analysis based on the wall moment of inertia equal to 0.8 I_g indicate the wall will crack due to flexure, the analysis should be repeated with I = 0.4 I_g in those stories where cracking is expected.

For analyses of service-load deflections, φ would generally be taken equal to 1.0 and the moment of inertia can be taken as 1.25 times the values given. As a result, the EI values for computing service-load deflections are 1.25/0.875 = 1.43 times the values given previously.

The Commentary to ACI 318-89, Section 10.11 recommends two sets of EI values for use in calculating the effective length factor k. For kl_u/r less than 60, the Commentary suggests the use of 0.5 E_cI_g for beams and 1.0 E_cI_g for columns when computing ψ, used to compute k. Reference 9 shows that this set of EI values is safe but conservative for longer columns. The use of 0.35 E_cI_g for beams and 0.70 E_cI_g for columns will give the same values of ψ and k as 0.5 E_cI_g and 1.0 E_cI_g and hence is recommended.

SWAY AND NONSWAY FRAMES

Traditionally, frames have been classified as braced frames and unbraced frames when evaluating effective length factors. Since all practical frames deflect laterally under lateral loads there is no such thing as a truly braced frame. For this reason the revised slenderness provisions refer to sway and nonsway frames rather than unbraced and braced frames. A nonsway frame is defined as one in which the second-order magnification of sway moments is 5 percent or less. This is checked by determining if

$$Q = \frac{\Sigma P_u \Delta_o}{V_u l_c} \qquad (2)$$

is equal or less than 0.05 where V_u is the lateral shear in the story and Δ_o is the first-order relative lateral deflection of the top and bottom of the story due to V_u. As shown in References 5 and 10, the sway magnifier δ_s is approximated closely by $1/(1 - Q)$ giving rise to the limit on Q of 0.05 for nonsway frames. The Commentary to ACI 318-89 set a limit of 0.04 on Q corresponding to a 4 percent permissible increase in sway moments. A slightly more liberal limit is given in the proposed revisions because the lateral deflection Δ_o is based on the values of EI given earlier. The 1990 CEB-FIP Model Code[11] requires consideration of second-order effects if lateral deflections result in more than a 10 percent increase in sway moments.

The Commentary to ACI 318-89 also defined a braced story as one in which the sum of the lateral stiffnesses of the bracing elements exceeded six times the sum of the lateral stiffnesses to the columns. This definition can be unconservative if $\Sigma P_u/P_{crf}$ is high, where P_{crf} is the critical load of the entire frame.

DESIGN OF NONSWAY FRAMES

Slenderness limit

Section 10.11.4.1 of ACI 318-89 allows the effects of slenderness to be neglected if

$$kl_u/r < 34 - 12M_{1b}/M_{2b} \qquad (3)$$

This equation was derived from Code Eq. (10-7), assuming δ_b was limited to 1.05. Two things are wrong with this equation. First, the original derivation was carried out in the late 1960s using a form of Eq. (10-7) which did not include the φ factor.[2] As a result, the code slenderness limit corresponds to a magnifier considerably greater than 1.05. Second, the equation ignores the effect of the axial load level on the moment magnification. In the proposed revision, Eq. (3) is replaced by Eq. (4)

$$\frac{kl_u}{r} \le \frac{25 - 10(M_1/M_2)}{\sqrt{\dfrac{P_u}{f_c' A_g}}} \qquad (4)$$

The shaded bands in Fig. 2 show kl_u/r values corresponding to a magnification factor of 1.05 for two axial load levels for braced hinged columns. Eq. (4) is shown by dashed lines and Eq. (3) by the broken line. For a column with $f_c' = 3$ ksi, $f_y = 60$ ksi, 2 percent steel, and $\gamma = 0.75$, $e/h = 0.10$ corresponds to $P_u/A_g f_c' = 0.68$ and the balanced eccentricity corresponds to $P_u/A_g f_c' = 0.27$. As a result, the kl_u/r value beyond which a column is classed as slender will tend to increase compared to the 1989 Building Code.

EI equations for slenderness calculations

The *EI* equations in ACI 318-89 are

$$EI = \frac{0.4\, E_c I_g + E_s I_{se}}{(1 + \beta_d)} \tag{5}$$

and

$$EI = \frac{0.4\, E_c I_g}{(1 + \beta_d)} \tag{6}$$

Eq. (5) and (6) have been retained, but in the draft they are multiplied by φ_s to maintain consistency with the format of the *EI* values proposed for structural analysis. For preliminary design of nonsway frames, Eq. (6) could be replaced with

$$EI = 0.25\, E_c I_g \tag{7}$$

This is equivalent to assuming $\beta_d = 0.60$. When lateral load moments govern the design, β_d will be zero and Eq. (7) will be excessively conservative.

Strength reduction factor φ_s

The 1971 and subsequent codes have taken the strength reduction factor φ in the moment magnifier equations equal to 0.7 or 0.75 for tied and spiral columns. This increases to 0.9 for the pure moment case. These values were originally derived for axially loaded short columns.

Mirza, Lee, and Morgan[12] suggest that for the practical range of variables for tied columns, φ could be taken equal to 0.80, while for the extreme range of variables, φ_s should be between 0.7 and 0.75. A flat value of 0.75 has been used in the magnifier equations in the proposed revisions. This remains constant throughout the whole range of eccentricity ratios e/h and applies to tied or spiral columns alike. To distinguish it from the regular φ factors for columns it has been called φ_s.

DESIGN OF SWAY FRAMES

In the proposed revisions, the design of sway frames for slenderness consists of three steps:

1. The magnified sway moments $\delta_s M_s$ are computed. This may be done in one of three ways which will be discussed in the next part of this paper.

2. The magnified sway moments $\delta_s M_s$ are added to the unmagnified nonsway moments M_{ns} at each end of each column.

$$M_1 = M_{1ns} + \delta_s M_{1s} \tag{8}$$

$$M_2 = M_{2ns} + \delta_s M_{2s} \tag{9}$$

where M_2 is the larger of the two end moments.

3. If the column is slender and the loads on it are high, it is checked to see whether the moments between the ends of the column exceed these at the ends of the column. This is done using the nonsway frame magnifier δ_{ns}, with P_c computed assuming $k = 1.0$ or less.

This is an extension of the separation of δ_b and δ_s and nonsway and sway moments proposed by Ford, Chang, and Breen[13] and introduced in the 1983 Code. The procedure has been changed because in most sway frames the possibility of the maximum moment occurring between the ends of the column is greatly reduced by the presence of the large double-curvature moments due to the lateral loads.

Determination of whether maximum moment is at the end of the column

In most columns in sway frames, the maximum column moment will occur at one end of the column, and the third step listed in the previous section will not be required. It is useful to have a simple way of determining when this will occur so that Step 3 is avoided when not required.

Galambos[14] has shown that the maximum moment M_c, in an elastic beam column loaded with an axial load and end moments M_1 and M_2 is

$$M_c = M_2 \delta \tag{10}$$

where

$$\delta = \sqrt{\frac{1 + \left(\frac{M_1}{M_2}\right)^2 - 2\left(\frac{M_1}{M_2}\right)\cos\alpha}{\sin\alpha}} \tag{11}$$

and $\alpha^2 = Pl^2/EI$. It will be assumed that stability effects can be disregarded if M_c is not more than $1.05M_2$, i.e., $\delta \leq 1.05$. Eq. (11) can be solved for the combinations of M_1/M_2 and α corresponding to $\delta = 1.05$. These are plotted with the solid line in Fig. 3. Combinations of M_1/M_2 and a falling below this line can be designed for the second-order end moments without further magnification. For sway frames the range of M_1/M_2 of interest is approximately −0.5 to −1.0, in the double curvature range. In this region, the solid line may be approximated by

$$\frac{M_1}{M_2} = 0.6 - \frac{P_u l^2}{5.25EI} \tag{12}$$

Substituting $EI = 0.4E_c A r^2$ and the ACI code value of E_c, and solving for the case of $M_1/M_2 = -0.5$ and $f_c' = 8000$ psi (55 MPa) gives

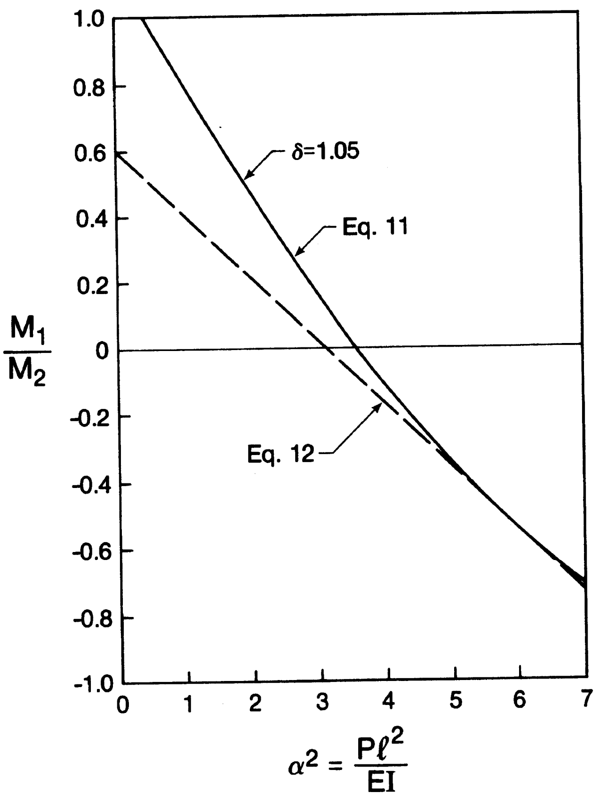

Fig. 3—Comparison of Eq. (11) for δ = 1.05 and Eq. (12)

$$\frac{l}{r} \leq \frac{34.9}{\sqrt{\frac{P_u}{F_c' A_g}}} \qquad (13)$$

If l/r is less than this, the maximum moment will be at one end of the column. The constant in Eq. (13) increases as M_1/M_2 approaches -1 and increases as f_c' decreases. Eq. (13), with the numerator rounded off to 35, is given in the proposed revision.

If l/r exceeds this value, it is necessary to magnify the M_2 obtained from Eq. (9). This is done using the nonsway moment magnifier.

Computation of magnified sway moment

The magnified sway moment $\delta_s M_s$ may be computed in one of three ways:

a. It may be computed using second-order elastic frame analysis based on the member stiffnesses given for structural analysis.

b. Alternately $\delta_s M_s$ may be taken as

$$\delta_s M_s = \frac{M_s}{1-Q} \geq M_s \qquad (14)$$

This can be shown to be the solution to the infinite series resulting from the iterative P-Δ calculation.[5,10] Reference 10 shows that Eq. (14) closely predicts the second-order moments in an unbraced frame until δ_s exceeds 1.5.

c. The third alternative is to use the sway moment magnifier from the 1989 ACI Building Code.

Sidesway instability under gravity loads

The classical case of sidesway buckling under gravity loads must be checked for sway frames. This is checked in different ways depending on which of the three methods was used to compute $\delta_s M_s$.

When $\delta_s M_s$ is calculated using a second-order elastic frame analysis, the possibility of sidesway buckling is checked by analyzing the structure loaded with the factored dead and live loads plus an arbitrarily chosen lateral load applied to the frame. The designer is free to choose any lateral load or set of lateral loads he or she wishes provided the load is large enough that the increase in lateral deflections due to second-order effects is distinguishable in the results. Thus, for example, the lateral load could be the factored wind loads used in designing the frame or it could be a single load applied at the top of the frame. For unsymmetrical frames for which gravity loads cause a lateral deflection, the arbitrary lateral load should be applied in the direction that increases the gravity load deflection. The frame is analyzed twice for this lateral load, once using a first-order elastic analysis and again using a second-order elastic analysis, and the ratio of lateral deflections is computed. If this ratio exceeds 2.5, the frame is too flexible laterally and hence will be close to failing due to sidesway buckling. The values of EI used in these analyses should be divided by $(1 + \beta_d)$ corresponding to the factored gravity loads.

When $\delta_s M_s$ is calculated using $\delta_s = 1/(1 - Q)$, the test for possible sidesway buckling is carried out by setting an upper limit on Q where Q is calculated using ΣP_u for 1.4 dead load and 1.7 live loads. The shear V_u is due to any assumed lateral loading (the wind loads used in design, for example), and Δ_o is the first-order relative lateral displacement of the top and bottom of the story caused by V_u, computed using EI values divided by the corresponding $(1 + \beta_d)$ for the gravity-load case. The limit on Q of 0.6 corresponds to a magnified deflection of 2.5 times the first-order deflection.

When $\delta_s M_s$ is calculated using the traditional sway magnifier equation, the sway magnifier must be positive and should not exceed 2.5, again with β_d based on the ratio of factored axial loads. For higher δ_s values, the frame will be very susceptible to changes in EI, foundation rotations, and other weakening factors.

SUMMARY

Revisions are proposed to the slender column design provisions of the ACI Building Code to simplify the design of slender columns and to recognize the use of second-order analysis. The most significant change concerns the superposition and magnification of nonsway and sway moments.

ACKNOWLEDGMENT

This paper and the proposed code revisions were developed on behalf of ACI Committee 318, Subcommittee D, Flexure and Axial Loads. They have been balloted through ACI Committee 318 and have been revised in response to the ballot. They are currently undergoing a second letter ballot in Committee 318. The author particularly wishes to thank R. F. Mast, Chairman of ACI 318 Subcommittee D, who reviewed the various drafts to check if the code clauses followed a logical order comments. His design examples flagged sections needing change. The author also wishes to thank Professors R. W. Furlong and R. Green who worked with the author in developing the proposed revisions.

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NOTATION

A_g = gross area of cross-section, in.

E_c = modulus of elasticity of concrete, psi

E_s = modulus of elasticity of reinforcement, psi

EI = flexural stiffness of compression member

f_c' = specified compressive strength of concrete, psi

I_g = moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement

I_{se} = moment of inertia of reinforcement about centroidal axis of member cross section

k = effective length factor

K = flexural stiffness of a beam

l_c = length of a compression member in a frame, measured from center to center of the joints in the frame

l_u = unsupported length of compression member

M_n = nominal moment capacity of cross section

M_1 = smaller factored end moment on a compression member, positive if member is bent in single curvature, negative if bent in double curvature

M_{1ns} = factored end moment on a compression member at the end at which M_1 acts, due to loads that cause no appreciable sidesway

M_{1s} = factored end moment on compression member at the end at which M_1 acts, due to loads that result in appreciable sidesway, calculated using a first-order elastic frame analysis

M_2 = larger factored end moment on compression member, always positive

$M_{2,min}$ = minimum value of M_2

M_{2ns} = factored end moment on compression member at the end at which M_2 acts, due to loads that result in no appreciable sidesway

M_{2s} = factored end moment on compression member at the end at which M_2 acts, due to loads that cause appreciable sidesway, calculated by a first-order elastic frame analysis

P_{crf} = critical load of a frame

P_u = factored axial load in a column

Q = stability index for a story

r = radius of gyration of cross section of a compression member

V_u = factored shear in a story

w = uniform load on a beam

β_d = a. for nonsway frames, β_d is the ratio of the maximum factored axial dead load to the total factored axial load

b. for sway frames, except for gravity load stability checks, β_d is the ratio of the maximum factored sustained lateral load to the maximum total factored lateral load in that story

c. for gravity load stability checks of sway frames, β_d is the ratio of the maximum factored axial dead load to the total factored axial load

δ_b = moment magnification factor for braced frame

δ_s = moment magnification factor for sway frame

Δ_o = first order relative lateral deflection of the top and bottom of a story due to V_u, computed using the specified stiffness values

μ = ratio of each moment due to lateral loads to the nominal moment capacity

η = ratio of maximum end moment due to gravity loads to the nominal moment capacity

φ = strength reduction factor

APPENDIX A—PROPOSED CHANGES TO ACI 318 SLENDERNESS PROVISIONS

1—Slenderness effects in compression members

1.1—Except as allowed in Section 1.2, the design of compression members, restraining beams, and other supporting members shall be based on the factored forces and moments from a second-order analysis considering material nonlinearity and cracking, as well as the effects of member curvature and lateral drift, duration of the loads, shrinkage and creep, and interaction with the supporting foundation. The dimensions of the cross sections used in the analysis shall be within 10 percent of the dimensions of the members shown on the design drawings, or the analysis shall be repeated. The analysis procedure shall have been shown to result in prediction of strength in substantial agreement with the results of comprehensive tests of columns in indeterminate reinforced concrete structures.

1.2—In lieu of the procedure prescribed in Section 1.1, it is permissible to base the design of compression members, restraining beams, and other supporting members on axial forces and moments from the analyses described in Section 2.

2—Magnified moments—General

2.1—The factored axial forces P_u, the factored moments M_1 and M_2 at the ends of the column and, where required, the first-order relative lateral story deflections Δ_o, shall be computed using an elastic first-order frame analysis with the section properties determined taking into account the influence of axial loads, the presence of cracked regions along the length of the member, and effects of duration of the loads. Alternatively, it is permissible to use the following properties for the members in the structure:

a. Modulus of elasticity = E_c from Section 8.5.1

b. Moment of inertia

Beams: 0.35 I_g

Columns: 0.70 I_g

Walls—Uncracked : 0.70 I_g

Cracked : 0.35 I_g

Flat plates and flat slabs: 0.25 I_g

c. Area: 1.0 A_g

The moments of inertia used in Section 2.1, and Section 4.3 shall be divided by $(1 + \beta_d)$ when (a) sustained lateral loads act, or for (b) stability checks made according to Section 4.5.

2.2—It is permissible to take the radius of gyration r equal to 0.30 times the overall dimension in the direction stability is being considered for rectangular compression members and 0.25 times the diameter for circular compression members. For other shapes, it is permissible to compute the radius of gyration for the gross concrete section.

2.3—Unsupported length of compression members

2.3.1—The unsupported length l_u of a compression member shall be taken as the clear distance between floor slabs, beams, or other members capable of providing lateral support in the direction being considered.

2.3.2—Where column capitals or haunches are present, the unsupported length shall be measured to the lower extremity of the capital or haunch in the plane considered.

2.4—Columns and stories in structures shall be designated as nonsway or sway columns or stories. It is permissible to assume a story within a structure is nonsway if

$$Q = \frac{\Sigma P_u \Delta_o}{V_u l_c} \leq 0.05 \qquad \text{(A)}$$

where ΣP_u and V_u are the total vertical load and the story shear, respectively, in the story in question, and Δ_o is the first-order relative deflection of the top and bottom of that story due to V_u.

2.5—The design of columns in nonsway frames or stories shall be based on the analysis given in Section 3.

2.6—Frames or stories which do not satisfy the definition of nonsway frames in 2.4 shall be designed as sway frames or stories. The design of columns in sway frames or stories shall be based on the analysis given in Section 4.

2.7—Where an individual compression member in the frame has a slenderness kl_u/r of more than 100, Section 1.1 shall be used to compute the forces and moments in the frames.

2.8—For compression members subject to bending about both principal axes, the moment about each axis shall be magnified separately based on the conditions of restraint corresponding to that axis.

3—Magnified moments—Nonsway frames

3.1—For compression members in nonsway frames, the effective length factor k shall be taken as 1.0, unless analysis shows that a lower value is justified. The calculation of k shall be based on the E and I values used in Section 2.1.

3.2—In nonsway frames it is permissible to ignore slenderness effects for compression members which satisfy

$$\frac{kl_u}{r} \leq \frac{25 - 12\,(M_1/M_2)}{\sqrt{\dfrac{P_u}{f_c' A_g}}} \qquad \text{(B)}$$

where M_1/M_2 is not taken less than -0.5. The term M_1/M_2 is positive if the column is bent in single curvature.

3.3—Compression members shall be designed for the factored axial load P_u and the moment amplified for the effects of member curvature M_c as follows

$$M_c = \delta_{ns} M_2 \qquad \text{(C)}$$

where

$$\delta_{ns} = \frac{C_m}{1 - \dfrac{P_u}{\varphi_s P_c}} \geq 1.0 \qquad \text{(D)}$$

where $\varphi_s = 0.75$

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} \qquad \text{(E)}$$

EI shall be taken as

$$EI = \frac{(0.2 E_c I_g + E_s I_{se})}{1 + \beta_d} \qquad \text{(F)}$$

or

$$EI = \frac{0.40 E_c I_g}{1 + \beta_d} \qquad \text{(G)}$$

3.3.1—For members without transverse loads between supports, C_m shall be taken as

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \geq 0.4 \qquad \text{(H)}$$

where M_1/M_2 is positive if the column is bent in single curvature. For members with transverse loads between supports, C_m shall be taken as 1.0.

3.3.2—The factored moment M_2 in Eq. (C) shall not be taken less than

$$M_{2,min} = P_u(0.6 + 0.03h) \qquad \text{(J)}$$

about each axis separately, where 0.6 and h are in inches. For members for which $M_{2,min}$ exceeds M_2, the value of C_m in Eq. (H) shall either be taken equal to 1.0, or shall be based on the ratio of the computed end moment $M_1$to $M_{2,min}$. [In Eq. (J) becomes $M_{2,min} = P_u\,(15 + 0.03h)$, where 15 and h are in mm.]

4—Magnified moments—Sway frames

4.1—For compression members not braced against sidesway, the effective length factor k shall be determined using E and I values in accordance with Section 2.1 and shall be greater than 1.0.

4.2—The moments M_1 and M_2 at the ends of an individual compression member shall be taken as

$$M_1 = M_{1ns} + \delta_s M_{1s} \qquad \text{(K)}$$

$$M_2 = M_{2ns} + \delta_s M_{2s} \qquad \text{(L)}$$

where $M_s M_{1s}$ and $\delta_s M_{2s}$ shall be computed according to Section 4.3.

4.3—Calculation of $\delta_s M_s$

4.3.1—The magnified sway moments $\delta_s M_s$ shall be taken as the column end moments calculated using a second-order elastic analysis based on the member stiffnesses given in Section 2.1.

4.3.2—Alternatively, it is permissible to compute $\delta_s M_s$ as

$$\delta_s M_s = \frac{M_s}{1 - Q} \geq M_s \qquad \text{(M)}$$

If δ_s computed in this way exceeds 1.5, $\delta_s M_s$ shall be computed using Section 4.3.1 or 4.3.3.

4.3.3—Alternatively, it shall be permissible to calculate the magnified sway moment $\delta_s M_s$

$$\delta_s M_s = \frac{M_s}{1 - \dfrac{\Sigma P_u}{\varphi_s \Sigma P_c}} \geq M_s \qquad \text{(N)}$$

where ΣP_u is the summation for all the vertical loads in a story and ΣP_c is the summation for all sway-resisting columns in a story, P_c is computed using Eq. (E) using k from Section 4.1 and EI from Eq. (F) or Eq. (G), and $\varphi_s = 0.75$.

4.4—If an individual compression member has

$$\frac{l_u}{r} > \frac{35}{\sqrt{\dfrac{P_u}{f_c' A_g}}} \qquad \text{(P)}$$

it shall be designed for the factored axial load P_u and the moment M_c, computed using Section 3.3 in which M_1 and M_2 are computed in accordance with Section 4.2, β_d as defined for the load combination under consideration and k as defined in Section 3.2.

4.5—In addition to load cases involving lateral loads, the strength and stability of the structure as a whole under factored gravity loads shall be considered.

a. When $\delta_s M_s$ is computed from Section 4.3.1, the ratio of second-order lateral deflections to first-order lateral deflections for 1.4 dead load and 1.7 live load plus lateral load applied to the structure shall not exceed 2.5.

b. When $\delta_s M_s$ is computed according to Section 4.3.2, the value of Q computed using ΣP_u for 1.4 dead load plus 1.7 live load shall not exceed 0.60.

c. When $\delta_s M_s$ is computed from Section 4.3.3, δ_s computed using ΣP_u and ΣP_c corresponding to the factored dead and live loads shall be positive and shall not exceed 2.5.

In the preceding Cases a, b, and c, β_d shall be taken as the ratio of the factored sustained axial dead load to the total factored axial load.

4.6—In sway frames, flexural members shall be designed for the total magnified end moments of the compression members at the joint.