JOURNAL OF THE
STRUCTURAL DIVISION

Design of Reinforced Concrete Spandrel Beams

By Thomas T. C. Hsu[1] and Kenneth T. Burton,[2] Members, ASCE

Introduction

In the 1971 ACI Building Code (2) new provisions have been incorporated for the design of reinforced concrete members subjected to torsion. These provisions can be readily applied if the torsional moments are known. For a spandrel beam in an indeterminate structure, calculation of the torsional moment requires not only statics but also compatibility conditions. In other words, the torsional stiffness of the spandrel beam must be considered. In such cases, the ACI Code tacitly implies that the torsional moment be determined by elastic analysis and the calculation of torsional stiffness be based on the uncracked sections (7). This ACI philosophy is questionable. It is well known that after cracking the torsional stiffness of a reinforced concrete member drops significantly to a small fraction of its value before cracking (6). Such a reduction will significantly affect the distribution of moments in an indeterminate structure after cracking (1,3). To take into account this moment redistribution it has recently been suggested (3) that elastic design is still valid except that the torsional stiffnesses of the spandrel beams be assumed as zero. In addition, a specified minimum amount of web reinforcement must be added to provide torsional ductility. In this design method the torsional strengths of the spandrel beams are completely neglected.

With this background we may ask such questions as: Would the torsional strength be reached if the spandrel beam is designed by the ACI philosophy? Can limit design replace elastic design to provide suitable torsional reinforcement? If so, how serious is the cracking at service load, and how much web reinforcement

Note.—Discussion open until June 1, 1974. To extend the closing date one month, a written request must be filed with the Editor of Technical Publications, ASCE. This paper is part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 100, No. ST1, January, 1974. Manuscript was submitted for review for possible publication on December 18, 1972.

[1] Prof., Dept. of Civ. Engrg., Univ. of Miami, Coral Gables, Fla.

[2] Struct. Engr., Carl Walker and Assocs., Inc., Consulting Engrs., Elgin, Ill.

is needed to ensure ductility? How do we detail the joint between spandrel beam and floor beam, etc.? To study these problems 10 T-shaped specimens representing a floor beam framing monolithically into a spandrel beam were tested. Based on the observed behavior new methods using limit design concept are suggested for the design of spandrel beams in frames.

Test Specimens

General Description.—Fig. 1 shows a portion of a three-dimensional structural frame. It includes the columns, the spandrel beam, and the floor beams. If a load, P, is applied to the floor beam it will produce a rotation at the ends which in turn induces a torsional moment in the spandrel beam. The interaction

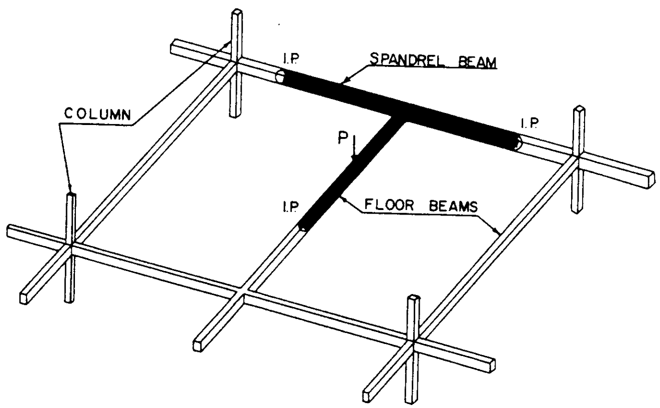

FIG. 1.—Spandrel Beams in Frame

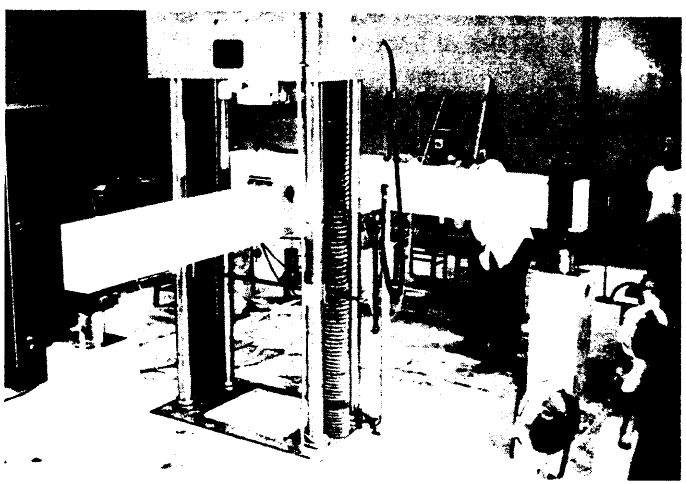

FIG. 2.—General View of Test Setup

of floor beam and spandrel beam can be studied using the shaded portion in the shape of a T-specimen. This T-specimen is cut at the inflection points which can be simulated by hinges.

Fig. 2 shows a T-specimen under test. The specimen was resting on three hinge supports and was loaded by a universal testing machine. In order to provide the torsional restraint a steel torsion arm was attached to each end of the spandrel beam and fastened to the floor. The ends of the spandrel beam were maintained torsionally fixed during the tests. This is not exactly the situation in a frame but it was adopted to simplify the analysis and testing procedures. Similar test specimens were also used in Ref. 3.

A total of 10 specimens were tested, each 9 ft (2.7 m) long from support to support for both the spandrel and the floor beams. The spandrel beam had a cross section of 6 in. × 12 in. (150 mm × 300 mm). The specimens were

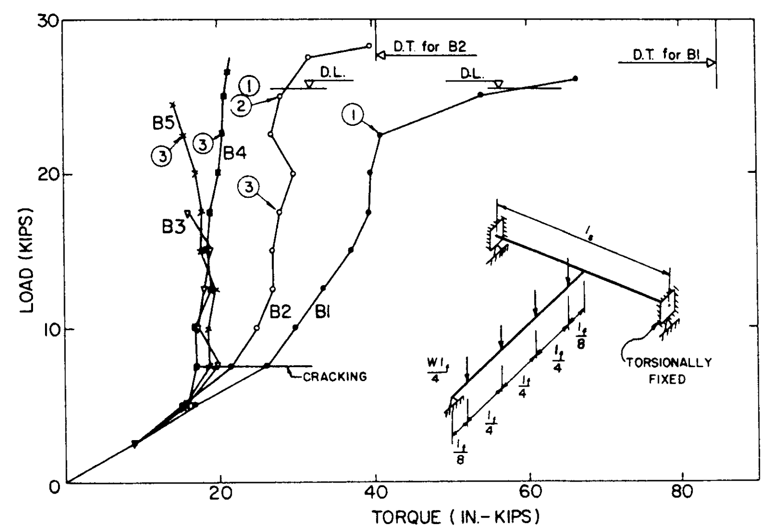

FIG. 3.—Load-Torque Curves—Series B (1 kip = 445 kN; 1 in-kip = 0.113 kN/m)

divided into two series, A and B, based on the loading and the cross section of floor beams given as follows

Series	Loading	Floor beam cross section
A	Concentrated load at midspan	6 in. × 12 in. (150 mm × 300 mm)
B	Uniform load (simulated by four concentrated loads)	6 in. × 9 in. (150 mm × 230 mm)

For all specimens in series B, the top surface of each of the floor beams was level with the top surface of the spandrel beam.

Elastic Analysis of Test Specimens.—In series A, the floor beams are loaded at midspan by a concentrated load, P, and the ends of the spandrel beams

TABLE 1.—Specimen Details—Series A (1 ksi = 6.89 MN/m²; 1 in. = 25.4 mm)

SPECIMEN	REINFORCEMENT			f_y ksi	f'_c psi
	FLOOR BEAM, MIDSPAN	FLOOR BEAM, JOINT	SPANDREL BEAM		
A1	2 #3 #3 @ 5-3/8" 3 #6	3 #3 #3 @ 5-3/8" 2 #6	2 #3 #3 @ 5-3/8" 3 #5	#3 70.3	5000
A2	2 #3 #2 @ 5-3/8" 3 #6	2 #3 #2 @ 5-3/8" 2 #6	2 #2 #2 @ 4-3/4" 2 #5 + 1 #3	#2 48.0 #3 71.2	3200
A3	2 #3 #2 @ 3-1/2" 3 #6	2 #3 #2 @ 3-1/2" 2 #6	2 #2 #2 @ 4-3/4" 2 #5 + 1 #3	#2 50.6 #3 69.1	5540
A4	2 #2 #2 @ 3-1/2" 3 #6	2 #2 #2 @ 3-1/2" 2 #6	NONE NONE 2 #5	#2 50.6	5780
A5	2 #2 #2 @ 4-1/4" 3 #6	2 #2 #2 @ 4-1/4" 2 #6	NONE 5 #2 @ 3-1/2" close to joint 2 #5	#2 50.0	4300

All cross sections are 6 in. x 12 in. All stirrup dimensions are 5 in. x 11 in. o.to o.(½"cover)
Yield strength of steel: #6 - 60.0 ksi. #5 - 62.5 ksi. Others as indicated in Table.

TABLE 2.—Specimen Details—Series B, (1 ksi = 6.89 MN/m²; 1 psi = 6.89 kN/m²; 1 in. = 25.4 mm)

SPECIMEN	REINFORCEMENT			f_y ksi	f'_c psi
	FLOOR BEAM, MIDSPAN	FLOOR BEAM, JOINT	SPANDREL BEAM		
B1	2 #2 #2 @ 4" 2 #5	2 #4 #2 @ 4" 2 #5	2 #3 #3 @ 5-3/8 2 #6	#2 50.0 #3 69.9	4380
B2	2 #2 #2 @ 4" 2 #5 + 1 #3	2 #3 #2 @ 4" 2 #5	2 #2 #2 @ 5" 2 #5	#2 50.0 #3 69.9	4120
B3	2 #2 #2 @ 4" 2 #5 + 1 #4	2 #2 #2 @ 4" 2 #5	NONE 5 #2 @ 3-1/2" close to joint 1 #5 + 2 #3	#2 50.0 #3 75.7	4030
B4	2 #2 #2 @ 4" 2 #5 + 1 #4	2 #2 #2 @ 4" 2 #5	2 #2 D1 @ 4" 3#2@4" at jt. 1 #5 + 2 #3	D1 72.0 #2 51.7 #3 77.0	4200
B5	2 #2 #2 @ 4" 2 #5 + 1 #4	2 #2 #2 @ 4" 2 #5 + 1 #4	See Fig. 6	D1 72.0 #2 53.2 #3 73.8	3610

Cross section of spandrel beam 6 in. x 12 in. Stirrup dimensions 5 in. x 11 in. o. to o. (½" cover)
Cross section of floor beam 6 in. x 9 in. Stirrup dimensions 5 in. x 8 in. o. to o. (½" cover)
Yield strength of steel: #6 - 60.0 ksi, #5 - 62.5 ksi, #4 - 80.0 ksi. Others indicated

were torsionally fixed making the structure indeterminate to the first degree. By equating the flexural end rotation of floor beam to the torsional rotation at the center of the spandrel beam the moment, M, at the joint, could be computed by

$$M = \frac{\frac{3}{16} P l_f}{1 + \frac{3}{4} \frac{K_f}{K_{ts}}} \quad \text{. . . . . (1)}$$

in which K_f = flexural stiffness of floor beam; K_{ts} = torsional stiffness of spandrel beam; and l_f = length of floor beam.

Once the moment at the joint, M, was known, all the bending moment, shear force, and torque diagrams could be drawn. Note that from conditions of equilibrium and symmetry the torque, T, in the spandrel beam is

$$T = \frac{M}{2} \quad \text{. . . . . (2)}$$

in which T is uniform on both halfs of the spandrel beam.

In Eq. 1 it can be observed that the joint moment, M, is sensitive to the ratio, K_f/K_{ts}. If $K_{ts} = \infty$, $M = (3/16)\ Pl_f$. If $K_{ts} = 0$, $M = 0$. Consequently, a drastic drop in torsional stiffness after cracking will cause a large drop in both joint moment and torsional moment.

The loading scheme of specimens in series B is shown in the insert in Fig. 3. In this series of specimens the floor beams were subjected to four concentrated loads approximating a uniform load. The joint moment for this series can be obtained from Eq. 1 if the numerator is replaced by $0.129\ wl_f^2$.

Comparison of Series A and B.—Since both the torsional moment and the angle of twist in spandrel beams were induced by the end rotation of the floor beam, the severity of the torsion problem in the spandrel beam could be observed by comparing the end rotations of floor beams in series A and B assuming simply supported at both ends. The end rotation of a floor beam in series B is much larger than that in series A for two reasons: (1) For a constant bottom fiber strain at the midspan of the floor beam, a uniform load as in series B should induce a 33% higher end rotation than the concentrated load as in series A; and (2) for a constant strain at midspan, the end rotation increases proportionally with decreasing depth of the floor beam. Since the depth of floor beam in series B was 33% smaller than those in series A, the end rotations in series B would be increased by 33% over those in series A. Combining these two effects, the spandrel beam in series B theoretically will be twisted 78% more than those in series A.

The spandrel beams in series B were not only more severely twisted, they were also more practical because a uniform load is more frequently encountered than a concentrated load. Furthermore, the depth of floor beam in series B was chosen such that the joint moment calculated by elastic analysis using uncracked sections became close to that given by the ACI moment coefficients. In the ACI Code the maximum span moment is $(1/14)\ wl'^2$ and the joint moment is $(1/24)\ wl'^2$, giving a ratio of joint moment to span moment of 0.584. For series B, computation shows that this ratio is 0.602.

Reinforcement.—All reinforcement, except no. 2 bars and D1 deformed wires, satisfies ASTM A-432 having a minimum yield strength of 60 ksi (410 MN/m^2). The deformations conformed to ASTM Designation A-305. The no. 2 bars were intermediate grade plain bars having a yield strength of about 50 ksi (340 MN/m^2). The D1 deformed steel wires used in specimens B4 and B5 had a diameter of 0.113 in. (2.87 mm) and a cross-sectional area of 0.010 sq in. (6.4 mm^2). They were specially annealed to gain a long yield plateau with a yield stress of 72 ksi (496 MN/m^2). The tensile strength, however, was not significantly higher than the yield stress. Details of reinforcement for each specimen are recorded in Tables 1 and 2.

Concrete.—The ready-mix concrete used in this investigation was 1:2.34:2.78 with a 3/8-in. maximum aggregate. The water-cement ratio was roughly 0.45 and the slump was about 3 in. (approx 80 mm). The compression strength was aimed at 5,000 psi (34 MN/m^2) at the time of testing. However, the actual strength varies from 3,200 psi (21 MN/m^2) to 5,780 psi (40 MN/m^2). The test specimens were cast in plastic-coated plywood forms. For each specimen three companion 6-in. × 12-in. (150-mm × 300-mm) cylinders for compression tests were cast in steel molds. All specimen were cured under polyethylene sheets, stripped at the age of 4 days and tested at the age of about 2 weeks to 3 weeks. The compressive cylinder strengths for each specimen at the time of testing are also recorded in Tables 1 and 2.

Instrumentations

As shown in Fig. 2, a T-specimen resting on three hinge supports was loaded by a 200-kip (890-kN) Baldwin universal machine. The hinge under the floor beam consisted of a roller and two plates. The two hinges under the spandrel beam were made of 1-1/4-in. (30-mm) diam high strength steel balls with the top plates grooved to permit longitudinal movement of spandrel beam under flexure. The hinges were placed on three compression load cells which monitored the three reactions.

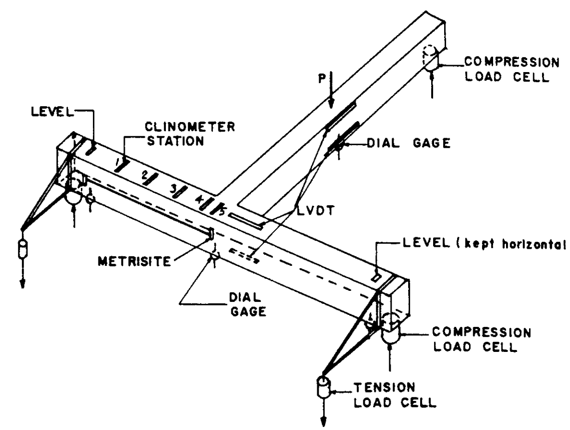

FIG. 4.—Locations of Various Instruments

TABLE 3.—Test Program and Load

Specimen (1)	Method of Design (2)	τ_u, in pounds per square inch: Design for f'_c = 5,000 psi (3)	τ_u, in pounds per square inch: Actual (4)	Actual rf_y, in pounds per square inch (5)	Designed ultimate load,[a] in kips (6)	Actual ultimate load, in kips (7)	Mode of failure[b] (8)
(a) Series A							
A1	ACI philosophy	$8.3\sqrt{f'_c}$	$8.3\sqrt{f'_c}$	478	30.4	36.8	F
A3	Limit Design	$4.0\sqrt{f'_c}$	$3.8\sqrt{f'_c}$	177	28.7	33.6	F
A5	Limit Design	$1.6\sqrt{f'_c}$	$1.7\sqrt{f'_c}$	0	28.0	19.5	T
A2	Identical to A3, except stirrups of floor beam to study shear failure of floor beam				28.7	28.5	S
A4	Identical to A5, except no stirrups at joint to study joint failure				28.0	22.7	J
(b) Series B							
B1	ACI philosophy	$8.3\sqrt{f'_c}$	$8.9\sqrt{f'_c}$	475	25.6	26.1	F
B2	Limit design	$4.0\sqrt{f'_c}$	$4.4\sqrt{f'_c}$	165	25.6	28.2	F
B3	Limit design	$1.6\sqrt{f'_c}$	$1.8\sqrt{f'_c}$	0	25.6	17.4	T
B4	Same as B3, except providing minimum stirrups			60	25.6	26.5	?
B5	Same as B4, except cutoff excessive longitudinal bars			60	25.6	24.5	T

[a]Designed for f'_c = 5,000 psi; f_y = 60 ksi for all reinforcement, except f_y = 40 ksi for No. 2 bars.
[b]F = Flexural failure of floor beam (ductile); S = Shear failure of floor beam (brittle); T = Torsional failure of spandrel beam (brittle); J = Joint failure (brittle).
Note: 1 kip = 4.45 kN; 1 psi = 6.89 kN/m^2.

A steel torsion arm was attached to each end of the spandrel beam. Each torsion arm was controlled by a hydraulic pump and the force was measured by a 10-kip (44-kN) tension load cell. The torsional moment in the spandrel beam was the product of the force and the length of the torsion arm. In order to simulate a torsionally-fixed end, an extremely sensitive level was placed on the top surface at each end of the spandrel beam as close to the torsion arms as possible, i.e. 3 in. (76 mm) from the center line of support. The locations of the levels are shown in Fig. 4. These two levels were kept horizontal throughout the test.

The torsional rotation on spandrel beam was measured by a transformer called a "Metrisite" (5) located at the middepth of spandrel beam and measured the total rotation over a length of 51 in. on one side of the spandrel beam. The output of the metrisite was continuously recorded on a Hewlett-Packet strip-chart recorder. The torsional rotation was also measured by a clinometer, which measured the angle change of the top surface. Five stations for measurement were installed as shown in Fig. 4. The stations were 12 in. (300 mm) apart except that between stations 4 and 5 which were 3 in. (76 mm) apart (half the width of floor beam). Since the spandrel beams were subjectêd to variable bending moment, the effect of bending moment on the torsional rotations could be observed by the clinometer measurements.

The concrete compressive strains and the flexural rotations at the maximum moment region of the floor beams and spandrel beams were measured by linear variable differential transformers (LVDT) and monitored by the strip chart recorder. Each LVDT measured the displacement over a length of 11 in. (280 mm), which was equal to the effective depth of the spandrel beams and floor beams in series A. The flexural rotation was computed by the sum of the diplacements of a pair of LVDT divided by the distance between them (see Fig. 4).

The stresses in the reinforcement were measured by SR-4 strain gages at various critical locations. Each specimen required about two dozen gages.

Test Program and Results

Series A.—The purposes for testing each specimen were tabulated in Table 3. Specimen A1 was designed according to the 1971 ACI Code philosophy. That means the joint moment was calculated by Eq. 1 using *uncracked* sections

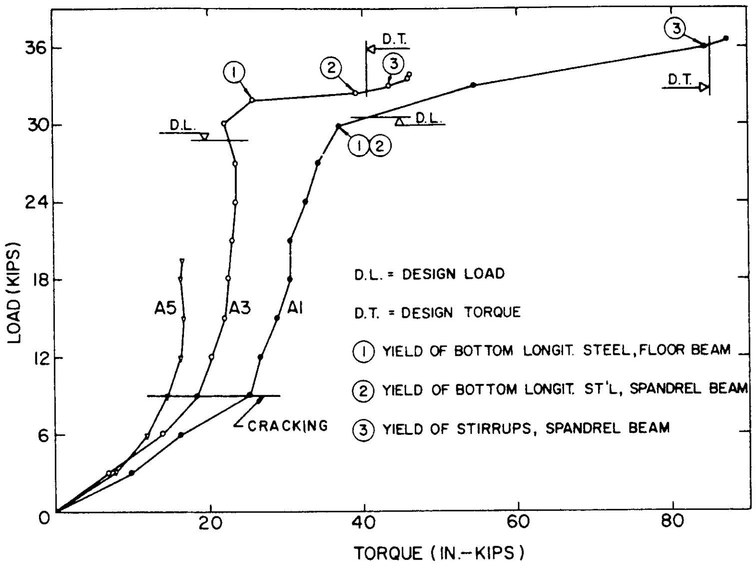

FIG. 5.—Load-Torque Curves—Series A (1 kip = 445 kN; 1 in-kip = 0.113 kN/m)

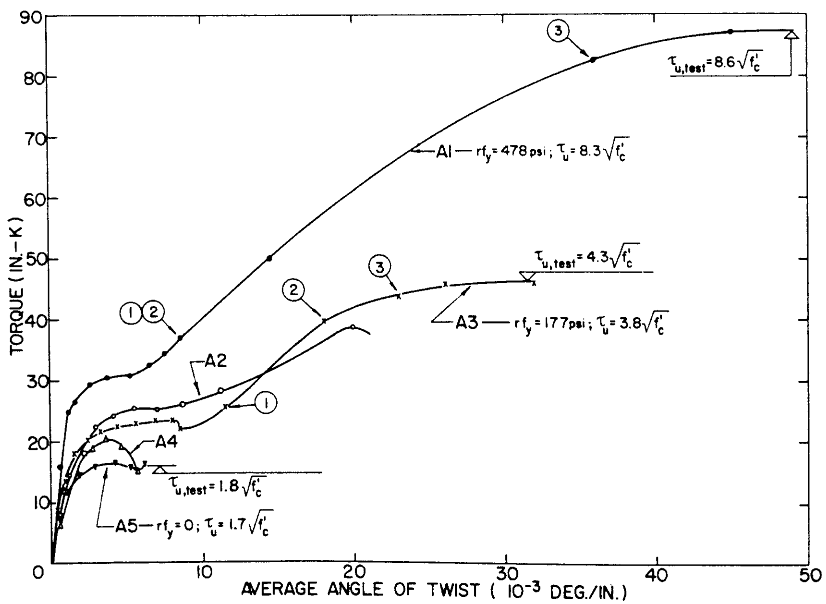

FIG. 6.—Torque-Twist Curves of Spandrel Beams—Series A (1 in-kip = 0.113 kN/m; 1°/in. = 0.688 rad/m; 1 psi = 6.89 kN/m²)

TABLE 4.—Crack Width

Specimen (1)	rf_y, in pounds per square inch (2)	Maximum Crack Width on Vertical Faces of Spandrel Beams, in inches: Service load (3)	Maximum Crack Width on Vertical Faces of Spandrel Beams, in inches: Load when stirrups yield (4)	Crack width at Junction of Two Beams, in inches: Service load (5)	Crack width at Junction of Two Beams, in inches: At 85% P_u (6)
(*a*) Series A					
A1	478	0.004	0.012	—	—
A3	177	0.005	0.011	—	—
A5	0	—	—	—	—
A2	177	0.004	No yielding of stirrups	—	—
A4	0	—	—	—	—
(*b*) Series B					
B1	475	0.005	No yielding of stirrups	0.003	0.007
B2	165	0.005	0.010	0.005	0.011
B3	0	—	—	—	—
B4	60[a]	0.007	0.020	0.010	0.045
B5	60[a]	0.016	0.055	—	—

[a] Excess longitudinal flexural steel remained in B4 but was cut off in B5.
Note: 1 psi = 6.89 kN/m²; 1 in. = 25.4 mm.

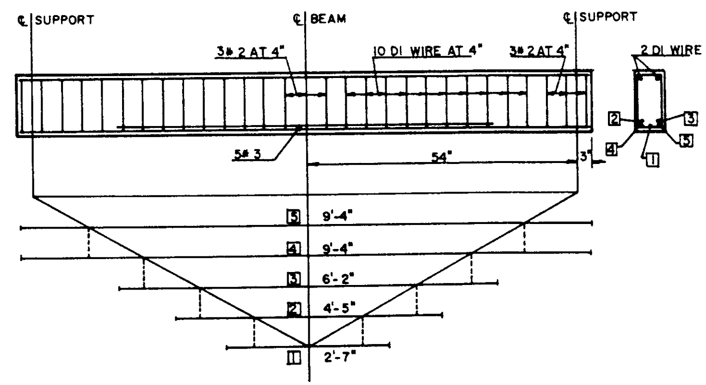

FIG. 7.—Reinforcement Arrangement in Spandrel Beam of Specimen B5 (1 in. = 25.4 mm)

for stiffnesses K_{ts} and K_f. Based on such elastic analysis the designed torsional stress, τ_u, was found to be $8.3\sqrt{f'_c}$ (τ_u was computed by code equation $\tau_u = 3T_u/x^2y$). In view of the fact that the actual concrete strengths may differ from the target strength of 5,000 psi (34 MN/m^2), the actual torsional stresses are also given. The closed stirrups of spandrel beam were designed using 1971 ACI torsion provisions. The number of stirrups was expressed by the common index, rf_y, to be 478 psi, (3,290 kN/m^2) and the design ultimate load was found to be 30.4 kips (135 kN).

Specimens A3 and A5 were designed by the limit design concept. Using this method a torsional moment was assigned to the spandrel beam by means of a specified torsional stress, τ_u. It is assumed that the spandrel beam will twist indefinitely under the specified torque until flexural failure occurs at midspan of the floor beam. For specimen A3, a torsional stress, $\tau_u = 4\sqrt{f'_c}$ was chosen for two reasons: (1) It resulted in rf_y = 177 psi (1,220 kN/m^2) which was close to 147 psi (1,000 kN/m^2) required by ACI Code for minimum reinforcement; and (2) it produced a simple arrangement of torsional steel in spandrel beams. For specimen A5, $\tau_u = 1.6\sqrt{f'_c}$. This stress was chosen because it could be resisted by concrete alone and required no web reinforcement ($rf_y = 0$). Thus, by comparing the three specimens A1, A3, and A5, the effect of web reinforcement in the spandrel beam on the behavior of test specimens can be observed. These comparisons are shown in Figs. 5 and 6.

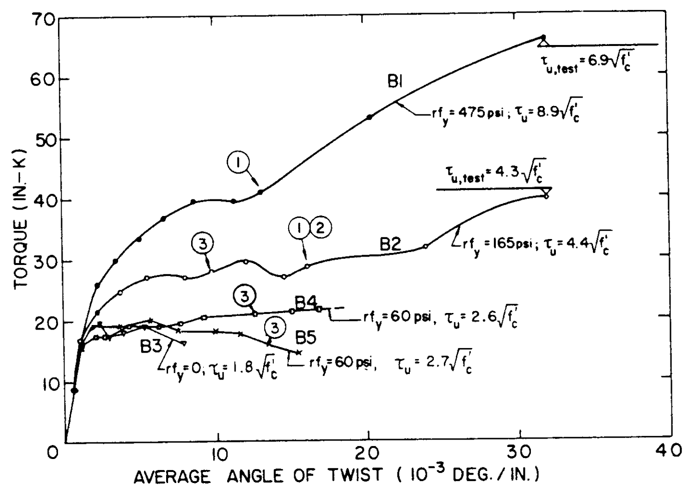

FIG. 8.—Torque-Twist Curves of Spandrel Beams—Series B (1 in-kip = 0.113 kN/m; 1°/in. = 0.688 rad/m; 1 psi = 6.89 kN/m²)

Specimen A2 was identical to A3 except for the stirrups in the floor beam. These stirrups were designed by the ACI shear provisions assuming the contribution of concrete to be $2\sqrt{f'_c}$. Since this specimen failed in shear, it was used to study the applicability of ACI shear provisions for shear design near the joint.

Specimen A4 was identical to A5 except that no stirrups were used to strengthen

the joint. Since this specimen failed at the joint, it was used to help establish a design method for the joint.

Series B.—As indicated in Table 3, specimen B1 was designed by the ACI philosophy which resulted in a high value of stirrup index, rf_y = 475 psi (3,270 kN/m^2). This corresponds to an actual torsional stress of $8.9\sqrt{f'_c}$. Specimen B2 was designed by limit design concept assuming a torsional stress of $4\sqrt{f'_c}$ which resulted in an actual stirrup index of rf_y = 165 psi (1,140 kN/m^2). Similarly, specimen B3 was designed by limit design concept but assuming a torsional stress of $1.6\sqrt{f'_c}$. This small amount of torsional stress required stirrups.

Since specimen B3 exhibited a brittle failure below the design load, a small amount of stirrups, rf_y = 60 psi (414 kN/m^2), was provided in both specimens B4 and B5. This amount of stirrups corresponds to torsional stresses of $2.6\sqrt{f'_c}$ and $2.7\sqrt{f'_c}$ for Specimens B4 and B5, respectively. In specimen B4 (as well as in specimens B1-B3) longitudinal bottom bars necessary to resist maximum moment at midspan were extended throughout the beam. However, in specimen B5, five no. 3 longitudinal bottom bars were cut off according to moment and development length requirements as shown in Fig. 7. The cutoff points satisfied ACI 318-71, section 12.1.6.1 governing cutting off reinforcement in the tension zone.

Test data for series B are given in Tables 3 and 4 and in Figs. 3-8.

Analysis

An analysis of the test results of the 10 specimens permit us to examine six questions.

Question 1.— Is the 1971 ACI design philosophy applicable to spandrel beams? This question can be answered by examining the strength and behavior of specimen A1 and B1. These two specimens were designed by the ACI philosophy. For specimen A1 the relation between load and torque is shown in Fig. 5. The curve for A1 can be idealized into three stages. The first stage represents the elastic behavior before cracking. In the second stage, the development of cracks caused the torsional stiffness to decrease drastically and the torsional moment remained essentially constant while the load increased. When the load reached the level where yielding of the bottom longitudinal steel of floor beam occurred as indicated by 1, the third stage began. In the third stage a plastic hinge developed under the load and accelerated the twisting of the spandrel beam. The torsional moment again increased. This increase of torsional moment was mainly responsible for the increase in load. (Strain hardening of longitudinal bars in the floor beam may also be involved.) Fig. 5 showed that both the design torque and the design load have been reached. The stirrups and the longitudinal bars in the spandrel beam also yielded before failure.

The behavior of specimen B1 was essentially the same as that of A1. However, Figs. 3 and 8 showed that the design torque was not reached and that neither the stirrups nor the longitudinal steel in the spandrel beam yielded. Since the ultimate torque for B1 was not too far below the design value and that the design load has indeed been reached (see Fig. 3), the design may be considered safe from the viewpoint of ultimate load.

Although the determination of torsional moment by the ACI philosophy gave safe designs, it had two disadvantages. First, the method resulted in large torsional stresses and a correspondingly large amount of web reinforcement for spandrel beams. The torsional stresses were 8.3 $\sqrt{f'_c}$ and 8.9 $\sqrt{f'_c}$ for specimens A1 and B1, respectively, which were very high when compared to the maximum allowable stress of 12 $\sqrt{f'_c}$. From another point of view, the web reinforcement index, rf_y, of 478 psi (3,290 kN/m^2) and 475 psi (3,270 kN/m^2) were used for A1 and B1, respectively. These are also very high when compared to the minimum shear stirrup index of 50 psi (340 kN/m^2). It is well known that in resisting moment, torsional reinforcement is many times less efficient than flexural reinforcement. Thus, the use of a large amount of torsional reinforcement to carry external load in a structural system is certainly not economical. Second, the ultimate torsional strength of a spandrel beam was obtained at a large angle of twist. Whether such torsional strength can be relied upon is a matter of conjucture.

In conclusion, although the ACI philosophy is safe from an ultimate load point of view, it may be uneconomical. Thus, it is desirable to have another design method which will provide a more reasonable amount of torsional reinforcement in the spandrel beam.

Question 2.—Can the limit design concept be used to provide a reasonable amount of torsional reinforcement in spandrel beams?

Examination of torque-twist curves in Figs. 6 and 8 shows that for all specimens after cracking the torque will reach a plateau at which the angle of twist increases rapidly without an increase in torque. For specimens A5 and B3 in which no torsional reinforcement was provided in the spandrel beams, the spandrel beam failed abruptly at a small angle of twist. However, if sufficient torsional reinforcement was provided as in specimens A3 and B2 a long plateau could be expected. This plateau can be utilized as a plastic hinge in limit design.

The applicability of the limit design concept was confirmed by specimens A3 and B2. For example consider specimen A3 which was designed by the limit design concept assuming a torsional stress of 4 $\sqrt{f'_c}$. The torque-twist curve shows that the ultimate torque can easily be reached and the load-torque curve shows that the design load has been exceeded. The angle of twist of the spandrel beam was large enough so that the specimen failed by flexure at midspan of the floor beam. Examination of cracks also revealed no excessive crack width at any load stage (see Table 4). Finally, this specimen has a stirrup index of rf_y = 177 psi (1,220 kN/m^2) which was only 37% of 478 psi (3,290 kN/m^2) used in specimen A1 designed by ACI philosophy.

Now that the limit design concept assuming a certain torsional stress has been shown to be both feasible and economical we proceed to ask the third question.

Question 3.—What is the minimum amount of stirrups necessary to provide torsional ductility for spandrel beams and to prevent excessive cracking?

To consider this problem a brief historical review seems appropriate. In 1966, based on PCA tests in pure torsion (6), the minimum amount of closed stirrups necessary to provide torsional ductility was found to be about 0.5% by volume for both longitudinal steel and stirrups. This percentage of stirrups can be represented by a stirrup index, rf_y, equal to 200 psi (1,400 kN/m^2), a value which is four times that of the stirrup index required for shear only. We now have two stirrup indices, end values covering the two cases of shear without torsion at the lower end and torsion without shear at the upper end. To complete the rf_y index between the end values, the ACI Committee on torsion proposed an empirical equation (9) making the minimum stirrups a function of the ratio of torsion to shear.

Shortly after the presentation of this proposed equation, Ferguson (1) suggested that rf_y = 60 psi (414 kN/m^2) was sufficient for all ratios of torsion to shear. In examining his tests the ACI Committee on torsion observed that all Ferguson's specimens included an excessive amount of longitudinal steel which helped to compensate for the lack of stirrups. Thus, the ACI Code equation was further modified to permit the use of rf_y = 50 psi (350 kN/m^2) for stirrups, provided the longitudinal steel is increased so that the total amount of torsional reinforcement is unchanged (2). Recently, Collins and Lampert (3) suggested that spandrel beams be designed assuming zero torsional stiffness. To ensure torsional ductility a minimum stirrup index of rf_y = 50 psi (350 kN/m^2) should be provided regardless of the torque to shear ratio or the longitudinal steel. However, their specimens so designed have utilized steel meshes as web reinforcement. The maximum flexural steel requirement at the midspan was often liberally increased and was extended throughout the spandrel beams.

The five specimens in series B provided good insight into the problems of minimum stirrups in spandrel beams. All five specimens were designed for an ultimate design load of 25.6 kips (114 kN). The difference lay in the amount of stirrups in the spandrel beams. An evaluation of the behavior of these five specimens should be based on the following three considerations:

1. Mode of failure—Table 3 shows that specimens B1 and B2 with rf_y of 475 psi (3,270 kN/m^2) and 165 psi (1,140 kN/m^2), respectively, exhibited ductile failure in flexure of floor beams. On the other hand, specimens B5 and B3 with rf_y of 60 psi (410 kN/m^2) and 0 psi exhibited brittle failure in torsion of spandrel beam. It was unfortunate that specimen B4 with rf_y = 60 psi (410 kN/m^2) and with excess longitudinal steel failed prematurely at the simple support of floor beam due to a mistake in detailing the reinforcement. Judging from the fact that the ultimate design load was reached and that yielding was imminent in the flexural steel at the maximum moment location of the floor beam, it was evident that specimen 4 would have failed in flexure of the floor beam. However, the information on the amount of ductility was lost.

Examination of the failure modes of specimens in series B thus shows that: (1) rf_y = 60 psi (410 kN/m^2) is definitely insufficient to maintain ductility without excess longitudinal steel (specimen B5); and (2) rf_y = 165 psi (1,140 kN/m^2) provides sufficient ductility (specimen B2).

2. Excessive crack width at service load—Service load is defined herein as one-half the ultimate design load or actual ultimate test load, whichever is larger. Table 4 showed that maximum crack width on the vertical faces of spandrel beams at service load was about 0.005 in. for specimens B1 and B2. In other words, the maximum crack width at service load remained approximately the same for rf_y above 165 psi (1,140 kN/m^2). However, when rf_y was reduced to 60 psi (410 kN/m^2) as in specimen B4, maximum crack width at service load increased to about 0.007 in. (0.18 mm). Maintaining rf_y = 60 psi (410 kN/m^2) but cutting off the excess longitudinal steel as in specimen B5 increased the maximum crack width at service load to 0.016 in. (0.41 mm). Thus, rf_y = 165 psi (1,140 kN/m^2) is approximately the stirrup index below which the

maximum crack width at service load begins to increase.

3. Premature yielding of stirrups in spandrel beam—It has been observed that yielding of stirrups in a spandrel beam causes a sudden and rapid increase in crack width. To avoid such excessive cracking it is desirable that yielding of stirrups in a spandrel beam (represented by 3 in Figs. 3-8) should not occur before the yielding of flexural steel in floor beam (represented by 1 in Figs. 3-8). Examination of Figs. 3 or 8 show that in specimens B4 and B5, rf_y = 60 psi (410 kN/m^2), yielding of stirrups in the spandrel beam occurs long before the yielding of flexural steel in floor beam. This behavior is undesirable. On the other hand, for specimen B1, rf_y = 475 psi (3,270 kN/m^2), stirrups in the spandrel beam do not yield even at failure of the floor beam. In this case the behavior is justified but the overdesign of stirrups in spandrel beam is uneconomical.

For specimen B2 with rf_y = 165 psi (1,140 kN/m^2) Figs. 3 or 8 show that 3 occurred before 1. However, it must be realized that the No. 2 plain bars used for stirrups in the spandrel beam of specimen B2 had a yield stress of 50.0 ksi (340 kN/m^2) which is 20% less than the yield stress of 62.5 ksi (431 kN/m^2) for the No. 5 bars used as the flexural steel of the floor beam. If identical steels had been used for both stirrups and flexural reinforcement, yielding would have occurred at approximately the same load stage. Thus, to avoid premature yielding of stirrups in a spandrel beam, it is desirable that rf_y should be about 165 psi (1,140 kN/m^2). This is roughly the same as the minimum stirrup index, rf_y = 143 psi (985 kN/m^2) required by the 1971 ACI Code for specimen B2.

In view of the preceding three considerations it is felt that specimen B2 represents the most acceptable design. In conclusion, for the limit design method a torsional stress of 4 $\sqrt{f'_c}$ may be specified. This torsional stress should give sufficient strength and ductility as well as adequate crack control.

Question 4.—How can we design the negative steel of floor beam at the joint?

In all specimens the negative steel of the floor beam was designed using the joint moment $M = 2T$ required by Eq. 2. Tests however, revealed that none of the negative steel yielded at failure. For some specimens these stresses actually reached a maximum at some load stage before failure and decreased noticeably as failure was approached. The maximum stresses varied in the range of 10 ksi to 40 ksi (69 MN/m^2 to 280 MN/m^2).

These interesting observations can be easily explained. Eq. 2 is based on the assumption that the spandrel beam is simply a straight line. In reality a spandrel beam has width, and the reaction of the floor beam, R, will induce an additional moment, $R\,e$, with respect to the longitudinal axis of the spandrel beam, e being the eccentricity of the reaction. Statics then requires that

$$M + R\,e = 2\,T \quad (3)$$

This additional moment, $R\,e$, is usually quite large and causes drastic reduction of the moment, M, to be carried by negative steel. This explains the small stresses in the negative steel observed in tests.

In the limit design concept the torque specified will be small. Therefore, use of Eq. 2, in which $R\,e$ is neglected, should be acceptable as illustrated by specimens A3 and B2. since statics usually requires small percentage of steel, the design of negative steel for a floor beam should also consider crack control.

To study the crack width at the end of floor beam due to negative moment, a comparison of specimen B1, B2, and B4 as shown in Table 4 is enlightening. For specimen B1 which has two No. 4 negative steel bars (see Table 2), the crack width at service load was 0.003 in. (0.08 mm) and increased slowly to 0.007 in. (0.18 mm) at about 85% of ultimate load. These crack widths are smaller than either the maximum flexural cracks in the floor beam or the maximum torsional cracks in the spandrel beam for all load stages. For specimen B2 with two No. 3 negative bars, the crack width at service load was 0.005 in. (0.13 mm) and increased somewhat faster to 0.011 in. (0.28 mm) at about 85% ultimate load. These crack widths are approximately the same as the maximum crack widths in the floor beam and the spandrel beam at various load stages. Finally, specimen B4 has two No. 2 plain bars and the crack width at service load was 0.010 in. (0.25 mm) which is excessive. Moreover, this crack increased rapidly to 0.045 in. (1.14 mm) at 85% ultimate load.

The preceding comparison shows that two No. 3 bars, which corresponds to 0.46% of steel, are roughly the lower limit for adequate crack control. It is interesting to compare this minimum steel with that of $200/f_y$ specified by the ACI Code. First, tests have shown that the negative steel at the floor beams did not yield. Thus, for crack control the minimum percentage of steel should not be a function of yield strength. Second, it was observed that the maximum stress did not exceed 40 ksi (approx 280 MN/m^2). Using 40 ksi (approx 280 MN/m^2) as the yield strength the minimum steel specified by the ACI Code became 0.50%, which is slightly higher than 0.46%, as indicated by specimen B2. For these reasons it is suggested that a minimum percentage of negative steel in floor beams may be taken as 0.45%. More research may show that this minimum steel can be somewhat reduced.

Question 5.—Is the ACI shear provisions applicable to the floor beam near the joint?

Two specimens A2 and A3 have been designed which were identical except for the shear design of floor beams. In specimen A2 the shear reinforcement was designed according to the ACI Code where the contribution of concrete was taken as 2 $\sqrt{f'_c}$. The calculated nominal shear stress was 263 psi (1,800 kN/m^2). Surprisingly, this beam failed in shear with a test shear stress of 227 psi (1,560 kN/m^2). The ratio of test value to calculated value is 0.86 which means 14% understrength. The failure region is at the end of floor beams near the joint as shown in Fig. 9.

In view of this shear failure, the floor beam of specimen A3 was designed by reducing the contribution of concrete to 1 $\sqrt{f'_c}$. Specimen A3 then failed in flexure which was desirable.

The reason for the shear failure of A2 became quite obvious if the condition of support for the floor beam was examined. In a normal support, the reaction was supplied underneath the beam through bearing. The ACI Code shear provision is based mainly on tests using this type of support. For the floor beam in this investigation the reactions were supplied by shear from the spandrel beam. Such reactions created a more severe diagonal tension problem in the floor beams. In other words, the so-called compression arch at the support became shallower and the shear carring capacity was reduced.

The inadequacy of the shear provision for beams framing monolithically into another beam has been pointed out in a series of tests by Ferguson (4) in 1956. It seems appropriate to again call attention to this problem for more study. In the meantime, for the monolithic floor beam, it may be wise to follow the European philosophy by neglecting completely the contribution of concrete in resisting shear.

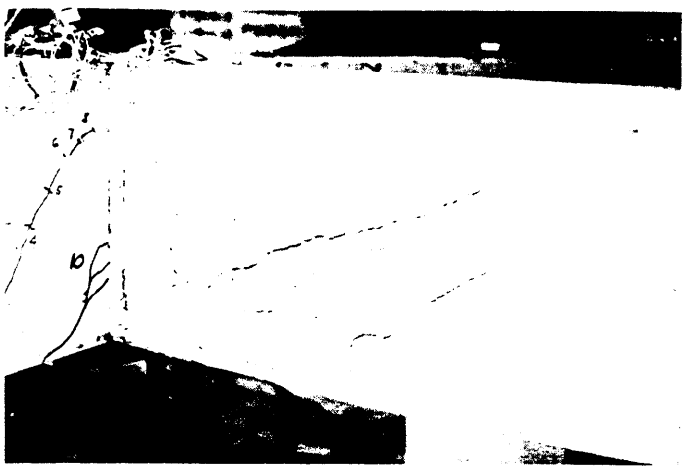

FIG. 9.—Shear Failure of Specimen A2

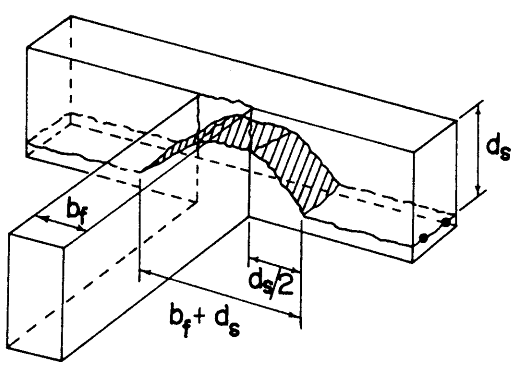

FIG. 10.—Failure Surface at Joint of Specimen A4

Question 6.—How can we detail the joint between floor beam and spandrel beam?

Specimen A4 was designed to fail at the joint by providing no shear or torsional reinforcement in the spandrel beam or joint. The failure surface is shown in perspective in Fig. 10 as indicated by the shaded surface. It can be seen that a vertical crack first occurred between the floor beam and spandrel beam due to negative moment at the end of the floor beam. The crack extended about

half way down the depth and caused failure to occur suddenly along the failure surface indicated. A secondary failure followed by tearing the bottom layer of concrete along the longitudinal bars. The failure surface consisted of a horizontal plane and two inclined planes. The horizontal projection of each inclined plane can be conservatively taken as one-half of the effective depth of spandrel beam, $d_s/2$.

In order to prevent this type of joint failure, stirrups should be provided near the joint to intersect the failure surface within a distance $b_f + d_s$, in which b_f is the width of floor beams (see Fig. 10). The number of stirrup should be sufficient to carry the reaction from the floor beam. Thus

$$V_u = \frac{b_f + d_s}{s} A_v f_y \qquad (4)$$

in which V_u = ultimate joint force transferred from the floor beam to spandrel beam; s = the spacing of stirrups within the failure zone; A_v = area of one stirrup (including two legs if exist); and f_y = yield strength of stirrups.

Specimen A3 was designed according to this formula and V_u was found to be 18.1 kips (80.5 kN). Tests of specimen A3 showed that the joint can at least carry a joint force of 17.2 kips (76.5 kN) without joint failure (as shown in Table 3 this specimen failed by flexure at midspan of floor beam). This joint force was only 5% less than the calculated value.

A practical rule was also observed in detailing the joint. When steel bars intersect each other at the joint, those coming into the joint from the floor beam are placed above those from the spandrel beam. This rule is really common sense in engineering.

It is interesting to mention that Leonhardt (8) has suggested placing closely spaced stirrups in the joint to "hang-up" 100% of the load transmitted from the floor beam. If this suggestion is followed, it is often necessary to place several stirrups in the joint. Apparently such severe requirement is not followed by practicing engineers and is not supported by tests in this investigation.

It is also interesting that Collin's and Lampert's test specimens (3) have incorporated Leonhardt's suggestion. Even with such severe requirement for stirrups, it was reported that failure occurred at the joint of one specimen. After carefully examining their photographs, it was found that the specimen which supposedly failed at the joint appeared to have failed by shear near the joint without destroying the integrity of the joint.

Conclusions

1. The application of ACI design philosophy to reinforced concrete spandrel beam is safe but uneconomical. According to this philosophy torsional moments are calculated by the elastic method based on uncracked sections and the cross sections are designed by ultimate strength method.

2. Design of spandrel beams using the limit design concept is both feasible and desirable. In using the limit design concept, the spandrel beam may be assumed to carry a torsional stress of $4\sqrt{f'_c}$.

3. The negative flexural steel in the floor beam at the joint can be determined from statics assuming a no-width spandrel beam. The minimum steel ratio may be taken as 0.45% for crack control.

4. A simple formula (Eq. 4) is suggested for the design of stirrups to prevent joint failure.

5. More research is needed to study the shear design of a floor beam framing into a spandrel beam. However, for design purpose, it is safe to design shear reinforcement of the floor beam by neglecting the contribution of concrete in resisting shear.

Suggested Design Criteria for Spandrel Beams

1. If torsion in a member arises from maintaining compatibility with other flexural members framing into it, such as the spandrel beams in a statically indeterminate structure, a design torsional stress of 4 $\sqrt{f'_c}$ may be assumed at the sections located less than a distance, d, from the face of the support. The moment transferred from a slab or closely spaced beams may be assumed uniform along the length of the torsional member.

2. When a floor beam is framed monolithically into a spandrel beam, the cross-sectional area of stirrups in the spandrel beam at the joint within a distance, $d_s/2$ from the faces of the floor beam should not be less than

$$A_v = \frac{V_u s}{(b_f + d_s) f_y} \quad (5)$$

in which V_u = ultimate joint force transferred from the floor beam to spandrel beam; s = spacing of stirrups; b_f = width of floor beam; d_s = effective depth of spandrel beam; and f_y = yield strength of stirrups.

Acknowledgment

This investigation was sponsored by the National Science Foundation under Grant No. GK-17797 to the University of Miami. The writers wish to thank Flavio Gomez, and Nguyen-Tien Thinh for helping in the experimental work.

Appendix I.—References

1. Behera, U., Rajagopalan, K. S, and Ferguson, P. M., "Reinforcement for Torque in Spandrel L-Beams," *Journal of the Structural Division*, ASCE, Vol. 96, No. ST2, Proc. Paper 7103, Feb., 1970, pp. 371-379.
2. *Building Code Requirements for Reinforced Concrete*, ACI 318-71, American Concrete Institute, Detroit, Mich., 1971.
3. Collins, M. P., and Lampert, P., "Redistribution of Moments at Cracking—The Key to Simpler Torsion Design?," *Civil Engineering Publication, 71-21*, University of Toronto, Toronto, Canada, Feb., 1971.
4. Ferguson, P. M., "Some Implications of Recent Diagonal Tension Tests," *Journal of the American Concrete Institute*, Vol. 28, No. 2., *Proceedings*, Vol. 53, Aug., 1956, pp. 157-172.
5. Hsu, T. T. C., and Mattock, A. H., "A Torsion Test Rig," *Journal of the Portland Cement Association Research and Development Laboratories*, Vol. 7, No. 1, Jan., 1965, pp. 2-9.
6. Hsu, T. T. C., "Torsion of Structural Concrete—Behavoir of Reinforced Concrete Rectangular Members," *Torsion of Structural Concrete*, Special Publication SP-18, American Concrete Institute, Detroit, Mich., 1968, pp. 261-306.
7. Hsu, T. T. C., and Kemp, E. L., "Background and Practical Application of Tentative Design Criteria for Torsion," *Journal of the American Concrete Institute*, Vol. 66, No. 1, Jan., 1969, pp. 12-23.
8. Leonhardt, F. "Die Verminderte Schubdeckung Bei Stahlbetontragwerken," *Der Bauingenieur*, Vol. 40, No. 1, Jan., 1965.
9. "Tentative Recommendations for the Design of Reinforced Concrete Members to Resist Torsion," ACI Committee 438, *Journal of the American Concrete Institute, Proceedings*, Vol. 66, No. 1, Jan., 1969, pp. 12-23.

Appendix II.—Notation

The following symbols are used in this paper:

A_v = cross-sectional area of one stirrup, including two legs if exist;
b_f = width of floor beam;
b_s = width of spandrel beam;
d_s = effective depth of spandrel beam;
e = eccentricity of floor beam reaction with respect to longitudinal axis of spandrel beam;
f'_c = cylinder compressive strength of concrete;
f_y = yield strength of reinforcement;
K_f = flexural stiffness of floor beam;
K_{ts} = torsional stiffness of spandrel beam;
l_f = length of floor beam;
l_s = length of spandrel beam;
l' = clear span of floor beam;
M = joint moment between floor beam and spandrel beam;
P = concentrated load on floor beam in Series A;
r = A_v / b_s = stirrup ratio;
R = reaction of floor beam at joint;
s = spacing of stirrups in spandrel beam;
T = torque in spandrel beam;
T_u = ultimate torque in spandrel beam;
V_u = ultimate joint force transferred from floor beam to spandrel beam;
x = smaller dimension of rectangular cross section;
y = larger dimension of rectangular cross section;
w = uniform load on floor beam in Series B; and
τ_u = $3T_u/x^2 y$ = design torsional stress.