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Unified Design Provisions for Reinforced and Prestressed Concrete Flexural and Compression Members

by Robert F. Mast

Presents proposed code changes which are presently under consideration by ACI Committee 318. The key concept is the definition of the strength reduction factor ϕ in terms of the maximum steel tensile strain at nominal strength. Permissible moment redistribution is also defined in terms of maximum steel strain at nominal strength. The proposed method applies equally to beams and columns, reinforced and/or prestressed. It also applies to composite sections, sections of any shape, and sections with steel distributed throughout the section depth. Nominal strength calculations are not changed, but the ϕ factor applied to nominal strength is affected in the transition region between compression and tension controlled sections.

Keywords: building codes; compressive strength; ductility; **flexural strength;** moment distribution; prestressed concrete; reinforced concrete; standards; strains; strength reduction factor; structural design; **structural members; unified design.**

Historically, different design methods and rules were developed for beams and for columns. Also, design procedures for prestressed concrete were different from those for conventionally reinforced concrete. In recent editions of ACI 318, attempts were made to partially unify these design requirements. The purpose of this paper is to propose modifications to the code requirements that would unify and simplify the design requirements for reinforced and prestressed flexural and compression members. This is to be accomplished without complicating the cases that are now simple, i.e., the design of a rectangular section with one type and layer of reinforcement. The proposed modifications do not alter nominal strength computations. They unify the determination of the appropriate strength reduction factor for reinforced and prestressed compression and tension controlled sections, as well as for intermediate cases. They also unify the treatment of moment redistribution in continuous beams of reinforced and prestressed concrete.

Discussion

"Unified Design Provisions for Reinforced and Prestressed Concrete Flexural and Compression Members," by Robert F. Mast, has been reviewed by members of Committee 318, and is published at the request of Committee 318 to elicit discussion from practicing engineers before it is considered for changes to the ACI Building Code for Reinforced Concrete Construction (ACI 318).

Discussion will appear in the September-October 1992 *ACI Structural Journal* if received by June 1, 1992.

SUMMARY OF PROPOSED CODE CHANGES

The key change proposed is a modification of Section 10.3.3, which defines the maximum reinforcement limit for flexural members. The concept of flexural members is replaced by the concept of tension controlled sections. A concept of compression controlled sections is also created. Compression and tension controlled sections are defined in terms of the tensile strain in the reinforcement at nominal strength. A compression controlled section is defined as one having a maximum net tensile strain in the steel of 0.0025 or less, and a tension controlled section is defined as one having a maximum net tensile strain in the steel of 0.0050 or more.

The provisions of Section 9.3.2 defining the capacity reduction factor ϕ are also revised. Currently, there is one factor (0.90) for flexure without axial load, and another factor (0.70 or 0.75) for axial load and axial load combined with flexure, with a transition based on the ratio $\phi P_n/f_c A_g$. This is replaced by the use of ϕ = 0.90 for tension controlled members and ϕ = 0.7 (or 0.75) for compression controlled members, also with a transition region for intermediate cases.

Moment redistribution provisions of Sections 8.4.3 and 18.10.4 are also redefined in terms of the maximum net tensile strain in the steel.

The new provisions apply equally to beams and columns of rectangular or nonrectangular section, reinforced and/or prestressed, with one or many layers of steel.

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EFFECT OF PROPOSED CODE CHANGES

The ACI Building Code and Commentary (ACI 318/ACI 318R), and computations based on the code, can be substantially simplified by using strain conditions to define the boundaries of tension controlled behavior and compression controlled behavior, and by a change in the definition of the depth used in setting those boundaries. These new definitions are also used to set the strength reduction factor. With these changes, the new provisions apply equally to:

1. Flexural members and compression members.
2. Conventionally reinforced sections, prestressed sections, and sections with both types of reinforcement.
3. Sections with steel located at various depths within the section.
4. Sections of any shape.
5. Composite sections with more than one type of concrete.

THE CURRENT SITUATION

The 1989 ACI 318 Building Code[1] (hereafter referred to as the "Code"), Section 10.3.2, defines balanced strain conditions as follows:

> Balanced strain conditions exist at a cross section when tension reinforcement reaches the strain corresponding to its specified yield strength f_y just as concrete in compression reaches its assumed ultimate strain of 0.003.

This is a time-honored definition based on the concept that balanced conditions occur when concrete crushing and steel yielding occur simultaneously.

The primary use of the definition of balanced strain conditions is to set limits on reinforcement in flexural members and to set limits on moment redistribution. In both cases, the ratio of provided-to-balanced reinforcement is used as a measure of ductility.

The balanced reinforcement provisions of the Code are easily applied to rectangular, noncomposite sections with one layer of tension reinforcement. The provisions are more cumbersome for flanged and other nonrectangular sections, and sections with compressive reinforcement (see the formulas in the 1983 commentary[2] to Section 10.3). Separate equations must be developed for each individual shape of cross section. The reinforcement limits for prestressed flexural members (Code Section 18.8.1) are more complex, particularly when both prestressed and nonprestressed reinforcements are used. And, the Code provisions for prestressed flanged sections are not consistent with the provisions for conventionally reinforced flanged sections. When one considers composite sections, sections with more than one layer of tension reinforcement, and members that are borderline flexural or compression members, further questions arise, specifically:

1. The Code (Section 10.3.3) prohibits tensile reinforcement ratios in excess of 0.75 ρ_b for members with axial loads less than $0.10 f_c' A_g$, but seems to permit higher tensile reinforcement levels for members with larger axial loads. This does not seem logical. For prestressed concrete, the Code (Section 18.8.1) gives a maximum reinforcement limit that is closer to balanced reinforcement than to the maximum limit of 0.75 of balanced given in Section 10.3.3. But, the Code allows one to use higher amounts of prestressed reinforcement provided "design moment strength shall not exceed the moment strength based on the compression portion of the moment couple." This is an unusual statement, since a couple is defined as two parallel and opposite forces of *equal magnitude.* These problems point to the need for a better definition of tension controlled (flexural) members and compression controlled (compression) members.

2. The current limits on tensile reinforcement in flexural members involve the tensile reinforcement ratio ρ for nonprestressed sections and the reinforcement index ω for prestressed sections. Both of these quantities require the use of the correct width b and depth d in their computation. This is simple enough for rectangular sections with one layer of tensile reinforcement. The computation is more difficult for flanged and other nonrectangular sections, and is very difficult for sections in which the compressive stress block falls partially within a tapered web or flange, as sometimes happens in prestressed members. The determination of ρ and ω for circular and polygonal sections is particularly challenging.

3. The definitions of the depth d and d_p are different for nonprestressed and for prestressed sections. For nonprestressed sections, d is defined as "the centroid of the tension reinforcement." This definition is easy to apply for the usual case in which the tensile reinforcement is concentrated near the tension face. But for deep sections with reinforcement distributed through the depth, it is not readily apparent which reinforcement is tension reinforcement. And, when checking for maximum ρ, it is neither correct nor conservative to neglect the reinforcement above the main reinforcement, because its inclusion increases the steel area and decreases the depth and thus increases ρ.

For prestressed concrete, d_p is defined as "the centroid of prestressed reinforcement." This means *all* the prestressed reinforcement, including that in the compression zone. But, A_{ps} is defined as the "area of prestressed reinforcement in the *tension* zone" (emphasis added). And ρ_p is defined as A_{ps}/bd_p. This is clearly inconsistent. Unfortunately, it is not totally correct to simply redefine d_p to relate to prestressed reinforcement in the tension zone only. It could be unconservative to ignore the prestressed reinforcement in the compression zone, for it is normally in tension at nominal strength. (This contrasts to nonprestressed compressive reinforcement, which may be conservatively disregarded.) The current Code definitions cause many uniformly prestressed sections used in bending to be above the reinforcement limits of Section 18.8.1, even though very large strains may be present on the tension face at nominal strength.

4. The definition of balanced strain conditions for nonprestressed sections is based on the reinforcement reaching yield simultaneously with crushing of the concrete at maximum strain. And, the maximum reinforcement limits are designed to insure that reinforcement strains well beyond yield will occur prior to failure in a flexural member, to give warning of failure. In a prestressed section, there is no definite yield point in the steel. However, the Code-defined yield strength of the steel is about 90 ksi above the usual effective prestress level (for Type 270K strand). But, if one substitutes 90 ksi into the Code formulas for nonprestressed concrete, the resulting reinforcement limit is substantially different from the Code limit for prestressed concrete.

5. The treatment of flanged sections is different for nonprestressed and prestressed sections. For prestressed flanged sections, the compressive force in the concrete flange is treated in the same manner as that in compressive reinforcement, and the compressive force in the flange may be offset by additional tensile reinforcement with the same nominal strength. This results in the same neutral axis depth (and the same strain on the tension face) at nominal strength as for rectangular prestressed sections. But the Code requirements for maximum reinforcement in nonprestressed sections with a stress block depth greater than the flange thickness allow only 75 percent of the compressive force in the flange to be offset by additional tensile reinforcement. This results in smaller neutral axis depths for flanged sections, as pointed out in Reference 3.

A CHANGE IN TERMINOLOGY AND A CHANGE IN THINKING

The terms "flexural members" and "compression members" are used in the Code. For the determination of reinforcement limits and the appropriate strength reduction factor, the author believes it would be more appropriate to think in terms of "tension controlled" and "compression controlled" behavior. Tension controlled behavior may be defined to include members that have large tensile strains in the steel at nominal strength. Compression controlled behavior may be defined to include members whose nominal strength is controlled primarily by crushing of concrete at the assumed maximum compressive strain of 0.003. Also, an intermediate range of behavior may be defined. The advantage of this way of thinking is that it clarifies the proper treatment of members with small axial loads. Furthermore, the determination of the strength reduction factor ϕ may be based on the type of behavior.

The Code uses the term "members" at times, and "cross sections" or "sections" at other times. The determination of behavior at nominal strength is done at specific cross section, and it would therefore be appropriate to use the term "cross section" in most cases. This paper uses the terms "tension controlled sections" and "compression controlled sections," with the word "cross" omitted for brevity.

The definitions of the boundaries of tension controlled and compression controlled behavior are important because they also may be used to define the value of the strength reduction factor ϕ. Tension controlled sections are designed with $\phi = 0.9$, whereas compression controlled sections are designed with $\phi = 0.7$ or 0.75. Tension controlled members may be designed with the higher ϕ factor because they are expected to:

1. Give adequate warning of impending failure prior to actual collapse.
2. Be less sensitive to variations in material strengths, particularly to the concrete strength.

Tension controlled sections may give warning prior to failure by excessive deflection and/or excessive cracking. Not all tension controlled sections will give both types of warning, but it is expected that tension controlled sections would give at least one type of warning.

Both types of warning, deflection and cracking, are functions of strain, particularly strain on the tension side, because tensile strains are larger than compressive strains in tension controlled sections at failure.

THE CONCEPT OF EXTREME DEPTH

The concept of effective depth is used in calculation of the nominal strength of a section. The effective depth d of a section is defined in the Code as the "distance measured from extreme compression fiber to centroid of tension reinforcement." This definition requires one to determine the centroid of the tension steel. The current definition is useful for the usual case in reinforced concrete, in which the steel is concentrated in a single layer, or in two layers close together. When the steel is distributed as in deep sections, some prestressed sections, and column-type members used in flexure, finding the centroid of the tension steel presents some difficulties. The location of the neutral axis, which determines which steel is in the tension zone, is not immediately known. Also, with distributed steel, not all of which is at yield, the location of the tensile force differs from the location of the centroid of the tensile reinforcement.

Problems associated with the effective depth concept occur more often in prestressed sections where draped tendons are located well away from the tension face. To overcome these problems, the Code uses the overall height h to define certain limits for prestressed sections, whereas the effective depth d is used to define corresponding limits for reinforced sections. The Canadian code (CAN3-A23.3-M84)[4] defines maximum reinforcement limits in terms of h for prestressed sections, but uses d in the definition for reinforced sections. The use of d for reinforced sections and h for prestressed sections creates inconsistencies between their design methodologies. One solution, proposed in Reference 3, would be to use h for reinforced sections also. A disadvantage of this proposal is that h is not currently used in the computation of the strength of reinforced sections, and thus an additional parameter would be required because d is needed for strength computations.

This author believes that a better solution would be to devise a new parameter, extreme depth. Extreme depth d_t may be defined as the "distance from extreme compression fiber to extreme tension steel." Reinforcement limits may then be defined using d_t. In the vast majority of cases, d_t will be equal to d (or d_p) and no additional parameter is required. And, for members with steel at more than one level, the designer may conservatively take d_t equal to d or d_p if one so chooses.

The effective depth d or d_p will still be used in nominal strength computations, for this proposal does not alter nominal strength computations. The use of the extreme depth d_t is in the definition of tension controlled and compression controlled sections, as described subsequently. Another way of looking at this is that the strain limits for tension and compression controlled sections will be defined using the maximum steel strain (measured at extreme depth d_t), instead of the average steel strain (measured at d or d_p).

THE CONCEPT OF NET TENSILE STRAIN

The Code defines the compressive strain at nominal strength to be 0.003 for all sections. The tensile strain and nominal strength will be determined by the properties and reinforcement of a section, and by the axial load, if any. The tensile strains in the steel may be used as a measure of the type of behavior (compression or tension controlled) at nominal strength.

For prestressed sections, the strain that is significant in evaluating behavior at nominal strength is the net strain, exclusive of the effects of prestress. It is not correct to call this strain the flexural strain, since it is the result of axial load effects (if any) as well as flexure. The term "net tensile strain ϵ_t" is defined as "the net tensile strain in extreme tension steel at nominal strength due to factored loads, exclusive of effective prestress strain." The net tensile strain may then be used to define the limits of compression controlled and tension controlled behavior, for both prestressed and reinforced sections.

The current Code limit of 0.75 ρ_b for flexural members produces a net tensile strain of 0.00376 at the centroid of Grade 60 reinforcement at nominal strength, for rectangular sections. For flanged sections, the net tensile strain is considerably higher. For prestressed sections, the limit on ω of 0.36 β_1 produces a net tensile strain of 0.00408 at the Code limit for flexural members, for both rectangular and flanged sections.

PROPOSED DEFINITION OF TENSION CONTROLLED AND COMPRESSION CONTROLLED SECTIONS

The author proposes that tension controlled sections be defined as those in which the net tensile strain is not less than 0.005 at nominal strength, for both prestressed and reinforced sections. This would give the same minimum amount of curvature at nominal strength (for a given depth) for all tension controlled flexural members. Furthermore, it is proposed that the 0.005 strain be measured at the extreme depth, not at the effective depth. The author believes that the strain at extreme depth is a better indication of ductility, cracking potential, and crack width. (For example, steel near the neutral axis has an effect on d, but little, if any, effect on ductility and cracking.)

The reasons for choosing the 0.005 strain limit, which is higher than that implied by the present Code, are as follows:

1. Because the net tensile strain is measured at d_t instead of d (or d_p), the limit should be slightly higher when more than one layer of tension steel is used.
2. The use of the 0.005 limit produces more reasonable-looking interaction diagrams. See Example Problems.

Balanced failure conditions can be used to define the boundary of compression controlled failure. Balanced conditions are defined in terms of strain. The Code definition uses a strain of f_y/29,000 psi (= 0.0021 for Grade 60 steel) for the tensile strain at balanced conditions in reinforced concrete, and balanced conditions are not defined for prestressed concrete. The author proposes a simplified definition of compression controlled sections as those in which the net tensile strain at nominal strength at the extreme depth d_t is less than or equal to 0.0025. These are the reasons for choosing a net tensile strain limit of 0.0025 instead of 0.0021.

1. For simplicity, it is desirable to have a single limit for all grades of steel. The proposed limit of 0.0025 is a compromise between the yield strain for Grade 60 and for higher grades of reinforcement.
2. The limit of 0.0025 may also be applied to prestressed sections. See Appendix B for a discussion of how the limit applies to prestressed sections.
3. For members with more than one layer of tension steel, it is more conservative to apply a higher strain limit at the extreme depth d_t than is currently being applied at the centroid of the tension reinforcement.

The preceding net tensile strain limits may be used to define the capacity reduction factor ϕ.

A ϕ factor of 0.9 applies to tension controlled sections. A ϕ factor of 0.7 (or 0.75) is used for compression controlled sections. Sections having net tensile strains between these limits are designed with an intermediate ϕ factor. These provisions apply equally to reinforced and prestressed concrete sections.

The preceding net tensile strain limits correspond to a limiting ratio of neutral axis depth to extreme depth c/d_t at nominal strength of 0.375 for tension controlled sections, and 0.545 for compression controlled sections. This is stated in the Commentary. The author prefers defining the limiting net tensile strain in the Code as he believes this to be more fundamental.

Limits on net tensile strain ϵ_t may also be used in defining the limits for moment redistribution in Chapter 8 of the Code. Limits defined in this way are applicable to both reinforced and prestressed concrete continuous members. The development of the provisions for redistribution of negative moment is given in Appendix C.

PROPOSED CODE CHANGES

Chapter 2—Definitions

Add:

Compression controlled section—A cross section in which the net tensile strain in the extreme tension steel at nominal strength is less than or equal to 0.0025.

Tension controlled section—A cross section in which the net tensile strain in the extreme tension steel at nominal strength is greater than or equal to 0.005.

Extreme tension steel—The reinforcement (prestressed or nonprestressed) which is farthest from the extreme compression fiber.

Net tensile strain—The tensile strain at nominal strength exclusive of effective prestress strain.

Chapter 8—Analysis and design—General considerations

8.0—Notation

Delete definitions of f_y, β_1, ρ, ρ', and ρ_b—no longer needed in this chapter.

Add ϵ_t = net tensile strain in extreme tension steel at nominal strength, exclusive of effective prestress strain

8.4 Delete "nonprestressed" and footnote in title.

8.4.1 Replace the formula with the following:

1000 ϵ_t percent, with a maximum of 20 percent.

8.4.2 No change.

8.4.3 Replace text with the following text: Redistribution of negative moments shall be made only when ϵ_t is equal to or greater than 0.0075 at the section at which moment is reduced.

Delete Eq. (8-1)

Chapter 9—Strength and serviceability requirements

9.0—Notation

Delete definitions of d', d_s, h, and P_b

Add:

c = distance from extreme compression fiber to neutral axis

d_t = distance from extreme compression fiber to extreme tension steel

ϵ_t = net tensile strain in extreme tension steel at nominal strength, exclusive of effective prestress strain

ρ = ratio of nonprestressed tension reinforcement A_s/bd

ρ_b = reinforcement ratio producing balanced strain conditions. See 10.3.2

Replace Sections 9.3.2.1 and 9.3.2.2 with the following:

9.3.2.1 — Tension controlled sections 0.90

9.3.2.2 — Compression controlled sections:

(a) Members with spiral reinforcement conforming to 10.9.3 .... 0.75

(b) Other reinforced members 0.70

For sections in which the net tensile strain in the extreme tension steel at nominal strength is between the limits for compression controlled and tension controlled sections, ϕ shall be linearly increased from that for compression controlled sections to 0.90 as the net tensile strain in the extreme tension steel at nominal strength increases from 0.0025 to 0.005.

Chapter 10—Flexure and axial loads

10.0—Notation

Add:

ϵ_t = net tensile strain in extreme tension steel at nominal strength, exclusive of effective prestress strain

10.3.2 This section is left intact for historical reasons. It would no longer be used in Code provisions. Replace "ultimate strain" with "strain limit."

10.3.3 Replace with the following:

Sections are compression controlled when the net tensile strain in the extreme tension steel is equal to or less than 0.0025 at the time the concrete in compression reaches its assumed strain limit of 0.003. Sections are tension controlled when the net tensile strain in the extreme tension steel is equal to or greater than 0.005 just as the concrete in compression reaches its assumed strain limit of 0.003. Sections with net tensile strain in the extreme tension steel between 0.0025 and 0.005 constitute a transition region between compression controlled and tension controlled sections.

Chapter 18—Prestressed concrete

18.0 Delete definitions of ω_p, ω_w, ω_{pw}, and ω_n' —no longer needed in this chapter.

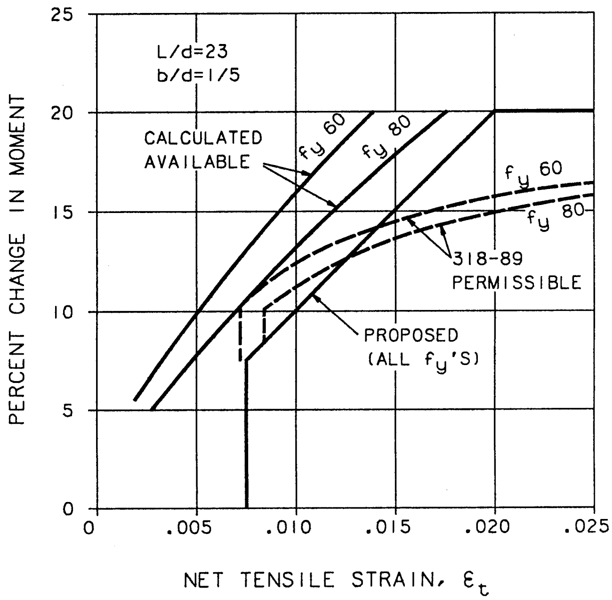


Fig. 8.4—Permissable moment redistribution for minimum rotation capacity

18.1.3 Delete Sections 8.4 and 10.3.3 from exclusion lists. These sections would now apply to prestressed concrete also.

18.8.1 Replace with the following:

Prestressed concrete sections shall be classified as tension controlled and compression controlled sections in accordance with 10.3.3. The appropriate ϕ factors from 9.3.2.2 shall apply.

Delete Section 18.8.2. Renumber Section 18.8.3.

18.10.4.1 Replace with the following:

Where bonded reinforcement is provided at supports in accordance with 18.9.2, negative moments calculated by elastic theory for any assumed loading may be increased or decreased in accordance with 8.4.

18.10.4.3 Delete.

PROPOSED COMMENTARY CHANGES

Chapter 8—Analysis and design—General considerations

R8.4 Second paragraph: Replace "varying from 10 to 20 percent" with "up to 20 percent." Delete last sentence.

R8.4 Add the following paragraphs:

Previous editions of the code specified the permissible redistribution percentage in terms of reinforcement indexes. This edition specifies the permissible redistribution percentage in terms of the net tensile strain ϵ_t. See Reference [*to this paper*] for a comparison of current and previous moment redistribution provisions.

The concept of net tensile strain is discussed in R10.3.3.

Fig. 8.4 Replace with new Fig. 8.4.

Chapter 9—Strength and serviceability requirements

R9.2 Add to last paragraph:

. . . This loading case may also be critical for tension controlled column sections. In such a case, a reduction in axial load and increase in moment may give a critical load combination.

R9.3.1 Delete second paragraph, and replace with the following:

The purposes of the strength reduction factor ϕ are (1) to allow for the probability of understrength sections due to variations in material strengths and dimensions, (2) to allow for inaccuracies in the design equations, (3) to reflect the degree of ductility and required reliability of the section under the load effects being considered, and (4) to reflect the importance of the member in the structure.[9.2,9.3]

R9.3.2 Delete, and replace with the following:

R9.3.2—Prior to 1995, the Code specified the magnitude of the ϕ factor for cases of axial load and/or flexure in terms of the type of loading. For these cases, the ϕ factor is now determined by the strain conditions at a cross section, at nominal strength.

A lower ϕ factor is used for compression controlled sections than for tension controlled sections because compression controlled sections have less ductility, are more sensitive to variations in concrete strength, and generally occur in members that support larger loaded areas than members with tension controlled sections. Members with spiral reinforcement are assigned a higher ϕ than tied columns since they have greater ductility or toughness.

For sections subjected to axial load with flexure, design strengths are determined by multiplying both P_n and M_n by the appropriate single value of ϕ. Compression controlled and tension controlled sections are defined in Chapter 2 as those which have net tensile strain in the extreme tension steel at nominal strength less than or equal to 0.0025 and equal to or greater than 0.005, respectively. For sections with net tensile strain in the extreme tension steel at nominal strength between the preceding limits, the value of ϕ may be determined by linear interpolation, as shown in

Fig. 9.3.2. The concept of net tensile strain is discussed in R10.3.3.

Since the compressive strain in the concrete at nominal strength is defined in 10.2.3 as 0.003, the net tensile strain limits for compression controlled members may also be stated in terms of the ratio c/d_t, where c ($= a/\beta_1$) is the depth of the neutral axis at nominal strength, and d_t is the distance from the extreme compression fiber to the extreme tension steel. The c/d_t limits for compression controlled and tension controlled sections are 0.545 and 0.375, respectively. Fig. 9.3.2 also gives equations for ϕ as a function of c/d_t.

The net tensile strain limit for tension controlled sections may also be stated in terms of the ρ/ρ_b ratio as defined in previous editions of the Code. The net tensile strain limit of 0.005 corresponds to a ρ/ρ_b ratio of 0.633 for rectangular sections with Grade 60 reinforcement. For a comparison of these provisions with those of previous editions of the Code, see Reference [*to this paper*].

The ϕ factor for bearing on concrete in this section does not apply to post-tensioning anchorage bearing plates (see R18.13).

Add Fig. 9.3.2.

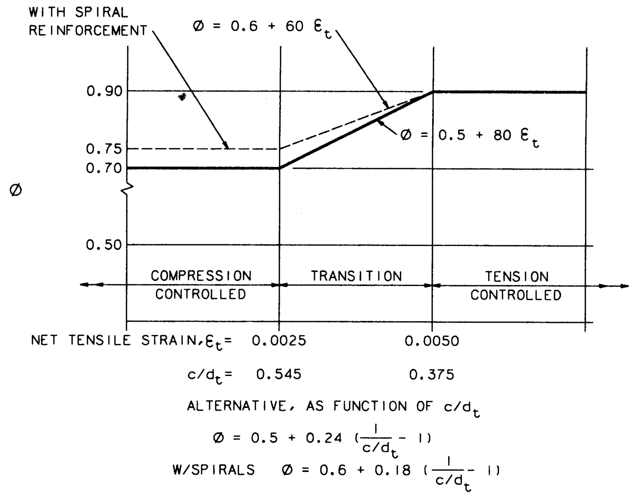


Fig. 9.3.2—Variation of ϕ with net tensile strain

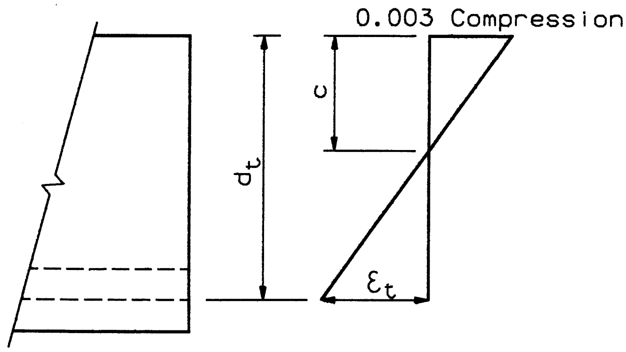


Fig. 10.3.3—Strain distribution and net tensile strain

Chapter 10—Flexural and axial loads

R10.3.2 Add at end: . . . The balanced reinforcement ratio ρ_b was used in Code editions prior to 1995, but is no longer required.

R10.3.3 Delete all but first paragraph, and replace with the following:

The nominal flexural strength of a member is reached when the strain in the extreme compression fiber reaches the assumed strain limit of the concrete. The net tensile strain in the extreme tension steel is determined from the strain distribution at failure, shown in Fig. 10.3.3, using similar triangles. The neutral axis depth c is equal to a/β_1, where a is the depth of the rectangular stress block.

When the net tensile strain in the extreme tension steel is sufficiently large (equal to or greater than 0.005), the section is defined as tension controlled and a failure condition with extensive deflection and cracking may be expected, giving ample warning of failure. When the net tensile strain in the extreme tension steel is small (less than or equal to 0.0025), a brittle failure condition may be expected, with little warning of impending failure. Flexural members are usually tension controlled, whereas compression members are usually compression controlled. Some sections, such as those with small axial load and large bending moment, will have net tensile strain in the extreme tension steel between the preceding limits. These sections are in a transition region between compression and tension controlled sections. Section 9.3.2 specifies the appropriate strength reduction factors for tension controlled and compression controlled sections, and for intermediate cases in the transition regions. See Reference [*to this paper*] for a comparison of these provisions to those in previous editions of the Code.

The net tensile strain limit for compression controlled sections was chosen to be a single value to apply to members with more than one type of steel. The value of 0.0025 represents a compromise between the net tensile strain at balanced conditions for Grade 60 and higher grades of reinforcement, and prestressing steel. For more information on the choice of this limit, see Reference [*to this paper*].

Prior to 1995, the limiting tensile strain for flexural members was given as a fraction of ρ_b, which was dependent on the yield strength of the reinforcement. The new net tensile strain limit of 0.005 for tension controlled sections was chosen to be a single value which applies to all types of steel (prestressed and nonprestressed) permitted by this Code. Although it is consistent to base the limit on the yield strain, there are many advantages in using a single value. Therefore, the Committee selected a conservative tensile strain limit. It is important to note that the new net tensile strain limit of 0.005 is not an absolute limit (as was the 0.75 ρ_b limit in earlier editions), but only a point at which the capacity reduction factor begins to change. Thus, flexural members with reinforcement ratios of 0.75 ρ_b are still permitted, but with a slightly reduced design strength, to reflect the smaller tensile strain at which flexural capacity is reached.

Unless unusual amounts of ductility are required, the 0.005 limit will provide ductile behavior for most designs. One condition where greater ductile behavior is required is in design for redistribution of moments in continuous members and frames. Section 8.4 permits redistribution of negative moments. Since moment redistribution is dependent on adequate ductility in hinge regions, moment redistribution is limited to sections that have a net tensile strain of at least 0.0075.

For beams with compression reinforcement, or T-beams, the effects of the compression reinforcement and flanges are automatically accounted for in the computation of net tensile strain ϵ_t.

Add Fig. 10.3.3.

Chapter 18—Prestressed concrete

R18.1.3 Delete references to exclusion of 8.4 and 10.3.3.

Delete R18.8.1 and R18.8.2 and replace with the following:

> **R.18.8.1** The net tensile strain limits for compression and tension controlled sections given in 10.3.3 apply equally to prestressed sections. These provisions take the place of the maximum reinforcement limits in earlier editions of the Code.
>
> The net tensile strain limit for tension controlled sections given in 10.3.3 may also be stated in terms of ω_p as defined in previous editions of the Code. The net tensile strain limit of 0.005 corresponds to $\omega_p = 0.319\beta_1$ for prestressed rectangular sections.

Renumber R18.8.3 as R18.8.2.

R18.10.4 Delete all but the last paragraph, and replace with the following:

> The provisions for redistribution of negative moments given in 8.4 of this Code apply equally to prestressed members. See Reference [*to this paper*] for a comparison to research results and past code provisions.

FUTURE TOPICS

In Chapter 11 of the Code, which deals with shear, there are provisions that use d when applied to reinforced sections, but which use h when applied to prestressed sections. The concept of extreme depth d_t could be used to unify some of these provisions.

The concepts presented in this paper might also be applied to some of the seismic provisions of Code Chapter 21. However, this would require much further investigation. Some of the requirements of Chapter 21 are designed to result in concrete strains at nominal strength much in excess of 0.003 and other provisions are designed to limit the tensile forces in flexural members, even at large strains.

One could argue that some minimum should be placed on the layer of reinforcement used to define d_t. Obviously, in heavily reinforced or prestressed sections with the main steel well above the bottom, the addition of, say, two #3 bars in the bottom would not have a major effect on their behavior. This is actually a potential problem with the current Code. For instance, because of the interrelationship between shear and moment, it would seem that the provision of a minimum d of $0.8h$ for prestressed sections should require some minimum reinforcement at a depth of $0.8h$ or greater. For conventionally reinforced sections, the crack control provisions of Section 10.6.4 should cause a reasonable amount of reinforcement to be placed near the tension face. However, Section 6.4 of the Code states that "it is permitted to take f_s as 60 percent of the specified yield strength f_y" when applying crack control equations. This is not conservative for members in which a significant portion of the reinforcement is located away from the tension face. The problem of determining a minimum amount of steel at extreme depth, and crack control at service load, is currently under study.

EXAMPLE PROBLEMS

Fig. 1 shows the effect of the proposal on a rectangular concrete flexural member with 5000-psi concrete and Grade 60 reinforcement. All of the tension reinforcement is assumed to be in one layer, although this may not be realistic for higher steel percentages. The solid lines show the design moment capacity versus reinforcement ratio for tension reinforcement only. For reinforcement ratios ρ less than 0.021, the proposed provisions cause no change. The current Code essentially prohibits reinforcement ratios in excess of 0.025. The proposal allows higher ratios, but the transition in the ϕ factor has the effect of limiting the bending moment to that permitted by the current Code for reinforcement ratios in the range of 0.021 to 0.022. In this respect, the proposal is more similar to the current Code provisions for prestressed concrete, which do not prohibit excess reinforcement indexes, but limit design moment capacity to that at the maximum index.

The dashed lines on Fig. 1 show the design moment capacity versus total longitudinal steel area, when compression reinforcement is used. The present and pro-

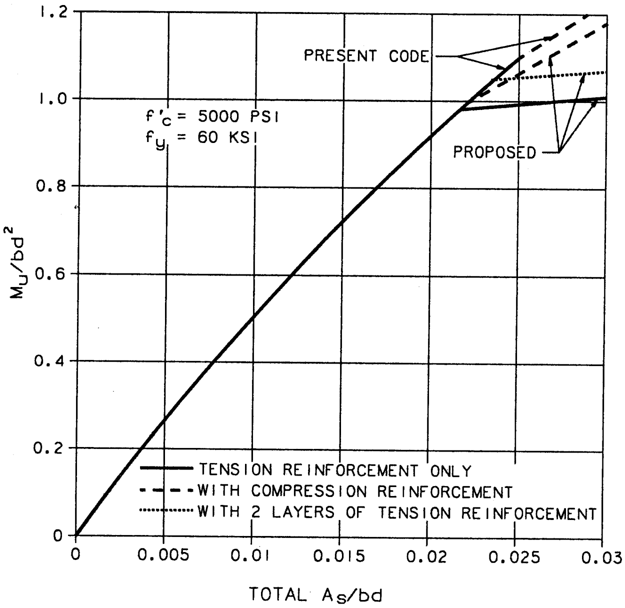


Fig. 1—Rectangular reinforced concrete beams

posed design moment capacities are almost identical. Thus, the proposed provisions that give a somewhat lower limit for tension-only reinforcement do not result in any real cost penalty, since higher design moment capacities may be obtained with virtually the same total reinforcement as in the current Code.

The dotted line in Fig. 1 shows the effect of the proposal when the tension reinforcement consists of two equal layers, 0.1d apart. This is probably more realistic for heavily reinforced members. Because ϕ is proposed to be defined in terms of d_t instead of d, the proposed design moment strengths are much closer to those given by the current Code. This is one justification for setting the tension controlled limit at a strain of 0.005 measured at the extreme depth d_t, as contrasted to the current Code which limits the strain to 0.0038, measured at depth d.

Fig. 2 and 3 show a 14-in. square prestressed pile. When this member is used as a flexural member (such as a fender pile on a marine pier), it is overreinforced by the current Code provisions, because d_p is only 0.5h. The commentary formula for members with $g\omega_p > 0.36\beta_1$ produces a design strength ϕM_n of 833.5 kip-in. But, if one adds a small axial load and calls the member a compressive member, then ϕM_n using $\phi = 0.7$ would be 1053 kip-in, which is 28 percent greater. Yet, strain compatibility shows the net tensile strain at d_t to be 0.00415, which is as large as the net tensile strain in a member prestressed with a single layer of strand conforming to the Code limits for flexural members with $\phi = 0.9$. Fig. 2 shows the interaction diagram using the current Code, and the proposed transition in ϕ between the net tensile strain limits of 0.0025 and 0.005. The author believes that the proposed definitions produce a more reasonable strength reduction factor in the lower portion of the interaction diagram.

Fig. 4 shows the interaction diagram for a reinforced column with approximately 3-percent reinforcement.

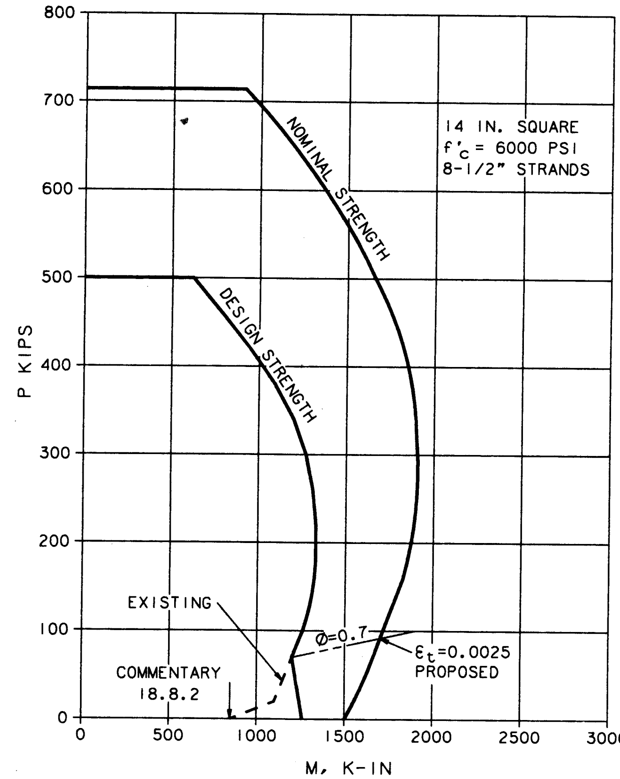
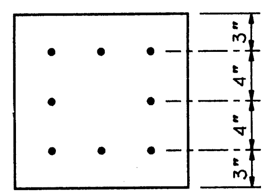


Fig. 2—Interaction diagram for prestressed pile

Given: 14 x 14 prestressed pile with 8-1/2 in. strands, used as a flexural member. f'_c = 6000 psi



By Current Code

d_p = 7 in.

Five strands are in tension zone

$\rho_p = A_{ps}/bd_p = 5 \times 0.153/14 \times 7 = 0.00781$

Assume f_{ps} = 230 ksi average for five strands in tension zone

$\omega_p = \rho_p f_{ps}/f'_c = 0.00781(230)/6 = 0.30$

$0.36\ \beta_1 = 0.36(0.75) = 0.27$

$\omega_p > 0.36\ \beta_1$

Use Commentary:

$\phi M_n = \phi\ [f'_c\ bd^2_p\ (0.36\ \beta_1 - 0.08\ \beta_1^2)$

$= 0.9\ [6(14)(7)^2\ (0.36\ (0.75) - 0.08\ (0.75)^2)]$

ϕM_n = 833.5 kip-in

But - if the member has axial load, however small, one can use $\phi = 0.7$

M_n = 1504 kip-in by strain compatibility

ϕM_n = 1053 kip-in

By Proposed Code Revision

$\varepsilon_t = 0.00415$

$\phi = 0.5 + 80\ \varepsilon_t = 0.832$

$\phi M_n = 0.832\ (1504) = 1251$ kip-in

Fig. 3—Computations for prestressed pile

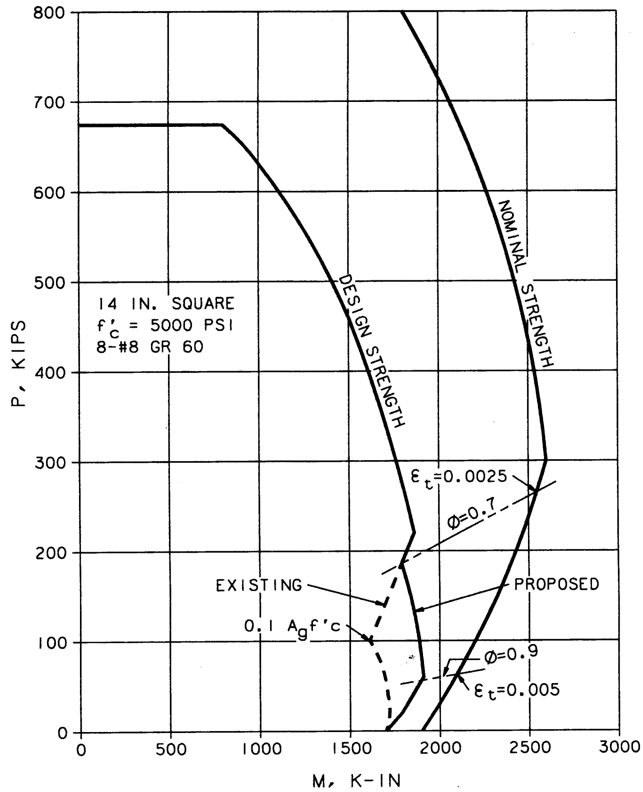


Fig. 4—Interaction diagram for column with 3.2 percent steel

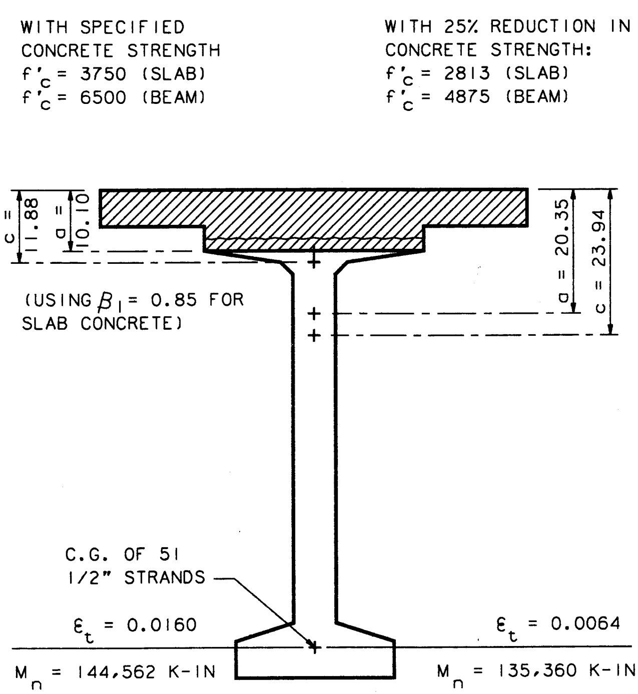


Fig. 5—Long span composite prestressed I-beam

The author had hoped to see the "kink" produced by the current Code removed when he drew this diagram, but he had not expected to see the "bulge." The intent of the proposed provisions is to make things more consistent, not to liberalize the design of tension controlled columns. However, the liberalization seems justified. What is more ductile than a section with equal reinforcement on both faces, loaded primarily in bending? Such members can be more ductile than singly reinforced flexural members.

A POSSIBLE PROBLEM

The present Code provision limiting reinforcement in flexural members to 0.75 ρ_b not only provides for ductile behavior at design strength, but also provides for ductile behavior even if the concrete should be somewhat understrength. As long as the concrete strength is greater than 75 percent of the specified strength, the reinforcement would reach its strain at specified yield prior to crushing of the concrete. The proposed provisions would provide similar (actually somewhat greater) protection for rectangular sections, but the situation could be different for flanged sections.

The current 0.75 ρ_b limit for reinforced concrete provides the same degree of protection against brittle behavior due to understrength concrete in both rectangular and flanged sections. But the current $0.36\beta_1$ limit on ω_p for prestressed concrete will result in a lower neutral axis in case of understrength concrete and greater likelihood of brittle behavior in a flanged section than in a rectangular section with the same ω_p.

The proposed provisions more resemble the current provisions for prestressed concrete. Some authors[3,5] have stated that the result of the reinforced concrete provisions is not what was intended. But, if the current provisions for flanged reinforced sections are "correct" for all flanged sections, then the author's proposal (and other proposals defining limits in terms of c/d or c/h) does not provide the same degree of protection against understrength concrete in flanged sections. (The use of material factors in the Canadian code[4] solves this difficulty.)

The design example shown in Fig. 5 is taken from real life. The prestressed I-beam was used in a bridge (albeit governed by an AASHTO, not ACI, code) on a 125-ft span. One question raised by a composite beam example pertains to the correct value of β_1. Should it be that used for the composite slab concrete, that for the beam concrete, or something in between? In this example, the β_1 for the slab is used, since most of the compression is in the slab. The left side of Fig. 5 shows the girder as designed. The neutral axis and the bottom edge of the stress block fall within the sloping portion of the flange. This makes the determination of ρ and ω difficult. But, the net tensile strain in the steel is 0.016, and thus the section is clearly tension controlled.

But what if the concrete strengths were 25 percent less than specified? In this case, the results are shown on the right side of Fig. 5. The neutral axis depth increases from 11.88 to 23.94 in. The net tensile strain decreases to 0.0064, but the section is still tension controlled. The nominal moment strength decreases 6.4 percent, but it is still much in excess of that required, as is usually the case in unshored composite beams. In this example, the section is clearly tension controlled, even though the neutral axis lies below the flange.

Heavily reinforced flanged sections are a rarity; most flanged sections are quite underreinforced. So the problem may be more academic than real. In any event, the problem is mitigated somewhat by the proposed higher 0.005 minimum net tensile strain limit for tension controlled sections.

STEELS OTHER THAN GRADE 60

It might be more logical to define the compression controlled limit using the yield strain ϵ_y and the tension controlled limit as ϵ_y plus a fixed quantity, say 0.0025. Then, for prestressed concrete, the compression controlled limit would be a net tensile strain of $\epsilon_{py} - f_{se}/E_{ps}$, and the tension controlled limit would be that strain plus 0.0025. Since f_{se} varies from one member to another, the limits for prestress sections would become more complicated. What does one do when both prestressing steel and reinforcement are used? And, if "sleeper" strands ($f_{se} = 0$) are added, surely it is not necessary to increase the limiting net tensile strain for compression controlled behavior to 0.0087.

In the interests of simplicity, the author strongly recommends that a single value of net tensile strain be used to define the compression controlled limit. A net tensile strain of 0.0025 is suggested, but this could be changed, particularly if Grade 80 should become more common.

Traditionally, ductility has been defined as the ratio of ultimate strain to yield strain, and the current Code provisions result in a minimum ratio of 1.82 for flexural members. The author believes that a fixed minimum net tensile strain at nominal strength provides better warning prior to failure, as cracking and deflection are functions of total net tensile strain, rather than the ratio to yield strain. This is discussed further in Appendix B.

DESIGN EXAMPLES

The proposed code and commentary changes do not alter nominal strength calculations. Therefore, nominal strength calculations will be done exactly as before, whether they are done longhand, by computer, or with charts and tables. Procedures that are changed are itemized as follows:

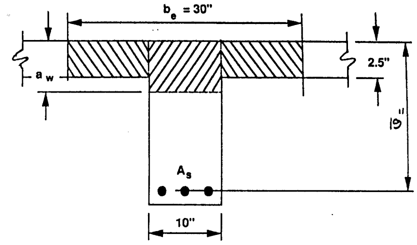
1. *Checking reinforcement limits for flexural members*—This procedure is replaced by a check to see whether the ϕ of 0.9 for tension controlled sections applies, or whether a reduced ϕ must be used.
2. *Determining ϕ for columns with small axial load*—The existing procedure, which is a function of f_c' and A_g, is replaced by a procedure using net tensile strain, or the c/d_t ratio.
3. *Moment redistribution*—The existing procedure using the ratio ρ/ρ_b is replaced by a procedure using net tensile strain or the c/d_t ratio.

Example 10.5—Design of Flanged Section with Tension Reinforcement Only

Select reinforcement for the T section shown, to carry a factored moment of M_u = 400 ft-kips.

f_c' = 4000 psi

f_y = 60,000 psi



Calculations and Discussion

6. Check maximum tension reinforcement permitted according to Section 10.3.3. See Part 6, Eqs. (8) and (11).

For flanged section with tension reinforcement only:

$$\rho_{max} = \left[0.75\frac{b_w}{b}(\bar{\rho}_b + \rho_f)\right] \quad \text{Eq. (11), Part 6}$$

$$\rho_f = 0.85\frac{f_c'}{f_y}(b - b_w)\,h_f/b_wd \quad \text{Eq. (8), Part 6}$$

$$= 0.85\frac{4}{60}(30 - 10)2.5/(10 \times 19) = 0.0149$$

From Table 10-1, $\bar{\rho}_b$ = 0.0285

$$\rho_{max} = 0.75\left[\frac{10}{30}(0.0285 + 0.0149)\right] = 0.0109$$

$A_{s(max)}$ = 0.0109 × 30 × 19 = 6.21 in.^{2} > 5.12 in.^{2} O.K.

Fig. 6—Design Example 1—Present code

```
6.   Check to see if section is tension controlled with φ = 0.9
     a      = 4.04 in. (a_w in PCA example, step 3)
     c      = a/β1 = 4.04/0.85 = 4.75 in.
     dt     = d = 19 in.
     c/dt   = 4.75/19 = 0.250 < 0.375
     ∴ tension controlled; φ = 0.9
```

Fig. 7—Design Example 1—Proposed method

The following two design examples are based on those in the Portland Cement Association (PCA) book, *Notes on ACI 318-89.*[6] The examples have been abbreviated, omitting calculations that are not affected by the proposed code changes. Refer to Reference 6 (hereafter "the PCA book") for the complete design example.

Example 1

Fig. 6 shows Example 10.5 from the PCA book. The design according to the proposed provisions may be done exactly as in Example 10.5, except for step No. 6, which is shown in Fig. 7.

Example 2

Fig. 8 shows Example 26.5 from the PCA book. Using the proposed methods, the problem would be stated, "For the single-tee section shown below, find the appropriate value of ϕ." The problem may be solved either using the c/d_t ratio or by the value of ϵ_t derived from strain compatibility, as shown in Fig. 9.

SUMMARY

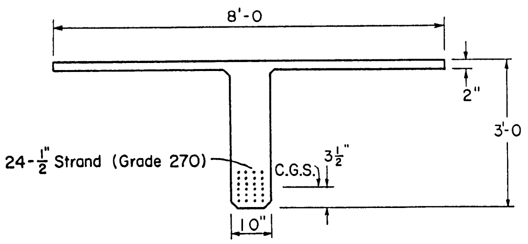
Existing code	Net tensile strain at balanced strain conditions	Net tensile strain at limit for flexural members
Reinforced concrete, rectangular sections, Grade 60 steel	0.0021	0.00376
Reinforced concrete, flanged sections, Grade 60 steel	0.0021	>>0.00376
Prestressed concrete, rectangular and flanged sections	Not defined	0.00408
Proposed	*Net tensile strain at limit for compression controlled sections*	*Net tensile strain at limit for tension controlled sections*
Limits for all cases	0.0025*	0.005*

*Measured at extreme depth d_t instead of at d or d_p.

Example 26.5—Limits for Reinforcement of Prestressed Flexural Member

For the single tee section shown below, check limits for the prestressed reinforcement provided.

$f'_c = 5000$ psi
$f_{pu} = 270{,}000$ psi (stress-relieved strands; $f_{py} = 0.85\, f_{pu}$)



Calculations and Discussion

Example No. 26.5.1

1. Calculate stress in prestressed reinforcement at nominal strength.

$$\omega_{pu} = \frac{A_{ps}f_{pu}}{bd_pf'_c} = \frac{24 \times 0.153 \times 270}{96 \times 32.5 \times 5} = 0.0636$$

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\omega_{pu}\right) = 270\left(1 - \frac{0.4}{0.8} \times 0.0636\right) = 261 \text{ ksi}$$

2. Calculate required depth of concrete stress block.

$$a = \frac{24 \times 0.153 \times 261}{96 \times 0.85 \times 5} = 2.35 \text{ in.} \; > \; h_f = 2 \text{ in.}$$

3. Calcuate area of reinforcement to develop flange.

$$A_{pf} = \frac{0.85h_f(b - b_w)f'_c}{f_{ps}} = \frac{0.85 \times 2(96 - 10)5}{261} = 2.8 \text{ in.}^2$$

4. Calculate area of reinforcement to develop web.

$$A_{pw} = A_{ps} - A_{pf} = 24 \times 0.153 - 2.8 = 0.88 \text{ in.}^2$$

5. Check $\omega_{pw} \le 0.36\beta_1 = 0.36(0.8) = 0.288$

$$\omega_{pw} = \frac{A_{pw}f_{ps}}{b_wd_pf'_c} = \frac{0.88 \times 261}{10 \times 32.5 \times 5} = 0.142 \; < \; 0.288 \qquad \text{O.K.}$$

Fig. 8—Design Example 2—Present code

2. Find depth of stress block and of neutral axis

$$A_c = \frac{T}{0.85\ f'c} = \frac{24 \times 0.153 \times 261}{0.85 \times 5} = 225.5 \text{ in}^2$$

$$A_f = 2\ (96-10) = \quad 170.0$$

$$A_w = 53.5 \text{ in}^2$$

$$a = A_w/b_w = 53.5/10 = 5.35 \text{ in}$$

$$c = a/\beta_1 = 5.35/0.8 = 6.69 \text{ in}$$

3. Compare to maximum c/d_t for tension controlled sections

$c/d_t = 6.69/32.5 = 0.206 < 0.375$ max
$\therefore$ tension controlled; $\phi = 0.9$

Using Strain Compatibility

1. Analyze by strain compatibility

$f_{ps} = 260.8$ ksi and $\varepsilon_t = 0.0116$

2. Check limit for tension controled section

$\varepsilon_t > 0.005 \qquad \therefore\ \phi = 0.9$

Fig. 9—Design Example 2—Proposed method

ACKNOWLEDGMENTS

The author wishes to express his appreciation to the many who have contributed ideas to this proposal, particularly the members of the ACI 318 and the 318 Subcommittees D and G.

NOTATION

a = depth of equivalent rectangular stress block as defined in Section 10.2.7, ACI 318
A_c = area of concrete stress block at nominal strength
A_f = area of concrete stress block in flanges
A_g = gross area of section
A_{pf} = area of prestressed reinforcement to develop flange
A_{ps} = area of prestressed reinforcement in tension zone
A_{pw} = area of prestressed reinforcement to develop web
A_s = area of nonprestressed tension reinforcement
A_s max = maximum reinforcement area for flexural members
a_w = depth of equivalent rectangular stress block in web
A_w = area of concrete stress block in web
b = width of compression face of member
b_w = web width
c = distance from extreme compression fiber to neutral axis
C = compressive force
d = distance from extreme compression fiber to centroid of tension reinforcement

d' = distance from extreme compression fiber to centroid of compression reinforcement
d_p = distance from extreme compression fiber to centroid of prestressed reinforcement
d_t = distance from extreme compression fiber to extreme tension steel
E_s = modulus of elasticity of reinforcement
E_{ps} = modulus of elasticity of prestressed reinforcement
f'_c = specified compressive strength of concrete
f_{ps} = stress in prestressed reinforcement at nominal strength
f_{pu} = specified tensile strength of prestressing tendons
f_{py} = specified yield strength of prestressing tendons
f_{se} = effective stress in prestressed reinforcement
f_y = specified yield strength of nonprestressed reinforcement
h = overall thickness of member
h_f = thickness of flange
jd = effective depth, i.e., distance between the centroid of the compressive force and the centroid of the tension force
l = span length
M = bending moment
M_n = nominal moment strength at section
M_u = factored moment at section
P = axial load
P_b = nominal axial load strength at balanced strain conditions
P_n = nominal axial load strength at given eccentricity
P_u = factored axial load
T = tensile force
T_{bal} = tensile force in reinforcement at balanced strain conditions
β_1 = factor defined in Section 10.2.7, ACI 318
γ_p = factor for type of prestressing tendon
ϵ = strain
ϵ_{cu} = ultimate (crushing) strain of concrete in compression
ϵ_{ps} = strain in prestressed reinforcement at nominal strength
ϵ_{se} = strain in prestressed reinforcement at effective prestress level
ϵ_t = net tensile strain in extreme tension steel at nominal strength due to applied loads, exclusive of prestress strain
ϵ_{ty} = net tensile strain at yield of extreme tension steel
ϵ_y = yield strain
μ = ductility ratio
ρ = ratio of nonprestressed tension reinforcement A_s/bd
ρ_b = reinforcement ratio producing balanced strain conditions
$\bar{\rho}_b$ = balanced reinforcement ratio for a rectangular section with tension reinforcement only
ρ_f = reinforcement ratio for tension steel to develop the compressive strength of the flanges
ρ_{max} = maximum ρ for flexural members
ρ_p = ratio of prestressed reinforcement A_{ps}/bd_p
ϕ = strength reduction factor
ω = $\rho f_y/f'_c$
ω_{max} = maximum ω for flexural members
ω_p = $\rho_p f_{ps}/f'_c$
ω_{pu} = $A_{ps}f_{ps}/(bd_pf'_c)$
ω_{pw} = $A_{pw}f_{ps}/(b_wd_pf'_c)$

CONVERSION FACTORS

1 in. = 25.4 mm
1 psi = 6.895 kPa
1 ksi = 6.89 MPa
1 kip-in. = 0.113 kN-M

REFERENCES

1. ACI Committee 318, "Building Code Requirements for Reinforced Concrete and Commentary (ACI 318-89/ACI 318R-89)," American Concrete Institute, Detroit, 1989, 353 pp.

2. ACI Committee 318, "Commentary on Building Code Requirements for Reinforced Concrete (ACI 318-83)," American Concrete Institute, Detroit, 1983, 155 pp.

3. Skogman, B. C.; Tadros, M. K.; and Grasmick, R., "Ductility of Reinforced and Prestressed Concrete Flexural Members," *PCI Journal,* V. 33, No. 6, Nov.-Dec. 1988, pp. 94-107.

4. "Design of Concrete Structures for Buildings with Explanatory Notes," CAN3-A23.3-M84, Canadian Standards Association, Rexdale, 1984.

5. Wang, C. K., and Salmon, C. G., *Reinforced Concrete Design,* 4th Ed., Harper & Row, New York, 1985.

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8. Warwaruk, J.; Sozen, M. A.; and Siess, C. P., "Strength and Behavior in Flexure of Prestressed Concrete Beams," *Engineering Experiment Station Bulletin* No. 464, University of Illinois, Urbana, 1962.

9. Naaman, A. E.; Harajli, M. H.; and Wight, J. K.; "Analysis of Ductility in Partially Prestressed Concrete Flexural Members," *PCI Journal,* V. 31, No. 3, May-June 1986, pp. 64-87.

Because of space limitations, the Supplementary Design Examples and portions of Appendix A are not presented here. They will be kept permanently on file at ACI headquarters, where photocopies will be available at the cost of reproduction and handling.

APPENDIX A

Fig. A-1 and A-2 show the proposed strain and stress provisions for tension controlled sections.

APPENDIX B
Net tensile strain limits for prestressed section

To properly set net tensile strain limits for prestressed sections, it is necessary to review the behavior of reinforced sections. The steel strains in a reinforced flexural member with a steel percentage ρ of 0.75 ρ_b—the current code limit for flexural members—are illustrated

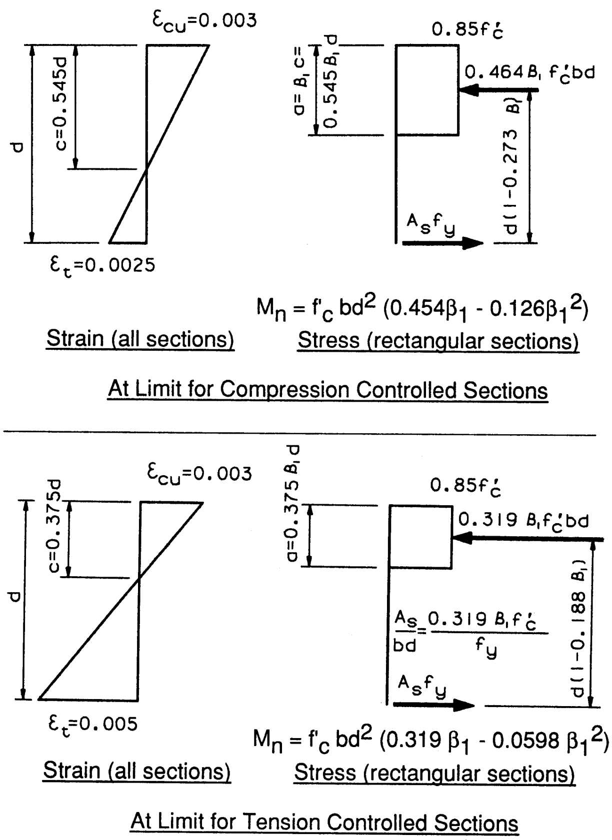
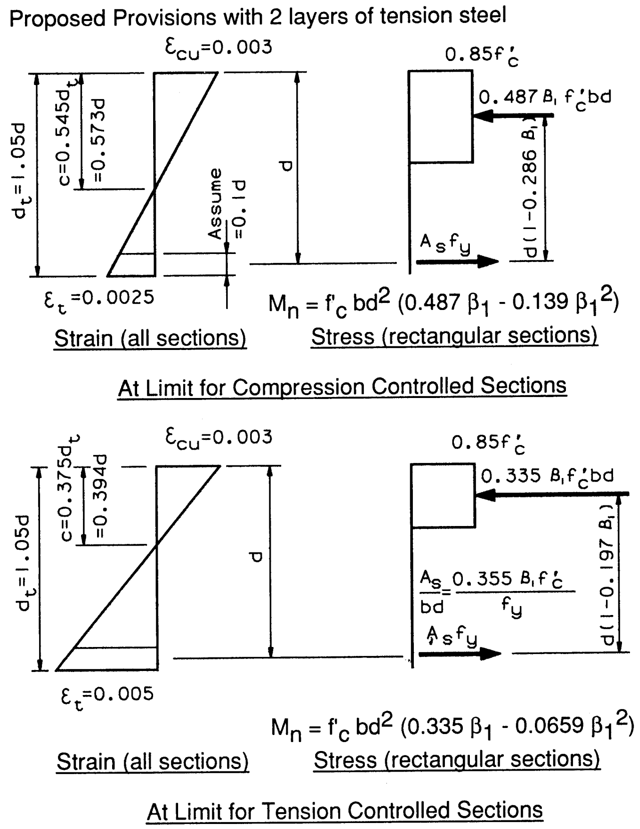


Fig. A-1—Proposed provisions



Note: The designer may conservatively assume d_e = d, and design as with one layer of c.g. of steel.

Fig. A-2—Proposed provisions, with two layers of tension steel

in Fig. B-1. What are the requirements for a prestressed flexural member to have equally desirable "ductile" behavior?

Ductility is commonly defined by the ductility ratio μ, which is the ratio of strain at nominal strength to yield strain. For the reinforcement in the beam described in Fig. B-1, the ductility ratio μ is 0.00376/0.00207 = 1.82. But, suppose a steel with a rounded yield point were used. Fig. B-2 shows steel strains for a beam identical to that illustrated in Fig. B-1, except that the reinforcement described in Fig. B-2 has a rounded yield point, and reaches yield at a strain of 0.0035. For this beam, the ductility ratio μ is 0.00376/0.0035 = 1.07. But is the performance of the beam of Fig. B-2 inferior to that of Fig. B-1? The author believes the performance of the two to be equal. Both beams would have the same deflection and the same degree of cracking at nominal strength, thus giving the same degree of warning prior to failure. Why? Because both beams have the same steel strain (i.e., the same ϵ_t) at failure. In particular, both beams have the same change in steel strain from service load to nominal strength. Since the beam is expected to have satisfactory cracking and deflection at service load, it is this additional strain that provides warning through unsatisfactory cracking and deflection at high load levels.

Now consider a prestressed beam using Type 270K low-relaxation strand. (Strand is chosen for the example because it is the highest strength material in common use. Other steels will have characteristics intermediate between strand and mild reinforcement.) Fig. B-3 shows the stress-strain diagram. A typical jacking load is 0.75 f_{pu} = 202.5 ksi and with 45 ksi loss, the effective prestress is 157.5 ksi, and the corresponding strain is 0.0056. The ASTM-defined yield strain is 0.01, and the yield strength is 243 ksi. The proposed net tensile strain limit for tension controlled sections is 0.005, which, added to the prestress strain of 0.0056, produces a total strain of 0.0106, barely beyond the ASTM-defined yield strain of 0.01. For the reasons cited previously, the author believes that this does not imply unsatisfactory behavior for a flexural member. In fact, since the steel strain caused by service loads is smaller in a prestressed member than in a reinforced member, the changes in strain (from service load to nominal strength) are greater in the prestressed member, for a given net tensile strain limit.

The compression controlled net tensile strain limit of 0.0025 is chosen to be the same as for reinforced concrete. When added to the prestress strain, the resulting strain is approximately that at the proportional limit of the prestressing steel.

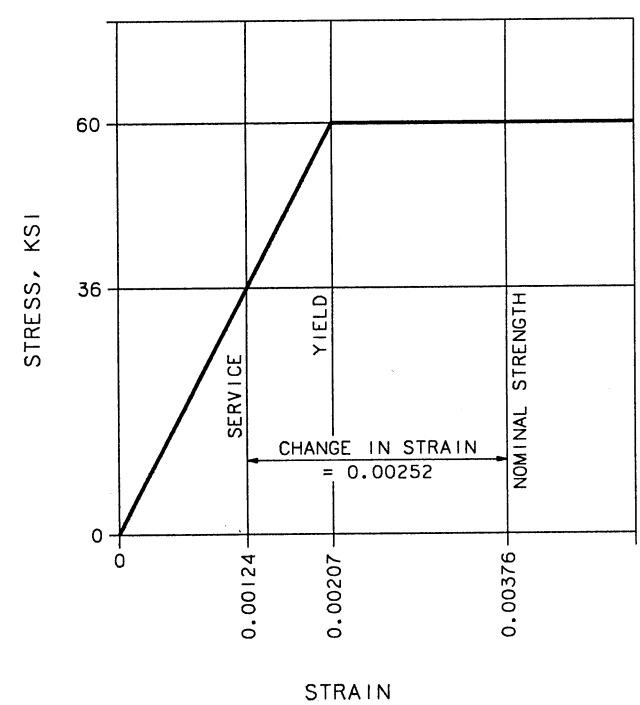


Fig. B-1—Reinforcement with sharp yield point

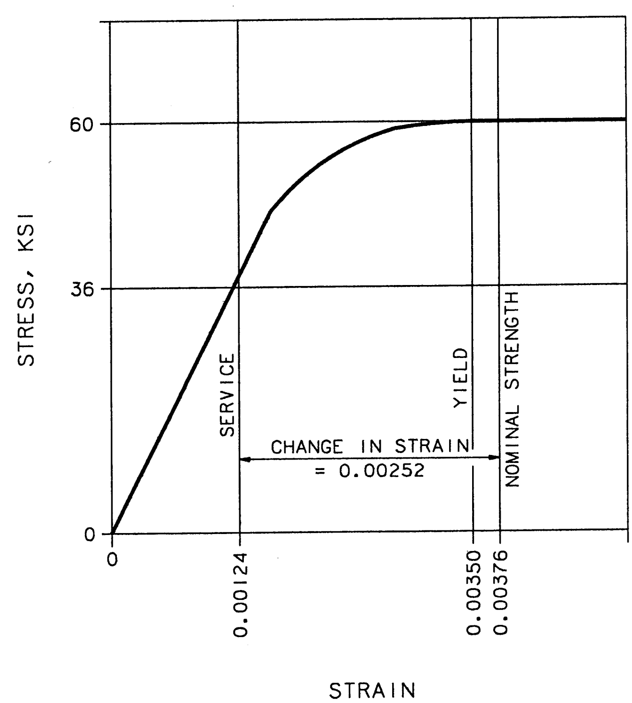


Fig. B-2—Reinforcement with rounded yield point

What if unstressed "sleeper" strands are added to a prestressed beam to improve its nominal strength? The strain in the sleeper strands at failure of the beams will often be well below the yield point—even below the proportional limit. Does this imply a lack of proper behavior? Not so long as the net tensile strain is the same as in the conventionally reinforced and/or prestressed beams that have satisfactory behavior.

It is the net tensile strain, not the ratio to yield strain, that is the valid measure of performance.

APPENDIX C
Provisions for redistribution of negative moments

The current Code provisions for reinforced concrete members are given in Section 8.4. These are stated in terms of the reinforcement ratios ρ. The research results on which the provisions are based are shown in Fig. 8.4 in the commentary. The provisions for prestressed concrete members are given in Section 18.10.4. The following formula may be used to convert from ρ to ϵ_t

$$\epsilon_t = \frac{0.003 + f_y/E_s}{\rho/\rho_b} - 0.003 \qquad \text{(C-1)}$$

The formula for percent moment redistribution given in Section 8.4 may be stated in terms of ϵ_t.

$$\text{Percent reduction} = 20\left(1 - \frac{f_y/E_s + 0.003}{\epsilon_t + 0.003}\right) \qquad \text{(C-2)}$$

The proposed commentary Fig. 8.4 shows the proposed permissible moment redistribution for reinforced sections and the calculated available capacity. The curves for calculated capacity are derived from those in the current commentary, restated in terms of ϵ_t. Note that because of the reciprocal relationship between ϵ_t and ρ, the hyperbola-like curves of the current commentary figure become almost straight lines. The proposed permissible redistribution is somewhat more conservative at low values of ϵ_t, to be consistent with requirements for prestressed concrete (see Fig. C-1). More redistribution is permissible at high values of ϵ_t, i.e., for lightly reinforced sections.

Fig. C-1 shows the proposed permissible moment redistribution for prestressed members. To understand the derivation of Fig. C-1, one must refer back to the derivation of the current provisions for redistribution for prestressed members, which is given in Reference 7. The equations on pages 31-35 of Reference 7 are rederived in terms of net tensile strain ϵ_t. This avoids the need to use β_1, ω's, and the definition of balanced conditions. The result is the following equation

$$\frac{x}{100 - x} = \frac{3}{2}(\epsilon_t/\epsilon_{ty} - 1)(d/l + 0.01) \qquad \text{(C-3)}$$

where x = available percent reduction in support moment and ϵ_{ty} = net tensile strain at yield of extreme tension steel. Eq. (C-3) is the equivalent of Equation 16 in Reference 7.

To create Fig. C-1, which shows percentage changes in moment versus net tensile strain, it is necessary to define the value of ϵ_{ty}. The author's reasoning parallels that given in Appendix B. Referring to Fig. B-3, which of Points 1, 2, and 3 is the best point to use to mark the beginning of inelastic behavior? Technically, inelastic behavior begins at Point 1, although the author would not advocate the use of Point 1. Point 3 has been used in the past[7,8] because it corresponds to the ASTM definition of the yield point for strand.

The author argues that Point 2 is the most reasonable point to mark the beginning of inelastic behavior for the following reasons. First, note that the concern is not for the behavior at Points 1, 2, or 3 when considering members for which moment redistribution is permitted. Redistribution is only permitted for members with strain at nominal strength greater than or equal to the strain at Point 4. When the strain in the member reaches Point 4, the member does not "know" whether the strain arrived at the point by the curved path

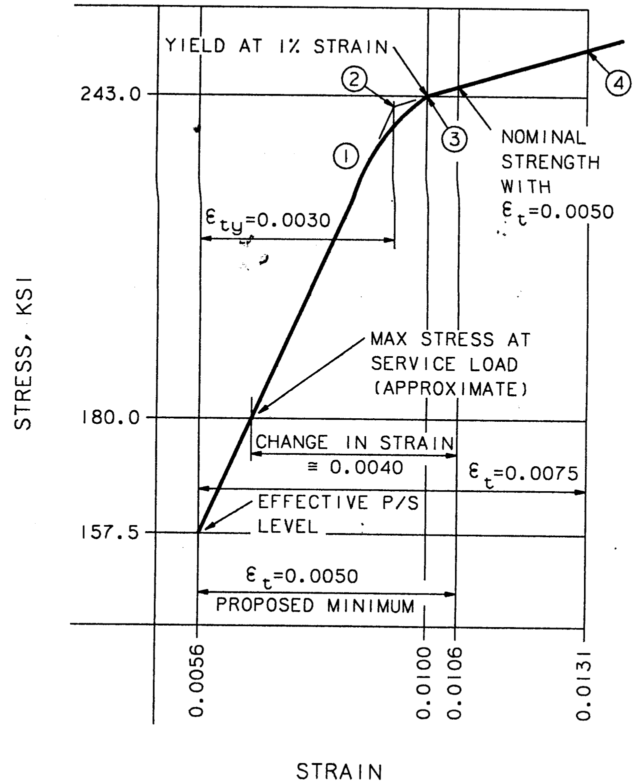


Fig. B-3—Prestressed reinforcement—Type 270K strand

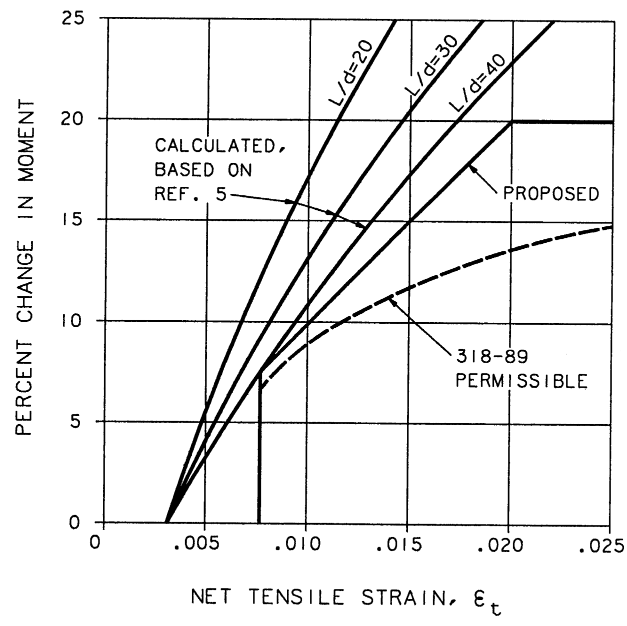


Fig. C-1—Permissible moment redistribution in prestressed members

between Points 1 and 3, or by the bilinear path of Points 1, 2, and 3. One is primarily concerned with the elastic behavior between Point 1 and the (approximately) straight line behavior in the vicinity of Point 4 and beyond. Point 2 represents the intersection of these two straight lines, and the author believes it is the most reasonable point to use in determining ϵ_{ty}. A similar argument is made in Reference 9. Assuming ϵ_{ty} to be 0.003, as shown in Fig. B-3. Fig. C-1 results.

Comparing new Fig. 8.4 and C-1, it may be seen that a common moment redistribution provision for reinforced and prestressed concrete members is workable.