

Safe Shear Design of Large, Wide Beams

Adding shear reinforcement is recommended

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igh-rise buildings often employ large, wide reinforced concrete beams to carry column loads from upper stories over required column-free space on the lower floors. Thus, in the new 400,000 ft² (37,000 m²) engineering building at the University of Toronto, called the Bahen Center, large, wide beams carry loads from the eight upper levels across a 278-seat lecture hall on the ground floor. In the design of such important members, the engineer must balance economy and safety while satisfying architectural constraints.

The trend to optimize structures to achieve the lowest possible construction costs, however, often means that only items specifically required by the governing building code will be provided. In this regard, there is concern that the shear design provisions for large, wide beams in the current ACI standard (ACI 318-02) can be unsafe and, hence, should not be used for such members. This article will use the specific situation of the Bahen Center beams to investigate the possibility of shear failure of large, wide beams. We will describe a recent laboratory experiment in which this type of member was loaded to failure and highlight situations where the current ACI 318 shear provisions can be unsafe. Finally, we will give recommendations for the safe shear design of large, wide beams.

BAHEN CENTER BEAMS

Figure 1(a) summarizes typical structural details of the large, wide beams in the Bahen Center. The beams which are made from 5000 psi (35 MPa) concrete with a maximum aggregate size of 3/4 in. (19 mm)—span 40 ft (12 m), have an overall depth of 71 in. (1800 mm), and



Fig. 1: Shear design of transfer beams at the Bahen Center (1 in. = 25.4 mm; 1 ksi = 6.89 MPa; 1 psi = 6.89 kPa; 1 k = 4.45 kN)

are 98 in. (2500 mm) wide. This chosen width resulted in the need for a large quantity of stirrups to resist the total factored shear force of about 1540 k (6800 kN). As an alternate design, the engineer might have considered increasing the width of the member until stirrups were no longer required. The ACI 318-02¹ standard specifies that "beams with total depth not greater than...0.5 of the width of the web ... " are exempt from the requirement to provide minimum shear reinforcement wherever the "factored shear force V_{μ} exceeds one-half the shear strength provided by concrete, ΦV_c ." To maximize the benefit of the increased concrete area, the engineer may have chosen a specified concrete strength of 10,000 psi (70 MPa). Figure 1(b) shows the resulting alternate design. Because the self-weight of the beam has increased, the member must now resist a total factored shear force of 1620 k (7200 kN). For this alternate beam. the shear capacity is calculated according to ACI 318-02, Section 11.3.1.1, as

$$\varphi V_c = 0.75 \times 2 \times \sqrt{1000} \times 165 \times 167 = 0.75 \times 2210 = 1660 \text{ k} (7370 \text{ kN})$$

Both designs shown in Fig. 1 meet the shear design requirements of ACI 318-02. The beam without stirrups, however, will be considerably simpler to construct and, hence, in some market areas, may offer cost savings. Bearing in mind that beams without stirrups can fail in shear with no warning and with a sudden loss of capacity, the key question becomes "Does the large, wide beam without stirrups have adequate safety?"

BACKGROUND TO ACI 318-02 SHEAR DESIGN PROVISIONS

The ACI 318-02 shear design provisions are, in most respects, similar to those first published in ACI 318-71.²



Fig. 2: Shear failure of 36-in.-deep (900 mm) beams in Air Force Warehouse, Shelby, OH (Photograph courtesy of Chester P. Siess)

These 1971 procedures were developed in the years immediately following the August 1955 partial collapse of a large warehouse at Wilkins Air Force Base in Shelby, OH (Fig. 2).³ Prior to this collapse, the ACI standard⁴ permitted stirrups to be omitted at locations where the shear stress under service loads was calculated to be less than $0.03 f_c'$. Thus, in the warehouse beams, the stirrups had been stopped when the calculated shear stress due to the 80 lb/ft² (4 kPa) dead load plus the 20 lb/ft² (1 kPa) live load was less than $0.03 \times 3000 = 90$ psi (0.6 MPa). The 36-in.-deep (900 mm) beams failed, under dead load only, at a shear stress less than 70 psi (0.5 MPa).

Experiments⁵ conducted by the Portland Cement Association (PCA) on 1/3 scale models of the warehouse beams indicated that, without axial tension, the beams could resist a shear stress of about 150 psi (1.0 MPa) prior to failure. The application of a tensile stress of about 200 psi (1.4 MPa), however, reduced the shear capacity by about 50%. PCA concluded that the tensile stresses caused by the restraint of shrinkage and thermal movements were the reason why the warehouse beams failed at such low shear stresses.

The shear stress at which stirrups could be omitted was reduced substantially in the 1963 ACI Code,⁶ and then, in 1971, the current provisions, which require at least minimum stirrups in nearly all beams, were introduced. These original minimum shear reinforcement provisions and the corresponding commentary sections are reproduced in Fig. 3. Note the reference to "unexpected tensile force" as the reason for providing minimum shear reinforcement. Also note that the commentary describes the only beams that are excluded from the requirement (to provide minimum shear reinforcement whenever v_u exceeds 1/2 of v_c) as "wide, shallow beams."

While the commentary clearly indicates that the 1971 Committee had wide, shallow beams in mind when formulating the exclusion of 11.1.1 (c), ACI 318 wording does not make this clear. In ACI 318-89,⁷ this apparent conflict between the code and the commentary was resolved by removing the reference to "wide, shallow beams" from the commentary. Essentially, the decision of ACI Committee 318 at that time was that if the web width of a beam is at least twice its total depth, the beam could be treated as a slab, irrespective of the depth of the beam.

Figure 4 summarizes the changes that have occurred over the last 50 years in the level of service load shear stress at which ACI 318 requires stirrups to be provided in flexural members. The ACI 318-02 basic expression for V_c , Eq. (11-3), is

$$V_c = 2\sqrt{f_c'} b_w d \tag{1}$$

and it is intended to conservatively estimate the shear failure load of sections not containing shear

ACI 318-71

Code 11.1 General reinforcement requirements 11.1.1 A minimum area of shear reinforcement shall be provided in all reinforced, prestressed, and non-prestressed concrete flexural members except: (a) slabs and footings

- (b) concrete joist floor construction defined by Section 8.8
- (c) beams where the total depth does not exceed 10 in., two and one-half times the thickness of the flange, or one-half the width of the web, whichever is greater
- (d) where v_u is less than one-half of v_c



reinforcement. This expression was formulated⁸ in 1962 and, at that time, it was not appreciated that for members without stirrups, the shear stress at failure decreases as the members become larger. This decrease in failure shear stress as member size increases is called the "size effect" in shear. Obviously, the 71 in. (1800 mm) deep alternate Bahen Center design will be influenced by this size effect.

SIZE EFFECT IN SHEAR

The most extensive experimental investigation of the size effect in shear was conducted by Shioya et al.9 and Shioya¹⁰ in Japan. The main results of this work are summarized in Fig. 5, where it can be seen that the shear stress at failure decreases, both as the member depth increases and as the maximum aggregate size decreases. The largest beam in this series spanned 118 ft (36 m), had an effective depth of 118 in. (3 m), a maximum aggregate size of 1 in. (25 mm), weighed nearly 500 tons (450 tonnes), and failed in shear at only 43% of the value predicted by Eq. (1) (Fig. 6). It is of interest that the beams tested by Shioya happened to have about the same concrete strength and the same percentage of longitudinal reinforcement as the Air Force Warehouse beams. Figure 5 plots the failure shear stresses for the prototype warehouse beam and the PCA model beam without axial tension. From this figure, it would seem that the more than 50% reduction in failure shear stress between the model and the prototype for the Air Force Warehouse beams was due primarily to the size effect in shear, rather than the influence of axial tensile stresses.

The simplest explanation of the size effect in shear is that the larger crack widths that occur in larger members reduce aggregate interlock. Crack widths increase nearly linearly both with the tensile strain in the reinforcement and with the spacing between cracks.¹¹ Shioya observed that the crack spacing at mid-depth of his beams was about equal to half the depth of the beams.¹⁰ Hence, for the same reinforcement strain, doubling the depth of the beam will double the crack widths at mid-depth.

Commentary

11.1.1 and 11.1.2 Stirrup reinforcement restrains the growth of inclined cracking and hence increases ductility and provides a warning in situations where in an unreinforced web the sudden formation of inclined cracking might lead directly to distress. Such reinforcement is of great value if a member is subjected to an unexpected tensile force or catastrophic loading. Accordingly, a minimum area of shear reinforcement not less than that given by Eq.(11-1) or (11-2) is required wherever the nominal ultimate shear stress v_e is greater than ½ of v_e. Three types of members are excluded from this requirement: slabs; floor joists; and wide, shallow beams.



Fig. 4: Service load stress at which stirrups are not required (1 in. = 25.4 mm; 1 psi = 6.89 kPa)

To maintain beam action, a shear stress equal to about $V/(b_w d)$ must be transmitted across these cracks. The shear stress that can be transmitted across such cracks, however, decreases as the crack width increases and as the maximum aggregate size decreases. In the development of the Modified Compression Field Theory (MCFT), Vecchio and Collins¹² suggested that for cracks transmitting only shear stress, the limiting stress would be

$$v_{ci} = \frac{2.16\sqrt{f_c'}}{0.31 + \frac{24w}{agg + 0.63}}$$
(2)

where *w* is the crack width and *agg* is the maximum aggregate size, both expressed in inches.

Using the parameters identified by the MCFT, Collins and Kuchma¹³ suggested the following expression for the shear capacity of members not containing stirrups

$$V_{c} = \frac{115}{50 + s_{e}} \sqrt{f_{c}'} b_{w} d$$
(3)

with $\sqrt{f_c}$ not to be taken greater than 100 psi (700 kPa). The effective crack spacing parameter s_e accounts for the influence of the crack spacing s_x and the maximum aggregate size *agg* in the following manner





Fig. 5: Influence of member depth and maximum aggregate size on shear stress at failure (tests by Shioya et al.⁹ and Shioya¹⁰) (1 in. = 25.4 mm; 1 ksi = 6.89 MPa; 1 psi = 6.89 kPa)

The crack spacing s_x is taken as 0.9d for members that have only concentrated reinforcement near the flexural tension face or as the maximum distance between the layers of longitudinal reinforcement (if the members contain intermediate layers of crack control reinforcement). To be effective, each layer of such reinforcement should have a total area of at least $0.003b_w s_x$ and the individual bars should not be spaced farther apart than 24 in. (600 mm).^{14,15}

Comparisons of test results from beams with and without intermediate layers of crack control reinforcement demonstrate that it is the distance away from the reinforcement, rather than the depth of the beam, that dictates the magnitude of the size effect in shear. Figure 7 illustrates this point, where it can be seen that the addition of three layers of intermediate longitudinal bars greatly reduced the crack spacing near mid-depth and increased the shear capacity by more than 50%. The ACI expression for V_c , Eq. (1), predicts that both beams shown in Fig. 7 would have a shear strength of 72 k (320 kN). The MCFT-based expression, Eq. (3), predicts shear failure loads of 45 k (200 kN) for the member without intermediate layers of reinforcement and 68 k (300 kN) for the member with such layers.

In members made from high-strength concrete, cracks pass through the aggregate rather than going around the aggregate and, hence, maximum aggregate size does not have the same effect on crack roughness. Thus, when f_c' exceeds 10,000 psi (70 MPa), it is appropriate to take the maximum aggregate size as zero when evaluating s_e . To avoid a discontinuity in predicted strengths, the aggregate size can be reduced linearly to zero as f_c' goes from 8500 psi (60 MPa) to 10,000 psi (70 MPa).¹⁶

The ACI expression for V_c was formulated using the observed diagonal cracking loads of relatively small beams, which had an average effective depth of 13.4 in. (340 mm) and a maximum aggregate size of 3/4 in. (19 mm). This gives an s_e value of 12 in. (300 mm). The authors suggest that the coefficients in Eq. (3) be changed so that the MCFT-based expression converges to the ACI expression when s_e is taken as 12 in. (300 mm). The resulting equation is



Fig. 6: Large beam failing in shear. Failure crack between S and X (photo courtesy of Shimizu Corp.)

$$V_c = \frac{100}{38 + s_e} \sqrt{f_c'} b_{\omega} d \tag{5}$$

Note that the V_c values given by Eq. (5) differ by only a few percent from those given by Eq. (3) for beams of practical size.

If Eq. (4) is applied to the alternate design for the Bahen Center beams in Fig. 1(b), one gets

$$s_e = \frac{1.38 \times 0.9d}{agg + 0.63} = \frac{1.38 \times 0.9 \times 67}{0 + 0.63} = 132$$
 in. (3350 mm)

Note that the skin reinforcement, consisting of three No. 6 bars on each face, would clearly not be capable of controlling the spacing of diagonal cracks across the 165 in. (4200 mm) wide web. To control crack spacing, a total area of 0.003 x $165 \times 11.2 = 5.54 \text{ in}^2$ (3600 mm²) of reinforcement would be required in each layer. This amount is 6.3 times greater than the area of skin referorcement provided.

Using Eq. (5), the shear capacity of the alternate design is

$$V_n = V_c = \frac{100}{38 + 132} \sqrt{10,000} \times 165 \times 67 = 650 \text{ k} (2900 \text{ kN})$$

Thus, the failure shear force predicted by Eq. (5) for this beam is only 29% of the failure shear force predicted by the ACI 318-02 expression for V_c , Eq. (1). As the unfactored shear due to the self-weight of the beam is about 160 k (700 kN), the MCFT-based expression predicts that the beam will fail when the applied column load reaches 980 k (4400 kN). That is, a sudden failure is predicted at just 45% of the service load that the beam is responsible for supporting safely.

TEST TO FAILURE OF A LARGE, WIDE BEAM

While many laboratory experiments conducted during the last 15 years have demonstrated the size effect in however, restricted the maximum width of the specimen to about 80 in. (2000 mm). Hence, a beam 79.1 in. wide x 39.6 in. deep x 234.6 in. long (2000 x 1000 x 6000 mm) was constructed (Fig. 8). This specimen, called AT-1, was cast using a standard concrete mixture from a local concrete supplier. The mixture was made using crushed limestone aggregate with a maximum aggregate size of 3/8 in. (10 mm). The beam was moist cured for 7 days and achieved a cylinder strength of 9300 psi (64 MPa) 47 days after casting at which time the beam was loaded to failure.

Beam AT-1, shown in Fig. 9, was about 1/2 the size of the alternate design beam for the Bahen Center. Because the effective depth of the beam did not exceed 36 in. (or



Fig. 7: Influence of distribution of longitudinal reinforcement on cracking pattern and shear strength of two companion beams

shear. none of these tests have been on large, wide beams satisfying the ACI 318 requirement of having a width at least twice as great as the overall depth of the beam. It would have been desirable to test a full-scale model of the Bahen Center beam. The available clearance under the largest test machine at the University of Toronto,



Fig. 8: Construction and loading of the large, wide beam, AT-1 under testing machine at the University of Toronto

1000 mm in the metric version of ACI 318-02), it did not require longitudinal skin reinforcement. The beam was designed in accordance with ACI 318-02 to resist an unfactored point load of 600 k (2700 kN) applied at midspan. Allowing for the self-weight of the beam, ACI 318 predicts that Beam AT-1 will fail in shear when the central point load reaches 1062 k (4730 kN).

Figure 10 shows the load-deflection response recorded as Beam AT-1 was loaded to failure. At nine load stages



Fig. 9: Design of test Beam AT-1 (1 in. = 25.4 mm; 1 ksi = 6.89 MPa; 1 psi = 6.89 kPa; 1 in² = 645 mm²; 1 k = 4.45 kN; and 1 ft-k = 1.354 kNm)



Fig. 10: Observed load-deformation response of Beam AT-1 (1 in. = 25.4 mm; 1 k = 4.45 kN)

during the test, the loading was stopped while cracks were marked and their widths measured. Figure 11 shows the crack development for the last three load stages, along with the appearance of the failure crack. Note that at 90% of the failure load, the maximum crack width measured was only 0.25 mm (0.010 in.). In an actual building, such a narrow crack would probably not be noticed and, if it was, would not be a cause of concern. Beam AT-1 exhibited a brittle shear failure, typical for large high-strength concrete beams,^{13,16} with a loud noise as the central load reached 549 k (2440 kN). This failure load is only 52% of the failure load predicted by the current ACI shear provisions and means that the beam would fail under the actual service loads. The midspan deflection at failure was only 0.374 in. (9.5 mm), which is less than 1/500 of the span.

At failure, the central portion of the beam moved downwards. Figure 12 shows the appearance of this central portion of the beam after removal of all concrete above the primary shear crack and after the reinforcing bars were cut. Note that the shape of the crack is the same across the width of the beam, supporting the observation that the failure shear stress is independent of the width of the beam. The shear failure surface was relatively smooth, with the crack cleaving nearly all of the aggregate in its path. For this 9300 psi (64 MPa) concrete, the effective aggregate size for use in Eq. (4) would be taken as $(700/1500) \times 0.375 = 0.175$ in. (4 mm).



Fig. 11: Crack development with crack widths in millimeters for Beam AT-1 $\,$

Hence, the effective crack spacing parameter s_e would be 55.5 in. (1400 mm) and so the predicted shear capacity by Eq. (5) would be 294 k (1300 kN). Allowing for self-weight of the beam, the predicted central, point load to cause failure using the MCFT-based equation is 549 k (2440 kN). It needs to be stressed that Eq. (5) was not derived by fitting to beam experiments and, hence, it is rather fortuitous that the beam happened to fail at precisely the predicted load.

Apart from its very large width, the characteristics of Beam AT-1 were very similar to those of Beam DB165 tested by Angelakos, Bentz, and Collins.¹⁶ These two beams were made from very similar concretes and had the same depth and span, but Beam AT-1 was 6.70 times wider than Beam DB165. The magnitude of the central point load required to fail AT-1 was 6.85 times larger than that required to fail DB165. Hence, the shear stress at the time of shear failure of these two beams was almost identical, differing by only 2%. This observation confirms the earlier work of Kani, Huggins, and Wittkopp¹⁷ who tested pairs of beams in which the only difference between the beams was the width, with the wider beams being four times the width of the narrower beams (24 in. [600 mm] versus 6 in. [150 mm]). For shear span-to-depth ratios of 6, 5, 4, and 3, the observed ratios between the shear failure load of the wider beam to the shear failure load of the narrower beam were 4.24, 4.26, 3.70, and 3.92, respectively, to give an average ratio of 4.03. These experimental results showed that the width of a beam does not change the shear stress at which shear failure occurs. Therefore, it is appropriate to use experimental results from large, narrow beams to investigate the shear safety of large, wide beams such as AT-1 or the Bahen Center alternate design beam.

SAFETY OF ACI 318-02 SHEAR DESIGN PROVISION

Because the ACI 318-02 shear design expression for members without stirrups, Eq. (1), neglects the size effect in shear, it assumes that if the depth of a member is doubled, the shear capacity will be doubled. The MCFTbased expression, Eq. (5), predicts that if the depth of a member is doubled, the shear capacity will less than double and that this strength ratio will become smaller as the size of the beam increases and as the aggregate size decreases. Figure 13 compares the shear strengths predicted by Eq. (1) and (5) for a range of member depths and a range of aggregate sizes. The predictions of the two equations begin to seriously diverge for depths greater than about 20 in. (500 mm), for members made from moderate-strength concrete, and greater than about 12 in. (300 mm), for members made from high-strength concrete (a = 0). For a high-strength concrete member as deep as the Bahen Center alternate design beam, ACI 318 predicted the beam's shear strength was 3.4 times as much as the MCFT-based prediction.

Very few experimental results are available for beams that are as deep as the Bahen Center beams, and none of these results are for members made from high-strength concrete. The results of 40 relevant beams in four different series have been plotted in Fig. 13 and are tabulated in Table 1. Note that only Beam AT-1 is a wide beam with the remaining 39 beams being regarded as narrow strips cut from wide beams. As has been shown, such strips fail at the same shear stress as wide beams. In Table 1, maximum aggregate sizes of 1.5 in. (40 mm),



Fig. 12: Failure surface of Beam AT-1. Note, reinforcing bars have been cut



Fig. 13: Safety of ACI shear design procedure for large, wide beams (1 in. = 25.4 mm; 1 lb/in./psi = 0.025 kN/m/kPa; and 1 psi = 6.89 kPa)

TABLE 1:

Experimental results from different size effect series involving large beams (1 in. = 25.4 mm; 1 ksi = 6.89 MPa; 1 psi = 6.89 kPa; and 1 k = 4.45 kN)

Series	SPECIMEN	agg* (in)	d (in)	b, (in)	<i>f</i> ," (psi)	р (%)	f, (ksi)	a _f d	V_{eq} (kips)	$\frac{V_{eq}}{b_e d \sqrt{f_i^2}}$	$\frac{V_{eq}}{b_e \sqrt{f_e^T}}$	s, (in)	$\frac{V_{\rm res}}{V_{\rm res}}$	$\frac{V_{\rm eq}}{V_{\rm MEFT}}$
Series 1														
Kawano Kawano Taylor Taylor	A-4A A-4B A1 B1	1.6 1.6 1.5 1.5	78.7 78.7 36.6 18.3	23.6 23.6 15.7 7.9	3220 3350 4170 3890	1.20 1.20 1.35 1.35	60.0 60.0 62.5 62.5	3.0 3.0 3.0 3.0	137.2 125.9 80.6 23.4	1.30 1.17 2.16 2.61	102.4 92.1 79.2 47.7	44.4 44.4 21.3 10.7	0.65 0.58 1.08 1.30	1.07 0.96 1.28 1.27
Series 2														
Shioya Shioya Shioya Shioya Bhal Bhal Bhal Bhal Bhal Bhal Bhal Bha	1-7 1-6 1-5 2-5 2-3 B1 B2 B3 B4 B5 B6 B7 B8	1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	118.1 78.7 39.4 23.6 7.9 11.7 23.6 35.4 47.2 23.6 23.6 23.6 35.4 35.4 35.4	59.1 39.4 19.7 11.8 6.2 9.4 9.4 9.4 9.4 9.4 9.4 9.4 9.4	3520 4130 3170 3130 3360 4300 3990 3660 3860 3860 3590 3950 4020	0.40 0.40 0.40 1.29 1.28 1.28 1.28 1.28 0.64 0.60 0.64 0.59	52.2 53.7 53.7 63.8 62.9 62.9 62.9 62.9 62.9 62.4 62.9 62.4	UDL UDL UDL 3.0 3.0 3.0 3.0 3.0 3.0 3.0 3.0 3.0 3.0	354 173.3 49.5 26.1 6.6 16.0 27.4 38.6 43.6 24.4 26.2 32.5 29.8	0.86 0.87 1.13 1.70 2.41 2.50 1.87 1.83 1.61 1.76 1.96 1.55 1.38	101.1 68.4 44.6 40.0 19.1 29.3 44.2 64.7 76.3 41.6 46.2 54.8 49.7	89.8 59.9 30.0 18.0 8.0 16.2 24.3 32.4 16.2 16.2 24.3 24.7	0.43 0.43 0.57 0.85 1.21 1.25 0.94 0.91 0.81 0.88 0.98 0.77 0.69	1.09 0.85 0.77 0.95 1.06 1.15 1.01 1.14 1.14 1.14 0.95 1.06 0.96 0.86
Series 3		112-0			3507			100				-		
Cao Yoshida Stanik Stanik Stanik Stanik Kuchma Shioya Shioya Kuzmanor Taylor Taylor	SB2003/0 YB2000/0 BN100 BN50 BN25 BN12 SE100A-45 1-4 1-3 vic B3 C2	3/8 3/8 3/8 3/8 3/8 3/8 3/8 3/8 3/8 3/8	75.8 74.4 36.4 17.7 8.9 4.3 36.2 39.4 23.6 24.2 18.3 9.2	11.8 11.8 11.8 11.8 11.8 11.8 11.6 19.7 11.8 37.8 7.9 3.9	4470 5370 5370 5370 5370 7250 3950 3060 6520 4130 3670	0.36 0.74 0.76 0.81 0.89 0.91 1.03 0.40 0.40 0.71 1.35 1.35	62.8 68.2 79.8 69.6 70.0 75.7 70.0 53.7 63.8 70.2 62.5 62.5	2.8 2.9 3.0 3.1 5.0 UDL UDL 3.0 3.0	56.3 63.2 46.0 30.0 16.6 9.0 45.0 45.0 44.4 20.8 109.8 19.2 5.4	0.94 1.03 1.46 2.16 2.41 1.26 0.91 1.35 1.49 2.07 2.47	71.3 76.7 53.2 34.7 19.2 10.4 45.6 35.9 31.9 36.0 37.9 22.6	93.8 92.0 45.0 21.9 11.0 5.4 42.1 48.6 29.1 29.9 22.6 11.3	0.47 0.52 0.73 0.98 1.08 1.21 0.63 0.46 0.68 0.74 1.03 1.23	1.24 1.34 1.21 1.06 1.05 1.01 0.79 0.91 1.01 1.25 1.22
Series 4														
Lubell Angelakos Kuchma Stanik Stanik Stanik Grimm Grimm Shioya Shioya Shioya	AT-1 DB165 BRL100 BH100 BH50 BH25 S3.1 S3.3 2-6 2-4 2-2	0.17 0.14 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	36.0 36.4 36.4 17.7 8.9 29.5 29.4 39.4 23.6 7.9	79.1 11.8 11.8 11.8 11.8 11.8 11.8 11.8 1	9300 9430 13600** 14300** 14300** 13200** 13700** 4090 3960 4130	0.76 1.01 0.50 0.76 0.81 0.89 0.42 0.83 0.40 0.40 0.40	66.7 79.8 79.8 69.6 70.0 68.2 70.6 53.7 63.8 63.8	3.0 2.9 2.9 3.0 3.0 3.5 3.5 UDL UDL UDL	294 42.9 38.1 44.7 29.9 19.2 33.3 45.8 37.1 17.2 6.4	1.07 1.03 0.88 1.04 1.43 1.83 0.95 1.32 0.75 0.98 2.01	38.5 37.4 32.2 37.8 25.3 16.2 28.2 38.8 29.4 23.2 16.0	55.5 58.6 71.8 34.9 17.5 58.2 57.9 59.0 40.1 14.6	0.54 0.51 0.44 0.52 0.71 0.92 0.48 0.66 0.37 0.49 1.01	1.00 0.99 0.97 1.14 1.04 1.02 0.92 1.27 0.72 0.77 1.06
AVERAGE C.O.V.			34.7										0.77 35.2%	1.04 14.7%

NOTES: "aggregate size linearly reduced to zero as f_0 ' goes from 8500 to 10,000 psi "maximum value of $\sqrt{f_0}$ ' of 100 psi applies to l_{w0} ' and l_{w00} ' methods

1 in. (25 mm), and 3/8 in. (10 mm) were selected for Series 1, 2, and 3, respectively, because for these aggregate sizes, some experimental results are available for beams deeper than 60 in. (1500 mm). Series 4, with maximum aggregate sizes close to zero, is directly relevant for high-strength concrete members. The small beams listed in the table and plotted in the figure formed part of the same experimental series as one of the larger beams. The 40 beams have an average depth of 34.7 in. (880 mm), with the largest depth being 118.1 in. (3000 mm) and the smallest depth being 4.3 in. (110 mm). The amount of longitudinal reinforcement ranges from 0.36 to 1.35%. The V_{exp} values were determined for the section at a distance *d* from the center of the support and include the self-weight of the beam. The experimental results plotted in Fig. 13 follow the MCFT-based predictions reasonably well and, hence, diverge strongly from ACI 318-02 predictions as the member depth increases and as the aggregate size decreases. For high-strength concrete members, dangerously unconservative results (for example, a failure shear less than 70% of the predicted value) can be expected for beams more than 18 in. (450 mm) deep. For members made from moderate-strength concrete, dangerously unconservative results can be expected for beams more than 28 in. (700 mm) deep if 3/8 in. (10 mm) aggregate is used, and more than 57 in. (1500 mm) deep if 1.5 in. (40 mm) aggregate is used.

The ACI 318-02 shear design expression for members without stirrups can be very unconservative not only because it was based on test results from beams that had rather small depths, but also because those test beams⁸ typically had very large amounts of longitudinal reinforcement ($\rho > 2\%$) to avoid any possibility of flexural failures. For a beam of a given size, given material properties, and given loading, there will be a critical amount of longitudinal reinforcement that dictates whether the beam will fail in flexure or in shear. If the beam contains less than this amount, it will undergo a ductile flexure failure. More than this amount will result in a brittle shear failure.

Figure 14 shows predictions for this critical reinforcement ratio for different beam depths and concrete strengths for simply supported beams subjected to point loads that have a shear span-to-depth ratio of 3.0. Thus, if the beam depth is 12 in. (300 mm) and the concrete strength is 5000 psi (30 MPa), a shear failure is predicted if the area of longitudinal reinforcement is more than 0.77% of the effective cross-sectional area of the beam. For a beam in a building, it is desirable to use somewhat less than the critical amount of longitudinal reinforcement to ensure a ductile flexural failure. For a laboratory investigation of shear, it is desirable to use somewhat more than the critical amount to ensure a brittle shear failure. If unrealistically large amounts of longitudinal reinforcement are used, however, unrealistically large values of shear strength will be observed. Increasing the amount of longitudinal reinforcement reduces the strain in this reinforcement at a given load and, hence, decreases the crack widths. This makes it possible to resist higher shear stresses.

Equation (3) and (5) were derived from the MCFT on the basis that the strain in the longitudinal reinforcement would be close to the yield strain (taken as 0.002) at failure and, hence, this expression will give conservative predictions for members that have amounts of longitudinal reinforcement greatly in excess of that required to prevent flexural yielding. Conversely, the equation may give unconservative predictions for members where the reinforcement has a yield strain significantly higher than



Fig. 14: Percentage of longitudinal reinforcement above which a beam without stirrups will fail in shear (1 in. = 25.4 mm; 1 ksi = 6.89 MPa; and 1 psi = 6.89 kPa)

0.002. If it is desired to evaluate the shear strength of such members more accurately, more comprehensive MCFT-based procedures have been given by Angelakos, Bentz, and Collins¹⁶ and Collins et al.¹⁴

SUGGESTED CHANGES TO ACI 318

To improve the safety of large, wide beams, Part (c) of Section 11.5.5.1 of ACI 318-02, which permits these beams to be excluded from the requirement to provide minimum shear reinforcement if V_u exceeds $0.5 \phi V_{c}$, should be deleted and replaced by:

(c) Beams with total depth not greater than 10 inches;

(d) Beams cast integrally with slabs, where the overall depth is not greater than one-half the width of the web nor 24 inches.

This change would bring the ACI minimum shear reinforcement requirements into agreement with the intent of the original authors of the provisions who wished to exclude "wide, shallow beams."

This change alone will not prevent unsafe designs of wide beams that do not contain stirrups. Similarly, large narrow beams without stirrups do not have an adequate and consistent factor of safety over the full range of commonly used depths. For example, from Eq. (5), a beam with an overall depth of 24 in. (600 mm) made from high-strength concrete is predicted to fail in shear at about 62% of the value given by the current ACI 318-02 expression, Eq. (1). As has been seen, a 71 in. (1800 mm) deep beam may fail at only 29% of the value given by Eq. (1) and, hence, even when this strength prediction is halved, it will still be unconservative, with failure expected at 58% of the permissible ultimate load.

To ensure the safety of these important members, the authors propose that Eq. (11-3) of ACI 318-02, which is Eq. (1) in this article, be replaced by Eq. (5) of this article. By more accurately predicting shear capacity



Fig. 15: Influence of minimum shear reinforcement on loaddeformation response of large beams (1 in. = 25.4 mm; 1 ksi = 6.89 MPa; 1 psi = 6.89 kPa; 1 in.² = 645 mm^2 ; and 1 k = 4.45 kN)



Fig. 16: Failure of large beam at 47% of ACI shear failure load

over the full range of beam depths, for members with practical amounts of longitudinal reinforcement, a more consistent factor of safety will be achieved.

For locations containing at least the minimum quantity of stirrups specified by Eq. (11-13) of ACI 318-02, the crack spacing parameter s_e can be taken as 12 in. (300 mm) and, for these cases, Eq. (1) and (5) become identical. The minimum amount of shear reinforcement specified in Eq. (11-13) of ACI 318-02 is

$$A_v = 0.75\sqrt{f_c'} \frac{b_w s}{f_y} \tag{6}$$

To ensure that this reinforcement controls crack spacing, the authors suggest that for large, wide beams, the spacing across the width of the beam between the legs of the stirrups should not exceed 24 in. (600 mm).

The shear equations of ACI 318-02 predict that a section containing the minimum amount of shear reinforcement will have a shear strength 1.375 times as much as a similar section without such steel. Because the provision of minimum stirrups controls the spacing of diagonal cracks, however, beams with such reinforcement do not display any significant size effect in shear. Hence, such reinforcement is even more beneficial for large beams than it is for small beams.

The very substantial improvement in the shear response of large beams caused by the introduction of minimum shear reinforcement was demonstrated by the results from two very large beams tested by Yoshida¹⁸ at the University of Toronto. Once again, these narrow beams can be regarded as typical strips cut from much wider beams. As shown in Fig. 15, these two beams were similar to the Bahen Center beams in terms of span-length and depth. One beam, called YB2000/0, did not contain stirrups, while the companion, YB2000/4, contained about the minimum amount of shear reinforcement specified by Eq. (6). Beam YB2000/0, which is one of the 40 beams listed in Table 1 and plotted in Fig. 13, failed with very little warning when the applied load at midspan reached 104 k (460 kN) (Fig. 16).

The addition of minimum stirrups increased the failure load by a factor of nearly three and increased the midspan deflection at failure by a factor of more than 10. Prior to final failure, the stirrups were extensively yielding throughout the span and the longitudinal reinforcement had reached yield near midspan. The load reached the calculated flexural capacity of the section. For large, wide beams, like those in the Bahen Center, behavior such as that exhibited by the beam with minimum stirrups is what an appropriate design should achieve. Beams without stirrups, which will behave like Specimen YB2000/0, should not be used for such critical members.

CONCLUSIONS

Fifty years ago, ACI 318 required stirrups to be provided in reinforced concrete beams only at those locations where the calculated shear stress under service loads exceeded $0.03 f_c'$. For the large beam without stirrups shown in Fig. 15 and 16, this procedure suggests that the safe service load for *P* is 202 k (900 kN), which is about twice the observed failure load. When this unsafe procedure was used to design the 3-ft-deep (900 mm) Air Force Warehouse beams, failure occurred at about 80% of service loads. While the allowable shear stress levels in ACI 318 have been reduced considerably since 1951, the experiment on the large, wide, high-strength concrete beam reported in this article shows that it is still possible for a beam designed by ACI 318-02 to fail under service loading.

The large, wide beam test reported in this article confirms Kani, Huggins, and Wittkopp's conclusion that the shear strength of wide beams is directly proportional to the width of the beam. Because of this, it is possible to use experimental results from narrow beams to investigate the safety of wide beams. These experimental results, shown in Fig. 13, demonstrate that there is a significant size effect in shear.

To provide a consistent level of safety, the ACI 318 shear provisions must recognize that beams without stirrups fail in shear at lower values of shear stress as the members become deeper and as the maximum aggregate size becomes smaller. Because the cracks in high-strength concrete beams pass through the aggregate, beams made from such concretes are more sensitive to the size effect in shear. Further, as seen in the test of AT-1, such beams can fail in shear with very little prior warning.

This article has suggested some simple modifications to the ACI 318 shear provisions so that large, wide beams designed by these provisions are appropriately safe. For all beams with an overall depth greater than about 30 in. (750 mm), the provision of a minimum amount of stirrups has a much larger beneficial effect than would be suggested by the current ACI 318 provisions. In view of the resulting increase in shear strength and ductility, it is recommended that all such beams contain at least minimum shear reinforcement.

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