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CODE PREVIEW PAPER
Background to material being considered for the next ACI Building Code

Shearhead Reinforcement for Slabs*

By W. GENE CORLEY and NEIL M. HAWKINS

Tests of concentrically loaded slab-column specimens containing either lightweight or normal weight aggregate concrete and shearhead reinforcement made from structural shapes are briefly described. Based on the results of tests on these 21 specimens, a design procedure for shearheads at interior supports is proposed and a design example is presented. Strengths implied by this design procedure are compared with measured loads from tests described here and also with loads from other tests. The proposed design procedure is shown to provide shear capacity in the slab that is consistent with load factors and strength reduction factors being considered for use in the 1970 ACI Building Code.

Keywords: building codes; concrete slabs; flat plates (concrete); flat slabs (concrete); lightweight aggregate concretes; loads (forces); reinforced concrete; reinforcing steel; research; shear tests; structural design.

SUGGESTED ULTIMATE STRENGTH DESIGN CRITERIA

The following criteria are being considered for the 1970 revision of the ACI Building Code by the various ACI committees concerned. They have not yet been adopted, and revisions may take place during future committee studies. Accordingly, these provisions are being published solely to solicit discussion and practical design studies. For convenience to the reader, reference is made to related code clauses in ACI 318-63.[1]

1—Shear reinforcement in slabs and footings

1.1 Shear reinforcement consisting of bars, rods, or wires may be provided in accordance with Sections 1702 to 1706, but such shear reinforcement shall be considered only 50 percent effective. Such reinforcement shall be considered entirely ineffective in members with a total thickness less than 10 in.

1.2 Shear reinforcement consisting of steel I or channel shapes shall be designed in accordance with the following provisions, which do *not* apply where shear is transferred to a column from an edge of a slab.†

1.2.1 Each shearhead shall consist of steel shapes so fabricated by welding that each pair of the four arms is continuous through the column. The ends of shearheads may be cut at angles up to 30 deg with the horizontal provided that the plastic moment capacity of the remaining tapered section is adequate to resist the shear force at-

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†Tests in progress indicate that, due to torsional effects and other peculiarities, the behavior of shearheads located at a slab edge differs substantially.

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tributed to that arm of the shearhead. The shear force will be assumed to be concentrated at the tip of the section. The ratio K* between the EI value for each shearhead arm and that for the surrounding composite cracked slab section of width $(c+d)$ shall not be less than 0.15. All compression flanges of the steel shapes shall be located within $0.3d$ of the compression surface of the concrete slab. The steel shapes shall not be deeper than 70 times their web thickness.

1.2.2 The full plastic moment of resistance M_p required for each arm of the shearhead shall be computed by:

$$M_p = \frac{V_u}{8\phi}\left[h + K\left(L_s - \frac{c}{2}\right)\right] \qquad (1\text{-}1)$$

where ϕ is the coefficient for flexure, h is the depth of the steel shape used, and L_s is the minimum length of each shearhead arm from the column center to its end required to comply with the shear stress requirements of Sections 1.2.3 and 1.2.4.

1.2.3 The critical section for shear to be used as a measure of diagonal tension shall be perpendicular to the plane of the slab. The section shall cross each shearhead arm at points three-quarters of the distance, $L_s - c/2$, from the column face to the end of the shearhead, and it shall be so located that its periphery is a minimum. However, the critical section need not approach closer than $d/2$ to the periphery of the column.

1.2.4 The ultimate shear stress v_u shall not exceed $4\phi\sqrt{f_c'}$ on the critical section, in which ϕ is the coefficient for shear. For lightweight aggregate concrete, the limiting stress v_u given by Section 1708 shall be used. The ultimate shear strength of a member with shearhead reinforcement shall not exceed 1.75 times that of the same member without shearhead.

1.2.5 The shearhead may be assumed to contribute a resisting moment M_s to each column strip of the slab:

$$M_s = \frac{\phi K V_u}{8}\left(L_s - \frac{c}{2}\right) \qquad (1\text{-}2)$$

in which ϕ is the coefficient for flexure, and L_s is the length of each shearhead arm actually provided. However, M_s shall exceed neither 30 percent of the total moment resistance required for each column strip of the slab, nor the change in column strip moment over the length L_s, nor the value of M_p given by 1.2.2.

DESIGN EXAMPLE

Description of structure

Following is an example of the design of shearhead reinforcement for a flat plate structure. An interior panel of this slab is supported by a 10 in. (25.4 cm) square column containing four #5 bars. The slab over the column has an average effective depth d of 6 in. (15.2 cm). It is necessary to transfer an ultimate shear V_u of 110 kips (49,900 kg) from the slab to the column. The column strip of the slab is designed to carry a negative moment of about 2100 in.-kips (24,200 kg-m). #5 bars spaced 5 in. (12.7 cm) on center provide this capacity.

Reinforcing bars with a yield stress of 60 ksi (4220 kg/cm^2) are used. The shearhead reinforcement will be fabricated from structural shapes having a yield stress of 36 ksi (2530 kg/cm^2). A single WF or I section will make up each arm of the shearhead.

Normal weight aggregate concrete will be used. The slab will be designed for concrete with a compressive strength of 3000 psi (210 kg/cm^2).

Design of shearhead

From the requirements of Section 1707 of ACI 318-63[1] and 1.2.4 of the suggested design criteria, the shear capacity of the slab is:

$$V_u = b_o d\, 4\phi\sqrt{f_c'}$$

Strength of the slab without shearhead is:

$$V_u = 4(10+6)\,(6)\,(4)\,(0.85)\sqrt{3000}$$

$$= 71.5 \text{ kips } (32{,}400 \text{ kg})$$

Since this is less than the required capacity of 110 kips (49,900 kg), shearhead reinforcement

*All symbols are defined in the list of notation in the Appendix.

must be used. The required minimum perimeter b_o of the critical section is:

$$b_o = \frac{V_u}{d\,4\phi\sqrt{f_c'}} = \frac{110{,}000}{(6)\,(4)\,(0.85)\sqrt{3000}}$$
$$= 98.4 \text{ in. } (250 \text{ cm})$$

In 1.2.3 it is required that the critical section be taken at three-quarters of the distance from the face of the column to the end of the shearhead. Consequently, for shearhead reinforcement with each arm made from a single structural shape, the perimeter is approximately defined by the following expression:

$$b_o = 4\sqrt{2}\left[\frac{c}{2} + \frac{3}{4}\left(L_s - \frac{c}{2}\right)\right]$$

Solving for L_s and substituting, the required length of each arm of the shearhead is:

$$L_s = \frac{4}{3}\left[\frac{b}{4\sqrt{2}} - \frac{1}{4}\,\frac{c}{2}\right] = \frac{4}{3}\left[\frac{98.4}{4\sqrt{2}} - \frac{5}{4}\right]$$
$$= 21.5 \text{ in. } (54.6 \text{ cm})$$

Now that the minimum required length of one arm of the shearhead has been determined, an estimate of the required moment capacity M_p can be made. By cutting off all bottom reinforcement short of the column center line, it would be possible to place the shearhead within ¾ in. (2 cm) of the compression face of the slab. This would permit the use of a section with a total depth of up to 4⅝ in. (12 cm). A section with a nominal depth of 4 in. will be used in this example.

Arbitrarily assuming a relative stiffness K of ¼, the value of M_p given by Eq. (1-1) is:

$$M_p = \frac{V_u}{8\phi}\left[h + K\left(L_s - \frac{c}{2}\right)\right] = \frac{110}{(0.90)\,(8)}$$
$$= \left[4 + \frac{1}{4}\,(21.5 - 5)\right]$$
$$= 124 \text{ in.-kips } (1430 \text{ kg-m})$$

A 4I7.7 with a yield stress of 36 ksi has a plastic moment capacity of about 126 in.-kips (1450 kg-m).

In 1.2.1 of the suggested design criteria it is required that the bottom flange of the shearhead reinforcement be located within $0.3d$ from the compression face of the slab. With ¾ in. (2 cm) cover over the bottom mat and assuming that #5 bars are used, this means that the flange must be kept below the top bars of the bottom mat. Consequently, it is convenient to locate the shearhead even with the bottom layer of bars, ¾ in. (2 cm) from the compression face of the slab.

The EI value for the 4I7.7 is about 174,000 kip-in.^{2} (509,000,000 kg-cm^2). For the section considered, center of gravity of the wide flange is located 2.75 in. (7 cm) above the compression face of the slab. The slab section used in this computation has a width of $c + d = 16$ in. (41 cm) and a reinforcement ratio of 0.0103. Assuming full composite action, the EI value of the cracked transformed section is about 790,000 kip-in.^{2} (2,310,000,000 kg-cm^2). The ratio K as defined in 1.2.1 is 0.22. This is greater than the minimum required value of $K = 0.15$.

With the necessary stiffness constant determined, it is possible to arrive at a final design for the shearhead reinforcement. For convenience, the length of one arm of the shearhead L_s

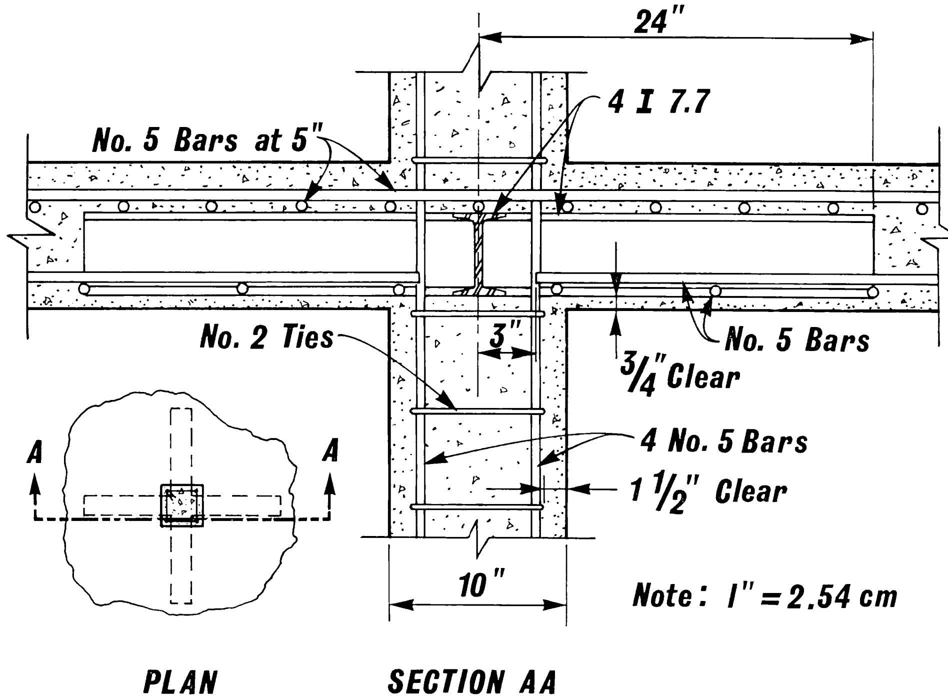


Fig. 1—Reinforcing details for design example

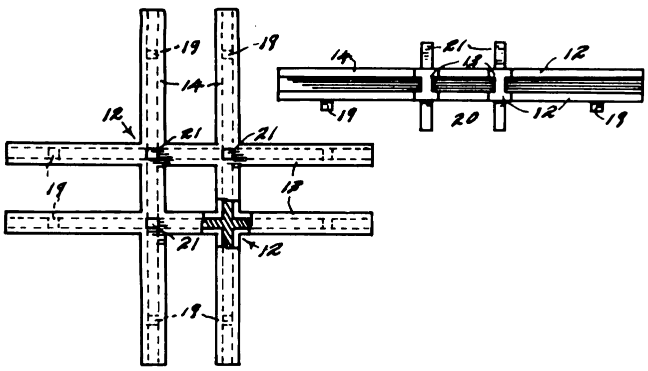


Fig. 2—W. H. Wheeler "frame for concrete columns"

will be taken as 24 in. (61 cm).* However, the required moment capacity from Eq. (1-1) is calculated using the required arm length of 21.5 in. (54.6 cm). Substituting into this equation, the necessary plastic moment capacity is:

$$M_p = \frac{110}{(0.90)\,(8)}\ [4 + 0.22\,(21.5 - 5)\,]$$
$$= 117 \text{ in.-kips } (1350 \text{ kg-m})$$

This is less than the 126 in.-kips (1450 kg-m) provided. No smaller 4 in. (10 cm) I section with adequate capacity is available. Similarly, all other dimensions satisfy the suggested criteria.

Reduction in column strip reinforcement

It is now possible to calculate the contribution of the shearhead to the resisting moment of the column strip. This calculation should be based on the shearhead arm length actually used, $L_s = 24$ in. (61 cm). The moment reduction then becomes:

$$M_s = \frac{\phi K V_u}{8}\left(L_s - \frac{c}{2}\right) = \frac{0.9\,(0.22)\,(110)}{8}\,(24 - 5)$$
$$= 52 \text{ in.-kips } (600 \text{ kg-m})$$

Since this is less than either M_p, 30 percent of the total resistance of the column strip or the change in column strip moment over the length L_s, the entire reduction can be made. In this example, approximately 2 percent of the column strip negative moment reinforcement can be eliminated. Reinforcement must be provided for about 2050 in.-kips (23,600 kg-m).

Reinforcement details for placement of this shearhead are shown in Fig. 1.

BACKGROUND

Since its introduction by Turner in 1905,[2] the reinforced concrete flat slab has continued to be an extremely popular structural system. In recent years, flat plate structures, a form of flat slab having neither capitals nor drop panels, have become quite popular.

In the last decade, several important research programs on flat plate structures have been completed. At the University of Illinois, five ¼-scale flat slab and flat plate structures were loaded to destruction.[3,4] As an extension of the same investigation, a near full-sized flat plate was tested to destruction at the PCA Laboratories.[5] Other tests in Europe[6,7] and Australia[8] were also carried out on large test specimens. More recently, tests of a full-sized waffle slab, a form of flat plate, were carried out at the site of the 1964-65 New York World's Fair.[9]

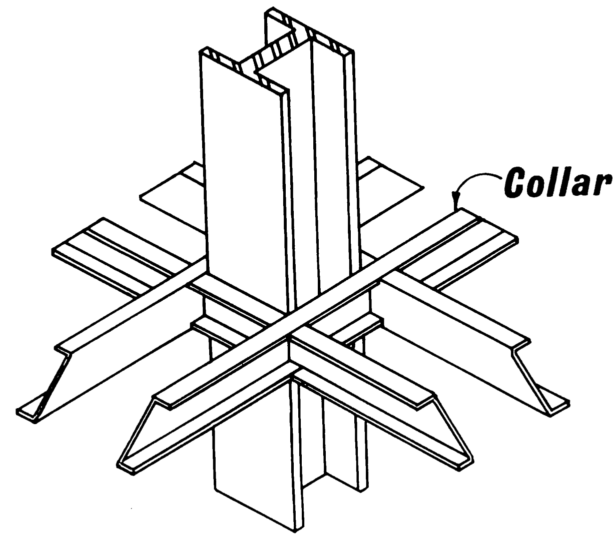


Fig. 3—Lift slab column assembly

These recent tests indicate that, in all practical cases, the capacity of a flat plate structure is governed by shear. Consequently, adequate shear capacity assured by proper design is critical.

Shearhead reinforcement

In 1930, Wheeler developed a "Frame for Concrete Columns."[10] Fig. 2 shows that the shearhead reinforcement Wheeler proposed was fabricated from two pairs of structural shapes. Although the Wheeler type of shearhead reinforcement has been used over the last four decades, little test data is available.[11,12]

Lift slabs are attached to columns by means of collars serving the same function as a shearhead. Fig. 3 shows a collar commonly used in this type of construction.[13] Similarity of this to the Wheeler type of shearhead reinforcement is evident.

Several tests carried out on lift-slab collars are reported in the literature. Andersson reported tests carried out at the Royal Institute of Technol-

*This also provides that the design shear capacity:

$$V_u = b_o d\, 4\phi\sqrt{f_c'} = 122 \text{ kips } (55{,}300 \text{ kg})$$

is greater than the shear needed to develop the flexural strength of the shearhead:

$$\frac{8M_p\phi}{h + K\left(L_s - \frac{c}{2}\right)} = 111 \text{ kips } (56{,}300 \text{ kg})$$

Consequently, the shearhead is assured of being "under-reinforcing" thereby providing a more ductile mode of failure.

ogy, Stockholm on eight specimens containing collars.[14] Tasker and Wyatt reported tests of several slab-column specimens.[15] Four specimens in this series of tests contained lift-slab collars.

PCA TEST PROGRAM*

Description of specimens

A test program intended to provide information to serve as the basis for design criteria for shearhead reinforcement at interior slab-column junctions was carried out at the PCA Research and Development Laboratories. A total of 21 specimens was tested under loads applied symmetrically around the edge of the slab.

Each test specimen was made up of a 7 ft (2.1 m) square, 5¾ in. (14.6 cm) thick slab supported by a centrally located square column. Fig. 4 shows limiting dimensions of the test slabs. A high percentage of deformed bar reinforcement having a nominal 60 ksi (4220 kg/cm^2) yield stress was used to assure that specimens without shearheads would fail in shear before their flexural capacity was reached. Deformations of the bars conformed to ASTM Designation A305. Table 1 lists properties and quantities of materials used in each test specimen. #5 bars were used to reinforce all slabs. This permitted a minimum ¾ in. (2 cm) cover both under the shearhead reinforcement and over the reinforcing bars.

Both lightweight and normal weight aggregate concrete were used. Table 1 lists the type of aggregate for each specimen. Detailed information about the properties of the aggregates designated 7 and 14 is reported elsewhere.[16,17,18] In the specimen designation, the letter S indicates a 50 percent replacement of lightweight fines with normal weight sand. The letter P indicates that the concrete was made entirely with lightweight aggregate. Elgin sand and gravel were used in all specimens containing normal weight aggregate.

Two types of shearhead reinforcement were used. Shearheads similar to Wheeler's design made of two pairs of channels were used in several specimens. One of these shearheads is shown in Fig. 5a. All other shearhead reinforcement was made from American Standard I-sections as shown in Fig. 5b. Table 1 lists material properties and overall length of each shearhead.

It should be noted that the shearheads consisting of I-sections were considerably easier to fabricate than those made from pairs of structural

*A detailed test report will be issued later.

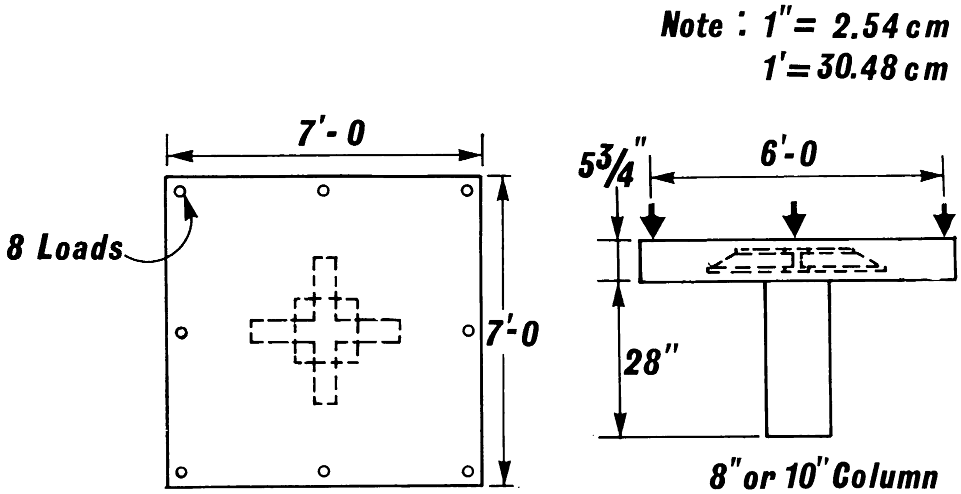


Fig. 4—Dimensions for PCA test specimens

(a) CHANNEL **(b) I-SECTION**

Fig. 5—Shearhead reinforcement

shapes. This ease of fabrication should result in lower costs.

Test setup

Loads were applied to the test specimens at eight locations around the edge of the slab as shown in Fig. 6. These loads were intended to produce shear and moment combinations similar to those at an interior column of a uniformly loaded slab. If the location of the line of contraflexure of the prototype slab is assumed to be where the loads were applied, 3 ft (91 cm) from the column center line, the specimens would represent a structure made up of square panels with columns about 15 ft (4.6 m) on center.

Loads were applied by a hydraulic system using procedures and equipment described elsewhere.[19]

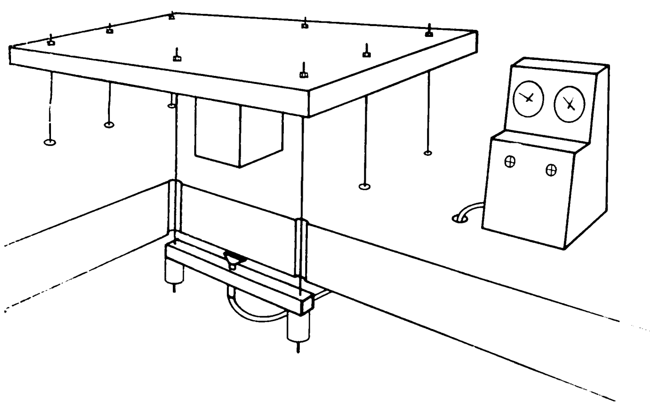


Fig. 6—Test setup

Approximately 12 increments of load were applied to each specimen to reach the ultimate load.

Instrumentation

Deflections at selected locations on the tension side of the slab were measured by means of a precision leveling instrument.[19] In addition, deflections at the column center line near the edge of the slab were measured using 10 in. precision linear potentiometers.

Electrical resistance strain gages were used to measure concrete strains on the slab near the column and steel strains at selected locations on the reinforcing bars and on the shearhead reinforcement. Gages on the shearhead reinforcement were placed at several locations along the sections. Both flanges and webs of the shapes were gaged at each location so that a measure of the magnitude and distribution of axial load, bending moment and shear could be obtained.

Behavior of specimens

To correlate results of these tests with those of other investigations, control slabs without shearhead reinforcement were tested. Identified by the letter N in the specimen designation, properties of these slabs are listed in Table 1.

Shown in Fig. 7a is the failure surface for Specimen AN-1. Typical for specimens without shearheads, the failure surface extends from the intersection of the column face and the compression face of the slab. It spreads toward the tension face of the slab on a surface inclined at about 20 to 30

TABLE 1—PROPERTIES OF TEST SPECIMENS

Mark	Slab reinforcement area, A_s, sq. in.‡	Slab reinforcement depth, d_{ave}, in.§	Slab reinforcement yield stress, f_y, psi**	Type of shearhead reinforcement	Length of shearhead reinforcement L_s, in.§	Aggregate*	Cylinder strength: Compression f_c' psi**	Cylinder strength: Split,† f_{sp} psi**	Column size, c in. x in.§
AN-1	5.58	4.38	58,500	None	—	Elgin	2710	349	10x10
AC-1	5.58	4.38	58,600	2—3x1⅞ [7.1	18	Elgin	2620	343	10x10
AC-2	5.58	4.38	59,900	2—3x1⅞ [7.1	24	Elgin	2660	346	10x10
AC-3	5.58	4.38	59,100	2—3[4.1	21	Elgin	3070	371	10x10
AH-1	4.96	4.38	63,500	3 I 7.5	20	Elgin	3300	384	10x10
AH-2	4.96	4.38	63,300	3 I 5.7	20	Elgin	3190	378	10x10
AH-3	4.96	4.38	63,800	3 I 5.7	12	Elgin	3190	378	10x10
BN-1	3.72	4.38	64,400	None	—	Elgin	2920	362	8x8
BC-1	3.10	4.38	60,900	2—3x1⅞ [7.1	14	Elgin	2870	359	8x8
BH-1	3.72	4.38	63,400	3 I 7.5	21	Elgin	2960	364	8x8
BH-2	3.72	4.38	61,400	3 I 5.7	9	Elgin	2620	343	8x8
BH-3	3.72	4.38	63,700	3 I 5.7	18	Elgin	3130	375	8x8
BN-1-P14	3.72	4.38	63,200	None	—	P14	3680	339	8x8
BH-2-P14	3.72	4.38	64,000	3 I 5.7	9	P14	2780	317	8x8
BH-3-P14	3.72	4.38	64,100	3 I 5.7	18	P14	3100	351	8x8
BN-1-S14	3.72	4.38	63,800	None	—	S14	3320	351	8x8
BH-2-S14	3.72	4.38	63,800	3 I 5.7	9	S14	2960	329	8x8
BH-3-S14	3.72	4.38	64,100	3 I 5.7	18	S14	3520	342	8x8
BN-1-S7	3.72	4.38	63,500	None	—	S7	3700	445	8x8
BH-2-S7	3.72	4.38	65,300	3 I 5.7	9	S7	3480	420	8x8
BH-3-S7	3.72	4.38	64,000	3 I 5.7	18	S7	3740	449	8x8

*Designation "Elgin" indicates all aggregate was normal weight Elgin sand and gravel. Designation S indicates 50 percent replacement of lightweight fines with normal weight Elgin sand. Designation P indicates all lightweight aggregate was used. Numbers 7 and 14 designate aggregate used.[16–18]

†Splitting strength, f_{sp}, calculated as $6.7\sqrt{f_c'}$ for normal weight aggregate concrete and taken as average value from tests on six or more cylinders for lightweight aggregate concrete.

‡To convert to cm² multiply by 6.45.

§To convert to cm multiply by 2.54.

**To convert to kg/cm² multiply by 0.0703.

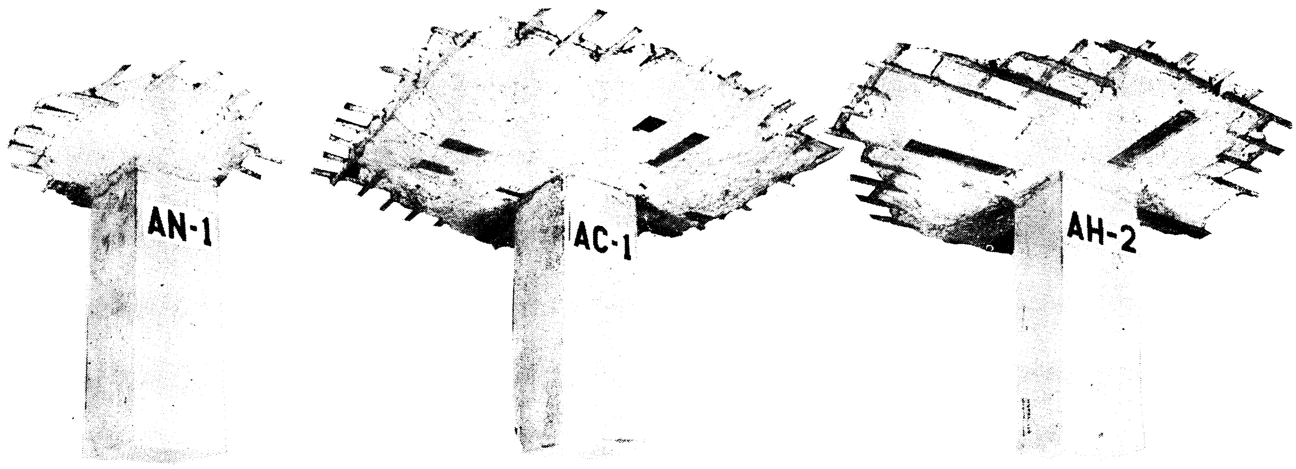


Fig. 7—Failure surface for selected specimens

deg to the horizontal until it reaches the level of the reinforcing bars. From here, the failure surface follows along the bars. Specimen AN-1 carried an ultimate load of about 75 kips (34,000 kg) as listed in Table 2.

The failure surface for a slab containing a very heavy or "over-reinforcing" shearhead is shown in Fig. 7b. For this amount of reinforcement, the failure surface generally follows the perimeter of the shearhead. Inclination to the horizontal varies from less than 20 to more than 45 deg. Some loss of the cover beneath the shearhead is evident in Fig. 7b. This specimen, AC-1, carried about 118 kips (53,500 kg), a 75 percent increase over the capacity of companion slab AN-1.

The failure surface for a third specimen, AH-1, is shown in Fig. 7c. This slab contained a light or "under-reinforcing" shearhead. For this amount of reinforcement, the failure surface falls inside the ends of the shearhead. Again the inclination to the horizontal is on the order of 30 deg. Although this specimen contained a shearhead longer than that in Specimen AC-1, it carried a somewhat lower load of 103 kips (46,700 kg).

This comparison illustrates the two types of behavior that were observed. When the flexural capacity of the shearhead at the face of the column was not exceeded before the end of this test, the failure surface generally followed the perimeter of the shearhead reinforcement. This was defined as an "over-reinforcing" shearhead. When its flexural capacity was exceeded, the failure surface fell inside the end of the shearhead reinforcement. This was defined as an "under-reinforcing" shearhead.

Analysis of test results

In Fig. 8, results of the 21 tests from this investigation are compared with strengths implied by the ultimate strength design procedures of ACI 318-63[1] for slabs without shearheads and with Moe's equation[20] for predicting the strength of slabs without shearheads. The five points representing specimens with c/d ratios ranging from about 2 to 2½ are for slabs without shearhead reinforcement. Data for specimens with shearheads lie to the right of these five points.

The ordinate to Fig. 8 represents the ratio of nominal shear stress, V_u/bd, to the cylinder splitting strength f_{sp}. Splitting strength of the normal weight concrete was assumed to equal $6.7\sqrt{f_c'}$ where f_c' is cylinder compressive strength.[21] Splitting strength of concrete containing lightweight aggregate was taken as the average of tests on six or more cylinders. Use of the cylinder splitting strength as a measure of relative tensile strength permitted comparison of specimens containing lightweight aggregate concrete in the same figure with those containing normal weight concrete.

For specimens without shearheads, the length b, was taken by Moe's definition as the perimeter of the column. When a shearhead was present, b was defined as the minimum perimeter connecting the ends of the shearhead reinforcement. The length c was taken as ¼b.

The curved line in Fig. 8 represents the strength implied at Moe's perimeter by Section 1707 of the 1963 ACI Code. The inclined line shows Moe's prediction of strength for slabs without shearheads, but where the shear capacity and flexural capacity are reached simultaneously. In a previous paper,[22] it was shown that Moe's equation can be described by the following empirical expression for shear stress:

$$v_u = \frac{V_u}{bd} = \left[2.24 \left(1 - 0.075 \frac{c}{d} \right) - 0.784 \frac{V_u}{V_{flex}} \right] f_{sp} \quad (1)$$

where

c = one-fourth of the perimeter of the column, in.

V_{flex} = calculated ultimate load for flexural failure, lb

All other terms have units of pounds and inches.

In Eq. (1) it can be seen that the ratio V_u/V_{flex} has an important influence on the ultimate shear stress. Consequently, it is necessary to adjust test strengths of slabs without shearheads for the effects of flexural yielding if a comparison is to be made in terms of c/d. Points plotted in Fig. 8 for test specimens without shearheads therefore represent the nominal shear stress, V_{test}/bd, corrected by subtracting the quantity:

$$0.784 \left(1 - \frac{V_{test}}{V_{flex}} \right) f_{sp}$$

No adjustment was made for slabs that contained shearhead reinforcement.

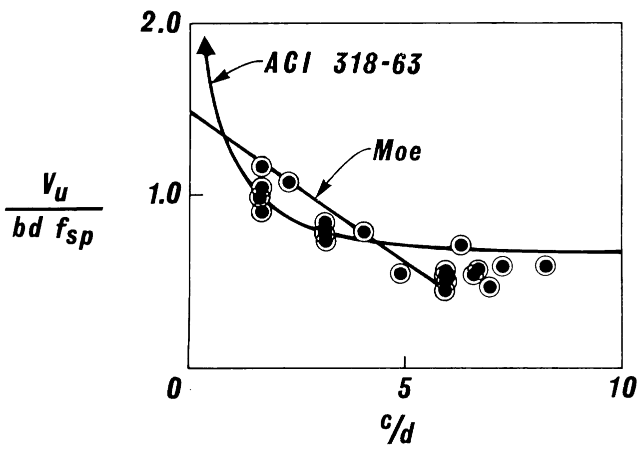


Fig. 8—Comparison of measured and calculated shear strength

The comparison shown in Fig. 8 shows that the shearhead reinforcement used in the test specimens increases shear capacity of a slab in much the same way a larger column would. However, for very long shearheads, the strength increase was somewhat less than that implied by provisions of ACI 318-63.

Several specimens containing shearhead reinforcement were instrumented at several locations to determine approximate values of bending moment along the shearhead. The difference in moment at two locations along the shearhead divided by the distance between these locations provided a measure of the distribution of shear. Fig. 9 shows a measured shear distribution representative of that observed in all specimens. The average shear between each of the four gage lines used is represented by the three points. It can be seen that, over most of the length, the shear is distributed approximately in proportion to the product of the inclined cracking shear V_c and a factor defined as the relative stiffness K of the shearhead to that of a composite section made up of a cracked section of the slab with a width $(c + d)$ and including the shearhead. However, near the

TABLE 2—TEST RESULTS

Mark	Ultimate shear, V_{test} kips‡	Calculated flexural capacity,* V_{flex} kips‡	$\frac{V_{test}}{V_{flex}}$	Maximum moment measured in shearhead at column face, M_{test} in.-kips§	Plastic moment capacity of shearhead M_p, in.-kips§
AN-1	75.1	92.4	0.81	—	—
AC-1	118.3	108.3	1.09	153.0	177
AC-2	126.2	109.4	1.15	182.8	170
AC-3	118.1	102.0	1.16	103.2	102
AH-1	110.1	103.6	1.06	94.8	93
AH-2	103.1	101.2	1.02	81.2	82
AH-3	91.2	102.2	0.89	48.6	82
BN-1	59.7	72.5	0.82	—	—
BC-1	71.8	76.4	0.94	63.0	191
BH-1	88.5	82.4	1.07	93.3	93
BH-2	67.7	77.0	0.88	29.7	84
BH-3	90.4	81.8	1.11	83.9	84
BN-1-P14	61.7	75.4	0.82	†	—
BH-2-P14	60.6	80.4	0.75	†	84
BH-3-P14	80.3	81.6	0.98	†	84
BN-1-S14	54.8	75.0	0.73	†	—
BH-2-S14	60.3	81.1	0.74	†	84
BH-3-S14	84.2	83.1	1.01	†	84
BN-1-S7	67.9	75.5	0.90	†	—
BH-2-S7	77.7	84.6	0.92	†	84
BH-3-S7	89.9	83.5	1.08	†	84

*Flexural capacity based on yield-line pattern.
†Data not adequate for determination of moment.
‡To convert to kg multiply by 454.
§To convert to kg-m multiply by 11.5.

column face a marked increase in shear force was observed. This concentration of shear was most noticeable just as failure occurred. Apparently, after inclined cracking, all added shear was carried by the shearhead reinforcement.

DESIGN CRITERIA

From these test data, design procedures for shearhead reinforcement in slabs at interior columns can be formulated. Three important basic criteria are evident. First, a minimum flexural capacity must be provided to assure that the required shear capacity of the slab is reached before the flexural capacity of the shearhead is exceeded. Second, the nominal shear stress in the slab at the end of the shearhead reinforcement must be limited. Third, after these two requirements are satisfied, the designer can somewhat reduce the negative slab reinforcement in proportion to the moment contribution of the shearhead at the design section.

Minimum flexural capacity

The assumed idealized shear distribution along an arm of a shearhead at an interior column is shown in Fig. 10. The shear along each of the four arms is taken as $KV_c/4$. However, the peak of shear at the face of the column was taken as the total applied shear per arm, $V_u/4$, minus the shear considered carried to the column by the concrete compression zone of the slab. The latter term was expressed as $(V_c/4)\ (1 - K)$, so that it approaches zero for a heavy shearhead and approaches $V_c/4$ when a very light shearhead is used. Based on the assumption that the inclined cracking load V_c is about one-half the ultimate load V_u, the following

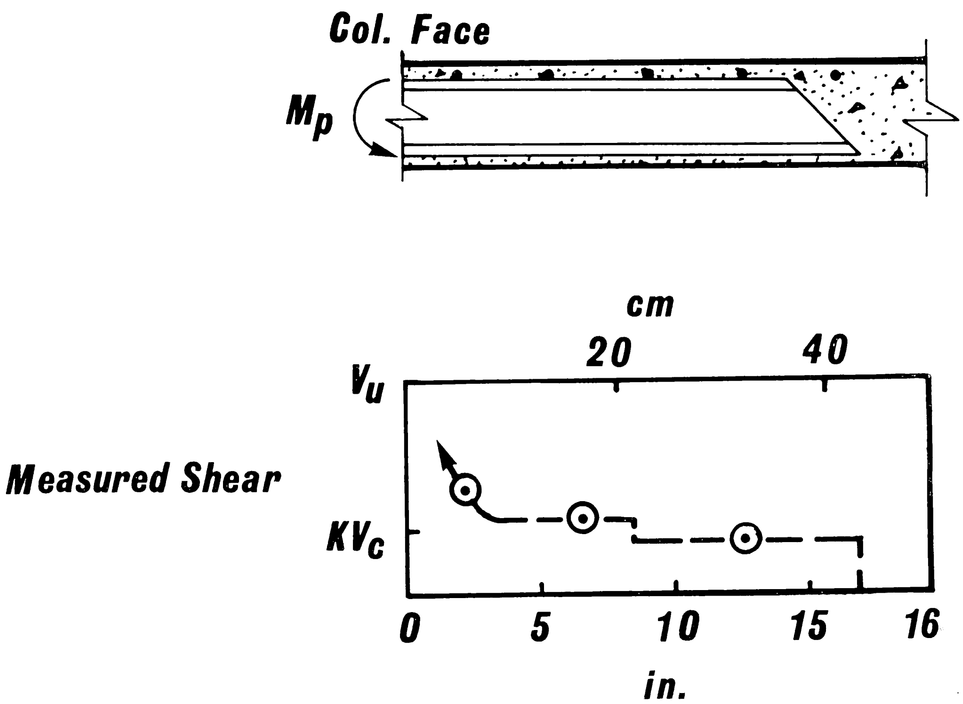


Fig. 9—Shear calculated from measured strains in shearhead reinforcement for Specimen BH-1

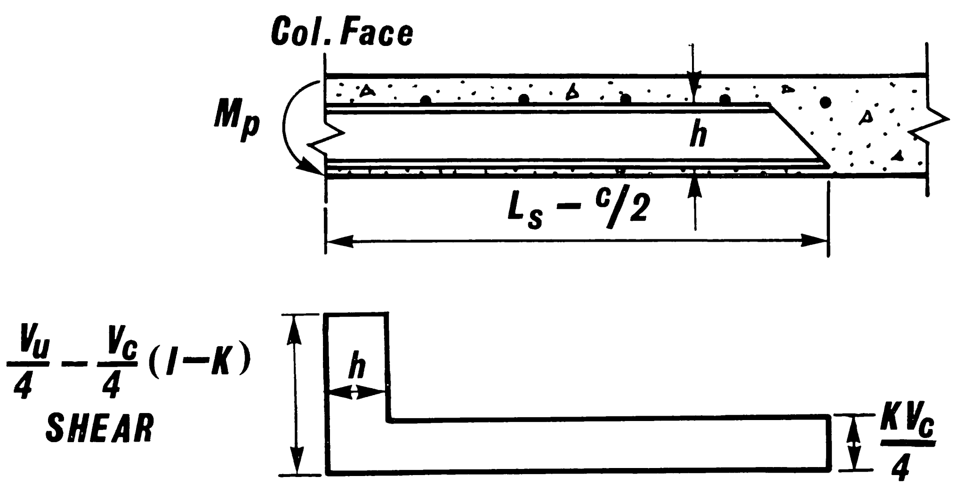


Fig. 10—Idealized shear distribution at ultimate load

equation for moment at the face of the column M_p is obtained:

$$M_p = \frac{V_u}{8\phi}\left[h + K\left(L_s - \frac{c}{2}\right)\right] \qquad (2)$$

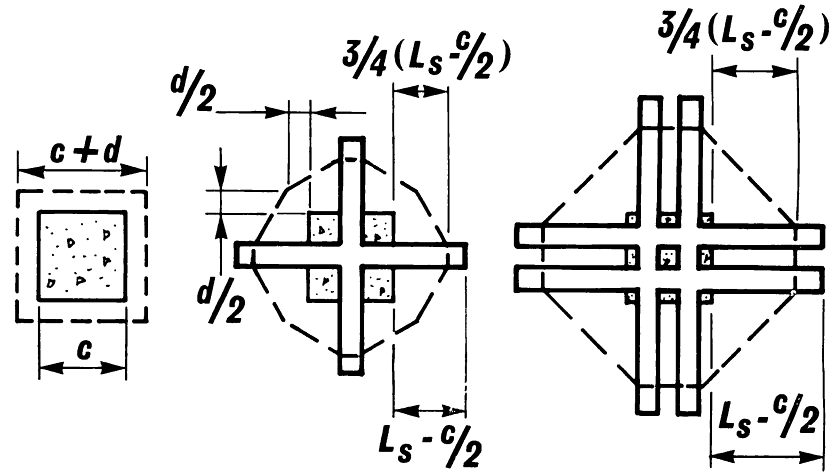
In this equation, ϕ is the strength reduction factor for flexure given by Section 1504 of ACI 318-63[1] and M_p is the required plastic moment capacity of each shearhead arm necessary to assure that ultimate shear is attained as the moment capacity of the shearhead is reached. The quantity L_s is the length from the center of the column to the point at which the shearhead is no longer required, and the distance $c/2$ is one-half the dimension of the column in the direction considered.

Nominal shear stress in slab

The test results indicate that several slabs containing shearhead reinforcement failed at a nominal shear stress less than the 1963 Code limitation (for normal weight aggregate concrete $4\sqrt{f_c'}$) on a critical section at the end of the shearhead. Although many test specimens reached a load equivalent to a stress of $4\sqrt{f_c'}$, the limited test data suggest that a conservative design is desirable. Therefore, it is proposed that the ultimate shear stress be calculated as $4\phi\sqrt{f_c'}$ on a fictitious critical section located inside the end of the shearhead reinforcement. In this relationship, ϕ is the coefficient for shear.

The proposed section is shown in Fig. 11. It is taken through the shearhead arms three-fourths of the distance $L_s - c/2$ from the face of the column to the end of the shearhead. However, this assumed critical section need not be taken any closer to the column than the section for a slab without a shearhead, the distance $d/2$.

For a practical case where the shearhead reinforcement extends beyond the column face a distance equal to the column width, the nominal stress on the section at the end of the shearhead becomes $3.3\sqrt{f_c'}$. For a very long shearhead, the minimum nominal shear stress at its end approaches the value of $3\sqrt{f_c'}$. When lightweight aggregate concrete is used in the slab, appropriate reductions in accord with Section 1708 of ACI 318-63 must also be included.



(a) No Shearhead **(b) Small Shearhead** **(c) Large Shearhead**

Fig. 11—Location of design section

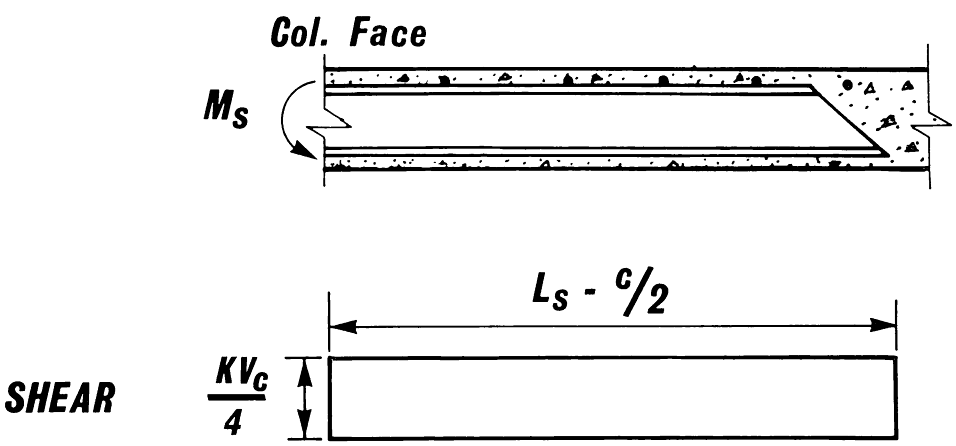


Fig. 12—Idealized shear distribution for calculating moment contribution of shearhead reinforcement

Contribution to slab flexural capacity

As shown in Fig. 9, test results indicate that shear was approximately uniformly distributed along the shearhead at low loads. This distribution may be idealized as shown in Fig. 12. The magnitude of the shear is again assumed proportional to the product of the inclined cracking load V_c and the relative stiffness K of the shearhead to that of a composite section made up of a portion of the cracked slab with a width equal to that of the column plus the effective depth of the slab including the shearhead. If the cracking load V_c is again assumed to be about half of V_u, the moment contribution of the shearhead M_s can be conservatively taken as:

$$M_s = \frac{\phi K V_u}{8}\left(L_s - \frac{c}{2}\right) \leq M_p \qquad (3)$$

in which ϕ is the coefficient for flexure.

COMPARISON OF DESIGN PROCEDURE WITH TESTS

Swedish and Australian lift slab collar tests

Andersson carried out tests at the Swedish Royal Institute on eight specimens containing lift slab collars fabricated from angles.[14] Slabs were circular and were about 6 ft (184 cm) in diameter and 6 in. thick (15 cm) thick. Fig. 13 shows both important dimensions and the loading arrangement for these test specimens. Properties of each specimen are listed in Table 3.

Tests by Tasker and Wyatt at the Commonwealth Experimental Building Station, Sydney, Australia, were carried out on octagonal slabs containing collars fabricated from angles.[15] These specimens were about 64 in. (163 cm) wide and 4 in. (10 cm) thick as shown in Fig. 14. Eight concentrated loads were applied symmetrically around the column to simulate shear and moment at an interior support of a uniformly loaded slab. Pertinent physical properties of the test specimens used in the comparison are listed in Table 3.

Shear strength

In the proposed design procedure, shear strength is limited either by the shear strength of the slab at the end of the shearhead, or by the flexural strength of the shearhead reinforcement at the face of the column. A comparison of test loads with strengths applied by this design procedure is presented. Shear strength in slabs made with normal weight aggregate is calculated by limiting the nominal stress v_u on the critical section to $4\sqrt{f_c'}$. For slabs containing lightweight aggregate concrete, v_u was modified by the ratio of splitting tensile strength f_{sp} to $6.7\sqrt{f_c'}$.

The shear capacity governed by the flexural strength of the shearhead reinforcement is calculated by Eq. (2). In this comparison, the plastic moment capacity M_p was calculated using experimentally determined yield stress and cross-sectional properties based on nominal dimensions. The shear force V_{calc} was then computed from M_p and other properties with $\phi = 1.0$. Pertinent values of M_p are reported in Tables 2 and 3.

Fig. 15 shows the camparison of test loads, on the ordinate, with calculated loads based on the proposed design procedure, on the abscissa. For specimens containing no shearhead reinforcement, test loads were modified for the effect of V_{test}/V_{flex} according to Eq. (1). Test loads for specimens containing shearheads were not modified. The solid line represents $V_{calc} = V_{test}$ while the dashed line represents the lower limit of the strength reduction factor, $\phi = 0.85$. The computed shear strength of eight specimens was governed by flexure of the shearhead.

In Fig. 15, all points representing test results fall near the line of equality. Without exception, the points fall above the line representing the strength reduction factor, $\phi = 0.85$. It is believed that the agreement between calculated loads and test loads is satisfactory for a design procedure.

TABLE 3—PROPERTIES OF SPECIMENS CONTAINING LIFT-SLAB COLLARS

Mark	Slab reinforcement area A_s, sq in.	Slab reinforcement depth, d_{ave}, in.	Slab reinforcement yield stress, f_y, ksi	Collar perimeter, in.	Cylinder strength: Compression* f_c', psi	Cylinder strength: Split† f_{sp}, psi	Collar plastic moment capacity M_p, in.-kips
Tests at Swedish Royal Institute, Stockholm, Sweden							
1 A1	3.1	4.9	62.5	66.2	4000	424	560
1 A2	3.1	5.0	65.0	66.2	4050	427	560
1 B1	3.1	4.8	64.0	66.2	4150	432	33
1 B2	3.1	4.8	64.0	66.2	4050	427	33
III C1	3.1	4.7	61.1	72.5	3550	400	29
III C2	3.1	4.8	61.1	72.5	3550	400	29
III D1	3.1	4.8	60.8	71.0	3700	407	29
III D2	3.1	4.8	61.7	71.0	3350	388	29
Tests at Commonwealth Experimental Building Station, Sydney, Australia							
D1	2.1	3.0	44.7	40	3300	384	—
E1	2.1	3.0	44.7	40	4000	423	—
F1	2.1	3.0	44.7	40	4000	423	—
Type A	2.1	3.0	44.7	40	3400	390	—

*For Swedish tests, cylinder strength calculated as 85 percent of cube strength reported.
†Splitting srength, f_{sp}, calculated as $6.7\sqrt{f_c'}$.

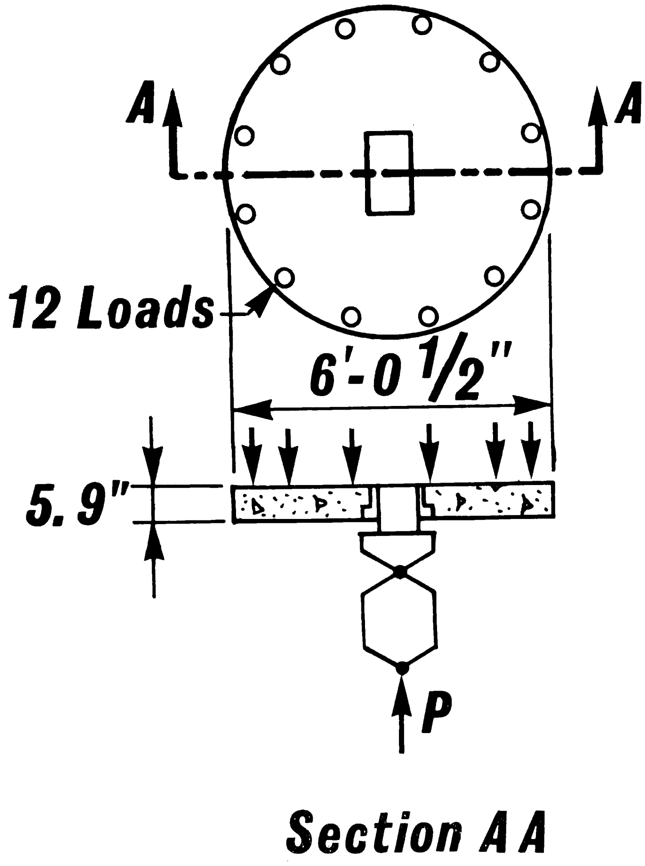


Fig. 13—Dimensions of lift slab specimens tested at Swedish Royal Institute, Stockholm

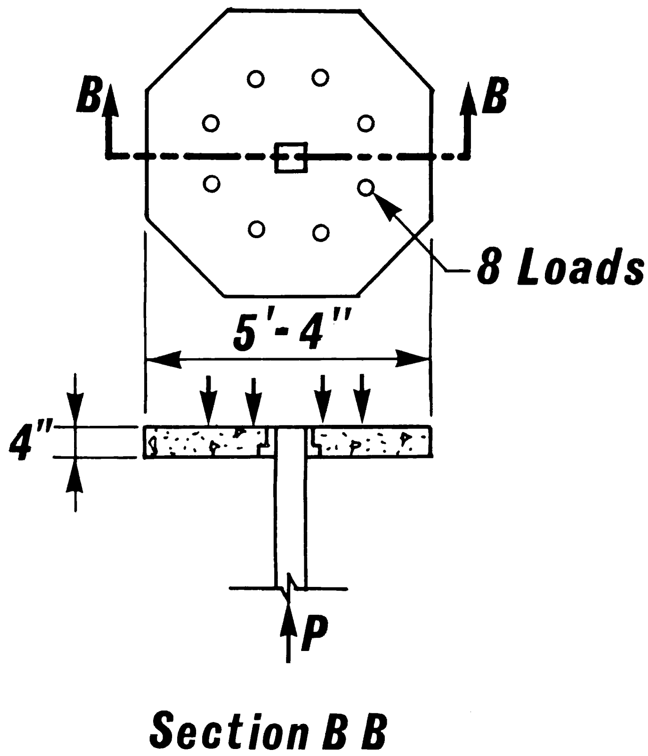


Fig. 14—Dimensions of lift slab specimens tested at Commonwealth Experimental Building Station, Sydney, Australia

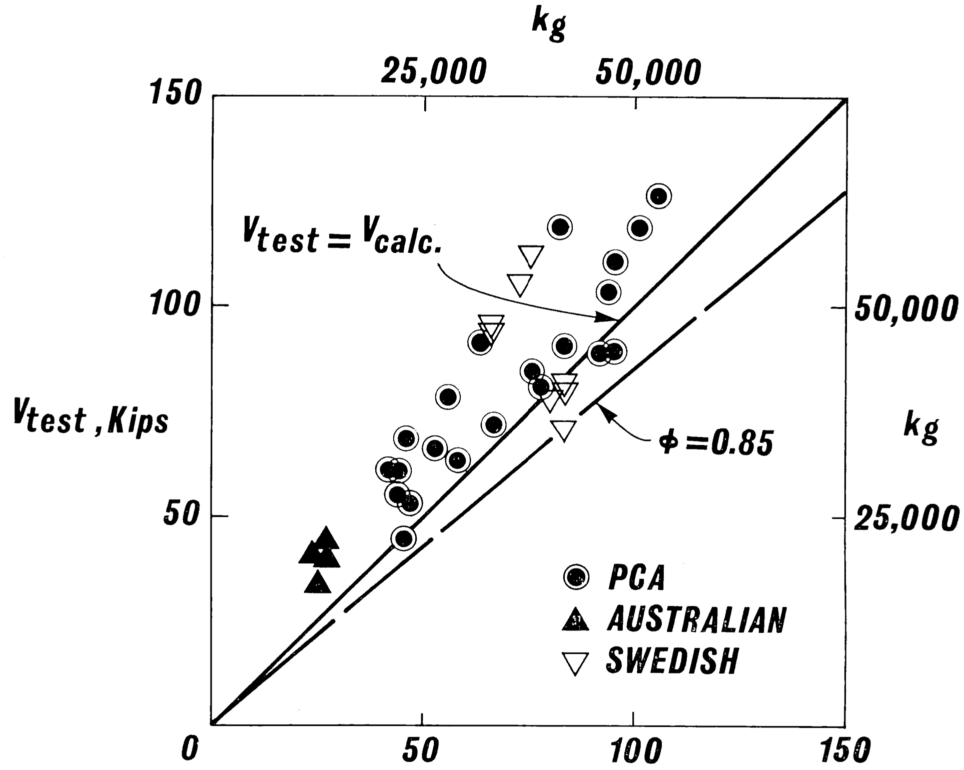


Fig. 15—Comparison of measured and calculated strength

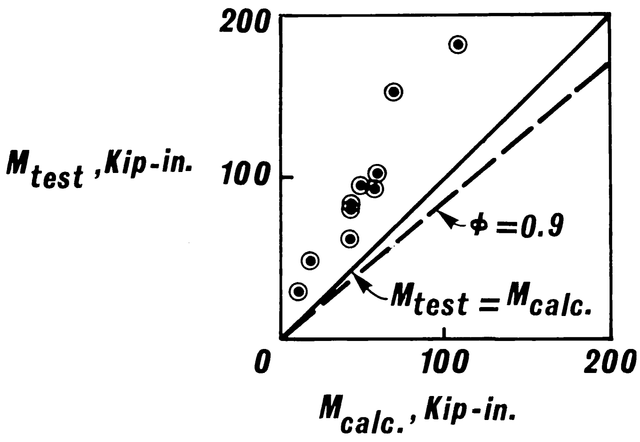


Fig. 16—Comparison of measured and calculated moment in shearhead reinforcement

Moment contribution of shearhead

Fig. 16 shows a comparison of moment at ultimate load measured in the shearhead reinforcement of PCA test slabs containing normal weight aggregate concrete with that calculated by Eq. (3) with $\phi = 1.0$. Measured values are plotted on the ordinate and calculated values on the abscissa. The dashed line represents $\phi = 0.9$. It can be seen that the proposed design procedure is quite conservative.

CONCLUDING REMARKS

Recent tests indicate that shearhead reinforcement fabricated from structural shapes is effective in thin slabs. In tests at the PCA Laboratories, up to 75 percent higher shear capacity was measured in specimens with shearheads compared to specimens without shearheads. The tests indicate that even larger increases may be possible. Based on these tests of 21 specimens, a design procedure has been developed as presented at the beginning of this paper.

ACKNOWLEDGMENTS

Tests described in this report were conducted at the Structural Development Laboratory, Portland Cement Association under the direction of Dr. E. Hognestad, Director of Development. Professor J. O. Jirsa of Rice University and Professor S. P. Shah of the University of Illinois, Chicago Circle Campus supervised parts of the test program.

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APPENDIX

NOTATION

A_s = area of slab tension reinforcement in each direction

b = perimeter of column or of shearhead reinforcement

b_o = perimeter of critical section used as measure of diagonal tension

c = one-fourth the perimeter of square column or of shearhead reinforcement

d = distance from extreme compression fiber to centroid of tension reinforcement

d_{ave} = average distance from extreme compression fiber to centroid of slab tension reinforcement

f_c' = cylinder compressive strength of concrete

f_{sp} = cylinder splitting strength of concrete

f_y = yield stress of slab reinforcement

H = indicates specimen with shearhead reinforcement fabricated from American Standard

h = depth of steel shapes in shearhead

K = relative stiffness of the shearhead to that of a composite section made up of a cracked section of the slab with width equal to that of the column plus the slab effective depth and including the shearhead

L_s = length of shearhead reinforcement measured from center of column to end of shearhead arm

M_p = plastic moment capacity of shearhead

M_s = moment in shearhead

M_{calc} = calculated maximum moment in shearhead

M_{test} = maximum moment measured in shearhead

N = indicates specimens with no shearhead reinforcement

P = indicates concrete containing all lightweight aggregate

ϕ = capacity reduction factor

S = indicates concrete with 50 percent replacement of lightweight fines with normal weight sand

V_c = total shear at diagonal cracking

V_{calc} = calculated total ultimate shear

V_{flex} = computed ultimate load for flexural failure of slab

V_{test} = measured total ultimate shear

V_u = total ultimate shear

v_u = nominal ultimate shear stress as a measure of diagonal tension

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Sinopsis — Résumé — Zusammenfassung

Refuerzo con Cabeza Cortante para Losas

Se describen brevemente los ensayos de muestras de losa-columna concéntricamente cargada conteniendo ya sea concreto con agregado de peso ligero o de peso normal y refuerzo de cabeza cortante hechos de acero estructural. Basado en los resultados de los ensayos de estas 21 muestras, se propone un procedimiento de diseño para cabezas cortantes en apoyos interiores, presentándose un ejemplo de diseño. Las resistencias implicadas por este procedimiento de diseño se comparan con las cargas medidas de ensayos descritos aquí y también con cargas de otros ensayos. El procedimiento de diseño propuesto demuestra que la losa provee capacidad de cortante, que es consistente con los factores de carga y los factores de reducción de resistencia considerados para ser usados en el Código de Construcción ACI de 1970.

Armature en Pointe de Cisaillement pour Poutre

Des essais de spécimens de colonnes poutres chargées concentriquement contenant des agrégats légers ou normaux de béton et une armature en pointe de cisaillement constituée de formes structurales sont brièvement décrits. Basés sur les résultats d'essais de ces 21 spécimens, une méthode de calcul pour pointes de cisaillement à supports internes est proposée et un exemple de calcul présenté. Les résistances imposées par cette méthode de calcul sont comparées avec les charges mesurées à partir d'essais décrits ici et également avec des charges mesurées à partir d'essais autres. Il est montré que la méthode de calcul proposée fournit une capacité au cisaillement dans les poutres compatible avec les facteurs de charges et les facteurs de réduction de résistance sont considérés pour utilisation dans la norme de construction ACI de 1970.

Schubbewehrung in Platten-Säulenverbindungen

Es werden Versuche an zentrisch belasteten Platten-Säulenproben beschrieben. Die Probekörper enthielten entweder Leicht- oder Normalzuschlagstoffe, und die Schubbewehrung bestand aus verschiedenen handelsüblichen Stahlquerschnitten. Aufbauend auf den Ergebnissen von 21 Versuchen werden eine Entwurfsmethode für die Schubbewehrung an Innenstützen vorgeschlagen und ein Rechenbeispiel gegeben. Die auf Grund dieser Entwurfsmethode rechnerisch ermittelte Festigkeit wird mit Ergebnissen der hier beschriebenen Versuche und mit den Bruchlasten aus anderen Versuchen verglichen. Es wird gezeigt, dass die vorgeschlagene Entwurfsmethode eine Schubfestigkeit der Platten erbringt, die in Übereinstimmung mit den Lastfaktoren und den Festigkeitsabminderungsfaktoren ist, die für die neue ACI Vorschrift 1970 in Betracht gezogen werden.