Title No. 59-1

Report of ACI-ASCE Committee 326

Shear and Diagonal Tension

The ACI-ASCE Committee 326 report will be published as follows:

Part 1—General Principles, Chapters 1-4: January 1962

Part 2—Beams and Frames, Chapters 5-7: February 1962

Part 3—Slabs and Footings, Chapter 8: March 1962

This report was submitted to letter ballot of the committee which consists of 15 members and was approved without a dissenting vote.

Received by the Institute August 14, 1961. Title No. 59-1 is a part of copyrighted Journal of the American Concrete Institute, Proceedings V. 59, No. 1, Jan. 1962. Reprints of the complete report will be available after publication of Part 3 in March 1962.

Discussion of this report (three parts) should reach ACI headquarters in triplicate by June 1, 1962, for publication in the September 1962 JOURNAL.

Esfuerzo Cortante y Tensión Diagonal

Se revisan los conocimientos científicos, ingeniería práctica y experiencias en construcciones relativas al esfuerzo cortante y tensión diagonal en vigas, armaduras (marcos rigidos), losas y cimientos de hormigón armado. Se recomiendan

nuevos procedimientos de diseño comprobados por los datos obtenidos por medio de ensayos extensivos.

Los capítulos 1 al 4 tratan de los antecedentes y principios generales. Los capítulos 5 al 7 presentan el desarrollo de nuevos métodos de diseño para miembros de hormigón armado con y sin refuerzo del alma y para miembros con y sin carga axial combinada con la flexión y el esfuerzo cortante. El capítulo 8 trata de las losas y cimientos incluyendo el efecto causado por agujeros y el traspaso de momentos de las columnas a las losas.

L'Effort Tranchant et la Contrainte Principale

On présente une revue de l'art scientifique, de la pratique du génie et des expériences dans la construction relatives aux efforts tranchants et à la contrainte principale dans les poutres, les portiques, des dalles et les semelles de fondations en béton armé. Les recommendations pour les nouvelles procédés de calcul sont justifées à l'aide de résultats nombreaux d'essais.

Les chapitres 1 à 4 concernent les bases et les principes généraux. Les chapitres 5 à 7 présentent l'évolution de nouvelles methodes de calcul d'éléments en béton armé avec et sans armatures de cisaillement et d'éléments soumis à la flexion simple et composée avec les efforts tranchants. Le chapitre 8 concerne les dalles et les semelles de fondations y compris l'influence des trous et la transmission de moments de flexion des colonnes aux dalles.

Schub- und Hauptzugspannungen

Es wird eine Übersicht gegeben über wissenschaftliche Erkenntnisse, technische Praxis und Bauerfahrungen bezüglich Schubsicherung in Stahlbetonträgern, Rahmen, Platten und Säulenfussplatten. Empfehlunger für neue Berechnungsverfahren werden durch umfassende Versuchsunterlagen erhärtet.

Kapitel 1 bis 4 behandeln die Vorgeschichte und die allgemeinen Grundsätze. Kapitel 5 bis 7 enthalten die Entwicklung von neuen Berechnungsmethoden für Stahlbetonteile ohne und mit Schubbewehrung und für Bauglieder unter Biegung und Schub ohne und mit gleichzeitiger Längskraft. Kapitel 8 behandelt Platten und Säulenfussplatten einschliesslich der Wirkung von Aussparungen und die Übertragung von Momenten von Säulen zu Platten.

ACI-ASCE Committee 326, Shear and Diagonal Tension, was formed in 1950 to develop methods for designing reinforced concrete members to resist shear and diagonal tension consistent with ultimate strength design. Several investigations and test programs were initiated, sponsored and conduced by numerous organizaions, including Committee 326, the Reinforced Concrete Council, many universities (especially the University of Illinois), the American Iron and Steel Institute, and the Portland Cement Association. Progress reports of Committee work were presented at the ACI 55th annual convention, February 1959, and the 56th convention, March 1960. This three-part report is the culmination of a 10-year study.

Shear and Diagonal Tension

Report of ACI-ASCE Committee 326

EIVIND HOGNESTAD
Chairman

EDWARD COHEN
Vice-Chairman

C. A. WILLSON
Secretary

RAYMOND ARCHIBALD
WALTER E. BLESSEY
BORIS BRESLER
RICHARD C. ELSTNER
CLYDE E. KESLER
WILLIAM J. KREFELD
SVEN T. A. ÖDMAN
ALFRED L. PARME
DOUGLAS E. PARSONS
RAYMOND C. REESE
FREDERIC ROLL
IVAN M. VIEST

Presents a review of scientific knowledge, engineering practice, and construction experiences regarding shear and diagonal tension in reinforced concrete beams, frames, slabs, and footings. Recommendations for new design procedures are substantiated by extensive test data.

Chapters 1 through 4 deal with background and general principles. Chapters 5 through 7 present the development of new design methods for reinforced concrete members without and with web reinforcement, and for members without and with axial load acting in combination with bending and shear. Chapter 8 deals with slabs and footings including the effect of holes and transfer of moments from columns to slabs.

FOREWORD

In submitting this report to the parent societies, Committee 326 wishes to express its profound gratitude and sincere appreciation for the enthusiastic leadership given the committee by the late Charles S. Whitney, Chairman from 1950 to his death in 1959. The surge of American research activities in the 1950's, which essentially form the basis of this report, came about largely by his dynamic guidance and support.

CHAPTER 1—PURPOSE AND DEVELOPMENT OF REPORT

100—Purpose of report

It is the purpose of this report to consolidate thoughts and knowledge gained from various experimental and analytical investigations into a form useful to practicing engineers, and also to formulate safe and workable design procedures. The committee terminated consideration of new researches with those available at the end of 1959, and committee efforts were then concentrated on development of design procedures based on knowledge available on January 1, 1960. The committee makes no attempt whatever in this report to consolidate knowledge with the high degree of detail and refinement required by research workers. An annotated bibliography was prepared for this purpose.†

101—Development of report

Joint Committee 326 of the American Concrete Institute and the American Society of Civil Engineers was formed in 1950 with the assignment of "developing methods for designing reinforced concrete members to resist shear and diagonal tension, consistent with the new ultimate strength design methods." The committee immediately commenced a comprehensive study of available information, and a program of tests of beams without web reinforcement was initiated at the University of Illinois under the sponsorship of the Reinforced Concrete Research Council. Although this initial program did not lead to a base for fulfillment of the committee's assignment, the results focused attention on the complexity of shear and diagonal tension, on a possible lack of safety by 1951 design procedures, and on a general lack of knowledge regarding the fundamental nature of effects of shear and diagonal tension on the behavior of reinforced concrete members.

Shear and diagonal tension therefore became the major research theme in the field of reinforced concrete for the 1950-1960 decade. A few structural failures added momentum to the intensive efforts devoted to the solution of the problems involved. Several investigations and test programs were initiated, sponsored and conducted by many interested organizations, including Committee 326, the Reinforced Concrete Research Council, various governmental agencies, several universities, the American Iron and Steel Institute, and the Portland Cement Association. The intensity of research efforts in the 1950's are illustrated by the number of articles appearing in technical literature.† Prior to 1950, published papers on shear and diagonal tension averaged about four per year, rarely exceeding six. In 1954 eight papers were published; the number increased to 17 in 1955, 23 in 1956, 35 in 1957, and over 40 in 1958.

The problems of shear and diagonal tension have not been fundamentally and conclusively solved. In some areas, research has amassed sufficient data from which reliable empirical design procedures may be established. Other areas, such as effects of torsion, have received relatively little research attention. Committee 326 wishes strongly to encourage further research work, not only to explore other areas of the problem, but also to establish a basically *rational theory* for effects of shear and diagonal tension on the behavior of reinforced concrete members.

---

†An extensive annotated bibliography on shear, diagonal tension, and torsion prepared by Committee 326 will be published in the bibliography series of the American Concrete Institute.

CHAPTER 2—BACKGROUND†

200—Early developments

Early pioneers of reinforced concrete before the year 1900 developed two schools of thought pertaining to the mechanism of shear failures in reinforced concrete members. One school of thought considered horizontal shear as the basic cause of shear failures. This seemed a reasonable approach at a time when scholars and engineers were familiar with the action of web rivets in steel girders and shear-keys in wooden beams, for which shearing stresses were computed using the classical equation

$$v = \frac{VQ}{Ib} \quad \text{(2-1)}$$

where‡

v = unit horizontal shear stress at a distance y from the neutral axis
V = total vertical shear at the section
Q = first moment of the part of the cross-sectional area cut off at distance y from the neutral axis, with respect to the neutral axis
I = moment of inertia of the cross-sectional area with respect to the neutral axis
b = width of the cross section at a distance y from the neutral axis

Reinforced concrete beams were treated as an extension of the older materials assuming that concrete alone could only resist low horizontal shearing stresses, and that vertical stirrups acted as shear-keys for higher shearing stresses.

The second school of thought, accepted by nearly all engineers today, considered diagonal tension the basic cause of shear failures. The origin of the concepts of diagonal tension is uncertain, but a clear explanation of diagonal tension was presented by W. Ritter as early as 1899. He stated that stirrups resisted tension not horizontal shear, and suggested design of vertical stirrups by the equation

$$V = \frac{A_v f_v jd}{s} \quad \text{(2-2)}$$

where

A_v = total cross-sectional area of one stirrup
f_v = allowable stress in the stirrups
jd = internal moment arm
s = spacing of stirrups in the direction of the axis of the member

Although Ritter's viewpoints were not widely accepted at the time, his design expression for vertical stirrups is identical to that appearing in modern design specifications of most countries.

---

†Hognestad, E., "What Do We Know About Diagonal Tension and Web Reinforcement in Concrete?", *Circular Series* No. 64, University of Illinois Engineering Experiment Station, Mar. 1952, 47 pp.

‡See Notation, Chapter 8.

Discussion between the proponents of horizontal shear and diagonal tension continued for nearly a decade until laboratory tests resolved the issue, mainly through the efforts of E. Mörsch in Germany. He pointed out that, if a state of pure shear stress exists, then a tensile stress of equal magnitude must exist on a 45-deg plane. Furthermore, he developed the equation for nominal shearing stress widely used today.

$$v = \frac{V}{bjd} \qquad \text{(2-3)}$$

Succeeding papers by Mörsch in 1906 and 1907 presented a clear explanation of the diagonal tension mechanism and listed several arguments against the horizontal shear concept:

1. The ultimate nominal shearing stresses in beams without web reinforcement, as computed by Eq. (2-3), are close to the tensile strength of concrete, while punching tests indicate that the shearing strength of concrete is considerably greater than its tensile strength. Hence, shear failure in beams is due to diagonal tension, not horizontal shear.

2. The effectiveness of stirrups far surpasses the values computed by the horizontal shear theory. The effectiveness of stirrups derived from the tensile force transmitted across a diagonal tension crack is in better accord with tests.

3. Eq. (2-3), which expresses the nominal shearing stress is intended to be only a nominal measure of diagonal tension.

Mörsch's data, supported by tests by F. von Emperger and E. Probst, terminated discussions of horizontal shear criteria. About 1910, a return to Ritter's pioneering concepts had been made, though the concepts of horizontal shear have reappeared periodically in the literature even in recent years. Today, however, most design codes and specifications throughout the world predicate their shear design procedures on the concepts of diagonal tension.

201—Development of the classical diagonal tension equation

Because of the widespread use and importance of Eq. (2-3), a critical review of its foundations and justifications is warranted. Its mathematical derivation, which can be found in any textbook on concrete structures, is based on the following assumptions:

1. Concrete and steel are homogeneous and isotropic.
2. Stresses do not exceed the proportional limits.
3. Beams have constant cross sections.
4. Distribution of the shearing stresses is uniform across the width of the beam.
5. Concrete carries no flexural tension below the neutral axis.

The inclusion of Assumption 5 distinguishes the derivation of shearing stresses in reinforced concrete beams from that for other beams of two materials. Because of this assumption, the shearing stress in reinforced concrete beams is found to be of constant magnitude below the neutral axis, having the value given by Eq. (2-3). Above the neutral axis its intensity diminishes parabolically to zero at the top surface. Since j normally varies within narrow limits about the value $j = 7/8$, it has become common American practice to express Eq. (2-3) as

$$v = \frac{8\,V}{7\,bd} \qquad \text{(2.3a)}$$

At any point in a homogeneous, isotropic beam the diagonal tension stress can be related to the shearing stress v, and the flexural tension stress f_t, through the principal stress equation

$$f_t(\text{max}) = \tfrac{1}{2}f_t + \sqrt{\tfrac{1}{4}f_t^2 + v^2} \qquad \text{(2-4)}$$

Since shear failures in concrete originate from a weakness in tension, it is important to establish a relation for reinforced concrete beams similar to that of Eq. (2-4). Most current texts on reinforced concrete adopt Eq. (2-4), either tacitly or with an explanation supporting its approximate applicability. They argue that in most regions of reinforced concrete beams where the shear stress v is relatively large, the flexural tensile stress f_t is relatively small, and, consequently, the diagonal tension stress f_t (max) is approximately equal to the shearing stress v. They assert that the magnitude of the shearing stress expressed by Eq. (2-3) is a *measure* of the diagonal tension stress.

In the development of the expression for shearing stress and in its generalization as a measure of diagonal tension, the ability of concrete to resist some degree of tension is first neglected and then acknowledged. Such an inconsistency suggests a basic weakness in Eq. (2-3).

202—Development of design equations for stirrups and bent-up bars

The analysis of stirrup action known as the "truss analogy" is presented here first in complete form, and then with simplifying assumptions.

The action of a reinforced concrete beam with stirrups may be represented as that of a truss in which the concrete compression zone is the top chord, the tension reinforcement is the bottom chord, the stirrups or bent-up bars are the tension web members, and portions of the concrete web of the beam are compression web members. This analogy involves four basic assumptions:

1. The compression zone carries only horizontal flexural compressive stresses.
2. The tension reinforcement carries only horizontal flexural tensile stresses.
3. All inclined and vertical tensile stresses are carried by the stirrups or bent-up bars.
4. A diagonal tension crack extends from the tensile reinforcement up to a vertical height equal to the effective moment arm jd.

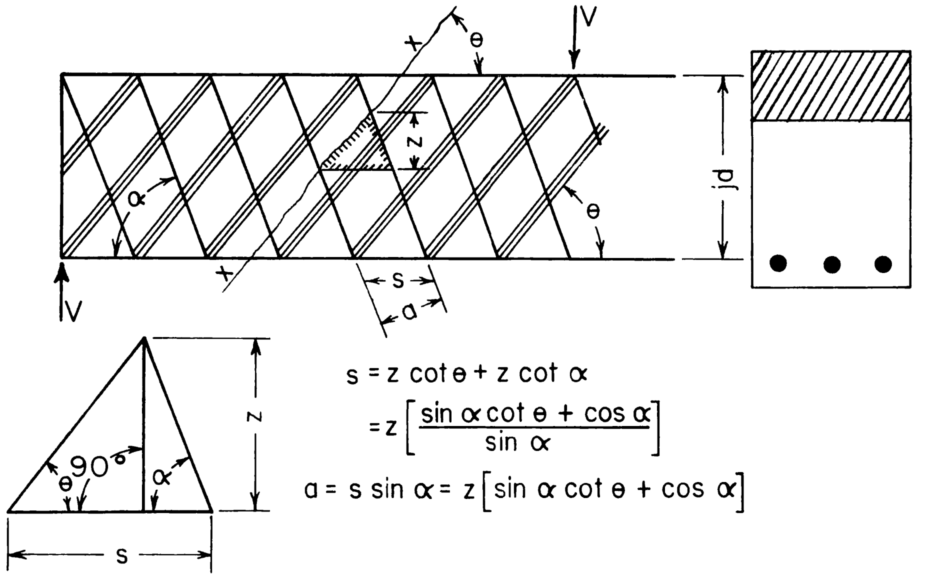


Fig. 2-1—Truss analogy

Based on this analogy and its implied assumptions, a rather general analysis of the stresses in stirrups inclined at an angle α with the axis of the member is shown in Fig. (2-1).† The inclination of the diagonal crack is generalized by assuming it forms at an angle θ with the axis of the member. The general equation expressing the force in a stirrup is then:

$$A_v f_v = \frac{Va}{jd(\sin\alpha\ \cot\theta + \cos\alpha)\ \sin\alpha} \qquad \text{(2-5)}$$

where

a = spacing of stirrups in direction perpendicular to the stirrups
α = angle of inclination of stirrups with respect to the axis of the beam
θ = inclination of diagonal crack
V = shearing force
jd = internal moment arm
$A_v f_v$ = area times stress of one stirrup

When the stirrups are vertical, $\alpha = 90$ deg; and if the diagonal crack has the angle $\theta = 45$ deg, the expression becomes:

$$A_v f_v = \frac{Va}{jd} \qquad \text{(2-6)}$$

†*Bulletin* 166, University of Illinois Engineering Experiment Station, University of Illinois, Urbana, Ill., June 1927.

Since $a = s$ when $\alpha = 90$ deg, Eq. (2-6) is identical to Eq. (2-2) proposed by Ritter in 1899.

Eq. (2-5) can be simplified by substituting $A_v = rab$, $V = vbjd$, and $K = (\sin\alpha \cot\theta + \cos\alpha)\sin\alpha$. Hence, the equation usually appears as

$$v = K\,r\,f_v \qquad \text{(2-7)}$$

where

v = nominal shearing stress, V/bjd
r = ratio of web reinforcement A_v/ab
f_v = tensile stress in web reinforcement
K = efficiency factor of web reinforcement

It has become common practice to further simplify the procedure by assuming a crack inclination $\theta = 45$ deg. Thus, $K = (\sin\alpha + \cos\alpha)\sin\alpha$. The factor K is then 1.00 for $\alpha = 90$ deg, and a maximum value of $K = 1.20$ is obtained when $\alpha = 67.5$ deg.

The development of this general analysis is logical and rational. However, the four assumptions implied in the analogy and the treatment of the angle θ pose questions which will be explored later in this report.

203—Review of ACI and JC design specifications

NACU report of 1908 — The first report of the National Association of Cement Users, forerunner of the ACI, was based on what is known today as ultimate strength design. It specified that "the shearing strength of concrete, corresponding to a compressive strength of 2000 psi, shall be assumed at 200 psi" and that " . . . when the shearing stresses developed in any part of a reinforced concrete constructed building exceeds, under the multiplied loads, the shearing strength as fixed by the section, a sufficient amount of steel shall be introduced in such a position that the deficiency in the resistance of shear is overcome." No formulas were presented either for the determination of shearing stress or for the design of web reinforcement.

NACU Standard No. 4 in 1910 — This standard introduced the concepts of working stresses as a design criterion and recommended, "In calculating web reinforcement the concrete shall be considered to carry 40 psi (assuming $f_c' = 2000$; could be increased to 50 psi proportionately with f_c'), the remainder to be provided by means of reinforcement in tension."

Progress report of First Joint Committee in 1909 — This report established the general methods followed by succeeding codes. It clearly indicated the principles of diagonal tension and the action of bent-up bars and stirrups:

> "Calculations of web resistance shall be made on the basis of maximum shearing stress as determined by the formulas hereinafter given ($v = V/bjd$). When the maximum shearing stresses exceed the value allowed for con-

crete alone, web reinforcement must be provided to aid in carrying diagonal tensile stresses. This web reinforcement may consist of bent bars, or inclined or vertical members attached to or looped about the horizontal reinforcement. Where inclined members are used, the connection to the horizontal reinforcement shall be such as to insure against slip.

"Experiments bearing on the design of details of web reinforcement are not yet complete enough to allow more general and tentative recommendations to be made. It is well established, however, that a very moderate amount of reinforcement, such as is furnished by a few bars bent up at small inclination, increases the strength of a beam against failure by diagonal tension to a considerable degree; and that a sufficient amount of web reinforcement can readily be provided to increase the shearing resistance to a value from three or more times that found when the bars are all horizontal and no web reinforcement is used. The following allowable values for the maximum shearing stress are therefore recommended (based on $f_c' = 2000$ psi; may be increased proportional to f_c' but this increase shall not exceed 25 percent):

(a) For beams with horizontal bars only, 40 psi

(b) For beams in which a part of the horizontal reinforcement is used in the form of bent-up bars, arranged with respect to the shearing stresses, a higher value may be allowed, but not exceeding 60 psi

(c) For beams thoroughly reinforced for shear, a value not exceeding 120 psi

"In the calculation of web reinforcement to provide the strength required in (c) above, the concrete may be counted upon as carrying 1/3 of the shear. The remainder is to be provided for by means of metal reinforcement consisting of bent bars or stirrups, but preferably both. The requisite amount of such reinforcement may be estimated on the assumption that the entire shear on a section, less the amount assumed to be carried by the concrete, is carried by the reinforcement in a length of beam equal to its depth.

"Stresses in web reinforcement may be estimated by means of the following formulas: vertical reinforcement, $P = Vs/jd$; reinforcement inclined at 45 deg, $P = 0.7\, Vs/jd$; in which P = stress in a single reinforcing member, V = proportion of total shear as carried by the reinforcement (2/3 of total V), and s = horizontal spacing of the reinforcing members."

Second Progress Report of First Joint Committee in 1913 — The second report retained the general methods of the first report in 1909 with modifications of allowable stresses. The allowable shearing stress for beams with horizontal bars only was set at $0.02\, f_c'$. A maximum ceiling of 66 psi is inferred in other sections of the report. Likewise the allowable shear stresses for beams thoroughly reinforced for shear was set at $0.06\, f_c'$, with a ceiling of 198 psi inferred in other sections of the report.

Also, the report includes a few philosophical concepts about shear and diagonal tension, many of which have subsequently become forgotten. They are quoted here not only because of their profound historical importance, but also because the same concepts have again been brought to the foreground by recent researches:

"For the composite structure of reinforced concrete beams, an analysis of the web stresses, particularly of the diagonal tensile stresses, is very complex, and when the variations due to a change from no horizontal tensile stresses in the concrete at remotest fibers to the presence of horizontal tensile stresses at some point below the neutral axis are considered, the problem becomes even more complex and indefinite. Under these circumstances, in designing, recourse is had to the use of the calculated shearing stress as a means of comparing or measuring the diagonal tensile stresses developed, it being understood that the vertical shearing stress is not the numerical equivalent of the diagonal tensile stress and even that there is not a constant ratio between them.

"Even after the concrete has reached its limit of resistance to diagonal tension, if the beam has web reinforcement, conditions of beam action will continue to prevail at least through the compression area, and the web reinforcement will be called on to resist only part of the web stresses . . . It is concluded that it is safe practice to use only 2/3 of the external vertical shear in making calculations of the stresses that come on stirrups . . .

"It is necessary that a limit be placed on the amount of shear which may be allowed in a beam; for when web reinforcement sufficiently efficient to give very high web resistance is used, at the higher stresses the concrete in the beam becomes checked and cracked in such a way as to endanger its durability as well as its strength.

"The section to be taken as the critical section in the calculation of shearing stresses will generally be the one having the maximum vertical shear, though experiments show that the section at which diagonal tension failures occur is not just at a support, even though the shear at the latter point be much greater."

ACI reports in 1916 and 1917 — These reports were never adopted as official codes, but they serve to illustrate the tendency toward the use of less and less web reinforcement. This trend continued in American specifications until the 1950's. The allowable shearing stress to be resisted by the concrete was set at 0.02 f_c', with 66 psi inferred to be the maximum limit. The excess shear up to a ceiling value of 0.075 f_c' (247 psi maximum inferred) could be resisted by web reinforcement.

These reports also introduced punching shear in slabs and footings having an allowable value of 0.075 f_c' based on the area having the width equal to the perimeter of the column or pier and depth equal to the depth from the top of the concrete to the centroid of tensile reinforcement.

In addition to punching shear, diagonal tension was calculated in footings on vertical sections a distance d from the face of the column or pier. The allowable stress was 0.02 f_c'.

Final Report of the First Joint Committee in 1916 — This final report paralleled the second progress report of 1913. The following maximum nominal shearing stresses were given:

No web reinforcement	0.02 f_c' (66 psi max)
Vertical stirrups or bent-up bars	0.045 f_c' (148 psi max)
Vertical stirrups and bent-up bars	0.05 f_c' (165 psi max)
Securely attached stirrups and bent-up bars	0.06 f_c' (198 psi max)

Web reinforcement should be designed for 2/3 of the shear. For combined stirrups and bent-up bars, however, the contribution of the bent-up bars should first be subtracted from the total shear (maximum of 0.045 f_c'); then 1/3 of the remaining shear should be carried by the concrete and 2/3 by the stirrups. Therefore, the maximum nominal shearing stress could be 0.105 f_c'.

The report revised the equation for the design of inclined stirrups and bent-up bars:

For bars bent up at angles between 20 and 45 deg with the horizontal and web members inclined at 45 deg, $T = 0.75\ V's/jd$, where T = total stress in single reinforcing member, and V' = total shear producing stress in reinforcement.

ACI Report of 1919 — The allowable nominal shearing stress for beams without web reinforcement was maintained at 0.02 f_c' (66 psi maximum implied), while for beams with properly designed web reinforcement it was increased to 0.075 f_c' (248 psi maximum implied). The equation for design of web reinforcement was rewritten to read: $A_s = (2/3)\ Vs/f_s jd$, in which A_s = area of one web bar or stirrup, s = spacing of bars normal to their direction, and V = total shear at section. Allowable punching stress in slabs, provided that the diagonal tension requirements were met, was set at 0.075 f_c' (248 psi maximum).

ACI Standard Specification No. 23 of 1920 — This specification represents an almost complete development of American design of web reinforcement. The specification permitted the following nominal shearing stresses: For beams without web reinforcement, 0.02 f_c' (60 psi maximum); for beams without web reinforcement, with special anchorage of longitudinal reinforcement, 0.03 f_c' (90 psi maximum).

Web reinforcement was designed by the equation $A_v f_v = V'a \sin \alpha / jd$, where V' = total shear minus 0.02 $f_c'\ bjd$ (or 0.025 $f_c'\ bjd$ with special anchorage), and a = spacing of shear steel measured perpendicular to its direction. The ceiling value for nominal shearing stress was 0.06 f_c' (180 psi maximum), or with anchorage of longitudinal steel 0.12 f_c' (360 psi maximum).

For bent-up bars in a single plane, the following equation was specified: $A_v f_v = V' \sec \alpha$, where V' = total shear minus 0.02 f_c' (60 psi maximum) bjd and α = angle between bent-up bar and the vertical.

Progress Report of Second Joint Committee 1921 — The proposed specification permitted nominal shearing stresses similar to those given in ACI Standard No. 23. It is interesting to note in the Joint Committee Report, however, that no limitation was placed on f_c'. Consequently the values 0.03 f_c', 0.06 f_c' and 0.12 f_c' had no ceilings.

This progress report first stated the allowable shearing stresses in flat slabs in the form used in recent years. The shearing stress is computed at approximately the distance d from the column face with an allowable shearing stress ranging up to 0.030 f_c' depending on the percentage of longitudinal steel passing through the area in question. Again f_c' was not limited.

Final Report of the Second Joint Committee in 1924 — The ACI Code of 1920 and the JC Progress Report of 1921 contained many special cases and limitations. This final report eliminated many of them by simplifying the design equations to $v = V/bjd = 0.02\ f_c' + (f_vA_v/bs \sin \alpha)$, for α greater than 45 deg and, $v = V/bjd = 0.02\, f_c' + (f_vA_v/bs)\ (\sin \alpha + \cos \alpha)$ for α less than 45 deg. The equations were limited to 0.06 f_c' when longitudinal reinforcement had no special anchorage, and 0.12 f_c' when there was special anchorage (in which case 0.03 f_c' was substituted for 0.02f_c' in the formulas). There were no limitations on f_c'. However, the quantity $f_vA_v\ (\sin \alpha + \cos \alpha)/bs$ was limited to 75 psi.

ACI Specification E-1A-27 of 1927 — This specification closely followed the JC Report of 1924. Design methods were simplified to one expression: $v = V/bjd = 0.02\ f_c' + (f_vA_v/bs)\ (\sin \alpha + \cos \alpha)$. When longitudinal reinforcement had no special anchorage, v was limited to 0.06 f_c'. With special anchorage, v was limited to 0.12 f_c' (in which case 0.03 f_c' was substituted for 0.02 f_c'). The last term in the equation was limited to 75 psi when longitudinal bars were bent up in a single plane. No limits were placed on f_c'.

ACI Specification E-1A-28T of 1928 — Equations for the design of web reinforcement reverted back to ACI Standard Specification No. 23 of 1920 except that there were no ceilings on allowable shear stresses resulting from limitations on f_c'. The maximum ceiling of 0.12 f_c' was also qualified; stresses greater than 0.09 f_c' were permitted only if the designer personally supervised construction. Allowable web steel stresses were also reduced.

ACI Standard 501-36T of 1936 — The conservatism of 1928 was removed by the 1936 Standard. The maximum ceiling again increased to 0.12 f_c' and allowable web stresses were raised. The definition of stirrup spacing s in the equations for web reinforcement was again changed.

Final Report of the Third Joint Committee in 1940 — The recommendations of the joint committee were essentially in agreement with ACI Standard 501-36T with one exception, viz., if the shearing stresses exceeded 0.06 f_c', web reinforcement should carry the entire shear.

ACI Building Regulations for Reinforced Concrete (ACI 318-41) — No changes were made from ACI Standard 501-36T except that the maximum shear in footings was limited to 75 psi.

ACI Standard 318-47 — No changes were made from ACI 318-41.

ACI Standard 318-51 — The requirements for special anchorage, included in the earlier codes, were replaced by the provision that all plain bars must be hooked and all deformed bars must meet the requirements of ASTM A 305. A maximum shearing stress of 0.03 f_c' was specified for all beams without web reinforcement and a ceiling of 0.12 f_c' was specified for beams with web reinforcement.

ACI Standard 318-56 — In this standard the shear requirements again became more conservative. As a result of recommendations by Committee 326, ceilings were placed on maximum stresses for the first time since 1920. Shearing stresses were limited to 90 psi for beams without web reinforcement, 240 psi for beams with stirrups or bent bars, and 360 psi for beams with both stirrups and bent bars. Shearing stresses in flat slabs were limited to 100 psi. Restrictions were added to continuous or restrained beams or frames not having T-beam action.

204—Review of foreign specifications

Germany — The provisions of the German specifications for reinforced concrete with respect to shear have developed largely from the work of Mörsch. The formula $v = V/bjd$ appeared in the first official German specifications in 1904. The maximum allowable stress v computed from the formula, was set at 64 psi for members without web reinforcement. This value could be exceeded by 20 percent if web reinforcement was provided. No method for the design of such web reinforcement was presented in these specifications.

The Prussian code of 1907 was based on a minimum concrete cube strength f'_{cu}, of approximately 1500 psi. Shearing stress up to 64 psi, as computed from this equation, was allotted to the concrete. Any excess shearing stress up to the ceiling value of 77 psi was to be resisted by web reinforcement.

A change in the provisions for web reinforcement was made in the German specifications of 1916 as a result of experimental studies conducted in Germany and in Austria. For concrete with a cube compressive strength, $f'_{cu} = 2100$ psi, the ceiling value of the nominal shearing stress v was raised from 77 to 200 psi, while the shear allotted to concrete was decreased from 64 to 57 psi. Wherever the nominal shearing stress exceeded 57 psi, all shear was to be resisted by web reinforcement alone. No equations for the design of web reinforcement were given.

The requirements for web reinforcement were revised again in the German specifications of 1925. If the nominal shearing stress v exceeded 57 psi, all shear in the corresponding half of the span was to be resisted by web reinforcement alone, without any contribution by the concrete.

The specifications of 1925 pertaining to web reinforcement remained in effect until 1943. Although the 1943 specifications, DIN 1045, retained the basic concepts of the 1925 specifications, the ceiling values of the nominal shearing stress v were adjusted in accordance with the introduction of four qualities of concrete with f'_{cu} ranging from about 1700 to 4300 psi. The ceiling values of v range from 0.116 to 0.067 times f'_{cu} when web reinforcement is provided; only nominal reinforcement to resist shear is required if v does not exceed from 0.033 to 0.027 times f'_{cu}. The percentage in either case decreases with increasing concrete strength. The web reinforcement may consist either of stirrups or bent-up bars or a combination of the two. However, the specifications recommend that a greater portion of the shearing stress be allotted to the bent-up bars.

USSR — In the Soviet Union the principles of limit design and of ultimate strength design were introduced into design specifications in 1938, and the "classical" theory of reinforced concrete was abandoned. Accordingly, the term "design load" is used to designate the working load times an overload factor, while the term "design stress" means the minimum probable strength of a material of a given quality class.†

The formula for the shear strength of a diagonal section given in the "Standards and Technical Specifications for the Design of Plain and Reinforced Concrete Structures (NiTU 123-55)" of 1955 is based on the condition of equilibrium of internal and external forces acting in the direction perpendicular to the axis of the member.

$$V = m\,[m_n\, m_s\, f_s\,(\Sigma A_o \sin\alpha + \Sigma A_v) + V_c] \qquad (2\text{-}8)$$

where

V = design shear force at the section in question

V_c = shear force resisted by the concrete compression zone

ΣA_v = total cross-sectional area of vertical stirrups crossing the diagonal section

$\Sigma A_o \sin\alpha$ = sum of the cross-sectional areas of bent-up bars crossing the diagonal section, multiplied by sines of the respective angles, α, of their inclination with respect to the axis of the element

f_s = design stress of the web reinforcement

m = coefficient reflecting various conditions (in general $m = 1$; for flexural elements of precast structures cast in plants or specially equipped yard $m = 1.10$)

m_s = coefficient reflecting the uniformity of the steel (e.g., for hot-rolled deformed bars $m_s = 0.9$)

m_n = coefficient introduced to take into account the possibility that web reinforcement does not always yield prior to failure ($m_n = 0.8$ for all types of steel with the exception of cold drawn wire, for which $m_n = 0.7$ is specified)

The magnitude of the shear force resisted by the concrete compression zone V_c is derived from experimental investigation and is given in the specifications by the empirical equation

$$V_c = 0.15\, f_c{}^{*}\, bd^2\, a \qquad (2\text{-}9)$$

---

†Yu, C. Y.; Corbin, Margaret; and Hognestad, E., "Reinforced Concrete Design in the USSR," ACI JOURNAL, *Proceedings* V. 56, No. 1, July 1959, pp. 65-69.

where

f_c^* = design stress of concrete in flexural compression

b = width of a rectangular section or of the web of a T-section

d = effective depth of the section

a = length of the projection of the inclined diagonal section on the axis of the member

The strength in shear of an inclined diagonal section, determined from Eq. (2-8) and (2-9), depends on its angle of inclination. When vertical stirrups are used without bent-up bars, the projected length of the *critical* diagonal section corresponds to a minimum value of the expression $(m_n m_s f_s \Sigma A_v + V_c)$. By substituting $\Sigma A_v = n A_v a/s$ and differentiating, the critical value of crack projection is:

$$a_o = \sqrt{0.15 f_c^* bd^2 q_v} \qquad \text{(2-10)}$$

where

q_v = $m_n m_s f_s A_v n/s$ = stirrup force per unit beam length

A_v = cross-sectional area of one leg of the stirrup

n = number of legs of stirrups in one section of the element

s = spacing of stirrups in the longitudinal direction of the element

The combined shear strength of the concrete compression zone and of the stirrups V_{vc} is then

$$V_{vc} = \sqrt{0.6 f_c^* bd^2 q_v} \qquad \text{(2-11)}$$

When the external shear force V exceeds mV_{vc}, additional stirrups or bent-up bars must be provided. In the latter case the required cross-sectional area A_o of bent-up bars distributed in one plane is determined from the equation:

$$A_o = \frac{\dfrac{V}{m} - V_{vo}}{m_n m_s f_s \sin \alpha} \qquad \text{(2-12)}$$

The distance between stirrups, as well as the distance between the end of a bent-up bar and the beginning of the next bend, should not exceed the value $s_{max} = m \, 0.1 \, f_c^* \, bd^2/V$, when stirrups and bent-up bars resist shear together.

Strength computations of diagonal sections may be omitted if the external shear V is smaller than $m \, f_t \, bd$, in which f_t is the design tensile stress of concrete. In this latter case, the disposition of bent-up bars and minimum amount of stirrups is governed by provisions concerning the detailing of reinforcement.

The British Standard Code of Practice CP 114 of 1957 follows the basic principles of the German code of 1916. Where the shear stress calculated from $v = V/bjd$ exceeds the permissible shear stress for concrete, all shear must be resisted by the web reinforcement alone. The permissible shear stress for the concrete varies with the composition of the concrete, hence with the concrete strength. When two or more types of web reinforcement are provided, the total resistance to shear is taken as the sum of the resistances computed for each type separtely. The maximum spacing of stirrups is specified to be jd.

The National Building Code of Canada of 1953 stipulates that shear in excess over that permitted on concrete be resisted by web reinforcement. The shearing v_c allotted to concrete is limited to 0.03 f_c'. With properly designed web reinforcement, the shear stress computed from $v = V/bjd$ can be raised up to 0.12 f_c'. No ceiling value is specified for the stress v.

Other foreign specifications are generally similar to the German specifications in Western Europe, similar to the British or United States specifications in English-speaking countries, and similar to the USSR specifications in Eastern Europe.

205—Summary of specifications

Design specifications for shear and diagonal tension in beams generally consist of four major parts: (1) Numerical values of shear stress below which further investigations of web stresses is not required. (2) In some specifications, a minimum stirrup reinforcement is required even for low web stresses. (3) Design methods for members that require web reinforcement. (4) In some specifications, maximum shearing stresses for members with web reinforcement.

A general and direct comparison of the four major specifications, those of Britain, Germany, United States, and USSR, is not feasible without simplifying assumptions. In ACI 318-56, concrete strength is given by a cylinder strength f_c', below 90 percent of which only one strength test in ten may fall. In the three other countries, concrete strength is given as the average cube strength f'_{cu}. For purposes of comparison, it is assumed the f_c' and f'_{cu} so defined are numerically equal. Furthermore, USSR's NiTU 123-55 is entirely based on ultimate strength design, while shear design methods of the other three countries are based on working loads and allowable stress. In this comparison a live-to-dead load ratio of one is assumed; the over-all USSR load factor for normal buildings is then 1.25.

Shear stress without web investigation — Shear stresses below which detailed examination of web stresses are not required are shown in Fig. (2-2) as a function of concrete cylinder strength. It is seen that the requirements of all four countries are closely similar in the cylinder strength range of 2000 to 3000 psi. For strengths in excess of 3000 psi,

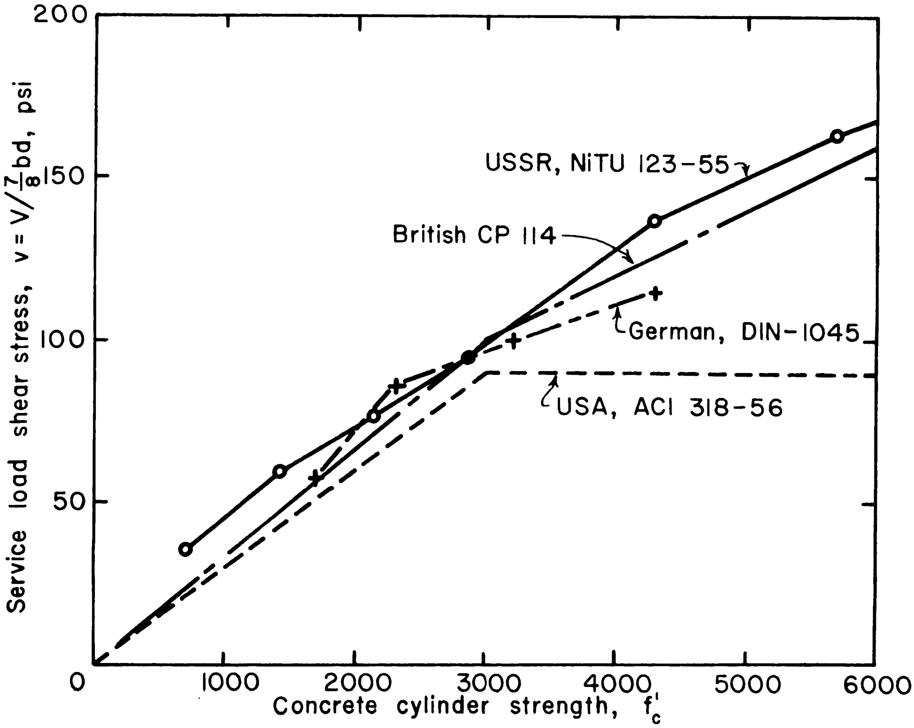


Fig. 2-2—Shear stress limits for beams without web investigation

ACI 318-56 calls for a ceiling stress of 90 psi, while the three other specifications continue the trend of approximately $v = 0.03\ f_c'$ which was also used in ACI 318-51.

Minimum stirrup reinforcement — ACI 318-56 calls for a minimum stirrup reinforcement, $r = 0.15$ percent, only when web reinforcement is required. The German DIN-1045 calls for stirrups in all beams, regardless of the magnitude of the shearing stress, though a numerical minimum amount is not given. USSR's NiTU 123-55 also calls for a minimum amount of stirrups in all beams, stirrup spacing not to exceed one-half of the effective beam depth for stirrup diameter at least one quarter of the diameter of the main reinforcing bars.

Design of web reinforcement — The German and British specifications both call for web reinforcement, when web investigation is required, to carry the total shear force by truss-analogy calculations. The ratio of web reinforcement required is indicated in Fig. (2-3) for $f_c' = 3000$ psi and $f_s = 20{,}000$ psi by the straight line $v = 20{,}000r$. The USA specification calls for web reinforcement to carry only the excess shear. This web ratio requirement is indicated (for the same parameters) by a line offset 90 psi from the British-German line in Fig. (2-3). The USSR specifications do not use the common form of truss analogy. The web ratio requirement of the equivalent of their design equation, Eq. (2-11), is shown by a parabola in Fig. (2-3) for $f_c^* = 1660$ psi and $m_n \, m_s \, f_s = 23{,}800$ psi.

Maximum shear stress — The German and British specifications both give maximum shear stresses as a function of concrete strength. The British maxima are four times the values permitted without web reinforcement. ACI 318-56 gives $v = 0.08 \, f_c'$ but not more than 240 psi for stirrups or bent bars alone; and $v = 0.12 \, f_c'$ but not more than 360 psi for combined web reinforcement. The USSR specification leads to high service load shear stresses as a result of the low over-all load factor of 1.25. Even so, no maximum values appear to be given.

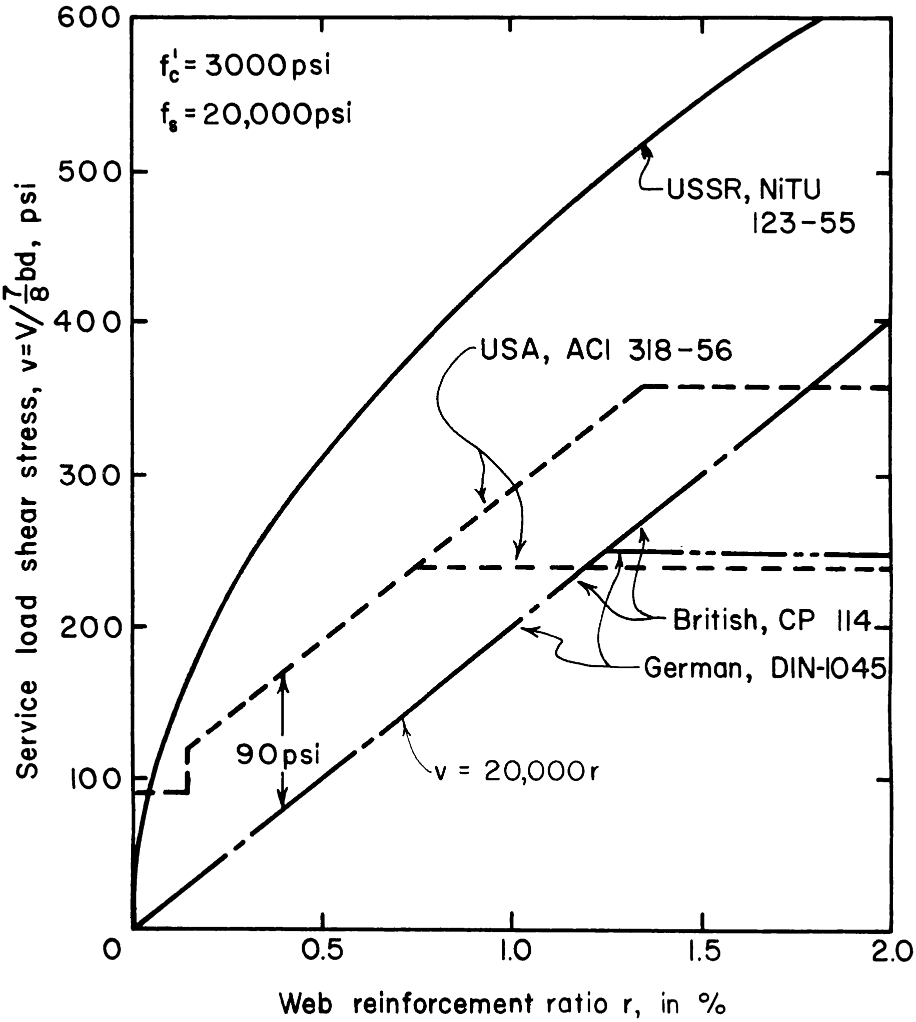


Fig. 2-3—Shear stress limits for beams with web reinforcement

CHAPTER 3—REVIEW OF STRUCTURAL FAILURES

One of the major difficulties in relating theoretical and laboratory investigations to failures of full-scale structures is that in actual structures designed with substantial factors of safety, failures occur infrequently and then usually from several contributory causes. It is often difficult to gather all of the pertinent facts and to determine the degree to which they contributed to the failure.

An outstanding exception was the warehouse failure at Wilkins Air Force Depot in Shelby, Ohio, which occurred in 1955. This failure intensified doubts and questions about calculation by ACI 318-51 of the diagonal tension strength of beams, doubts which had previously occurred to investigators reviewing laboratory test data then available.

The collapse of about 4000 sq ft of roof area at Wilkins followed a series of difficulties at other locations where the same warehouse design had been site adapted from standard plans by several different architect-engineers. Despite the relatively small failure area involved, all construction on new warehouses of this type was temporarily stopped immediately following this collapse, and an intensive investigation of the causes of the failure began.†

The warehouses involved consist of ten 400 x 200-ft units separated by transverse expansion joints. The roof of each unit is carried by five 6-span cast-in-place reinforced concrete continuous frames of six spans, 400 ft long and 33 ft on centers, and by two short-span frames at the transverse expansion joints. The main frame girders had only nominal web reinforcement. One longitudinal expansion joint runs the full 2000-ft length of the building near midwidth. At Wilkins, these frames support prestressed concrete block roof purlins, over which was cast a 4 in. gypsum slab with wire mesh and steel rails spanning between purlins. At other bases reinforced concrete purlins and sort-span channel slabs or channel slabs of 33-ft span were used for the roof deck.

At Wilkins AFD, each frame girder, 400 ft in length, was cast during one working day. Construction joints were made by use of steel plates set at the center of each span prior to casting. However, the girders were cast continuously across the plate joint, and the usefulness of these joints is doubtful. The length of the continuous girders would tend to aggravate the effects of shrinkage. At the other sites the sequence of casting varied widely and was generally more favorable. Cement, aggregates, and reinforcement used for construction were in accordance with the applicable federal specifications.

---

†For details see a series of three papers in the ACI JOURNAL, *Proceedings* V. 53, No. 7, Jan. 1957, pp. 625-678: Anderson, Boyd G., "Rigid Frame Failures," p. 625; Elstner, Richard C., and Hognestad, Eivind, "Laboratory Investigation of Rigid Frame Failure," p. 637; Lunoe Reinhart, R., and Willis, George A., "Application of Steel Strap Reinforcement to Girders of Rigid Frames, Special AMC Warehouses," p. 669.

The collapse at Wilkins originated in a diagonal tension failure in a frame girder. Failure occurred about 1 ft 6 in. beyond the cutoff point for the negative reinforcement. The computed dead load moment at this point was small and of positive sign. The frame in which the collapse was initiated had been cast on February 10, 1954, and no cracks had been reported until August 3, 1955, when timber cribbing was placed under the cracked girder.

On August 17, 1955, collapse took place. An observer states that the entire collapse occurred within 30 sec and without previous warning. A field inspection showed that the failure at Wilkins was not the result of an isolated weakness, but rather of a general condition which was evidenced by extensive cracking in many of the frame girders. The largest and most dangerous cracks were found to occur in the regions of low moment near the points of contraflexure. The tops of the cracks are located near or just beyond the points where the negative moment steel is substantially reduced in area or stopped completely. However, at some of the warehouses and other structures of similar type, large cracks were observed in the negative-moment region closer to the columns.

The design calculations and the plans were checked and found to conform to the minimum requirements of the building codes generally accepted in this country at the time of construction. In addition, as a result of experience during the construction program, the Office of Chief of Engineers revised the plans in the spring of 1954 to provide continuous top bars and nominal stirrups for the full length of the frames. However, this did not eliminate the basic difficulty in these structures.

In each warehouse building, cracks appear more prevalent in certain spans than in others even where the reinforcement is similar. At Wilkins AFD and certain other sites, cracks are much less frequent in frames where continuous top reinforcement and nominal stirrups were present throughout. In other cases, serious cracking was present even with the added reinforcement. One effect of reinforcing the region of low moment was to move the critical area back into the negative moment region near the columns.

Cores for compression tests, beams for flexural tests, and pull-out test specimens were taken from the collapsed spans at Wilkins and tested. Results indicate that the strength of the concrete and reinforcing steel generally exceeded the minimum requirements of the specifications. Moduli of rupture and bond strengths were more than adequate for the stresses computed by usual design procedures.

In evaluating the available data to find the factors responsible for the cracking and failures it became evident that the problem was not that of several unrelated failures, each the result of local conditions. Rather, any rational explanation of the failures must be based on unfavorable conditions which were consistently present at all the sites. Although at each building the cracks are more prevalent in certain spans, the failures were judged to be a result of a stress condition which is more or less typical of all spans, and that circumstances associated with the sequence of placing the concrete, methods of curing and decentering, variations in concrete strength, etc., tend to develop a general pattern at each particular warehouse. Thus the cracking in warehouses designed for a 40 lb per sq ft roof load was less severe than for the 20 lb per sq ft designs as the stresses under permanent load were lower. This is shown by crack patterns for the warehouse at Griffiths Air Force Base in New York State.

As a result of these failure experiences and laboratory test data then available, Committee 326 recommended some revisions for the 1956 ACI Building Code to ACI Committee 318. The maximum value of 90 psi was added to the shear stress of 0.03 f_c' permitted without web reinforcement, and limits of 240 and 360 psi were added for members with web reinforcement. Furthermore, provisions were added calling for web reinforcement to resist two-thirds of the total shear in certain regions of continuous or restrained free-standing beams and frames. These modifications were intended to be temporary measures until a more thorough knowledge of shear and diagonal tension could be developed. No failures involving beams or frames have been reported to Committee 326 for designs by ACI 318-56.

The failure of a four-story flat plate office building during construction in October 1956 called attention to the problem of shear in flat slabs and the effect of openings at the columns.

CHAPTER 4—PRINCIPLES OF SHEAR

400—Shear effect on beam behavior

One of the major contributions of recent shear studies to understanding of reinforced concrete is the phenomenological explanation of the effect of shear on the behavior of beams without web reinforcement. A simple beam that has two symmetrical point loads and is reinforced with horizontal tension bars only may be considered as an example. As load is applied to the beam, the first noticeable change is the formation of practically vertical tension cracks in the region of maximum moment. With increasing load, additional cracks form closer to the supports and some of the cracks become slightly inclined toward the load. These minor, although quite noticeable, changes in the direction of cracks are caused by the presence of shear but have no significant effect either on the magnitude of deflections or on the magnitude of steel strains in the tensile reinforcement.

If the beam is relatively long, or if the percentage of tensile reinforcement is low, further increase in load will cause failure by crushing of the concrete at or near the location of maximum moment. This final failure may or may not have been preceded by yielding of the tensile reinforcement, depending on the relative values of the percentages of tensile steel and the strength of the concrete. Such failures are typical flexural failures in which the ultimate capacity of the beam is reached gradually after large deflections and considerable yielding has taken place. For practical purposes, the load-carrying capacity and the mode of failure of this beam are not affected by shear.

Beams of intermediate length having normal percentages of flexural tensile reinforcement and relatively long beams having relatively high percentages of flexural tensile reinforcement exhibit the same behavior at low loads as described previously. As their loading is increased, however, before flexural tensile failure can occur, a characteristic inclined crack, the so-called critical diagonal tension crack, forms, as shown in Fig. 4-1. Such a crack often includes the slightly inclined tops of existing flexural tension cracks. Its propagation from the level of the tensile steel to the compression surface of the beam near the section of maximum moment is usually sudden and without warning, splitting the beam into two pieces and causing collapse.

In relatively short beams, diagonal tension cracking also forms as described above, but at a much slower rate. The propagation of the diagonal cracks is gradual as loading is continued, and one beam may sustain without collapse several cracks whose upper ends are only a few inches below the compression surface of the beam near the section of maximum moment. Barring yielding of the reinforcement, further loading of a beam in such a cracked condition is possible with little apparent extension of the cracks. Eventually, however, the concrete in the region at the end of the cracks or above them fails, and the beam collapses. Such failures are generally referred to as shear-compression failures.

It can be concluded that qualitatively shear affects the behavior of beams without web reinforcement through the formation of a diagonal tension crack. If a diagonal tension crack does not form, the effect of shear is negligible. Collapse of the beam *may* occur simultaneously with the formation of the diagonal crack. On the other hand, a beam *may* be capable of resisting loads in excess of those causing the formation of the critical diagonal tension crack, and in such cases the final collapse is caused by shear-compression or by some secondary cause brought about by the presence of the diagonal tension crack. Quantitatively, shear limits the ultimate strength of a beam if a diagonal tension crack forms before the ultimate load is reached.

When a critical diagonal tension crack forms, a redistribution of internal forces must take place. Since no force can be transmitted across the crack, the shear force must be carried partly by dowel action in the tensile reinforcement, but mainly by the concrete in the uncracked compression zone. Until diagonal tension cracks form, the stresses in the tension steel and in the concrete are distributed along the length of the beam in the same way as the external moments so that these stresses at any section are approximately proportional to the moment at that particular section. The formation of diagonal tension cracks changes these relationships. Such changes are called the redistribution of internal stress and were first discussed by Mörsch in 1907.

In Fig. 4-1(a) a beam without web reinforcement is shown after formation of diagonal tension cracks. The part of the beam located to the left of one crack is shown in Fig. 4-1(b) as a free body. Since no stresses can exist across the crack, the free body is subject to the action of the reaction of the support R, the force in the tension reinforcement T, the compressive force C' resisted by the concrete above the crack, and the vertical shear V. It is indicated in Fig. 4-1(b) that all vertical shear is transmitted through the concrete; in reality a part of the shear is transmitted through the dowel action of the reinforcement but the contribution of the tension reinforcement to the transfer of shear is believed to be negligible.

If the forces shown in Fig. 4-1(b) are in equilibrium, then

$$A_s f_s jd = Ra$$

that is, after the formation of the diagonal crack the steel stress at Section b-b depends on the moment at Section a-a. Consequently, the steel stress at Section b-b increases when diagonal cracking occurs, and the distribution of stresses in the tension reinforcement and in the concrete along the beam does not follow the distribution of external moments.

If the beam is not able to reach a force equilibrium after this redistribution, collapse occurs immediately. This is the case of beams with long shear spans (distance from load-point to nearest support) which fail in diagonal tension. On the other hand, if a new equilibrium of forces is possible, further increases of load may be carried without collapse. This is the case for beams with short shear spans which fail in shear-compression.

The ability of the beam to reach force equilibrium seems to depend primarily on the stability of the compression zone and may be influenced by several factors other than the length of shear span and the percentage of reinforcement. Certainly the depth of the compression zone above the diagonal crack is important. Experimental data indicate that this depth can be affected materially by random variations in the location and path of the critical diagonal crack. The location of the point of load application with respect to the location of the compression zone appears to be important. If the load is applied to the compression surface adjacent to the compression zone, the strength of the zone will be higher than if the load is applied beneath the compression surface as, for example, through secondary beams framing into sides of a girder.

Although most of the short specimens tested have failed in shear-compression at loads as much as 100 percent greater than loads causing the critical diagonal tension crack, enough of them have failed in diagonal tension to indicate that detailed knowledge is lacking regarding the ability of a beam to reach force equilibrium after redistribution. Furthermore, little is known about the long-time behavior of a diagonally cracked beam. Accordingly, *the load causing formation of a critical diagonal tension crack must ordinarily be considered in design as the usable ultimate load-carrying capacity of a reinforced concrete member without web reinforcement.*

401—Contribution of web reinforcement

Measurements of strain in web reinforcement indicate practically no stress in vertical stirrups and relatively small tensile stresses in inclined web reinforcement prior to the formation of diagonal tension cracks. Web reinforcement has little or no effect on the behavior of the beam before diagonal cracking, or on the cracking load.

Web reinforcement becomes effective only after the formation of diagonal tension cracks. When diagonal cracking occurs, web bars intersected by a crack immediately receive sudden increases in tensile stress in the vicinity of the crack, while web bars not intersected by diagonal cracks remain unaffected. The magnitude of tensile stress immediately absorbed by the web reinforcement seems to depend on many factors

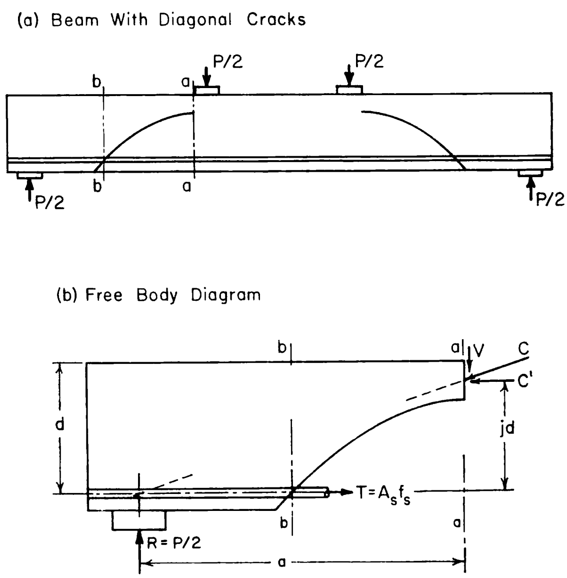


Fig. 4-1 — Redistribution of internal stresses such as the percentage and distribution of web reinforcement as well as on the factors influencing the ultimate strength of beams without web reinforcement. When diagonal cracking takes place, a sudden increase in stress will also take place for longitudinal reinforcement intersected by a diagonal crack. Neglecting effects of web reinforcement on equilibrium of the free body shown in Fig. 4-1 (b), the steel stress at Section b-b increases toward a magnitude corresponding to the bending moment at Section a-a. If some of the reinforcement at Section a-a is "not needed to cover the moment diagram" and is terminated between Sections a-a and b-b, therefore, redistribution of stresses may lead to premature yielding of the tension reinforcement at Section b-b. This must be avoided in design by extending positive and negative longitudinal reinforcement terminated in a zone of concrete tension a distance equal to the effective depth of the member plus an anchorage length beyond the point where it is no longer needed to resist flexural stress.

As additional load is applied to the beam, the tensile stresses in the web bars intersected by diagonal cracks continue to increase until the beam fails. The relationship between applied load and increased tensile stress is usually linear. Web reinforcement will usually yield before failure occurs if the percentage of web reinforcement is not too high. Beams with web reinforcement failing in shear generally fail by destruction of the compression zone, which mode of failure has previously been referred to as shear-compression failure. Sudden diagonal tension failures only occur if the percentage of web reinforcement is so low that the stresses in the web reinforcement go from practically no stress to full yielding at the formation of the diagonal crack.

Thus, the primary function of web reinforcement is to accommodate redistribution of internal forces when diagonal cracking occurs. This is accomplished in two ways. First, the web reinforcement will accept a portion of the redistributed internal forces through a sudden increase in tensile stress on formation of the diagonal crack. Secondly, the web reinforcement contains the diagonal crack, thus preventing deep penetration of the diagonal crack in to the compression zone. In general, the presence of web reinforcement assures a gradual development of a shear-compression failure, usually following large increases in diagonal crack width as the web reinforcement reaches its yield point.

Since web reinforcement becomes active after the formation of diagonal cracking and since failures of beams with web reinforcement normally occur gradually and with ample warning, *it seems logical and safe to base the design of web reinforcement on the ultimate load-carrying capacity.*

402—Shear effect on slab behavior

In flat slabs the connection between slab and column is generally the critical area as far as strength is concerned. Heavy bending moments and heavy shearing forces are concentrated there, and the economy of the structure is to a large extent governed by the degree to which the strength of this area can be predicted and utilized. In recent years there has been a tendency to eliminate or reduce drop panels, column capitals, and shear walls, and to place utility holes in slabs adjacent to columns. These factors intensify the importance of shear strength. Thus, it is not surprising to find that the effect of concentrated loads on slabs has received considerable attention in recent laboratory work.

In most laboratory studies, the behavior of a slab in the vicinity of a loaded area was difficult to determine because it was not possible to observe the formation of cracks within the slab. In a recent series of tests, however, Moe[†] placed utility openings in the slab adjacent to the loaded area. He was then able to observe the formation of diagonal tension cracks through these openings. The inclined cracks developed at approximately 60 percent of the ultimate strength. They usually developed from flexural cracks, and they extended rapidly to the proximity of the neutral axis. With increasing load, the cracks proceed rather slowly into the compression zone. In many cases only a narrow depth of compression zone remained intact prior to failure. At loads slightly below the ultimate, the diagonal cracks progressed to the tension surface of the slab and along the tensile reinforcement. Final failure always occurred when the loaded area punched through the slab, pushing ahead of it a plug of concrete which had the form of a cut-off cone or pyramid with a minimum cross section at least as large as the loaded area. It is believed that the behavior of slabs without holes follows the same pattern.

Thus, the formation of diagonal cracks in slabs takes place in approximately the same manner as in beams which fail in shear compression. However, the similarity ends here. Elstner and Hognestad[‡] have reported tests of beam strips representing center strips of comparative slab specimens to determine possible relationships between the behavior of such beam strips and of the corresponding slabs. They found no direct relationship in behavior or in mode of failure.

Beams bend in one direction and the internal stresses are two-dimensional. Slabs generally bend in two directions causing three-dimensional stresses, a condition generally referred to as "slab action." If there is slab action, the stresses in the third dimension influence the ability of the material to resist the stresses in the other two dimensions. Thus, the behavior of a slab cannot be directly compared to the behavior of a beam.

After the formation of a diagonal crack, a redistribution of stresses must occur in slabs which is similar to the redistribution that is known

---

†Moe, Johannes, "Shearing Strength of Reinforced Concrete Slabs and Footings Under Concentrated Loads," *Bulletin* D-47, Portland Cement Association Development Department, Apr. 1961, 130 pp.

‡Elstner, R. C., and Hognestad, E., "Shearing Strength of Reinforced Concrete Slabs," ACI JOURNAL, *Proceedings* V. 53, No. 1, July 1956, pp. 29-58.

to occur in beams. It has been pointed out previously that no increase in load capacity was possible in relatively long and slender beams after redistribution, and that such an increase is not reliable for practical design purposes in beams of average dimensions. However, slab action enhances the ability of the compression zone to resist compression and the combination of shear and compression. The sudden diagonal tension failure which is common in long and slender beams usually does not take place when slab action is present. Also, slab action seems to permit the compression zone of a slab to accept redistribution with reliability. Therefore, it appears reasonable to *select ultimate capacity, rather than cracking load, as the design criterion for shear strength of slabs.*

403—Inadequacies of classical diagonal tension design equation

The backgorund and development of the classical shear stress equation was discussed in Chapter 2. It was pointed out that $v = V/bjd$ was originally an expression of shear stress intended as a *measure* of diagonal tension. However, this equation has to some extent become regarded as a reliable means of determining diagonal tension stress. Actually, the equation is only a rough approximation because flexural tensile stress is neglected both in the development of the equation and in its generalization as a measure of diagonal tension.

Design specifications have related the *measure* of diagonal tension expressed by $v = V/bjd$ to the cylinder strength of concrete f_c', by restricting the stress v to values less than certain fractions of f_c', unless web reinforcement is used. To examine the accuracy of such design procedures, the nominal shear stress at critical diagonal tension cracking of several representative beams was plotted as a function of concrete cylinder strength f_c'. Fig. (4-2) shows that only a small portion of the variation in cracking shear stress is due to concrete strength variation. In other words, the classical design procedure does not correlate well with the test results. It appears, therefore, that an improved design approach is needed.

It seems reasonable to expect that calculation of diagonal tension strength is a problem of tensile strength that could be solved on a rational basis if the distribution of shear and flexural stresses were known or could be closely approximated. Even though hundreds of tests have been conducted in recent years, they have increased knowledge concerning the basic distribution of shear stress over the cross section of a reinforced concrete beam only to a limited extent. Furthermore, the distribution of flexural stress is also obscure because diagonal cracking usually takes place in a region of flexural cracking. In the absence of detailed knowledge regarding the stress distribution for both shear and flexure, a fully rational design approach to the problem does not seem possible at this time.

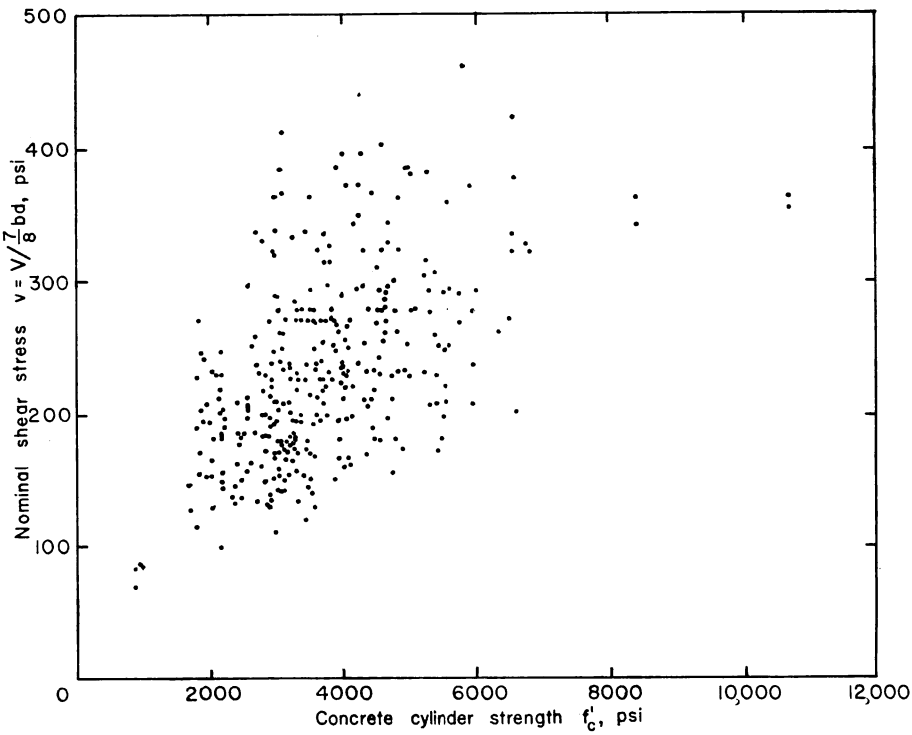


Fig. 4-2—Nominal shear stress in reinforced concrete beams at formation of diagonal tension cracks

It has been pointed out in this report that the classical procedures are questionable in their development, as well as misleading and sometimes unsafe in their application. Yet, the goals of a complete understanding and of a fully rational solution to the problem of computing diagonal tension strength have not been attained. In view of these circumstances, it appears necessary at this time to abandon the classical procedures in favor of a logical, though empirical, approach which takes into account the major variables affecting diagonal tension strength as shown by test results. Such procedures are presented in Chapters 5-8 of this report.

404—Inadequacies of design equations for stirrups and bent-up bars

A general analysis of stirrup action on the basis of the truss analogy was reviewed in Chapter 2. It was pointed out that the analogy implied four major assumptions. To simplify the general analysis, it was further assumed that diagonal cracks always form at an angle of 45 deg with the axis of the member.

In this analogy, all inclined tensile stresses are assumed to be carried by the web reinforcement. The ability of the compression zone and web concrete to resist diagonal tension is not considered. The distribution of shear force between the web reinforcement, the longitudinal reinforcement and the compression zone is not clearly understood. However, laboratory tests indicate that the two latter items contribute substantially because the total shear capacity of an average girder is greater than the contribution of the web reinforcement alone.

This difficulty has usually been overcome in design procedures by assuming either that the stirrups carry that portion of the total shear exceeding a fixed amount, or that they carry a fixed fraction of the total. ACI 318-56 makes use of both assumptions, using $Krf_v = v - A$, where $A = 0.03f_c'$, for one set of conditions, and $Krf_v = Cv$, where $C = 2/3$, for another set. In British and German procedures $A = C = 0$.

The fourth assumption of the truss analogy, that the diagonal tension crack forms up to a vertical height of jd, is an assumption of convenience without support from laboratory test results. Web reinforcement resists shear only after the formation of diagonal cracks. The height reached by a diagonal crack, which in turn governs the number of stirrups resisting diagonal tension, probably depends on many factors including the distribution of shear between web reinforcement, longitudinal reinforcement and compression zone. Therefore, this assumption is questionable even though the final development of the term Krf_v may be found useful in analyses of test data.

The assumption that the crack is inclined at an angle of 45 deg, also may be considered an assumption of convenience, and it may be erroneous especially in the cases of very short girders or very long girders. Although systematic studies of crack inclination are not available, the approximate nature of this assumption should be kept in mind.

It should finally be noted that the truss analogy analysis is based entirely on summation of vertical forces. Effects of web reinforcement on flexural moment capacity, or converse effects of moment on web reinforcement stress, are ignored. This point may be important, especially in girders with heavy shear reinforcement, for which ultimate strength in shear may be reached before the web reinforcement has yielded.

405—New design procedures

Chapters 1-4 of this report give a review of basic principles. Chapters 5-7 present Committee 326's studies and recommendations regarding improved design procedures for beams and frames, substantiated by extensive test data. Chapter 8 presents similar studies and recommendations for slabs and footings. These recommendations reflect modern knowledge regarding the principles of shear, and will thereby tend to correct the major shortcomings of older design methods.

Title No. 59-8

Report of ACI-ASCE Committee 326

Shear and Diagonal Tension

The ACI-ASCE Committee 326 report is being published as follows:

Part 1—General Principles, Chapters 1-4: January 1962 pp. 1-30

Part 2—Beams and Frames, Chapters 5-7: February 1962

Part 3—Slabs and Footings, Chapter 8: March 1962

This report was submitted to letter ballot of the committee which consists of 15 members and was approved without a dissenting vote.

Received by the Institute August 14, 1961. Title No. 59-8 is a part of copyrighted Journal of the American Concrete Institute, Proceedings V. 59, No. 2, Feb. 1962. Reprints of the complete report will be available after publication of Part 3 in March 1962.

Discussion of this report (three parts) should reach ACI headquarters in triplicate by June 1, 1962, for publication in the September 1962 JOURNAL.

Presents a review of scientific knowledge, engineering practice, and construction experiences regarding shear and diagonal tension in reinforced concrete beams, frames, slabs, and footings. Recommendations for new design procedures are substantiated by extensive test data.

Chapters I through 4 deal with background and general principles. Chapters 5 through 7 present the development of new design methods for reinforced concrete members without and with web reinforcement, and for members without and with axial load acting in combination with bending and shear. Chapter 8 deals with slabs and footings including the effect of holes and transfer of moments from columns to slabs.

Esfuerzo Cortante y Tensión Diagonal

Se revisan los conocimientos científicos, ingeniería práctica y experiencias en construcciones relativas al esfuerzo cortante y tensión diagonal en vigas, armaduras (marcos rigidos), losas y cimientos de hormigón armado. Se recomiendan

nuevos procedimientos de diseño comprobados por los datos obtenidos por medio de ensayos extensivos.

Los capítulos 1 al 4 tratan de los antecedentes y principios generales. Los capítulos 5 al 7 presentan el desarrollo de nuevos métodos de diseño para miembros de hormigón armado con y sin refuerzo del alma y para miembros con y sin carga axial combinada con la flexión y el esfuerzo cortante. El capítulo 8 trata de las losas y cimientos incluyendo el efecto causado por agujeros y el traspaso de momentos de las columnas a las losas.

L'Effort Tranchant et la Contrainte Principale

On présente une revue de l'art scientifique, de la pratique du génie et des expériences dans la construction relatives aux efforts tranchants et à la contrainte principale dans les poutres, les portiques, des dalles et les semelles de fondations en béton armé. Les recommendations pour les nouvelles procédés de calcul sont justifées à l'aide de résultats nombreaux d'essais.

Les chapitres 1 à 4 concernent les bases et les principes généraux. Les chapitres 5 à 7 présentent l'évolution de nouvelles methodes de calcul d'éléments en béton armé avec et sans armatures de cisaillement et d'éléments soumis à la flexion simple et composée avec les efforts tranchants. Le chapitre 8 concerne les dalles et les semelles de fondations y compris l'influence des trous et la transmission de moments de flexion des colonnes aux dalles.

Schub- und Hauptzugspannungen

Es wird eine Übersicht gegeben über wissenschaftliche Erkenntnisse, technische Praxis und Bauerfahrungen bezüglich Schubsicherung in Stahlbetonträgern, Rahmen, Platten und Säulenfussplatten. Empfehlunger für neue Berechnungsverfahren werden durch umfassende Versuchsunterlagen erhärtet.

Kapitel 1 bis 4 behandeln die Vorgeschichte und die allgemeinen Grundsätze. Kapitel 5 bis 7 enthalten die Entwicklung von neuen Berechnungsmethoden für Stahlbetonteile ohne und mit Schubbewehrung und für Bauglieder unter Biegung und Schub ohne und mit gleichzeitiger Längskraft. Kapitel 8 behandelt Platten und Säulenfussplatten einschliesslich der Wirkung von Aussparungen und die Übertragung von Momenten von Säulen zu Platten.

ACI-ASCE Committee 326, Shear and Diagonal Tension, was formed in 1950 to develop methods for designing reinforced concrete members to resist shear and diagonal tension consistent with ultimate strength design. Several investigations and test programs were initiated, sponsored and conducted by numerous organizations, including Committee 326, the Reinforced Concrete Council, many universities (especially the University of Illinois), the American Iron and Steel Institute, and the Portland Cement Association. Progress reports of Committee work were presented at the ACI 55th annual convention, February 1959, and the 56th convention, March 1960. This three-part report is the culmination of a 10-year study.

CHAPTER 5—DESIGN OF MEMBERS WITHOUT WEB REINFORCEMENT

500—Review of variables

Research work in the 1950's brought about a clear realization that shear and diagonal tension is a complex problem involving many variables. This is actually a return to forgotten fundamentals.

Following acceptance of Mörsch's concept that shear failure in reinforced concrete beams is a tensile phenomenon, early design specifications in the United States considered the nominal shearing stress, $v = V/bjd$, to be a *measure* of diagonal tension, and related it to the cylinder strength of concrete, f_c'. For members without web reinforcement, the allowable nominal shearing stress v of the early specifications was restricted to $0.02f_c'$ with maximum limiting values either stated or implied. The same procedures were in effect in the 1950's, although the fraction of f_c' used and the maximum values had been increased through the intervening years.

Thus, present design procedures neglect effects of flexural tension on diagonal tension and consider concrete compressive strength as the only principal variable. A. N. Talbot† pointed out the fallacies of such procedures as early as 1909:

> "It is seen that the value of the diagonal tensile stress depends upon the tensile stress in a horizontal direction at a given point as well as the amount of the horizontal and vertical shearing stresses there developed . . . It may be said that in the ordinary reinforced concrete beam the value of t (diagonal tensile stress) probably varies from one to two times v (nominal shearing stress).
>
> "It is evident that the value of diagonal tension is generally indeterminate. No working formulas are available. For this reason it is the practice, now becoming nearly universal, in beams without web reinforcement to calculate the value of the vertical shearing unit stress, and to use it as a measure or means of comparison of diagonal tensile stress developed in the beam; with the understanding, of course, that the actual diagonal tension is considerably greater than the vertical shearing stress. It will be found that the value of v (nominal shearing stress) will vary with the amount of reinforcement, with the relative length of the beam, and with other factors which affect the stiffness of the beam."

Talbot substantiated these statements with test results for 106 beams without web reinforcement, and he concluded as follows:

> "In beams without web reinforcement, web resistance depends upon the quality and strength of the concrete . . .
>
> "The stiffer the beam the larger the vertical shearing stress which may be developed. Short, deep beams give higher results than long slender ones, and beams with a high percentage of reinforcement than beams with a small amount of metal . . ."

†*Bulletin* No. 29, University of Illinois Experiment Station, Jan. 1909, 85 pp. For further references see the Committee 326 annotated Bibliography No. 4 of the ACI Bibliography Series.

Thus, Talbot demonstrated as early as 1909 that percentage of reinforcement and the length-to-depth ratio played an important role in shear and diagonal tension strength of beams without web reinforcement. Unfortunately, Talbot did not express his findings in mathematical terms. Consequently, his findings became lost as far as design equations were concerned. He did, however, state this added warning which was repeatedly expressed by nearly all investigators of the time: "Low working stresses in web resistance are to be commended and ample provisions for web strength should be made."

During and after World War I, extensive tests were conducted in connection with the concrete shipbuilding program of the Emergency Fleet Corp. The test specimens used were abnormally deep beams and exhibited abnormally high shearing stresses at failure. A paper by W. A. Slater in 1919 stressed this point, and a 1919 editorial appearing in *Engineering News-Record* made this comment:

> "In spite of a large number of tests, most of them of long standing, knowledge of shear in concrete is in a most unsatisfactory state. There seems to be little doubt that existing permissible safe values are much too low and that in some kinds of design, particularly in deep beams, this restriction operates with considerable hardship . . ."

In the interval between 1920 and the early 1950's, the conservative advice of Talbot and other pioneers was forgotten. Also forgotten were early experiments regarding effects on shear strength of two variables, the percentage of reinforcement and the length-to-depth ratio.

A return to the forgotten fundamentals began in the late 1940's when O. Morretto, in reporting a series of beam tests, adopted an empirical equation for shear strength which included the percentage of tensile reinforcement as a variable. Nearly all more recent investigators have used the percentage of longitudinal reinforcement as a variable in equations expressing shear and diagonal tension strength.

In the early 1950's, A. P. Clark introduced an expression for the span-to-depth ratio a/d involving the length of the shear span a and the effective depth of the beam d. The a/d term was immediately recognized as a mathematical means of expressing the effect of length to depth. Thus, Clark expressed Talbot's notions by a mathematical equation involving the three variables—percentage of longitudinal reinforcement, ratio of beam length to depth, and concrete strength.

Although the development of the a/d ratio was a definite step in the right direction, the term was handicapped because the shear span a, could not be defined for every cross section in a beam or for generalized cases of loading. In simple beams with a single point load or with two symmetrical point loads the term a is the distance from a load point to the nearest support. For numerous other loading conditions the term a has no direct physical meaning.

The difficulty was later overcome by a slight modification of the general concepts of diagonal tension. The length-to-depth ratio is in reality relating the effect of horizontal flexural tension on diagonal tension. This thought led to the development of theories based on the ratio M/Vd at the University of Illinois in the early 1950's, involving bending moment M, shear force V, and effective depth d. For the case of simple beams with a single point load or with two symmetrical loads, the terms M/Vd for a load-point section and a/d are synonymous; for any other loading condition M/Vd still has physical significance at any cross section of a beam. The development of the M/Vd-ratio concept may be considered a breakthrough toward an empirical solution of shear and diagonal tension as a design problem.

As a result of the multitude and variety of shear tests conducted in the 1950's, other variables are known to affect shear strength. Investigators have studied the effects of axial tension and axial compression on shear strength; variations in loading which have included simple beams, continuous beams, beams with overhangs, single concentrated loads, symmetrical two-point loads, multipoint loads, and uniform loads; a comparison of lightweight aggregates to sand and gravel aggregates; variations in cross section including rectangular beams, I-beams, T-beams, and beams with haunches; the effect of introducing load into beams by other means than through bearing plates; the effect of forcing failures to occur at various locations in a beam; and the effect of high strength reinforcement on shear strength. The investigators executing these projects have proposed several equations for shear strength which have involved considerations of the ratio of width to depth b/d; the modular ratio n; use of the full depth t, rather than the effective depth d; the ratio of length to depth L/d; and use of $\sqrt{f_c'}$ rather than f_c'.

501—Development of design criteria

The following concepts have been presented and discussed earlier in this report: (1) Diagonal tension is a combined stress problem in which horizontal tensile stresses due to bending as well as shearing stresses must be considered. (2) Failure due to shear may occur with the formation of the critical diagonal crack or, if redistribution of internal forces is accomplished, failure may occur by shear-compression destruction of the compression zone at a higher load. (3) The load causing the formation of the critical diagonal tension crack must ordinarily be considered in design as the ultimate load carrying capacity of a reinforced concrete member without web reinforcement. (4) Distributions of shear and flexural stress over a cross section of reinforced concrete are not known.

Since the actual distribution of shear stress over a cross section has not yet been clarified, continued use of the average shear stress seems advisable, though the refinement involving the internal moment arm jd is not warranted. Therefore, in succeeding portions of this report the unit shear stress is expressed as the average stress on the full effective cross section

$$v = \frac{V}{bd} \qquad (5\text{-}1)$$

Using the average shear stress and the criterion that the critical diagonal tension cracking represents the usable ultimate strength of beams without web reinforcement, a systematic study of data from more than 440 recent tests indicates that the shear capacity depends primarily on three variables, viz., the percentage of longitudinal reinforcement p, the dimensionless quantity M/Vd, and the quality of concrete as expressed by the compressive strength f_c'. Other variables have minor effects on shearing strength.

The location and inclination of diagonal tension cracks indicate that they are caused by excessive principal tensile stress. Thus, a rational analysis of the diagonal tension strength should logically be based on the equation for principal stress at a point

$$f_t(\max) = \tfrac{1}{2} f_t + \sqrt{(\tfrac{1}{2} f_t)^2 + v^2} \qquad (5\text{-}2)$$

Such an approach has been tried in the past, but it was usually unsuccessful because of difficulties in expressing the tensile bending stress f_t and the shearing stress v.

The magnitude of the tensile bending stress f_t in reinforced concrete is influenced by the presence of tensile cracks. Hence, it cannot be computed on the assumption of uncracked sections, neither can it be computed directly from the assumption of cracked sections. In the present approach the tensile stress f_t is assumed *proportional* to the tensile steel stress f_s, computed by the cracked section theory

$$f_t = \text{constant} \times \frac{f_s}{n} = C \frac{M}{npjbd^2} \qquad (5\text{-}3)$$

Neglecting variations in the moment arm factor j and designating by a new constant $F_1 = C/j$, the tensile stress is

$$f_t = F_1 \frac{M}{npbd^2} \qquad (5\text{-}4)$$

The magnitude of the shearing stress in concrete cannot be expressed directly either. It is assumed *proportional* to the average shearing stress on the cross section

$$v = F_2 \frac{V}{bd} \qquad (5\text{-}5)$$

A substitution for the tensile stress f_t and shear stress v, and rearrangement of the equation for principal stress gives the relationship between the external shear V and the principal stress $f_t(\max)$ in the form

$$\frac{V}{bd} = \frac{f_t(\max)}{\frac{1}{2} F_1 M/nVpd + \sqrt{(\frac{1}{2} F_1 M/nVpd)^2 + F_2^2}} \quad \text{(5-6)}$$

Diagonal tension cracking occurs when the principal tensile stress $f_t(\max)$ exceeds the diagonal tensile strength of concrete f_t'. Assuming that the factors F_1 and F_2 are constants, and observing that

$$f_t(\max) = f_t' \quad \text{(5-7)}$$

$$n = E_s/E_c \quad \text{(5-8)}$$

the following expression for shear at diagonal tension cracking may be obtained by rearrangement of Eq. (5-6)

$$\frac{V}{bd\, f_t'} = \frac{1}{C_1 \dfrac{E_c}{E_s p} \dfrac{M}{Vd} + \sqrt{\left(C_1 \dfrac{E_c}{E_s p} \dfrac{M}{Vd} \right)^2 + C_2}} \quad \text{(5-9)}$$

where C_1 and C_2 are new dimensionless constants.

Eq. (5-9) was derived by I. M. Viest in an attempt to develop a rational relationship for diagonal tension cracking load. It is based on the work of J. Morrow. This analysis relates the nominal shearing stress, $v = V/bd$, to the three major variables known to influence it: (1) the nominal shearing stress v increases with increasing concrete strength expressed in Eq. (5-9) in terms of f_t'; (2) v decreases with increasing M/Vd; and (3) v increases with increasing p. Furthermore, Eq. (5-9) groups the major variables into two dimensionless parameters: V/bdf_t' and $(E_c/E_s p)(M/Vd)$.

These two parameters may be further simplified. The modulus of elasticity of steel may be assumed a constant, E_s = constant. The resistance of concrete to the principal tensile stress should be comparable to the tensile strength of concrete, which in turn may be approximated as a function of $\sqrt{f_c'}$. Thus, it appears reasonable to assume that f_t' equals a constant times $\sqrt{f_c'}$. Finally, the modulus of elasticity of concrete may be expressed approximately as a function of $\sqrt{f_c'}$, so that E_c equals a constant times $\sqrt{f_c'}$. With these three approximations, the two parameters of Eq. (5-9) may be expressed as: a parameter $A = V/(bd\sqrt{f_c'})$ representing the strength in diagonal tension, and a second parameter $B = (\sqrt{f_c'}/p)(M/Vd)$ representing the properties of the cross section considered.

Thus, evaluation of available test data and the mathematical model based on the concept of principal stress both indicate that the three major variables affecting diagonal cracking are M/Vd, p, and $\sqrt{f_c'}$. These three variables may be grouped logically into the two parameters, A and B, permitting correlation of the test data on diagonal tension strength of beams without web reinforcement in a two dimensional plot, from which a quantitative relationship between the two parameters may be derived.

The initial derivation of these constants was based on test data for beams of constant cross section subjected to one or two concentrated loads in any one span. Such beams have constant shear in the region in which a diagonal tension crack forms. There were 194 beams from References 1, 2, 3, 4, 6, and 7 that satisfied these limitations. The references are listed in Section 507 of this report.

In evaluation of the test data, the ratio of moment to shear, M/V, was taken at the section of diagonal tension cracking. It was observed in the tests involved that: (1) diagonal tension cracks extended approximately from midway between the sections of zero and maximum moment to the section of maximum moment; (2) the location of diagonal tension cracks was influenced by the length of the shear span and the effective beam depth; and (3) diagonal tension cracks usually began to form near middepth of the beam, at the top of an existing tension crack, or near the tension reinforcement.

Because of these variations of the exact locations at which diagonal tension cracking began, the following simplifying assumptions were made in evaluation of the beam test data: (1) In shear spans longer than twice the effective beam depth d, a diagonal tension crack will begin at distance d from the section of maximum moment, and (2) in shear spans shorter than two effective beam depths a diagonal tension crack will begin at the center of the shear span. These two conditions give the following expression for the ratio of moment to shear at the critical diagonal cracking section of beams subjected to one or two concentrated loads in any one span

$$\frac{M}{V} = \left(\frac{M_{max}}{V} - d\right) \text{ but not less than } \left(\frac{M_{max}}{V} - \frac{a}{2}\right) \qquad (5\text{-}10)$$

where

M_{max} = maximum moment in the shear span considered
V = external shear in the shear span considered
d = effective beam depth
a = length of shear span defined as the distance between a concentrated load and the nearest reaction, that is, length of a region of constant shear

The values of parameter $A = V/bd\sqrt{f_c'}$, and of the inverse of parameter $B = pVd/M\sqrt{f_c'}$, for all 194 beams are listed in Tables 5-1 through 5-4, and are plotted in Fig. 5-1. It is seen that the diagonal

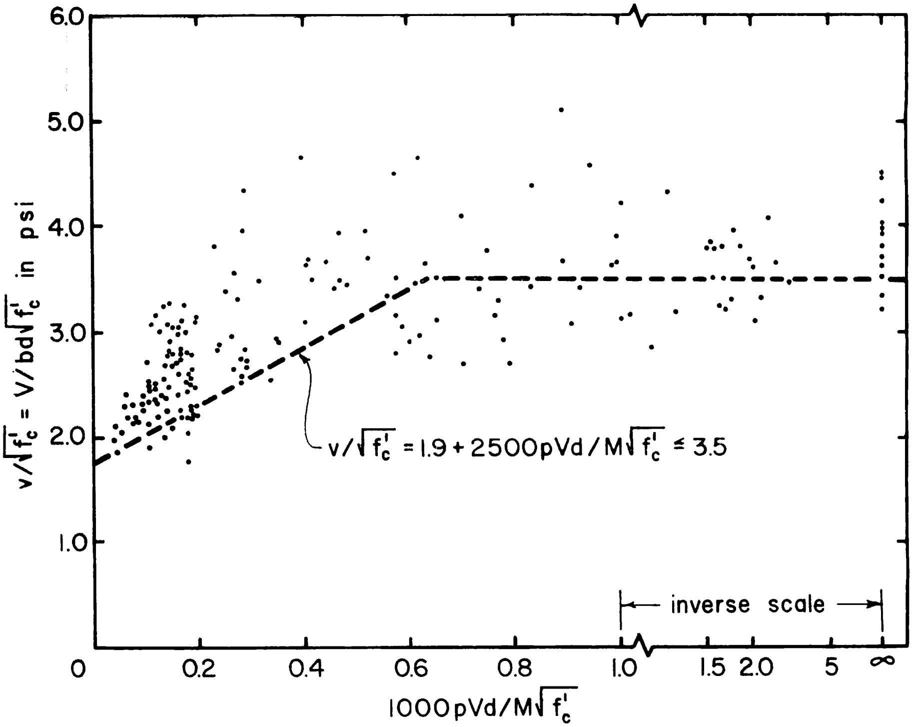


Fig. 5-1—Derivation of design equation [Eq. (5-11)]

tension strength represented by the parameter $V_{test}/bd\sqrt{f_c'}$ increases from a minimum of about 2 to a maximum of about 4 as the second parameter $pVd/M\sqrt{f_c'}$ increases from zero to about 0.8.

The trend of the test data shown in Fig. 5-1 may be expressed by two straight lines corresponding to the following expression

$$\frac{V}{bd\sqrt{f_c'}} = 1.9 + (2500 \text{ psi}) \frac{pVd}{M\sqrt{f_c'}} \text{ but not greater than } 3.5 \quad \text{(5-11)}$$

where

V = external shear at diagonal tension cracking of the section considered
b, d = dimensions of the cross section
p = ratio on tension reinforcement A_s/bd
V/M = ratio of shear to moment at section considered
f_c' = cylinder strength of concrete in psi; the units for $\sqrt{f_c'}$ are also psi, so that for $f_c' = 3600$ psi, the $\sqrt{f_c'} = 60$ psi.

The committee's choice of the lines represented by Eq. (5-11) was influenced by two major considerations. First the equation should be simple to facilitate every-day design work, and second, the equation should be such that the ultimate strength of beams resulting from practical design will be governed by flexure rather than by shear.

TABLE 5-1 — DEVELOPMENT OF EQ. (5-11) FOR SIMPLE BEAMS

Source	Beam No.	$\frac{Vpd}{M\sqrt{f_c'}}$†	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{calc}}$
Moody, Elstner, Hognestad, and Viest[1]	III24a	0.7031	3.500	2.679	0.765
	24b	0.6531	3.500	3.110	0.889
	25a	0.7634	3.500	3.149	0.900
	25b	0.9071	3.500	3.061	0.875
	26a	0.9943	3.500	3.642	1.041
	26b	1.0189	3.500	3.110	0.884
	27a	0.6414	3.500	2.749	0.785
	27b	0.6198	3.450	2.952	0.856
	28a	0.7802	3.500	2.925	0.836
	28b	0.7956	3.500	2.685	0.767
	29a	0.9927	3.500	3.636	1.039
	29b	0.9260	3.500	3.392	0.969
	A- A1	0.1585	2.296	2.718	1.184
	A2	0.1602	2.301	3.042	1.132
	A3	0.1667	2.317	2.826	1.220
	A4	0.1782	2.346	2.982	1.271
	A- B1	0.1461	2.265	2.812	1.241
	B2	0.1463	2.266	3.268	1.442
	B3	0.1542	2.286	3.053	1.336
	B4	0.1727	2.332	3.226	1.384
	C1	0.1349	2.237	2.009	0.898
	C2	0.1436	2.259	2.475	1.096
	C3	0.1306	2.227	2.395	1.076
	C4	0.1367	2.242	2.387	1.065

Source	Beam No.	$\frac{Vpd}{M\sqrt{f_c'}}$†	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{calc}}$
Moody, Elstner, Hognestad, and Viest[1]	B-1	0.1078	2.170	2.488	1.147
	2	0.1598	2.300	2.406	1.046
	3	0.1286	2.221	2.968	1.336
	4	0.1665	2.316	2.841	1.226
	5	0.1178	2.195	2.484	1.132
	6	0.1643	2.311	2.474	1.070
	7	0.1175	2.194	2.358	1.075
	8	0.1869	2.367	2.626	1.109
	9	0.1018	2.154	2.349	1.090
	10	0.1335	2.234	2.545	1.140
	11	0.1057	2.164	2.335	1.079
	12	0.1452	2.263	3.062	1.353
	13	0.1062	2.166	2.132	0.984
	14	0.1375	2.244	2.677	1.193
	15	0.1068	2.167	2.465	1.138
	16	0.1615	2.304	2.594	1.126
Bower and Viest[6]	IA-1a	0.1857	2.364	2.622	1.109
	1b	0.1646	2.312	2.245	0.971
	IIB-1	0.1084	2.171	1.894	0.872
	2	0.0972	2.143	2.262	1.055
	3	0.0856	2.114	2.138	1.011

Average $V_{test}/V_{calc} = 1.076$

Coefficient of variation = 15.8 percent

†In thousandths.

TABLE 5-2 — DEVELOPMENT OF EQ. (5-11) FOR STUB BEAMS

Source	Beam No.	$\frac{Vpd}{M\sqrt{f_c'}}$†	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{calc}}$
Morrow and Viest[2]	B14B2	0.8326	3.500	4.369	1.248
	E2	0.2814	2.604	3.951	1.518
	A4	0.8894	3.500	5.113	1.461
	B4	0.6202	3.450	4.649	1.347
	E4	0.3954	2.888	4.661	1.614
	A6	0.9431	3.500	4.582	1.309
	B6	0.4656	3.064	3.490	1.139
	B21B2	0.5700	3.325	4.505	1.355
	E2	0.1987	2.397	3.139	1.310
	A4	0.5158	3.190	3.935	1.233
	B4	0.4076	2.919	3.667	1.256
	E4	0.2880	2.620	4.353	1.661
	E4R	0.2508	2.527	3.378	1.337
	F4	0.2295	2.474	3.814	1.542
	G4	0.1189	2.197	3.143	1.431
	A6	0.6297	3.474	3.635	1.046
	B6	0.3146	2.686	3.477	1.294
	B28B2	0.4153	2.938	3.485	1.186
	E2	0.1338	2.235	2.526	1.131
	A4	0.4026	2.906	3.093	1.064
	B4	0.2799	2.600	2.518	0.968
	E4	0.1847	2.362	2.281	0.966
	A6	0.4540	3.035	3.411	1.124
	B6	0.2403	2.501	2.883	1.153
	B40B4	0.1482	2.271	2.813	1.239

Source	Beam No.	$\frac{Vpd}{M\sqrt{f_c'}}$†	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{calc}}$
Morrow and Viest[2]	B56B2	0.1401	2.250	2.802	1.245
	E2	0.0441	2.010	1.868	0.929
	A4	0.1434	2.259	2.911	1.129
	B4	0.1029	2.157	2.515	1.166
	E4	0.0673	2.068	2.194	1.061
	A6	0.1660	2.315	3.098	1.338
	B6	0.0796	2.099	2.151	1.025
	B70B2	0.0991	2.148	2.381	1.108
	A4	0.1021	2.155	2.720	1.262
	A6	0.1185	2.196	2.949	1.343
	B84B4	0.0613	2.053	2.316	1.128
	B113B4	0.0396	1.999	1.976	0.988
Baron and Siess[4]	A1	0.1568	2.292	2.691	1.174
	2	0.0740	2.085	2.318	1.112
	3	0.0628	2.057	2.417	1.175
	4	0.0401	2.000	2.111	1.055
	11	0.5206	3.202	3.683	1.150
	12	0.2679	2.570	3.550	1.381
	13	0.1961	2.390	3.104	1.298
	14	0.1319	2.230	3.245	1.456
	15	0.1106	2.177	3.071	1.411
Baldwin and Viest[3]	OB 28	0.2612	2.553	2.970	1.163
	OF 28	0.2729	2.582	3.308	1.281

Average $V_{test}/V_{calc} = 1.239$
Coefficient of variation = 13.5 percent

†In thousandths.

TABLE 5-3 — DEVELOPMENT OF EQ. (5-11) FOR RESTRAINED BEAMS

Source	Beam No.	$\frac{Vpd}{M\sqrt{f_c'}}$†	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{calc}}$
Moody, Elstner, Hognestad, and Viest[1]	I-g	0.4040	2.910	3.653	1.255
	h	0.7005	3.500	4.087	1.168
	i	1.0267	3.500	4.220	1.206
	1a	1.6102	3.500	3.236	0.924
	b	1.5246	3.500	3.830	1.094
	2a	2.1819	3.500	3.330	0.951
	b	2.0329	3.500	3.102	0.886
	c	1.7254	3.500	3.292	0.940
	3a	2.8097	3.500	3.472	0.992
	b	2.4671	3.500	3.644	1.041
	j	0.5985	3.396	2.910	0.857
	k	0.9522	3.500	3.914	1.118
	4a	1.2274	3.500	4.316	1.233
	b	1.2692	3.500	3.188	0.911
	5a	1.6670	3.500	3.498	0.999
	b	1.6711	3.500	3.188	0.919
	6a	1.9933	3.500	3.605	1.030
	b	2.2503	3.500	4.070	1.163
	l	0.5836	3.359	3.405	1.014
	m	0.8929	3.500	3.670	1.049
	7a	1.1656	3.500	3.806	1.088
	b	1.1409	3.500	2.866	0.819
	8a	1.5347	3.500	3.513	1.004
	b	1.5510	3.500	3.787	1.082
	9a	1.8521	3.500	3.796	1.085

Source	Beam No.	$\frac{Vpd}{M\sqrt{f_c'}}$†	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{calc}}$
Moody, Elstner, Hognestad, and Viest[1]	9b	1.9717	3.500	3.684	1.053
	n	0.5758	3.340	2.799	0.838
	o	0.8325	3.500	3.422	0.978
	p	1.0441	3.500	3.147	0.899
	q	1.5205	3.500	4.061	1.160
	r	1.7460	3.500	4.210	1.203
	II-a	infinity	3.500	4.456	1.273
	b		3.500	4.515	1.290
	c		3.500	3.972	1.135
	d		3.500	3.522	1.006
	17a		3.500	4.012	1.146
	b		3.500	3.938	1.125
	18a		3.500	3.695	1.056
	b		3.500	4.240	1.211
	19a		3.500	3.335	0.953
	b		3.500	3.628	1.037
	20a		3.500	3.202	0.915
	b		3.500	3.796	1.085
	IV-g	0.1602	2.301	2.983	1.297
	h	0.2361	2.490	2.836	1.139
	i	0.3474	2.769	2.940	1.062
	j	0.4665	3.066	3.937	1.284
	k	0.6116	3.429	3.459	1.009
	l	0.7450	3.500	3.772	1.078

TABLE 5-3 (cont.) — DEVELOPMENT OF EQ. (5-11) FOR RESTRAINED BEAMS

Source	Beam No.	$\frac{Vpd}{M\sqrt{f_c'}}$†	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{calc}}$
Moody, Elstner, Hognestad, and Viest[1]	VI-a	0.1865	2.366	2.281	0.964
	b	0.2866	2.616	2.828	1.081
	c	0.4385	2.996	3.658	1.221
	d	0.5729	3.332	3.504	1.052
	e	0.7339	3.500	3.410	0.974
	f	0.3516	2.779	2.932	1.055
	g	0.4811	3.103	3.433	1.106
	h	0.6471	3.500	3.507	1.002
	i	0.7685	3.500	3.291	0.940
Bower and Viest[6]	IA-2a	0.1799	2.350	2.436	1.037
	2b	0.1812	2.353	2.044	0.869
	3a	0.1700	2.325	2.197	0.945
	3b	0.1784	2.346	2.825	1.204
	4a	0.1850	2.363	2.599	1.100
	4b	0.1914	2.379	2.235	0.940
	5a	0.1787	2.347	1.760	0.750
	5b	0.1890	2.373	2.198	0.926
	6a	0.1903	2.376	2.342	0.986
	6b	0.1767	2.342	2.207	0.942
	7a	0.1918	2.380	2.223	0.934
	7b	0.1856	2.364	2.510	1.062
	8a	0.1748	2.337	2.528	1.082
	8b	0.1852	2.363	2.789	1.180

Source	Beam No.	$\frac{Vpd}{M\sqrt{f_c'}}$†	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{calc}}$
Bower and Viest[6]	IB-1	0.2810	2.602	2.729	1.049
	2	0.2873	2.618	2.728	1.042
	3	0.2830	2.608	2.601	0.998
	4	0.2644	2.561	2.641	1.031
	IIA-1a	0.5664	3.316	3.327	1.003
	1b	0.5719	3.330	3.148	0.945
	2	0.2918	2.630	2.668	1.015
	3	0.1924	2.381	2.479	1.041
	4a	0.1512	2.278	2.087	0.916
	4b	0.1392	2.248	2.245	0.999
	5	0.0784	2.096	2.140	1.021
	6	0.0539	2.035	2.045	1.005
	8	0.1234	2.209	2.201	0.998
	9	0.0967	2.142	2.332	1.089

Average $V_{test}/V_{calc} = 1.041$

Coefficient of variation = 8.4 percent

†In thousandths.

TABLE 5-4 — DEVELOPMENT OF EQ. (5-11) FOR CONTINUOUS BEAMS

Source	Beam No.	$\frac{Vpd}{M\sqrt{f_c'}}$†	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{calc}}$
Rodriguez, Bianchini, Viest, and Kesler[7]	E6N1	0.6159	3.440	4.040	1.174
	2	0.7034	3.500	4.067	1.162
	3	0.6292	3.473	3.672	1.057
	C6N1	0.6136	3.434	3.376	0.983
	2	0.7289	3.500	4.522	1.292
	3	0.6381	3.495	4.298	1.230
	E3N1	0.4056	2.914	2.843	0.976
	2	0.4324	2.981	2.880	0.966
	C3N1	0.4441	3.010	3.036	1.008
	2	0.4364	2.991	3.059	1.023
	E2N1	0.3152	2.688	2.548	0.948
	2	0.4036	2.909	2.919	1.004
	3	0.3808	2.852	2.385	0.836
	C2N1	0.3545	2.786	2.639	0.947
	2	0.3960	2.890	2.477	0.857

Average V_{test}/V_{calc} = 1.031 Coefficient of variation = 13.0 percent

†In thousandths.

To satisfy the first consideration, a simple straight-line representation rather than Eq. (5-9) was chosen. To satisfy the second consideration, the lines were placed near the lower extremes of observed shear stress rather than as average values. Furthermore, the sloping line was chosen particularly close to the lower envelope line, because the data for beams with no strength in excess of the critical diagonal tension cracking were generally located on the left side of Fig. 5-1. On the other hand, the data located on the right side of Fig. 5-1 represent beams which usually carried ultimate loads substantially in excess of diagonal tension cracking. Hence, the horizontal line was chosen nearer the average value given by the test data.

It should be re-emphasized that the proposed design equation was originally determined from 194 tests of beams of constant cross section and subjected to one or two concentrated loads in any one span. The data for these beams are shown in Fig. 5-1 as dots. The observed values $V_{test}/bd\sqrt{f_c'}$, and the values of $V_{calc}/bd\sqrt{f_c'}$ calculated from Eq. (5-11), as well as the ratios of V_{test}/V_{calc}, are listed in Tables 5-1 through 5-4. The means and coefficients of variation for the four dif-

TABLE 5-5 — MEANS AND COEFFICIENTS OF VARIATION OF FOUR TYPES OF TEST BEAMS

Type of test beam	No. of test beams	$\frac{V_{test}}{V_{calc}}$	Coefficient of variation, percent
Simple	45	1.076	15.8
Stub	48	1.239	13.5
Restrained	86	1.041	8.4
Continuous	15	1.031	13.0
All beams	194	1.097	15.1

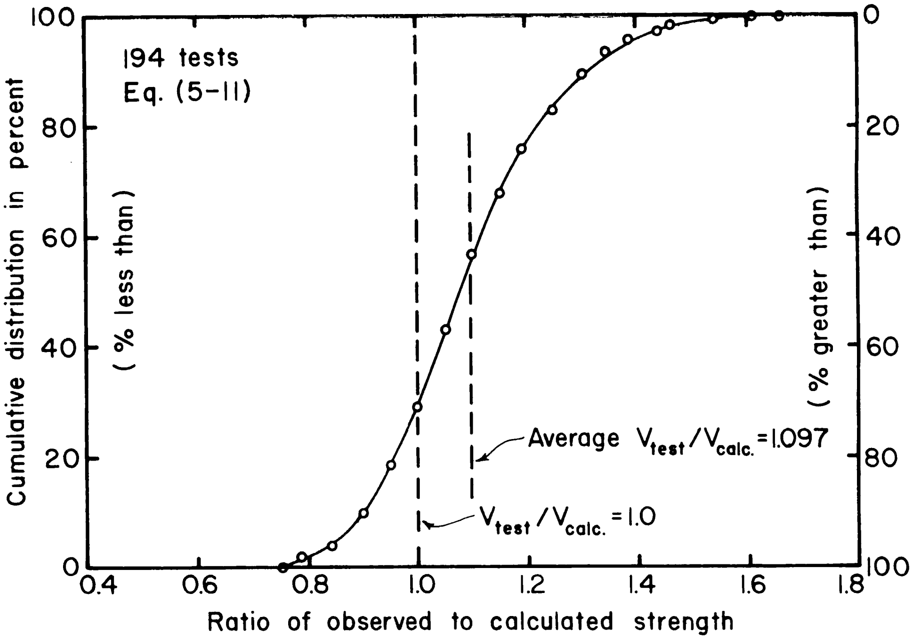


Fig. 5-2—Cumulative distribution V_{test}/V_{calc}

ferent types of test beams considered in this study are given in Table 5-5.

The cumulative distribution of V_{test}/V_{calc} is plotted in Fig. 5-2. It is clearly seen that the design equation [Eq. (5-11)] was purposely chosen on the safe side, V_{test}/V_{calc} is less than one only for about 30 percent of the tests.

502—Comparison of Eq. (5-11) with other test data

Eq. (5-11) was later compared with the results of several additional investigations. The observed diagonal tension cracking load (or shear) and the corresponding computed values are given in Tables 5-6 through 5-17. All test beams were without web reinforcement in the region of failure; though several specimens had reinforced webs in some regions to force diagonal tension cracking into the desired regions.

Comparison of Eq. (5-11) with these additional laboratory tests indicates that the equation is applicable to any loading condition, to any support condition, to any cross section, and to beams having high strength reinforcement. These variables have no tendency toward reducing the conservative nature of the proposed design equation. Thus, it seems reasonable to assume that the equation is applicable even when several of these variables are combined.

TABLE 5-6 — UNIFORM LOAD TESTS BY BERNAERT AND SIESS[5]

Specimen No.	P_{calc}, kips	P_{test}, kips	$\frac{P_{test}}{P_{calc}}$	Distance of DT crack† from support, ft	
				Computed	Measured
D-15	21.6	28.0	1.296	0.83	0.79
14	23.4	29.2	1.248	0.83	0.71
16	25.6	30.3	1.184	0.83	0.83
13	17.9	22.8	1.274	0.83	1.54
17	24.7	24.7	1.000	0.83	0.96
5	27.9	35.0	1.254	0.91	0.65
4	28.2	39.8	1.411	0.85	0.46
9	27.1	34.4	1.269	1.09	1.01
10	28.1	38.0	1.352	1.06	0.79
11	33.4	48.2	1.443	0.97	0.75
7	23.9	35.3	1.477	1.28	0.75
6	25.8	35.3	1.368	1.23	0.68
1	28.4	38.2	1.345	1.17	0.92
2	30.8	37.6	1.221	1.27	1.04
8	32.7	39.4	1.205	1.09	0.83
3	27.8	38.8	1.396	1.30	1.25
18	25.5	30.7	1.204	1.46	1.50

Average $P_{test}/P_{calc} = 1.291$
Coefficient of variation = 8.7 percent

†Distance from center of support to the point of intersection of the critical diagonal tension crack with tension reinforcement.

Uniform loads[5,11]—For beams with uniform loads, it was found necessary to add a limitation with respect to the location of diagonal cracks. It has been observed in tests of uniformly loaded beams that diagonal tension cracks always form some distance away from the end supports. It was assumed, therefore, that diagonal cracks would not form closer to the support than the effective beam depth.

The proposed equation for diagonal tension cracking was compared with the results of tests on uniformly loaded beams reported in References 5 and 11. The observed and calculated cracking loads, and their ratios are listed for all beams in Tables 5-6 and 5-7. The results are summarized in Table 5-18.

Columbia University tests with concentrated loads[11]—Simply supported rectangular beams were subjected to a concentrated load at midspan. The ratios V_{test}/V_{calc} are listed in Table 5-8 for all 58 beams for which diagonal tension cracking loads were reported. It will be noted that the lowest value of the ratio is 0.921 and the highest value is 1.486. The average for all beams is 1.241 and the coefficient of variation is 10.4 percent.

TABLE 5-7 — UNIFORM LOAD TESTS BY KREFELD AND THURSTON[11]

Specimen No.	P_{calc}, kips	P_{test}, kips	$\frac{P_{test}}{P_{calc}}$	Distance of DT crack from support, ft		Specimen No.	P_{calc}, kips	P_{test}, kips	$\frac{P_{test}}{P_{calc}}$	Distance of DT crack from support, ft	
				Computed	Measured					Computed	Measured
I-4A1	69.1	75	1.085	1.28	1.00	V-4EU	21.4	25	1.168	1.37	0.75
B1	67.5	85	1.259	1.28	0.92	5EU	22.3	24	1.108	1.52	1.12
5A1	77.3	95	1.229	1.28	0.75	6EU	24.1	28	1.162	1.64	0.92
B1	77.5	85	1.097	1.28	0.92	5GU	22.2	25	1.126	1.64	1.00
II-4A2	68.1	80	1.175	1.28	0.88	4JU	20.8	22	1.058	1.60	1.21
5A2	77.3	80	1.035	1.28	0.92	VII-6U	28.4	38	1.338	0.95	0.75
11A1	42.1	50	1.187	1.03	1.29	VIII-4AAU	32.1	40	1.246	0.83	0.42
IV-15A1	37.0	24	0.649	1.04	1.25	6AAU	36.3	40	1.102	0.95	1.04
16A1	21.9	26	1.187	0.79	0.71	4AU	29.2	34	1.164	1.03	0.88
17A1	22.2	26	1.171	0.82	0.96	5AU	31.2	42	1.346	1.12	1.00
17B1	23.3	28	1.202	0.80	0.83	6AU	34.3	40	1.166	1.20	1.21
18A1	35.5	42	1.183	1.04	1.42	IX-3AAU	21.3	26	1.221	0.87	0.67
19A1	27.3	38	1.392	0.95	1.08	4AAU	21.8	24	1.101	0.95	0.58
20A1	29.7	36	1.212	1.02	1.71	6AAU	27.3	25	0.916	1.10	0.75
21A1	40.9	54	1.320	1.05	1.87	3AU	20.2	22	1.089	1.08	1.92
V-3AU	23.2	30	1.293	0.94	1.37	4AU	20.6	20	0.971	1.21	0.71
4AU	22.9	31	1.354	1.09	1.11	6AU	25.0	28	1.120	1.41	1.21
5AU	25.8	32	1.240	1.15	1.13	3CU	17.5	20	1.143	1.22	1.12
6AU	27.8	35	1.259	1.25	1.81	4CU	21.2	22	1.038	1.27	1.85
V-2CU	19.3	22	1.140	0.93	0.96	5CU	21.4	24	1.121	1.42	0.90
3CU	19.4	22	1.134	1.11	1.96	6CU	22.5	24	1.067	1.56	1.46
4CU	22.6	25	1.106	1.22	0.79	4EU	18.8	19	1.011	1.47	1.00
5CU	24.0	30	1.250	1.34	1.57	5EU	20.4	22	1.078	1.61	2.00
6CU	25.7	33.2	1.292	1.45	1.50						

Average $P_{test}/P_{calc} = 1.156$

Coefficient of variation = 10.1 percent

TABLE 5-8 — COLUMBIA UNIVERSITY TESTS OF BEAMS WITH CONCENTRATED LOAD AT MIDSPAN[11]

Beam No.	$\frac{Vpd}{M\sqrt{f_c'}}$†	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{calc}}$
II-				
4A3	0.2309	2.477	2.748	1.109
5A3	0.3507	2.777	2.783	1.002
11A2	0.2706	2.576	3.056	1.186
12A2	0.2407	2.502	3.371	1.348
III-				
18A2	0.2674	2.568	3.292	1.282
18B2	0.2636	2.559	3.246	1.268
18C2	0.2470	2.518	2.807	1.115
18D2	0.2501	2.525	2.842	1.126
IV-				
13A2	0.0790	2.097	2.118	1.010
14A2	0.0690	2.073	1.910	0.921
15A2	0.1309	2.227	2.231	1.002
15B2	0.1292	2.223	2.691	1.210
16A2	0.1106	2.176	2.645	1.215
17A2	0.1339	2.235	2.778	1.243
18E2	0.2641	2.560	3.001	1.172
19A2	0.2299	2.475	3.072	1.242
20A2	0.2877	2.619	3.224	1.231
21A2	0.3326	2.731	3.478	1.273
V-				
2AC	0.0600	2.050	2.019	0.985
3AC	0.0959	2.140	2.713	1.268

Beam No.	$\frac{Vpd}{M\sqrt{f_c'}}$†	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{calc}}{V_{calc}}$
V-				
4AC	0.1418	2.254	2.898	1.285
5AC	0.1698	2.324	2.763	1.189
6AC	0.1929	2.382	3.526	1.480
3CC	0.0735	2.084	2.432	1.167
4CC	0.0965	2.141	2.748	1.283
5CC	0.1226	2.206	3.087	1.399
6CC	0.1547	2.287	3.096	1.354
4EC	0.0765	2.091	2.823	1.350
5EC	0.1010	2.152	2.805	1.303
6EC	0.1294	2.224	3.051	1.372
4GC	0.0644	2.061	2.414	1.171
5GC	0.0798	2.100	2.527	1.204
6GC	0.1026	2.156	2.763	1.281
VII-				
6C	0.2367	2.492	3.568	1.432
VIII-				
3AAC	0.1089	2.172	2.809	1.293
4AAC	0.1556	2.289	3.201	1.398
5AAC	0.1854	2.364	2.916	1.234
6AAC	0.2293	2.473	3.111	1.258
4AC	0.1042	2.161	2.758	1.276
5AC	0.1269	2.217	2.673	1.206

Beam No.	$\frac{Vpd}{M\sqrt{f_c'}}$†	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{test}}$
VIII-				
6AC	0.1578	2.294	2.883	1.257
4CC	0.0706	2.076	2.456	1.183
5CC	0.0904	2.126	2.730	1.284
6CC	0.1131	2.183	2.605	1.193
5EC	0.0729	2.082	2.048	0.984
6EC	0.0973	2.143	2.173	1.014
IX-				
3AAC	0.1807	2.352	3.495	1.486
4AAC	0.2342	2.486	3.469	1.396
5AAC	0.2709	2.577	3.728	1.447
6AAC	0.3677	2.819	3.838	1.361
3AC	0.1182	2.195	2.971	1.353
4AC	0.1602	2.301	3.083	1.340
5AC	0.1855	2.364	3.373	1.427
6AC	0.2616	2.554	3.187	1.248
3CC	0.0952	2.138	2.363	1.105
4CC	0.1058	2.164	2.627	1.214
5CC	0.1443	2.261	2.725	1.205
6CC	0.1897	2.374	3.229	1.360

Average $V_{test}/V_{calc} = 1.241$

Coefficient of variation = 10.4 percent

†In thousandths.

National Bureau of Standards tests[16,17]—Simply supported rectangular beams were subjected to two symmetrically located concentrated loads. The 31 tests included in Table 5-9 comprise data from two reports. One report[16] (first seven specimens) does not contain diagonal tension cracking loads, but extensive strain data permitted the determination of such loads for seven specimens.

It will be noted in Table 5-9 that the ratios of V_{test}/V_{calc} at diagonal tension cracking are close to 1.0 for the first seven beams, but are consistently in excess of 1.10 for all other beams. It is believed that at least in part this difference is the result of two different methods of determining the cracking load, determination from strains as opposed to visual observation. Because all of these beams had small a/d ratios, the two methods tend to give different values of the diagonal tension cracking load.

The mean of V_{test}/V_{calc} for all 31 beams is 1.289 and the coefficient of variation is 14.9 percent.

PCA tests of simple-span beams[14]—Five simply supported rectangular beams made with sand-and-gravel concrete were included in this study as control specimens. The specimens were loaded with two symmetrically located concentrated loads.

The ratios V_{test}/V_{calc} are listed in Table 5-10. It is noteworthy that Beam 8D had concrete strength of 10,680 psi—the strongest concrete for which comparisons were made with Eq. (5-11). The over-all mean of V_{test}/V_{calc} is 1.196.

TABLE 5-9 — NATIONAL BUREAU OF STANDARDS TESTS FOR BEAMS WITH TWO CONCENTRATED LOADS[16, 17]

Beam No.	$\frac{Vpd}{M\sqrt{f_c'}}$†	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{calc}}$	Beam No.	$\frac{Vpd}{M\sqrt{f_c'}}$†	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{calc}}$
A-15	0.2450	2.512	2.708	1.078	D-VA-27	0.1597	2.299	3.167	1.378
B-18-2	0.6878	3.500	3.406	0.973	D-VI-18	0.1629	2.307	3.239	1.404
C-18-1	0.4053	2.913	3.227	1.108	D-VI-23	0.1722	2.330	2.740	1.176
C-18-2	0.3989	2.897	3.176	1.096	D-I-8	0.6635	3.500	3.898	1.114
D-18-2	0.2528	2.532	2.578	1.018					
D-18-2	0.2466	2.516	2.514	0.999	D-I-29	0.6975	3.500	4.097	1.171
E-18-1	0.1699	2.325	2.068	0.890	D-II-21	0.4424	3.006	4.200	1.397
D-I-9	0.4008	2.902	3.248	1.119	D-II-32	0.4024	2.906	3.821	1.315
D-I-11	0.3934	2.883	3.826	1.327	D-III-5	0.4012	2.903	3.871	1.334
D-II-13	0.2533	2.533	3.952	1.560	D-III-31	0.4022	2.906	3.882	1.336
D-II-20	0.2522	2.531	3.935	1.555	D-IV-22	0.4157	2.939	3.997	1.360
D-III-16	0.2684	2.571	4.043	1.572	D-IV-30	0.4093	2.923	3.935	1.346
D-III-24	0.2646	2.562	3.985	1.556	D-VA-14	0.2656	2.564	3.404	1.328
D-IV-19	0.2568	2.542	3.350	1.318	D-VA-28	0.2459	2.515	3.781	1.504
D-IV-25	0.2594	2.548	4.061	1.593	D-VI-7	0.2540	2.535	3.248	1.281
D-VA-17	0.1743	2.336	3.456	1.480	D-VI-26	0.2526	2.532	3.230	1.276

Average V_{test}/V_{calc} = 1.289 Coefficient of variation = 14.9 percent

†In thousandths.

TABLE 5-10 — PCA TESTS[14]

Specimen	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{calc}}$
8A-X	2.598	3.139	1.208
8A	2.569	3.248	1.264
8B	2.479	3.084	1.244
8C	2.826	3.266	1.156
8D	2.722	3.018	1.109

Average $V_{test}/V_{calc} = 1.196$

TABLE 5-11 — UNIVERSITY OF TEXAS TESTS[12]

Specimen	V_{calc}, kips	V_{test}, kips	$\frac{V_{test}}{V_{calc}}$
F_2	4.35	3.75	0.862
F_3	4.35	5.50	1.264
F_5	3.89	4.00	1.028
F_7	5.07	5.00	0.986

Average $V_{test}/V_{calc} = 1.035$

University of Texas tests with concentrated loads[12]—Simply supported rectangular beams were tested with two and with four symmetrically located concentrated loads. Beams F_3 and F_5 were subjected to four loads. All beams were 4 in. wide. Only ranges are given for the diagonal tension cracking loads in Reference 12. The comparisons in Table 5-11 are based on medians for each range. The average value of V_{test}/V_{calc} is 1.035.

University of Illinois fatigue study[15] — Twenty-five simply supported rectangular beams were tested with static loads as a part of a fatigue study. One or two loads were applied symmetrically with respect to the supports. The beams were 4 x 5.38 in. The ratios V_{test}/V_{calc} are listed in Table 5-12. The mean is equal to 1.519 and the coefficient of variation is 10.1 percent. The ratios of this series are exceptionally high.

University of Illinois continuous beams[7]—Most results of these tests of two-span continuous beams of rectangular cross section were used

TABLE 5-12 — UNIVERSITY OF ILLINOIS TESTS OF BEAMS WITH CONCENTRATED LOADS[15]

Beam No.	$\frac{Vpd}{M\sqrt{f_c'}}$†	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{calc}}$	Beam No.	$\frac{Vpd}{M\sqrt{f_c'}}$†	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{calc}}$
I-A1	0.1569	2.292	3.284	1.433	4-21a	0.0961	2.145	2.943	1.372
B1	0.2075	2.419	3.233	1.337	21b	0.0961	2.145	3.440	1.604
C1	0.1735	2.334	3.240	1.388	22a	0.1080	2.170	3.295	1.518
C2	0.1735	2.334	2.939	1.259	22b	0.1082	2.171	3.611	1.663
II-A1	0.1701	2.325	3.518	1.513	23a	0.1074	2.169	3.305	1.524
A2	0.1701	2.325	3.563	1.532	23b	0.1074	2.169	3.441	1.587
B1	0.1613	2.303	3.426	1.487					
C1	0.4020	2.905	3.674	1.265	5-21a	0.1665	2.316	4.413	1.905
					21b	0.1665	2.316	4.202	1.814
III-A1	0.1756	2.340	3.829	1.637					
A2	0.5487	3.272	4.649	1.421	22a	0.1693	2.323	3.483	1.500
					22b	0.1693	2.323	4.030	1.734
B1	0.1852	2.363	3.450	1.460					
B2	0.1852	2.363	3.479	1.472	23a	0.1669	2.317	3.755	1.620
C1	0.4377	2.994	4.129	1.379	23b	0.1669	2.317	3.578	1.544

Average $V_{test}/V_{calc} = 1.519$ Coefficient of variation = 10.1 percent

†In thousandths.

in the derivation of Eq. (5-11). However, four beams which failed between two concentrated loads in the same span and one beam without web reinforcement were not used in the derivation. They are therefore included in these comparisons. All beams were loaded with concentrated loads located symmetrically with respect to the center support. The first four beams in Table 5-13 had two concentrated loads in each span and had web reinforcement everywhere except between the loads. The average of the ratios of P_{test}/P_{calc} for this series is 1.013.

PCA tests of restrained beams[8.18]—Beams with two symmetrical overhangs and symmetrical haunches were tested with eight or seven evenly spaced concentrated loads between the supports. Three beams had a T-section and eight had a rectangular cross section. The longitudinal reinforcement was in several layers and was discontinued according to the moment diagram. Regular embedded stirrups were provided at some locations, and exterior stirrups were used in some portions of some beams. However, no web reinforcement was present in the cracking locations studied in Table 5-14.

The test loads at diagonal tension cracking are compared with the computed values in Table 5-14. The load at cracking, P, was computed by applying Eq. (5-11) to every section of abrupt change, except in regions where failure was avoided by stirrups. The region along the beam length corresponding to the minimum total load P_{calc} is the predicted location of diagonal tension cracking. For every test beam, the predicted location was in agreement with the actual location of the critical diagonal tension crack.

The mean ratio of P_{test}/P_{calc} is 1.159 and the coefficient of variation is 15.5 percent. It may be noted that two of the three T-beams (No. 24 and 25) have high P_{test}/P_{calc} ratios. However, the ratio for the third T-beam (No. 21) is in the same range as the ratios for rec-

TABLE 5-13 — UNIVERSITY OF ILLINOIS TESTS OF CONTINUOUS BEAMS[7]

Specimen	P_{calc}, kips	P_{test}, kips	$\frac{P_{test}}{P_{calc}}$
E6A1	170.7	160	0.937
E6A2	158.7	160	1.008
C6A2	167.3	180	1.076
C3A1	107.8	110	1.020
B2N1	43.9	45	1.025

Average P_{test}/P_{calc} = 1.013

TABLE 5-14 — PCA RESTRAINED BEAM TESTS[8,18]

Specimen	P_{calc}, kips	P_{test}, kips	$\frac{P_{test}}{P_{calc}}$
1	34.2	38.7	1.132
2	30.4	32.0	1.053
2a	62.4	53.7	0.861
5	29.6	41.2	1.392
6	61.2	57.6	0.941
7	33.0	39.2	1.188
7a	36.2	45.7	1.262
18‡	40.3	41.4†	1.032
21§	38.4	41.6†	1.083
24§	34.9	49.7†	1.424
25§	33.9	46.7†	1.378

Average P_{test}/P_{calc} = 1.159
Coefficient of variation = 15.5 percent

†Values furnished by PCA laboratories, not published.
‡7-point loading of beam similar to Beam 1, all other beams had 8-point loading.
§T-beams similar to Beam 2 with 2-in. flanges overhanging 16 in. on each side.

TABLE 5-15 — PCA SPECIAL TESTS[18]

Specimen	V_{calc}, kips	V_{test}, kips	$\frac{V_{test}}{V_{calc}}$
A†	17.2	23.1	1.345
B‡	19.2	23.1	1.215

Average V_{test}/V_{calc} = 1.280

†Restrained beam.
‡Simple beam.

TABLE 5-16 — CORNELL UNIVERSITY TESTS[13]

Specimen	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{calc}}$
IA-1m	3.50	3.901	1.115
IC-1m	3.50	4.325	1.236
IIA-1m	2.407	3.044	1.265
IIC-1m	2.712	3.733	1.377
IIIA-1m†	3.220‡	4.750‡	1.475
IIIC-1m†	3.922‡	5.427‡	1.384

Average V_{test}/V_{calc} = 1.309

†Uniform loading.
‡Shear at the support.

tangular beams. For T-beams, the width of the stem was used in computations.

PCA special tests[18] — One restrained and one simple beam were tested to check the relative effects of the M/Vd ratio and the a/d ratio. Both beams were provided with web reinforcement except in regions in which diagonal tension failure was desired. The regions without web reinforcement had identical values of M/Vd for the two beams, but the a/d ratios were widely different. Both beams were of rectangular cross section. The shear forces V were calculated from Eq. (5-11) for the regions without web reinforcement. Table 5-15 lists ratios V_{test}/V_{calc}.

Cornell University tests[13]—Six restrained T-beams with one overhang were tested with concentrated and uniform loading. The mean ratio of V_{test}/V_{calc} given in Table 5-16 is 1.309. The width of stem was used in the computations.

University of Texas tests of T-beams[10]—Twenty-five simply supported T-beams were tested with two symmetrically located concen-

TABLE 5-17 — UNIVERSITY OF TEXAS TESTS OF T-BEAMS[10]

Beam No.	$\frac{Vpd}{M\sqrt{f_c'}}$†	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{calc}}$	Beam No.	$\frac{Vpd}{M\sqrt{f_c'}}$†	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{calc}}$
6	0.2359	2.490	3.188	1.280	22	0.0662	2.066	2.360	1.142
12	0.2906	2.626	3.697	1.408	7	0.1276	2.219	2.568	1.157
11	0.1703	2.326	3.218	1.384	24	0.1206	2.202	2.531	1.150
2	0.0966	2.142	2.790	1.303	16	0.1217	2.204	2.630	1.193
21	0.1753	2.338	3.440	1.471	17	0.0900	2.125	2.636	1.240
15	0.1473	2.268	3.273	1.443	18	0.0977	2.144	2.650	1.236
3	0.0792	2.098	2.632	1.255	19	0.1405	2.251	2.441	1.084
10	0.1397	2.249	2.784	1.238	25	0.1033	2.158	2.541	1.177
4	0.0806	2.102	2.740	1.304	9	0.0727	2.082	2.556	1.228
13	0.1405	2.251	2.562	1.138	20	0.1218	2.204	2.449	1.111
5	0.0736	2.084	2.578	1.237	14	0.0674	2.068	2.641	1.277
18	0.1214	2.204	2.477	1.124	23	0.1075	2.169	2.502	1.154
1	0.0675	2.069	2.586	1.250					

Average V_{test}/V_{calc} = 1.191 Coefficient of variation = 21.9 percent

†In thousandths.

trated loads. The stem was 3 x 5 in. In applying Eq. (5-11), the width of stem was substituted for b. The resulting ratios of V_{test}/V_{calc} are listed in Table 5-17. The average is 1.191 and the coefficient of variation in 21.9 percent.

Summary of comparisons — The comparisons of loads at diagonal cracking predicted by Eq. (5-11) with the results of 172 miscellaneous tests described previously are summarized in Table 5-19.

A summary of the comparison of the proposed ultimate strength design equation, Eq. (5-11), with the results of all 430 tests of beams without web reinforcement is presented in Table 5-20.

In these 430 tests the major variables, p, M/Vd, and f_c' were covered throughout the ranges normally encountered in design. The majority of test specimens were simple rectangular beams with sym-

TABLE 5-18 — MEANS AND COEFFICIENTS OF VARIATION FOR UNIFORM LOADS

Investigation	No. of test beams	$\frac{P_{test}}{P_{calc}}$	Coefficient of variation, percent
Bernaert-Siess[5]	17	1.291	8.7
Krefeld-Thurston[11]	47	1.156	10.1
All beams	64	1.192	10.9

TABLE 5-19 — COMPARISON OF RESULTS OF 172 TESTS WITH LOADS PREDICTED BY EQ. (5-11)

Type of beam	No. of test beams	$\frac{\text{Test}}{\text{Calc}}$	Coefficient of variation, percent
Simple rectangular beams	124	1.300	14.9
Restrained and continuous rectangular beams	14	1.091	13.8
T-beams	34	1.221	19.6
All beams	172	1.267	16.1

TABLE 5-20 — COMPARISON OF RESULTS OF 430 TESTS WITHOUT WEB REINFORCEMENT AND EQ. (5-11)

Type of test beam	Cross section	Loading	No. of test beams	$\frac{\text{Test}}{\text{Calc}}$	Coefficient of variation, percent
Beams used in derivation of Eq. (5-11)					
Simple	Rectangular	Concentrated	45	1.076	15.8
Simple with stub	Rectangular	Concentrated	48	1.239	13.5
Restrained	Rectangular	Concentrated	86	1.041	8.4
Continuous	Rectangular	Concentrated	15	1.031	13.0
Other beams					
Simple	Rectangular	Uniform	64	1.192	10.9
Simple	Rectangular	Concentrated	124	1.300	14.9
Restrained and continuous	Rectangular	Concentrated	14	1.091	13.8
Simple and restrained	T-beams	Concentrated and uniform	34	1.221	19.6
All beams			430	1.180	16.2

metrical one- and two-point concentrated loads and simple beams with uniform loads. All combinations of loading, cross section and support conditions have not been investigated. However, sufficient test data are available to indicate that the proposals of this report for the design of beams without web reinforcement are adequate at any section along the length of a member, regardless of concrete strength used, the strength of the reinforcement, the manner of loading and supporting the beam, or the cross-sectional shape of the member involved.

It is again emphasized that the design procedures proposed are empirical because the fundamental nature of shear and diagonal tension strength is not yet clearly understood. Further basic research should be encouraged to determine the mechanism which results in shear failures of reinforced concrete members. With this knowledge it may then become possible to develop fully rational design procedures.

503—Comparison of proposed and ACI 318-56 procedures

The design method of the 1956 ACI Building Code for beams without web reinforcement considers concrete strength to be the only variable

$$v = \frac{8V}{7bd}\text{; maximum allowable} = 0.03 f_c',$$

$$\text{but not greater than 90 psi} \qquad (5\text{-}12)$$

To compare this expression with the proposed Eq. (5-11), it is necessary to eliminate $j = 7/8$ and to raise the expression to ultimate strength by applying a factor of safety of 2.0. Thus, the Eq. (5-12) becomes

$$v = \frac{V}{bd} = 0.0525 f_c'\text{, but not greater than 157.5 psi} \qquad (5\text{-}12a)$$

Eq. (5-12a) may be compared to the proposed Eq. (5-11) written as follows

$$v = \frac{V}{bd} = 1.9\sqrt{f_c'} + \frac{2500\,pVd}{M},$$

$$\text{but not greater than } 3.5\sqrt{f_c'} \qquad (5\text{-}11a)$$

The two equations are compared in Fig. 5-3. The proposed Eq. (5-11a) is shown in terms of its limits $v = 3.5\sqrt{f_c'}$ (deep, short beam), and $v_c = 1.9\sqrt{f_c'}$ (long, shallow beam). The proposed equation is more liberal than the present procedures for short, deep beams; but it is more conservative for long, shallow beams.

Also shown in Fig. 5-3 is a curve representing the ACI Code design equation, Eq. (5-12a), reduced by a factor of one-third. If this procedure were adopted, all beams would have an adequate safety factor against shear failure. Long, shallow beams would have safety factors of at least 2.0 while short, deep beams would have safety factors approaching 4.0. Therefore, though this procedure would be simple to use, it would be conservative with respect to beams of average dimensions and would be ultraconservative with respect to short, deep beams.

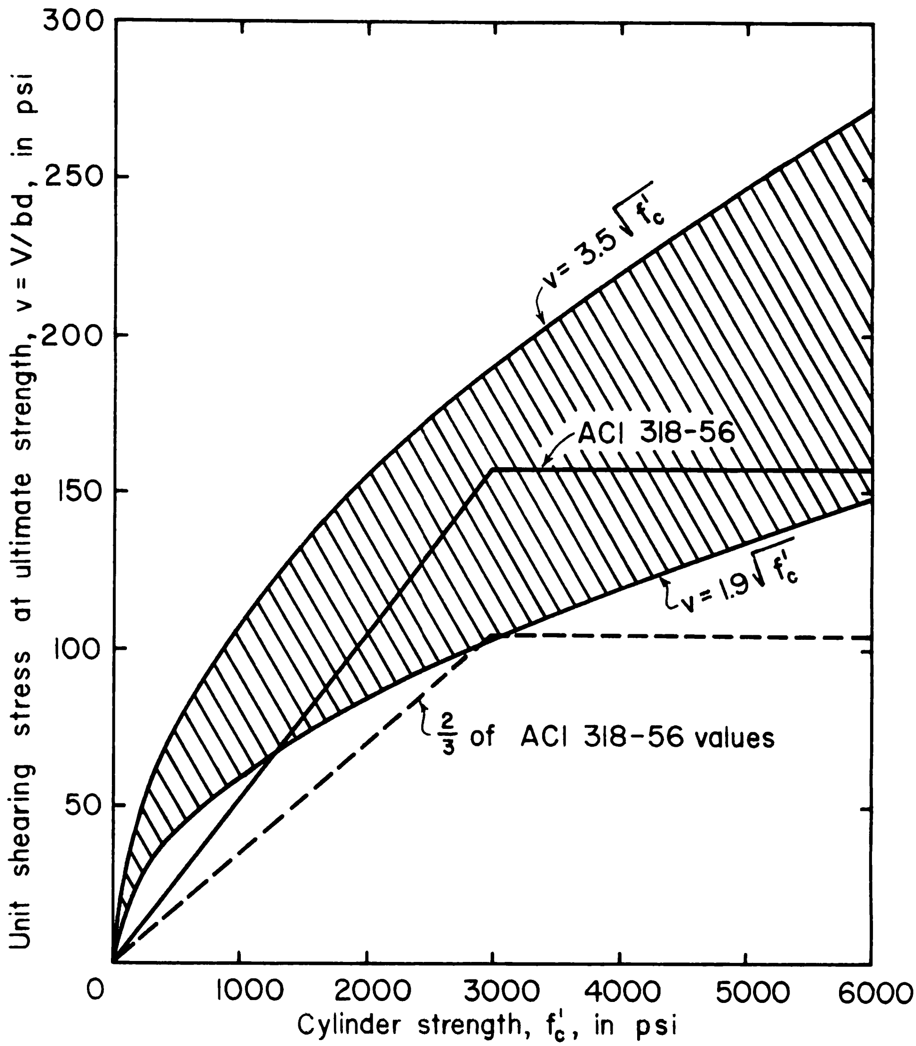


Fig. 5-3—Comparison of proposed procedure to ACI 318-56

504—Design recommendations

In the early days of reinforced concrete construction before 1900, all columns were designed for axial load only, and safety factors as high as 10 were used. The high safety factor was necessary to offset effects of flexural stresses induced by eccentric loading. For large eccentricities the high safety factor provided adequate strength, but obviously these design procedures were ultraconservative and uneconomical when eccentricities were small. Of course, this situation no longer exists. Design equations for columns consider both axial load and flexure.

The design methods of the 1956 ACI Code for shear and diagonal tension are analogous to the old column design procedures. Diagonal tension strength depends on two quantities, shear and flexure. So far, shear has been considered the only important quantity, and design produres consider shear alone. The neglected effects of flexure were considered sufficiently protected by the safety factor. Early laboratory tests substantiated this line of reasoning, and indicated that the safety factor was conservative. However, recent laboratory tests show that the effects of flexure can be important and, under certain normal design conditions, this effect may seriously reduce the margin of safety obtained by the 1956 design procedures. A few structural failures have verified that such a danger exists. This condition can be corrected by two alternative measures.

The first alternative is to continue the 1956 procedure of ignoring the effects of flexural stress. The safety factor must then be increased. If the present procedures are to be satisfactory for all girders, the present allowable nominal shearing stresses must be reduced by about one-third. Then, like in early column design for axial load only, the majority of girders would be unnecessarily penalized to insure safety under the worst condition.

The second alternative requires a revision of the fundamental philosophy of shear and diagonal tension design. That is, to consider in design equations effects of both shear and flexure. When the influence of flexural stress is small, the allowable nominal shearing stress, $v = V/bd$, can be even higher than allowed by ACI 318-56. However, when the influence of flexural stress is great, the allowable nominal shearing stress must be lower and more conservative than in the 1956 ACI Code.

Committee 326 considers the second alternative the best course to follow and, therefore, recommends basing the ultimate strength design procedure for beams without web reinforcement on Eq. (5-11). Since $v = V/bd$ and $f_s = M/jdA_s$, the term pVd/M in Eq. (5-11) may be expressed as v/jf_s, where f_s is the tension steel stress at the section considered. Assuming $j = 7/8$, substitution into Eq. (5-11) and rearrangement leads to an expression suggested to Committee 326 by M. P. van Buren

$$v = 1.9\sqrt{f_c'}\,\frac{f_s}{f_s - 2850\text{ psi}} \text{ but not greater than } 3.5\sqrt{f_c'} \qquad (5\text{-}13)$$

Another rearrangement of Eq. (5-11) is obtained by substituting $A_s = pbd$. This leads to a design formula suggested by Howard Simpson

$$v = \frac{1.9\sqrt{f_c'}}{1 - (2500\text{ psi})\dfrac{A_s d}{M}} \qquad (5\text{-}14)$$

Except for minor variations, Eq. (5-13) and (5-14) are identical to Eq. (5-11). However, while Eq. (5-11) is better suited for studies of experimental data, Eq. (5-13) with the coefficient 2850 psi rounded off to 3000 psi and Eq. (5-14) are possibly more convenient for everyday design practice.

Eq. (5-11) facilitates analysis of test data because the term pVd/M can readily be computed for test beams failing in shear, for which the steel stress at diagonal cracking, f_s in Eq. (5-13), is less than the yield point. In practical design, however, reinforcement will in most cases be terminated in accord with the moment diagram involved, so that the tension steel stress in flexural members at ultimate strength is approximately equal to the yield point f_y, of the reinforcement used. Eq. (5-13) can then be simplified by substitution of $f_s = f_y$. In some design cases, on the other hand, an excess amount of tension steel may be present at some cross sections so that f_s is less than the yield point. In such cases a shear stress v greater than that correponding to $f_s = f_y$ can be calculated by Eq. (5-13) or (5-14).

In the region near a point of inflection in continuou members, redistribution of reinforcing steel stress due to diagonal cracking must be considered. This may be done by considering a minimum value for M equal to Vd in Eq. (5-11) and (5-14), and also using such a minimum value for computation of the steel stress f_s in Eq. (5-13).

Ultimate strength design criteria—In view of the foregoing, Committee 326 recommends the following ultimate strength design criteria for beams without web reinforcement:

(a) The diagonal tension strength shall be determined on the basis of the average unit shearing stress v computed by the formula $v = V/bd$, where V equals shear force, b equals width of member, and d equals effective depth to centroid of tension steel.

(b) For beams of I- and T-section, or for floor joist construction, the web width b' shall be substituted for the width b.

(c) The ultimate diagonal tension strength v_c of a concrete section of an unreinforced web shall be computed by the formula

$$v_c = 1.9\sqrt{f_c'}\,\frac{f_s}{f_s - 3000\text{ psi}}$$

or

$$v_c = 1.9\sqrt{f_c'}\,\frac{1}{1 - (2500\text{ psi})\dfrac{A_s\,d}{M}}$$

except that in both cases v_c shall not exceed $3.5\sqrt{f_c'}$

In these expressions v_c equals ultimate diagonal tension strength in psi, f_c' equals concrete strength in psi and the units for $\sqrt{f_c'}$ are psi,† f_s equals tension steel stress at the section considered in psi but not less than that corresponding to a moment equal to Vd, A_s equals area of tension reinforcement, and M equals bending moment at the section considered but not less than Vd.

(d) If the ultimate load on the member produces a unit shearing stress v, which exceeds the calculated ultimate diagonal tension strength v_c, web reinforcement must be provided, or the cross section must be increased.

(e) The provisions of Paragraphs (c) and (d) are subject to the following limitations depending on the length from the edge of bearing a of any portion of the member within which the shear diagram retains the same sign: ‡

(1) If $a > 2d$, these provisions shall not apply within the distance d at either end of the length a.

(2) If $2d \geqq a \geqq \frac{3}{4}\,d$, these provisions shall apply only at the section located in the middle of length a.

(3) If $a < \frac{3}{4}\,d$, these provisions shall not apply.

(4) However, in all cases, v_c shall not exceed $3.5\sqrt{f_c'}$.

(f) When lightweight aggregate concretes are used, the modifications to these design criteria discussed in Section 505 shall apply.

(g) To apply these recommendations in ultimate strength design, suitable safety provisions must be combined with these recommendations. Development of such safety provisions is related to all phases of ultimate strength design and is considered beyond the scope of this Committee's mission.

---

†For example, for $f_c' = 3600$ psi, $\sqrt{f_c'} = 60$ psi

‡For example, in a simply supported beam loaded with one concentrated load at midspan, the distance a is measured between the support and the concentrated load. For members subject to concentrated loads, the distance a is often referred to as the shear span.

505—Lightweight aggregate concretes

The lightweight aggregate industry in the United States is growing rapidly and occupies an important position in building construction. This material was first manufactured by expanding shale and clay into pellets. Other materials that have since appeared are expanded slates and slags. Structural quality concrete may also be made using a few of the natural lightweight aggregates such as some pumices, scorias, or lavas. Indication of the growing importance of structural lightweight concrete, and the importance of providing design recommendations for this material, is provided by a recent tabulation by the Expanded Shale, Clay and Slate Institute listing some 220 structures that have involved structural lightweight concrete, ranging from single story height to 35 stories. Such wide use indicates that establishment of design procedures for lightweight concrete is desirable.

Results of recent diagonal tension tests of lightweight aggregate concrete beams have been studied by Committee 326, notably the results of 47 tests at the Portland Cement Association laboratories and 27 tests at the University of Texas.[†] Such test data clearly indicate that the general principles of shear strength presented in this report for normal-weight aggregate apply equally well to lightweight aggregate concrete members.

Thus, an equation similar to Eq. (5-11) was considered as a design criterion, $V/bd\sqrt{f_c'}$ = (Constant 1) + (Constant 2) $pVd/M\sqrt{f_c'}$. The two constants of this equation were derived independently from nine groups of beam test data involving nine different lightweight aggregates. It was found that the constants varied widely. Hence, if a single pair of constants were chosen to provide a single design equation that could be used safely for all aggregates involved in the 74 tests considered, then such an equation would be extremely conservative for some aggregates. Furthermore, Committee 326 has been informed that recent experimentation[†] has indicated an excellent correlation between shear stress at diagonal tension cracking of lightweight aggregate concrete beams and corresponding splitting tests of 6 x 12-in. cylinder specimens. In this manner, a design equation similar to Eq. (5-11) was developed in which the two constants are related to the ratio F_{sp}, between the split cylinder strength and $\sqrt{f_c'}$. This approach has been approved by ACI Committee 213, Properties of Lightweight Aggregates and Lightweight Aggregate Concrete.[†]

In view of these circumstances, Committee 326 does not wish to make specific design recommendations for lightweight aggregate concrete members.

---

†Hanson, J. A., "Tensile Strength and Diagonal Tension Resistance of Structural Lightweight Concrete," ACI Journal, *Proceedings* V. 58, July 1961, pp. 1-39.

506—Test data

The test data considered in this chapter on members without web reinforcement are presented in condensed form in Tables 5-1 through 5-4 and Tables 5-6 through 5-17. For more detailed information, the reader is referred to the list of references listed in Section 507.

507—References involving test data

1. Moody, K. G.; Viest, I. M.; Elstner, R. C.; and Hognestad, E., "Shear Strength of Reinforced Concrete Beams—Parts 1 and 2," ACI JOURNAL, *Proceedings* V. 51: No. 4, Dec. 1954, pp. 317-332; No. 5, Jan. 1955, pp. 417-434. See also *RCRC Bulletin* No. 6.

2. Morrow, J., and Viest, I. M., "Shear Strength of Reinforced Concrete Frame Members without Web Reinforcement," ACI JOURNAL, *Proceedings* V. 53, No. 9, Mar. 1957, pp. 833-869. See also *RCRC Bulletin* No. 10.

3. Baldwin, Jr., J. W., and Viest, I. M., "Effect of Axial Compression on Shear Strength of Reinforced Concrete Frame Members," ACI JOURNAL, *Proceedings* V. 55, No. 5, Nov. 1958, pp. 635-654.

4. Baron, M. J., and Siess, C. P., "Effect of Axial Load on the Shear Strength of Reinforced Concrete Beams," *Structural Research Series* No. 121, Civil Engineering Studies, University of Illinois, June 1956. See also Reference 9.

5. Bernaert, S., and Siess, C. P., "Strength in Shear of Reinforced Concrete Beams under Uniform Load," *Structural Research Series* No. 120, Civil Engineering Studies, University of Illinois, June 1956. See also Reference 9.

6. Bower, J. E., and Viest, I. M., "Shear Strength of Restrained Concrete Beams without Web Reinforcement," ACI JOURNAL, *Proceedings* V. 57, No. 1, July 1960, pp. 73-98.

7. Rodriguez, J. J.; Bianchini, A. C.; Viest, I. M.; and Kesler, C. E., "Shear Strength of Two-Span Continuous Reinforced Concrete Beams," ACI JOURNAL, *Proceedings* V. 55, No. 10, Apr. 1959, pp. 1089-1130.

8. Elstner, R. C., and Hognestad, E., "Laboratory Investigation of Rigid Frame Failures," ACI JOURNAL, *Proceedings* V. 53, No. 7, Jan. 1957, pp. 637-668.

9. Diaz de Cossio, R., and Siess, C. P., "Behavior and Strength in Shear of Beams and Frames without Web Reinforcement," ACI JOURNAL, *Proceedings* V. 56, No. 8, Feb. 1960, pp. 695-735.

10. Al-Alusi, A. F., "Diagonal Tension Strength of Reinforced Concrete T-Beams with Varying Shear Span," ACI JOURNAL, *Proceedings*, V. 53, No. 11, May 1957, pp. 1067-1077.

11. Krefeld, W. J., and Thurston, C. W., "Progress Report on the Shear and Diagonal Tension Investigation of Reinforced Concrete Beams," Columbia University, 1958. Unpublished report made available to Committee 326.

12. Ferguson, P. M., "Some Implications of Recent Diagonal Tension Tests," ACI JOURNAL, *Proceedings* V. 53, No. 2, Aug. 1956, pp. 157-172.

13. Guralnick, S. A., "High Strength Deformed Steel Bars for Concrete Reinforcement," ACI JOURNAL, *Proceedings* V. 57, No. 3, Sept. 1960, pp. 241-282.

14. Hanson, J. A., "Shear Strength of Lightweight Reinforced Concrete Beams," ACI JOURNAL, *Proceedings* V. 55, No. 3, Sept. 1958, pp. 387-403.

15. Chang, T. S., and Kesler, C. E., "Static and Fatigue Strength in Shear of Beams with Tensile Reinforcement," ACI JOURNAL, *Proceedings* V. 54, No. 12, June 1958, pp. 1033-1057.

16. Watstein, D., and Mathey, R. G., "Strains in Beams Having Diagonal Cracks," ACI JOURNAL, *Proceedings* V. 55, No. 6, Dec. 1958, pp. 717-728.

17. Mathey, R. G., and Watstein, D., "Shear Tests of Beams with Tensile Reinforcement of Different Properties," National Bureau of Standards, 1958. Unpublished report made available to Committee 326.

18. Communication to Committee 326 containing data on six tests at the PCA laboratories, 1958.

CHAPTER 6—MEMBERS WITH WEB REINFORCEMENT

600—State of knowledge

Recent research work regarding shear and diagonal tension has been devoted primarily to members without web reinforcement. Although several investigations have included members with stirrups, tests of members with web reinforcement are few in number, and the variables affecting shear strength have not been thoroughly and systematically studied.

There are three major reasons why web reinforcement has not recently received prominent research attention. First, the lack of safety which was apparent in certain members without web reinforcement demanded immediate attention. Second, the addition of stirrups added one more variable to the complex problem of understanding shear and diagonal tension. Third, neither laboratory tests, nor performance of members in the field, indicated a lack of safety for members with web reinforcement designed in accordance with presently accepted design procedures.

However, the extensive recent studies of beams without web reinforcement have shed new light on our understanding of the nature of shear and diagonal tension, and much of this understanding is applicable also to members with web reinforcement. But, because only limited test data are available, it is not reasonable at this time to depart radically from current design procedures which are leading to satisfactory field service.

In some cases, current design procedures are undoubtedly unduly conservative. Further research work should be encouraged to clarify the function and action of web reinforcement, and also to determine the relationship of web reinforcement to other variables affecting shear strength.

601—Function of web reinforcement

Prior to the formation of the diagonal tension cracks, web reinforcement contributes very little to the shear resistance of a member. Consequently, the location of the initial diagonal cracks and the magnitude of the external loads causing the initial diagonal cracking are not affected to any appreciable extent by the presence of web reinforcement.

When a diagonal crack occurs, there must be a redistribution of internal forces at the cracked section. If a beam has no web reinforcement, the external shear previously resisted by the concrete web must be redistributed partly to the tensile reinforcement through dowel action but mainly to the compression zone of the concrete. If such redistribution is successful, the beam is capable of carrying increased load until the compression zone finally fails under combined stresses.

Redistribution must also take place in members with web reinforcement. It is important to note that the web reinforcement serves two primary functions after the formation of a diagonal crack. First, a part of the external shear is taken up by tension in the web reinforcement. Second, the web reinforcement restricts the growth of a diagonal crack, thus reducing the penetration of the diagonal crack into the compression zone. Consequently, web reinforcement not only carries part of the shear load; the web reinforcement also increases the ability of the compression zone to resist shear.

602—Mechanism of failure

The failure mechanism of members with web reinforcement has not been established clearly by laboratory tests. However, drawing from our knowledge of members without web reinforcement, three mechanisms of failure seem logical.

First, it is possible that the member will not accept redistribution when the diagonal crack forms. In this case, the web reinforcement will yield immediately and the compression zone will be destroyed immediately. The result will be a diagonal tension failure without warning and without increase in external load. Such failures might occur in long, slender beams (low pVd/M ratios) having very small amounts of web reinforcement. Failures of this type would be considered dangerous and undesirable.

A second mechanism of failure is the most common one. It probably extends over most of the range covered by the current design requirements. As the external load increases after diagonal cracking, the web reinforcement and the compression zone continue to carry shear until the stress in the web reinforcement has reached the yield point. Further increase in external shear must then be resisted by the compression zone alone. Failure occurs when the compression zone is destroyed by the combined compression and shear stresses. This failure is not sudden because the yielding web reinforcement allows the diagonal crack to widen, thus giving sufficient warning of incipient failure.

A third mechanism of failure is a variation of the second one. If a member is heavily reinforced with web steel, the compression zone may be destroyed by combined stresses before the web reinforcement has reached its yield point. Such failures may occur with less warning than for the second mechanism.

The mechanisms of failure indicate that both the web reinforcement and the compression zone contribute to the ultimate shear capacity. The problem may therefore appear to be quite simple, requiring only the addition of the contribution of the web reinforcement to the contribution of the compression zone of a member without web reinforcement. However, such superposition is not strictly applicable because the contribution of the web reinforcement and the contribution of the compression zone are actually interrelated and interdependent.

603—Review of research work prior to 1945

The ability of the compression zone to carry a portion of the shear was discussed by A. N. Talbot in 1909: "The tests and calculations go to show that under the maximum loads applied to the beams the stirrups are not stressed to an amount necessary to take the entire shear." Talbot's tests indicated that the stirrups carried two-thirds to three-fourths of the total shear. Thus he recommended, "The use of a fractional part of the total vertical shear like two-thirds for the application in formulas seems to be warranted." He also included the following statement: "The amount of web resistance which may be developed even with carefully arranged stirrups is limited, the limit depending upon the quality of the concrete." Talbot did not recommend a specific limiting stirrup amount.

In 1927, F. E. Richart[†] stated: "In test beams with web reinforcement the shearing stress found at the ultimate load is somewhat more than that accounted for by the stress in the web reinforcement, and it is frequently stated that a portion of the shear is carried by the concrete. Obviously the strength of the concrete web is largely destroyed when diagonal cracks have formed and the web reinforcement has been brought into action, but undoubtedly some portion of the vertical shear is carried by shearing stress over the uncracked compression area at the top of the beam."

Richart further stated: "Two types of common formulas are in use. In one of them it is assumed that one-third of the total shear will be carried by the concrete and two-thirds by the web reinforcement. The other involves the assumption that concrete will carry a constant portion of the working shearing unit stress and that the web reinforcement must carry the remainder."

---

†*Bulletin* No. 166, University of Illinois Engineering Experiment Station, June 1927, 103 pp.

The two types of formulas referred to by Richart may be expressed as

$$v = C_1 + rf_v \qquad (6\text{-}1)$$

and

$$v = C_2 rf_v \qquad (6\text{-}2)$$

Richart's tests indicated that C_1 varied from 90 to 200 psi, and he reasoned "Undoubtedly the value of C_1 depends on the quality of the concrete, as well as on the amounts of longitudinal and web reinforcement." Richart did not relate these variables to C_1 in mathematical terms, and he did not feel that C_2 should be a fixed proportion. He called attention to the formula

$$v = (0.005 + r) f_v \qquad (6\text{-}3)$$

which was developed by Slater, Lord and Zipprodt in connection with the Emergency Fleet Corporation tests of reinforced concrete beams during and shortly after World War I. Richart stated, "Instead of a fixed proportion, such as two-thirds of the shearing stress being considered as producing stress in the web reinforcement, this equation implies that the proportion shall vary as $(r/0.005 + r)$. . . . . . While the equation [referring to Eq. (6-3)] does not agree with the shape of the observed load-stress curves, it does give values that agree fairly well with observed stresses near ultimate load, and at lower loads seems to err on the side of safety."

Richart's last statement reveals a difference in philosophy as compared to this committee's concepts of ultimate shear strength. Due to the influence of the working stress concepts, earlier investigators were more concerned with web reinforcement *stress* than with ultimate shear *strength*. As a result their equations were intended primarily to express a relationship between applied load and web steel stress. An allowable shear stress could then be determined from the equations for a given allowable stress in the web reinforcement. Both Slater and Richart agreed that Eq. (6-3) related shear stress and web steel stress reasonably well at ultimate load. However, their findings do not infer that the equation can be used to predict ultimate strength.

It is of interest to examine the tests of Slater, Lord and Zipprodt. Their test specimens were very short, deep beams with relatively thin webs. The yield point of the web reinforcement exceeded 60,000 psi. Thirty specimens having only vertical stirrups as web reinforcement failed in diagonal tension, and the relationship between measured web steel stresses and measured shear stresses at failure by Eq. (6-3) was good. However, it should be pointed out that more than half of the beams failed in diagonal tension before the web reinforcement had reached its yield point. Therefore, though Eq. (6-3) is a good relationship between the two measured quantities, it cannot be used to predict ultimate shear strength unless the web steel stress f_v is a known value. It cannot always safely be assumed that the value of f_v at ultimate strength is equal to the yield point of the steel f_y. Even if the equation were used as a working stress equation with f_v fixed at 20,000 psi, a consistent safety factor against failure would not always result. Nearly all of Richart's test beams failed in flexure. They add little quantitative information about ultimate shearing strength.

In general, the Talbot tests, the Slater, Lord and Zipprodt tests, the Richart and Larson tests, and most other tests prior to 1945 have primarily historical significance as far as the development of ultimate shear strength formulas is concerned. Although the early investigations pointed out possible variables and possible trends for the shear strength of members with web reinforcement, there are several reasons why their test data should not be strongly considered in the present studies of ultimate shearing strength:

1. Since the reinforcing bars did not have deformations in accordance with present-day standards, the effects of poor bond and anchorage may have led to conclusions which are no longer valid.
2. Most of the test specimens were very short, deep beams with relatively high percentages of web reinforcement. Conclusions developed from such beams are not necessarily applicable to beams of normal practical proportions.
3. Most of the specimens failed by modes other than diagonal tension. In such cases it is, of course, impossible to determine ultimate shear strength.

604—Historical development of design

American design specifications have long been characterized by the assumption that part of the shear is carried by the concrete and that web reinforcement is designed to carry only the excess shear. European specifications have been more conservative, requiring web reinforcement designed to carry the total shear. American specifications have also been characterized by periodic changes in the design formulas for web reinforcement. These changes have affected not only the allowable stresses but also the considerations of the slope of the web reinforcement and the proportioning of shear between web reinforcement and concrete. Some of these changes were based on, or at least supported by, laboratory test data or mathematical analyses.

In 1910, the *NACU Standard* No. 4 recommended: "In calculating web reinforcement, the concrete shall be considered to carry 40 psi, the remainder to be provided for by means of web reinforcement in tension." It seems doubtful that there was any justification for this provision other than logical reasoning. On the other hand, the Joint Committee Report of 1909, following Talbot's recommendation which was based on laboratory tests, proposed web reinforcement to be designed to resist two-thirds of the total shear.

In 1921, the Joint Committee and the ACI specifications reached agreement on a procedure for the design of web reinforcement. Both accepted the procedure of designing shear reinforcement to carry the excess shear over that assumed to be carried by the concrete alone. This procedure has persisted from 1921 to the 1956 ACI Code.

Design criteria of the 1956 ACI Code are based on allowable stresses. If the unit shearing stress v is less than $0.03f_c'$ (but not greater than 90 psi) no web reinforcement is required. If the unit shearing stress exceeds this value, web reinforcement must be provided to resist the excess shear. Assuming a safety factor of 2.00 these provisions may be expressed mathematically on an ultimate strength basis as

$$v_u = \frac{V}{bjd} = 0.06\,f_c' + Krf_y \qquad (6\text{-}4)$$

where

v_u = ultimate shearing stress
f_c' = concrete cylinder strength
K = $(\sin\alpha + \cos\alpha)\sin\alpha$
α = angle between inclined web bars and axis of beam
r = A_v/ab
A_v = cross-sectional area of web reinforcement
a = spacing of web reinforcement, measured at right angles to their direction
b = width of beam
d = effective depth of beam
f_y = yield point of web steel

This procedure is subject to limitations and maximum values, some of which are as follows:

$0.06f_c'$ cannot exceed 180 psi.

v_u cannot exceed 0.16 f_c' and maximum 480 psi in beams having perpendicular stirrups only.

v_u cannot exceed 0.16 f_c' and maximum 480 psi in beams having parallel bent up bars only.

v_u cannot exceed $0.24f_c'$ and maximum 720 psi in beams with a combination of stirrups and bent-up bars (the latter bent up suitably to carry at least $0.08f_c'$).

r cannot be less than 0.0015.

a cannot exceed $\frac{1}{2}d\sin\alpha$.

Although the nature of this design procedure has not been changed significantly since 1921, the limitations and maximum values have been altered in nearly every revision of the ACI Code. Like the basic design principles, the limitations and maximum values have tended to be products of logical reasoning rather than systematic laboratory tests.

In general, the design criteria for web reinforcement of the 1956 ACI Code were not developed directly from laboratory tests. They are not in full accord with our basic understanding of the function of web reinforcement and the mechanisms of failure. However, it is a fact that the design criteria have withstood the test of time. No structural failures known to Committee 326 can be directly traced to possible inadequacies in these design provisions for members with web reinforcement.

605—Review of recent tests

While the laboratory investigations before 1945 were concerned primarily with relationships between the applied load and stresses in the web steel, after 1945 the emphasis was placed on the ultimate strength in shear.

The first of the recent tests was reported in 1945 by O. Moretto.† It was concerned primarily with the advantages of welded stirrups. The concrete strength f_c' ratio of tensile reinforcement p, ratio of web reinforcement r, and inclination of the stirrups α, were varied. The beam dimensions were constant. Moretto compared his tests with welded stirrups to the series of tests with loose stirrups reported earlier by Slater, Lord and Zipprodt. He concluded that "it seems reasonable to expect a beam made with welded stirrups to have on the average about 20 percent greater resistance to diagonal tension than one containing similar, but loose, stirrups." Neither Moretto's tests, nor the ones he used for comparison, contained deformed bars conforming to ASTM A 305.

Moretto proposed two empirical equations based on his test data. For the load at which the web reinforcement was stressed to the yield point, the shearing stress was given by

$$v_y = \frac{V_y}{bjd} = Krf_y + 0.04 f_c' + 5000\,p \qquad (6\text{-}5)$$

At ultimate load, the shearing stress was given by

$$v_u = \frac{V_u}{bjd} = Krf_y + 0.10 f_c' + 5000\,p \qquad (6\text{-}6)$$

These expressions have the form of Eq. (6-1) with C_1 expressed as a function of the quality of concrete and the ratio of longitudinal steel.

Moretto's investigation included two pairs of beams without web reinforcement. It is interesting to note that the ultimate shearing

†Reference 19 in Section 610.

strength v_u, of these two beams was much less than the contribution of the concrete, $v_u - Krf_y$, of similar beams with web reinforcement. This fact supports the earlier suggestion that web reinforcement increases the ability of the compression zone to resist shear.

Moretto studied systematically the effect of the inclination of stirrups on the shear strength. His results indicated strongly that the truss-analogy expression $K = (\sin \alpha + \cos \alpha) \sin \alpha$ is substantially in agreement with the test results.

In 1951, A. P. Clark[20] reported on an investigation in which the ratio of the beam depth to the shear span and the ratio of web reinforcement were the principal variables. The effects of varying the amount of tensile reinforcement and the strength of concrete were also studied. Clark's tests are noteworthy not only because he recognized the major variables affecting the ultimate strength in shear but also because his test specimens had dimensions and percentages of longitudinal and web reinforcement comparable to practical conditions. This investigation was a major step toward a modern understanding of the ultimate strength of reinforced concrete beams in shear.

Clark presented an empirical expression which included the four major variables: ratio of tensile reinforcement p, concrete strength f_c', ratio of depth to shear span d/a, and ratio of web reinforcement r. Clark stated that his formula was intended to point out the relative effects of the four variables rather than to be used for general design purposes.

The majority of experimental investigations of shear carried out in the United States in the 1950's were concerned primarily with beams without web reinforcement. The only exceptions were the experiments reported by Elstner, *et al*, by Rodriguez, *et al*, and by Guralnick. Guralnick reported tests of T-beams, both simple and with overhangs, reinforced with high-strength steel. Elstner reported tests of simply-supported beams with symmetrical overhangs, while Rodriguez reported tests of two-span continuous beams. The analyses of both series of tests departed from the earlier studies in that they were based on the moment at shear failure rather than on the ultimate shear.

Evaluations of the strength in shear on the basis of the ultimate moment were suggested by Laupa, Moody and Zwoyer. Independent studies by these three investigators regarding the behavior of reinforced concrete beams after formation of a diagonal tension crack led to the hypothesis that a shear-compression failure occurs when the moment at the critical section reaches a certain limiting value. Several further studies were concerned with evaluation of the limiting shear-moment. Morrow, Walther and others based their analyses on certain assumptions concerning the effect of diagonal tension cracking on the condition of compatibility, while Bresler and Guralnick focused their attention on the combined stress conditions in the compression zone. However, development of the shear-moment concept has not progressed far enough to have a direct influence on the recommendations contained in this report.

Early studies by Committee 326 revealed that only few test data were available on the strength in shear of beams with small amounts of web reinforcement. Two investigations, one at the University of California and one at Columbia University, were designed to provide such information. Preliminary results of these investigations were made available to Committee 326 for studies leading to this report.

606—Development of design criteria

The preceding discussion indicated the following characteristics of beams with web reinforcement:

1. Both the web reinforcement and the concrete compression zone contribute to the shear capacity of a member.

2. The web reinforcement not only helps to carry part of the total shear, but it also increases the ability of the compression zone to resist shear.

3. In members of normal proportions having normal amounts of tensile and web reinforcement, failure occurs by destruction of the compression zone after the web reinforcement crossing diagonal cracks has yielded.

4. In members having high percentages of web reinforcement, and particularly in short, deep members, the compression zone may be destroyed before the web reinforcement has yielded.

5. In members having low percentages of web reinforcement, and particularly in long, slender members, failure may occur with the formation of the first diagonal crack without any apparent strengthing from the web reinforcement.

6. The observed behavior of beams with web reinforcement is more complicated than is indicated by the truss analogy on which the current design procedures are based.

7. Neither laboratory tests nor field observations of performance in service have indicated a lack of safety resulting from current design methods, such as those of ACI 318-56.

Furthermore, an examination of the available test data leads to the following additional considerations:

8. The ultimate strength in shear of beams with web reinforcement is influenced by four major factors: (1) compressive strength of concrete f_c'; (2) ratio of tensile reinforcement p; (3) ratio re-

lating moment to shear M/V, at the section considered; and (4) ratio, distribution, and yield point of web reinforcement.

9. Test data obtained prior to 1945 are not fully applicable to the present design practices because the test specimens were abnormal in size and in percentages of web reinforcement, and the bond properties of reinforcement have undergone radical changes.

10. Tests of beams with web reinforcement made after 1945 are limited both in number and scope.

Three mathematical approaches were available to Committee 326 for the development of quantitative design criteria: the truss analogy, the additive equations advanced by Moretto and Clark, and the shear-moment hypothesis. Although promising, the shear-moment approach has not yet been developed sufficiently to provide a basis for a reliable design procedure. The numerical coefficients in the additive equations were found to be constant only within relatively narrow ranges of variables. On the other hand, the truss analogy equation

$$v = \frac{V}{bd} = Krf_v + v_{con} \qquad \text{(6-7)}$$

has been found to provide a simple and safe design procedure even though its oversimplified representation of the actual beam action must be compensated by several restrictions.

An adaptation of Eq. (6-7) for ultimate strength design requires consideration of the stress in the stirrups f_v, and of the contribution of the concrete compression zone v_{con}. The terms K and r are given by the geometry of the beam and its reinforcement.

In beams of normal proportions, the web reinforcement usually yields before the beam fails in shear. Thus, the yield point of the web reinforcement f_y should be substituted for the steel stress f_v. However, it has been shown that some conditions lead to failures in shear before yielding of the web reinforcement. Limitations must therefore be placed on the values of f_y, on the total shear or on both.

In the 1956 ACI Building Code, it is assumed that the shear carried by concrete v_{con} is proportional to the cylinder strength f_c'. European codes involve the assumption that v_{con} is zero. A. N. Talbot proposed that web reinforcement be designed to resist 2/3 or 3/4 of the total shear. Recent test data have shown that v_{con} is neither proportional to the concrete strength nor a fixed fraction of the total shear. The ratio v_{con}/f_c' was found to decrease with increasing concrete strength and the ratio v_{con}/v_u was found to vary between 0 and 0.8. Although the European value, $v_{con} = 0$, probably would guarantee safety in all members, designs would tend to be unduly conservative even for average members encountered in practice. The 2/3-recommendation would be more realistic for the average case. On the other hand, it would also be unduly conservative for small amounts of web reinforcement, and it would tend to be unsafe for very large amounts of web steel.

Studies of test data have indicated that the shear carried by the concrete compression zone v_{con} is a function of f_c', p, M/V, and also Krf_y. They have indicated further that expressing v_{con} as equal to the diagonal tension strength v_c approximated properly the effects of f_c', p, and M/V, and also resulted in a better correlation with test data than any of the methods discussed in the preceding paragraph. Therefore, the following ultimate strength equation was selected by Committee 326 as the basis for design criteria

$$v_u = \frac{V_u}{bd} = Krf_y + v_c \qquad (6\text{-}8)$$

where

$$v_c = \frac{V_c}{bd} = 1.9\sqrt{f_c'} + 2500\,\frac{pVd}{M} \leq 3.5\sqrt{f_c'} \qquad (6\text{-}9)$$

Eq. (6-9) is the formula for the diagonal tension strength of members without web reinforcement derived in Chapter 5 of this report as Eq. (5-11). It may also be expressed by conservative approximations as Eq. (5-13) or (5-14).† Thus Eq. (6-8) indicates that the contribution of web reinforcement Krf_y is related to the difference between the ultimate shear v_u and the shear causing diagonal tension cracking v_c. It is noteworthy that investigations of the shear strength of prestressed concrete beams, in progress at the University of Illinois and at the PCA laboratories at the time this report was written, indicated an analogous finding.

When Eq. (6-8) is compared with the available test data, it becomes apparent that four limitations are needed. If the percentage of web reinforcement is very high, the compression zone fails before the web reinforcement has been developed fully. Therefore, there must be an upper limit for Krf_y or v_u. Fortunately test data are abundant in this region. The data indicate that $v_u = 8\sqrt{f_c'}$ represents a safe limit. The limit is based primarily on tests of relatively short, deep beams with rectangular cross sections and with vertical stirrups. A few tests of T-beams and beams with diagonal stirrups indicate that such beams can develop a higher ultimate strength v_u. For T-beams and beams with diagonal stirrups $v_u = 10\sqrt{f_c'}$ represents a safe limit.

If the percentage of web reinforcement is very low, the web reinforcement may not be sufficient to permit redistribution of internal forces when the diagonal crack forms; a lower limit on Krf_y is needed.

†Eq. (5-13) and (5-14) are close approximations to Eq. (5-11) only when V in the term pVd/M corresponds to the cracking load. For members with web reinforcement, V corresponds to an ultimate load greater than the cracking load, so that Eq. (5-13) and (5-14) are conservative approximations.

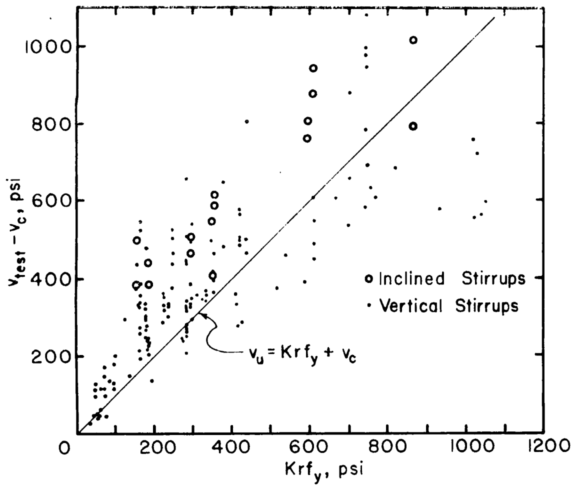


Fig. 6-1—Strength of beams with web reinforcement

The 1956 ACI Code design procedure requires a minimum r of 0.0015 which is equivalent to $Krf_y = 60$ psi for $f_y = 40{,}000$ psi. Tests recently completed at Columbia University and the University of California indicate the $Krf_y = 60$ psi is indeed a safe lower limit below which web reinforcement should not be considered effective.

Recent tests at Cornell University included beams which contained stirrups having very high yield points. The test data indicate that the stirrups were not capable of developing their yield strength even though the Krf_y values were moderate. Therefore, a limit of 60,000 psi is proposed on the value of f_y in Eq. (6-8).

The results of 166 beam tests are given in Tables 6-1 and 6-2 of Section 609. Ultimate shearing stresses calculated from Eq. (6-8) and observing the four limitations given above are compared to the test values. In general, the calculated values are conservative.

Comparisons of test findings with Eq. (6-8) are summarized in Fig. 6-1. It is seen that $(v_{test} - v_c)$ values fall below the line corresponding to $v_u = Krf_y + v_c$ only for beams with vertical stirrups and relatively high values of Krf_y. Most of these cases are covered safely by the limit of $v_u = 8\sqrt{f_c'}$.

The cumulative distribution of V_{test}/V_{calc} is plotted in Fig. 6-2. It is clearly seen that the design equation, Eq. (6-8), with the proposed limitations is safe. The observed shear strength V_{test}, was less than the calculated value V_{calc} for only about five percent of the tests.

607—Bent bars and anchorage

Early investigators used reinforcing bars that are considered smooth by modern standards. Bond and anchorage, both of which are associated with shear, were found to be major problems. Many failures of test beams reported by early investigators as shear failures may have

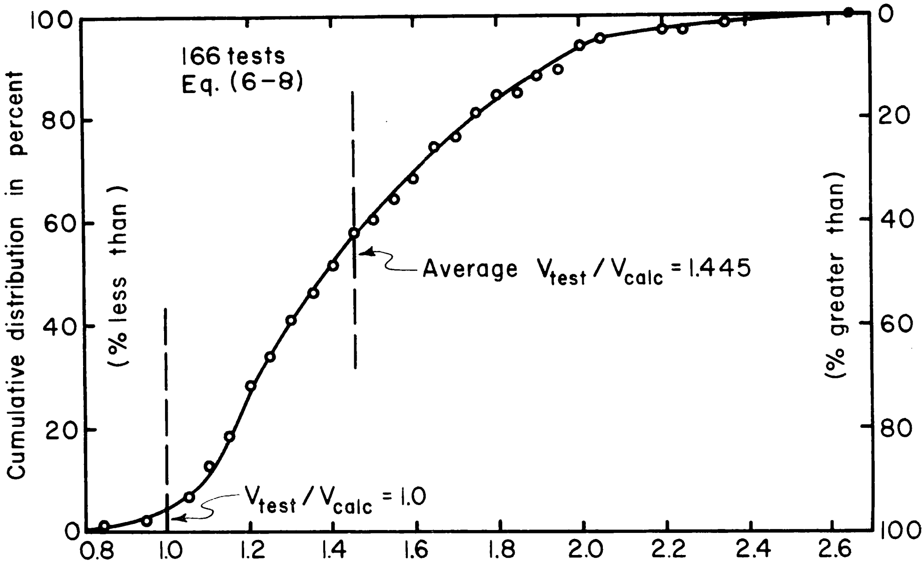


Fig. 6-2—Cumulative distribution V_{test}/V_{calc}

been anchorage or bond failures. The investigators recommend rigid attachment of stirrups, especially when inclined, to the longitudinal reinforcement and encouraged the use of bent bars less susceptible to bond and anchorage distress.

The adoption of the truly deformed reinforcing bar meeting ASTM A 305 has altered the problem of bond and anchorage. A few investigators have reported splitting failures, but in general bond and anchorage are no longer primary sources of weakness for members with web reinforcement.

No recent test data are available for beams with bent bars or for beams with combined bent bars and stirrups. However, a few tests of beams with inclined stirrups tied rather than welded to the longitudinal reinforcement have shown high shear resistance without bond or anchorage distress.

The 1956 ACI Building Code requires that inclined stirrups be welded to the longitudinal reinforcement. Recent experiences indicate that all reinforcing bars cannot be welded satisfactorily. The strength of high-carbon bars may be impaired by improper or careless welding. Since anchorage is no longer a major problem, the need for welding of stirrups to longitudinal reinforcement no longer exists. Therefore, proper welding of stirrups may be permitted, but welding should not be required.

In present design practice, longitudinal reinforcement may be terminated when it is no longer needed to resist stress. The 1956 ACI Code

TABLE 6-1 — TESTS OF SIMPLE BEAMS WITH WEB REINFORCEMENT

Beam	p	$\frac{M}{Vd}$	Calc v_o	Krf_y	Calc v_u	Test v_u	$\frac{\text{Test}}{\text{Calc}}$
Moretto[19]							
1V¼	0.0400	0.88	206	154	360	582	1.617
2V¼	0.0400	0.88	242	154	396	674	1.702
1I ¼	0.0400	0.88	216	185	401	603	1.504‡
2I ¼	0.0400	0.88	240	185	425	660	1.553‡
1D¼	0.0400	0.88	206	154	360	592	1.644‡
2D¼	0.0400	0.88	206	154	360	702	1.950‡
1V⅜	0.0400	0.88	195	297	446†	734	1.646
2V⅜	0.0400	0.88	225	297	514†	718	1.397
1I ⅜	0.0400	0.88	209	356	565	795	1.407‡
2I ⅜	0.0400	0.88	227	356	583	840	1.441‡
1D⅜	0.0400	0.88	187	297	484	648	1.339‡
2D⅜	0.0400	0.88	212	297	509	718	1.411‡
1aV¼	0.0188	0.82	170	129	299	465	1.555
1aV⅜	0.0188	0.82	167	322	462†	514	1.112
Clark[20]							
A1-1	0.0310	1.34	171	182	353	406	1.150
A1-2	0.0310	1.34	169	182	351	383	1.091
A1-3	0.0310	1.34	169	182	351	407	1.160
A1-4	0.0310	1.34	172	182	354	446	1.260
B1-1	0.0310	0.98	189	178	367	510	1.390
B1-2	0.0310	0.98	194	178	372	469	1.261
B1-3	0.0310	0.98	190	178	368	521	1.416
B1-4	0.0310	0.98	189	178	367	489	1.332
B1-5	0.0310	0.98	192	178	379	441	1.164
B2-1	0.0310	0.98	189	350	464†	551	1.188
B2-2	0.0310	0.98	196	350	494†	589	1.192
B2-3	0.0310	0.98	193	350	481†	612	1.272
B6-1	0.0310	0.98	214	178	392	693	1.768

Beam	p	$\frac{M}{Vd}$	Calc v_o	Krf_y	Calc v_u	Test v_u	$\frac{\text{Test}}{\text{Calc}}$
Clark[20]							
C1-1	0.0207	0.78	182	163	345	508	1.472
C1-2	0.0207	0.78	183	163	346	568	1.642
C1-3	0.0207	0.78	178	163	341	449	1.317
C1-4	0.0207	0.78	189	163	352	522	1.483
C2-1	0.0207	0.78	177	331	469†	530	1.130
C2-2	0.0207	0.78	180	331	482†	550	1.141
C2-4	0.0207	0.78	185	331	500†	527	1.054
C3-1	0.0207	0.78	152	163	315	409	1.298
C3-2	0.0207	0.78	151	163	314	366	1.166
C3-3	0.0207	0.78	151	163	314	344	1.096
C4-1	0.0310	0.78	208	163	371	565	1.523
C6-2	0.0310	0.78	253	163	416	775	1.863
C6-3	0.0310	0.78	252	163	415	795	1.916
C6-4	0.0310	0.78	257	163	420	783	1.864
D1-1	0.0163	0.59	186	221	407	550	1.351
D1-3	0.0163	0.59	182	221	403	469	1.164
D2-1	0.0163	0.59	181	293	472†	530	1.123
D2-2	0.0163	0.59	185	293	478	570	1.192
D3-1	0.0244	0.59	224	442	511†	722	1.412
D4-1	0.0163	0.59	179	585	463	571	1.233
D1-6	0.0342	0.94	211	221	432	529	1.224
D1-7	0.0342	0.94	212	221	433	542	1.252
D1-8	0.0342	0.94	212	221	433	563	1.300
E1-1	0.0342	1.02	210	350	530†	671	1.266
D2-6	0.0342	1.43	184	293	477	510	1.069
D2-7	0.0342	1.43	182	293	475	476	1.002
D2-8	0.0342	1.43	177	293	470	510	1.085

TABLE 6-1 (cont.) — TESTS OF SIMPLE BEAMS WITH WEB REINFORCEMENT

Beam	p	$\frac{M}{Vd}$	Calc v_c	Krf_y	Calc v_u	Test v_u	$\frac{\text{Test}}{\text{Calc}}$
Clark[20]							
D4-1	0.0342	1.43	180	235	415	510	1.229
D4-2	0.0342	1.43	176	235	411	476	1.158
D4-3	0.0342	1.43	167	235	402	500	1.244
D5-1	0.0342	1.43	180	178	358	443	1.237
D5-2	0.0342	1.43	183	178	361	476	1.319
D5-3	0.0342	1.43	179	178	357	476	1.333
Guralnick (T-Beams)[13]							
IA2R	0.0075	1.99	106	280	256§	446	1.742
IB2R	0.0043	1.97	99	280	249§	370	1.486
IC1R	0.0138	2.06	150	740	450†	1125	2.500
IC2R	0.0133	1.95	150	280	300§	594	1.980
ID2R	0.0075	1.99	142	280	292§	579	1.983
Moody, Viest, Elstner, and Hognestad[1]							
III30	0.0425	0.76	212	246	458	738	1.611
III31	0.0425	0.76	200	418	456†	779	1.708
Bresler and Scordelis[21]							
B-2	0.0243	3.91	126	73	199	275	1.381
C-2	0.0366	3.93	135	100	235	335	1.423
A-2	0.0228	3.93	127	50	127	253	1.992
A-1	0.0180	2.92	128	49	128	238	1.859
B-1	0.0243	2.95	134	73	207	305	1.473
C-1	0.0180	2.95	140	98	238	316	1.331
A-3	0.0273	5.91	147	49	147	238	1.619
B-3	0.0306	5.95	155	73	229	247	1.080
C-3	0.0363	5.98	151	98	249	279	1.121

Beam	p	$\frac{M}{Vd}$	Calc v_c	Krf_y	Calc v_u	Test v_u	$\frac{\text{Test}}{\text{Calc}}$
Thurston[22]							
26-1	0.0222	3.00	164	78	252	259	1.072
29a-1	0.0222	3.00	161	53	161	200	1.242
29b-1	0.0222	3.00	159	53	159	200	1.258
213.5-1	0.0222	3.00	161	35	161	185	1.149
29-2	0.0222	3.00	158	62	220	272	1.236
213.5-2	0.0222	3.00	158	42	158	202	1.278
318-1	0.0222	3.00	164	93	257	276	1.073
321-1	0.0222	3.00	161	79	240	204	0.850
313.5-2	0.0222	3.00	162	86	248	294	1.183
318-2	0.0222	3.00	161	64	225	221	0.981
321-2	0.0222	3.00	160	55	160	209	1.306
218-2	0.0222	3.00	159	31	159	205	1.289
39-3	0.0222	3.00	168	97	265	300	1.132
313.5-3	0.0222	3.00	168	64	232	267	1.151
318-3	0.0222	3.00	169	48	169	219	1.296
321-3	0.0222	3.00	169	42	169	176	1.041
II-5	0.0222	3.00	164	72	236§	277	1.172
II-6	0.0222	3.00	161	60	161§	206	1.280
II-7	0.0222	3.00	161	37	161§	186	1.155

Average $v_{test}/v_{calc} = 1.368$ Coefficient of variation = 0.205

Note: v_u(calc) are based on design recommendations of Section 608 and Eq. (6-8) and (6-9).

† $v = 8\sqrt{f_c'}$ or $10\sqrt{f_c'}$.

‡ Beams had diagonal stirrups.

§ f_y = 60,000 psi in calculations.

TABLE 6-2 — TESTS OF RESTRAINED BEAMS WITH WEB REINFORCEMENT

Beam	p	$\frac{M}{Vd}$	Calc v_c	Krf_y	Calc v_u	Test v_u	$\frac{\text{Test}}{\text{Calc}}$	Beam	p	$\frac{M}{Vd}$	Calc v_c	Krf_y	Calc v_u	Test v_u	$\frac{\text{Test}}{\text{Calc}}$
Rodriguez, Bianchini, Viest and Kesler[7]								Elstner, Moody, Viest, and Hognestad[23]							
C6A1	0.0267	0.70	195	665	445†	799	1.796	I10a	0.0476	0.68	194	246	440	661	1.502
C3A2	0.0267	1.05	170	308	448†	523	1.167	I10b	0.0476	0.68	186	246	424†	560	1.321
E2A1	0.0267	1.23	168	186	354	400	1.130	I11a	0.0476	0.68	209	418	478†	776	1.623
E2A2	0.0267	1.23	153	186	339	360	1.062	I11b	0.0476	0.68	197	418	451†	704	1.561
E2A3	0.0267	1.23	156	188	344	381	1.108	I12a	0.0476	0.68	221	606	506†	770	1.522
C2A1	0.0267	1.23	163	191	354	296	0.836	I12b	0.0476	0.68	199	606	454†	649	1.430
C2A2	0.0267	1.23	160	187	347	362	1.043	Is	0.0476	0.68	206	819	471†	893	1.896
E6H1	0.0267	0.70	207	1026	474†	927	1.956	It	0.0476	0.68	213	1019	486†	972	2.000
E6H2	0.0267	0.70	212	768	485†	818	1.686	I13a	0.0476	0.68	206	350	556	750	1.349‡
C6H1	0.0267	0.70	207	1050	472†	805	1.706	I13b	0.0476	0.68	187	350	535†	589	1.101‡
C6H2	0.0267	0.70	204	752	467†	836	1.790	I14a	0.0476	0.68	207	594	592†	1013	1.711‡
E3H1	0.0267	1.05	184	533	505†	642	1.271	I14b	0.0476	0.68	210	594	600†	973	1.622‡
E3H2	0.0267	1.05	178	408	482†	536	1.112	I15a	0.0476	0.68	202	861	577†	993	1.739‡
C3H1	0.0267	1.05	167	512	433†	544	1.256	I15b	0.0476	0.68	213	861	609†	1232	2.023‡
C3H2	0.0267	1.05	167	410	432†	500	1.157	I16a	0.0476	0.68	210	606	601†	1089	1.812‡
E2H1	0.0267	1.23	159	415	440†	438	0.995§	I16b	0.0476	0.68	199	606	569†	1143	2.009‡
E2H2	0.0267	1.23	157	274	431	395	0.916§	Iu	0.0286	0.68	214	280	489†	648	1.325
C2H1	0.0267	1.23	161	421	449†	448	0.998§	Iv	0.0286	0.68	209	377	478†	690	1.444
C2H2	0.0267	1.23	168	270	438	415	0.947§	Iw	0.0286	0.68	227	435	519†	725	1.397
E6I2	0.0267	0.70	201	1019	460†	756	1.643	Ix	0.0286	0.68	217	700	495†	876	1.770
C6I2	0.0267	0.70	191	1037	436†	756	1.734	Iy	0.0476	0.68	242	280	522	893	1.711
B2A1	0.0195	1.23	146	138	284	294	1.035	Iz	0.0476	0.68	244	377	557†	890	1.598
B2H1	0.0196	1.23	149	281	430	417	0.970	Iα	0.0476	0.68	249	435	570†	1056	1.853
B2H2	0.0196	1.23	154	182	336	394	1.173	Iβ	0.0476	0.68	251	700	573†	1130	1.972

TABLE 6-2 (cont.) — TESTS OF RESTRAINED BEAMS WITH WEB REINFORCEMENT

Beam	p	$\frac{M}{Vd}$	Calc v_c	Krf_y	Calc v_u	Test v_u	$\frac{\text{test}}{\text{calc}}$	Beam	p	$\frac{M}{Vd}$	Calc v_c	Krf_y	Calc v_u	Test v_u	$\frac{\text{Test}}{\text{Calc}}$
Elstner, Moody, Viest, and Hognestad[23]								Guralnick[13] (T-beams)							
II21a	0.0272	0.38	209	246	455	711	1.563	IA1	0.0077	0.77	133	740	399†	919	2.303
II21b	0.0272	0.38	211	246	457	649	1.420	IA2	0.0075	0.75	122	280	272††	450	1.654
II22a	0.0272	0.38	192	418	470†	688	1.464	IB2	0.0043	0.80	108	280	258††	358	1.387
II22b	0.0272	0.38	182	418	416†	666	1.601	IC1	0.0138	0.77	178	740	534†	1169	2.189
II23a	0.0272	0.38	199	606	454†	688	1.515	IC2	0.0133	0.74	178	280	328††	687	2.094
II23b	0.0272	0.38	197	606	450†	803	1.784	ID1	0.0077	0.76	158	740	474†	1238	2.612
IIe	0.0272	0.38	205	748	468†	895	1.912	ID2	0.0075	0.75	158	280	308††	691	2.243
IIf	0.0272	0.38	202	931	462†	780	1.688	IIA1	0.0077	2.05	117	740	351†	701	1.997
IVm	0.0476	1.06	187	280	428†	498	1.164	IIA2	0.0075	1.99	106	280	256††	364	1.422
IVn	0.0476	1.06	213	435	487†	675	1.386	IIB2	0.0043	1.97	99	280	249††	300	1.205
IVo	0.0476	1.06	205	700	468†	744	1.590	IIC1	0.0138	2.06	158	740	474†	1103	2.327
								IIC2	0.0133	1.95	158	280	308††	482	1.565

Average $v_{test}/v_{calc} = 1.547$ Coefficient of variation $= 0.220$

Note: v_u(calc) are based on design recommendations of Section 608 and Eq. (6-8) and (6-9).

† $v_u = 8\sqrt{f_c'}$ or $10\sqrt{f_c'}$.

‡ Beams had diagonal stirrups.

§ Beams failed by splitting.

†† $f_y = 60{,}000$ psi in calculations.

requires that such bars shall be extended at least 12 bar diameters beyond the theoretical cutoff point to insure proper anchorage. Previous discussion has indicated that redistribution of internal forces must occur in a member following the formation of a diagonal crack. This phenomenon has been well established by laboratory tests. In the process of redistribution, the tensile force carried by the longitudinal reinforcement is so affected that the tensile force calculated at a given section by flexural considerations may become redistributed to a section a distance d beyond the given section. Therefore, it is recommended to extend both positive and negative reinforcement a distance equal to the effective depth d, plus an anchorage length beyond the point where it is no longer needed to resist stress.

608—Recommendations for design

Tests of beams with web reinforcement are relatively few in number, and the variables affecting shear strength have not been studied systematically. Therefore, it is not possible at this time to develop new design procedures which agree fully with the observed actions of web reinforcement and with the observed mechanisms of failure. Since the current procedures have been used safely for many years, it seems desirable not to depart radically from these procedures. It is necessary, however, to modify the present procedures so that they become congruous with the proposed new procedure for the design of beams without web reinforcement and so that they become applicable to ultimate strength design. The following ultimate strength design procedures are recommended for members with web reinforcement:

(a) Web reinforcement shall be defined as

1. Simple U- or multiple stirrups properly anchored at both ends.

2. The center three-fourths of any bent-up longitudinal bar properly bent and anchored at both ends.

3. Any special arrangements of bar or wire which give the same factor of safety as similar specimens reinforced in conformity with the provisions of this design procedure as determined by comparative tests to destruction.

(b) Web reinforcement shall make an angle of 30 deg or more with the axis of the longitudinal reinforcement.

(c) The ultimate shearing stress v_u on a concrete section of a web having web reinforcement shall be computed by the formula

$$v_u = v_c + Krf_y$$

where

v_c = shearing stress permitted on the cross section neglecting web reinforcement (see Sections 504 and 606)
K = efficiency factor for web reinforcement $(\sin \alpha + \cos \alpha) \sin \alpha$
r = ratio of web reinforcement $= A_v / b\, s \sin \alpha$
f_y = yield point of the web reinforcement
α = angle of the web reinforcement with the axis of the longitudinal reinforcement.

(d) The preceding formula is restricted by the following limitations:

1. For rectangular sections or I-sections with vertical stirrups only, v_u shall not exceed $8\sqrt{f_c'}$.
2. For T-sections, for rectangular sections or I-sections having angle α less than 70 deg, or for rectangular sections or I-sections having combined vertical stirrups and bent longitudinal bars where at least 2/3 or the total shear is carried by bent-up longitudinal bars, v_u shall not exceed $10\sqrt{f_c'}$.
3. Krf_y shall not be less than 60 psi.
4. f_y shall not exceed 60,000 psi.

(e) Positive and negative longitudinal reinforcement terminated in a tensile zone of concrete shall be extended a distance equal to the effective depth of the member plus an anchorage length beyond the point at which it is no longer needed to resist flexural stress.

(f) Web reinforcement shall be continued a distance equal to the effective depth of the member beyond the point theoretically required.

(g) Development of safety provisions is beyond the scope of this Committee's mission.

609—Test data

The test data considered in this chapter on members with web reinforcement are presented in condensed form in Tables 6-1 and 6-2.

610—References†

19. Moretto, O., "An Investigation of the Strength of Welded Stirrups in Reinforced Concrete Beams," ACI JOURNAL, *Proceedings* V. 42, No. 2, Nov. 1945, pp. 141-162.

20. Clark, A. P., "Diagonal Tension in Reinforced Concrete Beams," ACI JOURNAL, *Proceedings* V. 48, No. 2, Oct. 1951, pp. 145-156.

21. Communication to Committee 326 containing data on nine recent tests at University of California by B. Bresler and A. C. Scordelis.

22. Communication to Committee 326 containing data on 19 recent tests at Columbia University by C. W. Thurston.

23. Elstner, R. C.; Moody, K. G.; Viest, I. M.; and Hognestad, E., "Shear Strength of Reinforced Concrete Beams, Part 3—Tests of Restrained Beams with Web Reinforcement," ACI JOURNAL, *Proceedings* V. 51, No. 6, Feb. 1955, pp. 525-539.

†References used in Chapter 6 refer to listing in Chapter 5, Section 507, with the above additions:

CHAPTER 7—MEMBERS SUBJECTED TO SHEAR, BENDING AND AXIAL LOAD

700—Introduction

Because diagonal tension is a principal stress problem, the addition of an axial load to a flexural member affects the diagonal tension strength of the member. Recent research investigations have included members having axial tension and axial compression in addition to bending and shear. The tests indicate that axial compression will increase diagonal tension strength while axial tension will cause a decrease.

The following analysis was developed primarily by I. M. Viest and is based on the concept of principal stresses. It closely follows the reasoning behind the development of the ultimate strength design equation for beams without web reinforcement which was presented in Chapter 5.

701—Development of design equation

The ultimate strength design equation for members without web reinforcement was developed from considerations of the principal stress at the point of diagonal tension cracking

$$f_t(\max) = \tfrac{1}{2} f_t + \sqrt{(\tfrac{1}{2} f_t)^2 + v^2} \qquad (7\text{-}1)$$

For members without axial load, the normal tensile stress f_t, was assumed proportional to the steel stress computed by the cracked section theory, and the shearing stress v, was assumed proportional to the average shearing stress

$$f_t = \text{constant} \times \frac{f_s}{n} = F_1 \frac{M}{npbd^2}$$

$$v = F_2 \frac{V}{bd}$$

If axial load is applied to the member in addition to the transverse loads, the expression for the normal tensile stress f_t must be modified to account for the axial load. The moment M and axial load N may be replaced by an eccentric force N acting at distance e from the tension reinforcement

$$e = \frac{M}{N} + d - \frac{h}{2}$$

Using the conventional straight-line theory, the equilibrium of moments about the centroid of the internal compressive force in the concrete gives the following expression for the steel stress

$$f_s = \frac{N(e - jd)}{A_s jd}$$

But

$$N(e - jd) = M + Nd\left(1 - \frac{h}{2d} - j\right)$$

and

$$A_s jd = jpbd^2$$

Thus the equation for the steel stress may be written as

$$f_s = \frac{M + \beta Nd}{jpbd^2}$$

where $\beta = 1 - (h/2d) - j$, which is usually negative in sign. The force N is taken as positive for compression, and negative for tension.

Using this expression for steel stress, and neglecting variations in j, the normal stress in concrete may be approximated as

$$f_t = F_1 \frac{M + \beta Nd}{npbd^2}$$

Hence, the effect of axial load is obtained by correcting the moment M, caused by external load, by adding βNd.

The ultimate strength design equation for diagonal tension cracking of members without axial load was given by Eq. (5-11) in Chapter 5 as

$$v_c = \frac{V}{bd\sqrt{f_c'}} = 1.9 + 2500 \frac{Vpd}{M\sqrt{f_c'}} \qquad \text{(7-2)}$$

The same equation is applicable when axial load is present if the equation is modified by correcting the moment M, for the effect of axial load

$$v_c = \frac{V}{bd\sqrt{f_c'}} = 1.9 + 2500 \frac{Vpd}{(M + \beta Nd)\sqrt{f_c'}} \qquad \text{(7-3)}$$

It has been shown in Chapter 5 that Eq. (7-2) for members without axial load is subject to the upper limit

$$\frac{V}{bd\sqrt{f_c'}} \leq 3.5$$

In essence, this limit implies that diagonal tension strength is independent of the normal tensile stress when the normal tensile stress due to moment is very small. Assuming that $M \approx 0$ and that the portion of the member considered is uncracked, the principal stress equation [Eq. (7-1)] becomes

$$f_t(\max) = -\frac{1}{2}\frac{N}{bd} + \sqrt{\left(-\frac{1}{2}\frac{N}{bd}\right)^2 + \left(F_2\frac{V}{bd}\right)^2}$$

This expression may be rearranged as

$$\frac{V}{bd} = \frac{f_t(\max)}{F_2}\sqrt{1 + \frac{N}{f_t(\max)\,bd}}$$

Assuming that the principal tensile strength is equal to

$$f_t(\max) = 7.5\sqrt{f_c'}$$

and noting that for beams with no axial load

$$\frac{f_t(\max)}{F_2} = 3.5\sqrt{f_c'}$$

the equation for nominal shearing stress may be expressed as

$$\frac{V}{bd\sqrt{f_c'}} = 3.5\sqrt{1 + \frac{0.133\,N}{\sqrt{f_c'}\,bd}}$$

For practical range of concrete strengths, the quantity $0.133/\sqrt{f_c'}$ varies between 0.0015 and 0.003. Therefore, it appears satisfactory to replace the term $0.133/\sqrt{f_c'}$ by a constant value of 0.002. Accordingly, the upper limit for the diagonal tension strength of members subjected to axial force may be expressed as

$$\frac{V}{bd\sqrt{f_c'}} = 3.5\sqrt{1 + 0.002\,\frac{N}{bd}} \qquad \text{(7-4)}$$

Thus the ultimate strength in diagonal tension of members subjected to combined shear, bending and axial load may be computed from

$$v_c = \frac{V}{bd\sqrt{f_c'}} = 1.9 + 2500\,\frac{Vpd}{(M + \beta Nd)\sqrt{f_c'}}$$

$$\leq 3.5\sqrt{1 + 0.002\,\frac{N}{bd}} \qquad \text{(7-5)}$$

where

V = external shear at diagonal tension cracking on the section considered, lb
b = width of the section, in.
d = effective depth of the section, in.
p = ratio of tension reinforcement A_s/bd
M = moment of external forces with respect to the centroidal axis of the gross uncracked section, in.-lb, taken always as a positive quantity
β = $1 - (h/2d) - j$
h = total depth of section
N = axial load, lb; for axial compression N should be taken as a positive quantity, for axial tension as a negative quantity
f_c' = cylinder strength of concrete, psi

In the derivation of Eq. (7-3) the assumption was made that the steel stress is below the yield point of the steel. If the steel stress reaches the yield point, flexural considerations automatically govern the design.

As for members without axial load, in shear spans longer than $2d$ the diagonal tension strength need not be investigated within the distance d at both ends of the shear span; and in shear spans shorter or equal to $2d$, the diagonal tension strength must be investigated only at the section half-way between the ends of the shear spans.

702—Comparison with test data

In Table 7-1, Eq. (7-5) is compared with the results of tests reported in References 2, 3, and 4. Tests of three types of members are included: symmetrical knee frames for which the ratio N/V was constant and equal to 1.0; unsymmetrical knee frames with the ratio N/V constant for any one frame but varying from specimen to specimen; and simple stub beams subjected to a constant axial load and a varying transverse load. Actual β-values were used in Eq. (7-5).

Reference 9 contains test data on diagonal tension cracking of 20 rigid frames without web reinforcement. The frames were simple bents subjected to vertical uniformly distributed load W and to horizontal loads N applied at the foot of both columns. The ratio N/W was kept constant for any one specimen. The computed diagonal tension cracking loads are compared with the test data in Table 7-2. To simplify the calculations, $\beta = -0.5$ was used in Eq. (7-5).

Eleven restrained beams were tested at the Portland Cement Association laboratories with several concentrated transverse loads and an axial tension. With one exception, the total transverse load P was applied first, and the axial load was then increased from zero until the specimen failed. The test results for three specimens are reported in Reference 8; the data for the remaining specimens have not been published. The results are compared with calculated values in Table 7-3. Although the failures had the appearance of diagonal tension failures, the computed failure loads for all 11 specimens were governed by *yielding* of the longitudinal reinforcement.†

Table 7-4 contains the mean values of the ratios of test/calc and coefficients of variations for the tests with axial compression. The tests of restrained beams with axial tension are not included because their strength was not governed by Eq. (7-5).

703—Recommendations for design

The diagonal tension strength of sections in flexural members is influenced by the presence of axial loads. Axial compression will increase while axial tension will decrease the diagonal tension strength of a section.

†Equation $N_{calc} = 2(M/d - 0.9\ A_s f_y)$ in every case gave a lower failure load than Eq. (7-5).

TABLE 7-1 — KNEE FRAMES AND STUB BEAMS

Specimen	$\frac{2500\,Vpd}{(M+\beta Nd)\sqrt{f_c'}}$	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{calc}}$
Reference 2				
F21B2	2.94	4.05	4.45	1.10
B2R	2.39	4.21	4.95	1.13
B4	1.81	3.71	4.89	1.3[illegible]
B4R	1.87	3.77	4.68	1.24
C4	1.66	3.56	4.58	1.29
C4R	1.57	3.47	4.77	1.33
D4	1.20	3.10	4.21	1.36
E4	0.91	2.81	4.04	1.44
F4	0.79	2.69	4.28	1.59
A6	1.79	3.69	4.36	1.18
B6	1.51	3.41	4.53	1.33
F38B2	0.83	2.73	3.51	1.29
E2	0.21	2.12	2.11	1.00
B4	0.55	2.45	3.26	1.33
D4	0.41	2.31	3.38	1.46
E4	0.19	2.19	2.73	1.25
A6	0.55	2.55	3.62	1.42
B6	0.40	2.40	2.78	1.16
F55B2	0.46	2.36	2.94	1.25
A4	0.33	2.23	3.09	1.39
B4	0.30	2.20	2.40	1.09
D4	0.26	2.16	2.55	1.18
E4	0.17	2.07	2.09	1.01
A6	0.40	2.30	3.29	1.43
B6	0.23	2.13	2.29	1.08

Specimen	$\frac{2500\,Vpd}{(M+\beta Nd)\sqrt{f_c'}}$	$\frac{V_{calc}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{bd\sqrt{f_c'}}$	$\frac{V_{test}}{V_{calc}}$
Reference 2				
F70B2	0.30	2.20	2.62	1.19
A4	0.24	2.14	2.87	1.34
A6	0.32	2.22	2.98	1.34
F84B4	0.21	2.53	2.53	1.20
Reference 3				
2F28	0.88	2.78	3.08	1.11
3F28	0.98	2.88	4.03	1.40
4F28	1.03	2.93	4.54	1.55
6F28	1.34	3.24	4.32	1.33
9F28	1.69	3.59	3.54	0.99
12F28	2.44	4.34	3.20	1.20
18F28	3.52	5.42	5.79	1.07
12F21	6.59	6.02	7.35	1.22
12F38	0.82	2.72	3.97	1.46
Reference 4				
B1	0.78	2.68	4.04	1.51
B2	0.29	2.19	3.03	1.38
B3	0.18	2.08	2.78	1.34
B4	0.13	2.03	2.46	1.21
B11	2.15	4.05	5.21	1.29
B12	0.87	2.77	3.93	1.42
B13	0.53	2.43	3.47	1.43
B14	0.38	2.28	3.04	1.33
B15	0.30	2.20	2.74	1.25

Average test/calc = 1.282 Coefficient of variation = 11.2 percent

Note: For references, see Section 507.

TABLE 7-2 — RIGID FRAMES†

Specimen	$\frac{N}{W}$	W_{calc}, kips	W_{test}, kips	$\frac{W_{test}}{W_{calc}}$	Location of DT crack‡	
					Computed	Observed
F- 1	0.165	53.3	61.6	1.16	P	P
2	0.165	57.1	66.0	1.16	P	P
12	0.165	47.7	72.0	1.51	N	N
15	0.165	43.3	46.2	1.07	N	P
16	0.165	46.4	54.6	1.18	P	P
21	0.165	43.5	60.0	1.38	P	P
F- 5	0.335	59.1	90.3	1.53	N	N
6	0.335	58.1	88.0	1.51	N	P
13	0.335	52.3	52.0	0.99	P	P
F-18	0.165	42.0	59.0	1.40	P	P
19	0.335	44.0	54.6	1.24	P	P
F- 3	0.133	35.1	57.2	1.63	P	P
4	0.133	39.4	51.7	1.31	P	P
22	0.133	39.6	46.2	1.17	P	P
24	0.133	40.5	46.2	1.14	P	P
F-17	0.265	35.5	50.4	1.42	P	P
F- 7	0.335	46.1	60.5	1.31	N	N
8	0.335	52.0	66.0	1.27	P	N
14	0.335	39.2	51.0	1.30	P	P
20	0.335	43.5	54.6	1.26	P	P

†See Reference 9, Section 507.

‡ P = DT crack starting at bottom reinforcement and extending up toward midspan.

N = DT crack starting at the top reinforcement and extending down toward the knee corner.

TABLE 7-3 — GIRDERS WITH AXIAL TENSION†

Specimen	Location of DT crack		Load at failure				$\frac{N_{test}}{N_{calc}}$
	Observed	Predicted‡	P_{test}	N_{test}	P_{calc}	N_{calc}	
8	4	4	30.9	−28.4	17.0	−17.1	1.66
9	4	4	17.7	−19.3	††	−16.5	1.17
10	4	4	21.7	−15.2	††	−13.6	1.18
14	4	4	31.6	− 9.4	††	− 6.3	1.49
15	4	4	33.4	− 5.3	††	− 4.9	1.08
16	4	4	31.2	−13.5	††	− 6.6	2.06
17	4	4	21.7	−19.8	††	−13.6	1.46
19	4	4	19.2	−13.5	††	−12.6	1.07
20	4	4	29.7	− 7.5	††	− 3.2	2.34
22§	4	4	26.8	−21.7	††	−14.2	1.53
23§	4	4	42.1	−11.6	††	− 5.3	2.19

†Data from Reference 8, Section 507, and unpublished test data made available to Committee 326 by the PCA laboratories.

‡Region 4 is located between the point of contraflexure and the nearest load toward midspan.

§T-beams; all other specimens had rectangular cross section.

††Transverse load equal to P_{test} (constant during the test).

TABLE 7-4—MEANS AND COEFFICIENTS OF VARIATION OF AXIAL COMPRESSION TESTS

Type of specimen	No. of specimens	$\frac{\text{Test}}{\text{Calc}}$	Coefficient of variation, percent
Knee frames and stub beams	47	1.28	11.2
Rigid frames	20	1.30	12.6
All specimens	67	1.29	11.6

Eq. (7-5) was found to predict safely the diagonal tension cracking loads of 67 laboratory tests with combined axial and flexural loads. However, a form more convenient for design purposes may be obtained by substituting $v_c = V/bd$ and $f_s = (M + \beta Nd)/jpbd^2$ into Eq. (7-5), assuming $j = 0.875$† and solving for v_c. The resulting expression

$$v_c = 1.9\sqrt{f_c'}\,\frac{f_s}{f_s - 2850 \text{ psi}} \qquad \text{(7-6)}$$

is the same as Eq. (5-13) for members without axial load.

A rearrangement of Eq. (7-5) similar to that used for Eq. (5-14) leads to

$$v_c = \frac{1.9\sqrt{f_c'}}{1 - (2500 \text{ psi})\dfrac{A_s d}{M - N\left(\dfrac{h}{2} - d + jd\right)}} \qquad \text{(7-7)}$$

Eq. (7-7) may be approximated by substituting M' for M in Eq. (5-14), where $j = 7/8$, so that

$$M' = M - N\left(\frac{4h - d}{8}\right) \qquad \text{(7-8)}$$

No test data are available on the strength in shear of members with web reinforcement subjected to an axial force in addition to shear and bending. Under these circumstances, the Committee considers it advisable to follow the design procedure recommended in Section 608 for beams without axial force, except that the contribution of the concrete compression zone to the shear strength should be computed as recommended in this chapter.

Committee 326 recommends the following ultimate strength design procedure for members subjected to combined shear, bending and axial force:

(a) The diagonal tension strength shall be determined by the procedure given in Section 504 except that the formulas for ulti-

†For small axial loads this assumption is a close approximation of the actual j-values. For large axial loads, actual j-values are smaller than 0.875; consequently the resulting equation will give a conservative estimate of v_c.

mate diagonal tension strength shall be those given in Paragraph (b) below.

(b) The ultimate diagonal tension strength of a concrete section of an unreinforced web subjected to an axial load in addition to shear and bending shall be computed by Eq. (7-5), (7-6), (7-7), or (7-8), except that in all cases v_c shall not exceed

$$3.5\sqrt{f_c' \ (1 + 0.002\, N/A_g)},$$

in which A_g is the gross area of the uncracked section in square inches.

(c) The web reinforcement shall be designed by the procedures given in Section 608 except that the shearing stress v_c carried by the concrete shall be computed as recommended in Paragraph (b) of this section.†

(d) Development of safety provisions is beyond the scope of this Committee's mission.

704—Test data

The test data considered in this chapter on members with axial load are presented in condensed form in Tables 7-1 through 7-4.

---

†In the design of web reinforcement, Eq. (7-6), (7-7), and (7-8) give conservative approximations of Eq. (7-5).

Title No. 59-9

Report of ACI-ASCE Committee 326

Shear and Diagonal Tension

The ACI-ASCE Committee 326 report is being published as follows:

Part 1—General Principles, Chapters 1-4: January 1962, pp. 1-30

Part 2—Beams and Frames, Chapters 5-7: February 1962, pp. 277-334

Part 3—Slabs and Footings, Chapter 8: March 1962, pp. 353-396

This report was submitted to letter ballot of the committee which consists of 15 members and was approved without a dissenting vote.

Received by the Institute August 14, 1961. Title No. 59-9 is a part of copyrighted Journal of the American Concrete Institute, Proceedings V. 59, No. 2, Feb. 1962. Reprints of the complete report are available at $2 each ($1 to ACI members).

Discussion of this report (three parts) should reach ACI headquarters in triplicate by June 1, 1962, for publication in the September 1962 JOURNAL.

Presents a review of scientific knowledge, engineering practice, and construction experiences regarding shear and diagonal tension in reinforced concrete beams, frames, slabs, and footings. Recommendations for new design procedures are substantiated by extensive test data.

Chapters I through 4 deal with background and general principles. Chapters 5 through 7 present the development of new design methods for reinforced concrete members without and with web reinforcement, and for members without and with axial load acting in combination with bending and shear. Chapter 8 deals with slabs and footings including the effect of holes and transfer of moments from columns to slabs.

Esfuerzo Cortante y Tensión Diagonal

Se revisan los conocimientos científicos, ingeniería práctica y experiencias en construcciones relativas al esfuerzo cortante y tensión diagonal en vigas, armaduras (marcos rigidos), losas y cimientos de hormigón armado. Se recomiendan

nuevos procedimientos de diseño comprobados por los datos obtenidos por medio de ensayos extensivos.

Los capítulos 1 al 4 tratan de los antecedentes y principios generales. Los capítulos 5 al 7 presentan el desarrollo de nuevos métodos de diseño para miembros de hormigón armado con y sin refuerzo del alma y para miembros con y sin carga axial combinada con la flexión y el esfuerzo cortante. El capítulo 8 trata de las losas y cimientos incluyendo el efecto causado por agujeros y el traspaso de momentos de las columnas a las losas.

L'Effort Tranchant et la Contrainte Principale

On présente une revue de l'art scientifique, de la pratique du génie et des expériences dans la construction relatives aux efforts tranchants et à la contrainte principale dans les poutres, les portiques, des dalles et les semelles de fondations en béton armé. Les recommendations pour les nouvelles procédés de calcul sont justifées à l'aide de résultats nombreaux d'essais.

Les chapitres 1 à 4 concernent les bases et les principes généraux. Les chapitres 5 à 7 présentent l'évolution de nouvelles methodes de calcul d'éléments en béton armé avec et sans armatures de cisaillement et d'éléments soumis à la flexion simple et composée avec les efforts tranchants. Le chapitre 8 concerne les dalles et les semelles de fondations y compris l'influence des trous et la transmission de moments de flexion des colonnes aux dalles.

Schub- und Hauptzugspannungen

Es wird eine Übersicht gegeben über wissenschaftliche Erkenntnisse, technische Praxis und Bauerfahrungen bezüglich Schubsicherung in Stahlbetonträgern, Rahmen, Platten und Säulenfussplatten. Empfehlunger für neue Berechnungsverfahren werden durch umfassende Versuchsunterlagen erhärtet.

Kapitel 1 bis 4 behandeln die Vorgeschichte und die allgemeinen Grundsätze. Kapitel 5 bis 7 enthalten die Entwicklung von neuen Berechnungsmethoden für Stahlbetonteile ohne und mit Schubbewehrung und für Bauglieder unter Biegung und Schub ohne und mit gleichzeitiger Längskraft. Kapitel 8 behandelt Platten und Säulenfussplatten einschliesslich der Wirkung von Aussparungen und die Übertragung von Momenten von Säulen zu Platten.

ACI-ASCE Committee 326, Shear and Diagonal Tension, was formed in 1950 to develop methods for designing reinforced concrete members to resist shear and diagonal tension consistent with ultimate strength design. Several investigations and test programs were initiated, sponsored and conducted by numerous organizations, including Committee 326, the Reinforced Concrete Research Council, many universities (especially the University of Illinois), the American Iron and Steel Institute, and the Portland Cement Association. Progress reports of Committee work were presented at the ACI 55th annual convention, February 1959, and the 56th convention, March 1960. This three-part report is the culmination of a 10-year study.

CHAPTER 8 — SLABS AND FOOTINGS†

800—Introduction

It is not economically feasible to test numerous slab floor systems to determine the shear strength of the column to slab connection or to determine the effect of a concentrated load. However, the shear strength problem in the vicinity of a concentrated load or reaction may be considered a localized condition involving only a portion of the slab around the loaded area. Consequently, laboratory specimens have generally been square slabs with column stubs at the center of the slab and with supports at the four edges. This specimen approximates the portion of a flat plate floor system which extends from the column out to the region of contraflexure of the slab.

This type of test specimen is similar to a column footing, except for the support conditions. In many respects reinforced concrete column footings are acting as a portion of an inverted flat slab. Therefore, the shear problem of a concentrated load on a slab, of a column to slab connection in a flat slab or flat plate, and of a column connection in a footing, are in reality similar problems. They will be considered simultaneously in this report unless otherwise noted.

801—Review of research

While the first theoretical and experimental investigations of the behavior of reinforced concrete beams was reported during the last years of the nineteenth century,[1] the results of the first extensive study of the shear strength of slabs was published in 1913 when *Talbot*[2] presented his well-known investigation of reinforced concrete footings. Altogether 114 wall footings and 83 column footings were tested to failure. Of the latter, approximately 20 specimens failed in shear. Talbot computed shear stress by the formula

$$v = \frac{V}{4(r + 2d)jd} \qquad \text{(8-1)}$$

in which

v = shear stress
V = shear force
r = side dimension of square column
d = effective depth of slab
jd = internal moment arm of slab

It was found that relatively high values of shear strength were obtained when large percentages of tensile reinforcement were used in the slabs.

† This chapter was developed largely on the basis of the work by Johannes Moe reported in Reference 20 of Section 811.

Talbot's study of reinforced concrete footings was reflected in the design practice of many countries throughout the world.

In 1915, *Graf and Bach*[3] reported a large number of slab tests which were undertaken mainly to study flexural strength. Most of the slabs were loaded simultaneously at eight or more points. A few slabs, loaded at the center only, failed in shear; a portion of the slab having the form of a truncated cone was pushed out underneath the load.

The shear strength of slabs loaded by concentrated loads near supports was studied by *Graf*[4] in 1933. A series of shear tests was carried out on each of three slabs. The results showed clearly that the shear capacity decreased as the load was moved away from the supports. Graf computed shear stress by the formula

$$v = \frac{V}{4\,rt} \qquad (8\text{-}2)$$

where t is the total depth of the slab.

Graf found that the shear strength increased with concrete strength, but at a lower rate than compressive and tensile strength. He suggested that flexural cracking may have some influence on shear strength.

Another series of tests was reported by *Graf*[5] in 1938. Eight very thick slabs, six of which had shear reinforcement, were tested. The slabs, which were approximately 5½-ft square, were supported along all four edges. The thicknesses varied between 12 and 20 in.

Richart and Kluge,[6] in 1939, reported an investigation of reinforced concrete slabs subjected to concentrated loads, which was undertaken to provide information on the design of highway bridge deck slabs. As part of the investigation, seven shear tests were made on two long rectangular slabs with an effective depth of 5.5 in. and a short span of 6 ft 8 in. The loaded areas were circular, 6 and 2 in. in diameter. This report also includes data on shear tests of eighteen 5-ft square slabs simply supported on two edges only. As shown in a recent study by *Elstner and Hognestad*,[7] many of these slabs seem to have failed in flexure, but some shear failures are also recorded. The main purpose of the latter series was to determine the effect of size and shape of the load-bearing area.

While the ultimate shear stresses, as computed by Eq. (8-1), were approximately 0.08 times the cylinder compressive strength f_c', for the long slabs, only about 60 percent of that stress was obtained in the latter series. The authors concluded that the stresses obtained by using Eq. (8-1) are only nominal and arbitrarily chosen, and that an increase in flexural strength of the square slabs would probably have increased shear strength.

Forsell and Holmberg,[8] in 1946, reported on extensive shear tests of slabs carried out from 1926 to 1928. One series of tests was made on circular slabs of sand-cement mortar. The slabs which had diameters of

11.5 to 17 in. were reinforced by spiral reinforcement and the edges were strengthened by heavy steel rings. The thickness of the slabs varied from 1½ to 3¼ in. Shear stresses, assumed to be parabolically distributed across the depth of the slabs, were computed by the formula

$$v = \frac{1.5\,V}{bt} \qquad \text{(8-3)}$$

The critical section determining the perimeter b was taken at a distance of $t/2$ from the edges of the loaded area. The diameter of the supporting ring, which varied between 6 and 12 in., was in many cases so small compared to the thickness of the slab that the base of the frustum of the cone forming at failure extended all the way to the supports. Considerable increase in shear strength due to the influence of the supports is probable under such conditions.

In a second series, twenty-two 4 ft square reinforced concrete slabs were tested. The slabs had a thickness of approximately 4¾ in. All slabs but one were supported along all four edges with the corners tied down. The main variables were concrete strength and manner of loading.

One reinforced concrete slab continuous over three spans was also tested by Forsell and Holmberg. Eleven shear tests were made at different points of this slab. The influence of bending moment on shear strength was clearly demonstrated.

At the University of Illinois, *Newmark, Siess, et al.,*[9,10,11] as part of an extensive investigation of highway bridges, reported a number of shear tests of mortar bridge deck slabs 1¾ in. thick. Their first report includes tests of 15 models of simple span, right angle I-beam bridges. All slabs failed in shear at loads P_{test}, considerably higher than those causing first yielding of the tensile reinforcement P_{yield}. The average value of P_{test}/P_{yield} was as high as 1.8, thus indicating that the ultimate flexural capacities of the slabs probably were almost exhausted at the time of shear failure. The authors stated that the loads at shear failure to a certain degree seemed to be dependent on the same factors as the loads at first yielding.

The second Illinois report describes tests on five skew simple-span I-beam bridges with angles of skew of 30 to 60 deg. Final failure in all cases took place by shear at loads higher than for the right-angle bridges.

In the third Illinois report, tests on three two-span continuous I-beam bridges are reported. Higher shear strengths were found at midspan than over the supports, probably as a result of the direct axial forces in the deck slabs, which acted as flanges in the composite I-beam sections.

Richart,[12] in 1948, reported the results of an extensive investigation on reinforced concrete footings. In all, 24 wall footings and 140 column footings were tested to failure. Of the latter, 128 were 7-ft square, the rest being of rectangular shapes, 6 x 9 ft, and 6 x 10 ft. The major variables were: amount, strength, bond characteristics, and end anchorage of tensile reinforcement; concrete strength; and effective depth of the footings. Apparently, 106 of the column footings failed in shear.

Richart concluded that shear stress rather than bond stress may frequently be a critical feature in the design of a footing. The shear stress at failure, calculated by Eq. (8-1) at a distance d from the faces of the columns generally varied from less than 0.05 f_c' to 0.09 f_c'. The value of v/f_c' increased consistently as the effective depth of the footing decreased. When high tensile stresses in the flexural reinforcement and extensive cracking of the footings were present, an early failure in shear evidently took place.

Hahn and Chefdeville,[13] in 1951, presented a short description of shear tests on three slabs. The specimens were approximately 7-ft square with thicknesses of 9 to 10 in., and were loaded through a 20 in. square column stub. Two of the slabs were strengthened in shear by heavy structural steel crossheads.

A few shear tests on deck slabs of I-beam bridges were reported by *Thomas and Short*[14] in 1952. One of these deck slabs was prestressed.

Hognestad,[15] in 1953, published the results of a re-evaluation of the shear failures of footings which were reported by Richart. Hognestad recognized the effect of superimposed flexure on ultimate shear strength, and he introduced the ratio $\phi = V_{test}/V_{flex}$ as one of the parameters in his statistical study of the test results. In this ratio, V_{test} is the observed shear force at shear failure, V_{flex} is the shear force at flexural ultimate strength as computed by yield-line theory. He also suggested that shear stress be computed at zero distance around the loaded area because this seemed to give the best measure of the shear strength.

The following ultimate shear strength equation was found to apply within the range of variables covered by Richart's tests

$$v = \frac{V}{\frac{7}{8}\,bd} = \left(0.035 + \frac{0.07}{\phi_o} \right) f_c' + 130 \text{ psi} \quad \text{(8-4)}$$

where $\phi_o = V/V_{flex}$.

Hognestad indicated that Eq. (8-4) was found to apply for values of r/d, the ratio of column width to effective slab thickness, between 0.88 and 2.63, but is probably unsafe for values of f_c' below 1800 psi. A new design method based on Eq. (8-4) was suggested and compared to the provisions of the 1951 ACI Building Code.

Elstner and Hognestad,[7] also in 1953, reported shear tests of twenty-four 6-ft square and 6 in. thick reinforced concrete slabs. The majority of these slabs were supported along all four edges. The results of these tests, as well as those reported by Forsell and Holmberg,[8] and Richart and Kluge[6] were analyzed and compared to the strengths predicted by Eq. (8-4).

Keefe,[16] in 1954, investigated the effectiveness of a special type of shear reinforcement known as "shearheads." Two pairs of octagonally shaped slabs were tested, one of each pair being furnished with a "shear-head," while the other had no shear reinforcement. The slabs with shearhead had an ultimate shear capacity approximately 40 percent higher than those without.

Elstner and Hognestad,[17] in 1956, reported tests of thirty-eight 6 ft square slabs which were loaded through column stubs in the center, and, with a few exceptions, were supported along all four edges. Twenty-four of these tests were also reported earlier.[7] The major variables were: concrete strength, percentage of flexural tension and compression reinforcement, percentage of shear reinforcement, and size of column. The effect of concentration of the flexural reinforcement was also explored. No effects on the ultimate shear strength were found due to variation in concentration of the tension reinforcement under the column or the amount of compression reinforcement. The new test results indicated that Eq. (8-4) gave unsafe values of the ultimate shear strength for high concrete strengths (4500-7300 psi). By statistical analysis of all of the slabs, except those with shear reinforcement, the following equation was found to be in better agreement with the test results

$$v = \frac{V}{\frac{7}{8}\, bd} = 333 \text{ psi} + 0.046\, f_c'/\phi_o \qquad \text{(8-5)}$$

For the slabs with shear reinforcement, the following equation was suggested on the basis of new tests and the tests reported by Graf[4,5]

$$v = 333 \text{ psi} + 0.046\, f_c'/\phi_o + (q_u - 0.050) f_c' \qquad \text{(8-6)}$$

where

$$q_u = \frac{A_v f_y \sin \alpha}{\frac{7}{8}\, bd\, f_c'} \qquad \text{(8-7)}$$

and

A_v = area of shear reinforcement
f_y = yield point of shear reinforcement
α = inclination of shear reinforcement

Eq. (8-6) indicates that the shear reinforcement is not fully effective.

Whitney,[18] in 1957, presented an ultimate strength theory for shear which is radically different from the earlier approaches to this problem. Whitney based his study on previously reported test results[12,17] of slabs and footings but excluded a number of tests which he believed involved bond failure. The excluded slabs had large amounts of tension reinforcement consisting of closely spaced bars. For the remaining slabs, Whitney assumed that the shear strength is primarily a function of the ultimate resisting moment m of the slab per unit width inside the "pyramid of rupture," i.e., the frustum of a cone or pyramid with surfaces sloping out in all directions from the column at an angle of 45 deg.

Whitney proposed the following ultimate shear strength equation

$$v = 100 \text{ psi} + 0.75 \frac{m}{d^2} \frac{d}{l_s} \qquad (8\text{-}8)$$

where v is computed at a distance of $d/2$ from the surfaces of the loaded areas, and l_s is the "shear span" which in the case of a slab supported along the edges is taken as the distance between the support and the nearest edge of the loaded area. In the case of a footing with uniform distribution of the reaction, l_s is taken as half of the distance between the edge of the footing and the face of the column.

Since the test results of specimens with relatively high flexural strengths were omitted in the study leading to Eq. (8-8), it can only apply in cases of nearly balanced design, i.e., when ϕ_o is close to unity.

According to Eq. (8-8) the shear strength of a slab can be effectively raised by increasing the amount of flexural reinforcement inside the pyramid of rupture. This also should apply if the increase of reinforcement through the pyramid of rupture is accomplished by shifting tensile reinforcement from outside the "pyramid" to the inside. Hence, Whitney's formula agrees with the 1956 ACI Building Code in that a concentration of the flexural reinforcement in a narrow band across the column is assumed to increase shear strength.

In computing the ultimate shear force V in footings, Whitney subtracted the support reaction inside a distance of $d/2$ from the faces of the column. Most other investigations have subtracted the total support reaction on the base of the "pyramid of rupture."

In a recent study, *Scordelis, Lin, and May*[19] investigated the shear strength of prestressed lift slabs by testing fifteen 6 ft square slab specimens. The slabs were supported along all four edges; twelve of the slabs were prestressed with unbonded cables. Major variables were concrete strength, amount of prestressing, size of steel collars, thickness of slabs, and amount of collar recess.

The ultimate shear strengths were compared to the predictions of Eq. (8-5) and (8-8), and reasonably good agreement was found in both cases. To apply these equations, it was necessary to evaluate the ultimate flexural moment capacities of the slabs. For this purpose it was necessary to make some assumptions regarding the values of the steel stress at ultimate moment and the effect of recesses in the slabs on the ultimate moment capacity. The assumptions made appear reasonable, but they could be questionable.

Moe[20] in 1961, reported tests of forty-three 6 ft square slabs which were similar to the test specimens of Elstner and Hognestad. Moe's principal variables were: effect of openings near the face of the column, effect of concentration of tensile reinforcement in narrow bands across the column, effect of column size, effect of eccentricity in applied load, and effectiveness of special types of shear reinforcement. He also included a statistical study of 260 slabs and footings tested by earlier investigators.

Moe's work represents a thorough, complete and up-to-date study of the shear strength of slabs based on practically all available data. Major portions of this chapter were taken almost verbatim from Moe's report, a draft copy of which was made available to Committee 326 by the Portland Cement Association.

Some of the more important conclusions arrived at by Moe are:

1. The critical section governing the ultimate shear strength of slabs and footings should be measured along the perimeter of the loaded area.
2. The shear strengths of slabs and footings depend on flexural strength.
3. The triaxial state of stress in the compression zone at the critical section influences the shear strength of that section considerably as described in Section 402 of this report.
4. The shear strength of the concrete is highest when the column size is small compared to the slab thickness.
5. The ultimate shear strength of slabs and footings is predicted with good accuracy by the formula

$$v_u = \frac{V_u}{bd} = \left[15\left(1 - 0.075\frac{r}{d} \right) - 5.25\,\phi_o \right] \sqrt{f_c'}$$

where

b = perimeter of the loaded area
d = effective depth of slab
r = side length of square loaded area
V_u = ultimate shear force
V_{flex} = ultimate shear force if flexural failure had occurred
ϕ_o = V_u/V_{flex}

For footings

$$V_u = \left[1 - \left(\frac{r + d}{a}\right)^2 \right] P_u$$

where

P_u = total load on loaded area
a = side length of square footing slab

6. Inclined cracks develop in the slabs at loads as low as 50 percent of the ultimate.
7. Loads 50 percent above the inclined cracking load, sustained for 3 months, did not affect the ultimate shear strength.

8. The effect of openings adjacent to the column may be accounted for by introducing the net value for the perimeter b into the equation in Conclusion 5.

9. Concentration of flexural reinforcement in narrow bands across the column did not increase the shear strength. However, such concentration increased the flexural rigidity of the test slabs, and also increased the load at which yielding began in the tension reinforcement.

10. Some increase in shear strength can be obtained by shear reinforcement. However, the anchorage of such reinforcement in the compression zone seems to be problematical, therefore the use of shear reinforcement in thin slabs was not recommended.

11. In cases of moment transfer between square columns and slabs, test results indicate it is safe to assume that one-third of the moment is transferred through vertical shear stresses at the perimeter of the loaded area distributed in proportion to the distance from the centroidal axis of the loaded area. Maximum shear stress due to the combined action of vertical load and moment should not exceed the value expressed by the equation in Conclusion 5.

802—Review of design specifications

The limited nature of knowledge previously available regarding the mechanism of failure in shear of slabs under concentrated loads is clearly reflected in the standard specifications of various countries. Quite different rules are applied to determine the critical shear or inclined tensile stresses, and the allowable stresses vary considerably.

In the *United States* the first standard specifications,[21] which were prepared by a Joint Committee appointed by a number of professional societies, stipulated an allowable shear stress in pure shear equal to 0.06 f_c'. The shear stress should be computed by $v = V/bt$, where the critical section was taken along the perimeter b of the loaded area, and t is the total slab thickness. In the revised version of 1917[22] it is also required that diagonal tension requirements be met, but no rules were given for determining diagonal tension stress.

The *ACI Standard of 1916*[23] allowed a stress in pure shear equal to 0.075 f_c' computed along the periphery of the loaded area.

In the *ACI Standard of 1920*[24] a clear distinction was made between the following two possible types of shear failure:

(a) A pure shear failure controlled by the allowable shear stress computed at zero distance from the periphery, and stipulated at 0.10 f_c'.

(b) A so-called "diagonal tension failure" controlled by shear stress computed by the formula $v = V/bjd$ at a distance of $d/2$ from the periphery, and limited to 0.035 f_c'.

In the report of the *Joint Committee of 1924*[25] it was specified that shear stress should be computed at a distance of (t — 1½ in.) from the periphery of the loaded area, and the allowable shear stress was given by

$$v = 0.02\,f_c'\,(1 + n) \leqq 0.03\,f_c' \qquad (8\text{-}9)$$

where n is the ratio of the area of the reinforcing steel crossing directly through the loaded area (column or column capital) to the total area of tensile reinforcement.

The Report of 1924 was also adopted by the American Concrete Institute as a standard, and only minor changes have been made later with respect to shear and diagonal tension in slabs and footings.

The *1956 ACI Building Code (318-56)*[26] allowed the following shear stresses, computed on a critical section at a distance d beyond the periphery of the loaded area:

$0.03f_c' \leqq$ 100 psi if more than 50 percent of the tensile reinforcement required for bending passes through the periphery
$0.025f_c' \leqq$ 85 psi when only 25 percent of the tensile reinforcement passes through the periphery
$0.03f_c' \leqq$ 75 psi for footings

In *Germany* a completely different approach to the design problem of shear in slabs has been practiced. In determining shear as well as flexural stresses, slab strips of certain widths are assumed. The widths given for shear computations are different from those in moment, and the widths also vary with the position of load on the slab. The *German Specification DIN 1045 of 1943*[27] gives the following formulas for the effective slab strip width in shear

$$b_1 = r + 2\,s \quad \text{and} \quad b_2 = \tfrac{1}{3}\left(l + \frac{r + 2\,s}{2}\right) \qquad (8\text{-}10)$$

where s is the thickness of a load-distributing layer on the top of the slab and l is the span of the slab.

The larger of the values b_1 and b_2 can be used. In the case of a load close to one of the supported edges, b shall be taken as $r + 5t$.

In some countries, such as Norway, a combination of the American and the German practice has been used. The *Norwegian Standard Specifications of 1939*[28] assumed the shearing stresses to be evenly distributed around the loaded area at a distance of $2d/3$ from the periphery. It is however, also necessary to consider a strip of the slab of a certain specified width as a beam and check the shearing stresses in this beam strip. If a load is placed close to one of the supported edges of a slab, this last check frequently gives the highest shearing stresses.

In the *British Code of Practice (CP114)*[29] the shear stresses in flat slabs are computed at a distance of $d/2$ from the periphery of the loaded area, while in footings the distance is taken equal to d.

803—Development of design recommendations

The evolution of the current concepts of shear action were pointed out in the review of research in Section 801. Three principal variables affect shear strength. They are: concrete strength, the relationship between size of loaded area and slab thickness, and the relationship between shear and moment in the vicinity of the loaded area. The most recent study by Moe led to the empirical ultimate strength design equation

$$v_u = \frac{V_u}{bd} = \left[15\left(1 - 0.075 \frac{r}{d} \right) - 5.25\,\phi_o \right] \sqrt{f_c'} \quad \text{..........(8-11)}$$

where

v_u = permissible ultimate shear stress
V_u = ultimate shear capacity
b = periphery around the loaded area
d = effective depth of the slab
r = side dimension of the loaded area
ϕ_o = V_u/V_{flex}
V_{flex} = ultimate shear force for flexural failure

Comparison of the equation with available data for 198 tests is presented in Table 8-1 of Section 810. The comparison is good as shown by the average ratio of V_{test} to V_{calc} and the standard deviation for each test series. The equation seems to be the best that has been developed to correlate laboratory shear tests of slabs and footings. However, the step from a research equation with limited ranges of applicability to a generally applicable practical design procedure is not an easy one in this particular case.

It was stated previously that shear failure is a local one, involving a portion of the slab structure around the loaded area. Therefore the simple slab specimens used in laboratory tests represent a portion of the slab from the loaded area out to the region of contraflexure. However, because of our limited knowledge concerning the distribution of moments in slab structures at loads near ultimate strength, it is difficult to determine which portion of the slab structure is involved in the shear failure.

Of course, this portion of slab area must be established only to determine the value of ϕ_o in Moe's equation. The variable ϕ_o is dependent on the ultimate flexural capacity of this portion. For simple laboratory specimens computation of flexural capacity is an easy task with the aid of the yield-line theory. For practical design cases, this is more difficult, and the yield-line theory has not yet become generally used in American practical design for flexure.

The variable ϕ_o is the ratio of shear capacity to flexural capacity. For balanced failure, shear capacity and flexural capacity are reached simultaneously, so that ϕ_o equals unity. If the slab fails in shear, ϕ_o is less

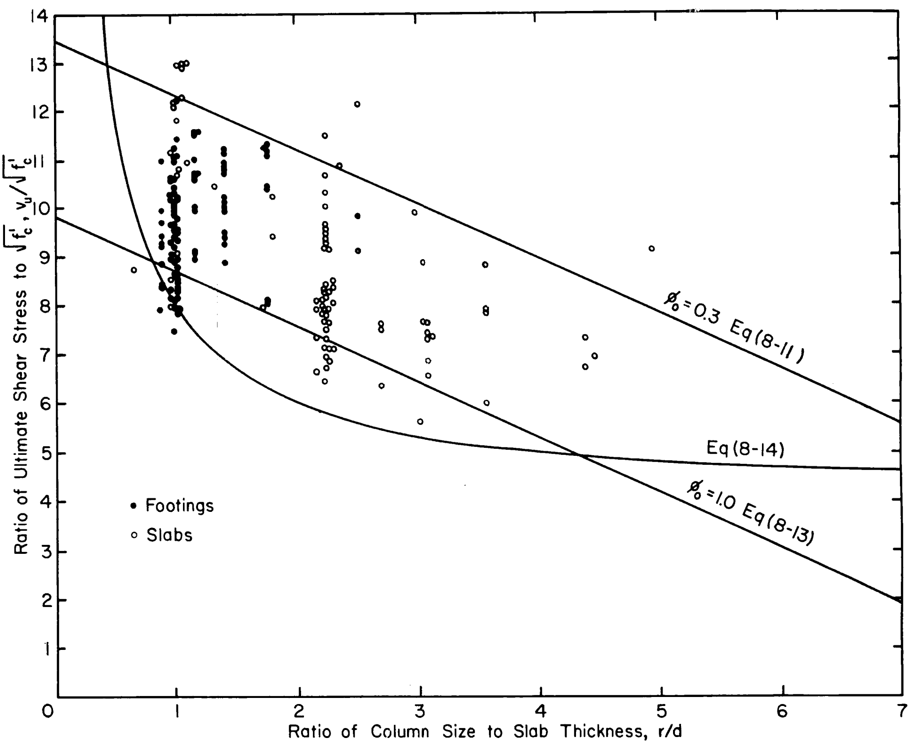


Fig. 8-1—Comparisons of design equations and test data

than unity. Since Moe's equation is only applicable to shear failures, ϕ_o cannot exceed unity.

As ϕ_o decreases, the shear strength of a slab increases. At first glance it would appear that increasing flexural capacity, which in turn decreases ϕ_o would be an efficient way of increasing shear capacity. However, substituting $\phi_o = V_u/V_{flex}$ into Moe's equation leads to

$$v_u = \frac{V_u}{bd} = \frac{15(1 - 0.075\,r/d)\sqrt{f_c'}}{1 + (5.25\,bd\sqrt{f_c'}/V_{flex})} \quad \text{(8-12)}$$

In Eq. (8-12) it is seen that V_{flex} must be increased substantially to affect a small increase in shear capacity. Although ϕ_o is an important variable in the shear stress equation, the interrelationship of ϕ_o, V_u, and V_{flex} is such that it is uneconomical to control shear capacity by the flexural capacity.

Generally speaking, the laboratory specimens were designed to fail in shear. The term ϕ_o was an important variable in the laboratory tests and it must be considered when laboratory test results are analyzed.

TABLE 8-1—COMPARISON OF TEST DATA WITH MOE'S EQ. (8-11)

Slab No.	r, in.	d, in.	$\sqrt{f_c'}$	ϕ_o	V_{calc}, kips	V_{test}, kips	$\frac{V_{test}}{V_{calc}}$
Richart's footings — Series 1							
105a	14.00	14.00	58.6	0.851	432	466	1.078
105b	14.00	14.00	48.9	0.760	379	359	0.948
106a	14.00	14.00	61.2	0.917	435	467	1.072
106b	14.00	14.00	60.2	0.906	430	421	0.979
107a	14.00	14.00	60.2	0.924	425	421	0.989
107b	14.00	14.00	58.4	0.908	417	378	0.906
108a	14.00	14.00	64.5	1.074	416	422	1.014
108b	14.00	14.00	57.3	1.003	386	400	1.036
109a	14.00	14.00	54.5	0.576	463	478	1.031
109b	14.00	14.00	55.5	0.583	470	409	0.868
110a	14.00	14.00	56.7	0.626	470	444	0.945
110b	14.00	14.00	51.9	0.592	438	462	1.053
111a	14.00	14.00	54.9	0.623	456	427	0.935
111b	14.00	14.00	58.3	0.648	479	509	1.063
112a	14.00	14.00	58.9	0.505	518	427	0.823
112b	14.00	14.00	53.1	0.471	474	462	0.974
109Ra	14.00	14.00	63.8	0.711	507	511	1.007
109Rb	14.00	14.00	64.6	0.718	512	489	0.953
110Ra	14.00	14.00	56.2	0.600	472	452	0.955
110Rb	14.00	14.00	59.2	0.621	492	527	1.070
						Average =	0.985
						Coefficient of variation =	0.068
Richart's footings — Series II							
201a	14.00	10.00	51.6	0.527	308	274	0.888
201b	14.00	10.00	51.7	0.527	308	312	1.013
202a	14.00	12.00	48.9	0.587	348	379	1.088
202b	14.00	12.00	46.7	0.570	335	362	1.078
Richart's footings — Series II (cont.)							
203a	14.00	14.00	51.3	0.687	413	373	0.904
203b	14.00	14.00	44.9	0.631	372	338	0.907
204a	14.00	12.00	50.8	0.566	385	362	0.937
204b	14.00	12.00	50.4	0.453	383	362	0.944
205a	14.00	14.00	47.5	0.498	420	409	0.974
205b	14.00	14.00	49.1	0.508	431	409	0.949
206a	14.00	16.00	54.0	0.600	526	454	0.862
206b	14.00	16.00	54.0	0.583	508	532	1.047
207a	14.00	8.00	65.2	0.595	289	335	1.158
207b	14.00	8.00	63.5	0.584	284	319	1.123
208a	14.00	10.00	63.3	0.706	344	328	0.950
208b	14.00	10.00	62.0	0.697	339	349	1.029
209a	14.00	12.00	53.7	0.756	353	398	1.126
209b	14.00	12.00	48.6	0.700	326	380	1.163
210a	14.00	10.00	65.7	0.611	376	413	1.099
210b	14.00	10.00	65.7	0.603	373	367	0.985
211a	14.00	12.00	60.5	0.674	412	434	1.051
211b	14.00	12.00	65.3	0.708	437	470	1.073
212a	14.00	14.00	62.4	0.779	478	462	0.966
212b	14.00	14.00	64.0	0.792	487	444	0.910
213a	14.00	8.00	67.1	0.605	296	317	1.069
213b	14.00	8.00	67.5	0.606	297	317	1.064
214a	14.00	10.00	69.1	0.745	368	422	1.148
214b	14.00	10.00	70.3	0.766	372	440	1.180
215a	14.00	12.00	71.1	0.887	431	434	1.007
215b	14.00	12.00	65.0	0.840	405	434	1.070
216a	14.00	8.00	67.5	0.606	297	335	1.126
216b	14.00	8.00	66.8	0.603	295	335	1.134

TABLE 8-1 (cont.)—COMPARISON OF TEST DATA WITH MOE'S EQ. (8-11)

Slab No.	r, in.	d, in.	$\sqrt{f_c'}$	ϕ_o	V_{calc}, kips	V_{test}, kips	$\frac{V_{test}}{V_{calc}}$
Richart's footings — Series II (cont.)							
217a	14.00	10.00	61.0	0.689	335	349	1.042
217b	14.00	10.00	68.8	0.744	367	422	1.151
218a	14.00	12.00	64.3	0.835	402	434	1.079
218b	14.00	12.00	66.5	0.852	412	398	0.965
						Average =	1.034
						Coefficient of variation =	0.088
Richart's footings — Series III-IV							
304a	14.00	14.00	58.6	0.854	431	466	1.080
304b	14.00	14.00	48.9	0.763	378	359	0.949
305a	14.00	14.00	59.3	0.864	434	498	1.145
305b	14.00	14.00	60.6	0.875	441	496	1.125
307a	14.00	14.00	57.6	0.785	441	462	1.048
307b	14.00	14.00	61.7	0.820	463	492	1.063
308a	14.00	14.00	61.8	0.936	434	476	1.098
308b	14.00	14.00	61.1	0.929	431	400	0.928
309a	14.00	14.00	62.2	0.925	440	421	0.958
309b	14.00	14.00	59.7	0.902	428	400	0.934
310a	14.00	14.00	64.1	0.840	475	533	1.121
310b	14.00	14.00	62.6	0.827	468	467	0.997
312a	14.00	14.00	60.2	0.888	535	455	1.045
312b	14.00	14.00	57.1	0.859	419	333	0.795
314a	14.00	14.00	56.6	0.809	429	489	1.139
314b	14.00	14.00	62.3	0.849	460	413	0.898
315a	14.00	14.00	53.6	0.928	378	333	0.880
315a	14.00	14.00	57.1	0.963	394	410	1.038
316a	14.00	14.00	62.4	0.946	436	467	1.070
316b	14.00	14.00	65.7	0.976	451	444	0.985

Slab No.	r, in.	d, in.	$\sqrt{f_c'}$	ϕ_o	V_{calc}, kips	V_{test}, kips	$\frac{V_{test}}{V_{calc}}$
Richart's footings — Series III-IV (cont.)							
317a	14.00	14.00	60.1	0.887	434	432	0.994
317b	14.00	14.00	61.0	0.896	438	462	1.053
319a	14.00	14.00	61.6	0.844	456	422	0.924
319b	14.00	14.00	59.2	0.823	443	480	1.083
321a	14.00	14.00	60.4	0.876	439	431	0.980
321b	14.00	14.00	61.9	0.888	447	459	1.026
323a	14.00	14.00	58.0	0.876	421	434	1.028
323b	14.00	14.00	61.3	0.907	438	454	1.036
324a	14.00	14.00	62.4	0.863	457	439	0.961
324b	14.00	14.00	63.3	0.870	460	448	0.969
326a	14.00	14.00	55.4	0.828	414	455	1.099
326b	14.00	14.00	61.8	0.887	446	461	1.032
403a	14.00	14.00	58.6	0.989	418	391	0.935
403b	14.00	14.00	44.3	0.765	342	320	0.935
						Average =	1.010
						Coefficient of variation =	0.081
Richart's footings — Series V-VII							
501a	14.00	10.00	60.7	0.682	335	365	1.089
501b	14.00	10.00	61.1	0.683	336	352	1.046
502a	14.00	16.00	59.4	0.959	478	490	1.025
502b	14.00	16.00	57.3	0.939	466	511	1.096
503a	14.00	16.00	59.6	0.959	479	518	1.082
503b	14.00	16.00	59.0	0.955	476	486	1.022
504a	14.00	10.00	60.1	0.644	334	299	0.894
504b	14.00	10.00	61.1	0.668	339	322	0.948
505a	14.00	16.00	60.6	1.031	467	479	1.025
505b	14.00	16.00	61.1	1.036	469	459	0.978

TABLE 8-1 (cont.) — COMPARISON OF TEST DATA WITH MOE'S EQ. (8-11)

Slab No.	r, in.	d, in.	$\sqrt{f_c'}$	ϕ_o	V_{calc}, kips	V_{test}, kips	$\frac{V_{test}}{V_{calc}}$
Richart's footings — Series II (cont.)							
506a	14.00	16.00	57.9	1.005	453	437	0.965
506b	14.00	16.00	61.8	1.042	473	437	0.924
701a	21.00	8.00	62.1	0.759	336	382	1.137
701b	21.00	8.00	59.6	0.741	326	395	1.208
702a	21.00	12.00	46.0	0.740	424	376	0.885
702b	21.00	12.00	53.7	0.976	428	440	1.026
						Average =	1.022
						Coefficient of variation =	0.087
Elstner-Hognestad slabs							
A-1a	10	4.63	45.2	0.840	68	68	0.996
A-1b	10	4.63	60.5	0.967	84	82	0.977
A-1c	10	4.63	64.9	1.000	88	80	0.908
A-1d	10	4.63	73.1	1.061	95	79	0.833
A-1e	10	4.63	54.2	0.913	78	80	1.025
A-2a	10	4.50	44.5	0.578	76	75	0.989
A-2b	10	4.50	53.2	0.605	89	90	1.007
A-2c	10	4.50	73.7	0.705	117	105	0.900
A-7b	10	4.50	63.6	0.655	104	115	1.107
A-3a	10	4.50	43.0	0.514	76	80	1.053
A-3b	10	4.50	57.3	0.499	102	100	0.981
A-3c	10	4.50	62.0	0.510	110	120	1.093
A-3d	10	4.50	70.8	0.538	123	123	0.998
A-4	14	4.63	61.6	1.041	98	90	0.920
A-5	14	4.50	63.5	0.720	124	120	0.971
A-6	14	4.50	60.2	0.567	129	112	0.865
A-7	10	4.50	64.3	0.939	87	90	1.028
A-8	14	4.50	56.4	0.964	94	98	1.071

Slab No.	r, in.	d, in.	$\sqrt{f_c'}$	ϕ_o	V_{calc}, kips	V_{test}, kips	$\frac{V_{test}}{V_{calc}}$
Elstner-Hognestad slabs (cont.)							
A-11	14	4.50	61.3	0.749	117	119	1.018
A-12	14	4.50	64.2	0.764	121	119	0.982
B-9	10	4.50	79.8	0.809	119	113	0.958
B-11	10	4.50	44.3	0.502	78	74	0.941
B-14	10	4.50	85.6	0.672	138	130	0.940
						Average =	0.980
						Coefficient of variation =	0.067
Graf slabs							
1362	10.8	10.66	47.6	0.682	242	262	1.082
1375	10.8	18.62	47.5	0.676	449	373	0.832
Forsell-Holmberg slabs							
1	4.34	3.98	43.6	0.342	36	41	1.147
2	4.34	4.37	43.6	0.442	38	40	1.042
3	4.34	4.17	43.6	0.455	36	39	1.071
4	4.34	4.33	43.6	0.477	37	40	1.069
5	4.34	4.37	43.6	0.467	38	45	1.182
6	4.34	4.21	43.6	0.395	37	41	1.103
7	4.34	4.17	43.6	0.294	39	42	1.089
8	0.00	4.37	52.1	—	—	26	—
9	0.00	4.21	52.1	—	—	26	—
10	10.90†	4.09	52.1	0.579	49	42	0.855
11	4.34‡	4.42	52.1	0.783	78	63	0.804
12	4.34‡	4.26	52.1	0.840	72	60	0.822
14	10.00§	4.26	49.6	0.501	41	46	1.128
15	4.34	4.33	49.6	0.529	41	40	0.962

TABLE 8-1 (cont.)—COMPARISON OF TEST DATA WITH MOE'S EQ. (8-11)

Slab No.	r, in.	d, in.	$\sqrt{f_c'}$	ϕ_o	V_{calc}, kips	V_{test}, kips	$\frac{V_{test}}{V_{calc}}$
Forsell-Holmberg slabs (cont.)							
16	4.34	4.26	49.6	0.521	41	43	1.066
17	4.34‡	4.26	49.6	0.672	75	66	0.879
18	4.34	4.42	49.6	0.539	42	42	1.010
19	4.34	4.37	57.1	0.596	46	36	0.784
20	10.00§	4.33	57.1	0.568	47	57	1.230
21	10.00§	3.94	57.1	0.517	42	55	1.288
22	4.34	4.26	57.1	0.583	45	45	0.998
						Average =	1.028
						Coefficient of variation =	0.145
Scordelis-Lin-May slabs							
S-1	13.00	4.25	53.0	0.684	93	105	1.124
S-2	13.00	4.25	63.7	0.728	109	109	1.001
S-4	13.00	2.63	62.9	0.630	53	80	1.517
S-5	13.00	3.00	54.2	0.797	50	60	1.193
S-6	13.00	3.63	68.5	0.928	79	79	0.994
S-7	13.00	4.38	53.7	0.690	98	122	1.234
S-8	13.00	3.63	66.0	0.831	82	100	1.210
S-9	16.00	3.63	66.2	0.901	82	105	1.286
S-10	16.00	3.63	68.3	0.840	89	118	1.321
S-11	13.00	7.63	71.6	0.985	225	225	1.001
S-12	13.00	5.63	70.1	0.820	166	172	1.030
S-13	13.00	3.63	72.3	0.595	107	109	1.017
S-14	13.00	5.63	69.3	0.928	153	168	1.100
S-15	13.00	3.63	71.5	0.701	98	120	1.219
						Average =	1.161
						Coefficient of variation =	0.152

Slab No.	r, in.	d, in.	$\sqrt{f_c'}$	ϕ_o	V_{calc}, kips	V_{test}, kips	$\frac{V_{test}}{V_{calc}}$
Moe's slabs							
R-1	††	4.50	63.2	0.882	87.0	88.5	0.973
R-2	6.00	4.50	62.0	0.601	64.5	70.0	1.085
S5-60	8.00	4.50	56.7	0.836	70.3	77.0	1.093
S6-60	10.00‡‡	4.50	55.8	0.897	78.2	72.0	0.921
S7-60	12.00‡‡	4.50	57.8	0.952	87.3	94.8	1.084
S5-70	8.00	4.50	57.8	0.751	75.3	85.0	1.128
S6-70	10.00‡‡	4.50	59.3	0.832	86.7	85.0	0.980
Elstner-Hognestad slabs (concentrated tensile reinforcement)							
A-9	10.00	4.50	65.8	0.704	104.2	100.0	0.959
A-10	14.00	4.50	65.6	0.779	122.5	110.0	0.897
Moe's slabs (concentrated tensile reinforcement)							
S1-60	10.00	4.50	58.1	0.911	80.7	87.5	1.084
S2-60	10.00	4.50	56.6	0.881	80.1	80.0	0.998
S3-60	10.00	4.50	57.3	0.902	80.0	81.8	1.021
S4-60	10.00	4.50	58.8	0.976	78.0	75.0	0.961
S1-70	10.00	4.50	59.6	0.832	87.1	88.2	1.011
S3-70	10.00	4.50	60.7	0.840	88.3	85.0	0.962
S4-70	10.00	4.50	71.4	0.943	97.0	84.0	0.865
S4A-70	10.00	4.50	54.5	0.930	74.7	70.0	0.936

† Rectangular load; area b = 25.60 in.

‡ Two load points $b = 8r$.

§ Line load $b = 2r$.

†† Column 6 x 18 in.

‡‡ Square steel plate.

TABLE 8-1 (cont.)—
COMPARISON OF TEST DATA WITH MOE'S EQ. (8-11)

Slab No.	r, in.	d, in.	b_o, in.	$\sqrt{f_c'}$	ϕ_o	V_{calc}, kips	V_{test}, kips	$\frac{V_{test}}{V_{calc}}$
Moe's slabs (slabs with openings)								
H1	10.00	4.50	40.00	61.5	1.000	80.2	83.5	1.041
H2	10.00	4.50	34.75	60.2	0.909	72.6	74.0	1.018
H3	10.00	4.50	29.75	58.6	0.813	64.5	73.0	1.130
H4	10.00	4.50	29.75	61.1	0.830	66.5	65.1	0.978
H5	10.00	4.50	24.60	60.2	0 724	57.9	56.1	0.968
H6	10.00	4.50	20.00	64.2	0.649	52.5	55.2	1.050
H7	10.00	4.50	36.14	60.4	0.931	74.4	70.1	0.941
H8	10.00	4.50	32.28	63.8	0.890	71.8	70.1	0.976
H9	10.00	4.50	36.40	59.1	0.924	73.6	70.3	0.955
H10	10.00	4.50	39.44	60.1	0.989	79.0	75.1	0.949
H11	10.00	4.50	40.00	61.5	1.000	80.2	76.1	0.948
H12	10.00	4.50	40.00	66.1	No slab action		60.4	—
H13	10.00	4.50	40.00	59.7	No slab action		45.1	—
H14	10.00	4.50	35.00	61.6	Flexure failure		56.8	—
H15	10.00	4.50	35.00	58.2	0.902	71.6	74.6	1.042
							Average =	0.992
							Coefficient of variation =	0.057

However, in practical design, ϕ_o is not an important variable because the shear capacity of the slab should exceed its flexural capacity, that is, ϕ_o should be at least equal to unity. Therefore ϕ_o may be eliminated from Moe's Eq. (8-11) by substituting $\phi_o = 1.0$

$$v_u = \frac{V_u}{bd} = \left(9.75 - 1.125 \frac{r}{d} \right) \sqrt{f_c'} \quad \text{(8-13)}$$

Fig. 8-1 shows test data in terms of the parameters $v_u/\sqrt{f_c'}$ versus r/d. The data are compared to Moe's general equation which produces a family of straight lines having ϕ_o as a parameter. The values of $\phi_o =$ 0.30 and $\phi_o = 1.00$ cover the limits of the test data. It is seen that Eq. (8-13) is conservative even when ϕ_o is less than 1.00. Unfortunately, however, Eq. (8-13) is not satisfactory over the full range of variables encountered in practical design.

If load is applied to a slab over a very small area, the perimeter b and the ratio r/d will be very small. According to Eq. (8-13) the ultimate shear stress v_u will approach $9.75\sqrt{f_c'}$; but the ultimate load capacity V_u will approach zero. This cannot be supported by logical reasoning. A thick slab can carry substantial load without failing in shear even if the load is applied over a very small area.

Likewise the equation obviously does not apply for very large values of r/d since it predicts $v_u = 0$ when $r/d = 8.67$. Drop panels or wall loads can give values of r/d exceeding 8.67. It is not reasonable to expect that slabs with drop panels or wall loads should have no shear resistance at all.

In the case of a continuous slab supported on a wall, the ratio r/d can, for the purpose of this discussion, be assumed to be infinite. The slab will have comparatively little slab action in the vicinity of the wall and will tend to behave like a wide, shallow beam. Therefore, as r/d approaches infinity, the value of v_u would approach the corresponding shear strength of a beam, which would be $1.9\sqrt{f_c'}$ plus a small term depending on the M/Vd ratio. In view of recent tests by Diaz de Cossio[30] showing the effect of ratio of width to depth on the shear strength of beams, as well as available results of tests on slabs, it appears that the shear strength v_u for slabs with large r/d ratios approaches a value substantially in excess of $1.9\sqrt{f_c'}$. It is consistent with available test results to take the shearing resistance of slabs as approaching the limiting value of $4.0\sqrt{f_c'}$ for large ratios of r/d when two-way slab action is present.

Therefore, the design expression for two-way slab action must satisfy the following conditions:

1. The ultimate shear stress v_u shall be a function of $\sqrt{f_c'}$ and r/d.
2. As r/d approaches zero, the ultimate shear load capacity V_u approaches a finite value.
3. Therefore, when r/d approaches zero, v_u approaches infinity.
4. When r/d approaches infinity, v_u approaches $4.0\sqrt{f_c'}$.
5. The shear stress v_u must decrease continuously to $4.0\sqrt{f_c'}$ as r/d increases.

The above conditions can be satisfied by a hyperbolic equation of the form $v_u = (Ad/r + B)\sqrt{f_c'}$, which for a conservative fit of the test data, gives

$$v_u = 4\left(\frac{d}{r} + 1\right)\sqrt{f_c'} \qquad \text{(8-14)}$$

Eq. (8-14) is plotted on Fig. 8-1 where it can be compared with Moe's Eq. (8-13) and with the test results given in Table 8-2.

The shear load capacity V_u can be evaluated from

$$V_u = v_u\, bd \qquad \text{(8-15)}$$

where b is the periphery at the edge of the loaded area.

Thus, for a square column, $b = 4r$, so that V_u can be expressed as

$$V_u = 16\, d^2\left(\frac{r}{d} + 1\right)\sqrt{f_c'} \qquad \text{(8-15a)}$$

TABLE 8-2 — RELATIONSHIP BETWEEN $v_u/\sqrt{f_c'}$ AND r/d

Slab No.	r, in.	d, in.	$\frac{r}{d}$	v_u test, psi	$\sqrt{f_c'}$ psi	$\frac{v_u}{\sqrt{f_c'}}$
Richart's footings — Series I						
105a	14.00	14.00	1.00	594	58.6	10.14
105b	14.00	14.00	1.00	458	48.9	9.37
106a	14.00	14.00	1.00	595	61.2	9.72
106b	14.00	14.00	1.00	537	60.2	8.92
107a	14.00	14.00	1.00	537	60.2	8.92
107b	14.00	14.00	1.00	480	58.4	8.22
108a	14.00	14.00	1.00	538	64.5	8.31
108b	14.00	14.00	1.00	510	57.3	8.90
109a	14.00	14.00	1.00	609	54.5	11.17
109b	14.00	14.00	1.00	520	55.5	9.37
110a	14.00	14.00	1.00	566	56.7	9.98
110b	14.00	14.00	1.00	588	51.9	11.33
111a	14.00	14.00	1.00	543	54.9	9.89
111b	14.00	14.00	1.00	649	58.3	11.13
112a	14.00	14.00	1.00	543	58.9	9.22
112b	14.00	14.00	1.00	588	53.1	11.07
109Ra	14.00	14.00	1.00	650	63.8	10.19
109Rb	14.00	14.00	1.00	623	64.6	9.64
110Ra	14.00	14.00	1.00	574	56.2	10.21
110Rb	14.00	14.00	1.00	672	59.2	11.35
Richart's footings — Series II						
201a	14.00	10.00	1.40	488	51.6	9.46
201b	14.00	10.00	1.40	556	51.7	10.75
202a	14.00	12.00	1.17	563	48.9	11.51
202b	14.00	12.00	1.17	563	46.7	11.52
203a	14.00	14.00	1.00	475	51.3	9.26
203b	14.00	14.00	1.00	430	44.9	9.58

Slab No.	r, in.	d, in.	$\frac{r}{d}$	v_u test, psi	$\sqrt{f_c'}$ psi	$\frac{v_u}{\sqrt{f_c'}}$
Richart's footings — Series II (cont.)						
204a	14.00	12.00	1.17	538	50.8	10.59
204b	14.00	12.00	1.17	538	50.4	10.67
205a	14.00	14.00	1.00	520	47.5	10.95
205b	14.00	14.00	1.00	520	49.1	10.59
206a	14.00	16.00	0.87	505	54.0	9.35
206b	14.00	16.00	0.87	593	54.0	10.98
207a	14.00	8.00	1.75	747	65.2	11.46
207b	14.00	8.00	1.75	710	63.5	11.18
208a	14.00	10.00	1.40	584	63.3	9.23
208b	14.00	10.00	1.40	623	62.0	10.05
209a	14.00	12.00	1.17	592	53.7	11.02
209b	14.00	12.00	1.17	565	48.6	11.63
210a	14.00	10.00	1.40	736	65.7	11.20
210b	14.00	10.00	1.40	654	65.7	9.95
211a	14.00	12.00	1.17	645	60.5	10.66
211b	14.00	12.00	1.17	699	65.3	10.70
212a	14.00	14.00	1.00	588	62.4	9.42
212b	14.00	14.00	1.00	566	64.0	8.84
213a	14.00	8.00	1.75	705	67.1	10.51
213b	14.00	8.00	1.75	705	67.5	10.44
214a	14.00	10.00	1.40	754	69.1	10.91
214b	14.00	10.00	1.40	785	70.3	11.17
215a	14.00	12.00	1.17	645	71.1	9.07
215b	14.00	12.00	1.17	645	65.0	9.92
216a	14.00	8.00	1.75	747	67.5	11.07
216b	14.00	8.00	1.75	747	66.8	11.18
217a	14.00	10.00	1.40	623	61.0	10.21
217b	14.00	10.00	1.40	754	68.8	10.96

TABLE 8-2 (cont.) — RELATIONSHIP BETWEEN $v_u/\sqrt{f_c'}$ AND r/d

Slab No.	r, in.	d, in.	$\frac{r}{d}$	v_u test, psi	$\sqrt{f_c'}$ psi	$\frac{v_u}{\sqrt{f_c'}}$
Richart's footings — Series V-VII (cont.)						
218a	14.00	12.00	1.17	645	64.3	10.03
218b	14.00	12.00	1.17	592	66.5	8.90
Richart's footings — Series III-IV						
304a	14.00	14.00	1.00	594	58.6	10.14
304b	14.00	14.00	1.00	458	48.9	9.37
305a	14.00	14.00	1.00	634	59.3	10.69
305b	14.00	14.00	1.00	632	60.6	10.43
307a	14.00	14.00	1.00	588	57.6	10.21
307b	14.00	14.00	1.00	627	61.7	10.16
308a	14.00	14.00	1.00	507	61.8	8.20
308b	14.00	14.00	1.00	510	61.1	8.35
309a	14.00	14.00	1.00	537	62.2	8.63
309b	14.00	14.00	1.00	510	59.7	8.54
310a	14.00	14.00	1.00	679	64.1	10.59
310b	14.00	14.00	1.00	595	62.6	9.50
312a	14.00	14.00	1.00	580	60.2	9.63
312b	14.00	14.00	1.00	424	57.1	7.43
314a	14.00	14.00	1.00	623	56.6	11.01
314b	14.00	14.00	1.00	527	62.3	8.46
315a	14.00	14.00	1.00	424	53.6	7.91
315b	14.00	14.00	1.00	521	57.1	9.12
316a	14.00	14.00	1.00	595	62.4	9.54
316b	14.00	14.00	1.00	566	65.7	8.61
317a	14.00	14.00	1.00	551	60.1	9.17
317b	14.00	14.00	1.00	588	61.0	9.64
319a	14.00	14.00	1.00	538	61.6	8.73
319b	14.00	14.00	1.00	611	59.2	10.32
Richart's footings — Series III-IV (cont.)						
321a	14.00	14.00	1.00	501	60.4	8.29
321b	14.00	14.00	1.00	502	61.9	8.11
323a	14.00	14.00	1.00	481	58.0	8.29
323b	14.00	14.00	1.00	483	61.3	7.88
324a	14.00	14.00	1.00	529	62.4	8.48
324b	14.00	14.00	1.00	530	63.3	8.37
326a	14.00	14.00	1.00	499	55.4	9.01
326b	14.00	14.00	1.00	502	61.8	8.12
403a	14.00	14.00	1.00	459	58.6	7.83
403b	14.00	14.00	1.00	446	44.3	10.07
Richart's footings — Series V-VII						
501a	14.00	10.00	1.40	650	60.7	10.71
501b	14.00	10.00	1.40	627	61.1	10.26
502a	14.00	16.00	0.87	545	59.4	9.18
502b	14.00	16.00	0.87	569	57.3	9.93
503a	14.00	16.00	0.87	578	59.6	9.70
503b	14.00	16.00	0.87	542	59.0	9.19
504a	14.00	10.00	1.40	532	60.1	8.85
504b	14.00	10.00	1.40	573	61.1	9.38
505a	14.00	16.00	0.87	534	60.6	8.81
505b	14.00	16.00	0.87	512	61.1	8.38
506a	14.00	16.00	0.87	488	57.9	8.43
506b	14.00	16.00	0.87	488	61.8	7.90
701a	21.00	8.00	2.51	568	62.1	9.15
701b	21.00	8.00	2.51	586	59.6	9.83
702a	21.00	12.00	1.75	373	46.0	8.11
702b	21.00	12.00	1.75	435	53.7	8.10

TABLE 8-2 (cont.) — RELATIONSHIP BETWEEN $v_u/\sqrt{f_c'}$ AND r/d

Slab No.	r, in.	d, in.	$\frac{r}{d}$	v_u test, psi	$\sqrt{f_c'}$ psi	$\frac{v_u}{\sqrt{f_c'}}$
Elstner-Hognestad slabs						
A1a	10.00	4.63	2.16	366	45.2	8.10
A1b	10.00	4.63	2.16	442	60.5	7.31
A1c	10.00	4.63	2.16	431	64.9	6.64
A1d	10.00	4.63	2.16	425	73.1	5.81
A1e	10.00	4.63	2.16	431	54.2	7.95
A2a	10.00	4.50	2.22	416	44.5	9.35
A2b	10.00	4.50	2.22	500	53.2	9.40
A2c	10.00	4.50	2.22	583	73.7	7.91
A7b	10.00	4.50	2.22	637	63.6	10.01
A3a	10.00	4.50	2.22	444	43.0	10.33
A3b	10.00	4.50	2.22	555	57.3	9.69
A3c	10.00	4.50	2.22	665	62.0	10.73
A3d	10.00	4.50	2.22	682	70.8	9.63
A4	14.00	4.63	3.03	347	61.6	5.63
A5	14.00	4.50	3.11	475	63.5	7.48
A6	14.00	4.50	3.11	444	60.2	7.38
A7	10.00	4.50	2.22	500	64.3	7.78
A8	14.00	4.50	3.11	388	56.4	6.88
A11	14.00	4.50	3.11	472	61.3	7.70
A12	14.00	4.50	3.11	472	64.2	7.35
B9	10.00	4.50	2.22	629	79.8	7.88
B11	10.00	4.50	2.22	410	44.3	9.26
B14	10.00	4.50	2.22	721	85.6	8.42
Graf slabs						
1362	10.80	10.66	1.11	521	47.6	10.95
1375	10.80	18.62	0.64	413	47.5	8.69

Slab No.	r, in.	d in.	$\frac{r}{d}$	v_u test, psi	$\sqrt{f_c'}$ psi	$\frac{v_u}{\sqrt{f_c'}}$
Forsell-Holmberg slabs						
1	4.34	3.98	1.09	597	43.6	13.70
2	4.34	4.37	.99	524	43.6	12.09
3	4.34	4.17	1.04	533	43.6	12.23
4	4.34	4.33	1.00	528	43.6	12.11
5	4.34	4.37	.99	588	43.6	13.49
6	4.34	4.21	1.03	566	43.6	12.98
7	4.34	4.17	1.04	583	43.6	13.37
8	0.00	4.37	0.00	∞	52.1	∞
9	0.00	4.21	0.00	∞	52.1	∞
10	10.90†	4.09	2.67	401	52.1	7.70
11	4.34‡	4.42	0.98	411	52.1	7.89
12	4.34‡	4.26	1.02	406	52.1	7.79
14	10.00§	4.26	2.35	540	49.6	10.89
15	4.34	4.33	1.00	528	49.6	10.64
16	4.34	4.26	1.02	586	49.6	11.81
17	4.34‡	4.26	1.02	447	49.6	9.01
18	4.34	4.42	0.98	554	49.6	11.17
19	4.34	4.37	0.99	480	57.1	8.40
20	10.00§	4.33	2.31	658	57.1	11.52
21	10.00§	3.94	2.54	698	57.1	12.22
22	4.34	4.26	1.02	613	57.1	10.73
Scordelis-Lin-May slabs						
S1	13.00	4.25	3.04	474	53.0	8.94
S2	13.00	4.25	3.04	492	63.7	7.74
S4	13.00	2.63	4.94	583	62.9	9.27
S5	13.00	3.00	4.44	383	54.2	7.09
S6	13.00	3.63	3.58	415	68.5	6.06

TABLE 8-2 (cont.) — RELATIONSHIP BETWEEN $v_u/\sqrt{f_c'}$ AND r/d

Slab No.	r, in.	d, in.	$\frac{r}{d}$	v_u test, psi	$\sqrt{f_c'}$ psi	$\frac{v_u}{\sqrt{f_c'}}$
Scordelis-Lin-May slabs (cont.)						
S7	13.00	4.38	2.97	532	53.7	9.91
S8	13.00	3.63	3.58	527	66.0	7.98
S9	16.00	3.63	4.41	450	66.2	6.80
S10	16.00	3.63	4.41	506	68.3	7.41
S11	13.00	7.63	1.70	567	71.6	7.92
S12	13.00	5.63	2.31	584	70.1	8.33
S13	13.00	3.63	3.58	577	72.3	7.98
S14	13.00	5.63	2.31	572	69.3	8.25
S15	13.00	3.63	3.58	635	71.5	8.88
Moe's slabs						
R-1	††	4.50	2.67	404	63.2	6.39
R-2	6.00	4.50	1.33	648	62.0	10.45
S5-60	8.00	4.50	1.78	533	56.7	9.40
S6-60	10.00‡‡	4.50	2.22	399	55.8	7.15
S7-60	12.00‡‡	4.50	2.67	437	57.8	7.56
S5-70	8.00	4.50	1.78	590	57.8	10.21
S6-70	10.00‡‡	4.50	2.22	472	59.3	7.96
Elstner-Hognestad slabs (concentrated tensile reinforcement)						
A-9	10.00	4.50	2.22	555	65.8	8.43
A-10	14.00	4.50	3.12	435	65.6	6.63

Slab No.	r, in.	d, in.	$\frac{r}{d}$	v_u test, psi	$\sqrt{f_c'}$ psi	$\frac{v_u}{\sqrt{f_c'}}$
Moe's slabs (concentrated tensile reinforcement)						
S1-60	10.00	4.50	2.22	486	58.1	8.36
S2-60	10.00	4.50	2.22	444	56.6	7.84
S3-60	10.00	4.50	2.22	453	57.3	7.91
S4-60	10.00	4.50	2.22	416	58.8	7.09
S1-70	10.00	4.50	2.22	489	59.6	8.20
S3-70	10.00	4.50	2.22	472	60.7	7.78
S4-70	10.00	4.50	2.22	465	71.4	6.51
S4A-70	10.00	4.50	2.22	388	54.5	7.12
Moe slabs (slabs with openings)						
H-1	10.00	4.50	2.22	462	61.5	7.51
H-2	10.00	4.50	2.22	473	60.2	7.86
H-3	10.00	4.50	2.22	544	58.6	9.28
H-4	10.00	4.50	2.22	486	61.1	7.95
H-5	10.00	4.50	2.22	505	60.2	8.39
H-6	10.00	4.50	2.22	612	64.2	9.53
H-7	10.00	4.50	2.22	432	60.4	7.15
H-8	10.00	4.50	2.22	486	63.8	7.62
H-9	10.00	4.50	2.22	431	59.1	7.29
H-10	10.00	4.50	2.00	417	60.1	6.94
H-11	10.00	4.50	2.22	411	61.5	6.68
H-12	10.00	4.50	∞§§	335	66.1	5.07
H-13	10.00	4.50	∞§§	251	59.7	4.20
H-14	10.00	4.50	2.22	360	61.6	Flex-ure
H-15	10.00	4.50	2.22	473	58.2	8.13

† Rectangular load area $b = 25.60$ in.
‡ Two load points $b = 8r$.
§ Line load $b = 2r$.
†† Column size 6 x 18 in.
‡‡ Square steel plate.
§§ No slab action.

It is apparent that Eq. (8-14) and (8-15) satisfy the previously stated conditions with $v_u = 4\sqrt{f_c'}$ for r/d equal to infinity and $V_u = 16d^2\sqrt{f_c'}$ for r/d equal to zero.

For practical design purposes, Committee 326 feels that slabs with very large r/d ratios should also be checked for action as a beam, in which case the recommendations of Chapters 5, 6, and 7 are applicable.

804—Comparison with procedures of 1956 ACI Building Code

The proposed concept of shear action is based on the premises that the shear area is the vertical section which follows the periphery at the edge of the loaded area, and that the ultimate shear stress is a function of $\sqrt{f_c'}$ and r/d. The concept of the design procedure in the 1956 ACI Code is different; the shear area is the vertical section which follows the periphery located a distance d beyond the edge of the loaded area, and the allowable shear stress is proportional to f_c' with maximum values of 100 and 75 psi for slabs and footings, respectively.

For purposes of comparison it is assumed that the maximum allowable shear stress by the 1956 Code is 0.03 f_c' and that a safety factor of 2.0 is used. In terms of ultimate shear load, the 1956 Code procedure is: $V_u/bjd = 0.06\, f_c'$. For square columns, $b' = 4(r + 2d)$. Thus, the comparable ultimate shear stress based on a section at the edge of the loaded area is, for $j = 7/8$

$$\frac{v_u}{\sqrt{f_c'}} = \frac{V_u}{4\,rd\sqrt{f_c'}} = 0.0525\sqrt{f_c'}\left(2\frac{d}{r} + 1\right) \qquad \text{(8-16)}$$

The 1956 Code requirements are compared with the proposed design equations in Fig. 8-2 and 8-3. In Fig. 8-2, Eq. (8-16) represents a family of hyperbolas having $\sqrt{f_c'}$ as a variable, while Eq. (8-16) is a family of straight lines in Fig. 8-3. The similarity between the 1956 Code design equation and the proposed equations is surprising in view of the fact that the basic concepts of the two procedures appear to be radically diferent. When the 1956 Code design criterion is written in the form of Eq. (8-16), however, it is seen that the variable r/d has been taken into account by assuming the shear area to be located at a distance d beyond the edge of the loaded area. Thus, the variable r/d may be accounted for either as a variable in expressing the shear stress v_u, or by the choice of the location of the shear area.

The 1956 Code procedure may be expressed in terms of the proposed concept as follows:

> The shear area shall be the vertical section which follows the periphery at the edge of the loaded area, and the ultimate unit shear stress shall be a function of f_c' and r/d.

$$v_u = \frac{V_u}{bd} = 0.0525\, f_c'\,(2\,d/r + 1)$$

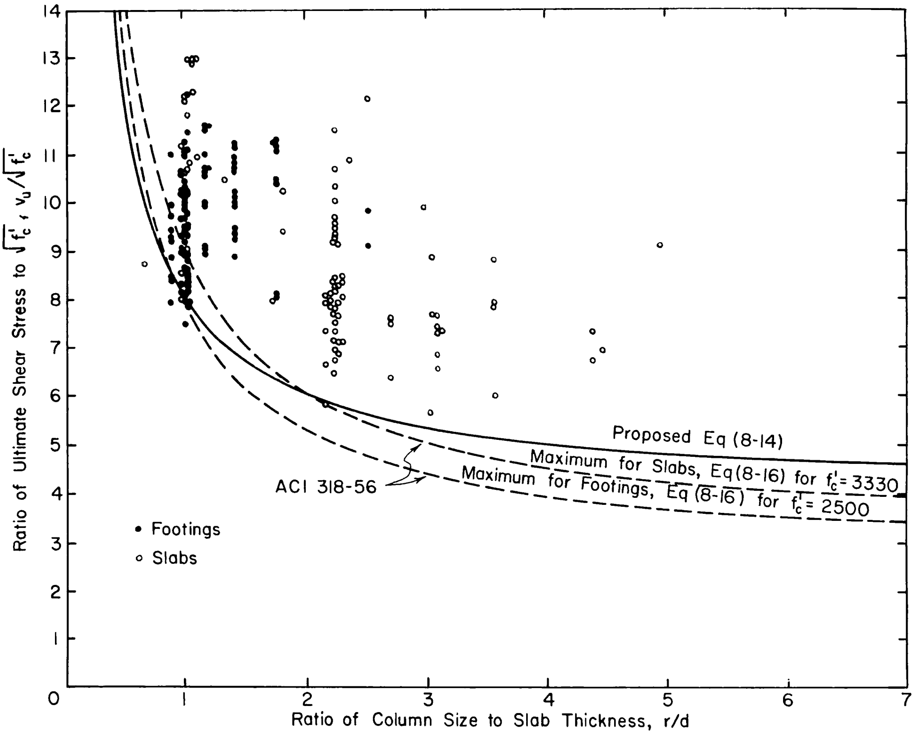


Fig. 8-2—Comparison of Eq. (8-14) to 1956 ACI Building Code

Similarly, the proposed design procedure may be expressed in terms of the 1956 Code concept:

> The shear area shall be the vertical section which follows the periphery at a distance d/2 beyond the edge of the loaded area, and the ultimate shear stress shall be

$$v_u = \frac{V_u}{bd} = 4.0\sqrt{f_c'}$$

Although the two concepts are different, the selection of one in preference to the other becomes a matter of personal choice. The proposed concept follows the line of reasoning adopted by laboratory research; it approaches the basic philosophy which explains the effect of slab action on shear strength. On the other hand, the 1956 Code concept is universally understood and accepted in everyday design practice.

The proposed design procedure offers several advantages over the 1956 Code procedure. In the region of normal flat slab or flat plate design they are more liberal than the 1956 procedures, but they are proven to be safe by laboratory tests. Although it would appear that the same re-

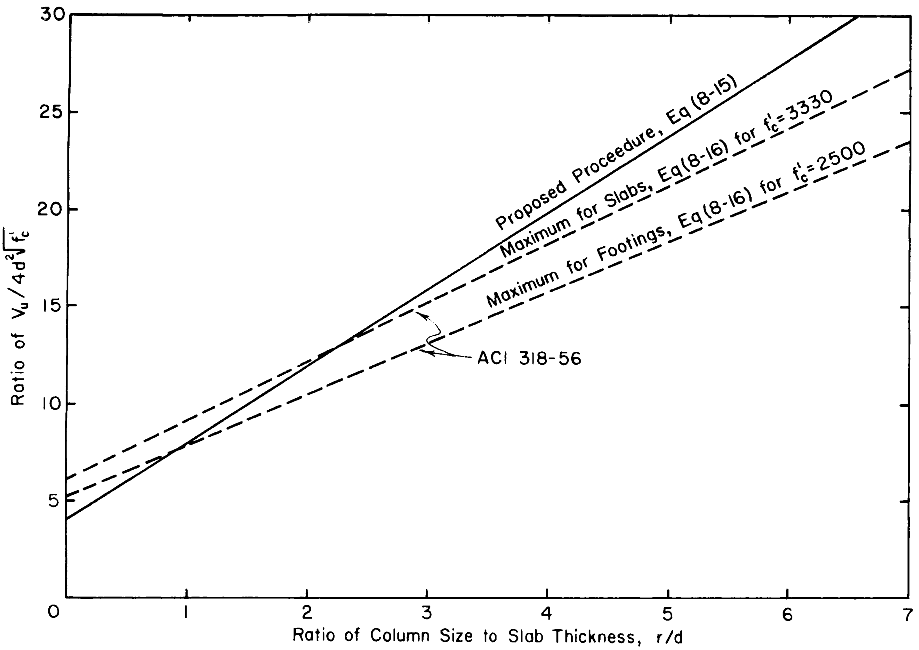


Fig. 8-3—Comparison of Eq. (8-15) to 1956 Building Code

sults may be achieved by increasing the ultimate shear stress of the present procedures, to do so would endanger safety at the extremes in r/d values. The proposed procedure also brings shear strength under two-way slab action in line with the general thoughts regarding the mechanisms of shear and diagonal tension. Furthermore, j is eliminated; and expressing ultimate shear stress in terms of $\sqrt{f_c'}$ facilitates effective use of concrete strengths exceeding 3000 psi.

There is merit, however, in expressing the proposed design procedure in terms of the 1956 Code concept. The proposed method requires expressing the ultimate shear stress as a function of r/d. For a loaded area of other shape than square, there may be doubt regarding the correct value of r to be used. On the other hand, by assuming the ultimate shear stress as equal to $4.0\sqrt{f_c'}$ and defining the critical section as that which is located at a distance $d/2$ from the periphery of the loaded area, regardless of its shape, the question of interpretation regarding the correct value of r does not arise. Furthermore, this gives a simple method of handling the case of openings in the vicinity of the loaded area.

The ultimate shear load capacity V_u can, therefore, be computed by

$$V_u = v_u bd \qquad \text{(8-17)}$$

where

$$v_u = 4.0\sqrt{f_c'} \quad \text{(8-18)}$$

and

b = the periphery of a pseudocritical section located at a distance $d/2$ from the periphery of the loaded area.

805—Concentration of reinforcement over a column

The 1956 ACI Building Code permits an increase in allowable shear stress if the tensile reinforcement of the column strip is concentrated over the periphery of the shear area. A thorough search of the technical literature failed to reveal the origin of, or the logic behind, this requirement. It was probably not based on laboratory tests.

Recent tests by Elstner and Hognestad and by Moe indicated no increase in shear strength due to concentration of tensile reinforcement through the shear area. However, Moe pointed out the following advantages from concentrating tensile reinforcement in the vicinity of the loaded area:[20]

1. Concentration increased the stiffness of the slabs; center deflections decreased as the amount of concentration increased.

2. Concentration reduced the stresses in the flexural tension reinforcement in the vicinity of the column and thereby raised the load at which first yielding took place.

3. Even with heavy concentrations, bond failure or splitting failure was not detected. The concentration reduced the violence of the shear failure.

These tests indicated advantages of concentrating tension reinforcement in the vicinity of the column, but the advantages were realized in the flexural behavior of the slabs, not in their shear behavior.

Because of its advantages in flexure, concentration of tensile reinforcement in the vicinity of the loaded area should be encouraged. However, Committee 326 feels that such encouragement should not be tied to the design requirements for shear.

806—Slabs with openings

Moe[20] reported 15 test specimens having different patterns of openings adjacent to the column. Fourteen of the specimens failed in shear. The tests showed that the ultimate shear strength of the slabs is affected by the size and location of the openings with respect to: the loaded area, the size of the loaded area, and the thickness of the slab.

The effect of the openings on the shear strength of the slabs can be accounted for by reducing the perimeter of the pseudocritical section which is assumed to be at a distance of $d/2$ from the loaded area. This

TABLE 8-3 — EFFECT OF OPENINGS[20]

Slab No.	Periphery, b_o† in.	Periphery, b_o† Percent of H-1	P_{test}, kips	$\sqrt{f_c'}$ psi	P_{test} corrected to $\sqrt{f_c'} = 61.5$ kips	P_{test} corrected to $\sqrt{f_c'} = 61.5$ percent of H-1	Test‡/Calc
H-1	58.00	100	83.5	61.5	83.5	100	1.00
H-2	50.75	87.5	74.0	60.2	75.5	90.5	1.03
H-3	43.50	75.0	73.0	58.6	76.5	91.7	1.22
H-4	43.50	75.0	65.1	61.1	65.5	78.5	1.05
H-5	36.25	63.4	56.1	60.2	57.4	68.7	1.08
H-6	29.00	50.0	55.2	64.2	53.0	63.5	1.27
H-7	52.88	91.2	70.1	60.5	71.3	85.5	0.94
H-8	47.76	82.5	70.1	63.8	67.5	80.9	0.98
H-9	52.80	91.0	70.3	59.1	73.0	87.5	0.96
H-10	54.00	93.0	75.1	60.2	76.8	92.0	0.99
H-11	55.20	95.2	76.1	61.5	76.1	91.4	0.96
H-12	29.00	50.0	60.4	63.5	58.5	69.0	1.38
H-13	19.36	33.4	45.1	59.8	46.4	55.6	1.66

† Periphery of critical section b_o, was calculated at $d/2$ from loaded area subtracting the projection of openings in accordance with the procedures recommended in Section 806.

‡ Test/Calc = ratio of percent reduction in corrected measured ultimate shear load, P_{test}, to calculated percent reduction in periphery b_o; in both cases percent reduction with respect to Slab H-1 which had no openings.

reduction in perimeter length depends on the size and location of the openings and the r/d ratio.

The ultimate shear load capacity V_u can be evaluated from Eq. (8-17) and (8-18) as

$$V_u = v_u b_o d = 4.0 \sqrt{f_c'} b_o d \quad \text{(8-17) (8-18)a}$$

reduction in perimeter length depends on the size and location of the pseudocritical section located at a distance $d/2$ from the periphery of the loaded area. Several examples of proposed methods for computation of reduced periphery b_o are shown in Fig. 8-4.

1. For an opening whose closest edge is located less than $d/2$ from the loaded area, it is proposed that the reduced perimeter be the length of the original pseudocritical section minus the radial projection of the opening on the pseudocritical section. The radial lines should be drawn from the centroid of the loaded area to the edges of the opening so that the radial lines lie completely outside the opening as shown in Fig. 8-4a. If there are several openings, the sum of the radial projections should be subtracted from the perimeter of the original pseudocritical section.

2. For openings whose closest edges are more than $d/2$ but less than $2d$ from the loaded area, the reduced perimeter should be taken as the smaller of the two given by the following criteria:

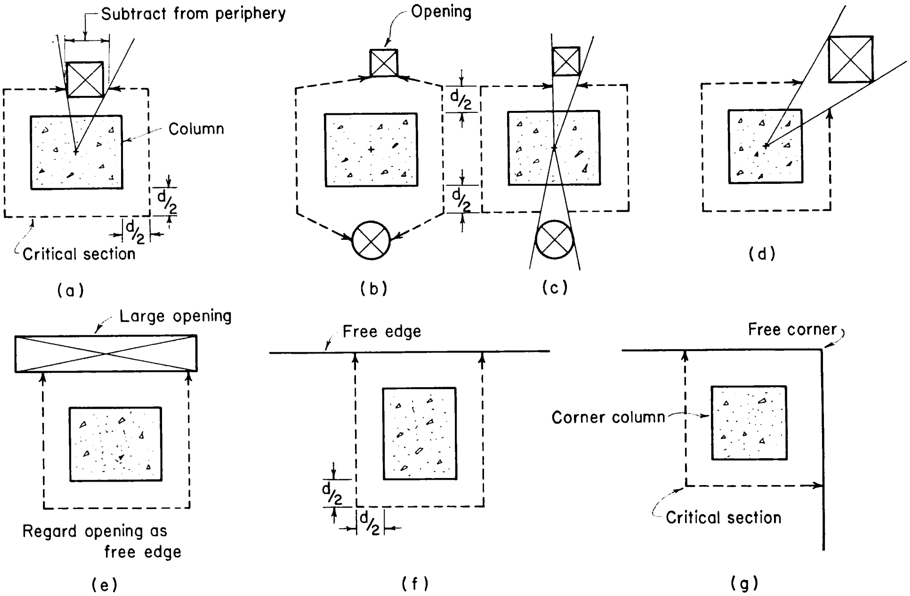


Fig. 8-4—Effect of openings and free edges

(a) The shortest of all possible sections lying not less than $d/2$ from the loaded area as shown in Fig. 8-4b.

(b) The original pseudocritical section minus the sum of the radial projections of the openings as shown in Fig. 8-4c.

3. For openings near the corners of the original critical section, such as the case shown in Fig. 8-4d, Criterion (a) gives no reduction. Criterion (b) probably overestimates the reduction.

4. For the effect of openings whose closest edges are more than $2d$ from the loaded area, only Criterion (a) and the original unreduced critical section need be investigated.

5. Openings that are large compared with the dimensions of the critical section, such as that shown in Fig. 8-4e, should be treated as free edges or corners as described below.

6. The shear capacity of the slab in the vicinity of free edges or corners, as shown in Figs. 8-4f and g, should be evaluated by applying Criterion (a). Particularly when the free edge is located at some distance from the column or loaded area, the original critical section should also be investigated.

7. If several openings are close together, Criteria (a) and (b) may be used provided the distance between openings parallel to the critical section is sufficient to maintain two-way slab action. The

TABLE 8-4 — TRANSFER OF AXIAL LOAD AND MOMENTS FROM COLUMN TO SLAB

Slab No.	r, in.	d, in.	e, in.	$\sqrt{f_c'}$ psi	ϕ_o	V_o,† kips	V_{calc},‡ kips	V_{test}, kips	$\frac{V_{test}}{V_{calc}}$
Moe slabs[20]									
M1A	12.00	4.50	0.00	55.0	0.691	99.5	99.5	97.3	0.978
M2A	12.00	4.50	7.30	47.4	0.631	84.4	52.5	47.8	0.912
M4A	12.00	4.50	17.10	50.6	0.671	92.8	38.3	32.3	0.844
M2	12.00	4.50	7.70	61.1	0.694	104.2	63.5	65.7	1.035
M3	12.00	4.50	13.30	57.4	0.701	103.1	48.9	46.6	0.954
M4	12.00	4.50	17.20	59.8	0.711	106.5	43.8	29.6	§
M5	12.00	4.50	24.20	62.5	0.726	110.3	36.6	22.7	§
M6	10.00	4.50	6.62	64.2	0.882	81.3	48.9	53.8	1.100
M7	10.00	4.50	2.40	60.2	0.916	83.6	67.4	70.0	1.039
M8	10.00	4.50	17.20	59.7	0.909	83.2	30.6	33.6	1.100
M9	10.00	4.50	5.00	62.1	0.897	81.4	54.3	60.0	1.105
M10	10.00	4.50	12.12	57.8	0.877	78.7	35.6	40.0	1.125

† Calculated ultimate capacity neglecting shear stress due to eccentric loading.
‡ Calculated by Moe's assumption that one-third of the column moment is transferred to slab by vertical shear.
§ Failed in negative bending near the column.

required distance is a function of slab depth, size of openings and other parameters. In extreme cases, a plurality of openings may create a free edge condition.

These criteria were applied to Moe's tests on slabs with openings as shown in Table 8-3. Slab H-1, which had no openings, was considered as the standard, and all slabs with openings were compared relative to this standard. As seen in Table 8-3, the use of the reduced length of the pseudocritical perimeter accurately predicts the reduction in capacity caused by the presence of openings. The strengths of Slab H-6 with openings adjacent to all four faces of the loaded area, and Slabs H-12 and H-13 with openings at all four corners are predicted conservatively. This is desirable because these latter three cases are extremes that should be avoided in practical design applications.

When relatively minor openings are present, it is safe to assume that the shear stress is uniformly distributed over the reduced critical section located at a distance $d/2$ from the column or loaded area. This ultimate stress v_u should then not exceed $4\sqrt{f_c'}$. When large openings, a plurality of openings, or free edges are present, however, it becomes necessary to consider transfer of bending moment. This leads to a nonuniform distribution of shear stress as described in Section 807.

807—Transfer of moment between columns and slabs

Only limited information is available regarding the shear strength of slabs near columns when both axial load and moment are transferred. Moment is transferred between column and slab by flexural moments, by torsional moments and by vertical shear as shown in Fig. 8-5. No experimental method has been found so far of directly measuring their individual contributions to the total transferred moment. The three quantities can be inter-related by mathematical analyses, but such analyses must be based on simplifying assumptions which may or may not be realistic.

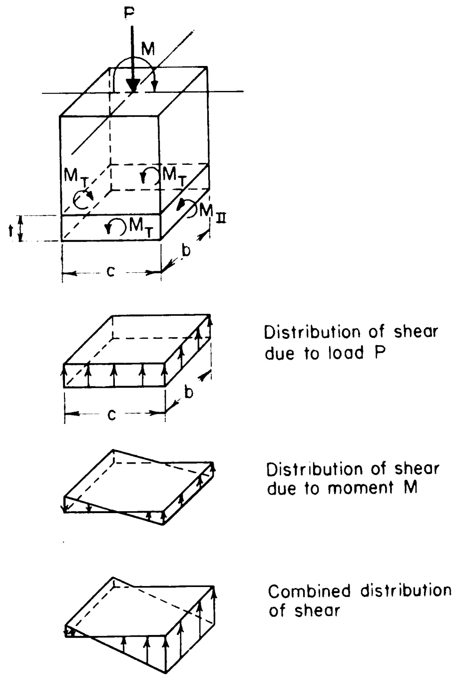


Fig. 8-5—Transfer of moment

Moe[20] reported tests of 12 specimens which had eccentricity of column load as the primary variable. His specimens were square slabs having a centrally located column stub. Eccentricity was varied from 0 to 24 in. on 12 and 10 in. square columns. In his analysis of the tests, Moe did not attempt to separate the transfer moment into its three components. He assumed that the vertical shear stresses were constant across the critical planes perpendicular to the plane of symmetry as shown in Fig. 8-5; and they were assumed to vary linearly along the other two critical planes. Secondly, he assumed that failure occurred when the maximum shear stress reached a value equal to the ultimate shear strength of the same slab loaded with zero eccentricity. Based on these assumptions, Moe worked backward from his test data and found that approximately one-third of the total moment M was transferred by vertical shear stresses. This finding is, of course, limited to the type and size of specimen used by Moe. The results of Moe's tests and analysis are summarized in Table 8-4.

In a study which is still in progress, Hanson[31] investigated the shear and moment transfer between slabs and columns by testing ten 3 in. thick slabs with 6 in. square columns. The rectangular slabs were 48 in. wide and 84 in. long, with the column centrally located. They were tested with line loads applied to the slab 36 in. from the column center to create various combinations of shear and moment. Five tests by Frederick and Pollauf[32] which were similar to Hanson's are summarized together with Hanson's tests in Table 8-5. In this group of 15 tests, the range of eccentricity of load was from zero to near infinity.

Calculation of ultimate shear strength for 25 tests of these three investigations at various distances x outside the column are summarized in Table 8-6. In these calculations, which follow the general form developed for interior columns by Di Stasio and van Buren[33] rather than the methods used by Moe, the shear stresses v_1 and v_2 in Fig. 8-6 are

$$\left.\begin{aligned} v_1 &= \frac{8t}{7d}\left[\frac{V}{A_c} + \frac{KM}{J_c}\left(\frac{c}{2}\right)\right] \\ v_2 &= \frac{8t}{7d}\left[\frac{V}{A_c} - \frac{KM}{J_c}\left(\frac{c}{2}\right)\right] \end{aligned}\right\} \qquad (8\text{-}19)$$

where

$$A_c = 2(c + b)t \qquad (8\text{-}20)$$

is the area subject to direct shear and

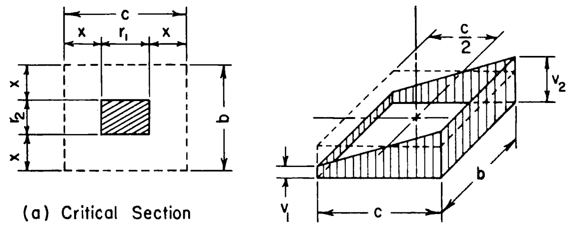
$$J_c = \frac{2\,tc^3}{12} + \frac{2\,ct^3}{12} + 2\,bt\left(\frac{c}{2}\right)^2 \qquad (8\text{-}21)$$

TABLE 8-5 — TRANSFER OF AXIAL LOAD AND MOMENTS FROM COLUMN† TO SLAB‡

Specimen No.	Voids in slab along side of columns	d, in.	$\sqrt{f_c'}$ psi	Transferred	
				V_{test}, kips	M_{test}, in.-kips
Hanson slabs[31]					
1	None	2.44	66.2	1.29	197.6
4	B§	2.44	73.2	0.92	213.3
5	C††	2.44	71.2	1.14	139.6
2	None	2.44	69.4	6.04	181.4
6	B	2.44	68.9	5.88	175.9
7	C	2.44	71.8	4.30	118.9
13	Column at slab edge	2.44	67.2	2.71	87.9
8	None	2.44	67.3	1.08	215.9
9	B	2.44	69.6	1.08	210.7
10	C	2.44	71.1	1.04	150.6
Frederick and Pollauf slabs[32]					
1	None	2.44	54.7	6.09	175.4
2	None	2.44	41.8	7.62	176.0
3	None	1.94	49.7	5.06	106.0
5	None	2.44	49.5	7.07	160.4
6	None	2.44	50.9	7.07	160.4

† All columns 6 x 6 in.
‡ All slabs 3 in. thick except Specimen 3 (t = 2.5 in.).
§ Voids along sides of column parallel to axis of stress symmetry.
†† Voids along sides of column perpendicular to axis of stress symmetry; all voids were 6 x 1 in.

Fig. 8-6 — Shear stress at distance x



is the polar moment of inertia of the surface described by the critical section passing through the slab thickness as shown in Fig. 8-6.

An increase in A_c and J_c may be made to account for dowel action of the steel crossing the area by multiplying the individual terms of Eq. (8-20) and (8-21) by $[1 + (n - 1)\,p]$.

The factor K in Eq. (8-19) is a reduction factor on the total moment transferred M, to obtain the moment transferred by torsional shear stress.

It is shown in Table 8-6 that the best correlation with the test data was obtained when the shear stresses were calculated at a distance $x = d$ outside the column with full dowel action considered. The value $K = 0.487$ gave the best coefficient of variation of 0.121 for the 25 tests available. The average value of the maximum shear stress at ultimate strength was $4\sqrt{f_c'}$ psi.

To be consistent the design procedures developed earlier in this chapter, Eq. (8-19), (8-20) and (8-21) can be written as

$$v_u = \frac{V}{A_o} + \frac{KM}{J_c}\left(\frac{c}{2}\right) \qquad \text{(8-22)}$$

where

$$A_c = 2(c + b)d = b_o\, d \qquad \text{(8-23)}$$

and

$$J_c = \frac{2\,dc^3}{12} + \frac{2\,cd^3}{12} + 2\,bd\left(\frac{c}{2}\right)^2 \qquad \text{(8-24)}$$

In Eq. (8-22) to (8-23), the critical section is taken at a distance $x = d/2$ from the face of the loaded area. When the perimeter of the critical section is reduced for exterior columns or when openings are present, Fig. 8-4, the polar moment of inertia should be computed on the basis of the remaining section and taken about the centroidal axis of the reduced perimeter. Furthermore, consistent with other parts of this report, no increase in shear resistance due to dowel action should be made for Eq. (8-23) and (8-24).

Using the results of the 25 tests available, the factor K was re-evaluated for Eq. (8-22), taking into account the reduction of the perimeter of the

TABLE 8-6 — SUMMARY OF CALCULATIONS OF ULTIMATE SHEAR STRESS FOR 25 SPECIMENS WITH MOMENT TRANSFER

Distance outside column, x	Dowel action of steel	Best coefficient, K	Coefficient of variation	Average $\frac{v_{test}}{\sqrt{f_c'}}$	Based on equation
0	Full	0.150	0.290	5.64	
0	None	0.148	0.272	6.62	
$t/2$	Full	0.331	0.143	4.32	(8-19)
$t/2$	None	0.312	0.182	5.04	
d	Full	0.487	0.121	3.99	
d	None	0.437	0.180	4.51	
$d/2$	None	0.200	0.259	4.47	(8-22)

critical section for those slabs which had openings. As shown in Table 8-6, a constant $K = 0.20$ gave a coefficient of variation of 0.26 and an average value of calculated ultimate shear stress, $v_u = 4.47\sqrt{f_c'}$ psi, so that the value $v_u = 4\sqrt{f_c'}$ used in other parts of this report appears to be a safe design value.

The r/d-ratio in the tests considered here was between 2.23 and 3.10. Additional tests are necessary to establish the strength for other values of the r/d-ratio. Until further experimental data are available, these procedures may serve as a general guide to design.

808—Shear reinforcement in slabs

Shear reinforcement transfers shear force across a diagonal tension crack. To accomplish this purpose, the shear reinforcement must be securely anchored at both ends. Generally speaking, anchorage of stirrups or bent-up bars fall into two categories. First, anchorage can be developed by transferring the force in the shear bar to other reinforcement, such as by rigidly attaching stirrups to longitudinal reinforcement or by tightly wrapping stirrups around the longitudinal reinforcement. Secondly, anchorage can be developed by transferring the force from the shear bar to the concrete by bond and bearing. The second category is exemplified by standard hooks on stirrups in which bond is controlled by embedment length and bearing is controlled by specifying minimum bend radii. Anchorage by bond and bearing should, of course, be confined to the compression zone of the concrete.

In beams of normal size, anchorage of shear reinforcement is rarely a serious problem. However, in slabs, it is a major problem which becomes more acute as the thickness of the slab diminishes. Consider a slab 6 in. thick. With normal percentages of tensile reinforcement, the depth of the compression zone will be 1 in. or less. Since the reinforcement must have at least a ¾-in. cover, it becomes impossible to anchor the shear reinforcement by bond within the compression zone of the slab.

Similarly, the bending of diagonal bars or stirrups presents difficulties in slabs. Generally speaking, the requirements for minimum radii of bends are dependent on the bending properties of the steel. However, minimum radii serve another purpose; i.e., limiting the direct compression or bearing on the concrete under the bend of the reinforcing bar. The bearing stresses on the concrete under the bend will depend on the radius of the bend and the load carried by the bar at the beginning of the bend. In beams, bends of minimum radii are easily located well above the neutral axis. A part of the load carried by the bar at a diagonal crack is transferred to the concrete by means of bond over the bar length from the diagonal crack to the beginning of the bend. Therefore, bearing stresses under bar bends are rarely a problem in beams.

However, two factors make bearing stresses an acute problem in slabs. Because of space limitations it is often difficult to have bends of minimum radii. Secondly, the diagonal cracks will cross the bar much closer to the beginning of the bend so that little of the load in the bar will be taken out by bond between the diagonal crack and the beginning of the bend. Both factors cause increased bearing stresses on the concrete under the bar bend.

Tests by Elstner and Hognestad[17] confirmed the difficulties which can be encountered with shear reinforcement in their slabs. Inability to anchor shear reinforcement within the compression zone caused the shear failure plane to pass around, rather than through, the shear reinforcement. Likewise, bearing failures of the concrete under shear bar bends were observed.

The test specimens of Elstner and Hognestad had effective depths of 4.5 in. Graf[5] reported six tests of slabs with shear reinforcement. Graf's slabs had effective depths of 10.7 and 18.7 in. Data from the 14 tests are presented in Table 8-7. The data are meager; the variables have not been fully explored. Unfortunately, the thin slabs of Elstner and Hognestad were limited to light shear reinforcement while the thick slabs of Graf were limited to heavy shear reinforcement. Therefore, it is not possible to determine the effect of poor anchorage in thin slabs since the two sources of data are not comparable on the basis of slab thickness being the only variable.

However, Table 8-7 does present interesting information. In all slabs except one, shear reinforcement increased the load capacity, and in all cases except one the ultimate load capacity was greater than either the load capacity of the concrete alone or the load capacity of the shear reinforcement alone. However, in no case was the ultimate load capacity of the slab equal to the sum of the load capacities of concrete and the shear reinforcement.

The conclusion may be drawn that the shear reinforcement was not fully effective, although this conclusion cannot be firmly supported by

TABLE 8-7 — SLABS WITH SHEAR REINFORCEMENT

Slab No.	r, in.	d, in.	$\sqrt{f_c'}$ psi	ϕ†	v_u,‡ psi	Krf_y, psi	v_u' test, psi	$\frac{v_u'}{v_u}$	$\frac{Krf_y}{v_u}$
				Elstner-Hognestad slabs[17]					
B-3	10.00	4.50	43.9	1.040	309	229	358	1.159	0.741
B-5	10.00	4.50	45.6	0.614	423	211	472	1.116	0.499
B-6	10.00	4.50	49.6	0.768	419	422	584	1.394	1.007
B-10	10.00	4.50	82.0	0.832	667	422	666	.999	0.633
B-12	10.00	4.50	81.5	0.856	653	574	982	1.540	0.879
B-15	10.00	4.50	84.2	0.743	724	638	860	1.187	0.881
B-16	10.00	4.50	81.0	0.798	673	854	932	1.385	1.269
B-17	10.00	4.50	45.8	0.888	359	229	445	1.240	0.638
				Graf slabs[5]					
1355	7.90	10.70	47.0	0.980	425	681	802	1.887	1.602
1356	7.90	10.80	47.0	1.042	410	669	846	2.063	1.632
1376	7.90	18.70	48.7	0.986	454	704	858	1.890	1.551
1377	7.90	18.70	47.0	0.966	443	704	840	1.896	1.589
1361	11.80	10.70	48.7	0.965	425	620	768	1.807	1.459
1363	11.80	18.50	48.7	0.852	479	617	778	1.624	1.288
				Moe slab with shearhead[20]					
S8-60	8.00	4.50	57.8	0.980	454	222	573	1.262	0.489

† $\phi = V_{test}/V_{flex}$
‡ v_u = shear stress calculated by Eq. (8-11), and neglecting the shear reinforcement.

test data. In the thin slabs there is little doubt that poor anchorage limited the effectiveness of the shear reinforcement. In the thick slabs the heavy shear reinforcement may have caused shear-compression failures before the shear reinforcement was fully effective.

In view of these circumstances, it is not possible at this time to recommend detailed design procedures for shear reinforcement in slabs. However, the following points can be emphasized:

1. Because of difficulties in anchorage and bending, shear reinforcement should not be permitted in slabs less than 10 in. thick.

2. Although shear reinforcement is beneficial, the required shear steel area may be abnormally large to increase load capacity even a small amount.

3. Restrictions on the use of shear reinforcement do not necessarily penalize flat plate design. The proposed design procedure is in many cases more liberal than that of the 1956 ACI Code. Furthermore, the proposed procedure permits higher shear stresses for concrete strengths exceeding 3000 psi. Therefore, it would seem more safe and practical, and possibly more economical, to increase shear capacity by using higher strength concrete rather than by using shear reinforcement.

809—Recommendations for design

Experimental investigations have indicated that the ultimate shear strength of slabs is dependent on three major variables: concrete strength $\sqrt{f_c'}$; ratio of column width to slab thickness r/d; and ratio of shear capacity to flexural capacity ϕ_o. In normal design practice, the shear capacity should be equal to or slightly greater than the flexural capacity. Therefore, ϕ_o was taken as unity in the development of design recommendations.

The variable r/d can be taken into account in two ways. First, the ultimate shear stress v_u can be expressed as a function of r/d in accordance with Eq. (8-14). The ultimate shear load capacity V_u can then be computed by Eq. (8-15), in which the critical section is the periphery around the loaded area. Secondly, the variable r/d can be taken into account by choosing a pseudocritical section which is located a distance $d/2$ from the periphery of the loaded area. The corresponding ultimate shear stress on this pseudocritical section is independent of r/d and is equal to $4.0\sqrt{f_c'}$ in accordance with Eq. (8-18). It has been pointed out that the latter method seems preferable because of its simplicity, especially for irregularly shaped loaded areas, and when openings, free edges or free corners are present in the vicinity of the loaded area. Furthermore, the second method involves a familiar concept similar to the method used in the 1956 ACI Building Code.

The following design recommendations are based on the previous discussion in this chapter:

(a) The shear strength of slabs and footings near a concentrated load or reaction is governed by the more severe of two conditions:

1. The footing or slab may act essentially as a wide beam with a potential diagonal crack extending in a plane across the entire width. This case shall be considered in accord with the recommendations made in Chapters 5, 6 and 7 of this report.

2. Two-way slab action may exist, with potential diagonal cracking along the surface of a truncated cone or pyramid around the concentrated load or reaction. This case shall be considered as described under Recommendations (b) through (j).

(b) Although the proposed concept of shear strength, when slab action is present, is based on the premises that the shear area is the vertical section which follows the periphery at the edge of the loaded area, and that the ultimate shear stress is a function of $\sqrt{f_c'}$ and r/d, an approximately equal shear strength can be evaluated by assuming that the shear area is a pseudocritical vertical section located at a distance $d/2$ from the periphery of the loaded area, and that the ultimate shear stress is a function of $\sqrt{f_c'}$ only.

It is therefore recommended that shear strength shall be computed by

$$v_u = \frac{V_u}{b_o d} \leq 4.0\sqrt{f_c'}$$

where

v_u = permissible ultimate shear stress
f_c' = compressive strength of 6 x 12 in. concrete cylinders
V_u = the ultimate shear force on a pseudocritical section of area $b_o d$
b_o = the effective periphery of the pseudocritical section at a distance $d/2$ from the periphery of the loaded area, taking into account the effect of openings, free edges or free corners in the vicinity of the loaded area
d = th effective depth of the slab at the periphery b_o

(c) Openings in slabs, free edges, and corners in the vicinity of the loaded area shall be considered by reducing the periphery of the pseudocritical section in accordance with the recommendations of Section 806. That part of the periphery of the pseudocritical section which is covered by radial projections of openings to the centroid of the loaded area shall be considered ineffective, or the shortest periphery of a critical section shall be used as outlined in Fig. 8-4.

(d) Flexural reinforcement shall be provided along the edges of all openings and extend as required for anchorage in both directions beyond the openings. However, all flexural reinforcement which would normally pass through the loaded area in slabs without openings, must be so rearranged that it continues to pass through the loaded area when openings are present.

(e) If the effective depth of the slab is less than 10 in., shear reinforcement consisting of bars, rods or wires shall not be considered effective.

(f) If the effective depth of the slab is greater than 10 in., shear reinforcement shall be permitted to carry the excess shear as described in Chapter 6, but the shear reinforcement shall be considered 50 percent effective.

(g) Concentration of flexural reinforcement in a slab over a column or column capital should be encouraged in flexural design, but the permissible ultimate shear stress shall not be increased because of concentration of reinforcement.

(h) If moment is transferred at a slab to column connection, it shall be assumed that, due to torsion, the vertical shear stresses are constant across the pseudocritical sections perpendicular to the plane of symmetry (parallel to the axis of torsion) and vary linearly on the other two pseudocritical sections parallel to the plane of symmetry (perpendicular to the axis of torsion.) The vertical shear stresses due to the total shear load shall be assumed uniformly distributed over the entire pseudocritical area $b_o d$. It shall further be assumed that the law of superposition applies so that the shear strength shall be computed by

$$v_u = \frac{V_u}{b_o\, d} + \frac{KM}{J_c}\left(\frac{c}{2}\right) \leqq 4.0\sqrt{f_c'}$$

where

M = the total joint moment on the pseudocritical peripheral section about its centroid

J_c = the polar moment of inertia of th pseudocritical peripheral section about its centroid equals $dc^3/6 + cd^3/6 + 2\,bd(c/2)^2$ for a critical section without openings or free edges

c = the side of the pseudocritical section perpendicular to the axis of torsion

b = the side of the pseudocritical section parallel to the axis of torsion

K = a reduction factor on the total moment to obtain the moment transferred by torsional shear stress, found to be 0.2 on the basis of the limited test data available, but may approach zero or take values greater than 0.2 under other conditions.

(i) It should be noted that no design recommendations are made by Committee 326 for lightweight aggregate concrete slabs and footings. Throughout this chapter, reference has been made to ordinary sand and gravel concrete only. The Committee has not considered a series of tests of lightweight aggregate concrete slabs carried out in 1961 at the PCA laboratories.

(j) To apply these recommendations in ultimate strength design, suitable safety provisions must be combined with these recommendations. Development of such safety provisions is considered beyond the scope of this Committee's mission.

810—Test data

The test data considered in this chapter on slabs and footings are presented in condensed form in Tables 8-1 through 8-7. For more detailed information, the reader is referred to the references listed in Section 811.

811—References

1. "Shear, Diagonal Tension and Torsion in Structural Concrete," to be published in the ACI Bibliography Series.

2. Talbot, A. N., "Reinforced Concrete Wall Footings and Column Footings," *Bulletin* No. 67, University of Illinois Engineering Experiment Station, Mar. 1913, 114 pp.

3. Bach, C., and Graf, O., "Tests of Square and Rectangular Reinforced Concrete Slabs Supported on all Sides," ("Versuche mit allseitig aufliegenden, quadratischen und rechteckigen Eisenbetonplatten"), *Deutscher Ausschuss für Eisenbeton* (Berlin), No. 30, 1915, 309 pp. (in German).

4. Graf, O., "Tests of Reinforced Concrete Slabs under Concentrated Load Applied Near One Support," ("Versuche über die Wiederstandsfähigkeit von Eisenbetonplatten unter konzentrierter Last nahe einem Auflager"), *Deutscher Ausschuss für Eisenbeton* (Berlin), No. 73, 1933, 28 pp. (in German).

5. Graf, O., "Strength Tests of Thick Reinforced Concrete Slabs Supported on all sides under Concentrated Loads," ("Versuche über die Widerstandsfähigkeit von allseitigen aufliegenden dicken Eisenbetonplatten unter Einzellasten"), *Deutscher Ausschuss für Eisenbeton* (Berlin), No. 88, 1938, 22 pp. (in German).

6. Richart, F. E., and Kluge, R. W., "Tests of Reinforced Concrete Slabs Subjected to Concentrated Loads," *Bulletin* No. 314, University of Illinois Engineering Experiment Station, June 1939, 75 pp.

7. Elstner, R. C., and Hognestad, E., "An Investigation of Reinforced Concrete Slabs Failing in Shear," *Mimeographed Report,* University of Illinois, Department of Theoretical and Applied Mechanics, Mar. 1953, 84 pp.

8. Forsell, C., and Holmberg, A., "Concentrated Load on Concrete Slabs," ("Stämpellast på plattor av betong"), *Betong* (Stockholm), V. 31, No. 2, 1946, pp. 95-123 (in Swedish).

9. Newmark, N. M.; Siess, C. P.; and Penman, R. R., "Studies of Slab and Beam Highway Bridges, Part I," *Bulletin* No. 363, University of Illinois Engineering Experiment Station, Mar. 1946, 130 pp.

10. Newmark, N. M.; Siess, C. P.; and Peckham, W. M., "Studies of Slab and Beam Highway Bridges, Part II, *Bulletin* No. 375, University of Illinois Engineering Experiment Station, Jan. 12, 1948, 60 pp.

11. Siess, C. P., and Viest, I. M., "Studies of Slab and Beam Highway Bridges, Part V," *Bulletin* No. 416, University of Illinois Engineering Experiment Station, Oct. 1953, 91 pp.

12. Richart, F. E., "Reinforced Concrete Wall and Column Footings," ACI JOURNAL, *Proceedings* V. 45: No. 2 and 3, Oct. and Nov. 1948, pp. 97-127 and 237-260.

13. Hahn, M., and Chefdeville, J., "Flat Slabs Without Column Capitals—Tests," ("Les Planchers-Dalles Sans Champignons—Essais"), *Annales,* Institut Technique du Bâtiment et des Travaux Publics (Paris), No. 167; Béton, Béton Armé (Paris), No. 16, Jan. 1951, pp. 23-31 (in French).

14. Thomas, F. G., and Short, A., "A Laboratory Investigation of Some Bridge Deck Systems," *Proceedings,* Institution of Civil Engineers (London), V. 1, Paper No. 5834, 1952, pp. 125-187.

15. Hognestad, E., "Shearing Strength of Reinforced Concrete Column Footings," ACI JOURNAL, *Proceedings* V. 50, No. 3, Nov. 1953, pp. 189-208.

16. Keefe, R. A., "An Investigation on the Effectiveness of Diagonal Tension Reinforcement in Flat Slabs," *Thesis,* Massachusetts Institute of Technology, June 1954, 43 pp.

17. Elstner, R. C., and Hognestad, E., "Shearing Strength of Reinforced Concrete Slabs," ACI JOURNAL, *Proceedings* V. 53, No. 1, July 1956, pp. 29-58.

18. Whitney, C. S., "Ultimate Shear Strength of Reinforced Concrete Flat Slabs, Footings, Beams, and Frame Members without Shear Reinforcement," ACI JOURNAL, *Proceedings* V. 54, No. 4, Oct. 1957, pp. 265-298.

19. Scordelis, A. C.; Lin, T. Y.; and May, H. R., "Shearing Strength of Prestressed Lift Slabs," ACI JOURNAL, *Proceedings* V. 55, No. 4, Oct. 1958, pp. 485-506.

20. Moe, J., "Shearing Strength of Reinforced Concrete Slabs and Footings Under Concentrated Loads", *Development Department Bulletin* D47, Portland Cement Association, Apr. 1961, 130 pp.

21. "Report on Concrete and Reinforced Concrete, Revised at the Meeting of the Joint Committee on Concrete and Reinforced Concrete, November 20, 1912," *Proceedings*, ASTM, V. 13, 1913, pp. 224-273.

22. "Final Report of the Joint Committee on Concrete and Reinforced Concrete," *Proceedings*, ASTM, V. 17, Part I, 1917, pp. 202-262.

23. "Report of the Committee on Reinforced Concrete Building Laws," ACI JOURNAL, *Proceedings* V. 12, 1916, pp. 171-180.

24. "ACI Standard Specification No. 23, Standard Building Regulations for the Use of Reinforced Concrete," ACI JOURNAL, *Proceedings* V. 16, 1920, pp. 283-302.

25. "Standard Specifications for Concrete and Reinforced Concrete—Joint Committee," *Proceedings*, ASTM, V. 24, Part I, Aug. 1924, pp. 312-385.

26. "ACI Standards, Building Code Requirements for Reinforced Concrete (ACI 318-56)," ACI JOURNAL, *Proceedings* V. 52, No. 9, May 1956, pp. 913-986.

27. "Specifications of the German Committee for Reinforced Concrete", ("Bestimmungen des Deutschen Ausschusses für Stahlbeton, Ausgabe 1943, DIN 1045"), Zentralblatt der Bauverwaltung, (Berlin), V. 63, No. 14/17, Apr. 1943, pp. 177-203 (in German).

28. "Standards for Reinforced Concrete Construction," (N.S. 427 Regler for utförelse av arbeider i armert betong"), *Den Norske Ingeniörforening* (Oslo), Nov. 1939, 83 pp. (in Norwegian).

29. *The Structural Use of Reinforced Concrete in Buildings,* British Standards Institution, British Standard Code of Practice CP114, General Series, Paragraph 316, 1957.

30. Correspondence between Committee 326 and R. Diaz de Cossio, National University of Mexico.

31. Correspondence between Committee 326 and N. W. Hanson, Portland Cement Association Laboratories.

32. Frederick, G. R., and Pollauf, F. P., "Experimental Determination of the Transmission of Column Moments to Flat Plate Floors," University of Toledo (Unpublished report).

33. DiStasio, J., and van Buren, M. P., "Transfer of Bending Moment Between Flat Plate Floor and Column," ACI JOURNAL, *Proceedings* V. 57, No. 3, Sept. 1960, pp. 299-314.

34. Kinnunen, S., and Nylander, H., "Punching of Concrete Slabs Without Shear Reinforcement," *Transactions,* Royal Institute of Technology (Stockholm), No. 158, 1960, 110 pp. in English.†

† This publication became available to Committee 326 too late to allow its consideration in development of this chapter.

NOTATION

The frequently used letter symbols of this report are summarized below:

A_c = area of peripheral section in slabs
A_g = gross area of the uncracked section
A_s = area of tensile reinforcement
A_v = area of shear reinforcement
a = spacing of web reinforcement in a direction perpendicular to web reinforcement; also length of shear span; also side length of square footing

b = width of cross section; also perimeter of critical peripheral section; also the side of the peripheral section parallel to the axis of torsion
b' = web width in I- and T-sections
b_o = effective perimeter of peripheral section
C, C', C_1, C_2 = constants
c = the side of the peripheral section perpendicular to the axis of torsion
d = effective depth
E_c = modulus of elasticity of concrete
E_s = modulus of elasticity of steel
e = eccentricity of axial load measured from centroid of tensile reinforcement
F_1, F_2 = constants
f_c' = compressive strength of 6 x 12-in. cylinders
f_c^* = design strength of concrete in flexural compression
f'_{cu} = compressive strength of cubes
f_s = tensile steel stress
f_t = flexural tension stress
f_t' = diagonal tension strength of concrete
f_v = stress in web reinforcement
f_y = yield point of steel
h = total depth of section
I = moment of inertia
J_c = polar moment of inertia of peripheral section about its centroid
jd = internal moment arm
K = $(\sin\alpha \cot\theta + \cos\alpha)\sin\alpha$ or $(\sin\alpha + \cos\alpha)\sin\alpha$; also a moment reduction factor
L = length of beam
l_s = shear span in slabs
M = bending moment
$M' = M - N\dfrac{4h - d}{8}$
m = ultimate resisting moment per unit width of slab
N = axial load
n = E_s/E_c
P_u = total load on loaded area
p = ratio of tensile reinforcement = A_s/bd $A_s/b'd$ for I- and T-beams
Q = first moment of part of a cross section
$q_u = \dfrac{A_v f_y \sin\alpha}{7/8\, bd f_c'}$
R = reaction
r = ratio of web reinforcement = A_v/ab; also side length of loaded area
s = spacing of web reinforcement along longitudinal axis of member
T = force in tension reinforcement
t = total depth of section
V = total shear force
V' = shear force carried by web reinforcement

V_c = shear force carried by concrete
V_{flex} = ultimate shear force for flexural failure
V_u = ultimate shear capacity
V_y = shear force at which web reinforcement yields
v = shear stress
v_c = shear stress allotted to concrete; also ultimate diagonal tension strength of beams without web reinforcement
v_{con} = shear stress in concrete compression zone
v_u = ultimate shear stress; also ultimate shear stress in members with web reinforcement
v_y = V_y/bjd
α = inclination of web reinforcement to longitudinal axis of member
β = $1 - (h/2d) - j$
θ = inclination of diagonal crack to longitudinal axis of member
ϕ_o = V_u/V_{flex}

CLOSING REMARKS BY THE CHAIRMAN

Committee 326 was formed 12 years ago and was given the assignment of "developing methods for designing reinforced concrete members to resist shear and diagonal tension, consistent with the new ultimate strength design methods."

This report consolidates thoughts, engineering judgement, and knowledge gained from engineering practice as well as extensive experimental and analytical investigations into a form believed to be useful to practicing engineers. Furthermore, safe and workable new design procedures are given. The Committee's original mission has been accomplished.

In closing this report, the Chairman wishes to express his personal appreciation to the Committee members for over a decade of active work. Engineers at home and abroad beyond the Committee membership, too numerous to be listed here, have also contributed importantly to this report. Special recognition is due members of ACI Committee 318, Standard Building Code and the European Concrete Committee, as well as Prof. C. W. Thurston of Columbia University.