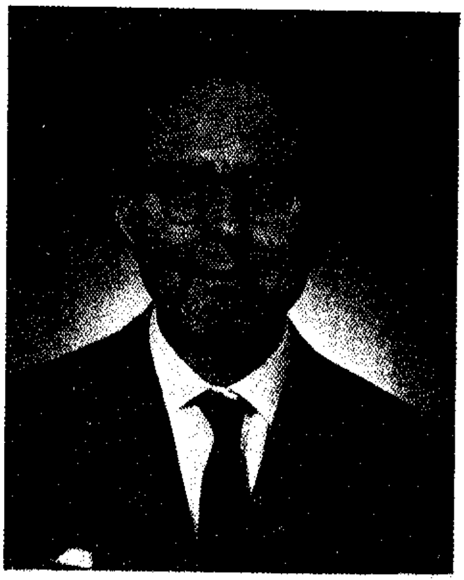
INTERNATIONAL ASSOCIATION FOR SHELL AND SPATIAL STRUCTURES

recommendations

working group nr 5



Ronald Stewart Jenkins, 1907-1975

This publication is dedicated to the memory of Ronald Stewart Jenkins, one of the founding members of the IASS, a long-time member of its Executive Council and Advisory Board, an analyst and a practical engineer *sans pareil*, an inspiration to shell designers everywhere.

RECOMMENDATIONS
FOR REINFORCED CONCRETE SHELLS AND FOLDED PLATES

Prepared by
The Working Group on Recommendations of
The International Association for Shell and Spatial Structures

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IASS, Madrid, 1979

RECOMMENDATIONS

For Reinforced Concrete Shells and Folded Plates

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FOREWORD

One of the aims of the IASS is the preparation of Recommendations for the various aspects of design, analysis and construction of thin shells and spatial structures. This important phase of the work of the Association is conducted through the activities of its Working Groups.

Because of their historical importance, both in the development of spatial systems and in the organization and growth of the IASS, a major effort extending over a number of years was devoted to the problem of thin shells. It is this effort which is culminating now in the publication of these Recommendations.

The work began in 1969 with the creation by the then president of the IASS, Professor Haas, of the IASS Working Group on Recommendations under the chairmanship of Dr. Rühle. Several meetings were held, and an outline of the Recommendations was developed, thus completing the first phase of the task.

In 1974, on recommendation of Dr. Rühle, I appointed Dr. Medwadowski as the new chairman, and the Working Group was enlarged by the addition of several experts. A well attended meeting was held at Udine, Italy, where several issues were discussed at length, resulting in assignment of tasks to all members. Following the Udine meeting, various chapters of the Recommendations were drafted, commented on, edited and re-edited. Completed text was presented to the Executive Council of the IASS which, at its meeting in Montreal in 1976, approved it for publication.

It is my pleasant duty to express my sincere thanks to all members of the Working Group who worked so hard preparing successive drafts, editing, attending meetings and participating in many a long discussion. My special thanks go to the chairman of the Working Group, Dr. Medwadowski, who through unceasing hard work, almost like a magician, managed to produce the final text of the Recommendations in an astonishingly short time.

I am confident that these Recommendations will prove helpful to all those throughout the world who work in the field of design, analysis and construction of concrete thin shell structures.

Prof. Dr. Ir. A. Paduart
President of the IASS

PREFACE

The development of Recommendations for the design, analysis and construction of concrete shells and folded plates constitutes a difficult and time-consuming task, made even more complex by the world-wide character of the sponsoring organization. IASS — the International Association for Shell and Spatial Structures — consists of individual and institutional members from some seventy five countries all around the globe. It is desirable, therefore, that a publication such as these Recommendations be applicable throughout the world. Yet the conditions under which shells are designed and built vary not only from continent to continent, but from country to country. The physical environment and climate, the degree of development of the technology of construction, the availability of skilled labor and of building materials, the cost of construction, even the sophistication of analytical tools available to a designer, all are likely to vary significantly between different locations. For these reasons, the decision was made to keep these Recommendations general in character; when dealing with specific, narrower problems reference should be made to the specialized literature of the given field. Here the published Proceedings of the IASS Symposia where specified topics are studied in detail may prove helpful.

The effort involved in the development of these Recommendations encompassed many people and extended over many years. The assistance of many individuals and institutions who provided informations about existing shell regulations in their countries was invaluable. It gives me very real pleasure, indeed, to take this opportunity to express my appreciation of their contribution to all members of the Working Group who labored so hard over a long period of time to make these Recommendations a reality. Particular thanks are due Dr. Rühle, the first chairman of the Working Group, and Prof. Scordelis, who contributed so much to the development of the ideas and the language of the text. Above all, the steady support and encouragement of Prof. Paduart, president of the IASS, who took a lively and active personal interest in the activities of the Working Group, is gratefully acknowledged.

Dr. Stefan J. Medwadowski, Chairman
IASS Working Group on Recommendations

LIST OF SYMBOLS

C	Coefficient proportional to extensional stiffness of Equation (3.1)
D	Dead Load
E	Earthquake load
E_c	Modulus of elasticity of concrete
E_{ci}	Creep reduced modulus of elasticity of concrete
E_s	Modulus of elasticity of reinforcing bars
$F.S.$	Factor of Safety
H	Relative humidity
K	Flexural stiffness of shell
L	Live load
R	Minimum radius of curvature of shell (Table 3.1)
U	Total load
W	Wind load
C_u	Coefficient in Equation (3.7)
f	Shell quantity in Equation (3.3)
f_c	Concrete strength at time of loading in Equation (3.7)
f_y	Yield point of steel
f_c'	Cylinder strength of concrete at 28 days
h	Shell thickness
p	Shell load
p_{cr}^{lin}	Linear critical load

LIST OF SYMBOLS

p_{cr}^{u}	"Upper" critical load
$p_{cr,reinf}^{u}$	Reduced "upper" critical load
p_{cr}^{plast}	Plastic critical load
p_{plast}	Plastic load, with buckling disregarded
r	Reduction factor in Equation (4.3)
w	Normal displacement
$w_o^{'}$	Computed or measured normal displacement (before buckling)
$w_o^{''}$	Amplitude of shape deviation
$\overline{w}$	Weight of concrete in Equation (2.1)
Φ	Airy stress function
$\alpha, \alpha_1, \alpha_2$	Shell coordinates in Equation (3.3) and (3.6)
$\beta_1, \beta_2, \beta_3$	Shell parameters in Equations (3.3) and (3.6)
ϵ_b	Flexural strain at extreme fiber
ϵ_m	Extensional strain at middle surface
ϵ_{sh}	Free shrinkage strain of hardened concrete
κ	Ratio of principal stresses
λ	"Wavelength" of the pattern of deformation of shells
μ	Coefficient in Figure 3.3
ν	Poisson ratio
σ	Stress
ϕ	Angle between a principal stress and reinforcing
ψ	Coefficient in Figures 3.3 and 3.4
Δ_k	2nd order differential operator which first occurs in Equation (3.1)

CHAPTER 1

INTRODUCTION

1.1. GENERAL REMARKS

Shell-like structures have been built by man since antiquity, when the arcuated forms of Roman architecture were extended into three dimensions. However, rational use of concrete thin shells is a relatively new development dating back to the early decades of the twentieth century. Since those early beginnings, the use of shells as roof structures and as elements of buildings has increased rapidly so that, at the three-quarter point of the century, concrete thin shells are being built with appreciable frequency on all continents.

With the increased use of shells has come an increased understanding of their behavior. Field observations, laboratory tests, development of theories of analysis and, in the last decade, development of rapid solutions aided by digital computers, all contributed to progress in successful realizations. However, because of the wide range of geometry possible with thin shell structures, the accumulated understanding is still limited. For some thin shell systems, such as cylindrical barrel vaults or spherical domes, sufficient past experience permits designs to be made with the same degree of accuracy as in conventional reinforced concrete construction. For other thin shells, less extensive experience and greater difficulty in performing accurate and thorough analyses, including all important effects, require sound structural knowledge and judgment to execute a safe and satisfactory design.

The purpose of the Recommendations is to provide assistance in the development of such knowledge and judgment to persons active in the design, analysis and construction of concrete thin shells which occur in building structures, and to facilitate their tasks as follows:

- by outlining problems common in concrete shell structures, and by discussing generally accepted solutions to these problems;
- by identifying those problems of design, analysis and construction which, perhaps, are not yet fully understood and for which generally accepted solutions are still lacking;
- by providing a selected bibliography of shells.

In view of the limited purpose of these Recommendations, and the fact that the design, analysis and construction of concrete thin shells requires thorough knowledge in this field, the recommendations contained herein are not sufficient of themselves for the satisfactory execution of shell structures. For additional information, designers are referred to specialists in the field, to the technical literature cited, and to the many other available texts and publications.

1.2. DEFINITIONS

1.2.1. Shells

Thin shell: A structural system which, in the large, has the form of a surface or a combination of surfaces in three-dimensional space. The thickness of a shell is small compared to its other dimensions. These geometrical properties endow shells with their characteristic three-dimensional load-carrying ability — loads being carried predominantly by in-plane stresses as determined by the shell geometry, the manner of its support, and the nature of applied loads.

Membrane: A thin shell which carries loads solely through in-plane stresses and is incapable of transmitting bending stresses.

Tension shell: A membrane capable of transmitting only tensile in-plane stresses.

Skeletal shell: A thin shell which consists of an assembly of one- or two-dimensional elements connected at nodal points which, in its entirety, forms a surface in space. Unlike a more typical thin shell, a skeletal shell does not necessarily enclose space.

Folded plate: A special type of thin shell structural system formed from a combination of two-dimensional planar elements connected at the edges. This structure forms a combination of surfaces in space. Because the elements are planar, significant bending action may occur.

1.2.2. Auxiliary Members

An auxiliary member is any structural element located along the boundary of a shell or shell segment with the capacity to stiffen the shell and to distribute or carry loads in composite action with the shell. Typically, an auxiliary member serves in a combination of capacities. In accordance with the established usage, the following types of auxiliary members may be distinguished.

Supporting members: Beams, arches, trusses, diaphragms, etc., along the edges of thin shells which serve both to support and stiffen the thin shell.

Edge members: Beams, trusses, etc., along the edges of thin shells which do not form a part of the main support structure but which serve to stiffen the shell and act integrally with it in transferring the load to supporting members.

Stiffening members: Ribs which serve to stiffen the thin shell and to control local deformations. The participation of stiffening members in transferring the load to supporting members is minor.

1.2.3. Geometrical Terms

Some of the more common geometrical terms which occur in shell design are defined as follows.

Middle surface: Surface which defines the form of a shell with a theoretical zero thickness. Usually, but not always, the middle surface is assumed to be equidistant from the two *faces* of the shell. If, as is usually the case, the middle surface is definable analytically, it is said to be of *geometric form.* If it is not so definable, it is termed a *free form* surface.

Curvature and radius of curvature: At any point on a surface, it is possible to construct a plane at that point through the normal to the surface. This *normal plane* intersects the surface along a curve. The radius of curvature of the curve at the point of intersection with the normal is the radius of curvature of the surface at that point, and the corresponding curvature is the surface curvature. Since the value of the radius may vary depending on the orientation of the normal plane, it is possible to establish the maximum and minimum values of the radii, called the *principal radii of curvature,* their orientation, and the associated *principal curvatures.*

Gaussian curvature: Product of principal curvatures at a point of the surface. The product may be positive, zero, or negative. *The Gaussian curvature index* takes on, respectively, the value of +1, 0, and -1, and the surface is called, respectively, *synclastic* (or *elliptic*) a *surface of zero curvature* (or *parabolic*), and *anticlastic* (or *hyperbolic*) at that point. The value of the Gaussian curvature index has a profound influence on the behavior of the shell, i.e., on the manner in which it transmits loads.

Mean curvature: Arithmetic mean of principal curvatures at a point of the surface.

Generatrix: A curve in a plane which defines a surface by moving in a prescribed manner through space.

Directrix: A curve in a plane which, in combination with others, serves to define the motion of the generatrix.

Developable surface: Any surface of zero Gaussian curvature. Such surfaces are capable of being unfolded onto a plane without stretching.

Ruled surface: Surface such that, and any point, a straight line may be ruled which lies in the surface. If one such line can be ruled, the surface is said to be *singly ruled;* if two lines can be ruled at any point of the surface, the surface is termed *doubly ruled.*

Minimal surface: Surface such that its mean curvature is zero everywhere. Minimal surfaces possess the property of having the least surface area for a given closed boundary contour.

1.3. CLASSIFICATION OF SHELLS

Shells may be classified in a number of ways. As an example, the system of classification might be based on the manner in which a concrete thin shell is constructed: cast in place, or precast. Another basis for classification might be the manner in which a shell transmits loads: a membrane, a tension shell, or a general shell capable of transmitting bending. However, the most meaningful classification is based on the geometry of the surface of the shell. The reason for this is that the geometry of the shell has decisive influence on essentially all aspects of shell behavior: the manner in which it transmits loads, the ease and economy of construction, and finally the aesthetic qualities of the completed structure.

In the following, thin shells are classified on the basis of the manner in which the surfaces which define these shells are formed: i.e., according to the shape and the manner of motion of the generatrix.

1.3.1. Shells of Revolution

Shells of revolution are formed by rotating the generatrix about an axis located in its plane. Some of the more common surfaces of revolution are as follows.

Cone: The generatrix is a straight line which intersects the axis of revolution. The surface is singly ruled, the Gaussian curvature is zero; hence the surface is developable. It is used principally for tanks, etc., but also for dome-like roofs and for covering of trapezoidal plans.

Circular cylinder: The generatrix is a straight line parallel to the axis of revolution. The surface is singly ruled; Gaussian curvature is zero, hence the surface is developable. This type of shell has found wide application in building construction. Segments of circular cylinders, alone and in combinations, are perhaps the single most common type of thin shell.

Sphere: The generatrix is a circle with the center at the axis of revolution. Gaussian curvature is positive and constant. In building construction, it is used most often as a spherical cap called a *spherical dome.* It is one of the oldest of thin shell forms, dating back to Roman times. Segments of spherical caps are sometimes used to cover areas which are polygonal in plan.

Dome: The generatrix is any plane curve segment extending from the axis of revolution. The surface is usually not ruled and the Gaussian curvature sign depends on the shape of the generatrix. Common shapes of the generatrix are: parabola, which forms a *paraboloid of revolution;* ellipse, which forms an *ellipsoid of revolution.* In these cases the Gaussian curvature is positive.

Hyperboloid of revolution of one sheet: The generatrix is a hyperbola, with the imaginary axis parallel to the axis of revolution. The surface is doubly ruled. The Gaussian curvature is negative and the surface is not developable. This is used most commonly in the construction of cooling towers, but occasionally to provide the form for whole buildings.

Hyperboloid of revolution of two sheets: The generatrix is a hyperbola rotated about its real axis. The Gaussian curvature is positive, and the surface is not developable. One sheet of this surface might be used as a domical roof.

Torus and toroids: The generatrix is a conic with the axis of symmetry not coincidental with the axis of revolution. In the particular case when the generatrix is a circle, the surface is called a torus. Torus is a closed surface, in contrast with the open surface of the toroid, which results from the rotation of the generatrix in the form of, say, a parabola. The surface is of double curvature. Gaussian curvature varies depending on the shape of the generatrix and the location of the point on the shell surface. As an example, in the case of a torus, Gaussian curvature varies from positive through zero to negative as one approaches the axis of revolution. Segments of torus have been used, in combinations, to form roof structures.

1.3.2. Catalan Surfaces

Catalan surfaces are formed by the motion of a straight generatrix in space in such a way that the generatrix remains always parallel to a given plane and in contact with two directrices. The directrices are plane curves located in planes parallel to each other and perpendicular to the datum plane. Catalan surfaces are suited, therefore, to covering areas rectangular in plan. Some of the more common shapes are as follows.

Hyperbolic paraboloid: The two directrices are straight lines, not parallel to each other. The surface is doubly ruled, Gaussian curvature is negative and the surface is non-developable. It has found wide application in building construction. This surface can be thought of also as a translational surface.

Conoid: One of the two directrices is a straight line. Depending on the shape of the second directrix, the conoid is termed *circular, parabolic, elliptic,* etc. In the special case when the generatrix is tangent to a prescribed director sphere, the conoid is said to be *spherical.* The surface is of double curvature, ruled, typically of negative Gaussian curvature, and is non-developable. The conoids have found wide use in building construction, particularly in combinations as north-light roofs.

Cylindroid: Both directrices are curves. The surface is singly ruled. The sign of Gaussian curvature depends on the shape and arrangement of the directrices. The surface is not developable.

1.3.3. Translational Surfaces

Translational surfaces are formed by motion of the generatrix along a directrix, the motion being parallel to a datum plane. The surface is suited for covering areas rectangular in plan. Many shapes are possible, some of the more common one being the following.

Cylindrical surfaces: The generatrix is a plane curve, the directrix a straight line. The surface is singly ruled, Gaussian curvature is zero, and the surface is developable. The special case in which the generatrix is in the form of a segment of a circle or a combination of such segments, forms a circular cylindrical barrel vault — perhaps the most common of all thin shell types used in roof construction.

Elliptic paraboloid: Both the generatrix and the directrix are parabolas with the curvatures of the same sign (they are "open" in the same direction). The surface is of double curvature. Gaussian curvature is positive and the surface is non-developable. This type of thin shell is commonly used in roof construction.

Hyperbolic paraboloid: Both the generatrix and the directrix are parabolas but their curvatures are of opposite sign (they are "open" in opposite directions). The surface is of double curvature, Gaussian curvature is negative and the surface is non-developable. This surface can be thought of also as a Catalan surface.

1.3.4. Helicoidal Surfaces

Helicoidal surfaces are formed when the motion of the generatrix is defined by means of two directrices: a helix and a straight line which is the axis of the helix and which is also the axis of the helicoid. The generatrix may be a straight line, when the surface is ruled, or a plane curve. If the generatrix intersects the axis, the helicoid is said to be *closed.* If the axis and the generatrix are skew in space, the helicoid is said to be *open.* A helicoidal surface is further modified depending on the type of helix directrix. The most common type is the *cylindrical helicoid;* other types are *conical helicoid, parabolic helicoid,* etc.

In the special case of a ruled, open, cylindrical helicoid with generatrices tangent to the director cylinder, the resulting surface has zero Gaussian curvature and is developable.

Common helicoid: The generatrix is a straight line, intersecting the axis at a right angle; the helix directrix is cylindrical. The resulting surface is a ruled, closed cylindrical helicoid of double curvature, the Gaussian curvature being negative. The common helicoid has the property of being a minimal surface, i.e., its mean curvature is zero everywhere.

1.3.5. Generalized Translational Surfaces

Translational surfaces were defined in Paragraph 1.3.3 as those surfaces defined by the parallel motion of the generatrix along a directrix perpendicular to the plane of the generatrix. This motion can be generalized in two ways. The motion might be no longer parallel, the angle that the generatrix makes with the datum plane changing in accordance with specified law. In addition, the shape of the generatrix might be permitted to vary, again in accordance with some specified law depending on the position of the generatrix. In each case, the laws of variation of the shape of the generatrix, and the variation of the motion, are constructed to suit a given problem. The resulting surface is said to be a generalized translational surface.

Parabolic surfaces: The generatrix is a parabola with a constant base width and a variable height. The law of height variation is such that the directrices are also parabolas. Parallel motion law is retained. This results in a surface of double curvature covering a rectangular plan, non-ruled and non-developable.

Elliptic surfaces: The generatrix is an ellipse with a constant base width and a variable height. The law of height variation is such that the directrices are also ellipses. Parallel motion is retained. The surface is of double curvature, covers a rectangular plan, is non-ruled and non-developable.

Sinusoidal surfaces: The generatrix and the directrix are segments of the sine curve so that the surface covers a rectangular plane, parallel motion being retained. The surface is of double curvature, non-ruled, and non-developable.

In all the three cases mentioned above, the sign of the Gaussian curvature may vary depending on the location of the point on the surface.

1.3.6. Affine Surfaces

Any surface of geometric form can be transformed into another surface through an affine transformation in space such that there exists a one-to-one correspondence between points of the two surfaces.

Under affine transformation a point, a line and a plane are transformed, respectively, into a point, a line and a plane, with the parallel properties of lines and planes retained. In general, the length of the metric ds is not preserved, nor are the angles between lines. For example, a circular cone can be transformed into an elliptical cone, and a circular conoid may be transformed into an elliptic conoid. Thus the technique of affine transformation can be used to adjust the geometry of a shell to given conditions.

1.3.7. Compound Surfaces

Segments of surfaces may be combined to create new shell forms. The new compound surface usually does not retain the continuity properties of the segments, making the number of possible forms of practical significance virtually unlimited. This variety is the source of the richness of architectural solutions attainable with shell structures. A few examples are as follows.

- Intersection of two circular cylinders of equal radius at a right angle. This type of a compound surface has been used since antiquity and, depending on the segments retained, is known as a cross vault (or groin vault), or a domical vault. New forms can be introduced by varying the angle of intersection, the profile of the cylinders, and the number of the intersection cylinders — which is by no means restricted to two.
- An assembly of cylindrical barrel vaults to cover a rectangular plan area. This structure has been used since the early days of the modern shell era. New forms can be achieved by varying the form of the segments, by changing the manner in which they are connected (concave, convex, north light), by making the segments straight lines (folded plates), and by changing the shape of the plan.

- An assembly of hyperbolic paraboloids to form an umbrella-type roof. This type of compound shell surface has been widely used, particularly in the last two decades. Variation of form results from changing the location of supports, the height, the number of segments, or the shape of the plan area.

1.3.8. Folded Plates

As defined in Paragraph 1.2.1, folded plates are assemblies of two-dimensional planar elements connected at their boundaries to form a structural system in space. Depending on the nature of the final assembly, two types of folded plates can be distinguished:

- folded plates which, in the limit as the size of the plane elements becomes smaller, tend to a segment of a surface of geometric form;
- folded plates which, in the limit, tend to a combination of surfaces of geometric form.

According to common usage, folded plates are classified on the basis of the shape of the plane elements of which they are formed, as follows.

Prismatic folded plates: Plane elements are rectangles with two opposite sides significantly longer than the other two sides. The rectangles are connected along the long sides to form a cylindrical surface which frames into transverse and supporting members. Prismatic folded plates can be thought of as translational surfaces, with the generatrix in the form of a chain of straight line segments, and a straight line directrix.

Non-prismatic folded plates: The plane elements are elongated trapezoids, or triangles, connected at the long edges and framing into transverse and supporting members.

Faceted folded plates: The plane elements are polygons with the length of all sides of the same order of magnitude, connected along the boundaries to form a surface-like assembly in space.

1.3.9. Minimal surfaces

From the definition of minimal surfaces, it follows that the principal curvatures at any point are of opposite sign and equal in absolute value. Therefore, they are particularly suited to the construction of tension shells, a subject outside the scope of these Recommendations.

1.4. SCOPE OF RECOMMENDATIONS

Except where stated otherwise, these Recommendations are applicable to the design, analysis and construction of concrete thin shells used in building construction; including the thin shell portions, the elements of skeletal shells (if any), and the auxiliary members of such structures.

In general, these Recommendations are not intended to apply to two special types of shell structures: tension shells and shells made of ferrocement. The characteristic material properties of these two systems require special consideration and reference to specialized literature.

Similarly, if shell structures other than those used in building construction are to be designed (such as tanks, cooling towers, bridges, etc.), full consideration should be given to the differing conditions of load, permissible stresses and deformation, proportioning of concrete and reinforcement, and details of construction characteristic of such structures.

These Recommendations are intended to supplement, not to supercede, any applicable codes or regulations. Accordingly, the provisions contained herein should apply to those conditions which are not covered in the codes, or where they are the more stringent of the two. Only in exceptional cases, and where departures from the applicable codes are permitted, should shell designers consider adhering to the provisions of these Recommendations even when those of the codes are more stringent.

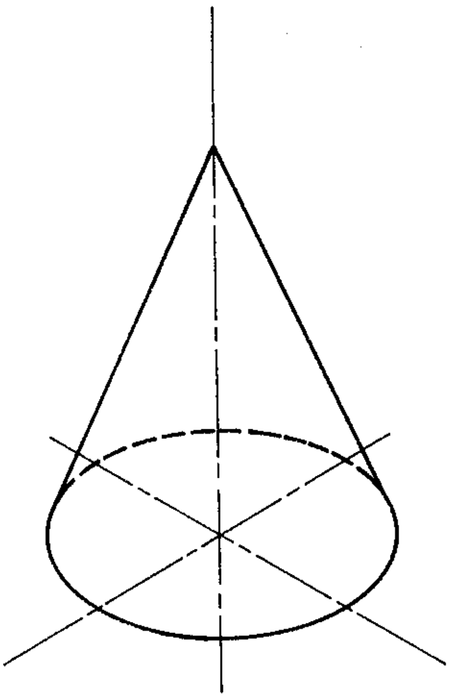


Fig. 1.1—Cone

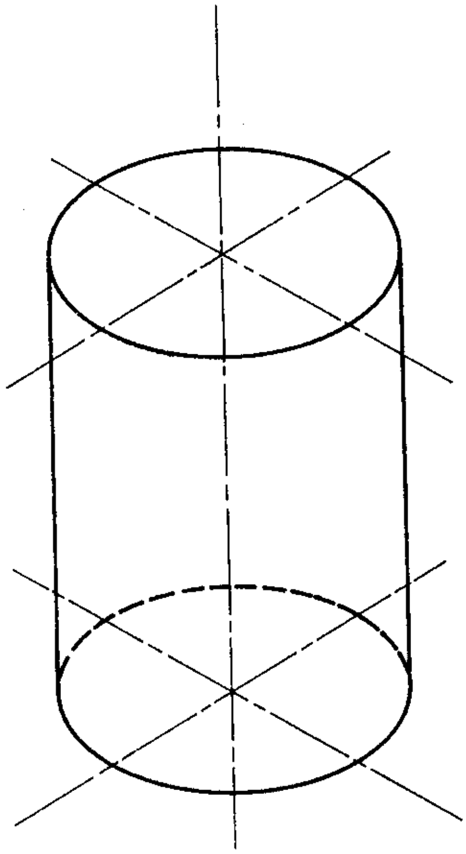


Fig. 1.2—Cylinder

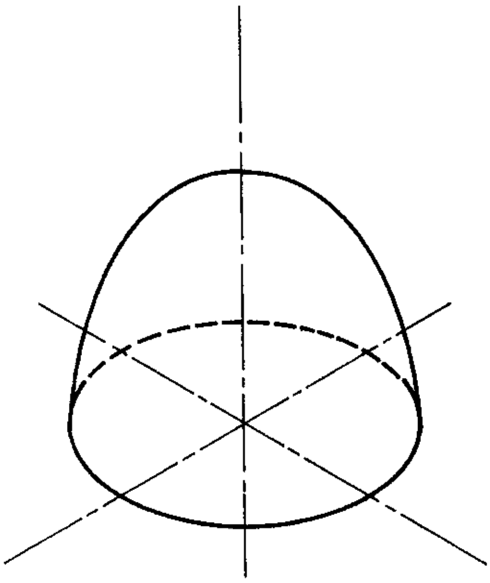


Fig. 1.3—A dome-like surface of revolution

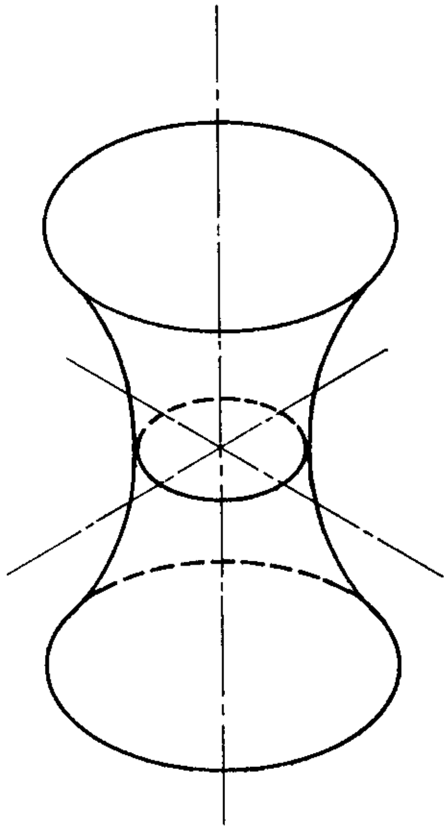
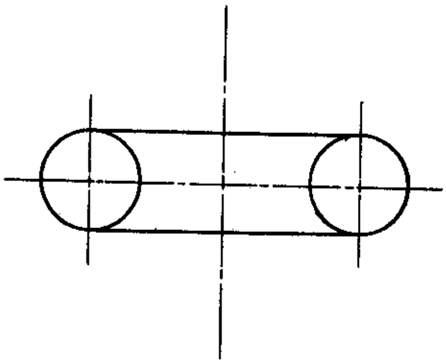
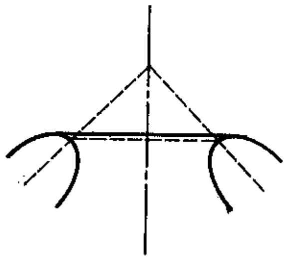


Fig. 1.4—Hyperboloid of revolution (one sheeted)



(a) Generating curve in form of a circle



(b) Generating curve in form of a parabola

Fig. 1.5 – Torus

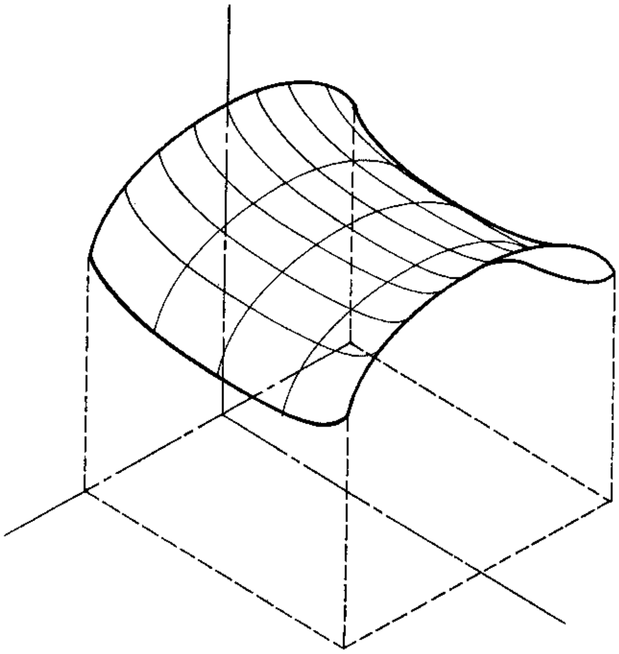


Fig. 1.6. (a)—Hyperbolic paraboloid

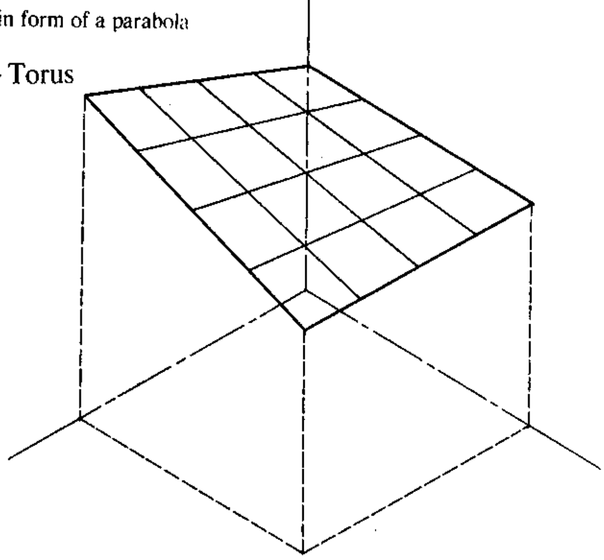


Fig. 1.6 (b)—Hyperbolic paraboloid

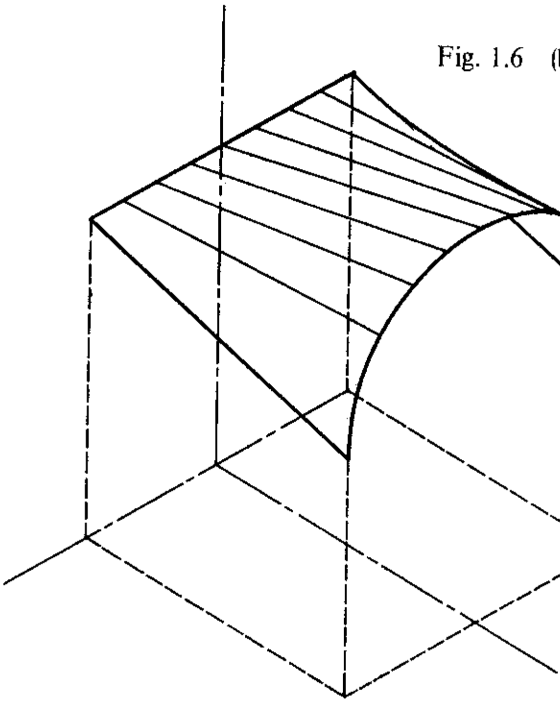


Fig. 1.7—Conoid

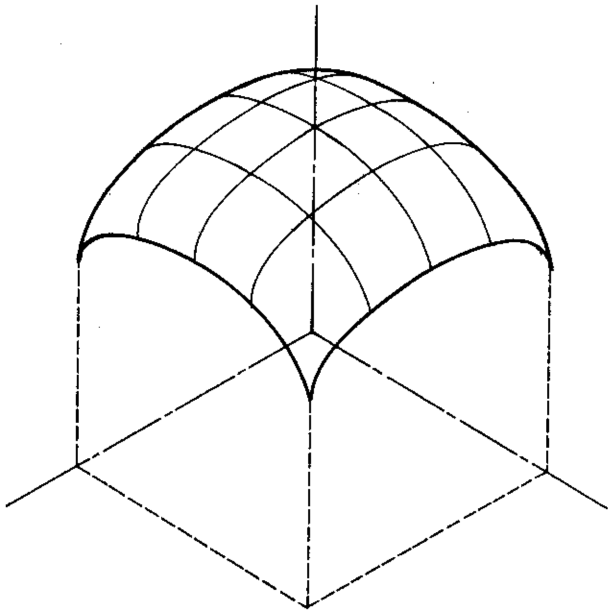


Fig. 1.8—Elliptic paraboloid

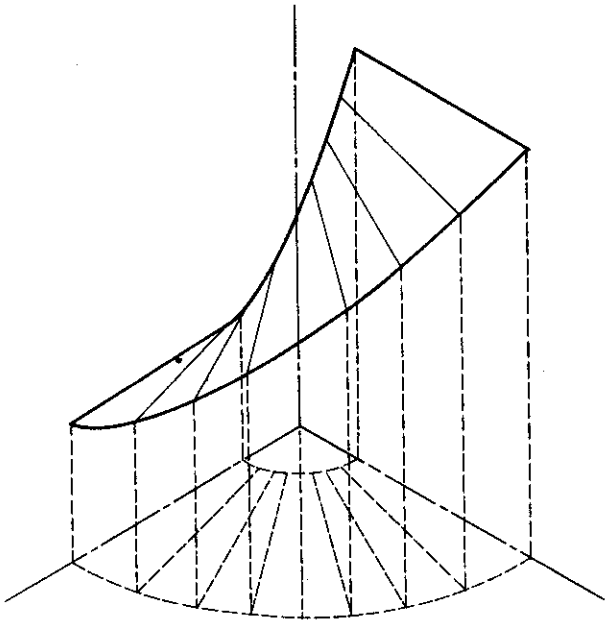


Fig. 1.9—Helicoid

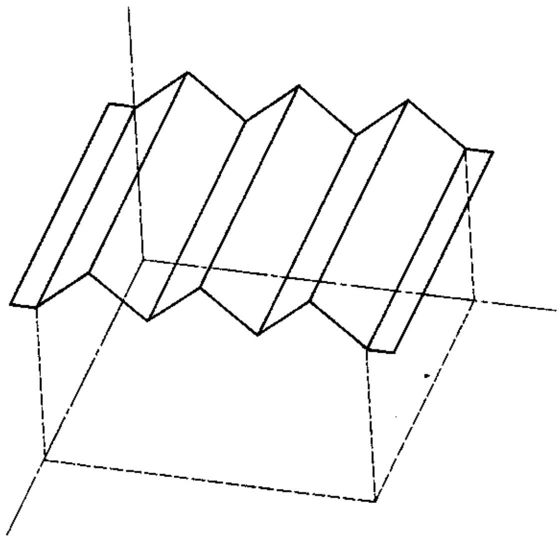


Fig. 1.10—Prismatic folded plate

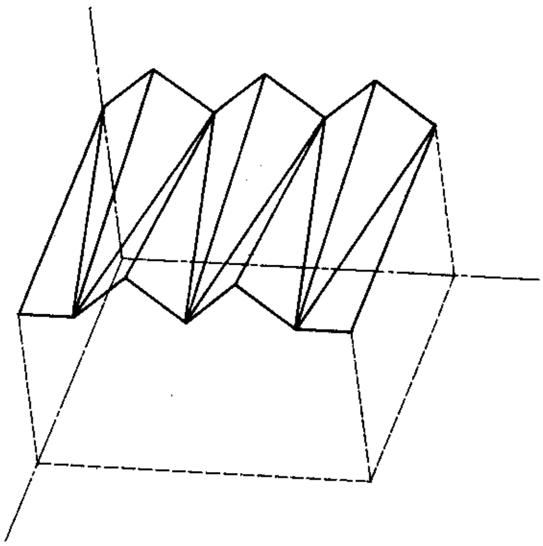


Fig. 1.11—Non-prismatic folded plate

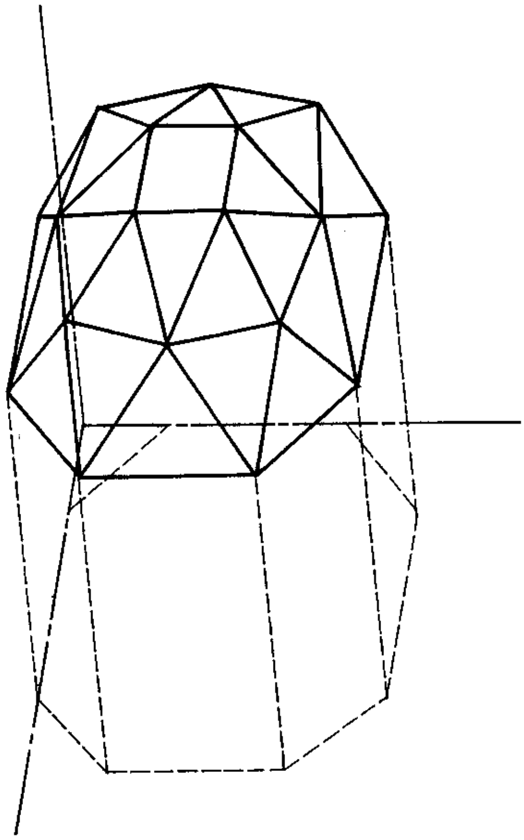


Fig. 1.12—Faceted folded plate

CHAPTER 2

STRENGTH AND SERVICEABILITY

2.1. GENERAL REMARKS

In common with other structural systems, concrete thin shells need to be analyzed for strength and serviceability.

Strength design: Consists of analysis of the shell, and proportioning of concrete and reinforcement so that allowable stresses are not exceeded. All shells should be designed for strength at working load, with elastic behavior the commonly accepted basis for analysis. Normally, it is also the basis for proportioning of concrete and reinforcement, although some exceptions to this are discussed in Chapter 4. Strength design at ultimate load should be used only if it is performed in addition to that at working load, and only if it has been shown to predict satisfactorily the behavior of the shell. Here, strength design at ultimate load means calculation of stresses based on a linear, elastic analysis for load combinations listed in paragraph 2.5 of these Recommendations.

Serviceability design: Consists of ensuring that allowable displacements are not exceeded, and that such cracking as may occur is controlled as to width, length, and frequency of distribution. Serviceability design should be performed at working load in all cases, whether the strength design is performed at working or at ultimate load.

Additional conditions to be investigated during shell analysis are discussed in Chapter 3 of the Recommendations.

2.2. LOADINGS

A number of loadings and load combinations need to be considered in the design of a shell structure. In general, the nature of the loads and their intensity should be as prescribed by the applicable regulations. In the following, some special considerations peculiar to shell structures are discussed.

2.2.1. Dead Load

In addition to dead loads typical of all structures, consideration should be given to the effect of the weight of edge members, such as the edge beams in hyperbolic paraboloids or cylindrical barrel vaults, as well as to the effect of any additional shell thickness which may be provided in some areas of the shell, such as the valleys of cylindrical barrel vaults or at shell boundaries. In many instances, the effect of these weights on shell stresses may be greater than the stresses due to the weight of the shell itself.

The sequence of decentering of form work should be considered in determining the manner in which the dead load comes onto the shell structure.

2.2.2. Live Load

Typically, shells are not sensitive to partial loading. The reason for this is that, unlike the more conventional structures such as arches, a shell of a given geometry is capable of transmitting loads primarily through in-plane stresses for any arrangement of distributed load. Accordingly, an investigation of the effects of partial loads is not normally required as far as the shell itself is concerned. However, it may still be necessary to investigate partial load effects on supporting members, columns and foundations.

2.2.3. Snow Load

Where snow might tend to collect, as in the valleys of barrel vaults, due consideration should be given to the load due to the added weight of the snow.

2.2.4. Wind Load

In general, the effects of wind load on shells are not severe and can be accounted for by the usual procedures. The possibility of suction over the entire shell surface should be recognized, as in the case of domes or cylindrical barrel vaults.

2.2.5. Earthquake Load

During an earthquake, foundations of the structure of which the shell is a part are subjected to horizontal and vertical accelerations. In consequence, the shell itself experiences motion and deformation, and hence internal stresses. The magnitude of these stresses and deformations should be investigated.

Typically, a shell structure is stiffer than a conventional framed structures and hence is subject to larger earthquake stresses.

Foundations of shell supports should be interconnected by a system of grade beams to minimize the relative horizontal movement which might occur between the supports during an earthquake. Similarly, to reduce the effect of the vertical component of ground motion, the foundations should be designed to minimize as much as possible the differential vertical motions of supports.

2.2.6. Temperature, Shrinkage and Creep

In general, loads are induced as a result of restraints placed on the motion of the structure, or as a result of differential movements due to these effects.

Careful consideration should be given to the forces induced in supporting or edge members having centroids which are eccentric to the shell so as to avoid progressive deflection and possible significant bending in these members.

2.2.7. Erection Loads and Movement of Supports

Each stage of construction should be studied to ascertain the erection loads and the structural system in existence capable of transmitting loads at that stage. Differential movement of supports may induce severe stresses in shells and thus should be minimized, or their possible effects should be included in the analysis.

2.2.8. Prestressing Loads

Local stress concentrations at anchorages should be investigated as in other structural systems, due regard being given to the thinness of the shell at these points and to the need to prevent the tendons from breaking out of the shell in the transverse direction as well as from splitting the shell in its own plane.

Tendon force components due to tendon curvature and normal to the shell surface, where they occur, should be allowed for in the analysis.

2.3. MATERIAL PROPERTIES

Some of the material properties of concrete of particular interest in shell design are discussed below. For detailed information, reference should be made to specialized publications such as the authoritative publications of the Comite Eur-International du Beton and the American Concrete Institute.

2.3.1. Concrete

Strength: The specified cylinder compressive strength at 28 days, f'_c, should be not less than 20 N/mm^2.

Modulus of elasticity: The modulus of elasticity of concrete, E_c, measured in N/mm^2, may be calculated from the formula:

$$E_c = \bar{w}^{1.5} \cdot 0.044\sqrt{f_c'}; \qquad (1440 \leqslant \bar{w} \leqslant 2480); \qquad (2.1)$$

where $\bar{w}$ is the mass of concrete in kg/m^3 and f_c' is the specified cylinder strength at 28 days in N/mm^2.

Poisson's ratio: The value of the Poisson ratio ν of concrete is usually taken as zero in the analysis of thin shells. In those cases where the effect of the Poisson's ratio is deemed significant, its value should be ascertained from experiment or, lacking experimental data, it may be taken as 0.20.

Variation of f_c' and E_c with time: Compressive strength and modulus of elasticity of concrete increases with time. The values shown in TABLE 2.1 represent range of experimental data obtained at 28 days, 1 year and 3 years.

TABLE 2.1 – RANGE OF ONE- AND THREE-YEAR RELATIVE INCREASE OF VALUES f_c' AND E_c

Relative Compressive Strength f_c'			Relative Modulus of Elasticity E_c		
28 days	1 year	3 years	28 days	1 year	3 years
1.00	1.2 to 1.4	1.26 to 1.48	1.00	1.40 to 1.53	1.40 to 1.58

Shrinkage: In the case of shrinkage properties, it is possible to generalize only within rather broad limits, and test data which incorporate the effects of local aggregates and conditions should be used where available. In the absence of such test data, the following shrinkage information may prove helpful. For design purposes, often the free shrinkage strain of hardened sand and gravel concrete (exclusive of plastic shrinkage) may be calculated from the formula:

$$\epsilon_{sh} = 0.0000125 \cdot (90-H); \qquad (2.2)$$

where ϵ_{sh} is the free shrinkage strain, and H is the relative humidity in percent.

Creep: The caveat on generalizations made in the preceding paragraph with regard to shrinkage applies also to creep properties of concrete. Accordingly, test data incorporating local conditions should be sought. In absence of such data, the approximate ultimate values of the creep coefficient for both normal weight and lightweight concrete under average design conditions given in TABLE 2.2 may be used. In this table, the smaller values tend to be applicable in all cases, except when the age of concrete at the time the sustained loads are applied is on the order of a few days to a week, or so. For load durations less than "permanent," the percentages of ultimate creep are approximately 12, 35 and 75 percent for load durations of 1 day, 1 month, and 1 year, respectively.

TABLE 2.2 – RANGE OF VALUES OF CREEP COEFFICIENTS C_u, RATIO OF ULTIMATE CREEP STRAIN TO INITIAL ELASTIC STRAIN

Concrete Strength	Average Relative Humidity			Average Values for Normal Conditions
	100%	70%	50%	
Ordinary ($20N/mm^2 \pm$)	1-2	1.5-3	2-4	2
High ($33N/mm^2 \pm$)	0.7-1.5	1-2.5	1.5-3.5	1.5

2.3.2. Reinforcing Steel

The maximum specified yield strength f_y of reinforcing steel should not exceed $420 N/mm^2$, unless allowable stress is suitably restricted to preclude large strains. The modulus of elasticity E_s of nonprestressed reinforcement may be taken at $210,000 N/mm^2$.

2.3.3. Prestressing Steel and Tendons

The tensile strength and the modulus of elasticity of prestressing steel and of the tendon assemblies should be determined by tests, or should be as prescribed by the applicable codes.

2.4. WORKING LOAD DESIGN (Strength and Serviceability)

2.4.1. Allowable Stresses

Allowable stresses in concrete and reinforcing steel used in strength design at working load should meet the provisions of the applicable regulations.

2.4.2. Allowable Displacements

Shell, edge members and stiffening members: In view of the great variety of the possible geometry of the shell surface and the arrangement of edge and stiffening members, a universally applicable prescription of allowable displacements does not appear practical. A number of factors should be considered by the designer when investigating allowable shell displacements. These factors include the desire to avoid unsightly sagging, the need to prevent cracking, the requirements of connections between the shell and non-structural elements such as window walls and finally, in the case of roofs, the need to ensure proper drainage. Additional comments on shell displacement limitations are contained in Chapter 3 of these Recommendations.

Supporting members: The displacement of supporting members should not exceed the limits of displacement of conventional structural elements set by the applicable regulations.

In general, the displacements of shells with substantial support along their periphery, such as domes, are seldom excessive. Shells more likely to develop significant displacements are the shells with free boundaries, or with boundaries at which only edge members or stiffening ribs occur, such as some types of hyperbolic paraboloids and cylindrical barrel vaults. This is particularly true in the case of large spans.

For long-term loads, creep and shrinkage may increase elastic displacements calculated at working load by a factor of 2 to 3; this effect should be recognized in the analysis.

Provisions should be made in attaching non-structural elements to shell structures to accommodate the anticipated displacements of the shell under live and long-term permanent loads.

2.4.3. Crack Control

Crack control can be aided by following the provisions discussed in Chapter 4 of these Recommendations, pertaining to proportioning of shell reinforcement. In general, the use of larger numbers of smaller sized bars with closer spacing to achieve the required steel area will be beneficial to crack control.

2.5. ULTIMATE LOAD DESIGN

Calculated ultimate load capacity of the shell and its auxiliary members should not be less than the load U calculated from the following formulae, where D is the dead load, L is the live load, W is the wind load, and E is the earthquake load.

Dead Load and Live Load:

$$U = 1.4D + 1.7L; \tag{2.3}$$

or

$$U = 0.9D + 1.7L; \tag{2.4}$$

Dead Load, Live Load and Wind Load:

$$U = 0.75\ (1.4D + 1.7L + 1.7W); \tag{2.5}$$

or

$$U = 0.9D + 1.3W; \tag{2.6}$$

Dead Load, Live Load and Earthquake Load:

$$U = 1.4(D + L + E); \tag{2.7}$$

or

$$U = 0.9D + 1.4E; \tag{2.8}$$

Effects of Creep, Shrinkage, Temperature and Motion of Supports:

These effects should be added to the effects of the Dead Load, and the load U should be calculated from the formula

$$U = 0.75\ (1.4D + 1.7L); \tag{2.9}$$

The value of the capacity reduction factor should be as prescribed by the applicable regulations, except that it should be taken as 0.70 for the calculation of the shell itself for in-plane compressive forces.

It should be noted that equations (2.3) through (2.9) are not unique; they are based on the publications of the American Concrete Institute. Similar equations contained in the applicable regulations should be used whenever available.

CHAPTER 3

ANALYSIS

3.1. INTRODUCTION

3.1.1. General Remarks

Analysis may be defined as that part of the design process during which the strength and serviceability of the shell together with its auxiliary members are investigated. Typically, the process is quite intricate, and it is made even more difficult by the non-homogeneity and anisotropy of the material. Further, it may be recalled that the response of a reinforced concrete structure is not uniform in that the pre-cracking (elastic), post-cracking (inelastic) and ultimate stages exhibit different behavior characteristics.

The purpose of analysis is to evaluate deformations and internal stresses in the shell. From a reasonable prediction of the intensity and distribution of internal forces the required thickness of concrete and the amount of reinforcing can be obtained, to ensure that the criteria of strength, serviceability, and stability are met, as follows.

Strength: The strength of the shell together with its auxiliary members should be such the stresses at working loads do not exceed the allowable stresses set in paragraph 2.4.1, and that the ultimate load capacity of the structure is greater than the load combinations listed in paragraph 2.5.

Serviceability: The displacements of the shell together with its auxiliary members, at working load, should meet the criteria discussed in paragraph 2.4.2.

Stability: The shell and its auxiliary members should remain stable, i.e., it should not experience large increase in deformations under small increase in applied loads. In addition, the shell should not be a mechanism, i.e., its geometry and manner of support should be such that significant changes of shape with but small changes in the magnitude of in-plane strains are precluded (see also a discussion of *inextensional bending* below).

3.1.2. Types of Analysis

Two general types of analysis may be distinguished.

Mathematical analysis: The shell is represented by a mathematical model derived from a rational set of assumptions consistent with the laws of structural mechanics, with the properties of materials, and with the observed behavior of structures. Any simplifying assumptions aimed at a reduction of the complexities of analysis should be consistent with the nature of the problem under consideration. The solution itself may be *analytical* or *numerical* in nature. An example of the latter is the computer-oriented finite element method. Mathematical analysis should be used in the design of all shell structures.

Experimental analysis: This type of analysis consists of measurements performed on a full size or scale model of the shell, constructed with due regard for the geometry of the shell and its supports, the nature of the materials and loads, and the laws of similitude. The model may represent either the prototype or the mathematical model. Experimental analysis should be used only as an aid in the derivation of a rational mathematical model, or as a check on the results obtained from mathematical analysis. With the advent of the powerful computer methods, model analysis lost some of its significance. Its primary field of application now appears to be investigation of shell behavior in the inelastic and ultimate phases where the corresponding mathematical analysis may not be practicable; it may be useful also in the conceptual design stage.

3.1.3. Checks

After the completion of analysis a statical check of equilibrium should be made at several sections of the shell selected for their overall importance to ensure that the internal forces obtained from analysis are in equilibrium with external forces. In the case of shells of unusual geometry and dimensions it may be desirable that the behavior of the shell as predicted by analysis be checked through the elastic, inelastic and ultimate ranges by means of an experimental study conducted on a properly scaled microconcrete model.

3.2. ANALYTICAL FORMULATIONS AND SOLUTIONS

3.2.1. General Remarks

A shell structure, however thin, is three dimensional. Therefore, the problem of shell analysis could be formulated as a problem of the three dimensional theory of elasticity, subject to prescribed geometry, loads, and material properties of the structure. However, even if the resulting system of equations were linear, shell analysis based on this approach would be prohibitively difficult and is not normally attempted. Thus it becomes necessary to make the problem more tractable through the introduction of some assumptions aimed at a simplification of one or more of its several aspects. The assumptions might be intended to simplify the formulations of the problem, to simplify its solutions (including numerical work), or to reduce the conceptual difficulties which may occur in shell analysis. In general, associated with each simplification is a loss of accuracy in the numerical solution.

Fundamental assumption of shell theory: The thickness of a shell is assumed small compared to its other dimensions. This assumption permits the reduction of the problem from three spatial dimensions to two, and allows the analyst to view the shell as a two dimensional elastic surface. There results a system of equations which, together with the equations of the boundary conditions, governs the behavior of shells. The system contains the *equations of equilibrium*, the *strain-displacements* relations, and the *constitutive relations* between stress resultants or couples, and the strains or changes of curvature of the middle surface. In addition, the latter must satisfy also a set of *compatibility equations* which reflect the condition that the shell, viewed as an elastic surface, experience no geometric discontinuities as a result of deformation. In an internally consistent shell theory, the equation which describes equilibrium of moments about the normal axis is identically satisfied. Further, a consistent shell theory exhibits a *static-geometric analogy*, in that the equations of equilibrium of forces from which the transverse shears have been eliminated with the aid of the remaining two equations of equilibrium of moments are analogous to the three equations of compatibility.

In addition to the fundamental assumption of the thinness of the shell, further simplifying assumptions can be made. Each additional assumption is associated with a new mathematical model and with a new governing system of equations. Ideally, it would be desirable to derive a single, universally accepted theory of shells valid for all types of shell geometry. This does not appear to be practical, however. The best approach may well be to consider each shell geometry and each type of load separately, using the simplest formulation appropriate to the given case.

3.2.2. Brief Review of Shell Theories

Several formulations of shell theory are briefly reviewed in the following.

First approximation theory: This is the simplest possible consistent shell theory. Initially proposed by Aron[1] (1874), a more complete version is due to Love,[2,3] (1888); for this reason it is often called *Love's first approximation.* Some inconsistencies in Love's formulation were corrected by Sanders[4] (1959), and independently by Koiter[5] (1959). Shell thickness is assumed

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[1]Numbers in superscript refer to similarly numbered references in paragraph 6.2 of Chapter 6, Selected Bibliography of Shells.

small compared not only to its other dimensions but also to the smallest radius of curvature of the undeformed middle surface. The effect of the transverse normal stress is assumed small compared to the effect of other stress components, and the shell is assumed to deform in such a way that straight lines normal to the middle surface before deformation remain unstretched. straight and normal to the middle surface after deformation *(Kirchhoff's assumption).* Four boundary conditions must be specified at the boundaries, involving combinations of stress resultants and couples. The first approximation theory does not predict correctly the behavior of shells near the singularities of load or boundary, much in the same manner as the Kirchhoffean theory of plates.

Higher approximation theories: Refined shell theories, presumably leading to more accurate results, can be obtained by re-examining one or more of the assumptions of the first approximation theory. In the *Flügge*[6]*-Lur's*[7]*-Byrne*[8] (FLB) theory the thickness to radius of curvature ratio is not assumed small compared to unity, (although the square of this ratio is dropped from the equations). Although this removes some inconsistencies from the Love formulation, boundary conditions are still the same as those of the first approximation, and the theory still fails at the singularities of load and boundary. The FLB formulation is significantly more complex and analytical solutions are significantly more difficult to obtain. In the *Reissner*[9] theory, in addition to the relaxation similar to that of the FLB formulation, the shell is assumed to deform so that lines straight and normal to the middle surface before deformation remain straight but not necessarily unstretched or normal to the middle surface after deformation. The resulting equations require prescription of five boundary conditions in terms of the five stress resultants and couples. This permits obtaining solutions near singularities of load and boundary.

Commonly, the analysis of reinforced concrete shells is performed on the basis of the linear first approximation theory or some simplified version of it.

3.2.3. Koiter's Classification of Approximate States of Stress in Shells.

The equations of shells, even in the relatively simple formulation of the first approximation theory, are quite involved and solutions of specific problems are difficult to obtain. Possible exceptions to this are shells of particularly simple geometry, such as spheres and some other shells of revolution. The difficulty in obtaining analytical solutions becomes even greater in the case of refined theories. To make the solutions easier to attain, additional approximations are evidently required.

Several such approximate shell theories have been devised. Among them, of particular interest are approximate theories of general type, i.e., theories applicable to shells of any geometry. Their importance is due also to the fact that they shed a light on the manner in which shells carry loads and deform.

A systematic classification of approximate states of stress in shells is due to Koiter.[10] The classification is based on two parameters. One of these is the ratio of flexural strain ϵ_b at the extreme fiber to the extensional strain ϵ_m of the middle surface, each calculated in the shell region of interest. The other parameter is the ratio $\lambda^2/(hR)$ where λ is the "wavelength" of the pattern of deformation of the shell, h is the shell thickness, and R is the minimum principal radius of curvature of the shell; all these quantities are calculated in the same region of the shell. The order of magnitude of the two parameters may be significantly less than, of the order of, or significantly greater than unity. Accordingly, nine possible cases of the state of stress in the shell region under consideration may be distinguished. These are listed in Table 3.1.

TABLE 3.1 — KOITER'S CLASSIFICATION OF APPROXIMATE STATES OF STRESS IN SHELLS			
	$\lambda^2/(hR) << 1$	$\lambda^2/(hR) \approx 1$	$\lambda^2/(hR) >> 1$
$\epsilon_b/\epsilon_m << 1$	plane stress	mixed	membrane
$\epsilon_b/\epsilon_m \approx 1$	mixed	shallow shell	mixed
$\epsilon_b/\epsilon_m >> 1$	bending of plates	mixed	inextensional bending

Of the nine states of stress listed in Table 3.1, three are of particular theoretical and practical interest. They are the shallow shell, membrane and inextensional bending states of stress, discussed in more detail in the following paragraphs.

3.2.4. Shallow Shell Theory

The importance of the theory in practical applications can be anticipated from its central position in Koiter's classification. Its development is usually associated with the names of Donnell,[11] v. Karman,[12] Jenkins,[13] and Vlasow.[14]

Assumptions: Assumptions additional to those of the first approximation theory are that the contribution of transverse shears to the in-plane force equilibrium, and the contribution of in-plane components of displacement to the changes of curvature of the middle surface are negligible.

Equations: Fundamental features of the first approximation theory are retained: the system is of eighth order, with both bending and in-plane effects being present; prescription of four conditions at the boundary is required. A significant simplification over the first approximation formulations is possible, in that all shell quantities can be expressed in terms of just two variables, the Airy stress function Φ and the normal displacement w. In absence of in-plane applied loads the equations, written in intrinsic coordinates of the shell surface, are:

$$\Delta\Delta\Phi + C\Delta_k w = 0; \qquad (3.1)$$
$$K\Delta\Delta w + \Delta_k \Phi = p;$$

where K is the flexural stiffness of the shell, p is the applied normal load, C is a parameter proportional to the extensional stiffness of the shell, Δ the Laplacian, and Δ_k is a second order partial differential operator which depends on the geometry of the middle surface, contains shell curvatures, and may be elliptic, parabolic or hyperbolic. In the special case of a flat plate this operator becomes identically zero and equations (3.1) reduce to the well known equations of *plane stress* and *bending of plates*. The structures of equations (3.1) makes their solution simpler than that of the first approximation theory. Boundary conditions are the same as in the case of the first approximation theory.

Range of validity: The shallow shell theory fails near singularities of load or boundary. A further loss of accuracy results from the two additional assumptions on which the theory is based. In particular, small rigid body motions result in physically inadmissible strains. Nevertheless, the range of validity of the shallow shell theory appears to be quite wide, particularly for shells of practical significance. In case of doubt, the validity of predictions of the theory should be tested by substituting the results into the more accurate equations of the first approximation theory, thus obtaining an estimate of error.

3.2.5. Membrane Shell Theory

Assumptions: The assumption additional to those of the first approximation theory is that resultant couples are small compared to in-plane forces. This is equivalent to taking flexural stiffness of the shell as vanishingly small compared to its extensional stiffness, or to taking the changes of curvature of the middle surface as small compared to its in-plane strains.

Equations: Accordingly, membrane theory equations can be obtained directly by setting K = 0 is the second of equations (3.1); the result is

$$\Delta\Delta\Phi + C\Delta_k w = 0; \quad \Delta_k\Phi = p; \tag{3.2}$$

The system is of fourth order — a reduction from the eighth of the first approximation theory. It is seen that integration can be performed in stages, first obtaining the Airy stress function Φ from the second equation, and then the normal displacement w from the first equation (3.2). The membrane is internally statically determinate, in complete analogy to the technical theory of beams.

The number of conditions which may be prescribed at the boundary depends on the nature of the operator Δ_k (but is never more than two). If Δ_k is elliptic, two conditions must be prescribed. If it is parabolic, one condition must be prescribed. Finally, in the case of a hyperbolic operator (shells of negative Gaussian curvature), two conditions must be prescribed. However, in general, a membrane shell of zero or negative Gaussian curvature is incapable of satisfying prescribed conditions all along its boundary. Literature should be consulted for a complete discussion of this problem.[15]

Range of validity: The assumption of vanishing stress couples carries with it a limitation on the type of load which may be applied to a membrane shell. Concentrated loads normal to the shell must be excluded, since their presence would imply a zone of bending at least near the vicinity of the point of application of the load.

Membrane theory predicts the behavior of shells with reasonable accuracy if the manner of support of the shell is compatible with the permissible boundary conditions. If this condition is not met, or if normal concentrated loads occur, membrane theory fails near the so-called *lines of distortion,* i.e., near the boundaries or near the lines of discontinuity of the shell or its loads. Starting from a line of distortion, the width of the zone where the membrane theory does not apply depends primarily on the geometry of the shell, and on the nature of the distortion. In shells which occur in building construction, the most important line of distortion is the edge of the shell.

The edge effect: It appears that for a relatively large class of shells, including shells of revolution, the zone where the membrane theory does not apply is relatively narrow, and the necessary analysis of bending in that zone can be performed with sufficient accuracy through an approximate analysis of the *edge effect.* The governing equation of the edge effect can be derived through an asymptotic procedure. The equation is[16]

$$\frac{\partial^4 w}{\partial \alpha_1^4} + \beta_1(\alpha_2)\, w = 0; \tag{3.3}$$

where w is the normal component of displacement of the middle surface, α_1 and α_2 are shell coordinates, and $\beta_1(\alpha_2)$ is a parameter which depends on shell geometry and is assumed (approximately) to be a function of only one of the two shell coordinates, α_2.

Membrane theory, together with the edge effect, is of significant value to designers and analysts. With its help, a formidable analytical problem is reduced to manageable proportions. The assumed manner of transfer of forces by means of in-plane stress resultants is consistent with the desirable behavior of shells which make them an efficient structural system — a shell designed to satisfy membrane boundary conditions will certainly prove a satisfactory structure. Finally, the solution of the membrane theory often constitutes an approximate particular integral of the bending theory, thus being an important first step in obtaining the complete solution of a shell problem.

3.2.6. Inextensional Bending

A shell is said to be in an inextensional state of stress when changes of curvature may occur without appreciable changes in in-plane strains. Accordingly, the assumption can be made that tangential strains, and hence also the in-plane forces, all vanish. It follows that the term $\Delta\Delta\Phi$ also vanishes in the first of equations (3.1), and the inextensional bending state of stress is governed by the equation

$$\Delta_k w = 0; \qquad (3.4)$$

Since the operator in equation (3.4) is the same as that which governs the behavior of membranes, the discussion of the boundary conditions to be placed in this case is the same as the corresponding discussion in paragraph 3.2.5.

An example of inextensional bending is afforded by a cylindrical shell supported along two directrices. However, the same cylindrical shell supported also along the two end generatrices will experience a change in in-plane strains, and the inextensional bending state of stress will not arise.

From the point of view of a shell designer, the importance of inextensional bending lies in the fact that in this state of stress the shell behaves in an undesirable manner, resisting loads through bending instead of in-plane action, and experiencing significant deformations. Accordingly, the design should be checked to make sure that the geometry of the shell, and the manner in which it is supported, preclude the inextensional bending state of stress from arising, particularly as regards the deformations.

3.2.7. Approximate Shell Theories of Specific Type

The shallow shell, membrane, and inextensional bending formulations discussed in the preceding paragraphs may be considered approximate shell theories of general type since, within the limits of their approximations, they apply to shells of all geometries.

It is possible on occasion to construct an approximate shell theory valid only for a limited class of problems. The nature of the additional assumptions introduced into the equation of the more general theory depends on the nature of the problem. Several examples of such approximate formulations are discussed in the following.

Approximate geometric theories: The geometry of the shell is approximated by a new geometry, possibly by assuming that it consists of a number of segments of simple surfaces. In the limit, as the size of segments becomes smaller, the assembly tends to the geometry of the prototype. As an example, a cylindrical surface may be approximated by a series of flat plane segments, and a surface of revolution by a series of conical frusta. Intuitively, it seems plausible to expect that the solution based on the approximate geometry will tend to the true solution based on that of the prototype. The obvious practical condition for the usefulness of this approach is that the solution based on the approximate model be simpler to obtain than the solution based on the true geometry.

Approximate static theories: For a shell of given geometry it may be possible to make additional assumptions regarding the relative magnitude of the various stress resultants and couples, with the corresponding simplifications of the governing system of equations. An example of this type of approximation is the semi-membrane theory of cylindrical shells.[17]

Approximate mixed theories: For a shell of given geometry it may be possible to make additional assumptions regarding the manner in which the shell will deform, as well as regarding the relative magnitude of the various stress resultants and couples. An example of this type of approximation is the beam theory of cylindrical shells, and the corresponding theory of prismatic folded plates.[18]

The range of validity of the approximate shell theories of specific type should receive careful consideration in each case. An estimate of the error can be obtained by substituting the results into the system of equations of a less restrictive formulation.

3.2.8. Folded Plates

The geometry of folded plates — an assembly of planar elements — introduces added simplification into analysis. Since the elements are planar all curvatures vanish, and the operator Δ_k is identically zero everywhere. Equations (3.1) become

$$\Delta\Delta\Phi = 0; \qquad K\Delta\Delta w = p; \tag{3.5}$$

where all symbols are as defined in paragraph 3.2.4. Equations (3.5) are recognized as those of plane stress and of Kirchhoffean bending of plates, the two modes of behavior no longer coupled. Solution of equations (3.5) can be obtained analytically only in some simple cases, such as the case of prismatic folded plates simply supported at the end generatrices, when the Levy-type solution is effective. Usually, recourse has to be made to an approximate method of analysis, or to a computer-oriented numerical procedure.

Prismatic folded plates: This type of folded plate structure occurs most often in practical applications. Two basic methods of analysis can be distinguished (although many variants of these methods exist in literature). The *beam method* assumes that the folded plate behaves like a beam. This presupposes that the cross-section of the folded plate does not change shape during deformation and rigid body motions are the dominant part of its displacements. This assumption makes the beam method valid for long folded plates only, i.e., those with the span significantly greater than the cross-sectional dimensions. The method can be applied to folded plates of all types of transverse sections, but numerical work is not simple when the cross-section is not symmetric. In the *ordinary theory,*[19] the cross-section of the folded plate is permitted to change shape during deformation under loads. Accordingly, its range of validity encompasses somewhat shorter folded plates.

Nonprismatic folded plates: Analysis becomes significantly more complex in the case when the folded plate is nonprismatic. For this reason, approximate methods of analysis established as firmly as those used in the case of prismatic folded plates are lacking. In the *extended ordinary theory*[20] the analysis proceeds as in the case of the ordinary theory, although it is more complicated due to the geometry of the structure. In a further modification of the procedure, transverse and longitudinal actions, including the effect of relative translation of joints, are matched simultaneously at a number of sections along the span. The range of validity of this modified formulation has not been fully established.

Faceted folded plates: No systematic methods for the analysis of faceted folded plates appear to exist. Recourse has to be made to numerical methods, of which the finite element method appears most suitable, since uncoupled planar finite elements represent the geometry of a faceted folded plate exactly.

3.2.9. Some Comments on Analytical Solutions

Because of the highly complex nature of the governing system of equations of shells the problem of obtaining solutions is typically very difficult. A discussion of the many techniques for arriving at the solution of a given boundary value problem is outside the scope of these Recommendations. It seems appropriate to note that, due to the wide availability of computers and packaged programs suitable for solving problems of shells, the importance of analytical solution methods *as a design tool* has greatly diminished, numerical methods having essentially replaced them for this purpose. Nevertheless, analytical procedures remain significant in that they provide the necessary insights into the *qualitative* behavior of shell structures.

General character of solution: Integrals of the governing homogeneous system of equations can be written approximately, in an asymptotic sense, in the form

$$f = \exp(\pm\beta_2\alpha)\,{}^{\sin}_{\cos}(\beta_3\alpha); \tag{3.6}$$

where f is any of the shell quantities sought, β_2 and β_3 are shell parameters, and α is a spatial coordinate. It is seen that the solution has the form of an exponentially decaying or growing wave.

Solution aids: For a number of shell geometries and loads, solution aids can be found in literature. Such aids are in the form of closed form solutions, and tabular and graphical representations of various shell quantities of interest in design. Some of these are listed in Chapter 6 of these Recommendations.

3.3. NUMERICAL METHODS

3.3.1. General Remarks

Since analytical solutions of the governing system of equations of shells are difficult to obtain, often the only practical approach is to resort to a numerical procedure. Two general categories of numerical methods may be distinguished.

Classical numerical methods:[21] These are procedures developed in the pre-computer era. Among them are the step-by-step integration procedures, and the finite difference method in its various forms, often combined with the use of a relaxation method. A number of the classical procedures have been adapted to the computer, often with great success.

Computer oriented methods: These are procedures which were developed largely because of the availability of the electronic computer. Examples are the finite difference energy method and the finite element method. It must be realized that the importance of these methods goes beyond the extremely rapid computational abilities of the computer.

Due to the prodigious development of the computer and of the numerical methods associated with its use, only a very brief discussion of this topic will be included here. For a detailed treatment the rapidly growing literature of the subject should be consulted.

3.3.2. Numerical Integration of Shell Equations[22]

This is a classical numerical integration procedure which has been adapted to the computer. It is applicable to those shell problems which are reducible to the form of an ordinary differential equation, such as shells of revolution. The boundary value problem is converted into an initial value problem in terms of a system of first order equations. The system is then integrated numerically using some suitable algorithm, such as that of the *Runge-Kutta method* (RK), usually in conjunction with the matrix transfer technique. The RK algorithm is self starting in that the value of the function at the next step is obtained from the value of the function at the current step. If desired, a refined algorithm can be used which utilizes the value of the function and/or its derivatives at several prior points. In addition, an iteration can be performed at every step to improve accuracy. Because of the exponential form of the solution, as discussed in paragraph 3.2.9, round-off error may require subdivision of the problem into several subproblems, which increases the amount of numerical work.

3.3.3. Classical Finite Difference Procedures[23]

The finite difference method (FD) is a discretization procedure in which the field problem of a continuum is converted into a discrete problem through the introduction of a system of nodal points so that the system of differential equations is converted into a system of algebraic equations in the values of the unknowns of the problem at the nodal points. The procedure requires that the value of a derivative of a function at a nodal point be expressed in terms of the values of the function itself at that nodal point, and at several neighboring points. The error of the method is due to the approximate nature of the resulting expressions. Accuracy can be improved by reducing the length of the interval between nodal points. A further loss of accuracy results from numerical differentiation. For this reason, the governing system of equations should be written in terms of equations of the lowest order possible: a system of eight equations of first order. However, this increases the number of the resulting algebraic equations. In the case of bending of shells, the usual approach is to use the basic system which consists of the first three equations of equilibrium written in terms of the components of displacement of the middle surface. To avoid certain inconsistencies, the equation which involves even order derivatives should be written at the regular nodal points, while the equations which contain odd order derivatives should be written at half-spacing nodal points.

Several alternate approaches to the formulation of the algebraic equations at the boundaries are possible. The approach which preserves the same accuracy as that at the interior points is to write at each point of the boundary the group of equations which is applicable at an interior point, and in addition the appropriate equations of the boundary conditions converted to the finite difference form. This requires the introduction of fictitious points outside the boundary of the shell, and increases the total number of the algebraic equations in the system.

In order to improve accuracy, an extrapolation procedure may be used, based on two or more solutions obtained for different sized nodal point grids. Richardson's extrapolation is an example of such a procedure; however, it should be used with caution, since it improves results reliably only in the case of monotonic convergence.

Finite difference procedures are capable of accommodating irregular grids, and boundary points located not on the grid of nodal points. However, this can be accomplished only at the cost of considerable increase in numerical work.

Several formulations of the finite difference procedure can be distinguished. The *standard FD formulation* utilizes terms such that the error is proportional to the square of the nodal point interval. Richardson's extrapolation procedure applies in this case. *Higher order FD formulation* utilizes formulae with an error proportional to the fourth power of the nodal point interval. At the boundaries, forward or backward FD expressions have to be used, resulting in decreased accuracy and an increase in numerical work; Richardson's extrapolation may not apply. Finally, the accuracy may be increased without increasing the number of nodal points through the use of the *Hermitian FD procedure* (French: *methode plurilocale;* German: *Mehrstellenverfahren);* error for the same number of points as in the standard FD procedure is proportional to the fourth power of the nodal point interval. The essence of the formulation lies in its utilization of the differential equation of the problem itself, in addition to the relations between the function and its derivatives, at each nodal point. The convergence of the Hermitian FD method is not necessarily monotonic, and Richardson's extrapolation may not apply. A special case of the procedure is *the method of lines* which consists of separating variables in a partial differential equation in such a way that these equations are satisfied simultaneously along several (typically three) lines. The Hermitian FD method is known in literature also as *the funicular polygon method.*

3.3.4. Finite Difference Energy Method[24]

This method is a numerical procedure which belongs to the category of variational methods in that it makes use of the variational principle of the minimum total potential energy of the system. All quantities which enter into the calculations are discretized through the introduction of appropriate FD expressions. In the calculation of the total strain energy, and the potential of the work done by external forces, integration is replaced by summation. Once the expression for the total potential energy of the system in terms of nodal displacements is obtained, it is extremized with respect to these unknown displacements, resulting in a system of algebraic equations. The method is applicable to linear as well as non-linear problems. Although this formulation of the FD energy method was introduced prior to the advent of computer, it received its widest application in computer-oriented solutions. Two computer programs, BOSOR and STAGS, the latter of particular power and generality, are based on the FD energy method; refer also to paragraph 3.3.6 below.

3.3.5. Finite Element Method[25]

The finite element method (FEM) is a discretization procedure in which the structure is divided into a finite number of component elements connected at a finite number of nodes and acted upon by a load system equivalent to the original load. The manner in which each element transmits loads and deforms, i.e., its elastic properties, is assumed known. This being the case, the solution proceeds using well established methods of structural mechanics cast in matrix form well suited to the use of digital computers. The displacement method is most commonly used, in which case the elastic properties of each finite element are described by its stiffness matrix. Thus the power of the finite element method lies in it generality, in that exactly the same procedure, utilizing the same computer program, can be employed to analyze structures of all kinds and of all geometries, subject to any load conditions.

In the case of structures composed of simple finite elements, such as a skeletal shell which consists of one-dimensional beam-like elements, the elastic properties of the component members are known exactly (to within the accuracy of the theory of beams), and the final element solution is the same as the classical solution. In the case of general shells, elastic properties of component elements considered as shell segments are very difficult to establish. For this reason an approximation is introduced. The usual procedure is to assume a pattern of deformation for each element, and to derive the elastic properties of the element from this pattern. In this sense, the finite element method may be considered a special case of the variational Ritz method.

Several sources of error exist in the finite element method as applied to shell analysis, apart form the approximations introduced by the shell theory itself. The elastic properties of the elements are approximate, and there is also the geometrical approximation involved in representing a shell as an assembly of elements. Intuitively, it seems plausible to assume the the solution resulting from the use of the finite element method converges to the "exact" solution as the size of the elements becomes smaller. One of the important aspects of the use of a variational procedure in deriving the elastic properties of finite elements lies in the fact that, if the condition of minimum potential energy of the system is met, monotonic convergence of solutions resulting from a consistent reduction in element size to the exact solution is assured.

To satisfy the minimum energy condition, the assumed displacement pattern of the element must satisfy several conditions. The continuity of displacements and rotations between elements must be preserved; the pattern of displacements must be capable of accommodating rigid body motions; finally, the constant strain states must be included. It appears that violations of the condition of continuity of inter-element displacements, by itself, need not preclude convergence, although monotonic convergence is no longer assured. Similar conclusion appears to hold if rigid body motions are not accommodated, although in each of these cases the rate of convergence may be slowed down. The violation of the condition that constant strain states be included may lead to erroneous results, in that convergence to an incorrect result may occur.

The shape of the finite element itself may vary depending on the nature of the problem under consideration. The element may be flat or curved, and it may be one or two dimensional. The elastic properties associated with the membrane state of stress and with the bending state of stress may be coupled or uncoupled.

As mentioned at the beginning of paragraph 3.3.5, the power of the finite element method is largely due to its generality, in that many types of structures can be analyzed using the same procedure. This characteristic makes FEM particularly suited to the analysis of shell structures, since elements representing the shell or wall or edge beams, supporting elements, etc. can be accommodated with equal facility.

One dimensional elements: As mentioned above, one-dimensional beam-like elements occur in the problems of skeletal shells. In addition, any continuum shell may be represented in an approximate fashion as a network of one-dimensional elements, in effect as a skeletal shell. In this case, the elastic properties of such elements should be derived through some rational process based on the laws of structural mechanics. Apart from these cases, many shells can be reduced to one dimensional; examples are shells of revolution and cylindrical shells. For cylindrical surfaces the element may be flat (a rectangle, usually elongated; this model represents exactly a prismatic folded plate, and approximately a curved cylindrical shell,) or it may be curved. Elements which exist in literature are segments of circular cylinders, with the stiffness matrix that of a shallow shell simply supported at the ends. Specialized computer programs based on this approach are available. For shells of revolution the corresponding flat element is a frustum of a cone, and the corresponding curved element is a frustum of a shell of revolution such that the geometry of the shell is fully preserved, including the curvatures at the nodal points. In each case the stiffness matrix is based on the prototype shell. Computer programs based on this model are available in literature.

Two dimensional elements: These are elements applicable to shells of any geometry. As in the case of one dimensional elements, there are flat and curved elements. Plan forms include triangles, quadrilaterals and other elements. Isoparametric elements are available in the form suited for analysis of moderately thick shells. Many different types of elements are available in literature, based on different functional representations of element displacements. In general, the higher the order of the element, i.e., the greater the number of degrees of freedom assigned to it, the larger is the size of the stiffness matrix of the structure. It appears that for problems of shells which occur in building construction flat elements of quadrilateral type, with twenty degrees of freedom per element, offer sufficient accuracy, convergence and a reasonably small computer cost.

3.3.6. Computer Programs*

A number of computer programs capable of analysis of complex shell structures have been implemented on commercially accessible computers. These programs are based usually on the finite element method, but also on the finite difference energy method and on the classical numerical integration procedure, particularly in connection with the analysis of shells of revolution. Some of these programs are: ANSYS (Swansons Analysis Systems, Inc.), ASKA (University of Stuttgart), BOSOR 4 (Lockheed Missiles and Space Company), EASE 2 (Engineering Analysis Corporation), ELAS (Duke University), MARC (MARC Analysis Research Corporation), NASTRAN (University of Georgia), SAP IV and NONSAP (University of California, Berkeley), STAGS (Lockheed Research Laboratories), and STRUDL II (ICES User's Group).

Computer programs and languages are subject to changes as rapid as the changes in the available equipment, both in the Central Processor Unit and in the peripherals. For this reason, the user should become throughly familiar with each program and language to be used, with its capabilities and limitations, and with the limitations of the method of analysis and the theory on which the given program is based. The importance of such familiarity cannot be overemphasized.

3.4. NONLINEAR BEHAVIOR OF SHELLS

3.4.1. General Remarks

As stated in paragraph 3.2, the design of reinforced concrete shells is based usually on a linear elastic analysis in which the structure is assumed uncracked, homogeneous, isotropic, linearly elastic and subject to small displacements. Section and material properties used in the analysis are based on the gross uncracked sections, the effect of reinforcement being neglected. The internal forces and moments found from this analysis are then used to determine the required concrete dimensions and reinforcement for the shell and its auxiliary members. This approach is satisfactory for most design problems.

In some cases, however, the behavior of the shell can be described with sufficient accuracy only if a refined model is used, based on the specific material properties, or on the nonlinear behavior of the shell.[26]

The problem of anisotropy is essentially the same as that which occurs in other reinforced concrete structures. In the following, the problem of nonlinear behavior is briefly discussed.

*Refer to paragraph 6.3.7 for a partial listing of computer program references.

3.4.2. Nonlinearities in Shells

Nonlinearity of the mathematical model which describes the behavior of a reinforced concrete shell may arise from two principal sources. These are material nonlinearities and geometric nonlinearities.

Material nonlinearities: In this case, nonlinear behavior is due to the cracking of concrete, due to the nonlinear stress-strain relations for concrete, steel, bond and aggregate interlock, or due to time dependent effects such as creep, shrinkage, temperature, and load history. Some recent nonlinear analyses have incorporated some of these effects, with the aid of the finite element method, in order to trace the response and crack propagation of a reinforced concrete shell through the elastic, inelastic and ultimate ranges. Two different approaches to modeling reinforced concrete shell material have been used. The first is based on an empirical relationship, with changing flexural and extensional stiffnesses assumed for the total shell thickness for different levels of loading and cracking. The second is based on dividing the shell thickness into several concrete and reinforcement layers so that the state of stress in each layer can be traced through the entire load history, allowing for the cracking of concrete, yielding of reinforcement, creep, shrinkage, and temperature changes. In both methods, the analysis is carried out by dividing the loading into a finite number of increments. For each load increment iterations are performed until the equilibrium and constitutive relations are satisfied to within some prescribed limits appropriate for current material and cracking states. Analyses of this type are useful at this time primarily as a research tool, since the required computer time is vastly greater than that required by the usual linear analysis used in design.

Geometric nonlinearities: Nonlinear behavior is due to large displacements, and due to large strains and changes of curvature of the middle surface. Recent advances in numerical methods based on the computer make possible an evaluation of these effects.

3.5. STABILITY ANALYSIS

3.5.1. General Remarks

Analysis of stability of traditional structural systems consists of the determination of the lowest value of the applied load at which the onset of buckling takes place. Such analysis is based on the linear buckling theory involving small displacements. In the case of a number of types of shells such analysis is not sufficient because the post buckling behavior has an important effect on the magnitude of the buckling load, an effect which the small displacement theory is incapable of predicting.

As a thin shell deforms under load, principal in-plane forces develop. If one of these forces is tensile, it tends to return the shell back to its original position, which generally enables the shell to carry in the post buckling range loads greater than the buckling load predicted by the linear small displacement theory. If, however, both principal in-plane forces are compressive, they tend to increase the deformation of the shell. In this case, in the post buckling range after the initial buckling load has been reached, the shell can transmit only loads smaller than the buckling load predicted by the linear theory. These two types of behavior are illustrated qualitatively in Figure 3.1.

Therefore, the first step in the analysis of stability of a shell is to calculate the initial buckling load (or critical load) predicted by the linear, small displacement theory; next, this critical load is modified as required to allow for the given case of geometry, type of support, type of load acting on the shell, and the consequent type of shell response. The basic modification, in effect, introduces into the calculation the influence of large displacements on the magnitude of the buckling load.

In addition, modifications should be made to take into account the material properties of concrete, the effect of reinforcement, and the deviation of the shell geometry as built from the idealized shape.[27]

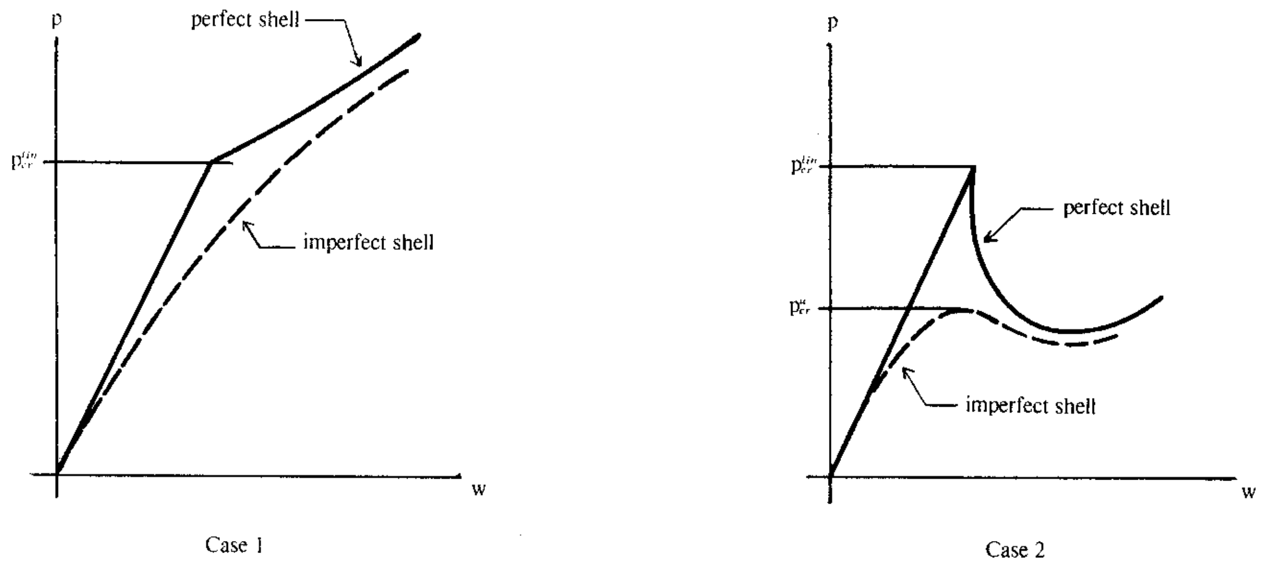


Fig. 3.1—Shell behavior in postbuckling range

3.5.2. Case 1: Post Buckling Behavior Does Not Govern

In the case when post buckling behavior does not govern, bucking load may be calculated using the linear, small displacement theory in which the shell is assumed to possess an idealized geometry. Material and sectional properties are based on uncracked and non-reinforced concrete. The value of this idealized linear critical load p_{cr}^{lin} is available in literature for a number of types of shells and applied loads.

No modification of the linear load need be made to account for the imperfections of the shell geometry as built. Modifications of the linear critical load due to other effects should be made as described in paragraph 3.5.4.

The linear buckling load should be used only when, on the basis of established analytical or experimental results, the shell is known to experience no decrease in the value of the initial buckling load in the post buckling range.

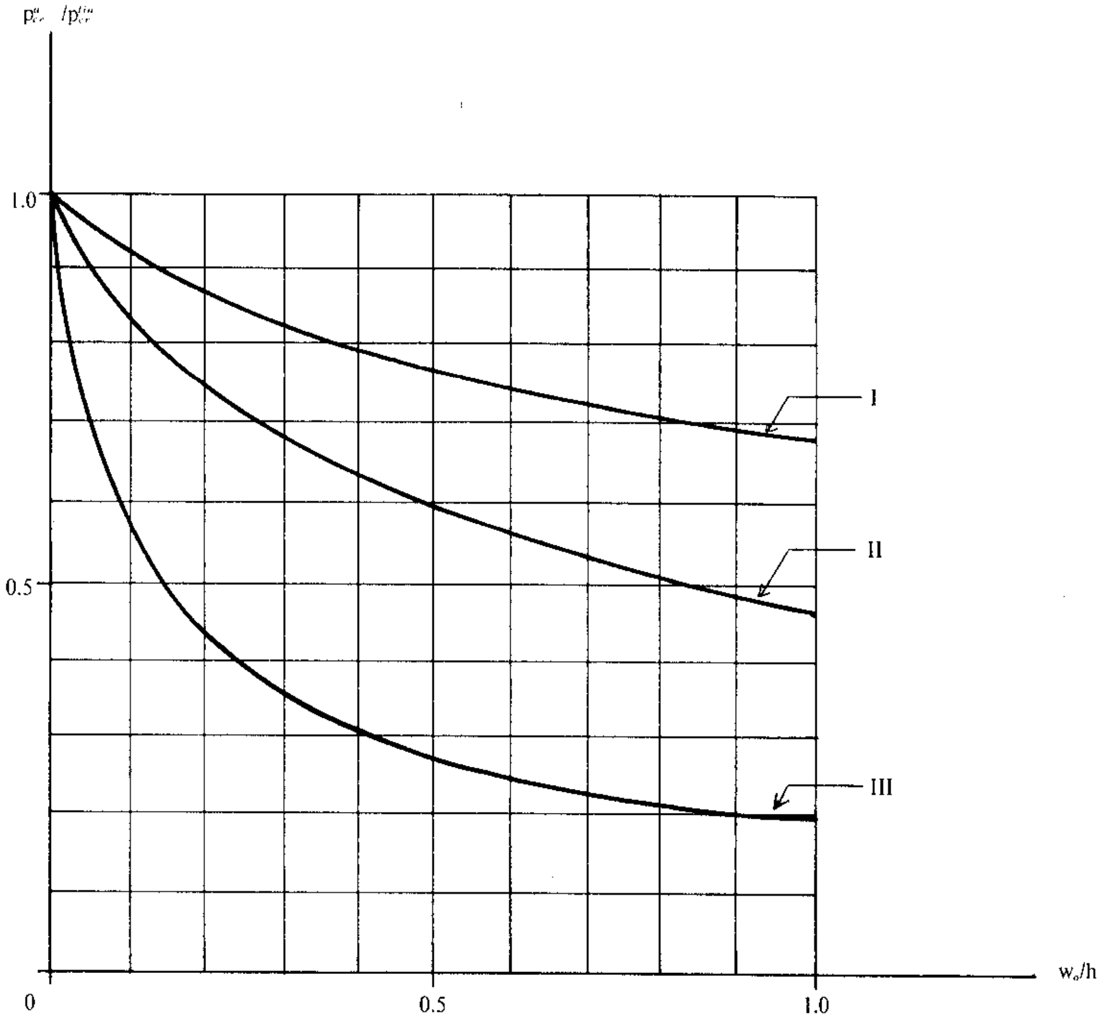
3.5.3. Case 2: Post Buckling Behavior Governs

This approach to stability analysis should be used whenever the shell geometry or its load are known to result in a decreased value of the initial buckling load, or whenever firm evidence to the contrary is lacking.

The linear buckling load p_{cr}^{lin} is used as the starting point, and the effect of large deflections is taken into account as follows. The deflection w_o of the shell is computed (or ascertained by other means, i.e., from the results of experiments). Next, the ratio of this deflection to shell thickness, or w_o/h, is calculated, and the reduced value of the upper critical load p_{cr}^{u} is obtained from Fig. 3.2. Graphs similar to that in Fig. 3.2 are available in literature for a number of shell geometries and configurations of the applied load. It is recommended that, in absence of other information, the lowest of the curves in Fig. 3.2 (i.e., the curve applicable to an axially compressed cylinder) be used.

3.5.4. Shape and Material Modifications

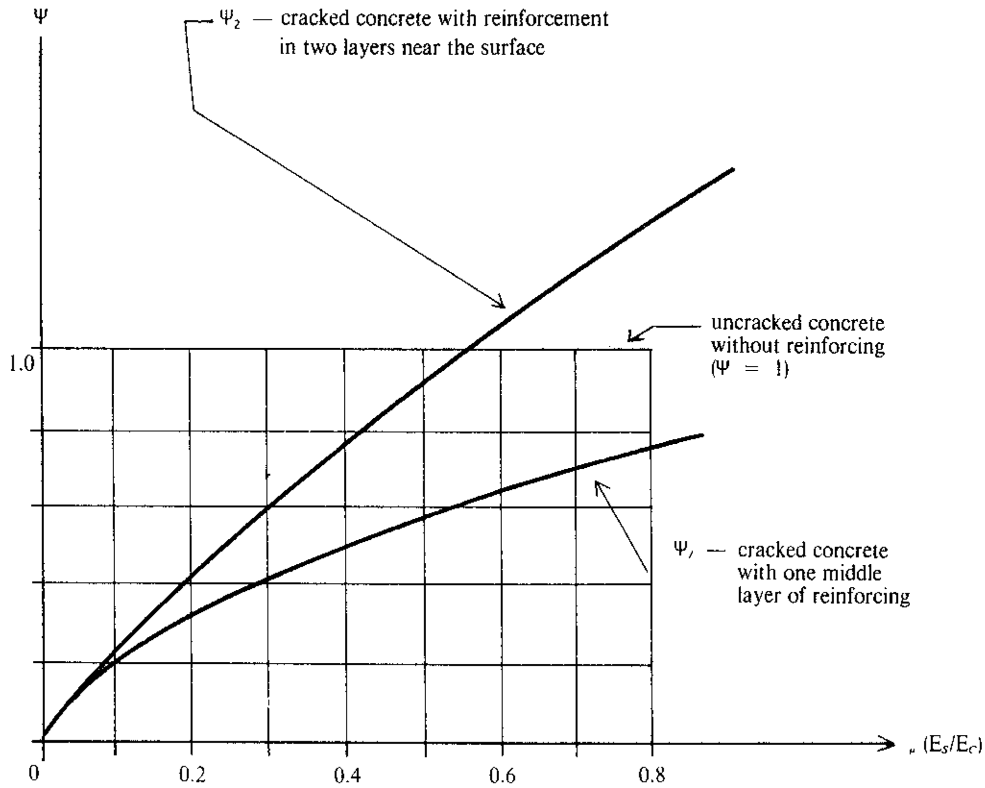
Geometric imperfections: The geometry of the shell as built differs, as a rule, from the idealized geometry postulated in the design. In the zones where the shell is flatter than the design shape, in-plane forces larger than the design forces will occur. As a result, for shells such that the post buckling behavior is important, a reduction in the value of the initial buckling load will occur. The magnitude of the reduced buckling load can be evaluated by first computing, or assuming, the value of the deviation w_o'' of the shell shape as built from the design shape, and by proceeding as if this were the deflection of the shell. The ratio w_o''/h is again formed, and the reduced value of the buckling load is obtained with the aid of Fig. 3.2. As before, the lowest curve in the figure should be used, unless specific information for the given shell geometry is available. It should be noted that if both w_o' and w_o'' terms are present then, because of the nonlinear nature of the problem, the above reduction procedure should be performed simultaneously.



Nomenclature: L_{ℓ} — length of cylinder
R_{ℓ} — radius of cylinder
h — shell thickness

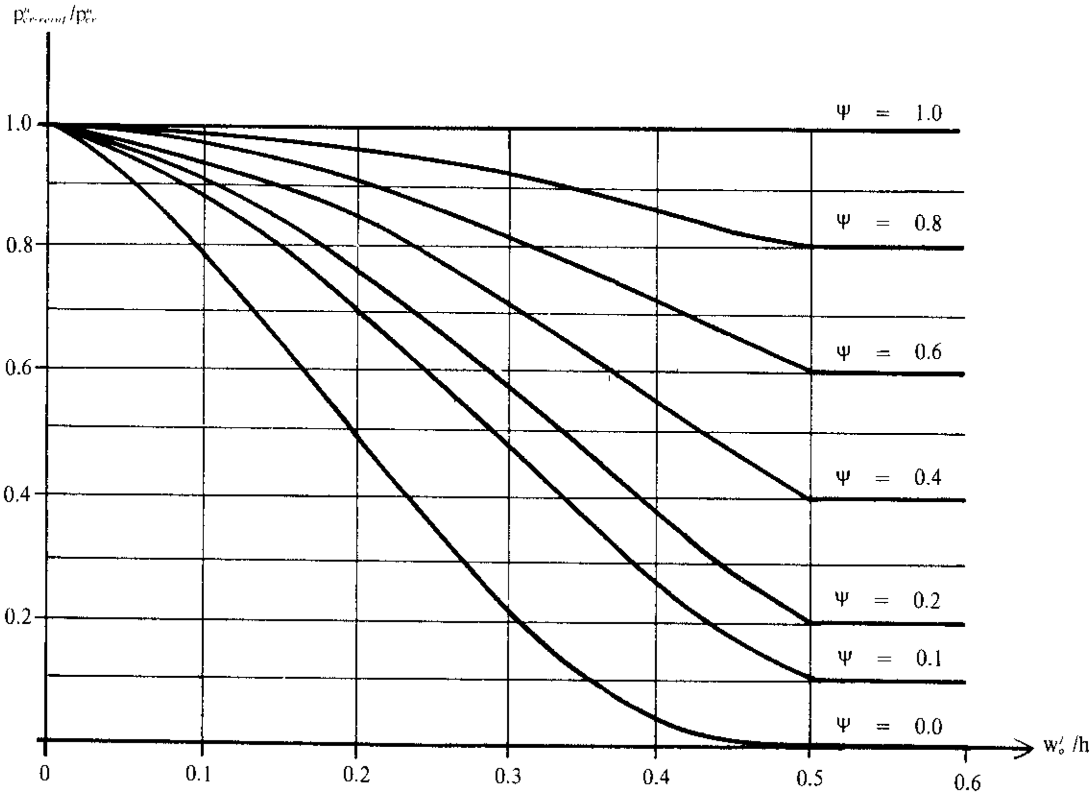
Curves: I — long cylinder ($L_{\ell}^2/R_{\ell}h$ = 1000), compressed in ring direction
II — short cylinder ($L_{\ell}^2/R_{\ell}h$ = 100), compressed in ring direction
III — sphere under radial pressure, and axially compressed cylinder

Fig. 3.2 — Upper critical load p_{cr}^{u} (based on Kollar & Dulacska[27])



Ψ — coefficient; enter Fig. 3.3 with known value of μ (E_s/E_c), known arrangement of reinforcing, and known cracking condition.
μ — ratio of area of reinforcing in one direction to concrete area.

Fig. 3.3 — Values of coefficient Ψ (based on Kollar & Dulacska[27])



Ψ — coefficient obtained from Fig. 3.3

Fig. 3.4 — Modification of upper critical load due to reinforcement and cracking (based on Kollar & Dulacska[27])

Effect of creep: The effect of creep may be estimated by reducing the value of the modulus of elasticity E_c of concrete in accordance with the formula

$$E_{cr} = \frac{E_c}{(1+C_u)}; \qquad C_u = 4-2\log f_c; \tag{3.7}$$

where E_{cr} is the reduced — due to creep — modulus of elasticity of concrete, and f_c is the strength of concrete at the time of loading, measured in N/mm^2. The previously calculated upper critical load p_{cr}^u must be reduced in the same proportion as E_c, since the latter occurs in the first power in the formulae for the linear critical load p_{cr}. Above reduction presupposes that all loads acting on the shell are long term loads. If some of the loads are short term, such as wind load or the loads due to seismic activity, the modulus of elasticity E_c should be reduced only partially, in proportion to the ratio of short term loads to all loads.

Effect of reinforcement and cracking: This effect may be taken into account as follows. Calculate the quotient $(E_s/E_{cr})\cdot(A_s/A_c)$, where the area of steel A_s is taken in one direction only. From Fig. 3.3 obtain the value of coefficient ψ appropriate to the stress range and to the arrangement of reinforcement. Then, for the given value ψ and of the deflection to thickness ratio w_o/h, the reduced value of the upper critical load $p_{cr,reinf}^u$ can be obtained from Fig. 3.4 where w_o is the sum of the two previously defined deflections w_o' and w_o''.

Effect of plasticity of concrete: So far, the material of the shell has been assumed elastic. Plastic properties of materials may be allowed for with the aid of the *semi-quadratic Dunkerley* interaction formula

$$\left(\frac{p_{cr}^{plast}}{p_{plast}}\right)^2 + \left(\frac{p_{cr}^{plast}}{p_{cr,reinf}^u}\right) = 1; \tag{3.8}$$

from which the critical load p_{cr}^{plast} of the plastic shell can be obtained. In equation (3.8), p_{plast} denotes the load that the shell with a computable eccentricity w_o' of the compressive in-plane forces can carry (buckling being disregarded), with both concrete and reinforcement at yield. It is recommended that in the calculation of p_{plast} the thickness of the shell be reduced by approximately 1 cm for shells thinner than 8 cm, as a means of allowing for the possible cross-sectional imperfections and weaknesses. The above calculation does not include the effect of shell imperfections w_o'', since these may be assumed to be associated with in-plane forces only, and since equation (3.8) yields results which are conservative compared with the results of the more exact quadratic formula.

3.5.5. Factor of Safety

The factor of safety F.S. is defined in the formula

$$F.S. = \frac{p_{cr}^{plast}}{p}; \tag{3.9}$$

where p is the real load. In the case of shells with critical load increasing in the post buckling range (paragraph 3.5.2), the value of the factor of safety may be taken at 1.75, the same as that used in the case of reinforced concrete columns, since in each case failure will occur through plastic collapse. In the case of shells where post buckling behavior governs (paragraph 3.5.3), the value of the factor of safety should be taken as a minimum at 3.5.

3.5.6. Experimental analysis of stability

Buckling loads may be obtained experimentally by testing microconcrete models, as discussed in paragraph 3.6.

3.6. EXPERIMENTAL ANALYSIS

3.6.1. General Remarks

As noted in paragraph 3.1, experimental analysis, by itself, should not be used as the basis for shell design. It is useful in confirming the validity of the mathematical model; in checking the validity of the results of analysis; and in obtaining a qualitative insight into the behavior of a shell and into the relative importance of some influence on the behavior of the shell.

3.6.2. Types of Experimental Analysis

Depending on the basis for classification, different types of experimental analysis may be distinguished.

Nature of the structure tested: Analysis of a model or the prototype.

Aim of experimental analysis: Testing of a model simulating the prototype, or the mathematical model.

Nature of the model: Elastic or inelastic; with the laws of similitude being obeyed (*direct experimental analysis*) or not obeyed (*indirect experimental analysis*); large scale or small scale model.

Nature of load: Loads may be static, dynamic, thermal, wind, etc.

Nature of shell behavior to be tested: Working or ultimate load; small or large deflections; stability; various special effects.

3.6.3. Stages of Experimental Analysis

Whatever its type or its purpose, the following three phases of experimental analysis may be distinguished.

Design of the experiment: At this phase decisions are made regarding the nature of the model and its size, the nature and magnitude of the applied loads, and the scope of the required instrumentation.

Testing: Behavior of the model under loads is observed, and the data are recorded.

Interpretation: Analysis and interpretation of the data is performed and conclusions and recommendations for design are made.

3.6.4. Laws of Similitude

The following laws of similitude must be observed in the case of direct experimental analysis (assuming time independence).

Geometrical similitude: All linear dimensions of the model are obtained by appropriately scaling using a constant scale factor the corresponding dimensions of the prototype.

Materials similitude: The stress-strain curve of the model material should correspond to that of the prototype in the range of loads under investigation.

Load similitude: The magnitude, point of application and direction of model loads are obtained using a proper scale factor from the loads of the prototype.

In the case of experimental analysis which includes time dependent effects the *time scale* should be obtained by means of the general laws of similitude (*Buckingham-Pi Theorem*).

3.6.5. Elastic Analysis

If both the prototype and the model materials are linearly elastic, homogeneous and isotropic, the material similitude laws will be satisfied if the Poisson ratio of the materials is the same. However, since the effect of the Poisson ratio on concrete shell behavior is usually assumed small, this requirement is often abandoned, permitting the use of metals or plastics to simulate concrete. Consideration should be given to this effect where warranted. Metals involve large moduli of elasticity, requiring large model loads to produce measurable strains. For this reason aluminum might be preferred to steel.

Plastics, either thermoplastic or thermosetting, are the most common material. Their disadvantages include large Poisson ratio, time dependent behavior, introduction of residual stresses during the manufacture of the model, disturbances in measurements due to local stiffening associated with the metal gage sensing units, and "hot spots" produced by the low thermal conductivity of the plastic in the readings of the electrical resistance gages. Some of these disadvantages can be avoided through the use of specialized techniques described in the literature of the subject.

Properly designed elastic models made of metal or plastic can be used also to study geometric nonlinearities due to large displacements. The important problems of buckling or post buckling behavior of shells can be studied using such models, but great care must be exercised in fabricating the model, and in making sure that the boundary conditions of the prototype are correctly simulated.

With the advent of sophisticated numerical methods based on the computer, such as the finite element method, which can be used to make elastic analyses of complex shell structures, the need for elastic model analysis has greatly diminished.

3.6.6. Inelastic and Ultimate Strength Analysis

Experimental analysis to study the behavior of a reinforced concrete shell through the elastic, inelastic and ultimate range requires a more complex model than that used purely for an elastic analysis. The concrete and steel reinforcement in the prototype must be simulated in the model with appropriate nonlinear materials having properties similar to those of the prototype throughout the entire range of behavior, from elastic to ultimate.

In general, it is difficult to simulate all the properties of the concrete in a prototype. A mixture of plaster, diatonites and water is sometimes used.

Microconcrete: The most suitable material appears to be microconcrete. Certain inherent difficulties arise. Complete scaling down of all materials is not possible, since a more finely ground cement is not available, and since the model mix requires relatively more water. The model material is a mixture of cement, aggregates and water, with the maximum size of the aggregates dictated by the practical requirements of the model shell thickness and the model reinforcement spacing. Usually, the important model material properties are compressive strength (f_c'), tensile strength, Poisson ratio, secant modulus of elasticity, maximum compressive strain, shrinkage and creep characteristics, and workability.

Some of the other difficulties which arise in working with microconcrete are as follows: its tensile strength increases as the size of aggregate decreases, which can influence the amount and extent of cracking visible at a particular load case; microconcrete contains relatively more water and its shrinkage is greater; some concrete characteristics depend on the size of the specimens tested and for this reason it may be desirable to consider the size of the test specimen as a function of the size of the model.

Microconcrete reinforcing presents problems in simulating prototype reinforcing; the shape of the stress-strain curve should be the same; bond characteristics are difficult to reproduce, as is the cracking pattern of concrete.

Microconcrete models usually need to be larger than the elastic models made of metals or plastic.

Notwithstanding its inherent difficulties, the microconcrete model analysis represents the only presently available practical procedure which can trace the behavior of a reinforced concrete shell through its elastic, inelastic and ultimate range. The use of a microconcrete model should be considered, in addition to mathematical analysis, as an aid in the design of large and important reinforced concrete shell structures.

3.6.7. Loads and Supports

Loading apparatus should be designed to permit loads up to ultimate stage. Supports should be designed to provide stiffness very much greater than the stiffness of the model.

3.7. ANALYSIS OF AUXILIARY MEMBERS

3.7.1. Supporting Members

Supporting members should be designed in accordance with the applicable regulations to satisfy strength and serviceability requirements.

Supporting members should be analyzed for partial loads, with due regard to the eccentricity of the element with respect to the shell, and to the participation of a portion of the shell in the action of the element, as discussed in Chapters 2 and 4 of these Recommendations.

3.7.2. Edge and Stiffening Members

These elements are a part of the shell structure and should be analyzed in conjunction with the analysis of the shell. A common procedure in incorporating the effect of stiffening members on the analysis of the shell is to consider it as possessing new extensional and flexural properties equivalent to those of the original shell but uniformly distributed throughout the middle surface. This procedure is known as *smearing*.

CHAPTER 4

PROPORTIONING AND REQUIREMENTS FOR DESIGN

4.1. GENERAL REMARKS

In addition to strength requirements, a number of factors influence the determination of concrete dimensions and the size and spacing of reinforcement of the shell and auxiliary members. These factors include the need for crack control, the requirements of placing and cover to reinforcement, the detailing of bars, splices and anchorages, and the design and detailing of prestressing.

Comments presented in this chapter pertaining to the proportioning and design of concrete shells are intended to apply generally to all types of shells and folded plates.

4.2. SHELL CONCRETE

4.2.1. Shell Thickness

In most cases shell thickness is dictated not by the requirements of strength but by other factors pertaining to the process of construction and to the performance of the shell. These factors include the quality of construction attainable for a given project, the requirements of placement and cover to reinforcement, the requirements of the stability of the structure, and the possible need to control the deformations of the shell and its auxiliary members.

Although shells as thin as 25mm have been successfully realized, a more practical range of the thickness appears to lie between 50mm and 100mm. This thickness permits a reasonably simple placement of reinforcement and concrete and allows for adequate cover to reinforcement.

In certain shell areas near edge and support members, additional thickness of concrete may be required in order to accommodate a somewhat greater quantity of reinforcement, particularly bending reinforcement. Experience indicates that the thickness of the shell in these areas may reach up to twice or even three times the typical thickness of the shell. This increase in thickness, where it occurs, should be obtained gradually, in a smooth transition, so as to avoid concentration and redistribution of stresses. The width of the zone where an increase in thickness occurs should be at least five or, if possible, up to ten times the additional shell thickness.

4.2.2. Concrete Cover to Reinforcement

In determining concrete cover to reinforcement, a balance must be reached between the need for protection of bars from injurious and unsightly rusting, and the desire to place the bars as close as possible to the surface of the shell, so as to reduce the width of the inevitable surface cracks. Based on these considerations, the following minimum cover to bars is recommended.

Edge members and supporting members: Use minimum cover as recommended in the applicable regulations for typical concrete elements, except that shell reinforcement which penetrates into these members may receive the same minimum cover as it does in the area of the shell itself.

Stiffening members: Use the same minimum cover to reinforcement as in the shell itself.

Shell: For shell surfaces protected from atmosphere, deleterious influences, or contact with ground, use minimum cover to reinforcement listed in TABLE 4.1.

TABLE 4.1 — MINIMUM COVER TO REINFORCEMENT		
Type of Construction	Bar Size (mm)	Cover (mm)
Cast in place	<16 ⩾16	12.5 18
Precast	<16 ⩾16	10 15
Prestressed (bars, wire, tendons,ducts,fittings)	<16 all other	10 15 but not less than bar, wire, tendon or duct diameter

For shell surfaces exposed to atmosphere, deleterious influences or ground, additional cover to bars should be provided. The location of the surface (whether exterior or interior) and the degree of exposure should be taken into account in the determination of the additional cover required. In addition, consideration should be given to increasing the minimum cover to reinforcement where the shell is steep, or where forming is used at both faces. In this case, minimum cover should be not less than the maximum size of the aggregate, and not less than the bar diameter.

If fireproofing requirements place more stringent demands on the minimum cover to reinforcement, such additional requirements should apply only to the principal stress reinforcement, and to bending reinforcement where its yielding would result in failure.

4.3. SHELL REINFORCEMENT

4.3.1. General Remarks

Design and detailing of reinforcement requires careful consideration of a number of factors pertaining not only to the requirements of analysis but also to the dictates of the practicalities of construction. Many of these requirements are common to all structural systems, and some are pertinent to shell construction only. The following procedures are believed to represent current practice among experienced shell designers.

Types of shell reinforcement: Three types of shell reinforcement may be distinguished: membrane reinforcement, bending reinforcement, and special reinforcement such as that at shell boundaries and shell openings. Accordingly, shell reinforcement is typically arranged in a number of layers. Membrane reinforcement usually occurs in two orthogonal layers placed near the mid-surface of the shell. Bending reinforcement is typically placed near the surfaces of the shell.

Bar size: In general, to reduce crack width, small bar diameters are preferred to large. Deformed bars should be used throughout, except in the case of welded wire fabrics. Spacing of bars should receive careful consideration, although it is recognized that even very dense bar spacing will not preclude the appearance of microcracks smaller than 0.2mm in width. However, such cracks may be considered insignificant.

High strength bars: As a rule, use of high strength bars (i.e., bars of steel with the yield point f_y greater than $280 N/mm^2$) should be avoided because of the increased tensile strains and cracking. If high strength bars are used, their effect on the deformation and cracking of the shell should be carefully considered, and a limit should be placed on the allowable steel stress.

Distribution of bars: The distribution of bars within the shell should generally follow the distribution of calculated stresses. This results in a relatively smooth variation of bar quantities within the shell and provides for a reasonably limited departure from the assumption of uniform distribution of elastic properties typically made in the analysis. Nevertheless, it appears that in some cases concentration of bars in small areas in the shell may be beneficial. These cases are discussed further in section 4.3.2, below.

4.3.2. Membrane Reinforcement

Membrane reinforcement should be provided to carry full calculated membrane tensions in the shell; no participation by concrete should be permitted.

In principle, it is possible to calculate principal stresses and their directions throughout the shell, and then to place the reinforcement along the lines of principal tensile stresses. In applications this approach is seldom practical since the magnitude and the direction of principal stresses vary depending on the load system, while the geometry of the shell surface may require that membrane reinforcement be shaped in three directions in space, generally a difficult undertaking for larger size bars. For these reasons, a more typical approach to the design and detailing of membrane reinforcement is to provide two orthogonal layers of bars located near the mid-surface of the shell and directed along some geometrically convenient system of lines, thus making the manufacture and placing of the bars relatively simple. In the case of circular cylindrical shells, as an example, the bars are typically placed along the straight lines of the generatrices, and along the circular lines of the transverse arches.

Nevertheless, in the shell areas subjected to high tensile stresses, a desirable common practice is to provide membrane reinforcement parallel, or nearly so, to the lines of principal tensile stresses. In practice this is sometimes done by providing a third layer of membrane reinforcement, appropriately directed, in the high stress areas only.

Minimum membrane reinforcement: Should consist of two layers of bars placed in orthogonal directions. The minimum cross sectional area of reinforcement per unit width of shell should be as specified by the applicable regulations for temperature and distribution reinforcement. Minimum diameter of deformed bars should be 6mm, and minimum diameter of wires in welded wire fabric should be 5mm in poured-in-place shells, and 4mm in precast shells. Minimum area of reinforcement in one direction should be 0.20%, and the sum of areas of reinforcement in two directions should be not less than 0.60%. In the case where shell thickness exceeds 14cm, two curtains of steel should be used.

Maximum membrane reinforcement: Should be calculated from the formula:

$$p = \frac{A_s}{A_c} = 0.6\frac{f_c'}{f_y}; \qquad \text{for } f_c' < 28\ N/mm^2; \tag{4.1}$$

$$= \frac{16.8}{f_y}; \qquad \text{for } f_c' \geqslant 28\ N/mm^2;$$

where A_s/A_c is the ratio of reinforcement area to concrete area for a given section of the shell, f_c' is the cylinder strength of concrete, and f_y the yield strength of steel, both measured in N/mm^2. Formula (4.1) applies to the case when the lines of reinforcement coincide with the lines of principal stresses. If the deviation is more that 10 degrees, the maximum areas of reinforcement should be reduced in the same manner as the allowable steel stress in accordance with formula (4.3).

Spacing of membrane bars: Deformed bars should be spaced not more than five times the shell thickness, nor more than 300mm. Welded wire fabric bars should be spaced not more than four times the shell thickness, nor more than 200mm.

Where the computed principal tensile stress in concrete is greater than the stress calculated from the formula

$$\sigma = 7.67\sqrt{f_c'}; \quad (N/mm^2) \tag{4.2}$$

where f_c' is the cylinder strength of concrete measured in N/mm^2, membrane reinforcement should be spaced not more than three times the shell thickness. This restriction need not apply in shell areas where bending reinforcement is provided at each surface, spaced in each direction not more than three times the shell thickness.

Deviation from lines of principal stresses: Membrane reinforcement deviating not more than 10 degrees from the direction of the lines of principal tensile stresses may be designed on the basis of the allowable stresses given in Chapter 2 of these Recommendations. If the deviation is more than 10 degrees, allowable stresses should be reduced. The reduction factor varies depending on the angle of deviation (which may be from 10 degrees to 45 degrees), and depending on the ratio κ of the principal tensile stress to the principal compressive stress at a given point of the shell. For the important in practical applications case of $\kappa = -1$, the stress reduction factor r may be taken as follows:

$$\begin{aligned} &\text{for } \phi \leqslant 10^\circ; && r = 1.00 \\ &\text{for } 10^\circ < \phi < 30^\circ; && r = 1.30-0.03\,\phi \\ &\text{for } \phi \geqslant 30^\circ; && r = 0.40 \end{aligned} \qquad (4.3)$$

where ϕ is the angle of deviation. For values of the reduction factor r when the ratio κ is other than -1, current research literature should be consulted.

The effect of bending moments on the magnitude of the angle of deviation need not be considered.

Concentration of reinforcement: Experience shows that in some cases the distribution of reinforcement in some section of the shell need not follow the calculated distribution of stresses in the same section, as recommended in 4.3.1. Two examples may be cited. In the case of cylindrical shells with edge beams, the longitudinal reinforcement resisting total tensions is sometimes placed concentrated at the bottom of the edge beams. Similarly, in the case of surfaces of revolution, reinforcement resisting hoop tensions is sometimes placed concentrated in the area of the tension ring.

A number of projects have been realized using this approach. Caution should be exercised to ensure adequate ultimate strength of the shell in such cases. If this procedure is used, the minimum ratio of steel area to concrete area in any portion of the tension zone of the shell should be not less than 0.0035.

4.3.3. Bending Reinforcement

In calculating the required amount of bending reinforcement, due regard should be given to any axial forces which may act together with the bending moments.

In bending zones of a shell, reinforcement should be provided at both the top and bottom surfaces. Draping of reinforcement by transferring it from one surface to the other to follow the bending moment curve should be avoided, since the point of the moment sign reversal may vary with the load system.

Design of bending reinforcement should follow the precepts of the applicable regulations.

Where a shell terminates in a bending zone near an edge or supporting member, the reinforcement normally governed by bending requirements may become governed instead by the torsional requirements of the edge or supporting members.

4.3.4. Special Shell Reinforcement

Careful consideration should be given to any additional reinforcement which may be required in the zone of the shell near the supporting members for the proper transfer of forces from the shell into the supporting members.

Shell reinforcement at the junction of the shell and auxiliary members should be anchored into or through such auxiliary members by means of an appropriate embedment, the use of hooks, or the use of mechanical anchoring devices. The length of the embedment, and the detailing of hooks and mechanical devices should meet the requirements of the applicable regulations.

Edges of shell openings should be always reinforced. As much as possible, edge members should be used. Reinforcement should extend beyond the end of the opening so that it might be lapped with the shell reinforcement. The amount of edge reinforcement should be calculated or, as a minimum, it should be twice the amount of reinforcement made discontinuous by the opening. In addition, careful consideration should be given to the possible need for diagonal bars across the reentrant corners, particularly in zones of high stresses.

4.3.5. Splices in Reinforcement

Splices should be kept to a practical minimum. Where splices are necessary, they should be staggered so that no more than every third bar in the same layer is spliced in the same section, with the minimum distance between splice locations of neighboring bars at least 100 times the larger bar diameter. Bars should be spliced within the same layer.

Splice length should be as prescribed by the applicable regulations, but not less than the lengths given in TABLE 4.2. Note that the splice lengths given in this table are based on bars made of steel with the yield point strength of $280 N/mm^2$, and must be adjusted proportionately for steel of higher yield strength.

TABLE 4.2 — MINIMUM SPLICE LENGTHS

Type of bar	Splice length (larger bar diameters)	Minimum (mm)
Bars in tension	45	600
Bars in compression	30	450
Reinforcement not required by calculation	24	300
Welded fabric	3 mesh sizes	300
Welded wire fabric not required by calculation	1 mesh size	150

Principal membrane tensile bars size 34mm and larger should be not spliced but welded using full penetration butt welds. Careful attention should be given to the possible need for preheating the bars, and for the use of low hydrogen electrodes to avoid brittle connections.

No hooks need be used in deformed bars of all sizes, nor in plain wire bars which are a part of a welded wire fabric. The splice lengths given in TABLE 4.2 apply to deformed bars. An adjustment of the splice length of plain bars should be made if hooks are not used.

4.4. REINFORCEMENT OF AUXILIARY MEMBERS

4.4.1. Reinforcement

Auxiliary members should be reinforced as required by calculations and by applicable regulations.

Consideration should be given to the possibility of extending bars from supporting members or edge members into the adjoining shell along the lines of principal stresses.

Reinforcement in edge beams may be designed on the basis of recognized theories of reinforced concrete. Thus, longitudinal bars may be designed concentrated in the area of largest tensile strains. If this approach is used, due consideration should be given to the possible need for provision of additional longitudinal reinforcement in the remainder of the edge beam, serving to reduce surface cracking. Alternatively, edge beam reinforcement distribution may be designed to follow the distribution of calculated stresses.

4.4.2. Shell Participation

A portion of the shell may be assumed to act as a part of the auxiliary member. The width of the zone of participation depends on the geometry of the shell, and may be less than the width of the flange for tee beams specified by codes.

4.5. PRESTRESSING REINFORCEMENT

4.5.1. Losses

The following losses should be considered in determining effective prestress:

Concrete losses: Elastic shortening, creep and shrinkage.

Tendon losses: Slip at anchorages, relaxation of steel stresses, and the frictional losses due to intended and/or unintended curvature in the tendons.

4.5.2. Tendon Force Components

Where prestressing tendons are draped within the thin shell, the tendon forms a curve in space which may not lie in a plane. The tendon will exert in this case a force on the shell which can be resolved into components; these components should be taken into account in the analysis of the shell.

4.5.3. Anchorages

At prestressing tendon anchorages, special reinforcement should be added to ensure that local overstressing does not occur, and that the tendon anchorage assembly is precluded from breaking out of the shell in a transverse direction.

Where tendon anchorage plates bear against thin shell section, care should be taken to ensure that adequate bearing area of concrete is available.

4.6. WELDED INSERTS

4.6.1. Design

Welded inserts are sometimes embedded in the shell or its auxiliary members to attach loads, or to joint precast shell elements. Such inserts should be adequately attached to concrete. The inserts and their attachments should be designed to transfer all shell membrane forces and bending moments to be developed. Due consideration should be given to the possible eccentricities and accidental loads which may arise during construction or under long-term loads.

Consideration should be given to the direction of anchors which attach the insert to concrete. Placement of anchors along the lines of principal stress is preferred. If the anchors are in form of reinforcing bars welded to the insert, consideration should be given to possible preheating of bars and to the use of low hydrogen electrodes to reduce brittleness in connections.

4.6.2. Protection

Whenever insert assembly is exposed to atmosphere or deleterious influence after construction, such assembly should be adequately protected by, for example, galvanizing by a hot dip process prior to installation. Areas of galvanizing damaged by field welding, and field welds, should be further protected by field galvanizing.

CHAPTER 5

CONSTRUCTION-ARCHITECTURAL DETAILS-ECONOMICS

5.1. GENERAL REMARKS

The behavior of thin concrete shells is affected markedly by the manner in which a shell is constructed and detailed. The term "behavior" includes in this case not only the manner in which a given shell transfers loads and deforms, but also its functional efficiency and the extent to which it is pleasing aesthetically. Similarly, the matter of economics of shell design and construction is all important; if the cost of a concrete shell structure cannot be justified by its presumed benefits, it will simply not be built. The project will remain unrealized, or it will be constructed using some other more efficient and economical structural system.

For these reasons, full consideration should be given from the very outset of conceptual design to the anticipated manner of construction and detailing of a given shell, and to its projected cost. In many cases, such considerations will prove decisive in selecting the type and geometry of the shell structure.

Some of the factors involved in construction, detailing, and economics of concrete thin shells are considered in this chapter. Discussion is limited to the factors pertinent to thin shell construction; those common to all structural systems are not discussed, or receive only a brief mention.

5.2. CONSTRUCTION

5.2.1. General Remarks

To an extent greater, perhaps, than is the case with other structural systems, the method of construction of a concrete thin shell has a strong effect on its behavior, its functional and aesthetic acceptability, and on its cost. Thus, in selecting the type and geometry of a shell it is necessary to consider all phases of the process of construction, including forming and scaffolding, fabrication and placing of reinforcement, placing and curing of concrete, the manner of erection, and the manner and sequence of stripping forms and decentering. In addition, the possibility of precasting, the possibility that prestressing might be used and, in general, all other pertinent characteristics of a given method of construction should be considered. Attention should be given also to the quality required of the finished product, and to the dimensional tolerances attainable under a given procedure.

5.2.2. Formwork and Scaffolding

Materials: Many different materials have been used successfully in the realization of shell projects. *Wood* (and *plywood*) is the traditional material. *Earth forms* have been used not only in the construction of shell-type foundations but also in the construction of a number of large span roofs. *Fiberglass* is used on occasion where a particular concrete surface pattern is desired. *Steel* forms are used primarily in conjunction with precasting. Special *paper,* reinforced with welded wire mesh, has been used as a form for a shell suspended from permanent reinforcement. Forms may be also made of *concrete,* in which case they may become a part of the completed structure, or they may be reused. Recently, progress has been made in the use of *inflatable forms* made of treated *canvas* or some other appropriate material, and in the use of *rigid polyurethane* shell forms which become the insulation of the finished structure. Each of these different types of forms has been used; the choice depends on the economics and on the desired type of surface treatment.

Single and double forms: The use of single (one sided) forms is recommended as much as possible. In the event that concrete is placed by traditional methods, the use of single forms permits a reduced water/cement ratio, easier placing and vibration, an hence stronger concrete and better finished surfaces. The single form can be used up to the shell slope of 30 degrees, possibly up to 45 degrees. If shell slopes exceed these angles over a substantial area of the structure, double (two sided) forms have to be used, or recourse has to be made to a different method of concrete placing, such as guniting.

Construction of forms: Forms should be built as for all concrete construction. They should be tight to prevent loss of cement paste, and they should be coated to facilitate stripping. Coating material itself should be non-deleterious to the finished product, should be non-staining, and should be applied prior to the installation of reinforcement so as to avoid a loss of bond between the bars and concrete. Consideration should be given to making forms battered to facilitate stripping, particularly where movable forms are used. A typical batter used in the construction of plane surfaces of edge beams varies from 1% to 5%, the upper limit being by no means restrictive.

Strength and stiffness of forms: Formwork and scaffolding should be sufficiently strong and stiff to support all dead and live loads which may be placed on it in the course of construction. Consideration should be given to the possible lateral loads which may arise as a result of the existence of unbalanced vertical loads, or due to wind or seismic loads. Designers should specify form camber required to accommodate shell deformations.

Formwork and scaffolding removal: Whenever practicable, and certainly in the case of thin shells of large span or unusual geometry, the engineer should determine the manner, the time and the sequence of form stripping and decentering. The time should be determined by specifying the minimum required strength of concrete, as measured by testing field cured standard specimens, and by specifying the minimum required stiffness, by measuring the modulus of elasticity E of concrete. The latter is measured by testing in flexure lightly reinforced beam specimens cured in the field. A typical dimension of the specimen beam may be 100mm x 150mm x 1m long. The manner and sequence of form stripping and decentering should be established before the method of scaffolding has been determined. Appropriate jacks should be incorporated into the scaffolding to facilitate removal. The sequence of decentering should be such that the thin shell is not subjected to concentrated loads, even temporarily. Generally, decentering should begin at the points of largest deflection and should progress towards the points of smallest deflection. Edge members and auxiliary members should be decentered at the same time as the shell; the scaffolding under the supporting members should not be removed until after the decentering of the shell proper. Where reshoring is to be used, its detailing, manner of installation and spacing should be determined by the engineer. On occasion, it may be useful to remove column forms first; this affords the opportunity for inspecting the quality of concrete as placed prior to shell form stripping.

Tolerances: The accuracy with which a form can be constructed and placed varies with the location of the project, the nature of the form material, and with the skill of the artisans and laborers who build it and erect it. Steel forms used in a precasting plant can be manufactured to closer tolerances than wood forms or earth forms built at the site. Further, the importance of maintaining close tolerances may also vary, depending on the nature of the project. For these reasons it is difficult to make a recommendation as to the attainable tolerances, and the designer should review the factors involved in each case, and should place tolerance limits in the light of the circumstances of the given project.

5.2.3. Reinforcement

A number of remarks regarding the preferred size, type and strength characteristics of bars occur in paragraph 4.3 of Chapter 4 of these Recommendations. Some additional comments are as follows.

Bar size: Although small bars are preferred to large, it should be noted that bars of small diameter are quite flexible and are subject to becoming permanently bent and displaced during the process of construction. For this reason, smaller bars should be supported more often on the forms than would be the case with bars of larger diameter. Since the thickness of shells is often determined by the requirements of reinforcement, a minor deviation from the design location of a bar may represent a relatively major deviation from the design strength of the shell. For this reason bars should be placed carefully and held in position firmly, particularly in the areas of shell which are subject to bending stresss.

Welded wire mesh: This type of reinforcement is economical for shells of single curvature, for ruled surfaces (such as hyperbolic paraboloid shells), and for shallow shells of double curvature where advantage can be taken of the rather simple installation procedures. The disadvantages of the bars of small diameter were mentioned above. In addition, the cold drawn wire of which a welded wire mesh is made has a tendency to spring back and hence the welded wire mesh should not be used for shells of large curvature. In general, the accuracy of placement of welded wire mesh is less than that of more rigid bars.

Trajectory bars: It is almost never practical to place bars in an orthogonal system along the principal stress trajectories, due to the high cost of manufacture of the bars and their placement. A more commonly used approach is to place an orthogonal system of bars compatible with the geometry of the shell, and utilize a third layer of bars placed as closely as possible to the trajectories of principal tensile stresses in those areas of the shell where such stresses are high.

5.2.4. Prestressing

In general, the design and construction of prestressing follows usual practice and applicable codes. Characteristic of shell construction are the possible limitations due to the small thickness of the shell. The thickness has to be sufficient to accommodate all tendons, and in the anchorage zones care must be taken to prevent out-of-plane spalling of concrete, as discussed in Chapter 4 of these Recommendations. Finally, consideration should be given to adequate protection of tendons against corrosion and against fire.

5.2.5. Concrete and Concrete Placement

Aggregate: The size of aggregate should not exceed one fourth of the thickness of the shell, nor the clear distance between reinforcing bars, nor one and one half the clear cover to reinforcement. In the cases where two-sided formwork is used, the maximum size of the aggregate should not exceed one fourth of the minimum clear distance between forms (or the thickness of the shell), nor the clear cover to reinforcement. These limitations on the maximum size of the aggregate are dictated by the desire to limit material variations due to the heterogeneous characteristics of concrete, to reduce the chance of formation of rock pockets or portions of the shell with excessive proportion of mortar, and thus avoiding creation of significant variation in concrete strength.

Concrete placement: As much as possible, concrete should be placed near its final position, in a symmetrical pattern which avoids the creation of unbalanced loads and hence the need for additional bracing of scaffolding. Construction joints should be detailed on drawings and should be placed, preferably, in compression zones of the shell. The effect of shrinkage stresses in the shell which is to be poured in several segments can be reduced by using a checkerboard sequence of pours, by scheduling an appropriate delay between pours, and by appropriate curing. Where deep beams or diaphragms are to be cast integrally with the shell, care should be taken to use the rate of concrete placing which will permit consolidation of concrete in the deep beams or diaphragms and preclude the formation of cold joints in the thin shell.

Vibration: Immersion vibrators commonly used in concrete construction should not be used, since the thickness of the shell is insufficient to accommodate the vibrators (the vibrator should never be laid flat, since this results in the splattering of concrete instead of its compaction). Vibration by means of vibrators attached to reinforcement is not desirable since it may result in the break of the bond between concrete and reinforcement, and cracking of concrete. Also, to be effective, vibrating reinforcement requires that the bars be fairly large, larger than typical shell reinforcement. Form vibrators are often used in the case of two-sided forms, particularly in the case of prefabricated elements. To avoid possible segregation due to the thinness of the shell, form vibrators should be moved as concrete placement progresses, so that energy dissipation is minimized. For satisfactory results the equipment used should be designed on the principle of small dissipation of energy together with normal amplitude of vibration. In the case when formwork is supported on rigid beams — such as steel beams — it is possible to attach the vibrators directly to these beams and thus vibrate the entire form. Usually in this case the damping of the system is sufficient to transmit only low energy levels to concrete. In the case of poured-in-place shells with one-sided forms, vibrating surface plates or screeds can be used more successfully than immersion vibrators, since the depth of the concrete to be compacted is shallow.

Concrete curing: Thin shells are susceptible to shrinkage cracking unless adequately cured. It is recommended that, as much as possible, fog spraying be used followed by the application of burlap mats which are kept continuously wet for at least 7 days. The use of sprayed curing membranes is less satisfactory, but may be found acceptable on occasion in moderate weather conditions. In cold weather, the use of accelerators and special provisions against freezing of concrete are recommended.

5.2.6. Precast Shells

The advantage of precast shells is that tighter tolerances and better quality of construction can be expected as a result of using factory methods. In addition, a reduction in cost may occur. More exacting tolerances permit the use of thinner elements, typically cast in two-sided forms. Reinforcement is often prefabricated in large-size panels of appropriate curvature, welded at intersection of bars to increase its rigidity and to reduce its thickness. The problems of concrete placement, vibration and curing are significantly simpler under factory conditions, leading to stronger, better finished and more accurately dimensioned shell elements. Otherwise, the problems of prefabrication of shell elements are the same as those encountered in the prefabrication of other concrete structures.

Prefabrication introduces two important elements which require careful consideration: the problem of transportation and erection of the shell elements after manufacture, and the problem of joints which are introduced into the structure in order to keep the size of the prefabricated elements to manageable and transportable proportions. The matter of joints should be given full consideration at the time of shell design; it is discussed further in article 5.3. The size of elements which can be prefabricated is limited by the requirements of transportation and by the capacity of the equipment used during the erection of the shell. Again, these factors should be given careful consideration at the time the shell is being designed.

In order to reduce stresses and deformations to which a prefabricated shell element might be exposed during transportation and erection, the shell designer may choose to thicken the edges of these elements or, conversely, the designer may place the boundaries of the prefabricated elements along the lines of already provided auxiliary members. Such auxiliary members may be incorporated into the precast element, or may be poured in the field, thus creating very effective monolithic joints. This solution offers the additional advantage of reducing the need for tight tolerances.

Environmental conditions which affect corrosion should be given consideration since they may control the design of prefabricated shells.

5.3. ARCHITECTURAL DETAILS

5.3.1. General Remarks

The importance of architectural detailing on the performance of concrete thin shell structures is generally recognized. All architectural details should be designed and executed to satisfy functional requirements, to enhance appearance, and to be consistent with the economic requirements of the project. In the following, some of the architectural details characteristic of concrete thin shell structures are considered. The problems which thin shells share with other structural systems are not discussed, or receive only brief mention.

5.3.2. Concrete Finishes

Due to their nature as a structural system, thin shells are often designed so that after completion large concrete surface areas are exposed to view. For the purpose of these Recommendations, such concrete may be termed *architectural.* Thus, because of the size of the exposed surfaces, the impact on the appearance of the project may be significant. Hence the need for careful design and control of the process of manufacture of these surfaces, aimed at the uniformity of texture and color within assigned areas, minimization of the visual impact of cracking, and reduction in the number of blemishes. It must be borne in mind that, due to the nature of concrete as a construction material, complete uniformity of color and texture is not attainable, and that cracking will occur. Accordingly, a prudent, pragmatic designer should attempt to detail well within the capabilities of the material and of the construction technology available for the given project.

Design of surfaces: Architectural surfaces of concrete should be designed at two levels. *Large scale design* involves the organization of the surfaces in the large. This may be achieved by subdividing the total exposed surface into smaller areas more nearly related to the human scale. As much as possible the subdivision should follow the lines which result naturally from the intrinsic structure of the shell, or from the manner in which the shell is built. Examples are the lines of change of geometry of the shell (such as the hip lines of a hyperbolic paraboloid umbrella), the lines along any stiffening ribs, the lines of joints between shell elements, or the lines of construction joints. Whatever the method of division of the surface, it should be related logically to the structure, to its function, and to the method of construction used, and it should be readily perceptible to an observer. *Small scale design* involves the design of surface area within the large scale design lines. It entails the design of concrete finishes, their texture, color and other characteristics.

Types of concrete finishes: Two general types of concrete finish may be distinguished. *Form imprint finish* results from leaving the surface essentially unfinished, the visible part of it being mortar; the texture may be smooth or rough depending on the nature of the form and its liner. *Treated surface finish* results from treating the surface after the removal of forms in any of a number of ways so as to remove a portion of the surface mortar and to expose the aggregates to a desired degree. As an aid in this process, retarder coating, surface washing, acid washing, sandblasting, mechanical treatment (bushhammering is an example), aggregate seeding, aggregate preplacing and aggregate transfer have been used. The procedures are essentially the same as in general concrete construction.

Color: Desired color can be achieved by adding an appropriate coloring agent to the mix, by exposing aggregate of desired color to the desired extent, or by painting, which changes color without changing the texture.

5.3.3. Watertightness

Shells are sensitive to water- and weather-tightness problems because of the size and degree of exposure of the concrete surfaces. It does not seem to be possible to ensure watertightness through the use of any concrete additives. Carefully designed and executed prestressing may provide a solution in some instances. Most often, however, tested roofing materials are used because of their proven reliability.

5.3.4. Joints

Joints between shell elements, and construction joints, should be designed to transfer loads as discussed elsewhere in these Recommendations. In addition, care should be exercised to ensure watertightness, and protection from corrosion. An appropriate caulking compound may have to be used, of a non-running type. When designing joints between prefabricated elements, consideration should be given to the possible rotations of the shell at the joints.

5.3.5. Mechanical and Electrical Services

Mechanical services do not present problems different from those encountered in other types of concrete construction. The possible exception is the attachment of various lines and ducts to the structure, where care should be taken to preclude significant concentrated loads from being attached directly to the shell,rather than to some auxiliary elements capable of distributing such loads. Electrical conduits should not be placed within the thickness of the shell, unless the diameter of the conduit is less than one tenth of the thickness of the shell. In addition, parallel conduits should be spaced a minimum of 6 shell thicknesses apart (within the plane of the shell surface), and care should be taken to ensure that the conduits do not interfere with and do not displace shell reinforcement.

5.4. ECONOMICS

5.4.1. General Remarks

The cost of design and construction of a concrete thin shell structure varies greatly not only with the type and dimensions but also with the location of the project. Availability of construction materials and equipment, availability of skilled labor, finally also the availability of design aids such as computers, all affect the final cost of the project to a very great extent. For this reason, it is not possible to consider the economics of shell design and construction so that the discussion would be applicable on all continents and in all countries. The remarks contained in this article are of necessity restricted in scope and applicability. It is assumed that, in each case, the economic aspects of a project involving shells need to be examined.

In general, concrete thin shell structures utilize materials effectively in that the ratio of material required to enclose space to the volume of space enclosed is smaller than for other structural systems. Further, the material itself is predominantly the rather inexpensive concrete, with a relatively small amount of reinforcing. However, successful construction of shells requires highly skilled design and construction personnel, and hence the labor-to-materials cost ratio tends to be higher, perhaps, than for other structural systems constructed in the same location.

5.4.2. Design Cost

The cost of design of a shell structure depends primarily on the geometry of the shell. In some cases, such as circular cylindrical shells, spherical domes, hyperbolic paraboloids of typical spans, and prismatic folded plates, the cost of design is of the same order of magnitude as the cost of design of other structures. Thin shells of more complex geometry cost significantly more to design, unless the solution can be obtained with the help of some existing computer program well understood by the user — it seems safe to say that the cost of development of a custom designed computer program intended for a specific project is prohibitive. It may be well worth it to change the geometry of a proposed shell structure even at the expense of the amount of material used so that it becomes one of the well established shapes, or shapes for which an established well functioning computer program is readily accessible.

5.4.3. Construction Cost

For a shell of a given geometry, the cost of construction materials — concrete and reinforcing — is essentially fixed to within some narrow limits. The primary component of cost over which the designer and the builder have a significant degree of control is the cost of formwork and scaffolding. Therefore the primary effort at reducing the cost of construction of concrete thin shells has been directed over the years at a reduction of the cost of forms. Several approaches are possible.

Geometry of the shell: The cost of shell can be reduced by using particularly convenient geometry so that forms can e constructed of plane elements, or of linear elements. As an example, the formwork for a hyperbolic paraboloid shallow shell can be constructed of small rectangular or square plane (flat) elements, and the formwork for the cylindrical shell can be built using boards placed in the direction of the directrix. As a rule, then, shallow shells and shells in the form of developable or ruled surfaces will offer some savings in construction cost.

Cost of form material: Typical form material is wood or plywood. Where this material is not available, or where it is expensive, attempts have been made to use cheaper, more readily available materials. A great many approaches have been tried. Earth forms were used in the construction of shell-like footings, in the construction of large-span domes, and in the construction of shells formed of a number of elements. Paper reinforced with welded wire mesh and suspended from permanent reinforcement has been used in the construction of inverted domes. Prefabricated reinforced concrete forms which are eventually incorporated into the finished structure have been used with great success in the construction of a number of ribbed shells of different geometries.

Re-use of forms: Where possible, the cost of forms can be reduced if the forms are reused. This approach applies to projects involving a number of shells of the same geometry, to large surfaces where the shell forms may be moved from one segment of a surface to another, and to shells prefabricated from a number of smaller precast identical elements. By reusing the form a great many times, the cost of it per unit of area of the shell surface may become significantly smaller than if the traditional methods of construction were used.

Method of depositing concrete: The cost of forms can be reduced on occasion by appropriately adjusting the method of depositing concrete. As an example, where the slope of the shell exceeds the limits of applicability of one-sided form, the need to use two-sided forms might be obviated through the use of pneumatically deposited concrete.

Precasting: Precasting represents a different approach to construction which may result in cost savings not only due to the possible reduction in the cost of forms, but also in reductions due to more industrialized methods of production. In addition, in some countries, the cost of factory labor is less than the cost of field labor. The necessary condition for applicability of prefabrication of shells is that their geometry be suitable; in addition, relatively heavy construction equipment must be available if full advantages of prefabrication are to accrue.

5.4.4. Detailing

The influence of detailing on the cost of construction is the same in shells as in other structural systems. Attempt should be made to use the same types of detailing repetitively, aimed at reducing the amount of material and labor while maintaining the efficiency of the detail.

CHAPTER 6

SELECTED BIBLIOGRAPHY OF SHELLS

6.1. GENERAL REMARKS

The literature of concrete shells is vast and no attempt is made here to present a comprehensive bibliography. References included here were selected on the basis of their importance, utility and accessiblity to concrete shell and folded plate designers. An attempt was made to include shell literature from many different countries and in many different languages. Because of the restrictions of space, to keep this bibliography within manageable proportions some references of unquestioned historical or theoretical importance are not necessarily included if the information contained in them is available in other, more readily accessible texts. The sole exception to this policy is the group of references cited in Chapter 3 of these Recommendations and listed in paragraph 6.2 of the bibliography. It should be noted that many of the references listed in this chapter contain extensive bibliographies.

Information on the design, analysis and construction of concrete shells and folded plates continues to appear in many technical, research and professional publications. A special mention should be made in this connection of the *Bulletin of the International Association for Shell and Spatial Structures,* published in Madrid, Spain; this *Bulletin of the IASS* is the only regularly appearing periodical devoted exclusively to shell and spatial structures.

The bibliography is organized as follows: references cited in Chapter 3 are grouped in paragraph 6.2; selected bibliography of paragraph 6.3 is further divided into separate groupings such as textbooks, and works devoted to some specific aspect of shell design, analysis or construction. Separate groupings are listed for the Proceedings of Shell Symposia organized by the IASS or by related organizations, and for the few existing shell regulations from various countries of the world.

The manner in which a publication is listed follows a generally recognized format. In the case of a periodical, its name is followed by the volume number, issue number (where available), the year of publication in parenthesis, and page numbers. The use of the original title proved impractical. English translation of the title, followed by the identification of the language of the original, is used instead. It should be noted that a great many of the publications listed have been translated into a number of languages; in such cases, only one reference is given, not necessarily to the original publication.

6.2. REFERENCES CITED IN THE TEXT

1. Aron, H., "The equilibrium and motion of an infinitely thin curved elastic shell" (in German), *Journal f. reine und ang. Math.* v. 78 (1874).
2. Love, A. E. H., "On the small free vibrations and deformations of thin elastic shells," *Phil. Trans. Royal Society (London),* v. 17A, pp. 491-546 (1888).
3. Love, A. E. H., *A treatise on the mathematical theory of elasticity,* fourth edition, New York, Dover Publications (1927).
4. Sanders, J. L., *An improved first-approximation theory for thin shells,* NACA TR R-24 (1959).
5. Koiter, W. T., "A consistent first approximation in the general theory of thin elastic shells," *Proc. of the IUTAM Symposium on the theory of thin elastic shells, Delft 1959,* North Holland, Amsterdam (1960), pp. 12-33.
6. Flügge, W., *Statics and dynamics of shells,* (in German), Berlin, Julius Springer-Verlag (1934).

7. Lur'e, A. I., "The general theory of thin elastic shells" (in Russian), *Prikl. Mat. Mekh.*, v. 4, no. 7 (1940).

8. Byrne, R., *Theory of small deformations of a thin elastic shell,* University of California (Los Angeles) Publications in Mathematics, N.S., v. 2, (1944), pp. 103-152.

9. Reissner, E., *Small bending and stretching of sandwich-type shells,* NACA Report 975 (1950).

10. Koiter, W. T., "A systematic simplification of the general equations in the linear theory of thin shells," *Proc. Kon. Acad. Wettenschapen afd. Nat. Amsterdam,* May 1961.

11. Donnell, L. H., "A new theory for the buckling of thin cylinders under axial compression and bending," *Trans. ASME,* v. 56, (1934), pp. 795-806.

12. v. Karman, T., and H. S. Tsien, "The bending of thin cylindrical shells under axial compression," *J. of the Aeronautical Sciences,* v. 8, (1941), p. 303.

13. Jenkins, R. S., *Theory and design of cylindrical shell structures,* The O. N. Arup Group of Consulting Engineers, London, May 1947.

14. Vlasov, V. Z., "Basic differential equations in the general theory of elastic shells" (in Russian), *Prikl. Mat. Mekh.,* v. 8, no. 2 (1944).

15. Goldenveizer, A. N., *Theory of elastic thin shells* (trans. from Russian), New York, Pergamon Press (1961).

16. Green, A. E., and W. Zerna, *Theoretical elasticity,* Oxford, Oxford University Press (1954).

17. Novozhilov, V. V., *The theory of thin shells* (trans. from Russian), Groningen, The Netherlands, P. Nordhoff Ltd. (1959).

18. Lundgren, H., *Cylindrical shells, Vol. I - Cylindrical Roofs,* The Institution of Danish Civil Engineers, Copenhagen, The Danish Technical Press, (1949).

19. Evans, H. R. and K. C. Rockey, "A critical review of the methods of analysis for folded plate structures," *Proc. Inst. Civ. Engrs.,* v. 49 (1971), pp. 171-192.

20. Johnson, C. D. and T. Lee, "Long nonprismatic folded plate structures," *J. Struct. Div. ASCE,* v. 49, ST 6 (1968), pp. 1457-1484.

21. Collatz, L., *The numerical treatment of differential equations,* third edition, New York, Springer-Verlag (1966).

22. Kraus, H., *Thin elastic shells,* New York, John Wiley & Sons (1967).

23. Soare, M., *Application of finite difference equations to the analysis of shells* (in French), Paris, Editions Eyrolles (1962).

24. Bushnell, D., B. O. Almroth and F. Brogan, "Finite difference energy method for nonlinear shell analysis," *Computers and Structures,* v. 1 (1971), pp. 361-387.

25. Zienkiewicz, O. C., *The finite element method,* third edition, New York, McGraw-Hill (1977).

26. Lin, C. S. and A. C. Scordelis, "Nonlinear analysis of R.C. shells of general form," *J. Struct. Div. ASCE,* v. 101, ST 3 (1975).

27. Kollar, L. and E. Dulacska, *Buckling of shells* (in German), Budapest, Akademiai Kiado, (1975).

6.3. SELECTED BIBLIOGRAPHY OF SHELLS

6.3.1. Proceedings of IASS Symposia

1. **Madrid, 1959:** "Non-traditional construction processes of shell structures," *Bulletin IASS,* nos. 1 through 9 (1960).
2. **Dresden, 1961:** "Precast shells," summary published in the *Bulletin IASS* no. 10 (1961).
3. **Delft, 1961:** "Shell research," A. M. Haas and A. L. Bouma, eds., North-Holland Publishing Co., Amsterdam (1962).
4. **Brussels, 1961:** "Simplified calculation methods," A. Paduart and R. Dutron, eds., North-Holland Publishing Co., Amsterdam (1962).
5. **Paris, 1962:** "Hanging roofs, continuous metallic shell roofs, and superficial lattice roofs," N. Esquillan and Y. Saillard, eds., North-Holland Publishing Co., Amsterdam (1963).
6. **San Francisco, 1962:** "World conference on shell structures," S. J. Medwadowski *et al.,* eds., Publication No. 1187, National Academy of Sciences-National Research Council, Washington, D.C. (1964).
7. **Warsaw, 1963:** "Non-classical shell problems," W. Olszak and A. Sawczuk, eds., North-Holland Publishing Co., Amsterdam (1964).
8. **Southampton, 1964:** "The theory of arch dams," J. R. Rydzewski, ed., Pergamon Press, London (1965).
9. **Budapest, 1965:** "Shell structures in engineering practice," Kultura Budapest, Budapest (1966); cf. *Bulletin IASS* Nos. 25, 26 and 27 (1965-66).
10. **Bratislava, 1966:** "Tower-shaped steel and reinforced concrete structures," R. Lopez Palanco *et al.,* eds., IASS, Madrid (1967).
11. **Leningrad, 1966:** "Problems of interdependence of design and construction of large span shells for industrial and civic buildings," Gosstroy USSR, Moscow (1967), vol. 1 and 2.
12. **Stuttgart, 1967:** "Pneumatic structures," Institut für Modellstatik, Technische Hochschule, Stuttgart (1968).
13. **Mexico City, 1967:** "The application of shell structures in architecture," (proceedings not published); cf. *Bulletin IASS* Nos. 38, 39, 40, 41 and 42.
14. **Weimar, 1968:** "Pipes and tanks," summary published in *Bulletin IASS,* No. 46 (1971).
15. **The Hague, 1969:** "Tower shaped structures," Institute T. N. O., Delft (1969).
16. **Madrid, 1969:** "Progress of shell structures in the last ten years and its future development," IASS, Madrid (1969-70).
17. **Vienna, 1970:** "Folded plates and prismatic structures," K. Krapfenbauer, ed., R. Krapfenbauer, Vienna (1973), vol. 1 & 2.
18. **Beirut, 1871:** "Future development in spatial structures; application of shells to low cost housing," (proceedings not published).
19. **Honolulu, 1971:** "Hydromechanically loaded shells," R. Szilard, ed., The University Press of Hawaii, Honolulu (1972).
20. **Tokyo, 1971:** "Tension structures and space frames," Y. Yokoo, T. Nakamura, K. Heki and S. Kawamata, eds., Architectural Institute of Japan, Tokyo (1972).
21. **Calgary, 1972:** "Shell structures and climatic influences," P. G. Glockner *et al,* eds., The University of Calgary, Calgary (1973).
22. **Delft, 1972:** "Pneumatic structures," E. Haug, ed., Stichting PDOB, Delft (1973).
23. **Kielce, 1973:** "Industrialized spatial and shell structures," Technical University of Kielce (1973).

24. **Haifa, 1973:** "Prefabricated shells," J. Glüch, ed., Technion City, Haifa.
25. **Udine, 1974:** "Folded plates and spatial panel structures," (proceedings not published).
26. **Bratislava, 1975:** "Cable structures," House of Technology, CSVTS, Bratislava.
27. **Montreal, 1976:** "Space enclosures," P. Fazio *et al.*, eds., Building Research Center, Concordia University, Montreal (1976).
28. **Alma Ata, 1977:** "Lightweight shell and space structures for normal and seismic zones," V. Raizer *et al.*, eds., MIR Publishers (1977).
29. **Darmstadt, 1978:** "Nonlinear behavior of reinforced concrete spatial structures," (in publication).
30. **Morgantown, West Virginia:** "Shells and spatial structures: the development of form," (in publication).

6.3.2. Proceedings of related conferences and symposia.

1. **London, 1952:** "Proceedings of a symposium on concrete shell roof construction," P. J. Witt, ed., Cement and Concrete Association, London (1954).
2. **Cambridge, Mass., 1954:** "Proceedings of the conference on thin concrete shells," MIT Press (1954).
3. **Oslo, 1957:** "Proceedings of the second symposium on concrete shell roof construction," A. Aas-Jacobsen *et al.*, eds., Teknisk Ukeblad, Oslo (1958).
4. **Delft, 1959:** "Proceedings of the I.U.T.A.M. Symposium on the theory of thin elastic shells," W. T. Koiter, ed., North-Holland Publishing Co., Amsterdam (1960).
5. **Langley Research Center, Hampton, Virginia, 1962:** "Collected papers on instability of shell structures," NASA TN C-1510, NASA, Washington, D.C. (1962).
6. **Houston, 1966:** "Proceedings of the symposium on the theory of shells to honor Lloyd Hamilton Donnell," D. Muster, ed., University of Houston, Houston, Texas (1967).
7. **London, 1966:** "Space structures, a study of methods and developments in three-dimensional construction resulting from the international conference on space structures, University of Surrey, September 1966," R. M. Davies, ed., Blackwell Scientific Publications, Oxford and Edinburgh (1967).
8. **Copenhagen, 1967:** "Proceedings, second I.U.T.A.M. symposium on theory of thin shells," F. I. Niordson, ed., Springer-Verlag, Berlin (1969).
9. **New York, 1970:** "Proceedings of the ACI symposium on concrete thin shells," S. J. Medwadowski, W. C. Schnobrich and A. C. Scordelis, eds., American Concrete Institute, SP-28, Detroit, Michigan (1971).
10. **Pasadena, 1972:** "Proceedings of the symposium on thin shell structures: theory, experiment and design," Y. C. Fung and E. E. Sechler, eds., Prentice-Hall, Inc., Englewood Cliffs, New Jersey (1974).

6.3.3. General texts

1. Ramaswamy, G. S., *Design and construction of concrete shell roofs,* New York, McGraw-Hill, (1976).
2. Rühle, H., *Spatial roof structures,* vol. 1, *Concrete, wood, ceramics;* vol. 2, *Steel* (in German), Berlin, VEB Verlag für Bauwesen (1969, 1970).

6.3.4. Conceptual design

1. De Syllas, L., "Domes, vaults and the development of shell roofing," pp. 3-17 of *Symposium on concrete shell roof construction,* London (1952).

2. Faber, C., *Candela: the shell builder,* New York, Reinhold Publishing Corporation (1963).
3. Huybers, P., "Development of standardized plastics building structures," *Bulletin IASS,* No. 54 (1974).
4. Joedicke, J., *Shell architecture,* (with contribution by W. Bauersfeld and H. Kupfer), New York, Reinhold Publishing Corporation (1963).
5. Medwadowski, S. J., "Conceptual design of shells," pp. 15-39 of *Proceedings, ACI Symposium on Concrete Thin Shells,* ACI Publication SP-28, Detroit (1971).
6. Michaels, L., *Contemporary structure in architecture,* New York, Reinhold Publishing Corporation.
7. Nervi, P. L., *Structures,* (trans. from Italian), New York, F. P. Dodge Corporation (1956).
8. Nervi, P. L., *Aesthetics and technology in building,* (trans. from Italian), Boston, Harvard University Press (1965).
9. Raafat, A., *Reinforced concrete in architecture,* New York, Reinhold Publishing Corporation.
10. Riani, P., *Kenzo Tange,* London, Hamlyn (1969).
11. Siegel, C., *Structure and form in modern architecture,* (trans. from German), New York, Reinhold Publishing Corporation (1962).
12. Slawinska, J., *Expression of forces in modern architecture* (in Polish), Arkady, Warsaw (1969).
13. Torroja, E., *The structures of Eduardo Torroja,* (trans. from Spanish), New York, Reinhold Publishing Corporation, (1958).
14. Torroja, E., *Philosophy of structures,* (trans. from Spanish by J. J. Polivka and Milos Polivka), University of California Press, Berkeley and Los Angeles (1958).

6.3.5. Analysis, detailing and construction.

In addition to the references listed below, refer to paragraphs 6.3.1, 6.3.2, 6.3.3 and 6.3.4; refer also to *Bulletin IASS* and to other professional and research journals. The state of art of design, analysis and construction of concrete shells is covered up to the date of publication by reference 16 of paragraph 6.3.1 (IASS Madrid Symposium of 1969), and reference 9 of paragraph 6.3.2 (ACI New York Symposium of 1970).

1. Ambartsumyan, S. A., *Theory of anisotropic shells* (in Russian), Moscow, GIMFL (1961).
2. Baumann, Th., "On orthogonal reinforcing of shells and folded plates" (in German), *Bauingenieur,* vol. 47 (1972), pp. 367-377.
3 Belluzzi, O., *Structural mechanics,* vol. III (in Italian), Bologna, Nicalo Zanichelli Editore (1958).
4. Billington, D. P., *Thin shell concrete structures,* New York, McGraw-Hill, (1965).
5. Brush, D. O. and B. O. Almroth, *Buckling of bars, plates and shells,* New York, McGraw-Hill, (1974).
6. Candela, F., "General formulas for membrane stresses in hyperbolic paraboloid shells," *Journal ACI,* vol. 32 (1960), pp. 353-371.
7. Cicala, P., *Linear shell theories,* Levrotto and Bella, Torino (1978).
8. Csonka, P., *Membrane shells* (in German), Berlin, Verlag von Wilhelm Ernst und Sohns, (1966).
9. Dabrowski, R., "Analysis of prestressed cylindrical shell roofs," *J. Struct. Div., Proc. ASCE,* vol. 89, ST 5 (1963), pp. 91-116.

10. Esquillan, N., "The shell vault of the Exposition Palace, Paris," *J. Struct. Div., Proc. ASCE,* vol. 86, ST 1 (1960), pp. 41-70.

11. Esquillan, N., "Olympic ice stadium at Grenoble," *Journal ACI,* vol. 66 (1969).

12. Flügge, W., *Stresses in shells,* Berlin, Springer-Verlag, (1960).

13. Gerard, G. and H. Becker, "Handbook of structural stability," Part III, *Buckling of curved plates and shells,* NACA TN 3783, and Part VI, *Strength of stiffened curved plates and shells,* NACA TN 3786, NACA, Washington, D.C., (1957 and 1958).

14. Girkmann, K., *Surface structures* (in German), fifth edition, Vienna, Springer-Verlag, (1961).

15. Goldberg, J. E. and H. L. Leve, "Theory of prismatic folded plate structures," *Publications of the International Association for Bridge and Structural Engineering,* vol. 17 (1957), pp. 59-86.

16. Gravina, P. B. J., *Theory and calculation of rotational shells* (in German), Berlin, Springer-Verlag, (1961).

17. Guerra, G., *Statics and technology of construction of ancient and modern domes* (in Italian), Naples, Instituto di Architettura Tecnica, (1958).

18. Haas, A. M., "Design of thin concrete shells," Vol. 1, *Positive curvature index,* Vol. 2, *Negative curvature index,* New York, John Wiley & Sons, Inc., (1967).

19. Jenkins, R. S. and O. N. Arup, "The design of a reinforced concrete factory at Brynmawr, South Wales," *Proceedings,* Institution of Civil Engineers (London), vol. 2, Part 3 (1953), p. 345 and following.

20. Khaidukov, G. K. and V. V. Shugaev, "The methods for testing models for thin-walled shell structures," *Proceedings,* RILEM Symposium on Methodology and technique of testing structures, Bucharest (1969).

21. Kollar, L., "On the behavior of shells in the postbuckling range," *Bulletin IASS,* No. 39 (1969), pp. 41-51.

22. Lin, T. Y. and B. W. Young, "Two large shells of post-tensioned precast concrete," *Civil Engineering Magazine - ASCE,* vol. 35, No. 7 (1965).

23. Medwadowski, S. J., "Bending of thin shell conical frustum segments," *Proceedings,* IASS Symposium on simlpified calculation methods, Brussels, 1961, North-Holland Publishing Co., Amsterdam (1962), pp. 398-415.

24. Manyhard, I., *Surface structures, statical calculations and dimensioning* (in Hungarian), Budapest, Müszki Könyvkiado, (1966).

25. Paduart, A., *Introduction to shell roof analysis*" (trans. from French), London, OR Books, Ltd., (1966).

26. Paduart, A., "Checking of the reinforcement sections of a concrete shell," *Bulletin IASS,* No. 53 (1973), pp. 39-44.

27. Parme, A. L., "Hyperbolic paraboloids and other shells of double curvature," *Proceedings, ASCE,* vol. 82, ST 5 (1956), pp. 1-32.

28. Pfluger, A., *Elementary statics of shells* (in German), second edition, Berlin, Springer-Verlag, (1957).

29. Popov, E. P. and S. J. Medwadowski, "Membrane stresses in hyperbolic paraboloidal shells circular in plan," *Publications, Int. Assoc. for Bridge and Struct. Eng.,* vol. 20 (1960), pp. 283-297.

30. Pozo, F. del, "Thin cylindrical shells formed by a triangular mesh" (in French), *Preliminary Publication,* 5th Congress of the Int. Assoc. of Bridge and Struct. Eng., Lisbon (1956).

31. Rosenblueth, E., "Shell reinforcement not parallel to principal stresses," *Journal ACI,* vol. 27, No. 1 (1955), pp. 61-71.

32. Rühle, H., "The use of gypsum models in experimental studies of doubly curved corrugated and conical shells, and applications" (in German), *Proceedings,* IASS Symposium Delft 1961, North-Holland Publishing Co., Amsterdam (1962).

33. Samartin, Q. A. and J. Martinez, "Dynamic analysis of translational shells," *Proceedings,* IASS Symposium of Alma Ata, 1977, MIR Publishers (1977), vol. 1, pp. 368-387.

34. Scordelis, A. C., "Analysis of cylindrical shells and folded plates," *Proceedings,* ACI Symposium on concrete thin shells, ACI Publication SP-28, Detroit (1971), pp. 207-236.

35. Tedesko, A., "Construction aspects of thin shell structures," *Journal ACI,* vol. 49, (1943), pp. 505-520.

36. Timoshenko, S. P. and S. Woinowsky-Krieger, "Theory of plates and shells," second edition, McGraw-Hill, New York (1960).

37. Yitzhaki, D., *Prismatic and cylindrical shell roofs,* Haifa Science Publishers, Haifa, Israel (1958).

6.3.6. Tabular and graphical aids to analysis.

1. Apeland, K. and E. P. Popov, *Design tables for translational shells,* University of California, Berkeley (1963).

2. Anon., *ASCE Manual No. 31, design of cylindrical concrete shell roofs,* New York, ASCE (1952; reprinted 1954 and 1956); see also ref. 25 below.

3. Baker, B. R. and C. B. Cline, Jr., "Influence coefficients for thin smooth shells of revolution subjected to symmetric loads," *J. Appl. Mech.,* vol. 29 (1962), pp. 335-339.

4. Baltrukonis, J. H., "Influence coefficients for edge loaded, thin, conical frustums," *J. Appl. Mech.,* vol. 26 (1959), pp. 241-245.

5. Brebbia, C. A. and A. J. Ferrante, "On the parabolic velaroidal shell," *Bulletin IASS,* No. 38 (1969), pp. 31-48.

6. Caligo, D., and M. J. Schwartz, eds., "Calculation of circular cylindrical thin shells," (in Italian), Part IV of *Manuali per applicazioni tecniche del calcolo,* Rome, Edizioni Cremonese (1960).

7. Casacci, B. and J. Bosc, *Calculation of conical shells in bending* (in French), Paris, Dunod (1959).

8. Chinn, J., "Cylindrical shell analysis simplified by beam method," *Journal ACI,* vol. 30 (1959), pp. 1183-1192; discussion by Parme & Connor, *Journal ACI,* vol. 31 (1959), pp. 1584-1591.

9. Galletly, G. D., "Influence coefficients for hemispherical shells with small openings at the vertex," *J. Appl. Mech.,* vol. 22 (1955), pp. 20-24.

10. Galletly, G. D., "Bending of open-crown ellipsoidal shells," *Welding Research Council Bulletin No. 54* (1959).

11. Galletly, G. D., "Influence coefficients for open-crown hemispheres," *Trans. ASME,* vol. 82A, (1960), pp. 73-81.

12. Galletly, G. D., "Influence coefficients for toroidal shells of positive Guassian curvature," *Trans. ASME,* vol. 82B (1960), 60-68.

13. Galletly, G. D., "Edge influence coefficients for toroidal shells of negative Gaussian curvature," *Trans. ASME,* vol. 82B (1960), pp. 69-77.

14. Holland, I., *Design of circular cylindrical shells,* Oslo, Oslo University Press (1957).

15. Parme, A. L. and H. W. Connor, "Design constants for ribless concrete cylindrical shells," *Bulletin IASS*, No. 18 (1964), pp. 11-56.
16. Portland Cement Association, *Design of circular domes*, PCA ST 55, Skokie, Ill., USA.
17. Portland Cement Association, *Design of barrel shell roofs*, PCA ST 11, Skokie, Ill., USA.
18. Portland Cement Association, *Elementary analysis of hyperbolic paraboloid shells*, PCA ST 85, Skokie, Ill., USA.
19. Portland Cement Association, *Design constants for interior cylindrical concrete shells*, PCA Advanced Engineering Bulletin 1, PCA, Chicago (1960).
20. Portland Cement Association, *Direct solution of folded plate concrete roofs*, PCA Advanced Engineering Bulletin 3, PCA, Chicago (1960).
21. Portland Cement Association,, *Design constants for circular arch bents*, PCA Advanced Engineering Bulletin 7, PCA, Chicago (1963).
22. Portland Cement Association, *Design constants for circular and elliptical arches of variable thickness*, PCA Advanced Engineering Bulletin 8, PCA, Chicago (1963).
23. Portland Cement Association, *Parabolic arches of variable thickness*, PCA Advanced Engineering Bulletin 9, PCA, Chicago (1963).
24. Portland Cement Association, *Design constants for ribless concrete cylindrical shells*, PCA Advanced Engineering Bulletin 11, PCA, Chicago (1964).
25. Portland Cement Association, *Coefficients for design of cylindrical concrete shell roofs* (extension of design tables 2A and 2B in ASCE Manual No. 31), PCA, Chicago (1959); refer also to reference 2 of this article.
26. Rüdiger, D. and J. Urban, "Circular cylindrical shells, a tabular compilation for computing circular cylindrical shell structures of arbitrary dimensions" (in English and German), Leipzig, B. G. Teubner Verlagsgeselschaft (1955).
27. Szmodits, K., "Design constants for spherical shells," *Bulletin IASS*, No. 19 (1964), pp. 33-46.
28. Taylor, C. E and E. Wenk, "Analysis of stresses in the conical segments of shell structures," *Proc. 2nd U.S. Nat'l Congress of Appl. Mech.*, ASME (1955), pp. 323-331.
29. Watts, G. W. and H. A. Lang, "Stresses in a pressure vessel with a conical head," *Trans. ASME*, vol. 74 (1952), pp. 315-331.

6.3.7. Computer software: surveys and programs.

1. Gallagher, R. H., "Analysis of plate and shell structures," in *Proceedings of the symposium on application of finite element methods in civil engineering, Nov. 1969, Nashville, Tenn.*, W. H. Rowan and R. M. Hackett, eds., American Society of Civil Engineers (1969), pp. 155-205.
2. Bushnell D., "Thin shells," in *Structural mechanics computer programs, surveys, assessments and availability*, W. Pilkey *et al.*, eds., University Press of Virginia, Charlottesville (1974), pp. 277-359.
3. Fredriksson, B. and J. Mackerle, *Structural mechanics finite element computer programs — surveys and availability*, Dept. of Mech. Eng., Div. of Solid Mech., Linköping Institute of Technology, Linköping, Sweden (1975).
4. ANSYS (Analysis System)
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5. ASKA (Automatic System for Kinematic Analysis)
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14. STAGS I and STAGS II
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6.3.8. Surveys and bibliographies

1. Nash, W. A., *Bibliography of shells and shell-like structures,* David Taylor Model Basin Report 863, U.S. Navy Dept., Washington, D. C. (1954); also *Bibliography of shells and shell-like structures 1954-1956,* Reports, Dept. of Eng. Mech., Eng. & Ind. Exp. Stn., University of Florida (1957).
2. Naghdi, P. M., "A survey of recent progress in the theory of thin elastic shells," *Applied Mechanics Reviews,* vol. 9 (1956).
3. Oniashvili, O. D., "Analysis of shells and some thin-walled spatial structures," Chapter 5 of *Structural mechanics in the USSR 1917-1957* (trans. from Russian), G. Herrman, ed., Pergamon Press (1960).
4. Nash, W. A., "Recent advances in the buckling of thin shells," *Applied Mechanics Reviews,* vol. 13, No. 3 (1960).
5. Naghdi, P. M., "Foundations of elastic shell theory," in *Progress in solid mechanics,* I. N. Sneddon and R. Hill, eds., North-Holland Publishing Company, Amsterdam (1963), vol. IV, chapter 1.
6. Habip, L. M., "A survey of modern developments in the analysis of sandwich structures," *Applied Mechanics Reviews,* vol. 18, No. 2 (1965).
7. Kalins, A., "Dynamic problems of elastic shells," *Applied Mechanics Reviews,* vol. 18, No. 11 (1965).
8. Hutchinson, J. W. and W. T. Koiter, "Postbuckling theory," *Applied Mechanics Reviews,* vol. 23, (1970).
9. Naghdi, P. M., "The theory of shells and plates," *Handbuck der Physik* vol. VI a/2, W. Flügge, ed., Springer-Verlag, Berlin (1972), pp. 425-640.

6.3.9. Shell codes, regulations and standards

1. *Instructions for design of reinforced concrete roofs* (in Russian), Academy of Construction and Architecture of USSR, Moscow (1961).
2. Indian Standard IS:2210 — 1962, *Criteria for the design of reinforced concrete shell structures and folded plates,* Indian Standards Institution, Manak Bhavan, New Delhi (1963); translated into Turkish, Mimarlik Fakültesi Ankara, Yavin No. 22 (1974).
3. New Zealand Standard Specification NZSS 1826:1964, *Design and construction of shell roofs,* New Zealand Standards Institute, Wellington (1964).
4. American Concrete Institute Committee 334, "Concrete shell structures — practice and commentary," *Journal of the ACI,* vol. 61 (1964), pp. 1091-1108.
5. American Concrete Institute Standard 318-77, *Building code requirements for reinforced concrete;* also *Commentary 318-77C,* American Concrete Institute, Detroit, Michigan (1977).
6. *Report 50, Shell Roofs* (in Dutch), CUR, Commissie voor uitvoering van research ingesteld door de betonvereniging, Zoetmeer, The Netherlands (1972).