TITLE NO. 64-54

Shear Capacity of Lightweight Concrete Beams

By DON L. IVEY and EUGENE BUTH

Twenty-six lightweight concrete beams were tested to provide additional information on the shear capacity of structural lightweight concrete and to evaluate the 1963 ACI Building Code requirements for shear. The beams tested in this program are compared with the present shear design formulas and with other design approaches that are being considered as modifications or changes to the 1963 ACI Code design procedure.

Keywords: beams (structural); building codes; diagonal tension; lightweight aggregate concretes; lightweight aggregates; reinforced concrete; shear; splitting tensile strength.

■ THE ACI STANDARD Building Code Requirements for Reinforced Concrete (ACI 318-63) includes specific recommendations for the first time for structural lightweight concrete. Shear studies of beams at the Portland Cement Association and the University of Texas showed that a shear capacity lower than that of normal weight concrete could be expected from some lightweight aggregate concretes. In the work by Hanson[1] an extremely good correlation was found between shear capacity of an unreinforced web and the tensile splitting strength of concrete cylinders.

In an endeavor to keep the lightweight shear equations in the same form as those of normal weight concrete, and to tie the shear resistance to the compressive strength, the physical property usually specified and controlled in construction, the factor F_{sp} was introduced. F_{sp}, the splitting ratio, is defined as an aggregate property in Section 505 of ACI 318-63. It is the average ratio of the splitting tensile strength to the square root of the compressive strength, found to be reasonably constant for most lightweight aggregates in the range of concrete compressive strengths of 3000 to 5000 psi.

The present ACI Code, ultimate strength design (USD) equation is:

$$v_c = \phi \left(0.28\, F_{sp}\sqrt{f_c'} + 2500\, \frac{pVd}{M} \right) \qquad (17\text{-}9)$$

For a value of F_{sp} of 6.8, it becomes identical to the normal weight concrete equation:

$$v_c = \phi \left(1.9\sqrt{f_c'} + 2500\, \frac{pVd}{M} \right) \qquad (17\text{-}2)$$

It should be noted that F_{sp} is merely an indirect method of putting f'_{sp} (tensile splitting strength) into the lightweight shear equations. Because of difficulties attending the determination of F_{sp} values, and of misunderstandings in the use of F_{sp}, there is a natural desire to relate shear capacity directly to measured tensile splitting strength of lightweight concrete. Also, because variations in F_{sp} (essentially a correction factor) have relatively little effect on shear design results, there is good argument to use a single correction factor

ACI member **Don L. Ivey** is associate research engineer in charge of the Structural Research Laboratory with the Texas Transportation Institute, Texas A&M University, College Station, Tex. He received a BS degree in civil engineering at Lamar State College of Technology in 1960, and received an NDEA (National Defense Education Act) fellowship to do graduate work at Texas A&M University where he received a ME degree in 1962 and a PhD degree in 1964. Dr. Ivey has engaged in research in structural concrete and concrete technology with the Texas Transportation Institute since completion of graduate work. Currently, he is a member of ACI Committee 408, Bond Stress; ACI-ASCE Committee 426, Shear and Diagonal Tension; and ACI Committee 212, Admixtures.

ACI member **Eugene Buth** is research associate, Texas Transportation Institute, Texas A&M University, College Station, Tex. He received a BS degree in civil engineering at Texas A&M University in 1963 and a ME degree at that institution in 1964. Since 1964, Mr. Buth has engaged in research in concrete technology. Currently, he is a member of ACI Committee 201, Durability of Concrete.

TABLE 1—AGGREGATE PROPERTIES

Properties		Aggregate 1		Aggregate 23		Aggregate 27		Brazos sand
		Coarse	Fine	Coarse	Fine	Coarse	Fine	
Unit weight, lb per cu ft (dry loose)		48.8	63.7	44.0	55.0	39.0	73.8	106.0
Sieve analysis, cumulative percent retained	¾ in.	0.0		0.0		0.4		
	½ in.	24.2		23.5		28.1		
	⅜ in.	49.2		75.0		69.5		
	#4	87.0	0.0	75.5	0.0	98.5	0.2	0.0
	#8	97.6	7.7	100.0	15.8	99.3	15.7	9.2
	#16	100.0	37.4		46.3	100.0	42.0	18.9
	#30		51.7		77.0		62.6	35.0
	#50		76.0		88.5		77.9	88.5
	#100		86.2		100.0		88.9	96.0
	#200		92.7				95.5	100.0
	Pan		100.0				100.0	
Absorption (percent of dry weight)	24 hr	6.0	5.9	6.5	6.3	6.5	5.0	1.0
	3 days	7.7	7.1	8.4	8.0	8.1	6.1	
Bulk specific gravity (dry)		1.63	1.98	1.52	1.75	1.10	1.94	2.62
Bulk specific gravity (saturated surface dry)	24 hr	1.73	2.10	1.62	1.86	1.17	2.04	2.63
	3 days	1.76	2.12	1.64	1.89	1.19	2.06	
Apparent specific gravity	24 hr	1.80	2.32	1.70	1.98	1.18	2.16	2.66
	3 days	1.85	2.38	1.74	2.04	1.20	2.21	

Note: Unit weight and gradation of lightweight aggregates determined by test methods of ASTM C330. Unit weight and gradation of Brazos sand determined by test methods ASTM C29 and C136-63, respectively. Absorption and specific gravities of lightweight aggregates determined by test method of Bryant.[2] Absorption and specific gravities of Brazos sand determined according to ASTM C127-59.

TABLE 2—BATCH DATA

Beam designation	Cement Type I, lb per cu yd	Water, lb per cu yd	Coarse dry weight, lb per cu yd	Fine dry weight, lb per cu yd	Coarse moisture,† percent	Fine moisture,† percent	Air, percent	Slump, in.	Unit weight wet, lb per cu ft
(1)* 1	668	527	658	1088	1.5	4.4	6.0	4	109
(1) 2	653	536	643	1039	1.4	6.6	5.8	3¾	106
(1) 3	660	522	651	1071	1.6	4.6	6.4	4	107
(23)* 1	552	605	680	1018	15.1	23.1	5.9	2	106
(23) 2	552	605	680	1018	15.1	23.1	5.6	3	106
(23) 3	552	605	680	1018	15.1	23.1	5.6	3¼	106
(23) 1S	485	452	700	1730	16.4	5.7	4.9	2¾	125
(23) 2S	485	438	700	1765	17.2	3.6	4.5	1¾	126
(23) 3S	485	420	700	1760	18.5	3.9	5.1	3	125
(27)* 1	538	515	735	860	16.4	14.6	2.8	3¾	98
(27) 2	542	524	736	866	17.2	14.1	3.1	3¾	99
(27) 3	572	532	737	880	18.5	14.0	4.0	3¾	100
(23) 4	572	602	695	1042	17.0	23.0	4.7	5	108
(23) 5	572	602	695	1042	17.0	23.0	4.7	5	108
(23) 6	572	602	695	1042	17.0	23.0	4.7	5	108
(23) 7	572	602	695	1042	17.0	23.0	4.7	5	108
(23) 8	427	547	635	973	7.9	9.9	5.5	2½	96
(23) 9	426	546	634	970	7.9	9.9	6.0	3½	96
(23) 10	431	552	641	981	7.9	9.9	5.5	3	96
(23) 11	420	474	646	987	3.5	6.0	7.0	2¾	94
(23) 12	420	478	652	995	3.5	6.0	6.0	2¼	94
(23) 13	420	477	650	992	3.5	6.0	6.2	2½	94
(23) 14	420	477	650	992	3.5	6.0	6.2	2½	94
(23) 15	472	622	690	1020	13.8	19.9	2.7	4	108
(23) 16	474	626	685	1040	14.6	18.7	4.9	3	107
(23) 17	474	590	689	1065	13.7	16.2	4.5	4	106

*Number in parenthesis, (1), (23), and (27) are the aggregate numbers, the next number designates the particular beam, and beam designations followed by an S were cast using the natural Brazos sand instead of lightweight fines.

†Stockpiles of Aggregate 23 were kept in a saturated condition for several weeks prior to batching beams. A considerable amount of surface moisture was present, accounting for the high moisture content. Exceptions to this were beams (23) 8 through (23) 14 which were batched with Aggregate 23 as supplied. Moisture contents shown for Aggregates 27 should not be confused with absorption (see Table 1) as these aggregates are stored in silos under hydrostatic pressure and are almost completely saturated with water as they are supplied.

TABLE 3—GEOMETRIC PROPERTIES OF TEST BEAMS

Beam designation	Cross section, in x in.	Shear span a, in.	Depth to steel d, in.	Reinforcement bar No. and size	Steel percentage p, percent
(1) 1	6x12	21	10.50	4#4	1.27
(1) 2	6x12	35	10.50	4#4	1.27
(1) 3	6x12	52	10.50	4#4	1.27
(23) 1	6x12	35	10.50	3#4	0.95
(23) 2	6x12	35	10.50	4#4	1.27
(23) 3	6x12	35	10.50	3#5	1.48
(23) 1S	6x12	35	10.50	3#4	0.95
(23) 2S	6x12	35	10.50	4#4	1.27
(23) 3S	6x12	35	10.50	3#5	1.48
(27) 1	6x12	21	10.50	4#4	1.27
(27) 2	6x12	35	10.50	4#4	1.27
(27) 3	6x12	52	10.50	4#4	1.27
(23) 4	4.25x9	24.7	7.42	2#4	1.27
(23) 5	6x12	35	10.50	4#4	1.27
(23) 6	7.5x15	43.7	13.10	4#5	1.27
(23) 7	8.87x18	51.9	15.55	4#6	1.27
(23) 8	6x12	21	10.50	3#4	0.95
(23) 9	6x12	21	10.50	4#4	1.27
(23) 10	6x12	21	10.50	3#5	1.48
(23) 11	6x12	21	10.50	3#6	2.10
(23) 12	6x12	31.5	10.50	3#6	2.10
(23) 13	6x12	42	10.50	3#6	2.10
(23) 14	6x12	52	10.50	3#6	2.10
(23) 15	6x12	35	10.50	3#4	0.95
(23) 16	6x12	35	10.50	4#4	1.27
(23) 17	6x12	35	10.50	3#5	1.48

in the normal weight shear formulas as applied to lightweight concrete that would be reasonably safe, thereby simplifying shear design. As design alternatives to this, a series of correction factors based on measured tensile splitting strength could be applied to the normal weight shear formulas, or the tensile splitting strength itself could be used directly in the present lightweight equations. The test data developed in this program are compared to these proposed modifications of lightweight concrete shear design.

MATERIALS

Aggregates

Three lightweight aggregates were chosen for the concretes in this study. These aggregates are described below.

Aggregate 1* is an expanded slate produced in a rotary kiln. Particles are angular and finer particle sizes result from crushing the coarse sizes.

Aggregate 23 is an expanded shale produced in a rotary kiln. Particles are angular, and both coarse and fine material are obtained by crushing.

*Aggregate numbers correspond to numbers used in National Bureau of Standards' Monograph 74, "Creep and Drying Shrinkage of Lightweight and Normal-Weight Concretes," by T. W. Reichard, Mar. 4, 1964, p. 30.

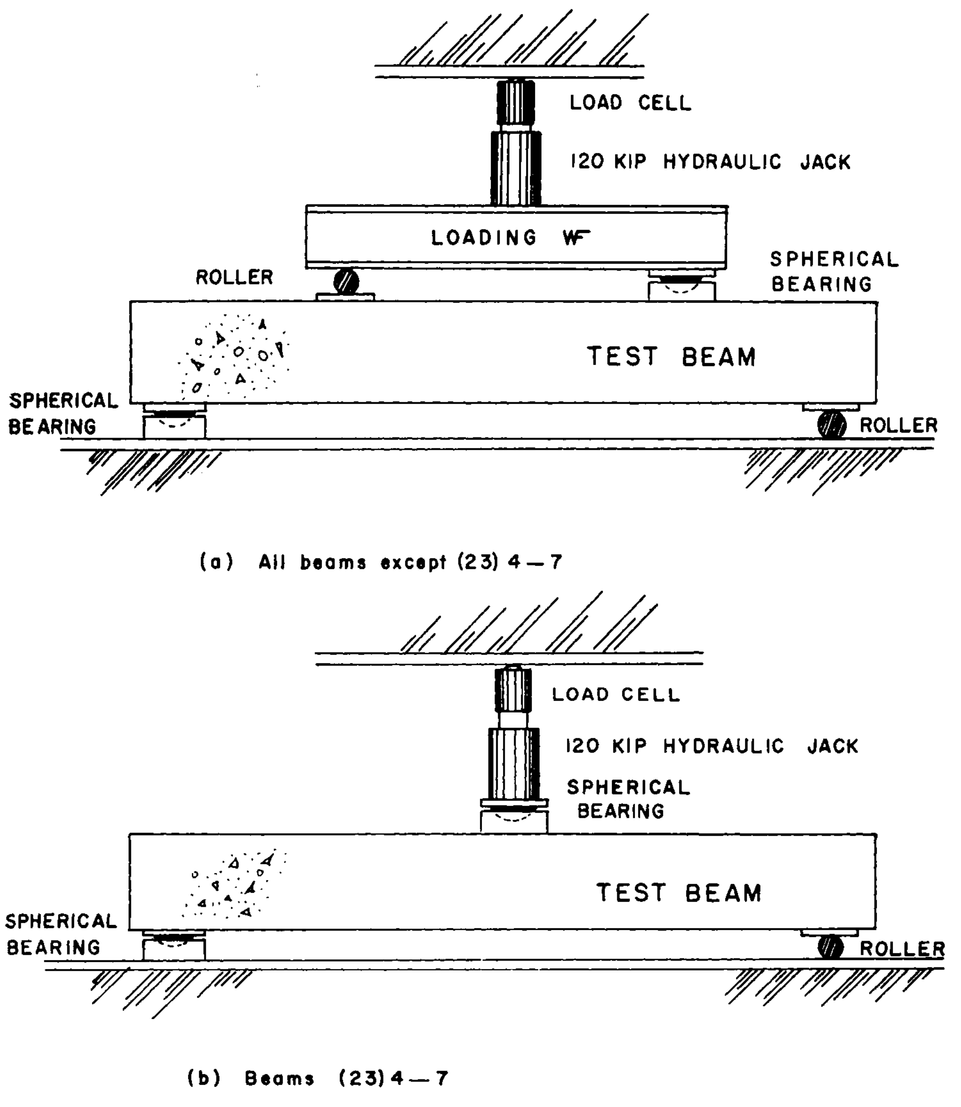

Fig. 1—Testing arrangement

Aggregate 27 is an expanded shale produced in a rotary kiln. All materials are presized before being fed into the kiln. The resultant particles are well rounded with a relatively impervious outer shell.

The natural sand used in Beams (23)1S through 3S is a high quality siliceous concrete sand from the Brazos River near Bryan, Texas. The physical properties of these aggregates are given in Table 1.

Cement

Type 1 cement, as manufactured by the Universal Atlas Cement Co., Waco, Texas, was used in all concretes.

Reinforcement

Deformed steel bars conforming to ASTM A432 were used for longitudinal tension reinforcement. The nominal yield strength of these bars was 60,000 psi. Deformations of all bars met the requirements of ASTM A305-56T.

with crack progression marked and center point deflection recorded after each increase in load.

TEST RESULTS

The test data on beams and cylinders are presented in Table 4. $V_{c_{DT}}$ is the total shear at formation of the diagonal tension crack. $V_{c_{ult}}$ is the maximum shear supported by the beam. This is greater than $V_{c_{DT}}$ for short shear spans where final failure is by compression of the arch structure formed after diagonal tension cracking. This difference in failure mechanisms[3] is illustrated by Fig. 2. An a/d ratio of approximately 3.5 appears to be the dividing line between those beams which have a shear compression mode of failure and those which fail completely at the formation of the diagonal tension crack. Beams with a/d ratios less than 3.5, each provide two points

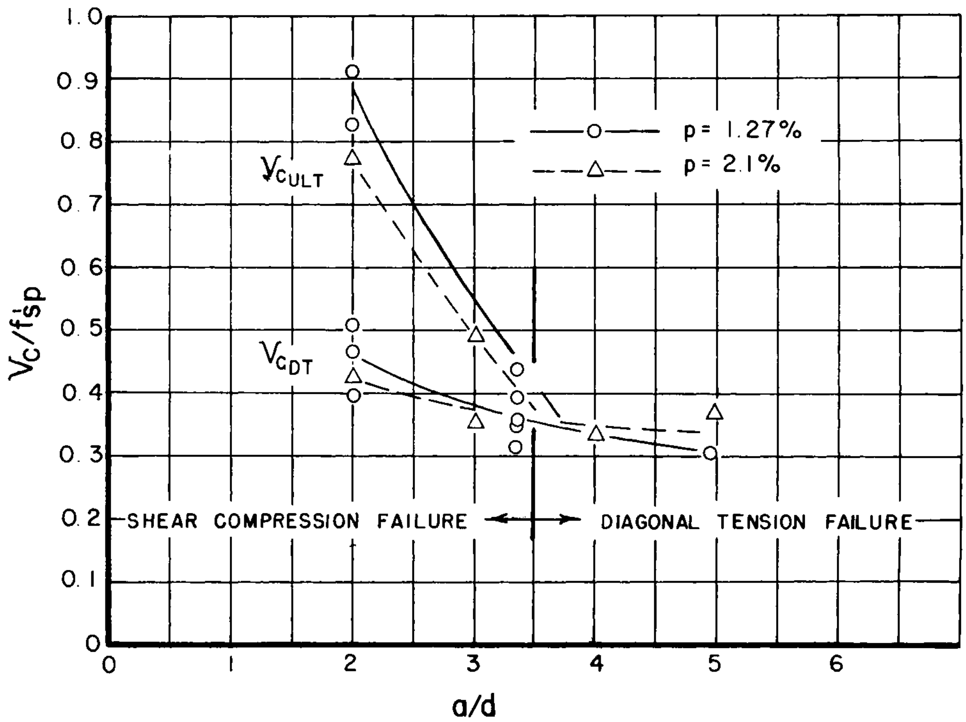

Fig. 2—Effect of shear span to depth ratios on shear capacities

FABRICATION AND TEST PROCEDURES

The concrete was mixed in a 6 cu ft horizontal drum mixer. The beams were cast in three lifts with each lift subjected to internal vibration. Reference cylinders were cast and tested in accordance with ASTM C192 and ASTM C496. They were subjected to the same 7 day moist cure, 21 days at 50 percent relative humidity environment that was imposed on the beams. The concrete batch data are given in Table 2.

Twenty-two of the 26 beams tested were 6 x 12 in. in cross section, with shear spans, percentage of reinforcement, and type of aggregate as the variables. The geometric properties of these beams are given in Table 3. These were tested in the loading jig shown in Fig. 1(a). The other four beams were scale models of each other and varied in depth from 9 to 18 in. The test arrangement for these beams is shown in Fig. 1(b). The beams were loaded to failure in 500-lb increments,

which are plotted in Fig. 2. The first point is determined during the test by the formation of a diagonal tension crack (at the shear, $V_{c_{DT}}$) which is accompanied by a loss in load on the beam due to its sudden increase in deflection. Load can then be further applied beyond the diagonal tension cracking load to a point where the remaining arch structure fails in compression near one of the load points (at the shear, $V_{c_{ult}}$). This is the ultimate shear the beam can support.

In this paper the failure load is considered the diagonal tension cracking load rather than the ultimate shear, even though the ultimate load at small values of a/d was much greater than the diagonal tension cracking load. It was recently

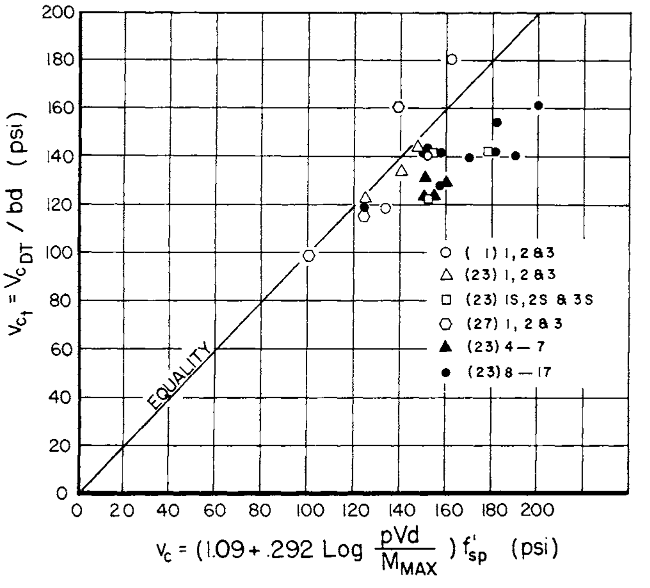

Fig. 3—Comparison of beam tests with Hanson's equation

pointed out by de Cossio and Loera[4] in a discussion of a paper by Kani, that, ". . . it has been established by Ferguson[5] and Taylor[6] that if beams with small a/d ratios are tested with the load not directly applied to the faces of the beam but through lateral stubs, these beams will fail at loads on the order of the inclined cracking load, since the confining action of the bearing plates is not present."

Fig. 3 shows the correlation of observed shearing strengths with the shear strength calculated by Hanson's equation [Eq. (6), p. 21, Reference 1]. It should be considered that Hanson's equation uses the splitting tensile strength, which is also subject to testing variation. The average of the observed shear strengths is 14 percent below the average of the corresponding values predicted by Hanson's equation.

Beams (23)4 through (23)7 were cast and tested in an effort to determine if there was any significant size effect on the shear resistance of these beams. Each beam is a scale model of the other three. They vary in cross section from 4¼ x 9 to 8⅞ x 18 in. The resulting values of v_{c_t} from the four beams were quite close (121, 130, 134, and 123 psi) and are probably within the test variation of identical beams. The tests indicate that the size effect is probably quite small within the range of beams tested.

COMPARISON OF DESIGN METHODS

The primary consideration at the present time is not in reiterating the previously proven effects of concrete strength, shear span, and reinforcement percentage on the shear capacity, but is the comparison of the test data developed in this program with the existing ACI 318-63 design requirements and the proposed alternatives to the present Code.

The values of observed shear strength (v_{c_t}) and calculated shear strengths (v_{c_1} and v_{c_2}) are tabulated in Table 5. The values shown as v_{c_1} are the shear strengths indicated by Eq. (17-9) of the ACI Code neglecting ϕ and using the test values of f'_{sp} and f_c' instead of the value of F_{sp}

TABLE 4—RESULTS OF BEAM AND CYLINDER TESTS

Beam designation	f'_{sp}, psi	f_c', psi	$V_{c_{DT}}$, kips	$V_{c_{ult}}$, kips	$\frac{pVd}{M\sqrt{f_c'}} \times 10^3$	$v_{c_t}/\sqrt{f_c'}$	$\frac{pVd}{M f'_{sp}} \times 10^3$	v_{c_t}/f'_{sp}
(1) 1	367 (3)*	4490 (2)*	11.35	22.65	0.188	2.67	0.035	0.491
(1) 2	394 (3)	4500 (2)	8.80	8.80	0.081	2.06	0.014	0.355
(1) 3	409 (3)	4690 (2)	7.50	7.50	0.048	1.76	0.008	0.291
(23) 1	366 (3)	4040 (2)	7.80	8.10	0.064	1.93	0.011	0.338
(23) 2	376 (3)	4170 (2)	8.40	8.40	0.085	2.06	0.015	0.354
(23) 3	368 (3)	4160 (2)	9.00	9.20	0.099	2.22	0.017	0.388
(23) 1S	445 (6)	3730 (3)	7.68	7.68	0.067	2.00	0.009	0.274
(23) 2S	405 (5)	3870 (3)	8.90	8.90	0.088	2.27	0.013	0.349
(23) 3S	440 (5)	4060 (3)	8.90	8.90	0.100	2.22	0.014	0.321
(27) 1	315 (3)	3360 (2)	10.10	16.40	0.219	2.76	0.040	0.509
(27) 2	325 (3)	3710 (2)	7.20	8.80	0.091	1.87	0.017	0.352
(27) 3	308 (3)	3420 (2)	6.20	6.20	0.054	1.66	0.010	0.320
(23) 4	396 (3)	3560 (1)	3.81	3.81	0.087	2.04	0.014	0.305
(23) 5	418 (3)	4290 (1)	8.20	8.20	0.087	1.99	0.013	0.311
(23) 6	394 (3)	3820 (1)	13.20	13.20	0.087	2.16	0.014	0.341
(23) 7	402 (3)	3760 (1)	17.00	17.00	0.087	2.00	0.014	0.307
(23) 8†	380 (7)	3030 (3)	8.13	11.21	0.173	2.35	0.025	0.340
(23) 9	355 (7)	2960 (3)	8.89	14.10	0.233	2.59	0.036	0.397
(23) 10	390 (7)	3250 (3)	9.81	13.98	0.259	2.72	0.038	0.399
(23) 11	390 (7)	3010 (3)	10.20	18.85	0.383	2.96	0.054	0.415
(23) 12	400 (6)	3270 (4)	8.73	12.21	0.184	2.44	0.026	0.346
(23) 13	400 (7)	3200 (3)	8.62	8.62	0.124	2.43	0.018	0.342
(23) 14	380 (7)	2780 (3)	8.94	8.94	0.101	2.70	0.014	0.373
(23) 15	375 (6)	3130 (3)	7.54	7.54	0.073	2.14	0.011	0.319
(23) 16	390 (5)	2780 (3)	8.82	8.82	0.103	2.66	0.014	0.359
(23) 17	450 (6)	3860 (3)	8.95	8.95	0.102	2.29	0.014	0.316

*Number of cylinders tested for each value of f'_{sp} or f_c'.
†The results from Beams (23) 8 through (23) 17 were used in an unpublished dissertation by Kazi Harun-ur-Rashid.[7]

determined for the aggregate used in each beam. This amounts to allowing $0.28\, F_{sp}\sqrt{f_c'}$ to be replaced by $0.28\, f'_{sp}$, where f'_{sp} is the observed splitting strength for each beam. It should be noted that this procedure is not in accordance with the specified use of this equation, since one of the purposes of using the splitting ratio (F_{sp}) was to eliminate the need for tensile splitting control tests. The tabulated ratios of v_{c_t} to v_{c_1} range from 0.904 to 1.333 with a mean of 1.074. The average of tests then is 7.4 percent higher than the value predicted by Eq. (17-9) used in the way previously defined. By including the factor ϕ, the test results would be about 26 percent higher than Eq. (17-9) predicts.

The values of v_{c_2} are found again by using Eq. (17-9) and neglecting ϕ, but with the manufacturers' recommended values of F_{sp} for each aggregate, and the observed value of f_c' for each beam. The ratios of v_{c_t} to v_{c_2} range from 0.992 to 1.474 with a mean of 1.209. The average test value is then 21 percent over that predicted by Eq. (17-9), or 42 percent over predicted values if ϕ is included.

The use of a constant proportionality, or correction factor for lightweight concrete, to be applied to the normal weight concrete shear equation (17-2), is being considered. The factor of 0.75 has been proposed* for an all-lightweight concrete and 0.85 for a sand-lightweight concrete.† Eq. (17-2) would be used as follows:

Normal weight

$$v_c = \phi \left(1.9\sqrt{f_c'} + 2500\, \frac{pVd}{M}\right)$$

All lightweight

$$v_c = 0.75\, \phi \quad 1.9\sqrt{f_c'} + 2500\, \frac{pVd}{M}\Big)$$

Sand lightweight

$$v_c = 0.85\, \phi \left(1.9\sqrt{f_c'} + 2500\, \frac{pVd}{M}\right)$$

The comparison of this proposal with the test beams, eliminating the factor ϕ, is illustrated by Fig. 4. Beams (23)1S, (23)2S, and (23)3S are the only ones with natural sand fines and should be compared to the 0.85 line. All other beams are compared to the 0.75 line. Under this design criterion, the ratio of test shear strengths to predicted shear strengths is from 1.101 to 1.671 with

*ACI Committee 213, "Lightweight Aggregate Concrete," in its recommendations to ACI Committee 318 for changes to the ACI Building Code (318-63) suggested the reduction factors for shear design of lightweight concrete, but included a waiver clause to permit an engineer to accept higher shear stresses when tests in accordance with the present design method using F_{sp} justified higher values. The minimum, $F_{sp} = 4$, when tests are not available, would no longer hold.

†100 percent replacement of natural sand for the lightweight fine aggregate.

TABLE 5—COMPARISON OF BEAM TESTS WITH (USD) CODE EQUATION (17-9)

Beam designation	$2\sqrt{f_c'}$	v_{c_1} (17-9),* psi	v_{c_2} (17-9),† psi	v_{c_t}, psi	v_{c_t}/v_{c_1}	v_{c_t}/v_{c_2}
(1) 1	134	135	141	180	1.333	1.277
(1) 2	134	124	123	140	1.129	1.138
(1) 3	137	123	120	119	.967	.992
(23) 1	127	113	108	124	1.097	1.148
(23) 2	129	119	113	133	1.118	1.177
(23) 3	129	119	115	143	1.202	1.243
(23) 1S	122	135	113	122	.904	1.080
(23) 2S	124	127	118	141	1.110	1.195
(23) 3S	127	139	123	141	1.014	1.146
(27) 1	116	120	116	160	1.333	1.379
(27) 2	122	105	102	114	1.086	1.118
(27) 3	117	94	93	98	1.043	1.054
(23) 4	119	125	106	121	.968	1.142
(23) 5	131	131	115	130	.992	1.130
(23) 6	124	124	109	134	1.081	1.229
(23) 7	123	126	108	123	.976	1.139
(23) 8	110	130	109	129	.992	1.183
(23) 9	109	131	116	141	1.076	1.216
(23) 10	114	146	125	156	1.068	1.248
(23) 11	110	162	137	162	1.000	1.182
(23) 12	114	138	114	139	1.007	1.219
(23) 13	113	130	105	137	1.054	1.305
(23) 14	105	120	94	142	1.183	1.511
(23) 15	112	115	96	120	1.043	1.250
(23) 16	105	123	95	140	1.138	1.474
(23) 17	124	142	112	142	1.000	1.268
				Average	1.074	1.209
				Range	1.333-0.904	1.474-0.992

*Neglecting ϕ and letting $0.28\, F_{sp}\sqrt{f_c'}$ be replaced by $0.28\, f'_{sp}$, where f'_{sp} is the observed splitting strength for each beam.

†Neglecting ϕ and using the commercially accepted value of F_{sp} for each aggregate. (No. 1, $F_{sp} = 5.8$; No. 23, $F_{sp} = 5.5$; No. 23 coarse and natural sand fines, $F_{sp} = 6.0$; No. 27, $F_{sp} = 5.2$).

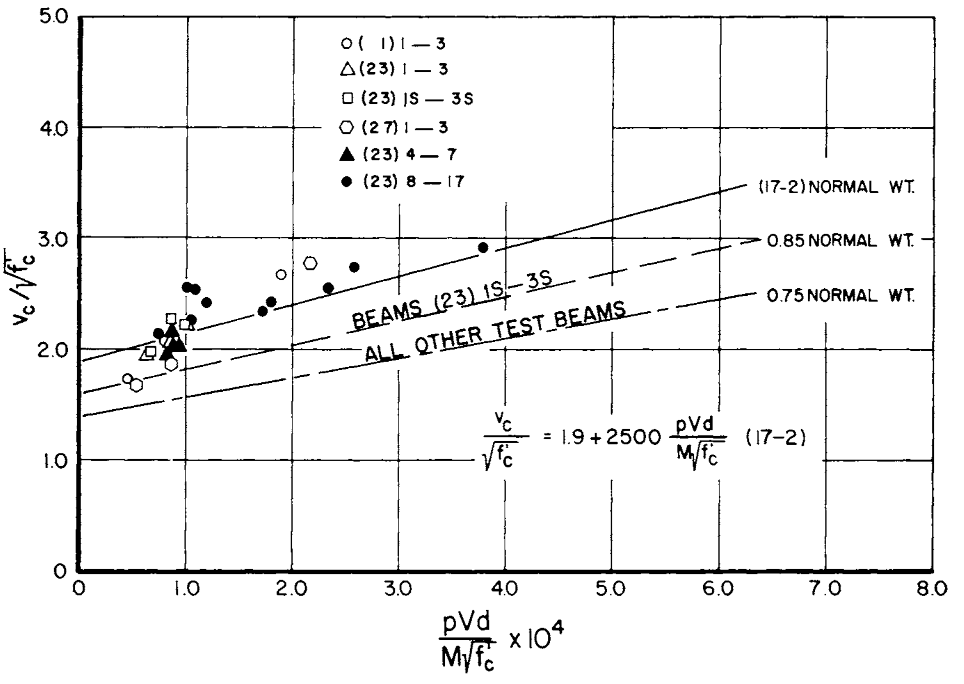

Fig. 4—Comparison of TTI test data with 0.75, 0.85 alternative

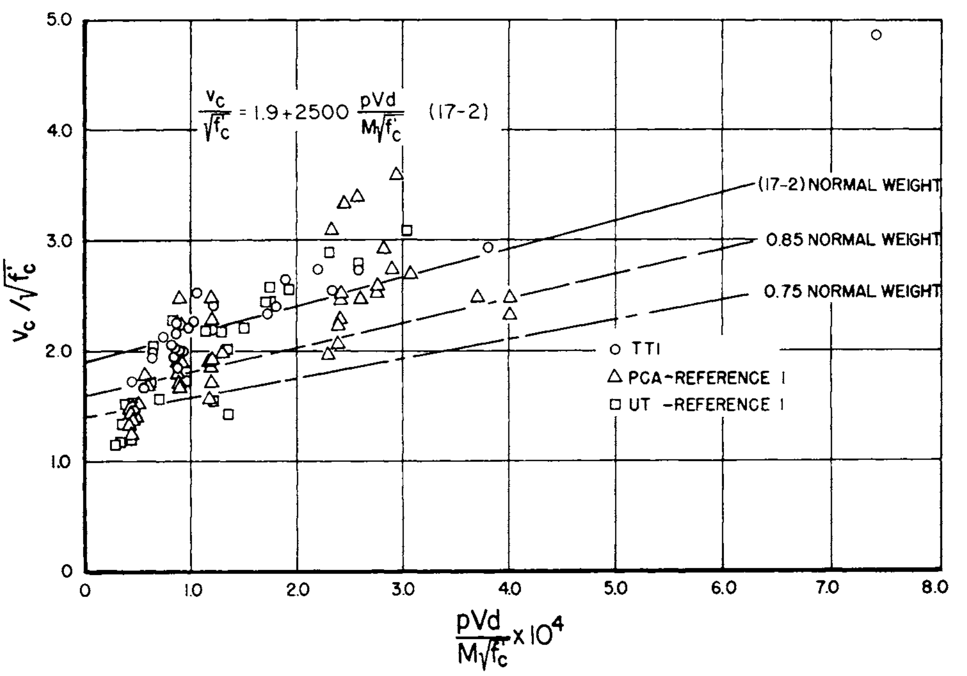

Fig. 5—Comparison of TTI, PCA, and University of Texas test data with 0.75, 0.85 alternative

a mean value of 1.358. For the four concretes tested in this program, this design approach is rather conservative, on the average 36 percent, or 60 percent considering ϕ, but when other lightweight concrete data are considered, it is seen these conservative values may be necessary because of the low shear resistance of some lightweight aggregate concretes. The data developed by PCA[1] and the University of Texas[1] are plotted along with the data developed in this program in Fig. 5, again with the factor ϕ not included. While the TTI beams show no tests below the proposed 0.75 to 0.85 lines, the PCA and UT beams show a number of tests below the 0.75 line.

Another alternative to the present code is the recognition of the splitting tensile strength (f'_{sp}) as the concrete strength parameter instead of continuing to disguise it within F_{sp}. It could be included as a design alternative to the 0.75, 0.85 procedure. One way in which the influence of f'_{sp} might be recognized would be by using Eq. (17-9) with f'_{sp} substituted for the product of F_{sp} and $\sqrt{f_c'}$. The equation would then take the form:

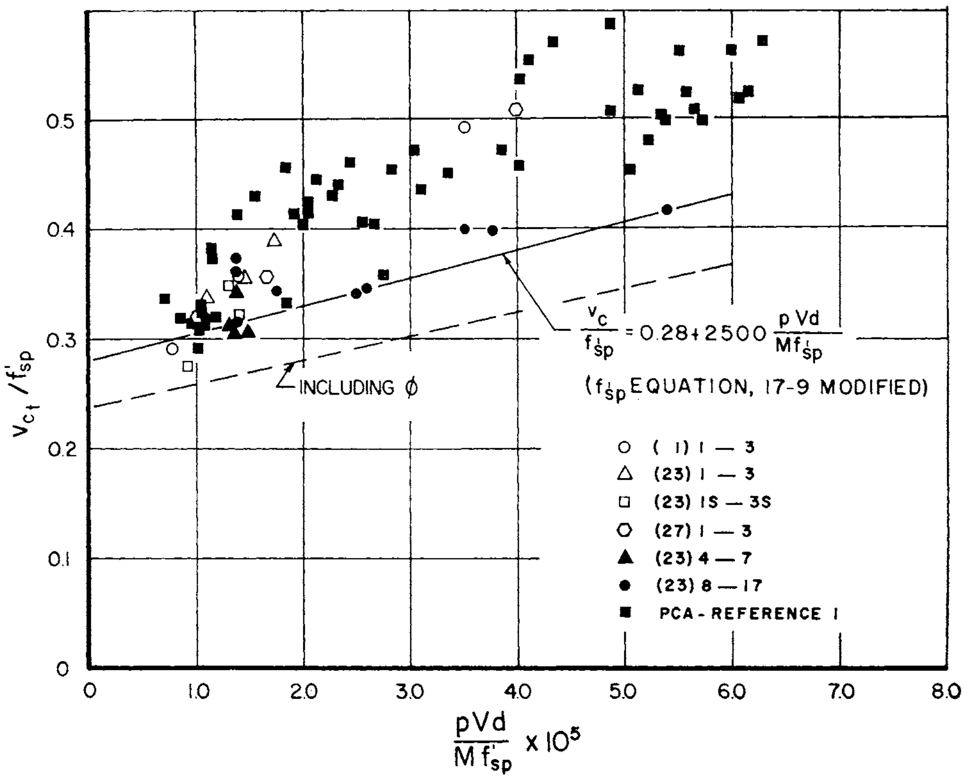

Fig. 6—Comparison of TTI and PCA data with modification of Eq. (17-9)

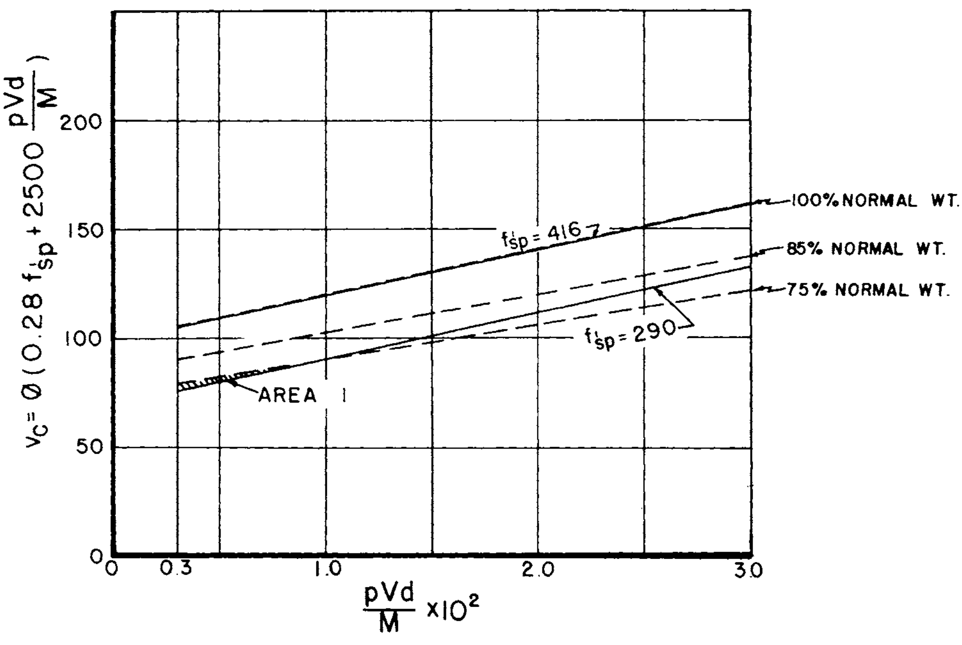

Fig. 7—Comparison of f'_{sp} alternative with 0.75, 0.85 method

$$v_c = \phi\left(0.28\, f'_{sp} + 2500\, \frac{pVd}{M}\right)$$

This will be referred to as the f'_{sp} equation. If this equation were used as a design alternative it could be required that f'_{sp} be determined* for a proposed concrete mix. The data developed by TTI and PCA are compared to this equation in Fig. 6. The equation very closely approximates the lower boundary of this data. The range of the conservative factor:†

$$v_{c_t}/v_{c_1},\ v_{c_1} = 0.28\, f'_{sp} + 2500\, \frac{pVd}{M}$$

shown by the TTI and PCA tests is from 0.90 to 1.47 with a mean of 1.18. Thus the average test is 18 percent over predicted or 39 percent considering ϕ. It should be noted that only five tests

*Previous work by the authors[8] has shown f'_{sp} is a reproducible value determined by a standard test method (ASTM C496).

†It should be emphasized that this is not the way to solve for v_c specified by ACI 318-63. Under 318-63, f'_{sp} is *not* a design parameter.

TABLE 6—SUMMARY OF DESIGN METHODS

Conservative factor $v_{c_{test}}/v_{c_{Design}}$ range	Average percent conservative	Average percent conservative including ϕ	Design equations
0.98-1.47	21	42	$v_{c_{Design}} = 0.28F_{sp}\sqrt{f_c'} + 2500 \frac{pVd}{M}$ Commercial value of F_{sp} TTI* beams
1.14-1.67	36	60	$v_{c_{Design}} = (0.75 \text{ or } 0.85)\left(1.9\sqrt{f_c'} + 2500 \frac{pVd}{M}\right)$ TTI beams
0.90-1.33	7	26	$v_{c_{Design}} = 0.28f'_{sp} + 2500 \frac{pVd}{M}$ TTI beams
0.90-1.47	18	39	$v_{c_{Design}} = 0.28f'_{sp} + 2500 \frac{pVd}{M}$ Combination of TTI and PCA beams

*Texas Transportation Institute.

out of 74 plotted in Fig. 6 fall below the f'_{sp} equation. If the 0.85 value of ϕ is applied, none of the data will fall below the prediction equation. The range of conservative factors and the average value for the TTI data only, will be identical to those calculated as v_{c_t}/v_{c_1} in Table 5, (0.904 to 1.333 with a mean of 1.074).

There is presently a proposal before Subcommittee III-f of Committee C-9, ASTM for a revision to C-330 to require that lightweight aggregate produce structural concrete having a minimum tensile splitting strength of 290 psi. This would mean that the f'_{sp} equation:*

$$v_c = \phi \left(0.28 f'_{sp} + 2500 \frac{pVd}{M}\right)$$

could be applied over a range of f'_{sp} from 290 to 416 psi (the value yielding a design shear stress equal to that of normal weight concrete assuming a value for f_c' of 3750 psi, the average of 3000 and 4500 psi from Section 505 of the Code). The comparison of this design method with the 0.75, 0.85 design is shown by Fig. 7. Values of v_c given by the f'_{sp} equation are plotted against pVd/M. The 0.75 and 0.85 lines are also shown on this figure. Therefore, it can be seen that for aggregates capable of producing an all-lightweight concrete with a splitting tensile strength of 312 psi or above, it would be less conservative to design by the f'_{sp} alternative. There is a small area (Area 1) at low values of pVd/M where it would be less conservative to design by the 0.75 method, but the difference is very slight. For sand-lightweight concrete, higher shear stresses would not be indicated by the f'_{sp} alternative as compared to the 0.85 method. This is true for f'_{sp} values less than about 320, for high values of pVd/M, or less than 350 for the lower values. One point should be emphasized, the f'_{sp} design alternative is the design value originally intended for use in shear computations for structural lightweight concrete, but without the indirect way of getting f'_{sp} into the equation through determinations of F_{sp} and the compressive strength.

A summary of the way the various design methods compare with test data and with each other is given by Table 6. From this table, it is seen that the 0.75, 0.85 method is the most conservative. The f'_{sp} equation [Eq. (17-9) modified] is the least conservative of the methods compared, and the present Code Eq. (17-9) falls between these two extremes. The Code could then be simplified, at the expense of some additional conservatism if the 0.75, 0.85 design procedure were adopted, while lightweight aggregates capable of producing high tensile strength concrete could be accommodated by the adoption of a design alternative such as the f'_{sp} equation.

SUMMARY AND CONCLUSIONS

1. The program encompassed the testing of twenty-six lightweight concrete simply supported beams. The major variables were shear span (21 to 52 in.), steel percentage (0.95 to 2.10), three different aggregates, and beam cross section (4¼ x 9 to 8⅞ x 18 in.). The compressive strengths were nominally from 3000 to 4500 psi.

2. The previously shown effects of tensile strength, shear span to depth (a/d) ratio and steel percentage were again demonstrated. The tests meant to show the effect of beam size on shear resistance proved inconclusive. The tests do seem to indicate that if such an effect is present, it is probably quite small.

*Again, this is not the way to solve for v_c under ACI 318-63.

3. The test data showed a reasonable correlation with a prediction equation derived previously by Hanson in a larger program at the Portland Cement Association, the basic data of which provided the major basis for the lightweight shear requirements in the 1963 ACI Code. The average of the TTI tests fell 14 percent below the value predicted by Hanson's equation.

4. The comparison of the existing and proposed design methods showed the proposed 0.75, 0.85 procedure is the most conservative of the three methods compared (36 percent, or 60 percent including ϕ). The use of the existing Eq. (17-9) for lightweight, but with f'_{sp} substituted for $F_{sp}\sqrt{f_c'}$ is the least conservative (7 or 26 percent including ϕ). The present Code Eq. (17-9), used as specified, yielded intermediate results between the two extremes (21 or 42 percent including ϕ).

ACKNOWLEDGMENTS

Appreciation is expressed to the Expanded Shale Clay and Slate Institute who sponsored this research, and to the members of the Structural Committee of that organization for their aid in the planning and reporting stages of this work.

The testing program was carried out using laboratory facilities of both the Structural Research Department of Texas Transportation Institute and the Civil Engineering Department of Texas A&M University.

Appreciation is also expressed to Mr. Kazi Harun-Ur-Rashid, who aided in the fabrication and testing stages.

REFERENCES

1. Hanson, J. A., "Tensile Strength and Diagonal Tension Resistance of Structural Lightweight Concrete," ACI JOURNAL, *Proceedings* V. 58, No. 1, July 1961, pp. 1-40.

2. Bryant, J. S., "The Determination of the Moisture Absorption Characteristics of Lightweight Concrete Aggregates," MS Thesis, Texas A & M University, 1959.

3. Kani, G. N. J., "The Riddle of Shear Failure and Its Solution," ACI JOURNAL, *Proceedings* V. 61, No. 4, Apr. 1964, pp. 441-468.

4. de Cossio, Roger Diaz, and Loera, Santiago, Discussion of the paper "Basic Facts Concerning Shear Failures," by G. N. J. Kani, ACI JOURNAL, *Proceedings* V. 63, No. 12, Dec. 1966, pp. 1511-1514.

5. Ferguson, P. M., "Some Implications of Recent Diagonal Tension Tests," ACI JOURNAL, *Proceedings* V. 53, No. 2, Aug. 1956, pp. 157-172.

6. Taylor, R., "Some Shear Tests on Reinforced Concrete Beams Without Web Reinforcement," *Magazine of Concrete Research* (London), V. 1, No. 4, Nov. 1960, pp. 145-154.

7. Harun-Ur-Rashid, Kazi, "Diagonal Tension Resistance of Structural Lightweight Aggregate Concrete Beams," an unpublished Dissertation submitted to the Graduate College of the Texas A & M University, Jan. 1966.

8. Ivey, Don L., and Buth, Eugene, "Splitting Tension Test of Structural Lightweight Concrete," *Journal of Materials,* V. 1, No. 4, Dec. 1966, pp. 859-871.

APPENDIX

NOTATION

v_c = shear stress carried by concrete

v_{c_1} = Eq. (17-9)* neglecting ϕ and replacing 0.28 $F_{sp}\sqrt{f_c'}$ by 0.28 f'_{sp}

v_{c_2} = Eq. (17-9) neglecting ϕ

v_{c_t} = shear stress at diagonal tension cracking

$V_{c_{DT}}$ = shear at diagonal tension cracking

$V_{c_{ult}}$ = shear at ultimate load

f'_{sp} = splitting tensile strength of concrete (Section 505)

f_c' = compressive strength of concrete (Section 301)

F_{sp} = ratio of splitting tensile strength to the square root of the compressive strength (Section 505)

p = A_s/bd (Section 1700)

d = distance from extreme compression fiber to centroid of tension reinforcement

a = shear span

V = total shear at section

M = bending moment

ϕ = capacity reduction factor (Section 1504)

*All equation and section numbers refer to ACI 318-63.

This paper was received by the Institute Apr. 24, 1967.

Sinopsis—Résumé—Zusammenfassung

Capacidad a Cortante de Vigas de Concreto Ligero

Se ensayaron 26 vigas de concreto ligero para obtener información adicional sobre la capacidad a cortante de concreto estructural de peso ligero y para evaluar los requisitos relativos a cortante establecidos en el Reglamendo de Construcciones ACI.

Las vigas ensayadas en este programa se comparan con las fórmulas actuales de cortante y con otros enfoques de diseño que están siendo considerados como modificaciones o cambios a los procedimientos de diseño del Reglamento ACI en uso.

Résistance à l'Effort Tranchant de Poutres en Béton Léger

Vingt-six poutres en béton léger ont été essayées en vue d'obtenir des renseignements complémentaires sur la résistance à l'effort tranchant du béton léger de structures et en vue de pouvoir porter un jugement sur les spécifications relatives à l'effort tranchant du Code ACI 1963. Les poutres de ce programme ont été comparées avec les formules actuelles de calcul à l'effort tranchant et avec d'autres procédés de calculs qui sont envisagés en vue de modifier ou de changer la méthode du Code actuel.

Die Schubtragfähigkeit von Balken aus Leichtbeton

Sechsundzwanzig Leichtbetonbalken wurden geprüft, um zusätzliche Information über die Schubtragfähigkeit von konstruktivem Leichtbeton zu erhalten und um die Gültigkeit der entsprechenden Abschnitte in den ACI Bauvorschriften aus dem Jahre 1963 zu überprüfen. Die im Rahmen dieser Untersuchung geprüften Balken werden mit den derzeitigen Entwurfsformeln und mit anderen Entwurfsmethoden verglichen, die z.Zt. als Verbesserungen oder Änderungen der gegenwärtig gültigen ACI Methode vorgeschlagen wurden.