Title No. 55-51 presents an investigation of flexural bond in beams pretensioned with seven-wire strand of 1/4, 3/8, and 1/2 in. diameter. Primary object was to clarify the factors affecting bond action and strength, and to study the influence of certain variables on strand slip. The principal factor investigated was variation of strand embedment length.

The experimental results are discussed in detail, and criteria are presented for adequate design with respect to general bond slip of pretensioned strand. A hypothetical shape for the flexural bond stress wave immediately before bond slip is derived from the experimental results reported.

FLEXURAL BOND TESTS

of pretensioned prestressed beams

NORMAN W. HANSON and PAUL H. KAAR

IN AMERICAN PRESTRESSING PRACTICE, pretensioning has developed more rapidly than post-tensioning and a dominating trend has developed in recent years toward use of the seven-wire prestressing strand. Associated with the use of strand for pretensioning, there has been an economy-dictated trend toward the use of increased diameter of strand. The 1/4 in. diameter strand was in common use in 1950. In 1956, extensive use was made of the 3/8 in. diameter strand, and some projects have recently utilized the 1/2-in. strand.

At this time there is no generally accepted method of predicting the bond performance of pretensioned strand; the effect on bond of these strand size increases is not fully understood. Bond studies of pretensioned concrete beams have, with few exceptions, been confined to prestress transfer bond, rather than flexural bond. The exceptions were studies by Janney[1] on flexural bond of single wire, and by Thorsen[2] and Norby and Venuti[3] on the relationship of flexural to transfer bond of pretensioned tendons.

The work reported in this investigation was done in the Research and Development Laboratories of the Portland Cement Association in 1955-57. The study was undertaken to assist in answering the often repeated query, "What is the maximum diameter of pretensioned seven-wire strand which one can safely use in a particular beam?" This question cannot be answered adequately without knowledge of the conditions under which bond failure may take place. Thus, the objectives of this investigation were primarily to

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develop a theory of bond action predicting ultimate strength in bond, and secondarily to study the influence of various factors on the bond performance of prestressing strand.

Scope of tests

The embedment length and diameter of strand were the principal variables investigated with respect to their influence on the bond performance of pretensioned prestressed beams. The influence of reinforcement percentage and reduction in concrete strength were investigated in a more limited manner. In addition, pilot tests checked on the influence of strand surface condition and the effect of embedded end anchorages on pretensioned strand.

Notation

The notation proposed by the Joint ACI-ASCE Committee 323, Prestressed Reinforced Concrete is used where applicable:

A_s = area of tension reinforcement
a = one-half the distance between layers of strand
b = width of rectangular section
c = depth of compression zone at ultimate moment
d = effective depth
Σo = total circumferential area per in. length of beam
ϵ_{cu} = tensile strain in concrete at level of reinforcement at ultimate moment
ϵ_{se} = tensile strain in reinforcement due to effective prestress
ϵ_{su} = tensile strain in reinforcement at ultimate moment
ϵ_u = ultimate compressive strain in concrete
F = $\frac{\epsilon_{su} - \epsilon_{se}}{\epsilon_{cu}}$
= ratio of strain increase in the steel relative to strain increase in the concrete at the steel level
f_c' = compressive strength of 6 × 12-in. cylinders
f_{sb} = stress in reinforcement at general bond slip
f_{se} = effective prestress in reinforcement
f_{su} = stress in reinforcement at ultimate moment
f_{yp} = yield strength of strand at 1 percent offset
k_1k_3 = coefficient determining average concrete stress in the compression zone at ultimate moment
k_2 = coefficient determining position of internal compressive force C
l = length of shear span from concentrated load to nearest support reaction
l_t = length of transfer zone
l_u = embedment length of strand
= distance from section of maximum steel stress to free strand end
M_c = cracking moment

M_{flex} = calculated ultimate flexural moment

M_{test} = measured moment at ultimate strength

M_{bond} = measured moment at general bond slip

p = A_s/bd = steel ratio

q_y = tension reinforcement index = pf_{yp}/f_c'

u_a = average bond stress over entire embedment length as defined by Eq. (10)

u_a' = average bond stress between end of transfer zone and a section of maximum strand stress as defined by Eq. (A1)

ANALYTICAL CONCEPTS OF BOND ACTION

In this discussion of bond in pretensioned beams, much use is made of Janney's paper[1] on the same subject.

Nature of bond

Bond in pretensioned concrete beams is of two types—transfer bond and flexural bond. Transfer bond utilizes a part of the available tensile strength of the steel to establish compression in the concrete. Flexural bond results from the action of external loads on beams. After cracking, the increase in steel stress above effective prestress develops flexural bond stress between the steel and concrete.

Transfer bond—Prestress transfer bond exists near beam ends after the load in the tensioned strand has been transferred to the concrete member. The length over which this transfer is made is termed the prestress transfer length, and depends mainly on the amount of prestress, surface condition of the strand, the strength of the concrete, and the method of steel stress release, which in these tests was gradual. Three factors which contribute to bond performance are adhesion between concrete and steel, friction between concrete and steel, and mechanical resistance between concrete and steel. In the transfer zone, reduction in the tensile strain of the steel does not equal the compressive strain in the concrete at the same point. There is relative movement of steel and concrete, and accordingly adhesion cannot contribute to prestress transfer. Friction is considered to be the principal agent causing stress transfer from pretensioning steel to concrete. As the tension in the strand is released, the strand diameter tends to increase, thus producing high radial pressure against the concrete, which in turn produces high frictional resistance in the transfer zone. Mechanical resistance probably contributes little to prestress transfer in the case of individual smooth wires, but it may be a factor of some significance in the case of strand.

Flexural bond—Flexural bond of significant magnitude exists only after the concrete beam has been loaded to cracking. When the concrete cracks, the bond stress in the immediate vicinity of the cracks rises to some limiting stress, slip occurs over a small portion of the strand length adjacent to the cracks, and the bond stress near the cracks is then reduced to a low value. With continued increase in load, the high bond stress progresses as a wave from the original cracks toward the beam ends. The bond stress remaining behind the wave is always lower than the maximum value at the peak of the bond stress wave.

If the peak of the high bond stress wave reaches the prestress transfer zone, the increase in steel stress resulting from the bond slip decreases the strand diameter, reduces the frictional bond resistance, and precipitates general bond slip. Following loss of frictional resistance, mechanical resistance is the only factor which can contribute to bond between concrete and steel. If the beam is prestressed with clean smooth wire this resistance will be slight, and the beam will quickly collapse after general slippage has occurred. If the wire is rusted, the resistance to slippage will be greater. If the beam is prestressed with strand, the helical shape of the individual wires will provide sufficient mechanical resistance that the beam can support additional load even after slip of the strand at the beam ends. This mechanical resistance of strand to continued slip will be discussed later.

Significance of the *F*-value

The ratio of steel flexure strain to apparent concrete flexure strain at the level of the steel has been designated F[4]: in an uncracked pretensioned prestressed beam the flexural strains in the steel and in the concrete at the level of the steel will be the same and F will equal 1.0. When a pretensioned beam cracks, a certain amount of local bond slip must occur on each side of each crack. From cracking load onward, the value of F will depend on the amount of local bond slip adjacent to the cracks. Where considerable local bond slip occurs, F can have values considerably below 1.0.

This concept of the relationship between steel strains and concrete strains is a useful one in interpreting beam behavior, but the variable nature of F for strand-reinforced beams seems almost to preclude its determination.

The ultimate flexural strength calculations for the test beams are based on an F of 1.0. Analysis of data[5] has confirmed this as a reasonable value for good bond conditions with ¼ to ½-in. tendons.

COMPUTATION OF ULTIMATE FLEXURAL STRENGTH

In analyzing the test data it was desirable to use the more precise methods of computing ultimate strength rather than simplified design equations, so that the relationships between test values and theoretical values might be determined as accurately as possible.

All ultimate moments given as M_{flex} in this paper are calculated using the ultimate strength factors, k_1, k_2, k_3, and ϵ_u presented by Hognestad, Hanson, and McHenry[6] in a paper concerning the stress distribution in the compression zone of structural concrete flexural members. The ultimate strength factors were expressed as functions of concrete strength as follows:

$$k_1 k_3 = \frac{3900 + 0.35 f_c'}{3200 + f_c'} \qquad (1)$$

$$k_2 = 0.50 - \frac{f_c'}{80{,}000} \qquad (2)$$

$$\epsilon_u = 0.004 - \frac{f_c'}{6.5 \times 10^6} \qquad (3)$$

Using these factors and a value of $F = 1.0$, the steel stress at ultimate flexural strength f_{su} was calculated by successive approximations as described by Janney, Hognestad, and McHenry[5]. Here, for beams containing steel at one level, we have from requirements of compatibility of strains:

$$\frac{c}{d} = \frac{F\epsilon_u}{\epsilon_{su} - \epsilon_{se} + F\epsilon_u} \qquad (4)$$

From condition of equilibrium of concrete and steel forces:

$$c = \frac{f_{su}\,A_s}{k_1\,k_3\,f_c'\,b} \qquad (5)$$

The flexural ultimate moment is given by:

$$M_{flex} = f_{su}\,A_s\,(d - k_2\,c) \qquad (6)$$

In this calculation, ϵ_{su} is first assumed, and f_{su} is then computed using Eq. (4) and (5). The values of ϵ_{su} and f_{su} so obtained are checked for compatibility with the stress-strain curve for the strand involved. Several trials may be necessary to establish this compatibility. The final value of f_{su} so obtained is substituted in Eq. (5) to give the neutral axis depth c, which in turn is substituted in Eq. (6) to obtain the ultimate moment of resistance of the section.

For beams containing steel placed in two layers, subscripts t and b are added to the steel notation to denote top and bottom layer. The strain compatibility equations are then:

$$\frac{c}{d} = \frac{F\epsilon_u\left(1 - \frac{a}{d}\right)}{\epsilon_{sut} - \epsilon_{se} + F\epsilon_u} = \frac{F\epsilon_u\left(1 + \frac{a}{d}\right)}{\epsilon_{sub} - \epsilon_{se} + F\epsilon_u} \qquad (7)$$

The condition of equilibrium of forces leads to:

$$c = \frac{f_{sub}\,A_{sb} + f_{sut}\,A_{st}}{k_1\,k_3\,f_c'\,b} \qquad (8)$$

The flexural moment is then given by the equation:

$$M_{flex} = f_{sub}\,A_{sb}\,(d_b - k_2\,c) + f_{sut}\,A_{st}\,(d_t - k_2\,c) \qquad (9)$$

The values of f_{su} and ϵ_{su} for the two layers of steel are obtained by the method of successive approximations as before. Eq. (8) and (9) are then used to calculate the depth of the neutral axis and the ultimate moment of resistance.

TEST SPECIMENS

Outline of tests

The test program involved 47 beam tests divided into four series as shown in Table 1:

Series 1—Eighteen prestressed beams containing clean, smooth pretensioned strands were tested over various shear-span lengths covering a wide range. These beams contained, separately, either ¼, ⅜, or ½ in. diameter strand. These tests were conducted to evaluate the effect of strand diameter and embedment length on bond performance.

TABLE 1—BEAM PROPERTIES

Beam series and No.	Strand No. and size, in.	Beam width, b, in.	Beam effective depth, d, in.	Shear span, l, in.	Embed-ment length, l_u, in.	Concrete strength, f_c', psi	Effective prestress, f_{se}, ksi	Per-centage steel, p
1-1	4 ¼	6.0	8.61	21	27	6040	141.0	0.279
1-2	4 ¼	6.0	8.54	31	37	6620	141.0	0.281
1-3	4 ¼	6.0	8.56	36	42	7800	141.0	0.280
1-4	4 ¼	6.0	8.76	42	48	6040	141.0	0.274
1-5	4 ¼	6.0	8.67	84	90	6040	141.0	0.277
1-6	4 ¼	6.0	8.68	168	174	5980	141.0	0.276
1-7	3 ⅜	6.0	8.56	21	27	5730	129.7	0.467
1-8	3 ⅜	6.0	8.63	42	48	5730	129.7	0.463
1-9	3 ⅜	6.0	8.65	84	90	5730	129.7	0.462
1-10	3 ⅜	6.0	8.60	168	174	5130	144.6	0.465
1-11	2 ½	6.0	8.40	21	33	5540	148.0	0.570
1-12	2 ½	6.1	8.50	30	36	5600	132.0	0.555
1-13	2 ½	6.1	8.50	34	40	6300	132.0	0.555
1-14	2 ½	6.0	8.41	42	54	5540	139.0	0.569
1-15	2 ½	6.1	8.44	60	66	5600	132.0	0.558
1-16	2 ½	6.1	8.31	71	77	5600	132.0	0.566
1-17	2 ½	6.1	8.69	84	90	5090	132.0	0.543
1-18	2 ½	6.1	8.40	168	174	5090	132.0	0.561
2-1	4 ⅜	6.0	8.62	54	60	3700	124.9	0.618
2-1R*	4 ⅜	6.0	8.47	54	60	3700	124.9	0.630
2-1A†	4 ⅜	6.0	8.41	54	60	3700	124.9	0.634
2-2	4 ⅜	6.0	8.62	54	60	5420	119.4	0.618
2-2R	4 ⅜	6.0	8.44	54	60	5420	119.4	0.632
2-2A	4 ⅜	6.0	8.44	54	60	5420	119.4	0.632
2-3	4 ⅜	6.0	8.69	54	60	7230	121.2	0.614
2-3R	4 ⅜	6.0	8.56	54	60	7230	121.2	0.623
2-3A	4 ⅜	6.0	8.45	54	60	7230	121.2	0.631
3-1	2 ¼	4.0	5.69	36	40	5020	146.0	0.316
3-2	4 ¼	4.0	5.66	36	40	5720	142.0	0.637
3-3	6 ¼	4.0	5.86	36	40	5900	113.0	0.921
3-4	2 ⅜	6.0	8.44	54	60	5300	134.0	0.316
3-5	2 ⅜	6.0	8.37	54	60	5400	131.0	0.318
3-6	4 ⅜	6.0	8.53	54	60	5300	136.0	0.625
3-7	4 ⅜	6.0	8.54	54	60	5900	119.0	0.624
3-8	6 ⅜	6.0	8.53	54	60	5450	132.5	0.938
3-9	6 ⅜	6.0	8.07	54	60	5700	127.0	0.991
3-10	2 ½	8.0	11.47	72	80	6450	120.0	0.313
3-11	4 ½	8.0	11.40	72	80	6050	135.0	0.631
3-12	6 ½	8.0	11.33	72	80	5420	132.0	0.963
4-1	1 ⅜	4.0	5.56	28	34	5750	128.0	0.359
4-1R	1 ⅜	4.0	5.56	28	34	5750	128.0	0.359
4-2	1 ½	4.0	5.59	28	32	5720	141.0	0.643
4-2R	1 ½	4.0	5.50	28	32	5720	139.0	0.654
4-3	1 ½	6.0	8.29	42	48	5430	147.0	0.289
4-3R	1 ½	6.0	8.40	42	48	5500	141.0	0.285
4-4	2 ½	6.0	8.38	42	48	5350	142.0	0.572
4-4R	2 ½	6.0	8.45	42	48	5350	147.0	0.567

*R indicates use of rusted strand.
†A indicates use of strand anchor embedded in concrete at beam ends.

Series 2—In nine beams with ⅜-in. strand, tested in three groups, the concrete strengths of the three groups were 3700, 5420, and 7230 psi. Within each group, one beam contained a rusted strand, one a clean, smooth strand, and one a clean, smooth strand with a "Strand-vise" anchor within the concrete at each end. These tests were designed to evaluate primarily effects of concrete strength on bond performance.

Series 3—Twelve test specimens comprising three sets of geometrically similar beams (b and d variable, but b/d constant) were prestressed with clean, smooth strands. The sets were reinforced with ¼, ⅜, and ½-in. strands, one strand size in each set, and the number of strands per beam was two, four, and six. The tests in this series were intended to demonstrate primarily the influence on bond performance of variation of reinforcement percentage.

Series 4—Eight beams were prestressed with ⅜ in. or ½ in. diameter strands. The beams were made in pairs, one with clean, smooth strand and one with rusted strand. The tests of this group together with Series 2, were expected to indicate possible effects on beam performance of strand surface condition.

AND TEST RESULTS

$q_y = \frac{p f_{yn}}{f_c'}$	Cracking moment, M_c, in.-kips	M_{bond} at bond slip, in-kips.	M_{test} at ultimate strength, in.-kips	Calculated flexural ultimate, M_{flex}, in.-kips	$\frac{M_{bond}}{M_{flex}}$	$\frac{M_{test}}{M_{flex}}$	Mode of failure‡
0.121	207.2	287.0	337.4	296.9	0.97	1.14	B
0.111	151.0	278.1	313.2	295.2	0.94	1.06	B
0.094	146.2	299.2	309.1	299.2	0.99	1.03	B
0.118	206.1	—	322.6	302.4	—	1.07	F
0.120	202.6	—	322.3	298.1	—	1.08	F
0.121	190.0	—	302.4	299.4	—	1.01	F
0.196	241.9	380.4	423.0	411.4	0.93	1.03	B
0.195	239.7	428.6	428.6	417.1	1.03	1.03	B
0.194	238.3	—	464.1	464.1	—	1.11	F
0.195	227.6	—	439.3	408.6	—	1.07	F
0.261	292.0	399.7	418.0	488.0	0.82	0.86	B
0.251	200.5	295.0	429.5	493.2	0.60	0.87	B
0.223	269.8	380.3	450.3	505.9	0.75	0.89	B
0.260	294.4	426.7	481.3	486.0	0.88	0.99	B
0.252	248.5	439.0	457.0	488.4	0.90	0.94	B
0.256	216.4	440.3	455.4	473.7	0.93	0.98	B
0.270	243.1	—	503.4	495.0	—	1.02	F
0.279	256.8	—	474.9	470.0	—	1.01	F
0.402	289.0	—	504.6	461.6	—	1.09	F
0.410	289.9	—	493.8	453.7	—	1.09	F
0.413	289.9	—	496.5	451.4	—	1.10	F
0.298	314.2	451.9	492.4	535.0	0.84	0.92	B
0.298	295.3	—	549.1	526.2	—	1.04	F
0.298	300.7	503.2	507.3	517.6	0.97	0.98	B
0.204	346.6	603.1	609.9	538.7	1.12	1.13	B
0.207	326.3	603.1	607.2	532.3	1.13	1.14	B
0.210	327.7	603.1	608.5	523.8	1.15	1.16	B
0.160	59.3	88.1	94.4	92.5	0.95	1.02	B
0.283	95.3	172.7	185.3	163.1	1.06	1.13	B
0.397	115.1	210.6	210.6	204.0	1.04	1.04	B
0.155	207.2	308.5	312.5	314.0	0.98	0.99	B
0.138	151.9	259.9	259.9	285.0	0.91	0.91	B
0.307	326.0	529.9	543.4	546.0	0.97	1.00	B
0.248	211.3	454.3	465.1	463.0	0.98	1.00	B
0.448	417.8	—	695.9	666.0	—	1.05	F
0.409	332.8	643.3	643.3	561.0	1.15	1.15	B
0.123	466.9	589.3	681.1	766.0	0.77	0.89	B
0.264	801.7	—	1330.9	1296.5	—	1.03	F
0.450	1035.7	1521.7	1602.7	1505.0	1.01	1.07	B
0.147	64.8	79.8	79.8	92.0	0.89	0.89	B
0.147	62.7	78.7	104.9	92.0	0.86	1.16	B
0.285	96.9	99.7	117.9	155.5	0.64	0.76	B
0.290	92.7	148.7	148.7	151.0	0.98	0.98	B
0.135	176.8	218.8	294.4	277.0	0.79	0.79	B
0.131	189.3	302.8	302.8	280.5	1.08	1.08	B
0.271	281.8	363.7	425.8	477.0	0.76	0.89	B
0.269	296.8	551.6	551.6	486.0	1.13	1.13	B

‡B designates bond slip to end of strand either before or at ultimate moment.
F designates flexural failure without bond slip to end of strand.

Materials

All beams were cast by internal vibration of a 2-in. slump concrete containing 1½-in. maximum size aggregate. The cement was a blend of four brands of Type I portland cement. Moist curing under burlap for the first 3 days was followed by storage at approximately 70 F and 50 percent relative humidity. Concrete cylinder strengths for each beam specimen are given in Table 1, based on tests of at least three 6 × 12-in. cylinders cast and cured with each beam specimen, or each group of beams cast simultaneously in the pretensioning bed.

The prestressing steel was seven-wire strand of ¼, ⅜, and ½ in. nominal diameter. Nominal cross-sectional areas and circumferences are given in Table 2.

The stress-strain curves to 1 percent offset are shown in Fig. 1. The stress is tension force divided by nominal strand cross section, and the strain was measured by three SR-4 gages mounted on separate single wires of the strand. For purposes of beam test analyses, strand strain measured in beams by SR-4 gages was converted into stress by the applicable curve of Fig. 1. Note that there are two different steels used in the beams with ¼-in. strand and three different steels used in the beams with ⅜-in. strand.

TABLE 2—STRAND PROPERTIES

Strand size, in.	Nominal circumference, 4/3 πd, in.	Nominal cross-sectional area, sq. in.
¼	1.05	0.036
⅜	1.58	0.080
½	2.10	0.144

All strand was cleaned of surface oil by washing in carbon tetrachloride. The rusted strands used in special beam tests were obtained by exposing the strand to a moist environment for approximately 1 week. The rust coating so obtained was uniform without pitting.

Instrumentation

As bond failure is progressive rather than instantaneous, the test beams were instrumented to record the continuous development of flexural strains and the subsequent evidence of bond slip. Bonded SR-4 type A-12-2 strain gages were cemented along the helical individual wires of the twisted strand. The beams in Series 3 were instrumented with gages at 12, 15, and 20-in. intervals for ¼, ⅜, and ½-in. strand size, respectively. To minimize reduction of surface area available for bond of any one strand, the gages were placed alternately on adjacent strands. Various gage spacings were used in beams of other series.

In addition to their use during the testing of the specimens, the strain gages were used to gage the proper strand tension before casting of the beams. Strain gage readings were taken immediately before and after tensioning, and also before and after stress release to determine immediate losses. These losses at the beam center ranged from 1.7 to 7.3 percent of the initial wire strain.

Dial indicators reading to 0.001 in. were used to measure beam deflections. Dial gages were also mounted at both ends of each beam to detect end slip of the projecting strand ends relative to the concrete.

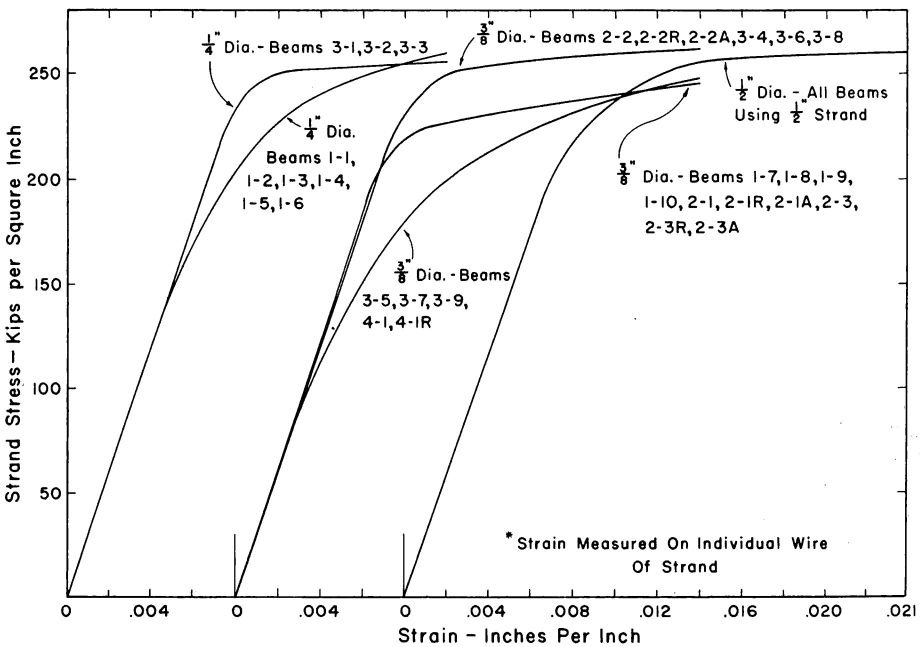

Fig. 1—Stress-strain curves of strand

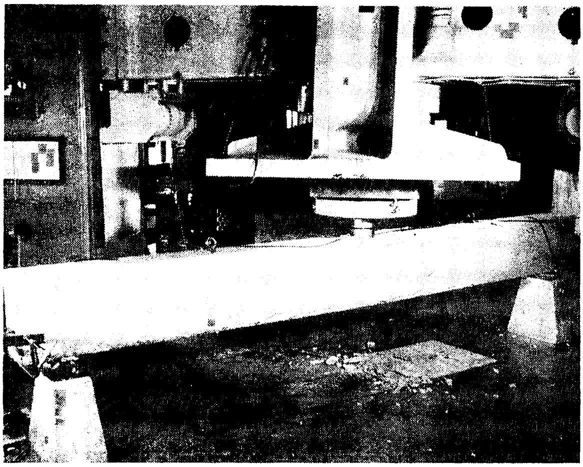

Fig. 2—Typical test arrangement

Test procedure

All beams were tested to failure in a hydraulic testing machine as shown in Fig. 2. Beams of Series 2 and 3 were loaded at midspan through either a ball or roller arrangement. All other beams were loaded at two points through rollers spaced twice the beam width on either side of midspan. The beams were supported on concrete piers with a free roller at one end and a pivot at the other end.

The strand tension was released at a concrete strength of 4500 psi except for the lower strength specimens of Series 2, which were released at a strength of 3500 psi. The beams were tested at the strengths shown in Table 1.

The individual beam tests were generally conducted in about 30 min, with a continuously increasing load. The steel strains as indicated by the SR-4 gages were recorded continuously, and dial gage readings of deflection and strand slip were made without interrupting the loading.

TEST RESULTS

Typical beam behavior

As an introduction to the discussion of test results, the test of a typical specimen of this project will be described in detail—Beam 3-10 of Series 3 shown in Fig. 3. Electrical resistance gages were placed 40 in. apart on the two ½ in. diameter strands, staggered so that there were three 20-in. intervals on either side of midspan. Since the 160 in. long beam was tested on a 12-ft span, the gages were spaced to within 20 in. of the beam end. Strains were continuously recorded during the entire test from zero load to ultimate strength. A load was applied at midspan of the beam through a spherical loading block bearing on a 4 in. wide plate.

Test results are plotted in Fig. 3. Strains have been converted to stresses, using the applicable stress-strain curve of Fig. 1. Strains before cracking are relatively small; they correspond to a stress increase above the prestress of less than 5000 psi, and flexural bond stresses are only 5 to 6 psi. These stresses are typical, however, of service conditions since service loads in pretensioned beams or slabs are ordinarily less than cracking loads.

In this test, with the beam center loaded, the initial crack formed at Gage 4, the center gage. At the cracking load of 12,450 lb, the steel strain at this crack location increased sharply and continued to increase until the end of the test. As the loading increased, the wave of high strains, caused by the progressing bond wave, enveloped Gages 5 and 3 at either side of the center. The bond wave intersected Gage 5 at 13,100 lb and Gage 3 at 15,400 lb load. The strand stress increases caused by the bond wave did not originate only with the midspan crack. Other cracks formed at either side of midspan during the test, and strain increases were caused by waves originating with the midspan crack and nearby cracks.

At a load of 15,500 lb, stress in Gage 5 began to decrease even though the beam load was increasing. This indicated bond slip; the initial slippage of the strand relative to the concrete caused some stress relaxation in the strand. This stress relaxation at slip reduced the strand stress only slightly, for fric-

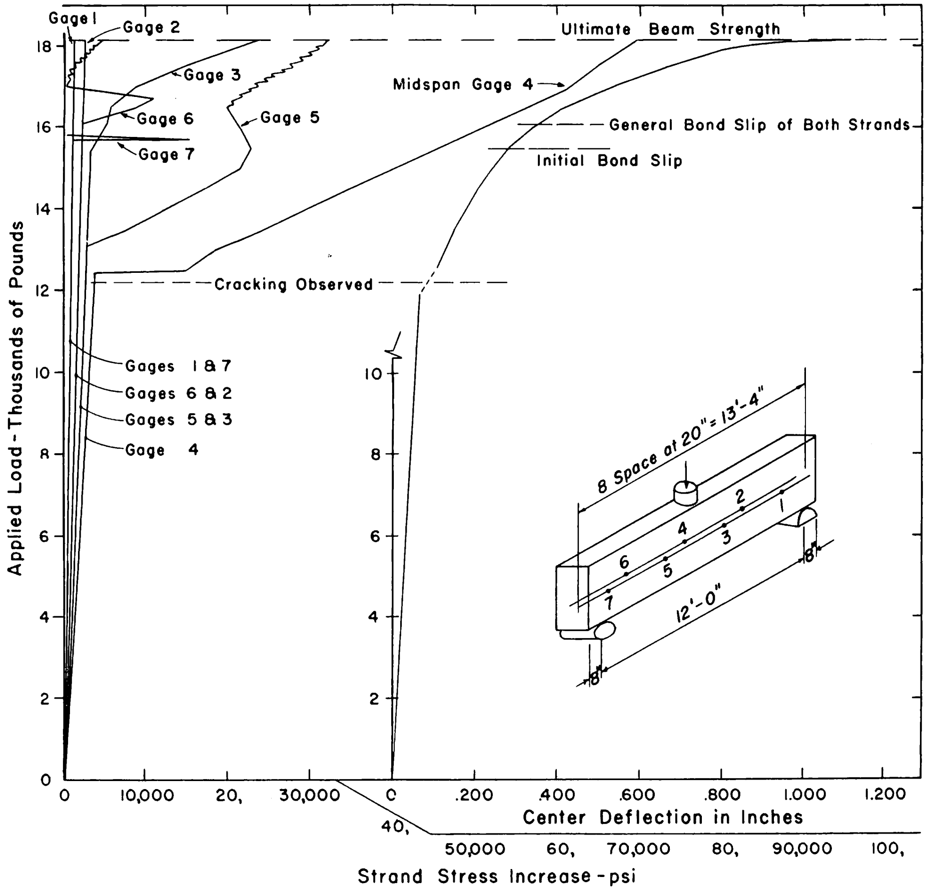

Fig. 3—Load versus strand stress increase and deflection, Beam 3-10

tional bond and mechanical resistance of the twisted strand to further movement in the beam replaced the now-destroyed initial adhesive bond. From this load to complete failure the strain trace of Gage 5, as recorded, moved in small jerks, showing the sawtooth trace indicative of the development of a mechanical interlock resisting slip.

Shortly after the bond slip indication at Gage 5, Gage 7, 40 in. away, showed a large strain increase followed by a reduction to a strain below the prestress level. The bond wave at this time had traveled to within 20 in. of the beam end. Since this location was within the prestress transfer length, resistance to flexural bond stress was only slight.

The same bond phenomena can be traced on the other of the two strands. At a load of 16,100 lb, slip occurred at Gage 6. This gage was near the prestress transfer zone, and bond slip here was not as sudden as at Gage 7 well within the transfer zone.

At the other end of the beam, it is apparent from the record of Gages 1 and 2 that the strands did not fail in bond. The bond wave traveled from the center to beyond Gage 3, but did not reach Gages 1 and 2.

General bond slip of both strands was indicated at a load of 16,100 lb by slip measured by dial gages at the protruding strand ends. Thereafter, the beam was loaded to 18,150 lb before crushing of the concrete at the top of the beam took place at midspan. Midspan deflections, shown in Fig. 3, increased rapidly after bond failure.

This characteristic beam behavior shows that seven-wire strand, in contrast to smooth individual wires, can develop additional beam strength even after general bond slip to the beam end has occurred. This result of mechanical bond resistance is an extremely important characteristic of strand performance.

Discussion of test results

Interrelation of variables—If a strand is to be stressed to fracture at ultimate load of a beam, then there is a critical embedment length for each size of strand, which must be provided if bond slip is to be avoided. However, if a beam contains a high percentage of steel, or if the concrete strength is low, then flexural failure may occur by crushing of the concrete while the steel is stressed below its ultimate strength. In this case bond slip will not necessarily occur even if the embedment length provided is less than the critical length needed for the size of strand used to develop the ultimate strand strength. Thus concrete strength, percentage of steel, and embedment length are interrelated in design.

Mode of failure—Thirteen of the beams failed in flexure without prior slippage of the strand along its entire embedment length. The remainder failed in flexure after a general bond slip of the strands. The moment sustained at general bond slip and the ultimate moment sustained were both of interest in this investigation. Comparison of moments sustained at general bond slip and at ultimate beam strength with the theoretical ultimate flexural strength is presented in Table 1.

Strand embedment length—The effect of variation of strand embedment length on the moment at general bond slip and on the ultimate moment of resistance of a given section, is illustrated in Fig. 4, in which M_{test} is the measured moment at ultimate strength, M_{bond} is the measured moment at general bond failure, and M_{flex} is the calculated ultimate flexural moment.

Beams having an embedment length from load to beam end of 80 in. or more failed in flexure by crushing of the concrete after yielding of the steel, before general bond slip could occur. As the embedment length decreased, failure occurred at progressively lower moments due to slippage of the strands. Failure by slippage of the strands occurred in two stages, (a) initial general slip of the strand along its whole embedment length, and (b) destruction of the mechanical interlocking effect between the strand surface and the surrounding concrete. In the case of strands with short embedment lengths, a considerable increase of load on the beam was possible between these stages. It should be emphasized, however, that the loading was of the static type. If such beams were subject to dynamic loading after the first stage of general bond slip, it is possible that the increase in load to reach stage two would be significantly reduced. Until dynamic loading tests can be carried out to cover this point, it appears desirable for *design* purposes to rely only on attaining a moment of resistance corresponding to general bond slip.

One reason for the scatter of data in these tests is initial slip of the center wire of the strand, which remained undetected by the strain gages. In several

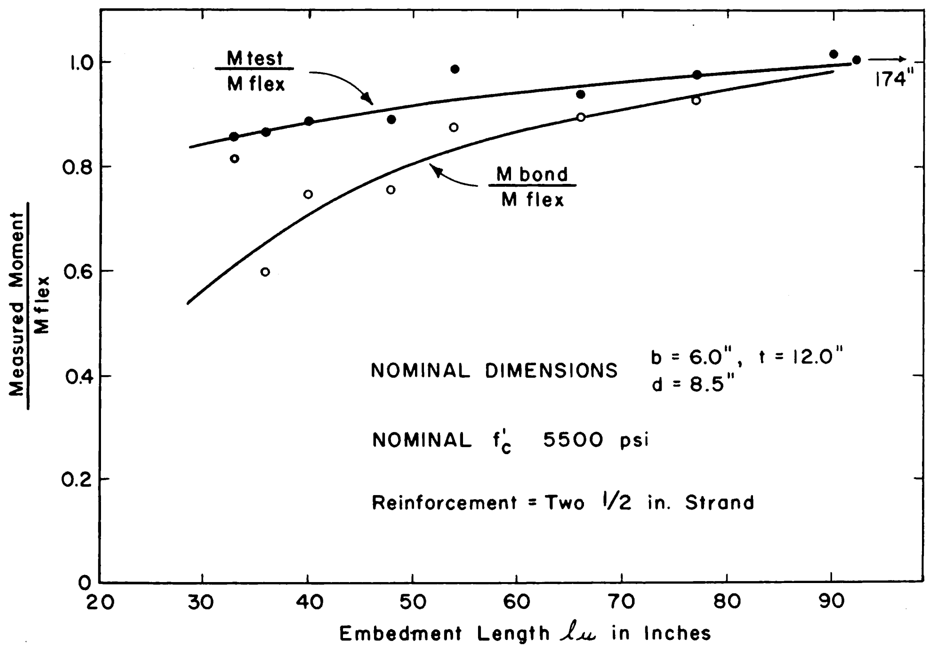

Fig. 4—Performance of beams with various embedment lengths

tests, the center wire had moved as much as 0.1 in. relative to the surrounding six wires at ultimate beam strength. In cases where the strand had been flame cut, thus fusing the strand end, the force exerted by the slipping center wire was in some cases enough to buckle the outside wires of the strand at the beam end.

TABLE 3—PERFORMANCE OF SERIES 3 BEAMS

Beams	Average steel percentage	Average $\frac{M_{bond}}{M_{flex}}$	Average $\frac{M_{test}}{M_{flex}}$
3-1, 3-4, 3-5, 3-10	0.31	0.90	0.95
3-2, 3-6, 3-7, 3-11	0.62	1.01	1.04
3-3, 3-8, 3-9, 3-12	0.95	1.06	1.07

Percentage of steel—High steel percentages reduce the possibility of bond failure for a given embedment length because the steel stress at flexural failure is less in a beam with a high steel percentage than in a beam with a low *p*. It follows that, for a given embedment length, the average bond stress at flexural failure will be less in a beam with high steel percentage than in a beam with low steel percentage. The risk of exceeding the average bond stress at which general slip occurs for a particular embedment length is therefore greater with a beam having a low percentage of steel. The beams of Series 3 illustrate this point (see Table 3).

Reduction of concrete strength—The effect of reduction of concrete strength in a particular beam is to decrease the steel stress at flexural failure, with a correspondingly lower average bond stress over the embedment length. If the reduction in average bond stress due to drop in steel stress is greater than the reduction in the bond strength due to drop in concrete strength, then a failure due to general bond slip will be less likely in the beam with reduced concrete strength. Tests in Series 2 indicate that this is the case. These beams of identical cross section were tested with the same strand embedment length. The three beams of high strength concrete failed by general bond slip. Of the beams made of intermediate strength concrete, two failed by general bond slip and one by flexure. The low strength concrete beams all failed in flexure.

From these tests it appears that reduction of concrete strength has more influence on ultimate flexural strength than on ultimate bond strength.

Strand surface condition—The beams prestressed by rusted strand performed as well as, or better than, the beams prestressed by clean, smooth strand. Moment at general bond slip compared with beams having clean strand was improved as much as 53 percent in the case of Beam 4-2, although one beam (2-3) showed no significant improvement. The average improvement for all Series 4 beams with rusted strand was 30 percent. This confirms Janney's findings.[1] However, the danger of careless rusting allowing localized pitting should be remembered.

Provision of end anchors—A comparison of anchored and unanchored beams in Series 2 for three concrete strengths used shows little difference in performance. The end anchors did not become effective until slip had occurred along the entire embedment length, and they were therefore unable to delay the onset of general bond slip. After the anchors became effective the beams acted as post-tensioned beams without bond, and their ultimate moments of resistance are no greater than those of the beams depending only on the mechanical interlocking effect between the strand and concrete after general bond slip.

Analysis of test results

Strand size and embedment length—These were the principal variables investigated with respect to their influence on the bond performance of pretensioned prestressed beams. In the case of these two variables a sufficient number of test results were obtained to warrant a detailed analysis. The influence of other less important variables has been discussed in general terms.

Calculation of bond stresses—At present it is not feasible to measure the magnitude and complex distribution of bond stresses along the entire length of a prestressing strand. However, to obtain suitable design criteria, we can calculate the average bond stress along the length of a strand from its free end to a section of maximum stress.

Consider the forces acting along the length of a prestressing strand between its free end and a section of maximum stress. The force due to tensile stress in the steel will then equal $f_s A_s$. The force due to bond stresses on the surface of the embedded strand will be given by $u_a \; l_u \; \Sigma o$. To satisfy the conditions of static equilibrium these forces must be equal: $u_a \; l_u \; \Sigma o = f_s A_s$. Therefore the average bond stress is given by:

$$u_a = \frac{f_s \; A_s}{l_u \; \Sigma o} \quad \text{.............................(10)}$$

Given the maximum steel stress at the time general bond slip occurs f_{sb}, and the embedment length l_u, we can calculate the average bond stress along the embedded length of strand:

$$u_a = \frac{f_{sb} \; A_s}{l_u \; \Sigma o} \quad \text{.............................(11)}$$

This average bond stress u_a is a convenient measure of the bond performance of prestressing strand, and has the merit of having a real physical significance. The value of u_a has been calculated for the various beams tested, and the results are plotted in Fig. 5.

For embedment lengths slightly greater than the transfer length, the combination of the peak zone of the flexural bond wave with the transfer bond stress region results in a high average bond stress just before general bond slip. For longer embedments, the flexural bond wave form includes a long "tail" stretching from a peak near the transfer zone to the section of maximum steel stress. The average bond stress just before general bond slip for long embedments is therefore less than the average bond stress at general bond slip for short embedments.

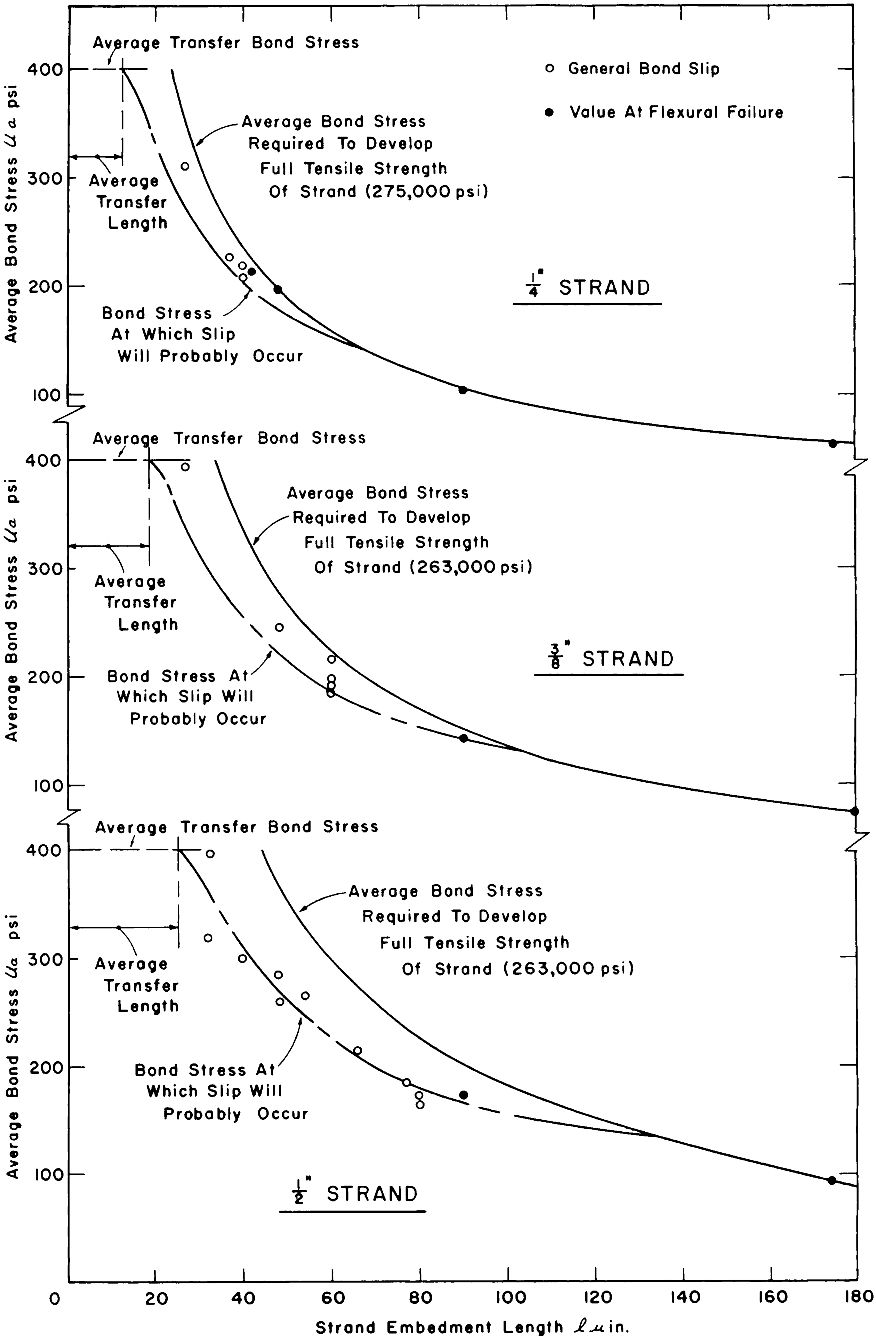

Fig. 5—Relation of average bond stress u_a to strand embedment length

TABLE 4—STEEL STRESS AT GENERAL BOND SLIP, psi

Embedment length, in.	¼-in. strand	⅜-in. strand	½-in. strand
20	194,000	160,000	—
30	218,000	187,000	166,000
40	234,000	201,000	180,000
50	250,000	211,000	192,000
60	264,000	220,000	200,000
70	—	229,000	206,000
80	—	238,000	213,000
90	—	247,000	219,000
100	—	257,000	226,000
120	—	—	244,000
140	—	—	272,000

In the Appendix a possible shape for the flexural bond stress wave immediately prior to general bond slip is deduced from the experimental results. This wave form has been used to calculate the average bond stress at which general slip will probably occur in the case of each of the three strand sizes considered in this report. These values of average bond stress have been plotted as broken lines in Fig. 5. These theoretical lines are substantiated by the experimentally determined average bond stresses at general bond slip for individual beam tests. The solid line represents the average bond stress necessary to develop the full ultimate strength of the strand considered for any particular embedment length. The intersections of the broken and solid lines give the minimum required embedment length if the ultimate strength of the strand is to be developed by beam flexure before general bond slip occurs. For the strand used in this investigation, these minimum embedment lengths are approximately as follows: 70 in. for ¼-in. strand (f_{yp} = 275,000); 106 in. for ⅜-in. strand (f_{yp} = 263,000); and 134 in. for ½-in. strand (f_{yp} = 263,000).

SUGGESTED CRITERIA FOR DESIGN

It is usually desirable in prestressed concrete design that ultimate strength should be governed by flexure rather than by general bond slip. To this end, curves of steel stress at which general bond slip will probably occur for given strand embedment lengths have been plotted in Fig. 6. These curves have been obtained directly from the curves of average bond stress at general bond slip contained in Fig. 5, using the equation $f_s = (\Sigma o/A_s)\ u_a\ l_u$. They are applicable only to strand initially tensioned to about 150,000 psi, embedded in concrete with a compressive strength of about 5500 psi.

The proposed method of use in design is as follows:

1. Calculate the steel stress at ultimate flexural strength, assuming that no general bond slip occurs.
2. Check the embedment length of strand, that is, the distance from the free end of the strand to the section of maximum steel stress.
3. Read from Fig. 6 the maximum steel stress that can be developed in the embedment length provided for the chosen size of strand. If this stress exceeds by an acceptable margin the steel stress at flexural ultimate strength already calculated, then general bond slip will not occur and the section is satisfactory. However, if this stress is less than the calculated steel stress at flexural ultimate strength, then a general bond slip is probable and the section or the strand size should be modified to avoid this.

To supplement Fig. 6, Table 4 gives values of steel stress at general bond slip for increasing values of embedment length of ¼, ⅜, and ½-in. prestress-

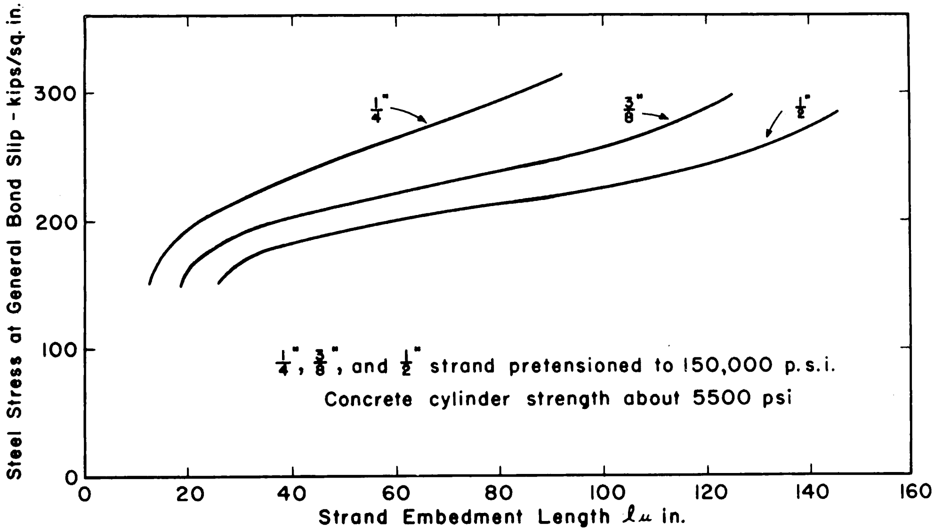

Fig. 6—Relation of steel stress at general bond slip to strand embedment length l_u

ing strand. Steel stresses for intermediate embedment lengths and strand sizes may be obtained with sufficient accuracy by use of linear interpolation between tabulated values.

SUMMARY

The primary object of this investigation was to gain a fuller understanding of the flexural bond behavior of pretensioned prestressed concrete beams, and to develop design criteria. Particular attention was paid to the influence of strand size and embedment length on the bond performance of pretensioned beams. The influence of the following variables was also investigated to a limited extent: (a) percentage of steel reinforcement, (b) reduction of concrete strength, (c) strand surface condition, and (d) use of embedded end anchorages on pretensioned strand.

The results of tests on 47 beams support the flexural bond wave theory proposed by Janney,[1] and confirm that a general bond slip occurs in a pretensioned beam when the peak of the flexural bond stress wave reaches the stress transfer zone.

It was found that strand size and embedment length have a considerable influence on the value of the average bond stresses at which general bond slip occurs; this is clearly demonstrated in Fig. 5. The test results enabled curves to be drawn of bond stress at which slip will probably occur for particular strand sizes and embedment lengths. From these curves design criteria for the avoidance of general bond slip were obtained. These are in the form of curves and a table relating steel stress at which general bond slip will probably occur to embedment length for ¼, ⅜, and ½-in. prestressing strand.

Using these curves in design, it is possible to check quickly whether or not general bond slip is probable in any particular beam.

An increase in reinforcement percentage or a reduction in concrete strength reduces the possibility of general bond slip, since the steel stress at flexural failure, and the corresponding bond stresses, are reduced.

Rusting the strand raised the moment at general bond slip, and the ultimate moment of resistance, relative to identical beams with clean smooth strand.

The seven-wire strand develops additional beam strength, due to mechanical bond resistance, even after general bond slip to the beam end has occurred.

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APPENDIX

SHAPE OF FLEXURAL BOND STRESS WAVE IMMEDIATELY BEFORE GENERAL BOND SLIP

The following is an attempt to deduce the general form of the flexural bond stress wave for smooth, clean strand, using the test results reported in this paper.

It is necessary to make assumptions as follows:

1. The average transfer bond stress is 400 psi.
2. The bond wave shape immediately before general bond slip is a smooth curve with its peak at the end of the transfer zone.
3. At a given distance toward beam midspan from the end of the transfer zone, the local flexural bond stress will have a particular value regardless of the embedment length provided.

The value of 400 psi for average transfer bond stress is an average deduced from measurements on many pretensioned beams at the Portland Cement Association Laboratories. The average transfer lengths for strand pretensioned to 150,000 psi were found to be 13, 19, and 26 in. for 1/4, 3/8, and 1/2-in. strand, respectively.

The average flexural bond stress between the end of the transfer zone and the section of maximum strand stress was calculated as follows. The area of the transfer zone bond stress block was subtracted from the product of the average bond stress u_u and the embedment length l_u; the result was divided by the embedment length minus the transfer length. Hence

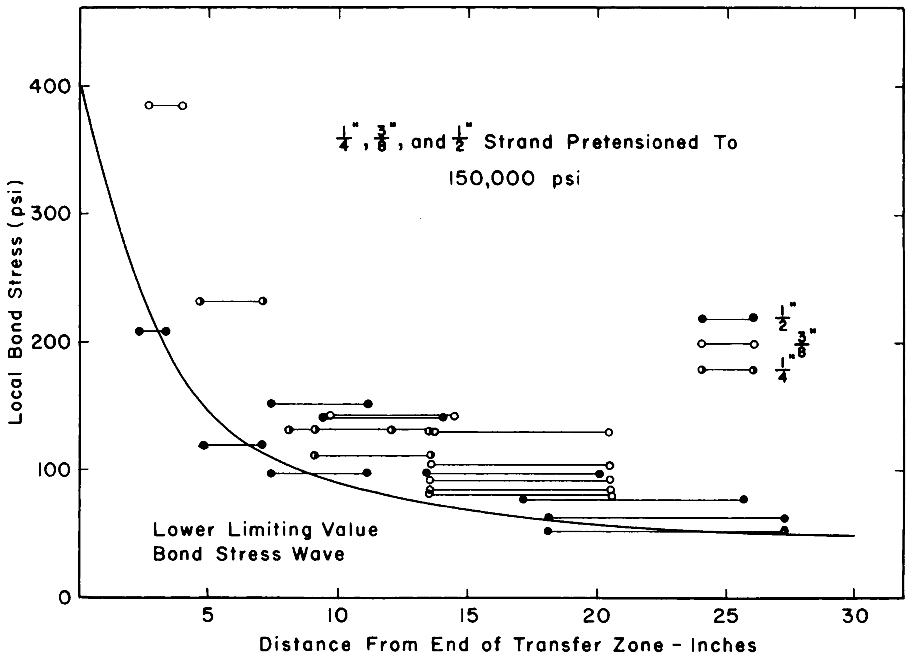

Fig. A-1—Relation of local bond stress to distance from end of transfer zone

the average bond stress between end of the transfer zone and a section of maximum strand stress may be expressed as follows:

$$u_a' = \frac{u_a \, l_u - 400 l_t}{(l_u - l_t)} \quad \text{(A1)}$$

Values of u_a' were calculated for various embedment lengths using the experimental values of u_a obtained in this investigation.

Consider a smooth curve represented by the equation $y = F(x) = Ax^n = B$ where x lies between zero and a finite value x_1, and where n is between 1 and 4. It can be shown by algebra that for a given value of n within this range, a horizontal line with ordinate equal to the average value of y will cut the curve $y = F(x)$ at some point between $x = 0.50x_1$ and $x = 0.67x_1$.

It was thought that, for any particular embedment length considered, the flexural bond wave curve would fall between curves $u = Al + B$ and $u = Al^4 = B$, where u = local bond stress at distance l from the section of maximum strand stress, measured toward the free end of the strand. Values of u_a' calculated using Eq. (A1) were therefore plotted in Fig. A-1 as horizontal lines extending from $l = 0.50\ (l_u - l_t)$ to $l = 0.67\ (l_u - l_t)$. The actual bond wave curve for each particular size strand should therefore at some point cut each of the horizontal lines which relate to the same strand size.

The experimental results for the different strand sizes overlap in Fig. A-1, and it is therefore reasonable to draw a single flexural bond wave curve for all strand sizes. The curve shown in Fig. A-1 was drawn so as to be a *lower limiting curve* for local flexural bond stress immediately prior to general bond slip.

Using this limiting curve of local bond stress it is possible to calculate the average bond stress for any particular strand size and embedment length as follows:

1. Measure the area under the curve of local bond stress over the embedment length considered, and add to this the area of the transfer bond stress block ($400l_t$).
2. Divide the area obtained by the total embedment length, and the result is the average bond stress over that embedment length immediately prior to general bond slip.

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