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# Behavior of a Continuous Slab Prestressed in Two Directions

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**Elastic behavior and ultimate strength of a continuous concrete slab prestressed in two directions were investigated. The slab, consisting of four panels, was supported at nine points and simulated a flat slab. Prestressing was accomplished by means of unbonded post-tensioned cables. Experimental values for moments, deflections, and reactions were compared with theoretical values obtained by the elastic plate theory and by approximate theories used in present design methods.**

IN THE PAST FEW YEARS, a large number of structures using flat slabs prestressed in two directions have been erected by the lift-slab method of construction. These slabs have performed well in service. An essentially crack-free slab having little or no deflection under working load can be obtained by proper prestressing. As yet few experimental data are available regarding the behavior of such slabs through the elastic and plastic ranges and finally under ultimate load. In an earlier study[1] the case of a square slab prestressed in two directions and supported at four points was considered. The investigation reported herein considers the case of a square slab simply supported at nine points, simulating nine column supports.

The purpose of this investigation was to determine the behavior, through and above the elastic range, of a continuous concrete slab prestressed in two directions. Some of the questions which the investigation endeavored to answer for this type of slab were: (1) Is the elastic plate theory valid up to the appearance of cracks? (2) Can the cracking load be predicted by the elastic plate theory using the flexural tensile strength as determined by plain concrete specimens? (3) What is the physical behavior of the slab through the plastic range and finally under ultimate load? Does it deflect excessively? Is failure sudden or gradual? (4) Can the ultimate strength be predicted by available theories for ultimate strength of slabs? (5) Are the present approximate methods of design sufficiently accurate for predicting the behavior of such a slab? If not how should they be changed or modified? (6) What is the distribution of moments in the slab in the elastic and plastic ranges, under various loading conditions and under prestress alone?

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Professors Lin and Scordelis have collaborated on a number of research problems in prestressed concrete, and have had several technical papers published on the subject in the ACI Journal. ● R. ITAYA, engineer, T. Y. Lin and Associates, Los Angeles, was formerly with the Division of Architecture, State of California, Sacramento. An ACI member, Mr. Itaya formerly served as research engineer at the University of California. In addition to slab study, he has participated in research work on prestressed columns and prestressed shells.

To answer the questions posed as well as to study other practical design problems involved in this type of slab, a 15 x 15-ft slab 3 in. thick and supported at nine points was first prestressed and then subjected to a series of loading tests under laboratory controlled conditions to ensure accuracy. In the final test the slab was loaded to failure so that information on its behavior at ultimate load was obtained.

## Notation

The letter symbols used in this paper are generally defined when they are introduced. The most frequently used symbols are listed below for convenient reference:

$A_s$ = area of prestressing steel

$a$ = side dimension of a single panel

$D$ = flexural rigidity of slab $= \frac{Eh^3}{12\,(1-\mu^2)}$

$E_c$ = modulus of elasticity of concrete

$E_s$ = modulus of elasticity of steel

$f_c'$ = compressive strength of 6 x 12-in. cylinders

$f_t'$ = modulus of rupture of concrete

$h$ = slab thickness

$M_x, M_y$ = bending moments per unit length acting on sections normal to the $x$- and $y$-axes, respectively

$M_{xy}\ M_{yx}$ = torsional moments per unit length acting on sections normal to the $x$- and $y$-axes, respectively

$n$ = number of subdivisions of Span $a$ for finite difference elements

$q$ = load per unit area on slab

$R$ = reaction at supports

$w$ = deflection

$x, y$ = Cartesian coordinates with the center of the slab as origin

$\lambda$ = length of finite difference elements $= a/n$

$\mu$ = Poisson's ratio

## EXPERIMENTAL PROGRAM

### Description of test slab

The test slab, Fig. 1, measured 15 x 15 ft in plan and was 3 in. thick. Supports were placed 7 ft on centers in both directions. The test slab represented about a one-third scale model of slabs in actual practice. A rocker and roller arrangement at each support point permitted the necessary rotations and horizontal movements so that no restraints were introduced at these points.

The slab was post-tensioned with 12 cables in each direction, spaced 15 in. on centers. Each cable consisted of a single ¼-in. high strength steel wire greased and placed in a plastic tube to provide for post-tensioning. The profile for all of the cables running in both directions is shown in Fig. 2.

In addition to the prestressing steel, two layers of wire mesh were placed over each support to cover an area 18 x 18 in. This steel was included to help prevent the possibility of local failures at the supports.

### Materials

Concrete for the slab was proportioned for a minimum strength of 5000 psi at 28 days as measured by 6 x 12-in. cylinders. The mix contained 7 sacks of Type I cement per cu yd of concrete. The water-cement ratio was 5.7 gal. per sack. The aggregate consisted of Livermore Valley sand and gravel having a maximum size of ¾ in. Batch proportions by weight based on saturated surface dry conditions were: water, 0.51; cement, 1.00; sand, 2.05; gravel, 2.56. Slump was 5 in. and placement of the concrete took approximately 35 min. The slab was cast in place on forms, cured moist with damp burlap for 10 days, and then left air dry until testing.

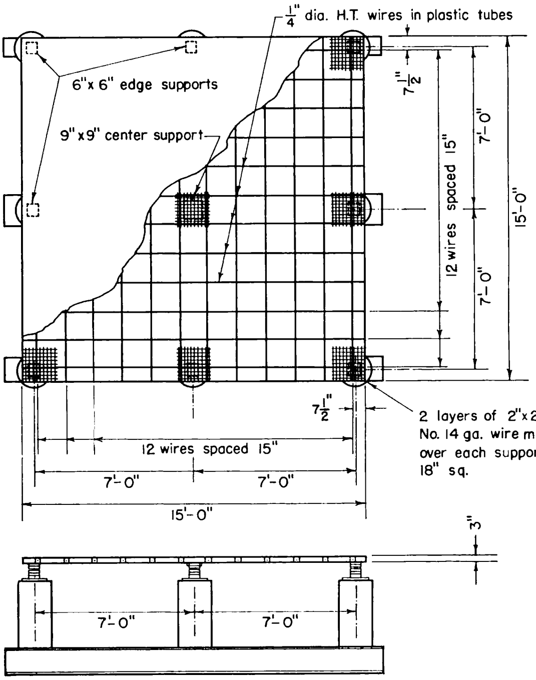



Fig. 1—Plan and elevation of test slab, showing steel arrangement

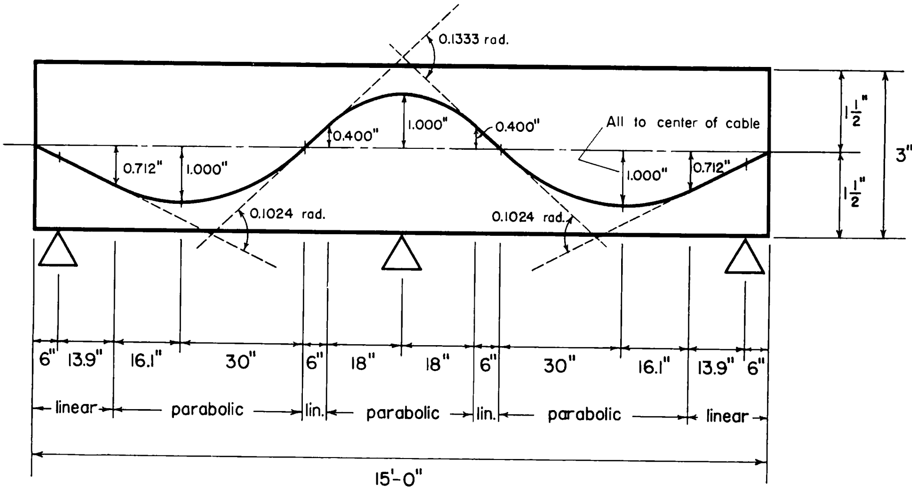



Fig. 2—Typical cable profile

Control specimens were cured in the same manner as the slab. Three cylinders were tested in compression at 7, 14, 22, and 28 days. One cylinder was tested at 43 days. Average values for the compressive strength, modulus of elasticity, and Poisson's ratio for these cylinders are given in Table 1. Three 6 x 6 x 20-in. beams were tested under third-point loading on an 18-in. span, at 14 and 28 days. Average modulus of rupture values obtained for these beams are also listed in Table 1.

Six samples of the prestressing wire were tested in tension on a 10-in. gage length and the following average values were found: proportional limit, 170 ksi; yield point as measured by the 0.2 percent offset method, 218 ksi; ultimate strength, 253 ksi; modulus of elasticity, 29,400 ksi; and ultimate elongation, 6.7 percent.

## Method of loading

Cable prestress was applied by a 30-ton capacity hydraulic jack which had been accurately calibrated.

For the slab loading it was desired to provide a uniform load on each of the four panels independently. This was accomplished by using four plastic air bags placed between the top of the slab and a ½-in. plywood sheathing which was supported by a heavy steel frame. Each of the bags covered one panel or one-quarter of the total slab. By introducing air pressure into each of the bags independently, the magnitude of the uniform load on each panel could be accu-

TABLE 1 — PROPERTIES OF CONCRETE

| Age, days | 7 | 14 | 22 | 28 | 43 |
|---|---|---|---|---|---|
| Compressive strength, psi, of 6 x 12-in. cylinders | 3590 | 4337 | 4985 | 5455 | 5940 |
| Secant modulus of elasticity at 1000 psi | — | $3.40 \times 10^6$ | — | $3.50 \times 10^6$ | — |
| Poisson's ratio | — | 0.14 | — | 0.14 | — |
| Modulus of rupture, psi, 6x6x20-in. beams on 18-in. span under third-point loading | — | 540 | — | 480 | — |

rately controlled. Air pressure in each of the bags was measured with a water manometer. With the arrangement used the load on the slab could be controlled and measured to the nearest 2 or 3 psf.

This method of loading had been successfully used in earlier laboratory tests[1] and again proved to be an excellent technique for producing uniform loadings.

### Instrumentation

The instrumentation was designed to measure the following quantities:

1. Strains on top and bottom surfaces of the slab.
2. Force at the ends of the prestressing cables.
3. Strains in the prestressing steel between the two ends.
4. Reactions at each support point.
5. Deflections at various locations.

Seventy SR-4 gages were used to measure strains on the top and bottom surfaces of the slab. Because of symmetry, it was only necessary to instrument one-eighth of the slab with strain gages to obtain all of the necessary strain data.

The initial prestress force at the jacking end was measured for all cables with the pressure gage attached to the hydraulic jack. On six of the cables a simple dynamometer, accurate to the nearest 20 lb, was attached at the nonjacking end so that the prestress force existing there could be measured. To distribute the prestress more uniformly the jacking and nonjacking ends were alternated along each side of the slab.

To get a record of the prestress force at several points along the cables, two SR-4 Type A12 gages were mounted directly onto each of the six cables which had dynamometers at the ends. These gages were waterproofed and properly protected and worked well through the entire test program.

Reactions at each support point were measured by pressure meters between the slab and the bearing plates. The pressure meters, calibrated in a testing machine prior to their use with the slab, could measure a reaction to the nearest 35 lb.

To obtain deflections at various points simple dial gages bearing on the bottom surface of the slab were used. The dial gages, which had a least count of 0.001 in., were used until the load approached its ultimate value after which they were removed. Deflections were then obtained at the center of each panel by level readings taken on graduated scales hung from the bottom of the slab.

### Tests conducted

The test program was designed to study the distribution of moments and deflections and the behavior of the slab under a variety of conditions.

The four panels of the slab were identified as PI, PII, PIII, and PIV, with PI being the instrumented panel. In each direction the center pair of cables and each pair of cables closest to an edge were designated as column strip cables while the remaining cables were designated as middle strip cables. Thus, in each direction there were a total of six column strip cables and six middle strip cables. Table 2 lists the various stages of loading in chronological order.

## ANALYTICAL STUDIES

Present design of prestressed concrete lift slabs is based on approximate procedures.[2] The procedure normally used is to divide the structure into a series of bents, each consisting of a row of columns or supports and strips of supported slabs, each strip bounded laterally by the

## TABLE 2 — TEST PROGRAM

| Date | Test No. | Stage | Load Data: Column strip cable Prestress, lb (at jack) | Middle strip cable Prestress lb (at jack) | Panel Live Load, psf: PI | PII | PIII | PIV |
|---|---|---|---|---|---|---|---|---|
| 2/4 | | Slab cast on forms | — | — | — | — | — | — |
| 2/17 | | 25 percent Prestress, nominal (forms still in place) | 1960 | 1960 | — | — | — | — |
| 2/18 | | 25 percent Prestress + dead load (forms removed, dead load on from now on) | 1960 | 1960 | — | — | — | — |
| 3/3 and 3/4 | 1a | 50 percent C.S.* —25 percent MS* | 3750 | 1960 | — | — | — | — |
| | b | 50 percent C.S. — 50 percent M.S. | 3750 | 3750 | — | — | — | — |
| | c | 100 percent C.S. — 50 percent M.S. | 7500 | 3750 | — | — | — | — |
| | d | 100 percent C.S. — 100 percent M.S. | 7500 | 7500 | — | — | — | — |
| 3/10 and 3/11 | 2a and 2b | Design load and repeat | 7500 | 7500 | 0 | 0 | 0 | 0 |
| | | | 7500 | 7500 | 80 | 80 | 80 | 80 |
| | | | 7500 | 7500 | 0 | 0 | 0 | 0 |
| 3/12 and 3/13 | 3 | Skip loading | 7500 | 7500 | 0 | 0 | 0 | 0 |
| | | | 7500 | 7500 | 60 | 60 | 60 | 60 |
| | a | | 7500 | 7500 | 120 | 60 | 60 | 60 |
| | b | | 7500 | 7500 | 60 | 120 | 60 | 60 |
| | c | | 7500 | 7500 | 60 | 60 | 120 | 60 |
| | d | | 7500 | 7500 | 60 | 60 | 60 | 120 |
| | | | 7500 | 7500 | 60 | 60 | 60 | 60 |
| | | | 7500 | 7500 | 0 | 0 | 0 | 0 |
| 3/14 and 3/17 | 4a and 4b | Cracking load and repeat | 7500 | 7500 | 0 | 0 | 0 | 0 |
| | | | 7500 | 7500 | 240 | 240 | 240 | 240 |
| | | | 7500 | 7500 | 0 | 0 | 0 | 0 |
| 3/18 | 5 | Load to failure | 7500 | 7500 | 0 | 0 | 0 | 0 |
| | | | 7500 | 7500 | 360 | 360 | 360 | 360 |

*C.S. indicates column strip; M.S. indicates middle strip.

center line of the panel on either side of the center line of columns or supports. A series of such bents is first taken longitudinally and then transversely through the building. Each bent is analyzed for the various loading conditions which may come on the slab. The slab moments obtained, which are for a full panel width, are then apportioned to the column strips and middle strips. The percentage going to each is dependent on edge and support conditions, but has usually been taken for uniform loads as 45 and 55 percent going to the middle and column strips, respectively.

From these analyses an envelope of maximum and minimum slab moments in the longitudinal and transverse directions is plotted. Using these moment envelopes the magnitude and location of the prestress force required at each section to keep the stresses within certain allowable limits can be determined.

Where the slab-to-column connection is not rigid or where the column stiffness relative to the slab stiffness is small, the problem described above reduces to the analysis of a continuous beam in each direction rather than a bent. This method of analysis, often called the "beam method," is an approximate one, since it reduces the two-way action involved in the actual slab to the one-way action of a beam in each direction.

A more precise determination of the slab moments and deflections can be found by using the elastic plate theory, which does take into account the two-way action of the slab. For continuous slabs this requires considerable mathematical effort and for this reason it is not generally used directly in design.

**Beam method**

The test slab was designed on the basis of the elastic beam theory. The bending moments for a two-span continuous beam uniformly loaded are obtained quite easily. The effect of prestress force alone can be obtained by converting the force in the cable into upward and downward equivalent loads.[2] By combining the effect of uniform load and prestressing, the design and cracking loads can be determined.

According to the beam theory, the live load design capacity for the test slab based on no tensile stress in the concrete was found to be 76 psf for a prestress force of 6840 lb in every cable. The 6840 lb per cable was the average prestress force along the length of the cable, not the jacking force. The live load for cracking based on a modulus of rupture of 480 psi and a prestress force of 6840 lb per cable was 194 psf.

Deflection at the center of a panel can be approximated by use of the beam theory. The usual method is to superimpose deflection due to beams in two directions. For example, the deflection at the center of a panel is calculated by adding the deflections at the center of two beams, a 1 ft wide beam passing over the columns and another 1 ft wide beam orthogonal to the first and passing through the center of the panel. Each beam is loaded uniformly.

For a slab similar to the test slab, the deflection at the center of the panel, under a uniform load $q$, calculated by the beam theory is:

$$\Delta = 0.125 \frac{qa^4}{Eh^3}$$

in which $a$ is the panel dimension, $E$ is the modulus of elasticity, and $h$ is the slab thickness. The same deflection from the plate theory, for Poisson's ratio, $\mu = 0.15$, is:

$$\Delta = 0.149 \frac{qa^4}{Eh^3}$$

A comparison between deflections from the beam theory and the plate theory is shown in Fig. 3 for slabs having various edge conditions. Data for the deflection of a panel supported on four columns and a typical interior panel may be found in References 1 and 3.

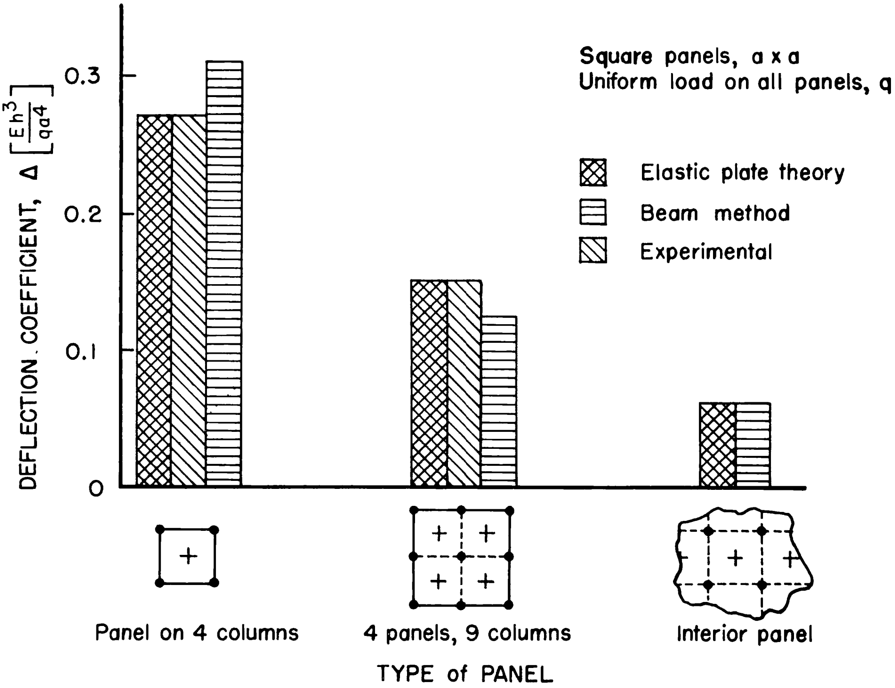



Fig. 3—Comparison of deflections at center of panel by plate theory and approximate beam method for uniform load

## Ultimate load

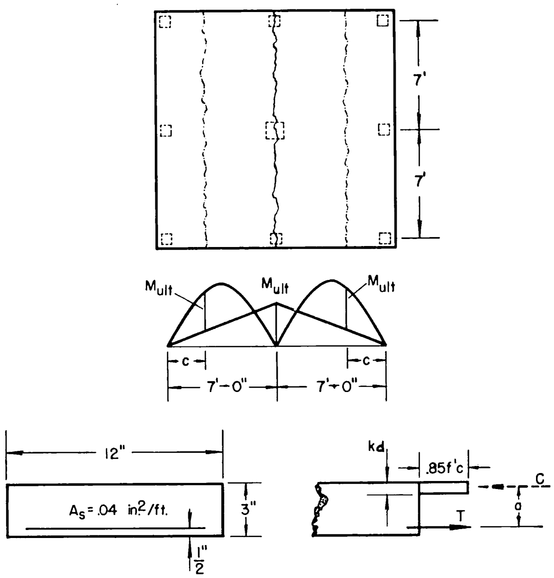
The ultimate load capacity of the slab can be estimated from the yield line theory for slabs. A study of possible yield line patterns indicates that the pattern shown in Fig. 4, which is the same as that for a continuous beam, is critical.

An assumption must be made as to the stress in the prestressing steel at ultimate. Because the cables are unbonded, it is unlikely that the steel stress will reach its ultimate or even its yield strength. Assuming a steel stress of 200 ksi at ultimate load, with $f_c' = 6$ ksi, the ultimate moment capacity of a 1 ft wide section of the slab in Fig. 4 is 19.5 in-kips. A yield line analysis gives the distance $c$ equal to 2.9 ft and an ultimate load capacity of 385 psf. Deducting the 38 psf dead load leaves an ultimate live load capacity of 347 psf.

## Elastic plate theory

A 14 x 14-ft plate on nine supports under uniform load was analyzed by the elastic plate theory to determine the general applicability of the theory to calculation of moments, deflections, and reactions in prestressed slabs. Note that the test slab differed slightly from the plate considered in this theory since a 6-in. overhang on all sides resulted in a 15 x 15-ft test slab. The slab analyzed by the theory had its sup-

Fig. 4 — Ultimate strength theory




ports at the edges and center of a 14 x 14-ft plate. The effect of the overhang is small when considering moments and deflections and is thus neglected in comparisons between these experimental and theoretical values.

Analytical determination of deflections and moments in an elastic plate involves solution of the Lagrangian differential equation for plates, subject to the appropriate boundary conditions imposed by its loading and supports. This differential equation and the means by which it can be solved are thoroughly discussed by Timoshenko.[3] For a plate supported by concentrated forces the necessary differential equations cannot be solved explicitly because of the rather complicated boundary conditions. Various approximate methods are available for the solution; one of the simplest in concept is the method of finite differences.

The finite difference method substitutes finite elements for infinitesimal elements and reduces the solution of a differential equation to the solution of a set of linear simultaneous equations. A detailed discussion of this method and finite difference expressions for various derivatives are given by Salvadori and Baron.[4]

Because of symmetry only one-eighth of the plate need be considered in the analysis. As a first approximation a coarse mesh with finite elements having a mesh size $\lambda = a/2$ was used to analyze the plate. As a second approximation a mesh size having $\lambda = a/4$ was then used. This second approximation involved the solution of 33 linear simultaneous equations, which was obtained with the aid of an IBM 701 computer. No programming was necessary as a standard routine was available for the solution of 45 or less linear simultaneous equations. Using a mesh size of $\lambda = a/8$ would have required solving 101 linear simultaneous equations and thus was not attempted.

The accuracy of the theoretical results obtained from the first and second approximations can be increased by Richardson's extrapolation procedure. This procedure is discussed by Salvadori and Baron.[4] Theoretical moments, deflections, and reactions obtained from the second approximation and by Richardson's extrapolation procedure are summarized in Table 3. Poisson's ratio was taken as 0.15.

## EXPERIMENTAL RESULTS AND DISCUSSION

A summary of the cases considered and the experimental and analytical studies made is given in Table 4. At the start of each live load test the average prestress force measured the same in all cables, 6840 lb per cable or 140 ksi.

Curves shown for $M_x$ represent the bending moment distribution along a section $x =$ a constant. The system of coordinates used is shown in Fig. 5. The gross bending moment at a section is therefore the area under the $M_x$ curve, $\Sigma\ (M_x \cdot \Delta y)$. Since each of the reactions was measured independently, the gross bending moment at a section could also be calculated by the principles of statics to obtain a check between the strain measurements and the reaction measurements. This latter value is represented by the term "Statics: $M$" in the moment curves. Experimental moments were obtained at sections $x = 0$, 1.75, 3.5, 5.25, and 7.0 ft.

### Uniform prestress case

The moments produced by the equal prestress of 6840 lb per cable are plotted for two sections in Fig. 6 and 7. $M_x$ is the bending moment distribution across the section whereas $M_y$ is the bending moment diagram along the section. The difference between the gross moments as meas-

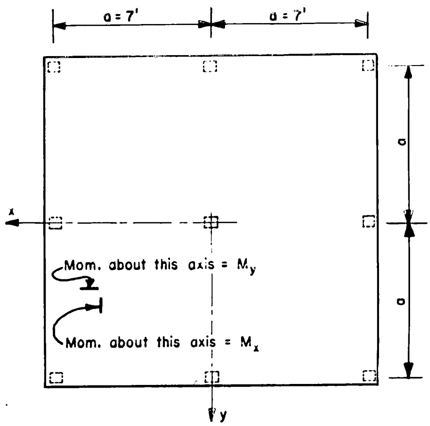



Fig. 5—System of coordinates

## TABLE 3 — THEORETICAL VALUES OBTAINED FROM PLATE THEORY FOR A UNIFORMLY LOADED PLATE SUPPORTED AT NINE POINTS

| Location (coordinates) | | Bending moment, $\frac{M_x}{qa^2}$ | | Bending moment, $\frac{M_y}{qa^2}$ | | Twisting moment, $\frac{M_{xy}}{qa^2}$ | | Deflection, $\frac{wD}{qa^4}$ | | Reaction, $\frac{R}{qa^2}$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | $y$ | Second approximation | Extrapolation | Second approximation | Extrapolation | Second approximation | Extrapolation | Second approximation | Extrapolation | Second approximation | Extrapolation |
| 0 | 0 | —0.2448 | —0.2842 | —0.2448 | —0.2842 | — | — | 0 | 0 | 1.392 | 1.427 |
| 0 | $a/4$ | —0.1035 | | —0.0209 | | 0 | | 0.0067 | | | |
| 0 | $a/2$ | —0.0470 | —0.0486 | 0.0950 | 0.0929 | 0 | 0 | 0.0110 | 0.0100 | | |
| 0 | $3a/4$ | —0.0628 | | 0.0932 | | 0 | | 0.0088 | | | |
| 0 | $a$ | —0.1657 | —0.1891 | 0 | 0 | — | — | 0 | 0 | 0.513 | 0.516 |
| $a/4$ | $a/4$ | 0.0099 | | 0.0099 | | —0.0243 | | 0.0101 | | | |
| $a/4$ | $a/2$ | 0.0126 | | 0.0736 | | —0.0058 | | 0.0129 | | | |
| $a/4$ | $3a/4$ | 0.0154 | | 0.0682 | | 0.0176 | | 0.0112 | | | |
| $a/4$ | $a$ | 0.0253 | | 0 | | 0.0382 | | 0.0053 | | | |
| $a/2$ | $a/2$ | 0.0607 | 0.0561 | 0.0607 | 0.0561 | —0.0019 | —0.0077 | 0.0143 | 0.0127 | | |
| $a/2$ | $3a/4$ | 0.0653 | | 0.0555 | | 0.0047 | | 0.0133 | | | |
| $a/2$ | $a$ | 0.0872 | 0.0844 | 0 | 0 | 0.0098 | 0.0112 | 0.0090 | 0.0079 | | |
| $3a/4$ | $3a/4$ | 0.0595 | | 0.0595 | | —0.0108 | | 0.0118 | | | |
| $3a/4$ | $a$ | 0.0811 | | 0 | | —0.0213 | | 0.0071 | | | |
| $a$ | $a$ | 0 | 0 | 0 | 0 | — | — | 0 | 0 | 0.139 | 0.127 |

ured by the strain gages and as obtained from statics using measured reactions and cable forces range from 5 to 26 percent. The moments calculated from the measured reactions and cable forces are sensitive to the cable location. A change in the cable location of 0.1 would eliminate the differences between the two calculated moments.

Also plotted in Fig. 6 and 7 are theoretical values for bending moments as obtained by the beam theory. There is good agreement between theoretical and experimental values at all sections indicating that the beam theory is fairly accurate in predicting the moments for this case.

Deflections at the center of a panel are plotted against prestress force in Fig. 8. Again agreement between experimental values and those obtained by the beam theory is relatively good.

## TABLE 4 — SUMMARY OF CASES CONSIDERED

| Case | Quantities obtained: Experimental moments, deflections and reactions | Beam theory theoretical moments and deflections | Plate theory theoretical moments, deflections, and reactions |
|---|---|---|---|
| Uniform prestress (equal prestress in all cables) | X | X | |
| Unequal prestress (1.8:1.0 for column to middle strip) | X | | |
| Skip loading (live load on one panel) | X | | |
| Uniform load (live load on four panels) | X | X | X |

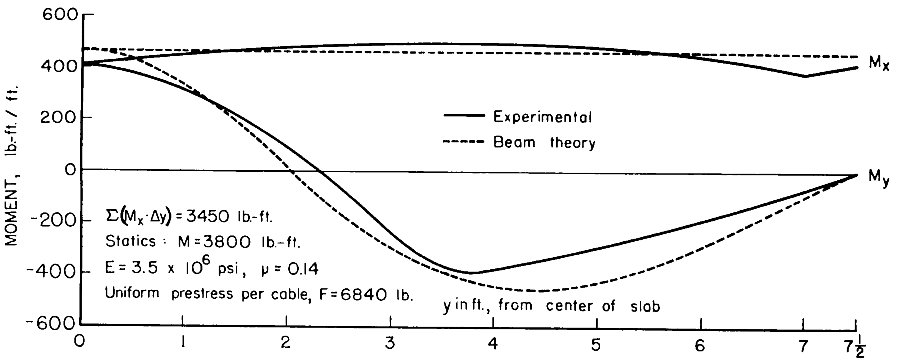



Fig. 6—Moments at $x = 0$ ft due to uniform prestress

## Unequal prestress case

Fig. 9 shows experimental values for the moments at two sections caused by prestressing the cables in the column strips nominally twice as much as the cables in the middle strips. The average force was 6840 lb each in the column strip cables, and 3320 lb each in the middle strip cables. The width of column and middle strips by definition is $\frac{1}{4} \times 14 \times 12 = 42$ in. With the cable spacing used it can be found that the total force delivered to the column strip over the outer row of columns, the middle strip, and the column strip over the center row of columns is 19.2, 10.0, and 16.4 kips, respectively. The net effect has been termed a nominal 1.8:1 ratio of prestress.

## Skip loading case

The moments due to a uniform load of 100 psf on one panel are shown in Fig. 10 and 11. A modulus of elasticity of 4,150,000 psi and a Poisson's ratio of 0.14 were used for the calculations. In both the skip load and the uniform load case, the modulus of 4,150,000 psi was used to maintain consistency between the gross moments as calculated from strain measurements and the gross moments as calculated from the equations of statics using the reaction measurements.

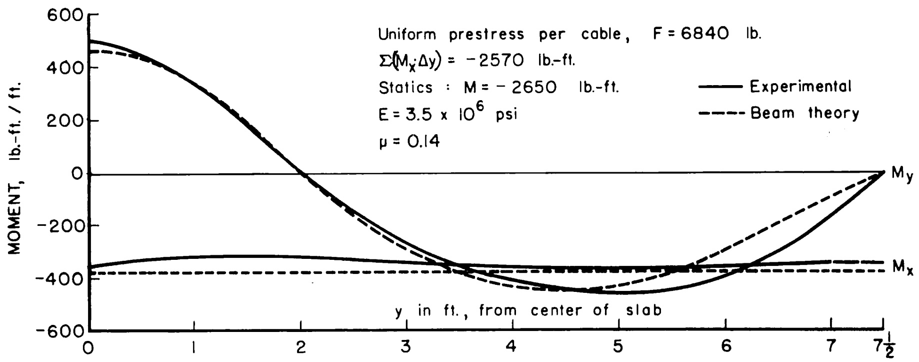



Fig. 7—Moments at $x = 3.5$ ft due to uniform prestress

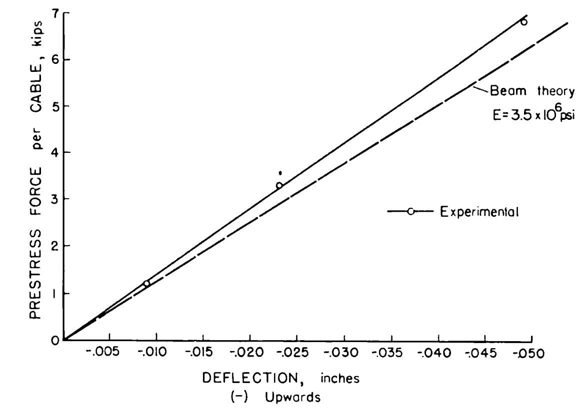



Fig. 8—Prestress force versus deflection at center of panel for uniform prestress case

Comparing Fig. 6 and 7 for uniform prestress with Fig. 10 and 11 for skip load, the skip load moments in the loaded panel are counteracted well by the prestressing moments. In the unloaded panels, however, the moments tend to add, causing an increase in tensile stresses already existing. At $x = -5.25$, $y = 3.5$, the top fiber stresses caused by full prestress, 100 psf live load on one panel, and dead load are +40, +50, and −65 psi, respectively, the total tensile stress being +25 psi, a relatively small value.

## Uniform load case

The final test on the slab consisted of subjecting it to an increasing uniform live load on all four panels until failure. Because of the test arrangement a visual inspection for cracks on the top surface of the slab during the test could not be made. However the first tensile crack, as indicated by strain readings, seems to have occurred over the center

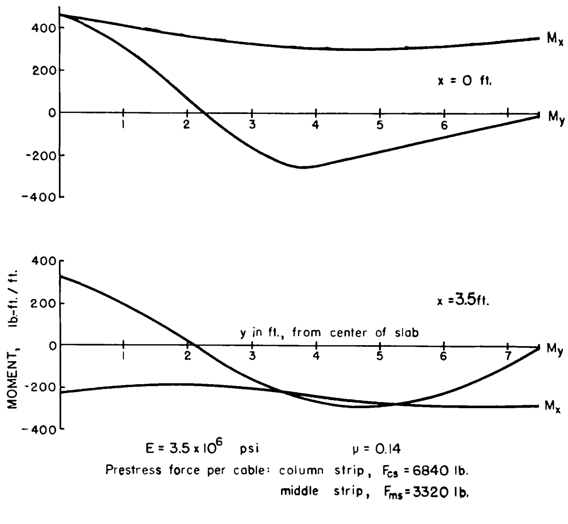



Fig. 9—Moments at $x = 0$ and 3.5 ft due to 1.8:1 prestress (experimental values only)

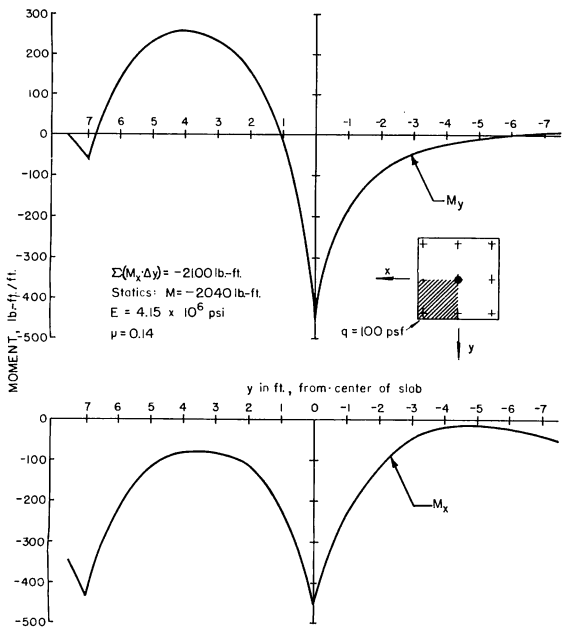



Fig. 10—Moments at $x = 0$ ft due to load on one panel (experimental values only)

support at a live load of 100 psf. This crack was localized and did not pass through the gage 6 in. away from the center support until the load reached 160 psf.

The first visual observation of cracks occurred at a load of 290 psf. Cracks were then observed at the edges of the slab at Points 1 and 2 shown in Fig. 12.

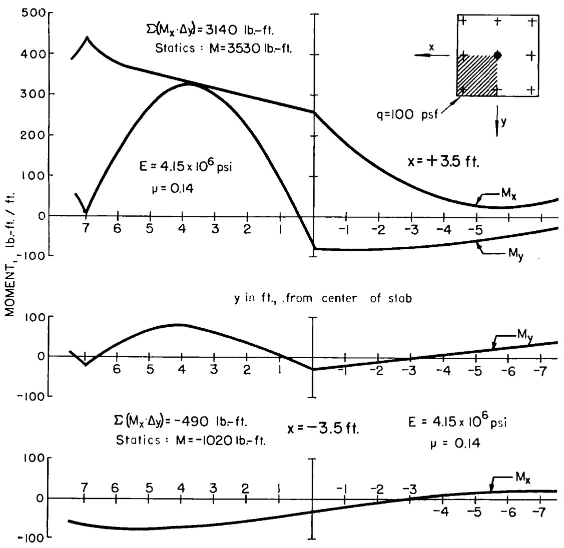
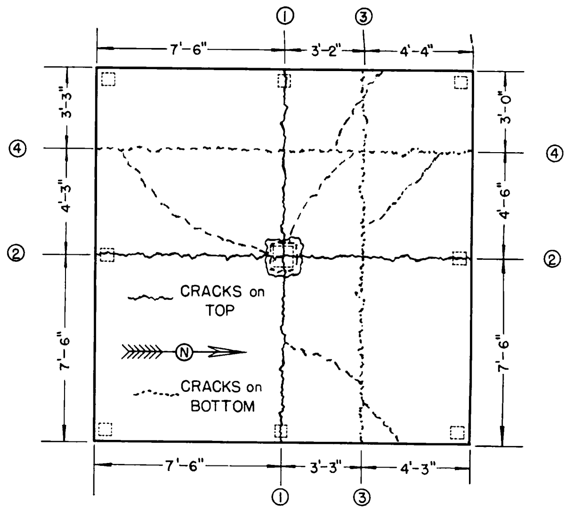



Fig. 11—Moments at $x = \pm 3.5$ ft due to load on one panel (experimental values only)

Fig. 12—Crack pattern at failure ($q_{LL} = 362$ psf) for uniform load on entire slab




The first cracks on the bottom of the slab were observed at a load of 330 psf. They began at the edges of the slab and extended inward about 2 ft on Lines 3-3 and 4-4. At a load of 347 psf the crack along Line 3-3 opened to 1/8 in. wide and extended across the width of the slab. At 356 psf a similar crack occurred along Line 4-4.

After extensive flexural cracking ultimate failure occurred at a live load of 362 psf with the center support punching through the slab. The failure occurred directly around the edges of the 9 x 9-in. center support at a shear angle of about 45 deg. The crack pattern at ultimate load is shown in Fig. 12.

The moments at two sections caused by a uniform load of 100 psf, which was within elastic range, are shown in Fig. 13 and 14. Agreement between the moments calculated from the strain measurements $\Sigma (M_x \cdot \Delta y)$, and the moments calculated by statics from the reaction measurements, "Statics: $M$", is very good.

Fig. 13 and 14 also show a comparison between the experimental moments obtained from strain measurements and theoretical moments obtained by the elastic plate theory. Agreement again is good, the only major discrepancy exists for the moments near the center support, i.e., at $x = 0$, $y = 0$. At this point it can be seen that the theoretical value obtained using Richardson's extrapolation procedure tends to approach the experimental value. This indicates that if a finer mesh were used in the finite difference solution the theoretical and experimental values would probably be in close agreement at this point. Based on the above comparisons it can be stated that the elastic plate theory can be used to accurately predict the magnitude and distribution of moments in a prestressed concrete slab loaded within the elastic range.

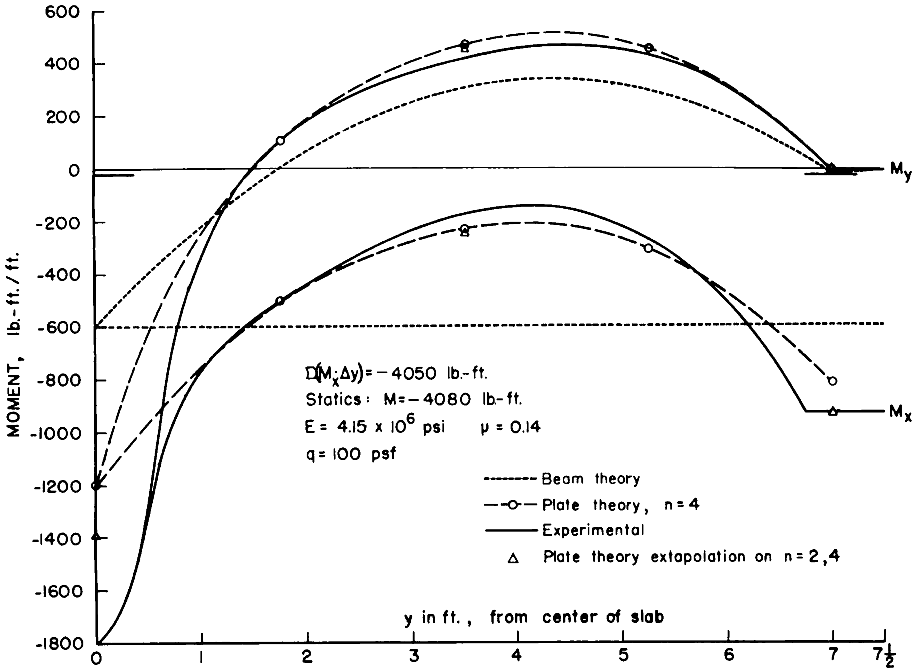



Fig. 13—Uniform load moments at $x = 0$ ft

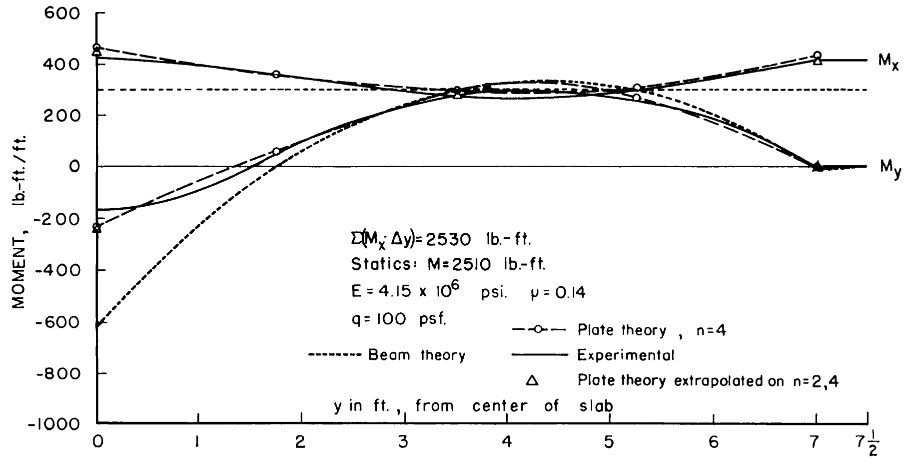



Fig. 14—Uniform load moments at $x = 3.5$ ft

The experimental moments may also be compared to the theoretical moments obtained by the beam theory. It is apparent that at sections of high moment such as at $x = 0$ the distribution of the experimental moments $M_x$ is not uniform across the section, and that some distribution of the gross moment obtained by the beam theory should be made, with a higher percentage going to the column strips than to the middle strips.

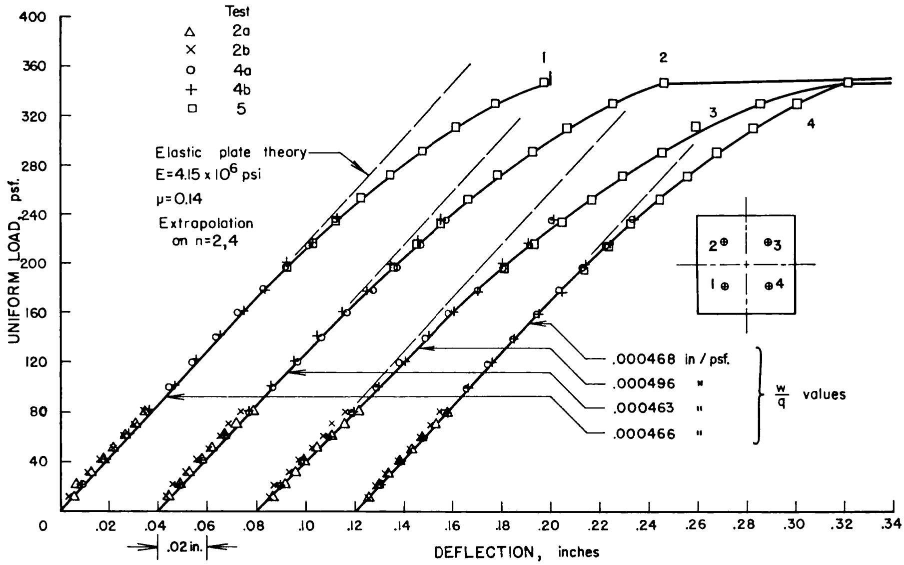



**Fig. 15—Uniform load-deflection curves for panel centers**

The load-deflection curves for the panel centers are shown in Fig. 15. Elastic behavior of the slab continued until the load reached around 160 to 200 psf. The maximum deflection just prior to the serious cracking at the load of 347 psf was 0.2 in. The deflections increased sharply when the large cracks occurred.

Fig. 16 shows the typical behavior of the prestressing steel for uniform live load on the slab. Note that the stress in prestressing steel increased sharply when serious cracking occurred at a load of 347 psf. Strain measurements in the cables at Line 1-1 of Fig. 12 indicated that just prior to failure the average stress in the steel at this section was 170 ksi.

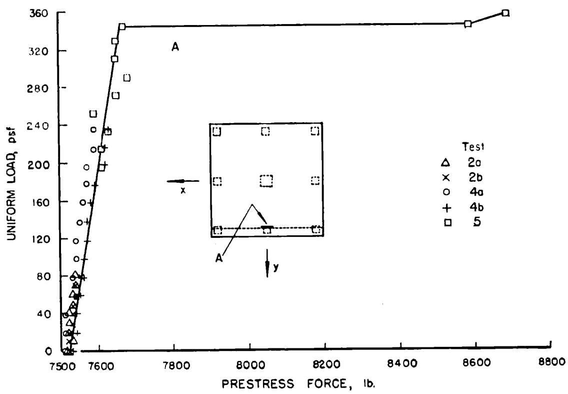



**Fig. 16—Typical behavior of prestressing steel under uniform load**

## SUMMARY AND CONCLUSIONS

The behavior of a 15 x 15-ft prestressed slab, 3 in. thick and supported at nine points was studied analytically and experimentally for a variety of loading conditions.

Using the beam theory as usually applied in present design methods, the uniform live load design capacity for the slab based on no tension in the concrete was 76 psf; the live load for cracking based on a modulus of rupture of 480 psi was 194 psf; and the ultimate live load based on a steel stress of 200 ksi was 347 psf. The actual behavior of the slab was ideal as a structure. Localized cracking occurred on the top surface near the center support at a live load somewhere between 100 and 160 psf. First cracks on the bottom surface occurred at a live load of 330 psf. The maximum deflection just prior to serious cracking at 347 psf was only 0.20 in. The slab failed at a live load of 362 psf. Just prior to failure the maximum deflection was 2.1 in.

On a basis of the studies made the following conclusions are advanced.

1. The elastic plate theory may be used satisfactorily to predict the behavior of a prestressed concrete slab loaded within the elastic range.

2. The cracking load has little practical significance since initial cracking is localized at points of high moment. The slab can sustain large increases in load before widespread cracking takes place.

3. Moments due only to equal prestress in all cables can be calculated with sufficient accuracy for design purposes by the beam method.

4. A quantitative study of the results indicates that for elastic behavior under uniform load the total negative moment calculated by the beam method should be distributed approximately 75 percent to the column strips and 25 percent to the middle strips, while the total positive moment calculated by the beam method should be distributed approximately 60 percent to the column strips and 40 percent to the middle strips. Thus the ratio of 55/45 common in present practice should be modified.

5. Deflections under uniform load obtained by the beam method are within 15 percent of those obtained experimentally or by the elastic plate theory.

6. For the design live load acting on one panel only, in combination with dead load and uniform prestress, small and relatively insignificant tensile stresses are produced in the slab.

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