

# Connections in Precast Concrete Structures—Strength of Corbels

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## SYNOPSIS

This paper describes a project directed toward development of design criteria for reinforced concrete corbels. Part 1 contains these design criteria, together with design aids and design examples. Part 2 describes the tests on which the proposed criteria are based, involving 124 corbels subjected to vertical loads only and 71 corbels subjected to combined vertical and horizontal loads. Part 3 contains the discussion and analysis of the experimental data and the derivation of the design equations. Detailed test data are given in an appendix.

## INTRODUCTION

A series of investigations of connections in precast concrete structures is in progress at the Research and Development Laboratories of the Portland Cement Association. The three previous papers in this series, collectively entitled "Connections in Precast Concrete Structures," have been concerned with the strength and behavior of continuity connections in double-tee floor construction<sup>1</sup>, with the bearing strength of column heads supporting precast beams<sup>2</sup>, and with the strength and behavior of scarf joints in beams and

columns<sup>3</sup>. This paper deals with the development of design criteria for the strength of corbels which protrude from the face of a column.

## PART 1—DESIGN OF CORBELS

### Background

Corbels projecting from the faces of columns are used extensively in precast concrete construction to support primary beams and girders. Typical applications of corbels may be found in the Prestressed Concrete Institute manual of connection details<sup>4</sup>.

Until recent years little research had been available on the strength of corbels. In the United States it has been customary to design them as short cantilevers, using the flexural and shear design equations derived for beams of more normal propor-

tions. Since the assumptions made in deriving these equations are not valid for deep beams, it is not surprising that corbel brackets designed by these equations can have varying safety factors. The tests described in Part 2 of this paper show that design on this basis will lead to questionably safe designs when the amount of tension reinforcement exceeds about one percent, and also if shear reinforcement is necessary and is provided in the form of vertical stirrups. In addition, corbels have in general been designed for vertical loads only, although horizontal forces caused by restrained creep, shrinkage, and temperature deformations of the beams supported by the corbels are often important indeed. Tests described in Part 2 of this paper have shown that such horizontal forces can substantially reduce the vertical load-carrying capacity of corbels. This effect has also been evidenced in the field where some corbels carrying light vertical loads were damaged by horizontal restraint forces.

In Europe the design of corbels has been based mainly on the investigations of Rausch<sup>5,6</sup>. These design procedures involve the "straight-line" method of design for flexure, and the provision for bent bars to resist all shear forces.

In 1961, Niedenhoff<sup>7</sup> suggested that a corbel acts essentially as a simple truss composed of two members: a horizontal tension member, i.e. the tension reinforcement, and an inclined concrete compression strut. On the basis of an experimental investigation, Niedenhoff proposed that the depth of the equivalent truss be taken as 0.8 times the total depth of the corbel. These assumptions form the basis of Niedenhoff's working load design procedure.

ture.

A series of tests conducted at the University of Illinois<sup>8,9,10,11</sup> involved the strength of deep beams. A deep beam, loaded by a concentrated load at midspan and supported by concentrated reactions at the ends, acts essentially as a double corbel protruding from opposite faces of a column. However, the number of specimens tested under concentrated loads was not sufficient to lead to design procedures for corbels. These tests, together with recent tests of short cantilevers made at the University of Texas<sup>12</sup>, will be referred to later.

The tests recently carried out in the PCA Structural Laboratory, and reported in this paper, have been specifically concerned with corbels in which the ratio of the shear span to the effective depth of the bracket at the column face was less than unity. One hundred ninety-five corbels were tested, of which 124 were subject to vertical load only and 71 to combined vertical and horizontal loads. The variables included in the tests were: size and shape of corbel, amount of main tension reinforcement and its detailing, concrete strength, amount of stirrups, ratio of shear span to effective depth, and the ratio of the horizontal force to the vertical force.

The design criteria set out below are based on a study of the results of these tests; they have also been checked against the results obtained from the tests at the Universities of Illinois<sup>8,9,10,11</sup> and Texas<sup>12</sup>. In the development of such design criteria, numerous plots and numerical computations were made to compare observed performance with various empirical expressions. Considerable use was made of electronic computation to arrive at suitable ultimate

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strength design equations.

## Proposed Criteria for the Design of Corbels

### 1. Notation

- $A_s$  = area of tension reinforcement, in.<sup>2</sup>  
 $A_v$  = total area of horizontal closed stirrups, in.<sup>2</sup>  
 $a$  = shear span, i.e. distance from column face to resultant of vertical load, in.  
 $b$  = width of corbel, in.  
 $d$  = effective depth of corbel measured at column face, in.  
 $f'_c$  = concrete cylinder strength, psi  
 $\sqrt{f'_c}$  = relationship expressed in psi, so that  $\sqrt{f'_c} = 60$  psi for  $f'_c = 3600$  psi  
 $H/V$  = ratio of horizontal load to vertical load  
 $p$  = reinforcement ratio at column face,  

$$p = \frac{A_s + A_v}{bd}$$
 when  $H/V = 0$ , i.e. vertical loads only,  

$$p = \frac{A_s}{bd}$$
 when  $H/V$  does not equal zero, i.e. combined vertical and horizontal loads  
 $v_u$  = nominal shear stress at ultimate strength, psi,  

$$v_u = \frac{V_u}{bd}$$
  
 $V_u$  = vertical load at ultimate strength, i.e. shear at ultimate strength, lb  
 $\phi$  = capacity reduction factor

### 2. Scope

(a) These provisions apply to corbel brackets having a shear span to depth ratio,  $a/d$ , of less than unity.

(b) Provisions of the ACI Building Code (ACI 318-63) not in conflict

with the provisions of these proposed criteria should be considered applicable to the design of corbels.

### 3. Safety Provisions and Design Loads

(a) Strength should be computed in accordance with the provisions of section 4.

(b) The coefficient  $\phi$  should be 0.85.

(c) The strength capacities of corbels so computed should be at least equal to the total effects of the design loads required by Section 3(d).

(d) The design loads to be used in the design of corbels should equal the design loads specified in Section 1506 of the ACI Building Code (ACI 318-63), multiplied by 4/3.

### 4. Strength Computations

(a) When special provisions are made so that a corbel is subject to vertical loads only, the ultimate design load capacity may be calculated by:

$$V_u = \phi [6.5bd \sqrt{f'_c} (1 - 0.5^{a/d}) (1000p)^{1/3}] \quad (1)$$

where  $p = (A_s + A_v)/bd$  does not exceed 0.02, and  $A_v$  does not exceed  $A_s$ .

(b) In all other cases the ultimate design load capacity may be calculated by:

$$V_u = \phi \left[ 6.5bd \sqrt{f'_c} (1 - 0.5^{a/d}) \frac{(1000p)^{(1/3 + 0.4H/V)}}{10^{0.8H/V}} \right] \quad (2)$$

where  $p = A_s/bd$  does not exceed 0.013.

### 5. Minimum reinforcement

(a) The amount of tension reinforcement  $A_s$  should be not less than  $0.004bd$ .

(b) Closed horizontal stirrups should be provided having a total

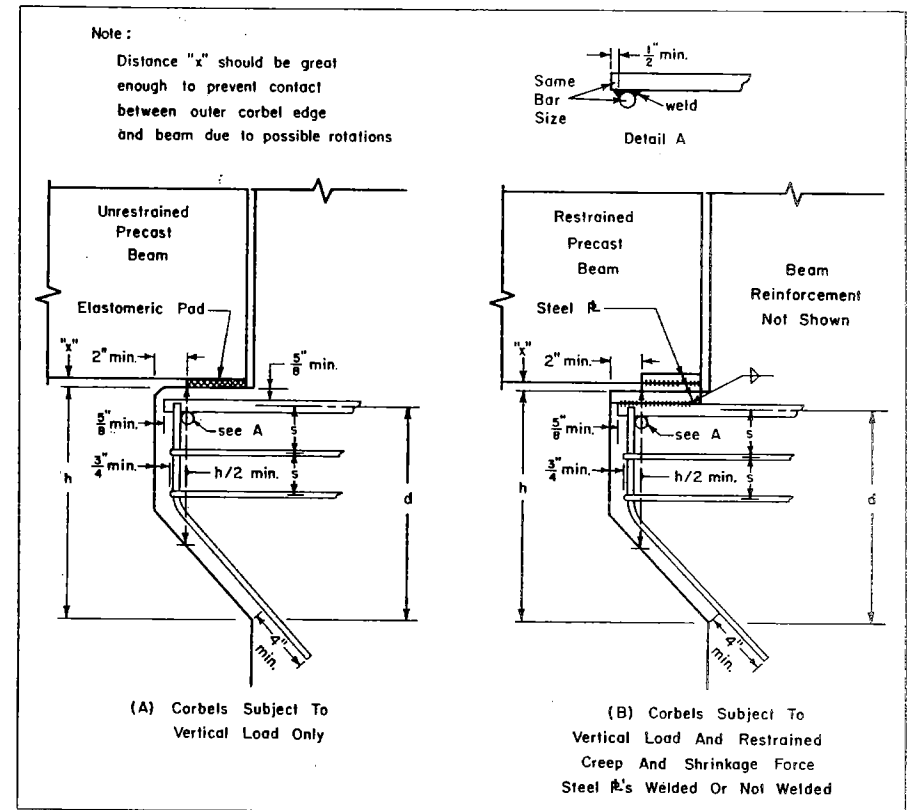


Fig. 1—Recommended Corbel Details

cross section  $A_v$  not less than  $0.5A_s$ .

### 6. Detailing of Corbels\*

(a) The tension reinforcement should be anchored as close to the outer face of the corbel as cover requirements permit, by welding a cross-bar to the ends of the tension reinforcing bars. The size of the cross bar should be at least equal to the maximum size for bar used as tension reinforcement.

(b) The closed horizontal stirrups should be distributed over the up-

per two thirds of the effective depth at the column face.

(c) The total depth of a corbel under the outer edge of a bearing plate resting on the corbel should be not less than half the total depth of the corbel at the face of the column.

(d) The outer edge of a bearing plate resting on a corbel should be placed not closer than 2 inches to the outer edge of the corbel.

(e) When corbels are designed to resist horizontal forces, steel bearing plates welded to the tension reinforcement should be used to transfer the horizontal forces directly to the tension reinforcement.

\* The requirements of Section 6 are illustrated in Fig. 1.

## 7. Bearing Stresses

(a) The bearing stresses at ultimate strength beneath a bearing plate resting on a corbel should be not more than  $0.5f'_c$ .

### Discussion of Proposed Design Criteria

#### Safety Provisions and Design Loads

The proposed safety provision and design loads are in agreement with the philosophy concerning safety provisions and design loads of Part IV-B, Ultimate Strength Design, of the ACI Building Code (ACI 318-63). Since a corbel is primarily a shear transfer device, and since its ultimate strength is governed by shear strength, it is considered appropriate to use the value  $\phi = 0.85$  specified in ACI 318-63 for ultimate strength governed by shear and diagonal tension.

The design loads specified for corbels are made one third greater than those specified for the design of members in ACI 318-63 for two reasons. First, in corbels having less than about one percent of tension reinforcement, yield of the reinforcement occurs before the ultimate strength of the corbel is developed. The ratio of the load at which yield occurs to the ultimate load can vary

between  $\frac{1}{3}$  and 1. The load factors proposed will provide an adequate factor of safety against yield of the reinforcement, thus insuring serviceability of the corbels under moderate overloads. Second, it is considered good practice that the strength of a precast concrete structure should be governed by the strength of the members and not by the strength of the connections between members. Since a corbel forms part of the connection between a beam and a column it should be made stronger than either the beam or the column. Use of the proposed design loads will assure this.

#### Strength Computations

The equations for ultimate strength presented in Section 4 and Part 3 are based on a study of the results of tests of 195 corbels carried out at the PCA Structural Laboratory. Eq. (2) reduces to Eq. (1) when  $H/V$  is zero. However, the different definitions of reinforcement ratio  $p$  in Eqs. (1) and (2) should be noted. Whereas stirrups make a considerable and consistent contribution to the strength of a corbel subject to vertical load only, their contribution to the strength of a corbel subject to combined vertical and horizontal loads is smaller and more variable. It is therefore considered sounder for the present not

to rely on their contribution when designing a corbel subject to combined loading.

Eqs. (1) and (2) have been used to calculate the strengths of 181 members tested at PCA, the University of Illinois, and the University of Texas. Tests involving local failures resulting from inadequate reinforcing details were excluded. A summary of the results of this application of the proposed equations is set out in Table 1. In these calculations,  $\phi$  was taken equal to 1.0, since accurate values of material properties and of dimensions were known.

The application of these equations is simplified considerably by the use of design aids which are presented following this discussion.

#### Minimum Reinforcement

The minimum amount of tension reinforcement is specified to insure against too rapid opening of cracks after first cracking. The lower the amount of tension reinforcement, the lower is the ratio of load at yield of tension reinforcement to ultimate load.

Closed horizontal stirrups are required in all corbels to eliminate the possibility of a sudden explosive-type failure of the corbel, which can occur in a corbel without stirrups.

#### Detailing of Corbels

The correct detailing of corbels is fully as important as the over-all design of the reinforcement. Almost invariably, distress of corbels in the field can be traced to poor detailing. If the tension reinforcement is not effectively anchored close to the outer face of the corbel, the full strength potential of the reinforcement

cannot be developed and failure will occur at a lower load than indicated by Eqs. (1) and (2). The recommended form of anchorage using a bar welded across the ends of the tension reinforcement is shown in Fig. 1. A frequently used detail for the main tension reinforcement is shown in Fig. 2(a). However, in order to conform to Section 801 of the ACI Building Code which specifies minimum bend radii for reinforcing bars, the bars are actually bent as shown in Fig. 2(b). Failure has then been observed, both in the field and the laboratory, to occur on the surface indicated in Fig. 2(b), the tension reinforcement being bypassed completely. Welding of the bearing plate to the main reinforcement when horizontal forces act is specified to eliminate the possibility of a local failure of the concrete between the bearing plate and the reinforcement.

The horizontal stirrups are located so that they will be as effective as possible, both from consideration of ultimate strength and for control of diagonal cracks. A suitable spacing of stirrups,  $s$ , is given by

$$s = \frac{2}{3} \left( \frac{d}{n+1} \right)$$

where  $n$  is the number of stirrups used. The stirrups should be placed in the corbel beginning at a distance  $s$  from the tension reinforcement. Horizontal stirrups are used rather than vertical stirrups because of the steep inclination of the diagonal cracks. These cracks can in some cases be almost vertical.

The limiting proportions of a corbel, and the limiting location of the bearing plate, are both recommended to insure against local failures of the concrete before the po-

Table 1—Comparison of Test and Calculated Strengths

Source	Type of Specimen	Number of Specimens	H/V	Average $V_u$ test $V_u$ calc	Standard Deviation
PCA	Corbels without stirrups	78	0	1.02	0.119
PCA	Corbels with stirrups	10	0	1.11	0.084
PCA	Corbels without stirrups	25	$\frac{1}{2}$	1.05	0.132
PCA	Corbels without stirrups	21	1	1.21	0.216
PCA	Corbels with stirrups	4	1	1.42	—
U of I <sup>9,10,11</sup>	Deep beams	23	0	1.01	0.134
U of I <sup>18</sup>	Beams with $a/d = 1.33$	14	0	1.14	0.168
U of T <sup>12</sup>	Short cantilevers $a/d < 1.10$	6	0	1.03	0.066

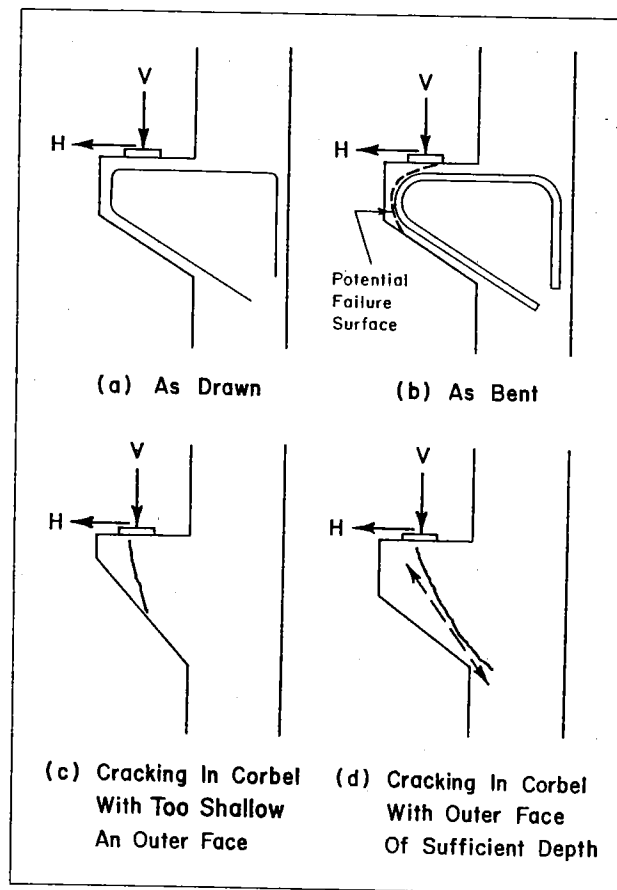


Fig. 2—Corbel Details

tential strength of the corbel has been developed. If the outer face of the corbel is made too shallow, the principal diagonal crack will take a course as shown in Fig. 2(c), and will intercept the sloping face of the corbel, resulting in instantaneous failure. If the outer face is sufficiently deep, however, the principal diagonal crack will take a course as shown in Fig. 2(d). In this case a diagonal concrete compression strut is formed as indicated, and a further increase in load may be possible after formation of the crack. Location of the bearing plate too close

to the outer face of the corbel can result in a bearing failure beneath the plate at relatively low intensities of stress. This is particularly the case if the load on the bearing plate becomes eccentric. It is essential to insure that rotation of the end of a beam due to deflection under load shall not result in the beam bearing on the outer edge of the corbel.

#### Bearing Stresses

Use of the maximum bearing stress of  $0.5f'_c$  is contingent upon compliance with the requirements of Section 6(d). Bearing failures were ex-

perienced at stresses lower than  $0.5f'_c$  in corbels loaded through bearing plates located closer to the outer face than two inches.

#### Design Aids and Design Examples

##### Design Aids

Design aids have been prepared to facilitate the use of Eqs. (1) and (2).

Eq. (1) may be written:

$$V_u = \phi b d \sqrt{f'_c} F_1 F_2 \quad (1a)$$

where  $F_1 = 6.5 (1 - 0.5^{d/a})$ , and

$$F_2 = (1000p)^{1/3}$$

Values of  $F_1$  and  $F_2$  are listed in Tables 2 and 3.

Similarly, Eq. (2) may be written:

$$V_u = \phi b d \sqrt{f'_c} F_1 F_3 \quad (2a)$$

$$\text{where } F_3 = \frac{(1000p)^{(1/3 + 0.4H/V)}}{(10)^{0.8H/V}}$$

Values of  $F_3$  are listed in Table 4. Using Eqs. (1a) or (2a), and Tables 2, 3, and 4,  $V_u$  may be readily evaluated for given values of  $b$ ,  $d$ ,  $f'_c$ ,  $p$ ,  $a/d$ , and  $H/V$ . The use of the tables is illustrated in the following examples.

Since both  $p$  and  $a/d$  can be varied independently, design of a corbel must be by successive trials. This process is simplified by use of the design chart given in Fig. 3. It is proposed that corbels be designed by successive trials using the design chart, and that the strength of the final design be checked using either Eq. (1a) or (2a), whichever is appropriate. Use of the chart and equations in this manner is illustrated in the examples.

#### Example 1

A typical interior corbel shown in Fig. 4(a) projects from a 14 x 14-in.

square tied column. It supports a 50-ft span prestressed girder carrying a live load of 1500 lb/ft and a dead load of 960 lb/ft. Design the corbel for the vertical reaction from the girder, assuming that suitable bearings are provided to eliminate horizontal restraint forces, and that the corbel does not have to resist wind or earthquake forces. Intermediate grade reinforcement is used and  $f'_c = 5000$  psi. Tolerance gap between beam end and column face is one inch.

#### Design Loads.

Dead load reaction = 24 kips

Live load reaction = 37.5 kips

Ultimate design load,

$$\begin{aligned} V_u &= \frac{4}{3} (1.5D + 1.8L) \\ &= 2.0D + 2.4L \\ &= 2.0(24) + 2.4(37.5) \end{aligned}$$

$$V_u = 138 \text{ kips}$$

- Determine shear span "a".  
 $a \approx 2$  (tolerance gap between beam and column)  
+  $\frac{1}{2}$  (bearing plate width)

$$\begin{aligned} \text{Bearing plate width} &= \frac{V_u}{b(f'_c/2)} \\ &= \frac{138,000}{14 \times 2500} = 3.9 \text{ in., say 4 in.} \end{aligned}$$

$$a = 2(1) + 4/2 = 4 \text{ in.}$$

- Estimate depth  $d$ .  
 $a/d$  is generally between 0.15 and 0.4; assume  $a/d = 0.3$ , hence  $d = 13.3$  in.
- Determine  $v_u = V_u/bd$ .

$$v_u = \frac{138,000}{14 \times 13.3} = 741 \text{ psi}$$

- Find required  $p$  from design chart. Enter chart at  $v_u = 741$  psi, proceed horizontally to  $f'_c = 5000$  psi, vertically to  $a/d = 0.3$ , horizon-

Table 2—Values of  $F_1 = 6.5(1 - 0.5^{d/a})$

$a/d$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50
0.1	6.49	6.49	6.48	6.47	6.45	6.44	6.41	6.39	6.36	6.33
0.2	6.30	6.26	6.22	6.18	6.14	6.09	6.05	6.00	5.95	5.90
0.3	5.85	5.80	5.75	5.70	5.65	5.60	5.55	5.50	5.45	5.40
0.4	5.35	5.30	5.25	5.20	5.15	5.10	5.06	5.01	4.97	4.92
0.5	4.87	4.83	4.79	4.74	4.70	4.66	4.61	4.57	4.53	4.49
0.6	4.45	4.41	4.37	4.34	4.30	4.26	4.22	4.19	4.15	4.12
0.7	4.08	4.05	4.02	3.98	3.95	3.92	3.89	3.86	3.83	3.80
0.8	3.77	3.74	3.71	3.68	3.65	3.62	3.60	3.57	3.54	3.52
0.9	3.49	3.46	3.44	3.42	3.39	3.37	3.34	3.32	3.30	3.27

Table 3—Values of  $F_2 = (1000p)^{1/3}$

$p$	$F_2$	$p$	$F_2$	$p$	$F_2$
0.0040	1.59	0.0095	2.12	0.0150	2.47
0.0045	1.65	0.0100	2.15	0.0155	2.49
0.0050	1.71	0.0105	2.19	0.0160	2.52
0.0055	1.76	0.0110	2.22	0.0165	2.54
0.0060	1.82	0.0115	2.26	0.0170	2.57
0.0065	1.87	0.0120	2.29	0.0175	2.60
0.0070	1.91	0.0125	2.32	0.0180	2.62
0.0075	1.96	0.0130	2.35	0.0185	2.64
0.0080	2.00	0.0135	2.38	0.0190	2.67
0.0085	2.04	0.0140	2.41	0.0195	2.69
0.0090	2.08	0.0145	2.44	0.0200	2.71

Table 4—Values of  $F_3 = \frac{(1000p)^{(1/3 + 0.4H/V)}}{(10)^{0.8H/V}}$

$\frac{H}{V}$													
$p$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	
0.0040	1.40	1.23	1.08	0.95	0.83	0.73	0.64	0.57	0.50	0.44	0.38	0.34	
0.0045	1.46	1.29	1.14	1.00	0.89	0.78	0.69	0.61	0.54	0.48	0.42	0.37	
0.0050	1.52	1.34	1.19	1.06	0.94	0.83	0.74	0.66	0.58	0.52	0.46	0.40	
0.0055	1.57	1.40	1.25	1.11	0.99	0.88	0.78	0.70	0.62	0.55	0.49	0.44	
0.0060	1.62	1.45	1.30	1.16	1.04	0.92	0.83	0.74	0.66	0.59	0.53	0.47	
0.0065	1.67	1.50	1.34	1.20	1.08	0.97	0.87	0.78	0.70	0.62	0.56	0.50	
0.0070	1.72	1.55	1.39	1.25	1.12	1.01	0.91	0.82	0.73	0.66	0.59	0.53	
0.0075	1.76	1.59	1.43	1.29	1.16	1.05	0.95	0.85	0.77	0.69	0.63	0.56	
0.0080	1.81	1.63	1.48	1.34	1.21	1.09	0.99	0.89	0.80	0.73	0.66	0.59	
0.0085	1.85	1.68	1.52	1.38	1.25	1.13	1.02	0.93	0.84	0.76	0.69	0.62	
0.0090	1.89	1.72	1.56	1.41	1.28	1.17	1.06	0.96	0.87	0.79	0.72	0.65	
0.0095	1.93	1.75	1.60	1.45	1.32	1.20	1.10	1.00	0.91	0.83	0.75	0.68	
0.0100	1.96	1.79	1.63	1.49	1.36	1.24	1.13	1.03	0.94	0.86	0.78	0.71	
0.0105	2.00	1.83	1.67	1.53	1.40	1.27	1.16	1.06	0.97	0.89	0.81	0.74	
0.0110	2.04	1.86	1.71	1.56	1.43	1.31	1.20	1.10	1.00	0.92	0.84	0.77	
0.0115	2.07	1.90	1.74	1.60	1.46	1.34	1.23	1.13	1.04	0.96	0.87	0.80	
0.0120	2.10	1.93	1.78	1.63	1.50	1.38	1.26	1.16	1.07	0.98	0.90	0.83	
0.0125	2.14	1.96	1.81	1.66	1.53	1.41	1.30	1.19	1.10	1.01	0.93	0.86	
0.0130	2.17	2.00	1.84	1.70	1.56	1.44	1.33	1.22	1.13	1.04	0.96	0.88	

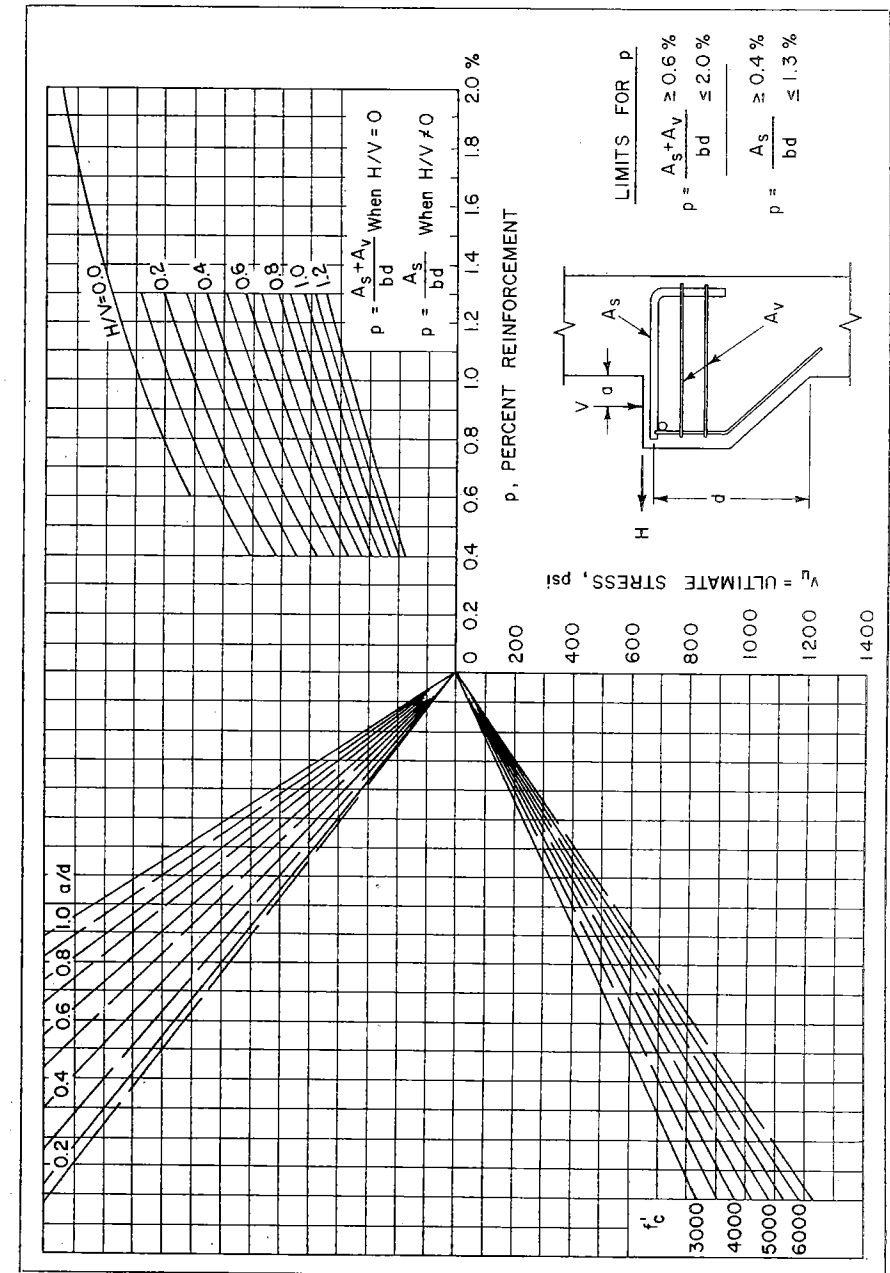


Fig. 3—Design Chart for Ultimate Strength of Corbels

tally to  $H/V=0$  and vertically downward to the  $p$  scale.

$p = 0.98\%$  OK since it is  $< 2.0\%$ ,

and  $> A_s + A_v = 0.4\% + \frac{0.4\%}{2} =$

0.6%.

- Select  $A_s$ ,  $A_v$ , and corbel dimensions.

$$p = \frac{A_s + A_v}{bd} = \frac{1.5 A_s}{bd},$$

if  $A_v$  is made equal to  $0.5A_s$ .

Hence

$$A_s = 0.0098 \times 14 \times 13.3 / 1.5 = 1.22 \text{ in.}^2$$

Use 4-#5 bars.

$$A_v = 0.61 \text{ in.}^2$$

Use 2-#4 bar closed stirrups.

Stirrup spacing (2-#4 stirrups)

$$s = \frac{2}{3} \left( \frac{d}{n+1} \right) = \frac{2}{3} \left( \frac{13.3}{3} \right)$$

$$= 2.96 \text{ in.}$$

Use 3 in. ctrs. from tension reinforcement.

Allowing 1 in. cover to reinforcement, over-all depth of corbel

$$= 1 + 0.3 + 13.3 = 14.6 \text{ in.}$$

Use 15 in.

Length of corbel

$$= 2 + (\text{bearing width}) + (\text{clearance})$$

$$= 2 + 4 + 1 = 7 \text{ in.}$$

Depth of outer face of corbel, say

$$\text{half over-all depth at column face,}$$

$$= 15 / 2 = 7.5 \text{ in.}$$

Use 8 in.

- Check design.

$$d = 15.0 - 1.0 - 0.62 / 2 = 13.7 \text{ in.}$$

$$a/d = 4.0 / 13.7 = 0.29$$

$$p = \frac{A_s + A_v}{bd} = \frac{2.04}{14 \times 13.7} = 1.06\%$$

$$V_u = \phi b d \sqrt{f'_c} F_1 F_2 \quad (1a)$$

Using Tables 2 and 3 to obtain  $F_1$  and  $F_2$

$$V_u = 0.85 \times 14 \times 13.7 \times \sqrt{5000} \times 5.90 \times 2.19$$

$$= 149 \text{ kips OK, greater than required design load.}$$

- The details of this corbel are shown in Fig. 4(a).

#### Example 2

Redesign the corbel of Example 1 assuming that a bearing shoe in the prestressed girder is welded to the corbel, and because of this, a horizontal force of 45 kips will occur due to restraint of creep and shrinkage deformation of the girder. This example is illustrated by Fig. 4(b).

- From Example 1,  $V_u = 138$  kips, and  $a = 4$  in.

• Section 1506(a)5, of the ACI Building Code (ACI 318-63), requires that the effects of creep and shrinkage be considered on the same basis as the effects of dead load, when calculating the design ultimate loads. Hence the load factor for the horizontal restraint force will be:

$$\frac{4}{3} (1.5) = 2.0$$

$$H_u = 2.0(45) = 90 \text{ kips}$$

therefore

$$H/V = 90 / 138 = 0.65$$

- From the design chart, the value of  $v_u$  corresponding to the maximum allowable  $p$  ( $= 1.3\%$ ),  $H/V$  of 0.65, an assumed  $a/d$  of 0.3, and  $f'_c$  of 5000 psi, is about 460 psi.

Therefore

$$d = \frac{V_u}{v_u b} = \frac{138,000}{460 \times 14} = 21.4 \text{ in.}$$

now

$$a/d = 4 / 21.4 = 0.19$$

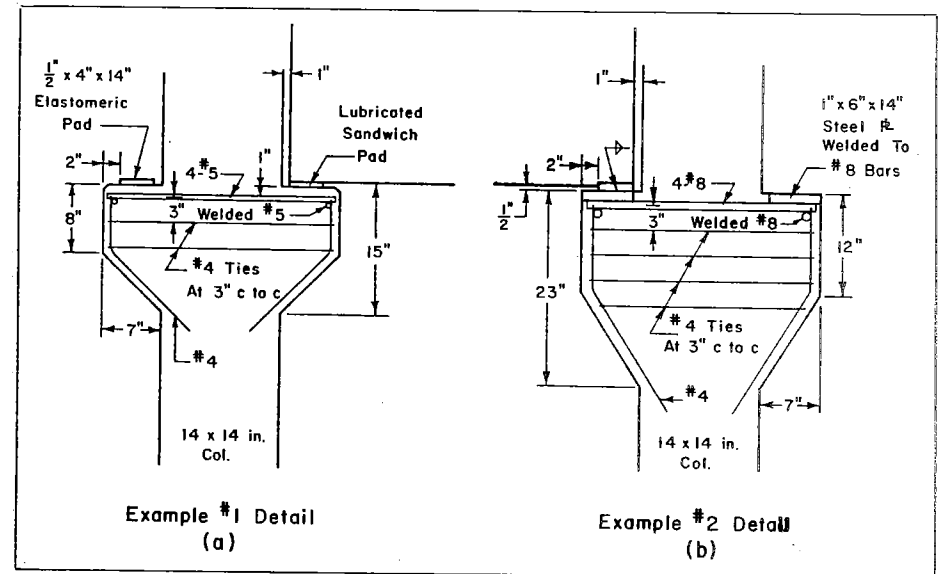


Fig. 4—Corbel Details for Example Problems

- Determine  $p$  from the design chart for  $v_u = 460$  psi,  $f'_c = 5000$  psi,  $a/d = 0.19$  and  $H/V = 0.65$ .

$p = 1.07\%$  OK since it is  $> 0.4\%$  and  $< 1.3\%$

- Select  $A_s$ ,  $A_v$ , and corbel dimensions.

$$p = A_s / bd$$

Hence

$$A_s = pbd = 0.0107 \times 14 \times 21.4 = 3.20 \text{ in.}^2$$

Use 4-#8 bars.

$$A_v = A_s / 2 = 1.60 \text{ in.}^2$$

Use 4-#4 bar closed stirrups.

Stirrup spacing (4-#4 stirrups)

$$s = \frac{2}{3} \left( \frac{d}{n+1} \right) = \frac{2}{3} \left( \frac{21.4}{5} \right)$$

$$= 2.88 \text{ in. Use 3-in. centers.}$$

Assuming a 1-in. thick bearing plate welded to the main tension reinforcement, over-all depth of corbel:

$$h = 1 + 0.5 + 21.4 = 22.9 \text{ in.}$$

Use 23 in.

Length of corbel will be as in Example 1, 7 in.

- Check design.

$$d = 23 - 1 - 0.5 = 21.5 \text{ in.}$$

$$a/d = 4 / 21.5 = 0.19$$

$$p = A_s / bd = 3.16 / (14 \times 21.5) = 1.05\%$$

$$V_u = \phi b d \sqrt{f'_c} F_1 F_3 \quad (2a)$$

Using Tables 2 and 4 to obtain  $F_1$  and  $F_3$

$$V_u = 0.85 \times 14 \times 21.5 \times \sqrt{5000} \times 6.33 \times 1.21$$

$$= 139 \text{ kips OK, greater than design load}$$

- The details of this corbel are shown in Fig. 4(b).

#### Special Note

It should be noted that the addition of the horizontal restraint force has necessitated an increase in depth of the corbel of 53 percent and an increase in main tension reinforcement of 162 percent. It is clear, therefore, that for safety, a realistic estimate must be made of any horizontal forces that may act on a corbel. If special provision is not

made to eliminate the horizontal restraint forces by using lubricated sandwich pads at one end of each girder, it is proposed that  $H/V$  should be assumed in design to be at least 0.5, unless the horizontal force is calculated.

## PART 2—TESTS OF CORBELS

### Scope

Three series of tests were made: (a) exploratory tests, (b) tests of corbels subjected to vertical loads only, and (c) tests of corbels subjected to combined vertical and horizontal loads. The exploratory tests involved testing procedures and reinforcing

detailing. The other two series involved a systematic investigation of the effect of different variables on the strength and behavior of corbels.

The variables considered in the tests were: reinforcement ratio, concrete strength, ratio of shear span to effective depth, amount and distribution of stirrup reinforcement, size and shape of corbel, and the ratio of the horizontal applied load to the vertical applied load. The range of the variables is indicated on Fig. 5.

### Test Specimens

All specimens consisted of a length of 8 x 12-in. column with two corbels

arranged symmetrically, as shown in Fig. 5. With the exception of certain specimens in series (a) the main tension reinforcement consisted of straight deformed bars anchored by bars of equal diameter welded across their ends, as shown in Fig. 6. Corbels with horizontal stirrups were detailed as shown in Fig. 6(b). Corbels to be subjected to combined vertical and horizontal loading were provided with grooved bearing plates welded to the tension reinforcement as shown in Fig. 6(c). The detailing of the reinforcement of the corbels in the exploratory series (a) was as indicated in Fig. 7.

The dimensions of the individual specimens and the material properties are set out in Tables A1 through A4 appended to this paper.

### Materials and Fabrication

All concrete was made with Type I portland cement. The coarse aggregate was a gravel of  $\frac{3}{4}$ -in. maximum size, and the fine aggregate was Elgin sand. The concrete slumps varied from 1½ to 3 in. An air-entraining agent was added to produce 4 to 6 percent air. One batch of concrete was used for each specimen, with the exception of two large specimens, which required two batches each. Three 6 x 12-in. cylinders were taken from each batch for determination of concrete strength. The specimens and test cylinders were moist cured for three days under a plastic cover, and then stored at 70°F and 50 percent relative humidity, and were tested at six days. The concrete cylinder strengths

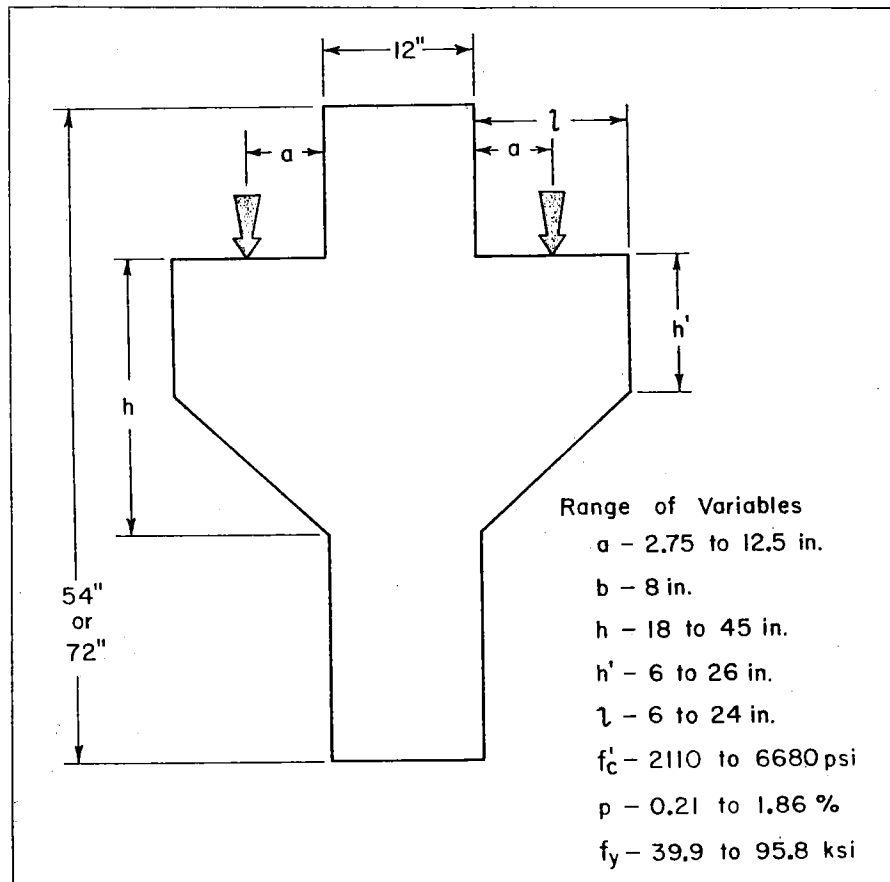


Fig. 5—Corbel Test Specimen

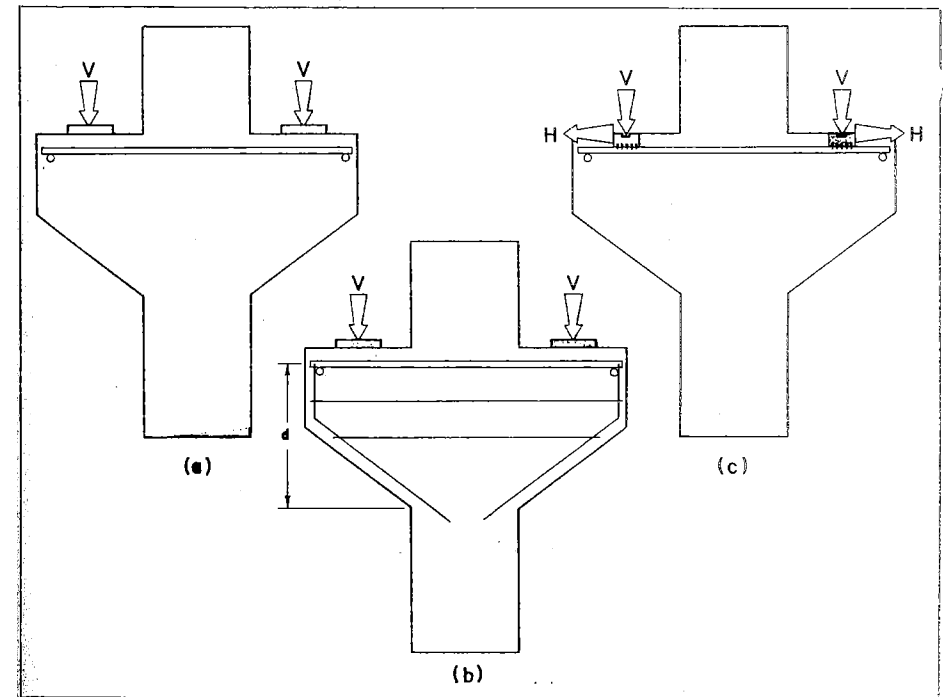


Fig. 6—Reinforcement Details of Test Corbels

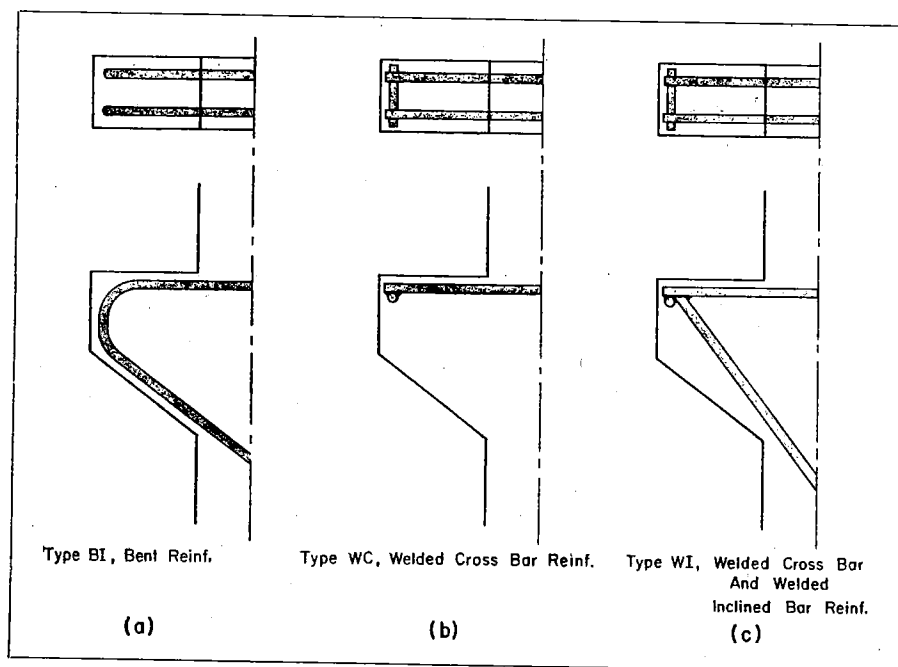


Fig. 7—Detailing of Corbel Reinforcement in Auxiliary Test Series

varied from 2110 psi to 6680 psi, as given in Tables A1 through A4.

The reinforcing steel conformed to ASTM Designation A305 for deformations. The steel yield strengths were determined from tension tests of 30-in. coupons taken from each reinforcing bar used; the yield strength varied from 39,900 psi to 95,800 psi, and are given in Tables A1 through A4.

#### Instrumentation

The corbels were instrumented with SR4-A-12 strain gages mounted on the reinforcement and with SR4-A-9-4 strain gages mounted on the concrete. This instrumentation varied according to the purpose of individual tests.

#### Test Procedures

For convenience all corbels were tested in an upside-down position. A heavily-reinforced U-frame cen-

tered under the loading platen of a million-pound testing machine was used to support the corbels. To assure adequate bearing capacity of the legs of the U-frame, the top of the legs was armored by steel plates. These plates were carefully aligned in the forms of the U-frame before placing the concrete to provide parallel bearing surfaces.

The corbels were subjected to various combinations of vertical and horizontal loads. The loads were increased in increments until failure. After each load increment the development of cracks was observed and marked on the specimens. All strain measurements were recorded continuously by strip-chart strain recorders

#### Vertical Loading Only

The corbels were loaded through steel bearing plates placed symmet-

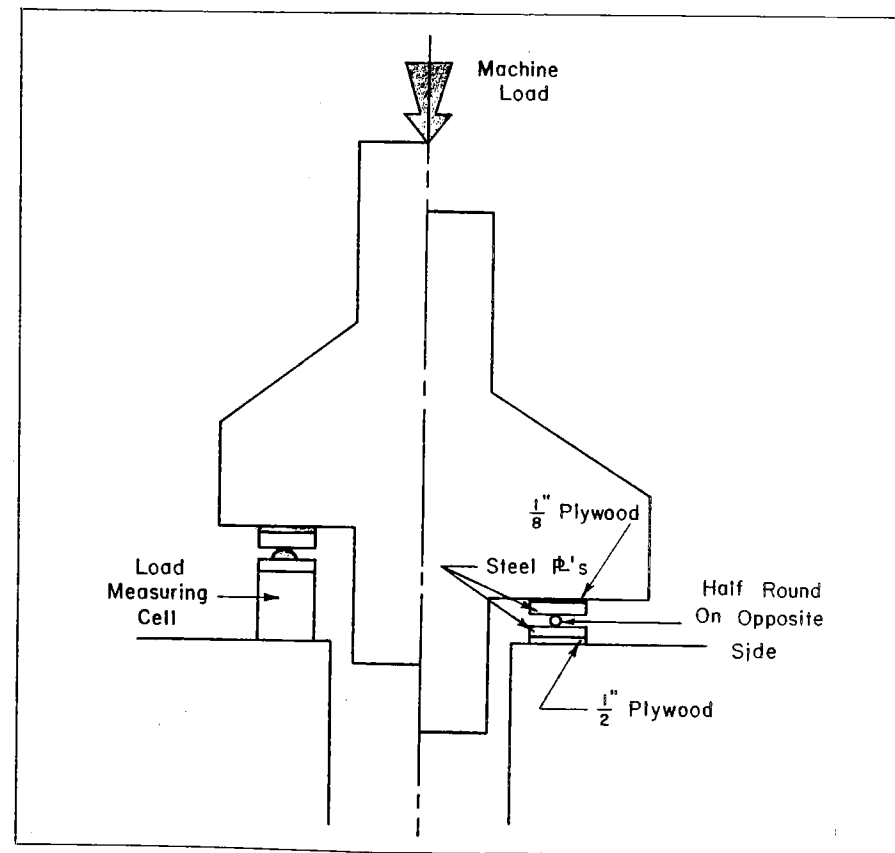


Fig. 8—Vertical Loading of Corbels

rically on the top\* of the corbels as shown in Fig. 8. The length of the bearing plates was equal to the width of the corbels. The width  $w$  was either 3 or 5 in. and the thickness of the plates was 1 or  $1\frac{1}{2}$  in. To eliminate restraint of deformations, a half-round and a round bar were placed between the bearing plates and another set of steel plates which rested on the supporting U-frame. The load was applied to the bottom of the column stub by the testing machine platen.

To assure uniform load distribu-

tion on all bearing areas, new plywood inserts were used in each test. A  $\frac{3}{4}$ -in. plywood sheet was placed between the column bottom and the testing machine platen,  $\frac{1}{8}$ -in. plywood sheets between the corbels and the bearing plates, and  $\frac{1}{2}$ -in. plywood sheets between the U-frame and the second set of steel plates. After the application of the first 10,000 lbs, the machine platen was blocked to prevent its rotation.

In the first five tests the load applied to the corbels was checked by load measuring cells to establish that the load was distributed equally to the two corbels. Since the two loads did not differ by more than two percent, the use of these load

\* Top refers to the position in a structure and not to the position of the specimen in the testing machine. This convention is used throughout this paper.



measuring cells was discontinued in further tests. The two test setups are shown schematically in Fig. 8. Fig. 9 shows the test setup used for the tests involving vertical load only.

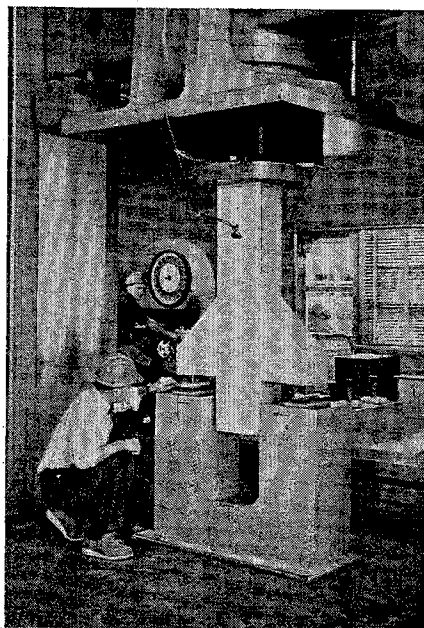


Fig. 9—Test Setup for Vertical Load Only,  $H/V = 0$

Three tests were made to determine whether a column load carried from upper floors influenced the strength of the corbels. In these tests a load was applied to the top of the column stub by a 100-ton hydraulic ram. A constant ratio of the machine load to the ram load was maintained throughout each of these tests. The loading of the ram was controlled by the oil pressure indicator but the load was also continuously monitored by a load-measuring cell placed between the ram and the column top. This test setup was similar to that shown in Fig. 9, except for the 100-ton hydraulic ram which was within the U-portion of the test frame.

### Combined Vertical and Horizontal Loading

The horizontal forces which develop in precast beams as a result of restrained volume changes were simulated by horizontal forces applied at the level of the top of the corbels. To permit a direct transfer of the horizontal forces to the tension reinforcement, the 3-in. bearing plates were welded to the reinforcing bars. The horizontal forces were applied by four or six hydraulic rams to a set of loading plates, and transferred to the bearing plates through milled shear keys. The hydraulic rams were positioned on each side of the corbels in such a manner that the resultant of the ram loads was at the level of the top of the corbel. The frictional restraint to lateral deformations was eliminated by placing 2-in. diameter round bars between the loading plates and the steel plates on the supporting U-frame.

The rams used for applying the horizontal forces were calibrated so that the loads could be correlated with the oil pressure. The operation of the rams during testing was checked by load measuring cells which indicated that the errors in the load as determined from the oil pressure were less than one percent. Therefore, the use of the load-measuring cells was discontinued.

The vertical load was applied in the same manner as in the tests of corbels subjected to vertical loads only. A constant ratio between the vertical and the horizontal loads was maintained throughout each test.

The loading system for combined horizontal and vertical loading is shown in Fig. 10.

### Test Results

The principal data obtained in



Fig. 10—Test Setup for Combined Horizontal and Vertical Loading,  $H/V$  does not equal zero

these tests have been listed in Tables A1 through A4 appended to this paper. Other data are reproduced where appropriate in the discussion of the behavior of the corbels set out in Part 3.

### PART 3—BEHAVIOR OF CORBELS

#### Series (a)—Exploratory Tests

##### Effect of Additional Column Loads

Three tests were made on pairs of identical specimens. One of each of the companion specimens was subjected to vertical loads applied to the corbels only, while the other specimen was subjected also to an additional load applied at the top of the column stub. The pertinent data are given in Table 5. These tests show that the strength of the corbels is not significantly influenced by the additional load carried by the

column. Therefore, subsequent tests were performed with loads applied to the corbels only.

#### Detailing the Corbel Reinforcement

Test of corbels reinforced conventionally according to Fig. 7(a) have shown the weakness of such detailing when loads were applied close to the outer edges of the corbels. These corbels failed along a surface following the bends of the reinforcement, Fig. 11, indicating that the

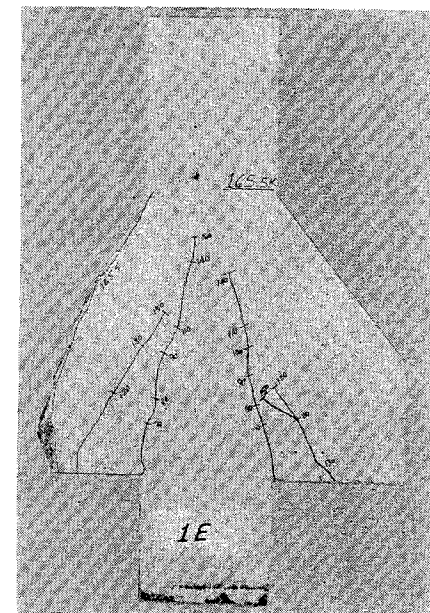


Fig. 11—Conventionally Reinforced Corbel, Type B1, Loaded Near Outer Edge. Failure Plane Follows Bend Radius

Table 5—Effects of Additional Column Load

Specimen	Concrete Strength, $f'_c$ , psi	Effective Depth Shear Span	Load per Corbel at Ultimate, kips	Additional Column Load, kips
4	3520	0.171	99.9	0
5E	4010	0.171	114.3	114.0
15	4500	0.370	72.0	0
6E	4140	0.370	63.9	48.0
24	4250	0.372	88.8	0
7E	4490	0.372	109.3	110.0

reinforcement was not fully effective and that it even created a possible source of weakness. Measurements of the strains in the reinforcement along the compression side indicated only small compressive stresses throughout the length of the reinforcement.

Previous tests of corbels and of deep beams<sup>8,9,11</sup>, and the tests reported herein, show that the stresses in the tension reinforcement of a corbel do not vary significantly along its length between the face of the column and the point of load application. Consequently, high bond stresses exist in the outer parts of the tension reinforcement and may lead to bond failures. Such bond failures were observed in tests of deep beams<sup>8</sup>. The anchorage of the bars can be assured by cross-bars welded to the ends of the tension reinforcement as shown in Fig. 7(b). This method proved satisfactory and subsequent tests were made on specimens reinforced with straight tension bars anchored by the welded cross-bars.

Tests of corbels with inclined compression reinforcement welded to the ends of the tension reinforcement, Fig. 7(c), show the compression reinforcement contributes little to the strength of the corbels. Therefore, compression reinforcement was not used in further tests.

The strength of corbels with the three types of reinforcement is compared in Table A1. The specimens designated by letters WC had tension reinforcement with welded cross bars, Fig. 7(b), specimens BI had bent reinforcement, Fig. 7(a), and specimens WI had compression reinforcement and cross-bars welded to the ends of the tension reinforcement, Fig. 7(c).

The arrangement and amount of

reinforcement in the column has little influence on the strength of the corbels projecting from the column, as may be seen in Table A1. Thus, the amount of column reinforcement used in subsequent tests was that which would prevent failure of the column portion of the test specimens.

### Series (b)—Corbels Subject to Vertical Loads Only

#### Behavior Under Load

Initially the corbels behaved elastically, and the stress in the main tension reinforcement was proportional to load. In all the tests, the first cracks to appear were flexural cracks starting at the junction of the horizontal face of the corbel and the face of the column. After formation of these cracks the tension reinforcement stress increased much more rapidly. Typical relationships between applied load and force in the tension reinforcement are shown in Fig. 12. Subsequent development of the cracks depended primarily on the reinforcement ratio and the ratio of the shear span to the effective depth, and was also closely related to the mode of failure.

Four principal types of failure were observed, as described below.

- **Flexural Tension**—A flexural tension failure occurs by crushing of the concrete at the bottom of the sloping face of the corbel after extensive yielding of the tension reinforcement. Such a failure is illustrated in Fig. 13(a). The appearance of a corbel after a flexural tension failure is characterized by very wide flexural cracks.

- **Flexural Compression**—A flexural compression failure occurs when crushing of the concrete takes place at the bottom of the corbel before

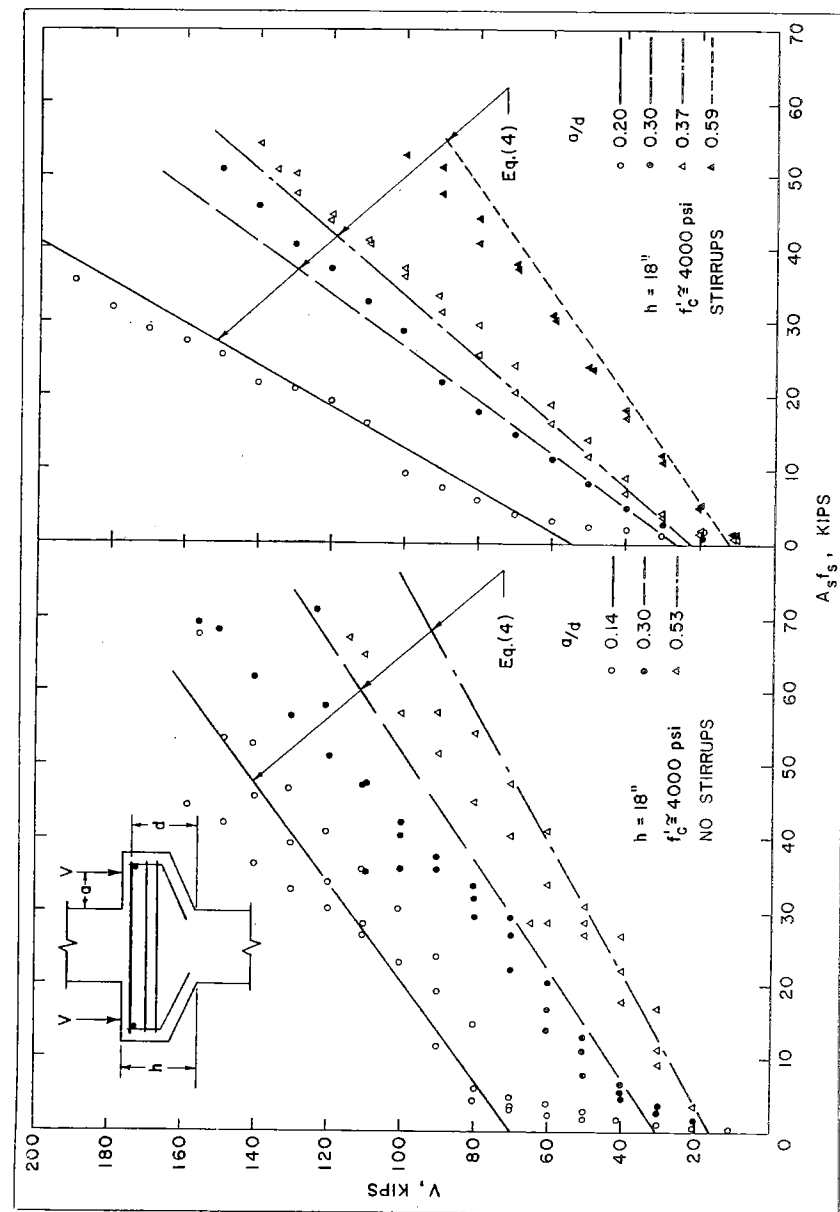
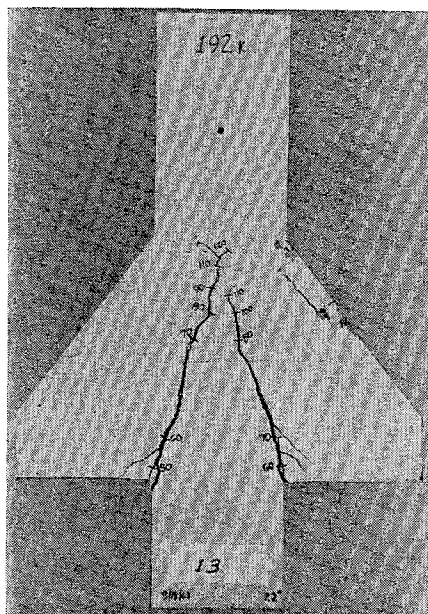
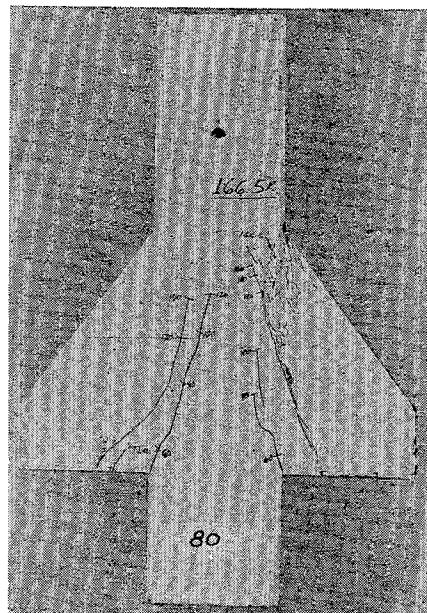


Fig. 12—Relationship Between Applied Load and Tension Steel Force, Vertical Load Only



(a) Tension Failure (FT)



(b) Compression Failure (FC)

Fig. 13—Flexural Failures,  $H/V = 0$

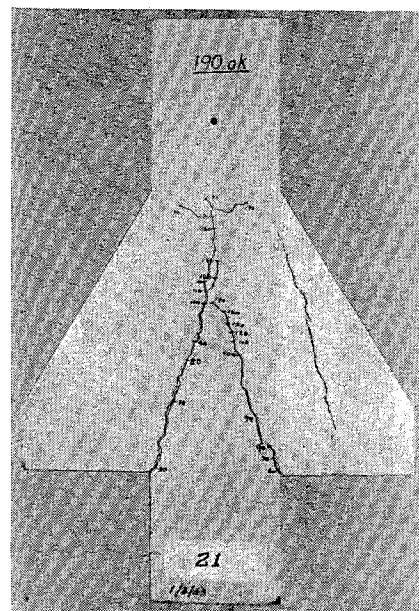
extensive yielding of the reinforcement has occurred. The tension reinforcement stress at failure is either below or just at the yield point and the flexural cracks, while well developed, have not opened excessively. Such a failure is illustrated in Fig. 13(b).

- **Diagonal Splitting**—The diagonal splitting mode of failure is shown in Fig. 14(a) and 14(b). The flexural crack pattern was well developed before the diagonal splitting of the concrete, which occurred along a line extending from the bearing plate toward the junction of the sloping face of the corbel and the face of the column. A corbel with such a crack usually fails by shear-compression of the concrete compression zone, as in the corbel shown in Fig. 14(b).

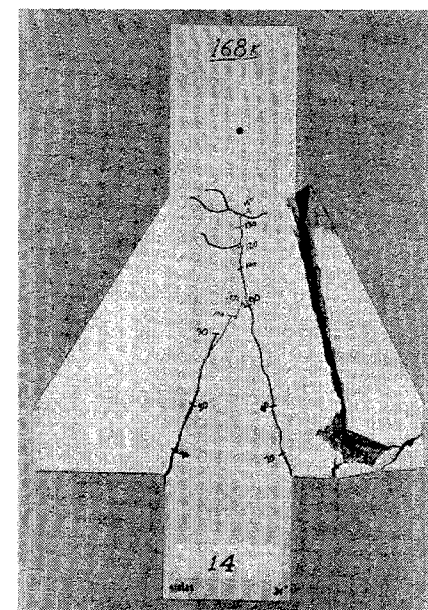
- **Shear Failure**—Shear failures were characterized by the develop-

ment of a series of short inclined cracks along the plane of the interface between the column and the corbel, as may be seen in Figs. 15(a) and (b). The final failure was by shearing along this weakened plane, and the appearance after failure can be seen in Fig. 15(b).

- **Secondary Modes of Failure**—Failures which did not involve the deepest section of the corbel at the column face were considered secondary modes of failure. These were of two types: (a) the splitting away of a portion of the concrete due to a major crack intersecting the sloping face of the corbel, as seen in Fig. 16(a), and (b) bearing failures of the concrete beneath the bearing plate, as seen in Fig. 16(b). Both types of secondary modes of failure occurred at loads lower than those at which failure would have occurred by one of the principal modes

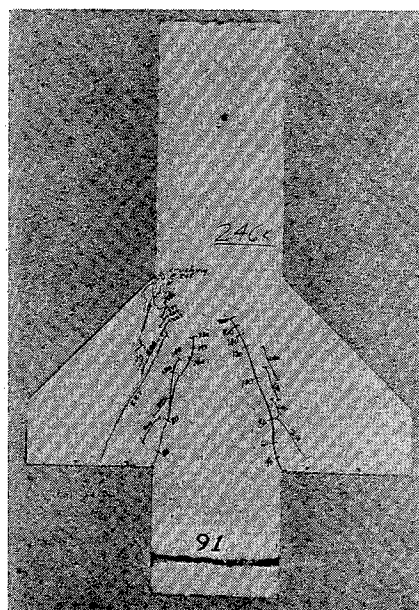


(a)

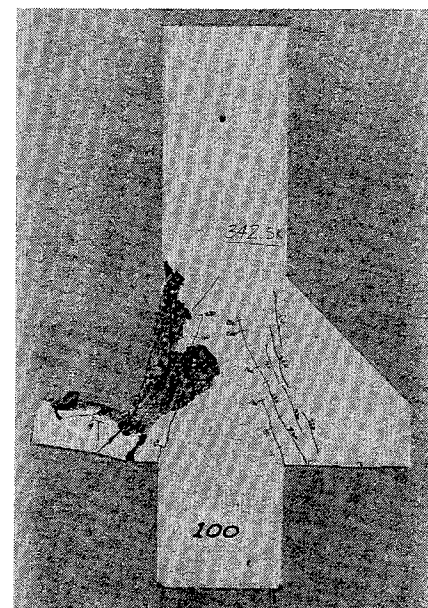


(b)

Fig. 14—Diagonal Splitting Failures (DS),  $H/V = 0$

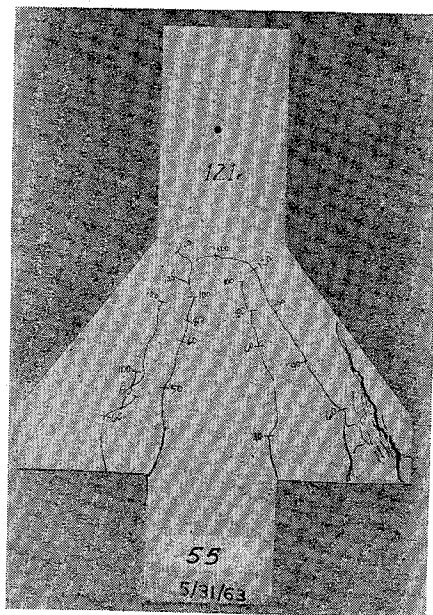


(a)

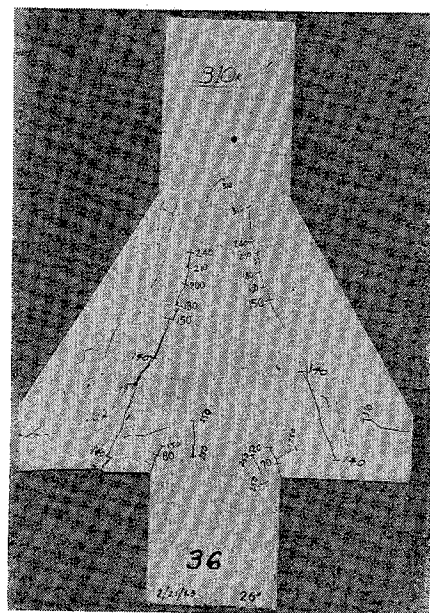


(b)

Fig. 15—Shear Failures (S),  $H/V = 0$



(a) Corbel End Failure (CE)



(b) Bearing Failure (B)

Fig. 16—Secondary Modes of Failure,  $H/V = 0$

of failure had the secondary failures been prevented.

### Discussion of Behavior

To understand the behavior of corbels and to arrive at design equations, extensive plotting of test data was made. During such studies, further tests were conducted to cover adequate ranges of the significant variables. Empirical design equations were gradually arrived at by numerous comparisons of observed properties to those computed by various expressions. An LGP-30 electronic computer was used in these studies.

The relationships between tension reinforcement force and applied load shown in Fig. 12 are for corbels made from concrete with a strength of about 4000 psi. Similar relationships were found to hold for corbels without stirrups made from 2000 and 6000-psi concrete. It was not con-

sidered necessary to test corbels with stirrups made from concretes having strengths other than 4000 psi. It was found that the tension reinforcement force,  $A_s f_s$ , is a function of the applied load,  $V$ , of the ratio of shear span to effective depth,  $a/d$ , and of the concrete strength  $f'_c$ . The relationship between load  $V$  and tension force  $A_s f_s$  can be idealized as shown in Fig. 17. The linear part of the

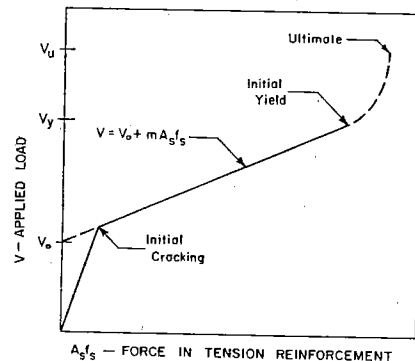


Fig. 17—Idealized Relationship Between Applied Load and Force in Tension Reinforcement

relationship between first cracking and yield of the reinforcement can be represented by the equation:

$$V = V_0 + m A_s f_s \quad (3)$$

where  $V$  = applied load

$V_0$  = nominal cracking load

$m$  = slope

The nominal cracking load,  $V_0$ , and the slope,  $m$ , are both functions of  $f'_c$  and  $a/d$ . These functions can be expressed as:

$$V_0 = bd \frac{4.4}{(a/d)^{1/2}} \left( \frac{f'_c}{a/d} \right)^{1/3} C_1$$

$$\text{and } m = \frac{2}{3} \sqrt{\frac{f'_c}{1000}} \frac{1}{C_2}$$

Substituting for  $V_0$  and  $m$  in Eq. (3) above yields:

$$v = \frac{V}{bd} = \frac{4.4}{(a/d)^{1/2}} \left( \frac{f'_c}{a/d} \right)^{1/3} C_1 + \frac{2}{3} \sqrt{\frac{f'_c}{1000}} \frac{pf_s}{C_2} \quad (4)$$

where  $C_1 = 1$  for vertical loads only  
 $C_2 = 0.8(10)^{a/3d}$  when there are no stirrups

$= 0.25(10)^{a/d}$  when there are stirrups

Eq. (4) may be used to calculate the nominal shear stress,  $v$ , at working load by substituting the allowable steel stress for  $f_s$ , and can be used to calculate the nominal shear stress at yield of the tension reinforcement,  $v_y$ , by substituting  $f_y$  for  $f_s$ .

Eq. (4) has been used to calculate the nominal shear stress,  $v_y$ , at yield of tension reinforcement in those corbels tested in which yielding occurred. The average value of  $(v_y \text{ test}/v_y \text{ calc})$  given in Table A2 is 1.06 and the standard deviation is 0.135. When the computed steel stress,  $f_{su}$ , given by Eq. (5) below was less than the yield point of the steel used, no value for  $v_y \text{ calc}$  is

given in Table A2.

Eq. (4) can also be used to define whether or not the tension reinforcement will yield prior to the corbel developing its ultimate strength. If the nominal shear stress at ultimate strength is  $v_u (= V_u/bd)$ , then transposing Eq. (4) and substituting  $v_u$  for  $v$  yields:

$$f_{su} = \left[ v_u - \frac{4.4}{(a/d)^{1/2}} \left( \frac{f'_c}{a/d} \right)^{1/3} C_1 \right] \times \frac{1.5C_2}{p\sqrt{f'_c/1000}} \quad (5)$$

in which the stress,  $v_u$  may be calculated from Eq. (7).

The tension reinforcement will yield if  $f_{su}$  calculated using Eq. (5) is equal to or greater than the yield point stress  $f_y$ .

To facilitate the use of Eqs. (4) and (5), values of  $C_1$  and  $C_2$  have been listed in Tables A7 and A8 appended to this report.

For purposes of practical design, it should usually not be necessary to check the stress in the tension reinforcement. As indicated in the discussion of design criteria in Part 1, yield of the tension reinforcement will usually take place at  $\frac{2}{3}$  to 1 times the ultimate load. The proposed ultimate strength procedure accounts for this by specifying load factors  $\frac{1}{3}$  greater than those used for the individual precast members.

### Ultimate Strength

The ultimate strength equation must of necessity be empirical because of the complexity of the state of stress in the corbel. Several conclusions concerning the effect of individual variables on the strength of corbels can be drawn on the basis of the experimental data presented herein. These conclusions, together with the requirements of the laws of



similitude, lead to a suitable form for the ultimate strength equation.

The ultimate strength of a corbel,  $V_u$ , is a function of its width  $b$  and effective depth  $d$ , of the reinforcement ratio,  $p$  ( $= A_s/bd$ ), of the concrete strength  $f'_c$  and of the ratio of the shear span to the effective depth,  $a/d$ . From the laws of similitude it is concluded that the ultimate strength,  $V_u$ , must be directly proportional to the width  $b$  and to the effective depth  $d$ . The tests have shown that the strength is also proportional to  $\sqrt{f'_c}$ . Accordingly, the strength may be expressed in terms of the non-dimensional ratio  $V_u/bd\sqrt{f'_c}$ . This ratio must be a function of the remaining two variables,  $a/d$  and  $p$ .

The tests show that increasing the  $a/d$  ratio lowers the corbel strength,  $V_u$ . The maximum strength is obtained for  $a = 0$ , while  $a = \infty$  represents the condition of pure bending. Hence,  $V_u = 0$  when  $a = \infty$ . The variation of the strength with  $a/d$  can be represented by a term of the form  $K_1(1 - K_2^{a/d})$ , where  $K_2$  is less than unity.

These tests also show that the strength increases when the reinforcement ratio increases. The effect of the reinforcement ratio can be expressed by the term  $K_3 p^{K_4}$ . The foregoing analysis leads to the expression:

$$\frac{V_u}{bd\sqrt{f'_c}} = K_1(1 - K_2^{a/d}) K_3 p^{K_4} \quad (6)$$

The constants  $K_1$  and  $K_3$  need not be known separately and may be combined into a single coefficient. Statistical analysis of the test data resulted in the following equation:

$$\begin{aligned} \frac{v_u}{\sqrt{f'_c}} &= \frac{V_u}{bd\sqrt{f'_c}} \\ &= 6.5(1 - 0.5^{a/d})(1000p)^{1/3} \end{aligned} \quad (7)$$

Multiplying both sides of Eq. (7) by  $bd\sqrt{f'_c}$  and introducing the strength reduction factor  $\phi$  yields Eq. (1) of the proposed criteria for design of corbels.

Eq. (7) was used to calculate the nominal shear stress at ultimate strength,  $v_u$ , for all corbels subjected to vertical loads only, and the results of these calculations are listed in Table A2. Excluding those specimens which experienced secondary failures by bearing or splitting off of the corbel end, the average value of ( $v_u$  test/ $v_u$  calc) was found to be 1.02, and the standard deviation 0.119.

Analysis of data from tests of corbels with horizontal stirrups shows that the stirrups are as effective in resisting vertical loads as is the main tension reinforcement. Accordingly, the strength of a corbel with horizontal stirrups and subject to vertical loads only can be calculated using Eq. (7) but calculating  $p$  on the basis of the total cross section of tension and stirrup reinforcement, i.e.  $p = (A_s + A_v)/bd$ . The calculated ultimate strengths of corbels with stirrups and subject to vertical loads listed in Table A4 were determined in this manner. The average value of ( $v_u$  test/ $v_u$  calc) was 1.11 and the standard deviation 0.084.

Fig. 18 shows a graphical representation of Eq. (7), together with the corresponding test values. The test results from corbels which experienced secondary failures are not included in this figure.

In Table A5 comparisons have been made between data obtained by other investigators at the Universities of Illinois and Texas, and the ultimate strengths calculated using Eq. (7). A satisfactory agreement is found.

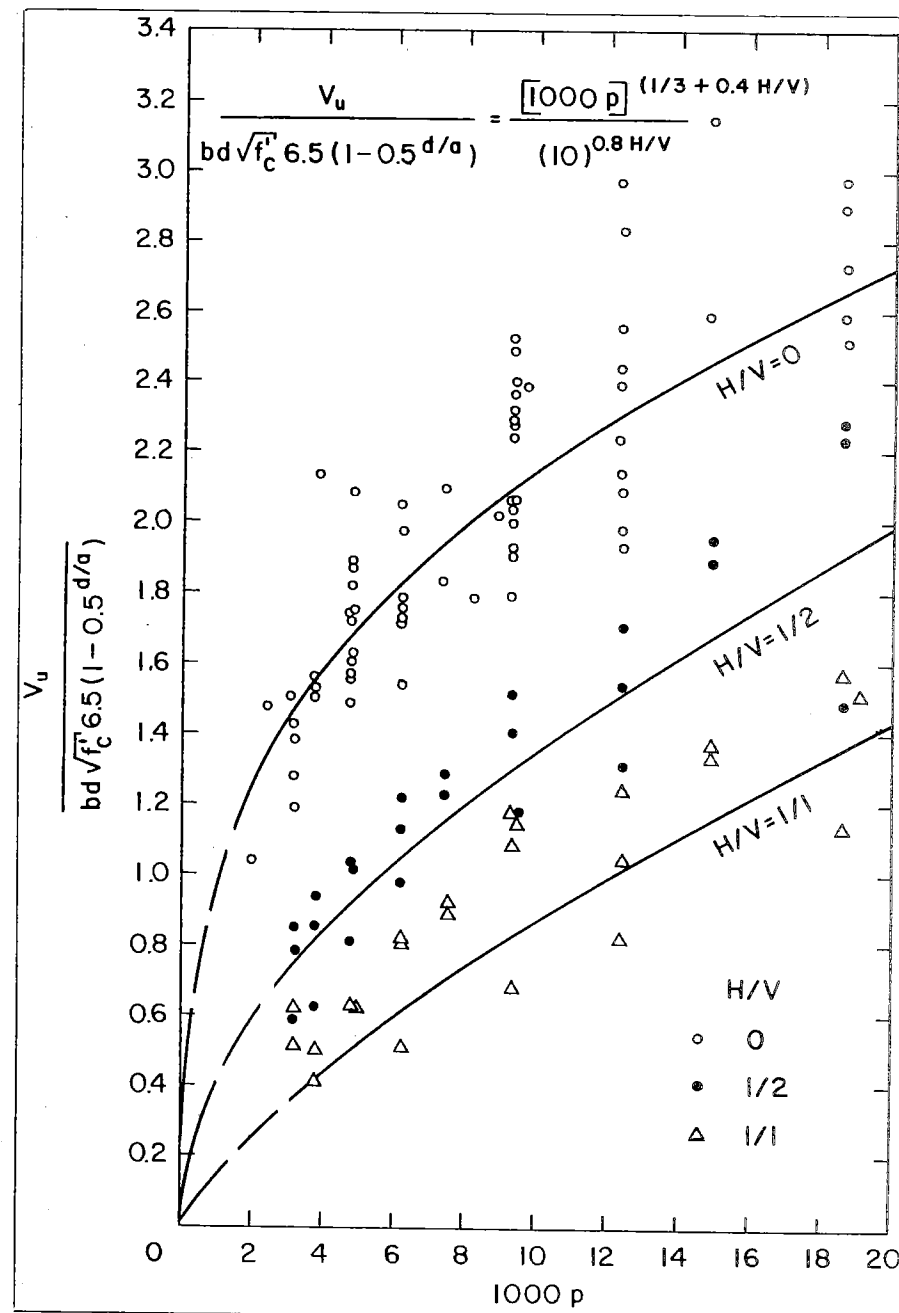


Fig. 18—Ultimate Strength of Corbels

## Series (c)—Corbels Subject to Combined Vertical and Horizontal Loads

### Discussion of Behavior

The addition of outward horizontal forces to the vertical loads does not change the essential characteristics of behavior, which can still be represented by the idealized diagram of Fig. 17. However, the functions for  $V_0$  and  $m$  must be modified to account for the lower values of the nominal cracking load  $V_0$  and of the slope  $m$  observed in data from tests of corbels subject to combined loading. Typical relationships be-

tween applied load and tension reinforcement force for corbels subjected to combined loading are shown in Fig. 19.

The function for  $v$  derived from the data shown in Fig. 19, and from other similar data not presented here, takes the form:

$$v = \frac{V}{bd} = \frac{4.4}{(a/d)^{1/2}} \left( \frac{f'_c}{a/d} \right)^{1/3} C_1 + \frac{2}{3} \sqrt{\frac{f'_c}{1000}} \frac{pf_s}{C_2} \left( 1 + \frac{2}{3} \frac{H}{V} \sqrt{\frac{f'_c}{1000}} \right) \quad (8)$$

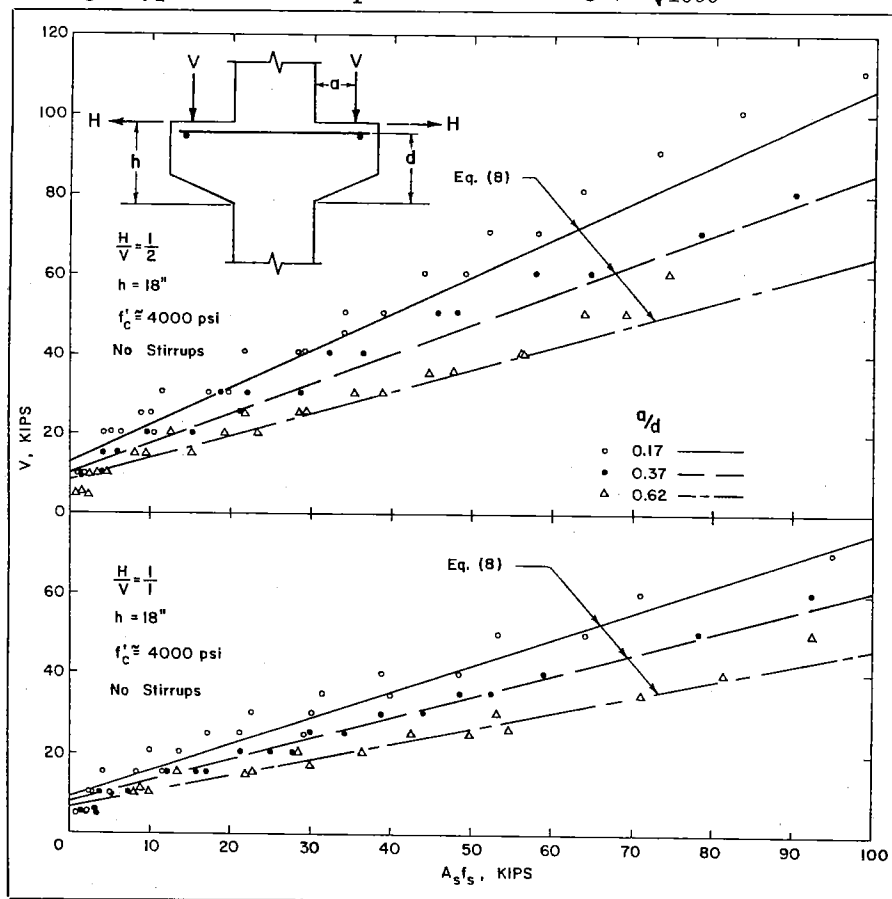


Fig. 19—Relationship Between Applied Load and Tension Steel Force, Combined Vertical and Horizontal Loading

where  $C_1 = 1.5 (a/d)^{2/3}$ , and  $C_2 = 0.7 (10)^{a/2d}$ , whether stirrups are present or not. Eq. (8) reduces to Eq. (4) when  $H/V = 0$ , i.e., for vertical load only. However, it should be noted that coefficients  $C_1$  and  $C_2$  must then be as defined earlier for Eq. (4).

Eq. (8) has been used to calculate the nominal shear stress,  $v_y$ , at yield of the tension reinforcement in those corbels tested in which yield of the tension reinforcement occurred. The results are given in Table A3. The average value of  $(v_y \text{ test}/v_y \text{ calc})$  was 1.04 for  $H/V = 1/2$ , and 0.92 for  $H/V = 1/4$ , the standard deviations being 0.088 and 0.084 respectively.

As before, by equating Eq. (8) to the nominal shear stress at ultimate strength,  $v_u$ , and transposing, the reinforcement stress at ultimate strength,  $f_{su}$  can be determined.

$$f_{su} = \left[ v_u \left\{ 1 + \frac{2}{3} \frac{H}{V} \sqrt{\frac{f'_c}{1000}} \right\} - \frac{4.4}{(a/d)^{1/2}} \left( \frac{f'_c}{a/d} \right)^{1/3} C_1 \right] \frac{1.5 C_2}{p \sqrt{f'_c/1000}} \quad (9)$$

where  $C_1$  and  $C_2$  are as defined for Eq. (8) above, and  $v_u$  is obtained from Eq. (10) below. Values of  $C_1$  and  $C_2$  are also listed in Tables A9 and A10 appended to this report.

For purposes of practical design, yield of the tension reinforcement may again be accounted for by the use of load factors  $1/2$  greater than those specified for individual members.

### Ultimate Strength

The principles used in the derivation of the ultimate strength equation for corbels subjected to vertical loads only apply also to the derivation of an ultimate strength equation for corbels subject to combined

horizontal and vertical loads. The ultimate strength  $V_u$  must again be proportional to  $b$  and  $d$ , and it may be assumed that it is also proportional to  $\sqrt{f'_c}$ . The ratio  $V_u/bd \sqrt{f'_c}$  is then a function of  $a/d$ ,  $p$  and  $H/V$ , which should reduce to Eq. (7) when  $H/V = 0$ , i.e. for vertical loads only. The following equation was established after study of the test data, having in mind the above requirements.

$$v_u = \frac{V_u}{bd} = 6.5 \sqrt{f'_c} (1 - 0.5^{d/a}) \frac{(1000p)^{(1/3 + 0.4H/V)}}{(10)^{0.8H/V}} \quad (10)$$

Eq. (10) was used to calculate the nominal shear stress at ultimate strength for all corbels subjected to combined vertical and horizontal loads, and the ultimate shear stresses so calculated are set out in Table A3. Eq. (2) of the proposed design criteria is based on Eq. (10). Excluding those specimens which experienced secondary failures (i.e., by bearing or by splitting off of the corbel ends), the average value of  $(v_u \text{ test}/v_u \text{ calc})$  was 1.05 for  $H/V = 1/2$ , and 1.21 for  $H/V = 1/4$ , the standard deviation being 0.132 and 0.216, respectively.

The appearance of typical corbels after failure under combined loading is shown in Figs. 20 and 21.

A limited number of corbels with stirrups were tested under combined loading, and the results are given in Table A4. It was found that the stirrups did not increase the resistance of a corbel to combined loading by as large a proportion as was the case with a corbel subject to vertical load only. Also, the contribution of the stirrups was more erratic, viz. corbels 13S and 14S with 0.34% and

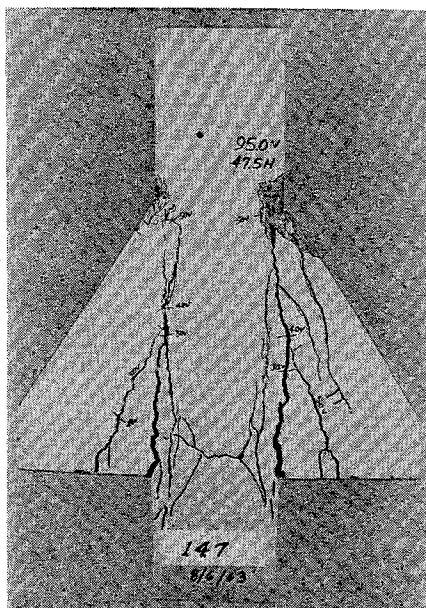


Fig. 20—Flexural Yielding Failure Followed by Crushing of the Concrete (FT),  $H/V = 1$

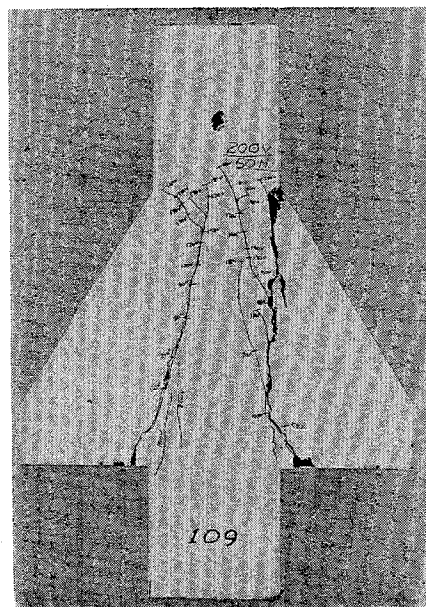


Fig. 21—Shear Failure (S),  $H/V = 1/2$

0.93% of stirrup steel, respectively, and all else the same, gave ultimate shear stresses of 260 and 273 psi. The effectiveness of the stirrups is also apparently a function of the  $a/d$  ratio and of the  $H/V$  ratio. A considerable program of tests would be necessary to assess the influence of the various factors which apparently influence the effectiveness of stirrups in a corbel subject to combined loading. For the present it was decided that any contribution from the stirrups should be regarded as reserve strength, and should not be taken into account in design. Stirrups do lead to a more ductile form of failure, and hence it was concluded that a minimum amount of stirrups should always be provided.

### Secondary Failures

The following comments apply to both vertical load only and to combined vertical and horizontal loading.

**Corbel End Failure**—In certain of the tests the depth of the outer face of the corbel was deliberately varied in order to determine the minimum depth necessary to prevent the occurrence of a secondary failure by splitting away of a portion of the concrete at the tip of the corbel. It was found that this type of failure, as shown by Fig. 22, did not occur in those corbels having a depth below the outer edge of the bearing plate greater than about 0.5 the depth of the corbel at the face of the column.

**Bearing Failure**—Crushing of the concrete below the bearing plate occurred in some of the tests. The bearing stress,  $f_{bu}$ , at ultimate strength of the corbels is listed in Table A6. Bearing failures occurred

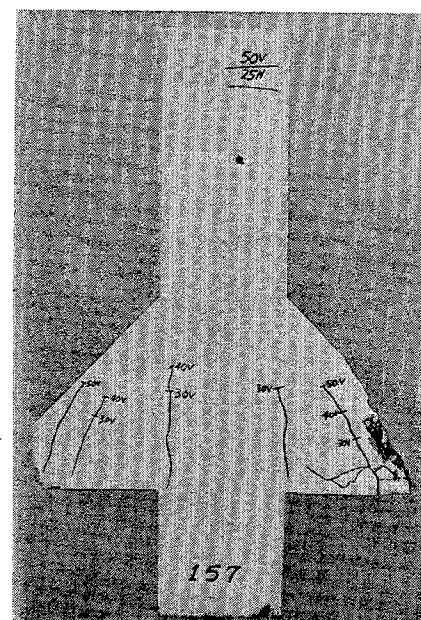


Fig. 22—Corbel End Failure (CE),  $H/V = 1$

at stresses as low as  $0.34f'_c$  when the load was applied near the outer edge of the corbel in a combined loading test. However, if the outer edge of the bearing plate was at least 2 in. from the outer face of the corbel, then bearing failures did not occur at bearing stresses less than  $0.5f'_c$ . A detailed study of bearing stresses was not made. It is believed that  $0.5f'_c$  is a suitably conservative value.

### CONCLUDING REMARKS

The experimental evidence presented in this paper indicates that the nominal ultimate shear stress,  $v_u$ , in corbels with a shear span to effective depth ratio less than one may exceed the maximum shear stress allowed by Chapter 17 of the ACI Code (ACI 318-63) for beams with  $a/d$  ratio greater than one.

The nominal ultimate shear stress in a corbel is a function of the ratio of the shear span to the effective

depth, of the reinforcement ratio, of the concrete strength, and of the ratio of the horizontal and vertical components of the applied loads.

Horizontal forces acting outward from the column significantly reduce corbel strength, and must be considered in the design of a corbel unless special provisions are made for free movements of the supported beams.

Tension reinforcement and horizontal stirrups are equally effective in increasing the strength of a corbel subject to vertical loads only. However, the effective amount of reinforcement is limited.

Loads carried by a column do not affect the corbel strength, nor does the amount or arrangement of column reinforcement.

The results of this investigation have been used as a basis for the formulation of "Proposed Criteria for the Design of Corbels" which is presented in Part 1 of this paper.

### ACKNOWLEDGMENTS

The work described herein was carried out in the Structural Laboratory of the Portland Cement Association under the direction of Eivind Hognestad and Alan H. Mattock. Contributions were made by several members of the laboratory staff. Particular credit is due Bernard J. Doepp, William Hummerich, Jr., David C. Yates and Kenneth Hirte for the laboratory work involved.

### NOTATION

The notation of the ACI Building Code (ACI 318-63) is used wherever applicable. The letter symbols used in this paper are defined below:

- $A_s$  = area of tension reinforcement, in.<sup>2</sup>
- $A_v$  = area of horizontal stirrups, in.<sup>2</sup>

$a$  = shear span measured from the face of the column to the resultant of applied load, in.  
 $b$  = width of corbel, in.  
 $d$  = effective depth of the centroid of tension reinforcement at the column face, in.  
 $f_{bu}$  = bearing stress at ultimate strength, psi  
 $f_s$  = stress in tension reinforcement, psi  
 $f_{su}$  = stress in tension reinforcement at ultimate strength, psi  
 $f_v$  = stress in horizontal stirrups, psi  
 $f_y$  = yield stress of reinforcement, psi  
 $f'_c$  = concrete cylinder strength, psi  
 $\sqrt{f'_c}$  = relationship expressed in psi, so that  $\sqrt{f'_c} = 60$  psi for  $f'_c = 3600$  psi  
 $H/V$  = ratio of horizontal and vertical components of applied loads  
 $h$  = over-all depth of corbel at column face, in.  
 $h'$  = depth of corbel outer face, in.  
 $n$  = number of horizontal closed stirrups  
 $p$  = reinforcement ratio =  $(A_s + A_v)/bd$  when  $H/V = 0$  =  $A_s/bd$  when  $H/V$  does not equal zero.  
 $s$  = center to center spacing of stirrups, in.  
 $V$  = applied vertical load, lb  
 $v$  = nominal shearing stress =  $\frac{V}{bd}$ , psi  
 $V_0$  = nominal cracking load, lb  
 $V_u$  = ultimate vertical load, lb  
 $v_u$  = nominal ultimate shearing stress =  $V_u/bd$ , psi

$V_y$  = vertical load at initial yielding of tension reinforcement, lb  
 $v_y$  = nominal shearing stress at initial yielding of tension reinforcement =  $V_y/bd$ , psi  
 $w$  = width of bearing plates, in.  
 $\phi$  = capacity reduction factor

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Table A1—Exploratory Test Results

No.	Type	b, in.	h', in.	a, in.	d, in.	P, %	a/d	f' <sub>c</sub> , psi	f <sub>y</sub> ', ksi	f <sub>su</sub> test, ksi	f <sub>y</sub> test, psi	v <sub>y</sub> calc, psi	v <sub>y</sub> test, psi	v <sub>y</sub> test/v <sub>y</sub> calc	v <sub>u</sub> test/ v <sub>u</sub> calc	v <sub>u</sub> test/ v <sub>u</sub> calc
Effect of Reinforcing Details																
42 <sup>a</sup>	WC	26	11	9.5	24.1	0.62	0.249	4850	52.5	*	*	605	778	689	-	1.13
12 <sup>a</sup>	BI	26	11	9.5	24.1	0.62	0.249	4190	48.5	34.0	-	539	429	640	-	0.67
22 <sup>a</sup>	WI	26	11	9.5	24.1	0.62	0.249	4440	44.9	44.9	726	520	751	659	1.40	1.01
29	WC	26	26	6.0	24.1	0.62	0.249	3730	47.5	45.3	-	611	640	684	-	0.94
32 <sup>a</sup>	BI	26	12	6.0	24.1	0.62	0.249	3980	43.0	43.0	648	480	648	624	1.35	1.00
42 <sup>a</sup>	WI	26	12	6.0	24.1	0.62	0.249	4200	44.9	44.9	726	507	755	641	1.43	0.98
Effect of Additional Column Load																
4	WC	18	9	2.75	16.1	0.93	0.171	3520	43.6	43.6	777	-	777	797	-	1.00
52 <sup>a</sup>	CL 100%	18	9	2.75	16.1	0.93	0.171	4010	44.5	*	*	-	889	851	-	1.07
15	WC	18	6	6	16.2	0.48	0.370	4500	48.1	48.1	405	472	556	622	0.86	1.00
62	CL 75%	18	6	6	16.2	0.48	0.370	4140	48.1	48.1	478	456	493	597	1.05	0.92
24	WC	18	9	6	16.1	0.93	0.372	4250	47.3	42.5	-	731	691	753	-	1.00
72 <sup>a</sup>	CL 100%	18	9	6	16.1	0.93	0.372	4490	44.5	44.5	770	716	850	774	1.08	1.20
Effect of Column Reinforcement																
82 <sup>a</sup>	WC	26	16	9.5	24.1	0.62	0.394	4580	46.5	*	*	540	726	669	-	1.08
92 <sup>a</sup>	CR 5-#9	26	16	9.5	24.1	0.62	0.394	4790	53.3	*	*	608	772	685	-	1.13
102 <sup>a</sup>	CR 6-#9	26	16	9.5	24.1	0.62	0.394	4750	46.5	*	*	549	659	681	-	0.97

NOTES:

λ = 12 in. and b = 8 in. for all specimens

WC = welded cross-bar tension reinforcement

BI = inclined bar formed by bending tension reinforcement

WI = inclined bar welded to WC tension reinforcement

CL-% additional column load, % indicates ratio of column load to corbel load

CR = No. Following CR indicates reinforcing bars in 8x12" column

λ v = 5 in. (in all other cases v = 3 in.)

Δ column failed

\* not measured or inconclusive test data

Table A2—Test Results for Vertical Load Series

No.	b, in.	h', in.	a, in.	d, in.	P, %	a/d	f' <sub>c</sub> , psi	f <sub>y</sub> ', ksi	f <sub>su</sub> test, ksi	f <sub>y</sub> test, psi	v <sub>y</sub> calc, psi	v <sub>y</sub> test, psi	v <sub>y</sub> test/v <sub>y</sub> calc	v <sub>u</sub> test/v <sub>u</sub> calc	v <sub>u</sub> test/v <sub>u</sub> calc	Type Failures
1	22	6	2.75	20.2	0.38	0.136	3790	45.3	45.3	619	616	619	623	1.00	0.99	S
2 <sup>a</sup>	22	6	2.75	20.2	0.38	0.136	6170	47.0	47.0	990	761	1090	792	1.30	1.38	S
3	26	6	2.75	24.2	0.32	0.114	3820	45.3	45.3	529	4	563	591	-	0.95	S
4	18	6	2.75	16.1	0.93	0.171	3520	43.6	43.6	777	4	777	797	-	0.97	S
5	26	6	2.75	20.1	0.75	0.137	3840	43.3	43.3	808	4	849	782	-	1.08	S
6	26	6	2.75	24.1	0.62	0.114	3970	47.0	47.0	687	4	715	751	-	0.95	S
7	18	6	2.75	16.1	1.86	0.171	3260	45.3	29.4	-	1090	967	-	1.13	1.06	S
8	22	6	2.75	20.1	1.49	0.138	4170	45.8	45.8	1060	4	1090	1030	-	1.06	S
9 <sup>a</sup>	26	6	2.75	20.1	1.49	0.138	6500	45.0	45.0	1650	4	1650	1280	-	1.29	S
10	26	6	2.75	24.1	0.124	0.114	4790	47.0	47.0	898	4	898	1040	-	0.86	S
11 <sup>a</sup>	14	8	4.0	12.1	1.24	0.330	3900	47.7	47.7	940	4	954	826	-	1.15	DS
12	18	6	6.0	16.2	0.31	0.370	4240	51.0	51.0	385	366	544	521	1.05	1.04	FT
13	22	6	6.0	20.2	0.25	0.298	4580	51.0	51.0	371	380	594	536	0.98	1.11	FT
14	26	6	6.0	24.2	0.21	0.248	4540	51.0	51.0	310	388	434	524	0.80	0.83	DS
15	18	6	6.0	16.2	0.48	0.370	4500	48.1	48.1	405	472	556	622	0.86	0.89	DS
16 <sup>a</sup>	18	6	6.0	16.2	0.48	0.370	3430	48.0	48.0	463	419	606	543	1.10	1.12	DS
17	18	6	6.0	16.2	0.48	0.370	3990	95.8	72.2	-	4	660	585	-	1.15	DS
18	18	18	6.0	16.2	0.48	0.370	4210	47.3	47.3	556	454	625	601	1.22	1.04	S
19	18	6	6.0	20.2	0.38	0.297	3790	45.2	45.2	326	405	572	565	1.30	1.01	DS
20	22	6	6.0	20.2	0.38	0.297	3590	95.8	67.8	-	4	533	547	-	0.97	DS
21	26	6	6.0	24.2	0.32	0.248	3920	43.2	43.2	426	411	491	563	1.04	0.87	DS
22	26	6	6.0	24.2	0.32	0.248	3740	95.8	66.2	-	4	542	549	-	0.99	DS
23	26	26	6.0	24.2	0.32	0.248	3950	45.0	43.9	-	420	457	566	-	0.81	DS
24	18	6	6.0	16.1	0.93	0.372	4250	47.3	42.5	893	731	691	753	-	0.92	DS
25	18	6	6.0	16.1	0.93	0.372	6410	46.6	46.6	875	675	1010	985	1.02	1.09	DS
26	18	18	6.0	16.1	0.93	0.372	4280	53.3	53.3	899	4	859	755	-	1.14	S
27	22	6	6.0	20.1	0.75	0.298	4320	47.3	47.3	653	683	715	733	0.96	0.95	DS
28	26	6	6.0	24.1	0.62	0.249	4630	47.3	47.3	648	670	648	762	0.97	0.85	DS
29	26	26	6.0	24.1	0.62	0.249	3730	47.3	45.3	-	611	640	684	-	0.94	S
30	22	6	6.0	20.0	0.99	0.300	4260	45.6	45.6	844	810	844	820	1.04	1.03	DS
31	26	6	6.0	24.0	0.82	0.250	4040	46.6	46.6	781	753	782	782	1.04	1.00	DS
32	26	6	6.0	24.0	0.82	0.250	4390	45.6	45.6	716	770	729	814	0.93	0.89	DS
33	18	6	6.0	16.1	1.86	0.372	3830	47.3	27.2	-	4	885	900	-	0.98	DS
34	18	18	6.0	16.1	1.86	0.372	4070	53.3	34.0	-	4	959	928	-	1.05	DS
35	26	6	6.0	20.1	1.49	0.298	3820	47.3	*	*	4	822	882	-	(0.92)	B
36	26	6	6.0	24.1	1.24	0.249	3960	47.3	*	*	4	804	889	-	0.90	DS
37	26	26	6.0	24.1	1.24	0.249	3770	54.3	28.3	-	4	809	867	-	0.93	S
38 <sup>a</sup>	18	9	9.5	16.1	0.93	0.590	4700	53.0	*	*	4	664	647	-	1.03	DS
39 <sup>a</sup>	18	9	9.5	16.1	0.93	0.590	4490	54.5	*	*	4	674	632	-	1.07	DS
40 <sup>a</sup>	18	12	9.5	16.1	0.93	0.590	4340	44.3	44.3	660	567	675	622	1.16	1.08	FC

Table A2—Test Results for Vertical Load Series (continued)

No.	h, in.	h', in.	a, in.	d, in.	p, %	$\frac{a}{d}$	f', ksi	f <sub>y</sub> , ksi	f <sub>su</sub> test, ksi	v <sub>test</sub> , psi	v <sub>calc</sub> , psi	$\frac{v_{test}}{v_{calc}}$	v <sub>test</sub> , psi	v <sub>calc</sub> , psi	$\frac{v_{test}}{v_{calc}}$	Type Failures
41 <sup>1</sup>	18	18	9.5	16.1	0.93	0.590	4200	44.4	44.4	606	560	1.08	612	606	0.99	DS
42 <sup>1</sup>	26	11	9.5	24.1	0.62	0.394	4850	52.5	*	778	605	1.13	689	778	0.98	S
43 <sup>1</sup>	26	16	9.5	24.1	0.62	0.394	4140	45.7	45.7	622	510	1.22	636	622	0.98	S
44 <sup>1</sup>	26	26	9.5	24.1	0.62	0.394	3840	45.4	45.4	581	491	1.18	613	581	0.95	DS
45 <sup>1</sup>	18	9	9.5	16.1	1.86	0.590	4280	50.5	*	932	780	1.19	780	932	1.19	DS
46 <sup>1</sup>	18	12	9.5	16.1	1.86	0.590	3840	44.3	32.5	814	758	1.10	758	814	1.10	DS
47 <sup>1</sup>	18	18	9.5	16.1	1.86	0.590	4060	44.4	36.8	811	759	1.07	759	811	0.97	DS
48 <sup>1</sup>	26	11	9.5	24.1	1.24	0.394	4280	45.4	*	716	807	1.08	807	716	0.88	CE
49 <sup>1</sup>	26	11	9.5	24.1	1.24	0.394	4180	48.0	*	716	807	1.08	807	716	0.89	B
50 <sup>1</sup>	26	26	9.5	24.1	1.24	0.394	4590	45.4	21.3	477	826	1.08	826	477	0.58	B
51 <sup>1</sup>	26	26	9.5	24.1	1.24	0.394	4490	45.7	22.7	563	835	1.08	835	563	0.67	B
52 <sup>1</sup>	18	6	10.0	16.2	0.48	0.617	3960	44.3	44.3	370	323	1.04	370	370	0.79	B
53 <sup>1</sup>	18	6	10.0	16.2	0.48	0.617	3560	44.3	44.3	420	588	0.87	588	420	0.71	B
54 <sup>1</sup>	18	18	10.0	16.2	0.48	0.617	3950	45.0	45.0	347	328	1.06	347	347	0.75	B
55 <sup>1</sup>	22	6	10.0	20.2	0.38	0.495	4010	45.3	45.3	355	324	1.10	355	374	0.77	CE
56 <sup>1</sup>	26	6	10.0	24.2	0.32	0.413	3770	45.3	45.3	301	314	0.96	301	301	0.65	CE
57 <sup>1</sup>	26	26	10.0	24.2	0.32	0.413	4130	47.5	47.5	337	335	1.00	499	435	0.87	B
58 <sup>1</sup>	18	6	10.0	16.1	0.93	0.621	3720	44.6	34.6	424	561	0.78	561	424	0.79	CE
59 <sup>1</sup>	18	6	10.0	16.1	0.93	0.621	3510	43.3	34.0	490	545	0.79	545	490	0.79	CE
60 <sup>1</sup>	18	12	10.0	16.1	0.93	0.621	3880	44.3	43.9	620	568	1.09	568	620	1.09	FC
61 <sup>1</sup>	18	18	10.0	16.1	0.93	0.621	4110	54.3	46.7	583	589	0.99	589	583	0.99	DS
62 <sup>1</sup>	22	6	10.0	20.1	0.75	0.497	3260	43.6	36.1	436	545	0.80	545	436	0.80	CE
63 <sup>1</sup>	26	6	10.0	24.1	0.62	0.415	3420	43.6	31.8	442	339	0.60	339	442	0.60	CE
64 <sup>1</sup>	26	6	10.0	24.1	0.62	0.415	3540	46.6	41.0	621	493	0.63	493	621	0.63	CE
65 <sup>1</sup>	26	16	10.0	24.1	0.62	0.415	3660	53.2	39.6	525	519	0.88	587	519	0.88	B
66 <sup>1</sup>	26	26	10.0	24.1	0.62	0.415	4040	44.1	38.2	480	481	0.78	617	481	0.78	B
67 <sup>1</sup>	26	26	10.0	24.1	0.62	0.415	4060	52.8	34.7	547	458	0.74	618	458	0.74	B
68 <sup>1</sup>	18	6	10.0	16.1	1.86	0.621	3380	43.0	23.4	418	673	0.62	673	418	0.62	CE
69 <sup>1</sup>	18	12	10.0	16.1	1.86	0.621	3680	44.3	29.7	668	702	0.95	702	668	0.95	DS
70 <sup>1</sup>	18	18	10.0	16.1	1.86	0.621	4010	53.3	25.2	621	733	0.85	733	621	0.85	S
71 <sup>1</sup>	22	6	10.0	20.1	1.49	0.497	4410	45.6	27.6	491	799	0.61	799	491	0.61	CE
72 <sup>1</sup>	26	6	10.0	24.1	1.24	0.415	4110	45.6	26.8	491	799	0.61	799	491	0.61	CE
73 <sup>1</sup>	26	16	10.0	24.1	1.24	0.415	4050	44.1	24.1	392	784	0.50	784	392	0.50	CE
74 <sup>1</sup>	26	26	10.0	24.1	1.24	0.415	4360	52.7	26.3	570	778	0.72	778	570	0.72	B
75 <sup>1</sup>	45	12	12.5	41.7	0.95	0.300	4110	45.4	37.4	599	807	0.81	807	599	0.81	DS
76 <sup>1</sup>	45	12	12.5	41.7	0.95	0.300	4090	46.7	35.3	770	794	0.81	794	770	0.81	DS
77 <sup>1</sup>	26	6	3.5	24.2	0.48	0.144	2210	45.3	34.6	785	749	0.94	792	749	0.94	DS
78 <sup>1</sup>	26	6	3.5	24.2	0.48	0.144	2210	45.3	34.6	785	749	0.94	792	749	0.94	DS
79 <sup>1</sup>	26	6	3.5	24.1	0.97	0.145	2200	44.3	39.6	546	636	0.86	636	546	0.86	S
80 <sup>1</sup>	26	6	3.5	24.1	1.24	0.145	2400	47.3	34.0	517	732	0.71	732	517	0.71	B
81 <sup>1</sup>	22	6	6.0	20.2	0.49	0.297	2430	43.5	43.5	365	515	1.04	493	365	1.04	FC

Table A2—Test Results for Vertical Load Series (concluded)

No.	h, in.	h', in.	a, in.	d, in.	p, %	$\frac{a}{d}$	f', ksi	f <sub>y</sub> , ksi	f <sub>su</sub> test, ksi	v <sub>test</sub> , psi	v <sub>calc</sub> , psi	$\frac{v_{test}}{v_{calc}}$	v <sub>test</sub> , psi	v <sub>calc</sub> , psi	$\frac{v_{test}}{v_{calc}}$	Type Failures
81 <sup>1</sup>	22	6	6.0	20.1	0.94	0.298	2570	44.6	44.6	672	610	1.10	627	672	1.07	FC
82 <sup>1</sup>	18	6	4.75	16.0	1.23	0.297	2110	45.1	36.3	659	478	1.06	623	659	1.06	FC
83 <sup>1</sup>	18	18	8.5	16.2	0.48	0.525	2310	45.8	45.8	397	386	1.03	386	397	1.03	FC
84 <sup>1</sup>	18	18	8.5	16.1	0.93	0.588	2290	47.3	42.5	470	543	1.14	478	470	1.14	FC
85 <sup>1</sup>	18	18	8.5	16.0	1.23	0.571	2170	44.6	31.1	405	510	0.97	510	405	0.97	FC
86 <sup>1</sup>	26	6	3.5	24.2	0.48	0.144	4180	46.3	46.3	804	878	1.19	703	804	1.19	S
87 <sup>1</sup>	26	6	3.5	24.1	0.93	0.145	3880	44.3	38.1	804	878	1.19	703	804	1.19	S
88 <sup>1</sup>	26	26	3.5	24.1	1.24	0.145	3880	47.5	34.0	804	878	1.19	703	804	1.19	S
89 <sup>1</sup>	22	6	6.0	20.2	0.94	0.297	4010	44.8	44.8	577	681	1.14	634	577	1.14	FC
90 <sup>1</sup>	22	6	6.0	20.1	0.94	0.298	4240	46.5	46.5	964	791	1.22	805	967	1.22	S
91 <sup>1</sup>	18	6	4.75	16.0	1.23	0.297	4060	46.7	45.3	961	864	1.11	864	961	1.11	S
92 <sup>1</sup>	18	18	8.5	16.2	0.48	0.525	4160	45.8	45.8	497	518	0.94	518	497	0.94	DS
93 <sup>1</sup>	18	18	8.5	16.1	0.93	0.588	3980	47.5	47.5	699	610	1.14	610	699	1.14	DS
94 <sup>1</sup>	18	18	8.5	16.0	1.23	0.571	3940	46.7	42.5	888	687	1.29	687	888	1.29	DS
95 <sup>1</sup>	26	6	3.5	24.2	0.48	0.144	6310	45.3	45.3	815	864	1.01	864	815	1.01	S
96 <sup>1</sup>	26	6	3.5	24.1	0.93	0.145	6430	46.5	46.5	1300	1090	1.19	1090	1300	1.19	S
97 <sup>1</sup>	26	26	3.5	24.1	1.24	0.145	6420	44.3	32.5	1110	1200	0.93	1200	1110	0.93	S
98 <sup>1</sup>	22	6	6.0	20.2	0.49	0.297	6610	44.3	44.3	619	603	1.03	615	619	1.03	DS
99 <sup>1</sup>	22	6	6.0	20.1	0.94	0.298	6570	46.5	46.5	902	967	1.14	1000	902	1.14	DS
100 <sup>1</sup>	18	6	4.75	16.0	1.23	0.297	6430	47.5	47.5	1340	1090	1.23	1090	1340	1.23	S
101 <sup>1</sup>	18	18	8.5	16.2	0.48	0.525	6370	46.3	46.3	602	641	0.85	641	602	0.85	FC
102 <sup>1</sup>	18	18	8.5	16.1	0.93	0.588	6680	46.5	46.5	754	817	0.91	817	754	0.91	DS
103 <sup>1</sup>	18	18	8.5	16.0	1.23	0.571	6590	47.5	47.5	922	889	1.04	889	922	1.04	DS

NOTES:

Types of Failure (see Figs. 13 to 16)

B - Bearing

CE - Corbel End, crack intersecting inclined face

DS - Diagonal Splitting

FC - Flexural Compression

FT - Flexural Tension

S - Shear

f' = 12 in. and b = 8 in. for all specimens unless otherwise noted

\* not measured or inconclusive test data

For 39 specimens Avg.  $\frac{v_{test}}{v_{calc}} = 1.05$ , Standard Deviation = 0.135;For 70 specimens Avg.  $\frac{v_{test}}{v_{calc}} = 1.02$ , Standard Deviation = 0.119. (Failure types B and CE excluded.)

A v = 5 in. (in all other cases v = 3 in.)

B f' = 6 in. and b = 16 in.

S f' = 24 in.

A f' calculated smaller than f<sub>y</sub>

Table A3—Test Results for Combined Load Series  
( $H/V = 1/2$ )

No.	$h$ , in.	$h'$ , in.	$a$ , in.	$e$ , in.	$d$ , in.	$P$ , %	$\frac{a}{d}$	$f'_c$ , ksi	$f_y$ , ksi	$f_u$ , ksi	$v_y$ , psi	$v_u$ , psi	$v_{u,calc}$ , psi	$\frac{v_y}{v_{u,calc}}$	$\frac{v_{u,calc}}{v_y}$	Type Failures
104	18	6	2.75	16.2	0.48	0.170	4210	45.7	45.7	309	294	434	380	1.05	1.14	S
105	22	6	2.75	20.2	0.38	0.136	3860	45.7	45.7	278	257	364	427	1.08	1.17	S
106	26	6	2.75	24.2	0.32	0.114	4040	47.3	47.3	258	243	358	305	1.06	1.17	S
107	18	6	2.75	16.1	0.93	0.171	4080	48.5	48.5	343	311	621	534	1.06	1.16	S
108	22	6	2.75	20.1	0.75	0.137	3860	47.7	47.7	463	431	515	446	1.07	1.10	S
109	26	6	2.75	24.1	0.62	0.114	4240	48.2	48.2	441	397	519	445	1.11	1.16	S
110	18	6	2.75	16.1	1.86	0.171	4250	47.5	47.5	932	793	932	788	-	1.18	S
111	22	6	2.75	20.1	1.49	0.137	3900	48.8	48.8	793	726	793	678	-	1.17	S
112	26	6	2.75	24.1	1.24	0.114	4310	48.7	48.7	726	650	726	650	-	1.12	S
113	18	6	6.0	16.2	0.47	0.370	4400	46.5	46.5	270	246	376	334	1.10	1.12	FT
114	22	6	6.0	20.2	0.38	0.297	4320	45.7	45.7	248	223	334	314	1.11	1.06	FT
115	26	6	6.0	24.2	0.32	0.248	4950	45.7	45.7	207	215	339	318	0.96	1.06	FT
116	18	6	6.0	16.1	0.93	0.372	3870	48.3	48.3	483	405	483	446	1.19	1.03	S
117	22	6	6.0	20.1	0.75	0.298	3880	44.7	44.7	404	344	451	424	1.17	1.06	S
118	26	6	6.0	24.1	0.62	0.249	4240	48.4	48.4	363	343	454	419	1.06	1.08	S
119	18	6	6.0	16.1	1.86	0.171	4210	48.5	48.5	776	700	815	674	-	1.21	S
120	22	6	6.0	20.1	1.49	0.298	4130	47.7	47.7	700	596	715	634	-	1.13	S
121	26	6	6.0	24.1	1.24	0.249	3970	48.2	48.2	596	518	596	586	-	1.02	S
122	18	6	10.0	16.2	0.48	0.617	3380	46.5	46.5	174	185	211	234	0.94	0.90	DS
123	22	6	10.0	20.2	0.38	0.495	4240	46.5	46.5	178	185	209	260	0.94	0.80	DS
124	26	6	10.0	24.2	0.32	0.413	4240	46.5	46.5	155	181	207	255	0.86	0.81	FT
125	18	6	10.0	16.1	0.93	0.621	3250	47.9	51.1	-	301	236	326	-	(0.72)	CE
126	22	6	10.0	20.1	0.75	0.621	4480	53.4	53.4	344	357	344	363	0.96	0.90	DS
127	26	6	10.0	24.1	0.62	0.497	3300	47.9	39.6	-	286	244	326	-	(0.83)	CE
128	18	6	10.0	16.1	0.93	0.415	3610	48.0	39.6	-	277	244	334	-	(0.73)	CE
129	22	6	10.0	20.1	0.62	0.415	4120	47.0	47.0	285	282	337	357	1.01	0.94	S
130	26	6	10.0	24.1	0.62	0.415	3930	47.7	22.7	-	324	357	518	-	(0.62)	CE
131	18	6	10.0	16.1	1.86	0.621	4220	45.0	31.0	-	505	426	537	-	0.79	DS
132	22	6	10.0	20.1	1.49	0.497	4120	44.7	26.2	-	505	280	527	-	(0.53)	CE
133	26	6	10.0	24.1	1.24	0.415	4180	43.4	35.4	-	469	279	521	-	(0.54)	CE
134	26	26	10.0	24.1	1.24	0.415	4290	43.9	38.4	-	469	458	528	-	0.87	DS

For 17 specimens Avg.  $v_y$  test/ $v_{u,calc}$  = 1.04, Standard Deviation = 0.088;  
For 25 specimens Avg.  $v_u$  test/ $v_{u,calc}$  = 1.05, Standard Deviation = 0.132.

Table A3—Test Results for Combined Load Series (continued)  
( $H/V = 1/1$ )

No.	h, in.	h', in.	a, in.	d, in.	P, %	$\frac{a}{d}$	$f'_c$ , psi	$f_y$ , ksi	$f_u$ test, ksi	$v_y$ test, psi	$v_u$ test, psi	$v_{u,calc}$ , psi	$\frac{v_y}{v_{u,calc}}$	$\frac{v_{u,calc}}{v_y}$	Type Failures	
136	18	6	2.75	16.2	0.48	0.170	3870	47.0	47.0	174	185	251	198	-	1.26	FT
137	22	6	2.75	20.2	0.38	0.136	4610	44.3	44.3	155	146	224	186	0.84	1.20	FT
138	26	6	2.75	24.2	0.32	0.114	3870	46.8	46.8	134	146	213	150	-	1.42	FT
139	18	6	2.75	16.1	0.93	0.171	4420	44.3	44.3	373	343	503	346	1.08	1.45	FT
140	22	6	2.75	20.1	0.75	0.137	3890	44.3	44.3	295	272	373	278	-	1.34	FT
141	26	6	2.75	24.1	0.62	0.114	4000	45.3	45.3	233	217	337	248	-	1.36	S
142	18	6	2.75	16.1	1.86	0.171	4270	44.3	44.3	660	660	660	564	-	1.17	S
143	22	6	2.75	20.1	1.49	0.137	4110	47.2	47.2	553	519	575	476	-	1.21	S
144	26	6	2.75	24.1	1.24	0.114	4250	48.8	48.8	519	483	532	425	-	1.25	S
145	18	6	6.0	16.2	0.48	0.370	3720	45.0	45.0	154	166	212	167	0.93	1.27	FT
146	22	6	6.0	20.2	0.38	0.297	4300	45.0	45.0	136	157	166	163	0.87	1.02	FT
147	26	6	6.0	24.2	0.32	0.248	4040	45.0	45.0	110	127	245	144	-	1.70	FT
148	18	6	6.0	16.1	0.93	0.372	4250	43.6	43.6	272	272	388	291	1.00	1.33	FT
149	22	6	6.0	20.1	0.75	0.298	4320	43.6	43.6	230	247	349	266	0.93	1.31	FT
150	26	6	6.0	24.1	0.62	0.249	4050	43.6	43.6	207	223	311	235	0.93	1.32	FT
151	18	6	6.0	16.1	1.86	0.372	4230	45.3	45.3	543	543	492	492	-	1.10	S
152	22	6	6.0	20.1	1.49	0.298	4130	48.5	48.5	482	453	513	433	-	1.18	S
153	26	6	6.0	24.1	1.24	0.249	3960	45.3	45.3	404	404	404	366	-	1.05	DS
154	18	6	10.0	16.2	0.48	0.617	4750	48.5	48.5	127	146	127	151	0.87	(0.84)	CE
155	22	6	10.0	20.2	0.38	0.495	4120	48.5	48.5	121	133	121	133	-	(0.90)	CE
156	26	6	10.0	24.2	0.32	0.413	3670	48.5	48.5	110	110	119	119	-	(0.92)	CE
157	18	6	10.0	16.1	0.93	0.621	4150	45.3	45.3	194	218	194	229	0.89	(0.85)	CE
158	22	9	10.0	16.1	0.93	0.621	4300	45.6	41.4	221	221	203	233	-	(0.87)	CE
159	18	18	10.0	16.1	0.93	0.621	4540	45.5	45.5	202	225	202	240	0.91*	0.84	DS
160	22	6	10.0	20.1	0.75	0.497	4200	45.3	45.3	155	208	155	219	0.75	(0.71)	CE
161	26	6	10.0	24.1	0.62	0.415	4090	45.3	45.3	130	195	142	204	0.66*	(0.70)	CE
162	26	11	10.0	24.1	0.62	0.415	4470	43.2	36.9	-	192	156	214	-	(0.73)	CE
163	26	26	10.0	24.1	0.62	0.415	4350	46.7	46.7	182	202	182	211	0.90	(0.86)	B
164	18	6	10.0	16.1	1.86	0.621	4080	48.3	38.2	-	378	272	378	-	(0.72)	CE
165	26	18	10.0	16.1	1.86	0.621	4520	45.4	38.4	-	396	337	397	-	0.85	DS
166	22	6	10.0	20.1	1.49	0.497	4110	42.5	42.5	212	344	212	360	0.62	(0.59)	CE
167	26	6	10.0	24.1	1.24	0.415	4440	42.5	39.6	-	326	192	554	-	(0.54)	CE
168	26	26	10.0	24.1	1.24	0.415	4550	46.7	46.7	272	355	295	358	0.77	0.82	DS

For 10 specimens Avg.  $v_y$  test/ $v_{u,calc}$  = 0.92, Standard Deviation = 0.084;  
For 21 specimens Avg.  $v_u$  test/ $v_{u,calc}$  = 1.21, Standard Deviation = 0.216.

For 10 specimens Avg.  $v_y$  test/ $v_{u,calc}$  = 0.92, Standard Deviation = 0.084;  
For 21 specimens Avg.  $v_u$  test/ $v_{u,calc}$  = 1.21, Standard Deviation = 0.216.

Table A3--Test Results for Combined Load Series (concluded)

No.	h, in.	h', in.	a, in.	d, in.	p, %	a/d	f' <sub>c</sub> , psi	f' <sub>y</sub> , ksi	f' <sub>su</sub> , ksi	f <sub>test</sub> , ksi	f <sub>test</sub> , psi	v <sub>calc</sub> , psi	v <sub>test</sub> , psi	v <sub>calc</sub> , psi	v <sub>test</sub> , psi	Type
																Failures
135 <sup>1</sup>	14	8	3.0	12.1	1.24	0.248	6430	46.8	46.8	542	533	775	605	1.02	1.28	S
										(H/V = 9/4)						
159 <sup>1</sup>	14	8	3.0	12.1	2.48	0.248	6650	46.8	46.8	692	2	983	722	-	1.36	S
										(H/V = 5/4)						

## NOTES:

Types of Failure (see Figs. 13 to 16)

B - Bearing

CE - Corbel End, crack intersecting inclined face

DS - Diagonal Splitting

FC - Flexural Compression

FT - Flexural Tension

S - Shear

 $\ell$  = 12 in. and b = 8 in. for all specimens unless otherwise noted

\* not measured or inconclusive test data

<sup>1</sup>  $\ell$  = 6 in.<sup>2</sup>  $f_{su}$  calculated smaller than  $f_y$ 

Table A4--Test Results for Corbels with Stirrups

No.	h, in.	h', in.	a, in.	d, in.	p <sub>s</sub> , %	p <sub>s</sub> , %	Stirrups and Tens. Reinf.	f' <sub>c</sub> , psi	f' <sub>y</sub> , ksi	f' <sub>su</sub> , ksi	f' <sub>y</sub> , ksi	Tension Reinf. psi	f <sub>test</sub> , psi	f <sub>test</sub> , psi	v <sub>calc</sub> , psi	v <sub>test</sub> , psi	v <sub>calc</sub> , psi	v <sub>test</sub> , psi	Type
																			Failures
134 <sup>1</sup>	18	9	9.5	16.1	0.34	1.27	0.590	4340	31/2	50.0	44.0	44.0	44.0	738	4	738	691	-	1.07
284 <sup>1</sup>	18	9	9.5	16.1	0.62	1.55	0.590	4590	31/2	46.2	44.0	44.0	44.0	844	4	844	759	1.18	1.11
384 <sup>1</sup>	18	9	9.5	16.1	0.93	1.86	0.590	4350	29/4	46.9	45.0	45.0	45.0	849	4	849	795	1.18	1.07
484 <sup>1</sup>	18	9	6.0	16.1	0.34	1.27	0.372	4330	31/2	56.8	44.2	44.2	44.2	932	4	932	843	-	1.10
584 <sup>1</sup>	18	9	6.0	16.1	0.62	1.55	0.372	4340	31/2	49.4	44.2	44.2	44.2	1050	4	1050	902	-	1.16
684 <sup>1</sup>	18	9	6.0	16.1	0.93	1.86	0.372	4480	29/4	49.1	44.2	44.2	44.2	1160	4	1160	974	-	1.20
784 <sup>1</sup>	26	11	9.5	24.1	0.34	1.27	0.394	4110	4	55.7	45.0	45.0	45.0	827	4	827	806	-	1.03
884 <sup>1</sup>	26	11	9.5	24.1	0.62	1.55	0.394	4300	4	50.0	45.0	45.0	45.0	959	4	959	881	-	1.07
984 <sup>1</sup>	26	12	9.5	24.1	0.93	1.86	0.394	4230	31/4	51.5	46.3	46.3	46.3	912	4	912	911	-	1.00
1084 <sup>1</sup>	18	9	4.75	16.1	0.62	1.55	0.295	4150	31/2	49.1	47.5	47.5	47.5	1210	4	1210	942	-	1.28
1184 <sup>1</sup>	26	12	4.75	24.1	0.62	1.55	0.197	4280	4	49.1	44.2	44.2	44.2	1030	4	1030	-	-	-
For 10 specimens Avg. v <sub>test</sub> /v <sub>calc</sub> = 1.11; Standard Deviation = 0.084																			
128	18	9	10.0	16.1	0.62	0.95 <sup>2</sup>	0.621	6120	31/2	50.2	47.5	47.5	47.5	388	352	536	444	1.10	1.21
(H/V = 1/2)																			
138	18	9	10.0	16.1	0.34	0.95 <sup>2</sup>	0.621	3900	31/2	49.0	47.5	47.5	47.5	259	4	260	222	-	1.17
148	18	9	10.0	16.1	0.93	0.95 <sup>2</sup>	0.621	4350	29/4	49.9	47.5	47.5	47.5	272	229	273	235	1.19	1.16
158	18	9	6.5	16.1	0.62	0.95 <sup>2</sup>	0.404	4110	31/4	49.1	47.5	47.5	47.5	408	4	432	277	-	1.56
168	26	12	4.75	24.1	0.62	0.95 <sup>2</sup>	0.197	4100	4	49.1	47.3	47.3	47.3	467	4	589	326	-	1.81
For 4 specimens Avg. v <sub>test</sub> /v <sub>calc</sub> = 1.42																			

 $\ell$  = 12 in. and b = 8 in. for all specimens<sup>1</sup> v = 15 in. (v = 3 in. for all others)<sup>2</sup> Test stopped at v = 1190 psi<sup>3</sup> Stirrups not included in p<sup>4</sup>  $f_{su}$  calculated smaller than  $f_y$

Table A5—Comparison with Test Results of Other Investigators

Source	No.	a, in.	d, in.	P, %	$\frac{a}{d}$	f' psi	f <sub>y</sub> ksi	f <sub>u test</sub> , ksi	v <sub>y test</sub> , psi	v <sub>y calc</sub> , psi	v <sub>u test</sub> , psi	v <sub>u calc</sub> , psi	$\frac{v_{y test}}{v_{y calc}}$	$\frac{v_{u test}}{v_{u calc}}$	b
U. of I. <sup>(4)</sup>	B-8	14.0	29.0	1.03	0.483	3390	46.0	*	*	626	621	628	-	0.99	4
	B-2-1	14.0	22.0	1.00	0.636	2910	39.9	*	*	440	511	501	-	1.02	4
	B-2-2	14.0	22.0	2.00	0.636	2290	45.9	*	*	1	502	560	-	0.90	4
	B-3-1	14.0	15.5	1.00	0.903	3740	50.9	50.9	403	1	427	459	-	0.93	4
	B-3-2	14.0	15.5	1.00	0.903	4940	51.4	51.4	516	1	532	527	-	1.01	4
	B-3-3	14.0	15.5	2.00	0.903	2960	42.4	*	*	1	645	514	-	1.25	4
	B-4-1	14.0	10.0	1.00	1.400	2800	45.0	45.0	237	261	237	289	0.91	0.82	4
	B-4-2	14.0	10.0	2.20	1.400	2520	54.5	*	*	1	317	357	-	0.89	4
	B-4-3	14.0	10.0	1.00	1.400	6460	47.3	47.3	285	404	372	439	0.70	0.85	4
Avg. $\frac{v_{y test}}{v_{y calc}} = 0.96$ ; Standard Deviation = 0.128															
U. of I. <sup>(5)</sup>	F4S1	6.0	6.0	0.83	1.000	4970	46.7	46.7	306	411	442	464	0.74	0.95	4
	F4S2	6.0	6.0	1.67	1.000	5030	48.6	48.6	614	1	854	589	-	1.45	4
	F3S2	6.0	8.0	0.83	0.750	3530	47.4	47.4	427	433	575	472	0.98	1.22	3
	F3S3	6.0	8.0	1.67	0.750	4980	47.4	47.4	700	1	1140	707	-	1.61	3
	F2S1	6.0	12.0	0.83	0.500	4920	46.0	46.0	610	616	902	693	0.99	1.30	2
U. of I. <sup>(6)</sup>	F2S2	6.0	12.0	1.29	0.500	4600	44.8	44.8	885	1	1150	844	-	1.36	2
	Specimens have compression reinforcement														
U. of I. <sup>(6)</sup>	G23S-11	6.0	13.0	0.83	0.462	3560	45.7	45.7	533	516	776	595	1.03	1.30	2
	G23S-21	6.0	13.0	0.46	0.462	3420	51.4	51.4	325	362	462	478	0.90	0.97	2
	G24S-11	6.0	13.0	0.83	0.462	5600	45.7	45.7	535	636	785	746	0.84	1.05	2
	G24S-21	6.0	13.0	0.46	0.462	5240	51.4	51.4	323	437	435	591	0.74	0.74	2
	G33S-11	6.0	9.0	1.67	0.667	3380	47.3	47.3	667	1	711	600	-	1.18	3
	G33S-21	6.0	9.0	0.83	0.667	3050	45.2	45.2	324	382	454	452	0.85	1.00	3
	G33S-31	6.0	9.0	2.58	0.667	2890	45.2	45.2	861	1	891	642	-	1.39	3
	G34S-11	6.0	9.0	1.67	0.667	5100	47.2	47.2	694	1	915	737	-	1.24	3
	G34S-21	6.0	9.0	0.83	0.667	4960	47.0	47.0	359	493	467	577	0.73	0.81	3
U. of I. <sup>(7)</sup>	G43S-11	6.0	7.0	1.67	0.857	3510	44.1	44.1	475	1	618	518	-	1.19	3
	G44S-11	6.0	7.0	1.67	0.857	3560	47.9	47.9	530	1	671	522	-	1.29	4
	Avg. $\frac{v_{u test}}{v_{u calc}} = 1.10$ ; Standard Deviation = 0.209														
U. of I. <sup>(7)</sup>	H0a	6.0	8.0	0.83	0.750	2930	45.0	45.0	338	381	367	430	0.89	0.85	3
	H0b	6.0	8.0	0.83	0.750	5800	51.0	51.0	421	581	448	605	0.72	0.74	3
	H0n	6.0	8.0	0.83	0.750	3580	51.0	51.0	351	463	382	476	0.76	0.80	3
Avg. $\frac{v_{u test}}{v_{u calc}} = 0.80$															

Table A5—Comparison with Test Results of Other Investigators (concluded)

Source	No.	a, in.	d, in.	P, %	$\frac{a}{d}$	f' <sub>c</sub> , psi	f' <sub>y</sub> , ksi	f <sub>su test</sub> , ksi	v <sub>y test</sub> , psi	v <sub>y calc</sub> , psi	v <sub>u test</sub> , psi	v <sub>u calc</sub> , psi	v <sub>y test</sub> / v <sub>y calc</sub>	v <sub>u test</sub> / v <sub>u calc</sub>	b
JACI <sup>(11)</sup>	24a	28.0	21.0	2.72	1.33	2580	45.7	24.8	-	1	452	402	-	1.12	7
	24b	28.0	21.0	2.72	1.33	2990	45.7	26.3	-	1	462	433	-	1.07	7
	25a	28.0	21.0	3.46	1.33	3530	45.4	18.4	-	1	408	510	-	0.80	7
	25b	28.0	21.0	3.46	1.33	2500	45.4	20.0	-	1	442	429	-	1.03	7
	26a	28.0	21.0	4.25	1.33	3140	43.8	26.0	-	1	643	513	-	1.25	7
	26b	28.0	21.0	4.25	1.33	2990	43.8	23.0	-	1	605	500	-	1.21	7
	27a	28.0	21.0	2.72	1.33	3100	45.7	28.5	-	1	531	441	-	1.20	7
	27b	28.0	21.0	2.72	1.33	3320	45.7	29.5	-	1	544	457	-	1.19	7
	28a	28.0	21.0	3.46	1.33	3380	45.4	21.0	-	1	462	499	-	0.93	7
	28b	28.0	21.0	3.46	1.33	3250	45.4	26.1	-	1	520	489	-	1.06	7
	29a	28.0	21.0	4.25	1.33	3150	43.8	22.4	-	1	595	514	-	1.16	7
	29b	28.0	21.0	4.25	1.33	3620	43.8	24.1	-	1	667	551	-	1.21	7
	30	28.0	21.0	4.25	1.33	3680	43.8	*	*	1	731	555	-	1.32	7
U.of T. <sup>(8)</sup>	31	28.0	21.0	4.25	1.33	3250	43.8	*	*	1	776	522	-	1.49	7
	1	21.0	33.4	0.55	0.629	5120	64.0	*	*	523	568	548	-	1.03	12.87
	2	21.0	33.7	1.12	0.623	4680	47.0	*	*	1	662	668	-	0.99	12.37
	3	15.0	33.4	0.37	0.449	6170	75.0	*	*	569	589	623	-	0.95	12.50
	4	36.25	33.0	0.46	1.10	2860	75.0	*	*	267	309	270	-	1.14	12.44
U.of T. <sup>(8)</sup>	5	36.25	33.0	1.01	1.10	4820	47.0	*	*	444	457	457	-	1.02	14.00
	6	20.87	33.0	1.02	0.632	4820	47.0	*	*	1	723	673	-	1.07	12.62
Avg. v <sub>u test</sub> /v <sub>u calc</sub> = 1.03; Standard Deviation = 0.066															

\* Not measured or inconclusive test data

† f<sub>su</sub> calculated smaller than f<sub>y</sub>

Table A6—Corbel Bearing Stresses at Ultimate Strength

No.	$f'_c$ psi	$f'_{bu}$ psi	$\frac{f'_{bu}}{f'_c}$	Type Failure	No.	$f'_c$ psi	$f'_{bu}$ psi	$\frac{f'_{bu}}{f'_c}$	Type Failure	No.	$f'_c$ psi	$f'_{bu}$ psi	$\frac{f'_{bu}}{f'_c}$	Type Failure
1	3790	4170	1.10	S	41	4200	1950	0.46	DS	81	2570	2700	1.05	FC
2	6170	4410	0.71	S	42	4850	3750	0.77	S	82	2110	2110	1.00	FC
3	3820	4940	1.29	S	43	4140	3000	0.72	S	83	2310	1290	0.56	FC
4	3520	4170	1.18	S	44	3840	2800	0.73	DS	84	2290	1750	0.76	FC
5	3840	5690	1.48	S	45	4280	3000	0.70	DS	85	2170	1580	0.73	FC
6	3970	5730	1.44	S	46	3840	2620	0.68	DS	86	4180	4250	1.02	S
7	3260	5860	1.80	S	47	4060	2610	0.64	DS	87	3880	3880	1.00	S
8	4170	4370	1.05	S	48	4920	3460	0.70	CE	88	3820	3750	0.98	S
9	6500	6640	1.02	S	49	4180	3450	0.82	B	89	4010	2750	0.68	FC
10	4790	7220	1.51	S	50	4390	2500	0.52	B	90	4240	3890	0.92	S
11	3900	3950	0.99	DS	51	4490	2710	0.60	B	91	4060	3080	0.76	S
12	4240	2940	0.69	FT	52	3960	2000	0.50	B	92	4160	1610	0.39	DS
13	4580	4000	0.87	FT	53	3660	2270	0.36	B	93	3980	2250	0.56	DS
14	4540	3500	0.77	DS	54	3950	1880	0.47	B	94	3940	2840	0.72	DS
15	4500	3000	0.67	DS	55	4010	2320	0.58	CE	95	6310	4750	0.75	S
16	3430	1960	0.57	DS	56	3770	2420	0.64	CE	96	6430	6260	0.97	S
17	3990	3560	0.89	DS	57	4130	3500	0.85	B	97	6420	5550	0.85	S
18	4210	3380	0.80	S	58	3720	2330	0.63	CE	98	6610	5180	0.48	S
19	3790	3950	1.02	DS	59	3510	2270	0.65	CE	99	6570	4610	0.70	DS
20	3590	3590	1.01	DS	60	3820	3330	0.87	FC	100	6430	4280	0.66	S
21	3920	3960	1.01	DS	61	4110	3130	0.76	DS	101	6370	1950	0.31	FC
22	3740	4370	1.17	DS	62	3260	2920	0.90	CE	102	6680	2430	0.36	DS
23	3990	3680	0.93	DS	63	3420	2720	0.80	CE	103	6590	2950	0.45	DS
24	4250	3710	0.87	DS	64	6940	3960	0.60	CE	104	4210	2340	0.56	S
25	6410	5420	0.84	DS	65	3660	4170	1.14	B	105	3860	2590	0.67	S
26	4280	4610	1.08	S	66	4040	3680	0.96	B	106	4040	2880	0.72	S
27	4320	4790	1.11	DS	67	4060	3680	0.91	B	107	4080	3350	0.82	S
28	4630	5210	1.12	DS	68	3380	2240	0.66	CE	108	3860	3450	0.89	S
29	3730	5140	1.38	S	69	3680	3590	0.98	DS	109	4240	4170	0.98	S
30	4280	3620	0.92	DS	70	4010	3330	0.83	S	110	4250	5000	1.18	S
31	4040	3750	0.93	DS	71	4410	3290	0.75	CE	111	3900	5310	1.36	S
32	4390	5830	1.33	DS	72	4110	3150	0.77	CE	112	4310	5850	1.35	S
33	3890	4750	1.24	DS	73	4090	4580	1.13	B	113	4400	2050	0.46	FT
34	4070	5140	1.26	DS	74	4360	4810	1.10	B	114	4320	2250	0.52	FT
35	3820	5510	1.44	B	75	4110	2680	0.65	DS	115	4950	2730	0.55	FT
36	3960	6460	1.63	DS	76	4090	3120	0.76	DS	116	3870	2590	0.67	S
37	3770	6500	1.72	S	77	2210	2300	1.04	S	117	3880	3020	0.78	S
38	4700	2140	0.46	DS	78	2200	2630	1.20	S	118	4240	3640	0.86	S
39	4490	2170	0.48	DS	79	2400	2490	1.04	B	119	4210	4380	1.04	S
40	4340	2180	0.50	FC	80	2430	2080	0.86	FC	120	4130	4790	1.16	S

Table A6—Corbel Bearing Stresses at Ultimate Strength (concluded)

No.	$f'_c$ psi	$f'_{bu}$ psi	$\frac{f'_{bu}}{f'_c}$	Type Failure	No.	$f'_c$ psi	$f'_{bu}$ psi	$\frac{f'_{bu}}{f'_c}$	Type Failure	No.	$f'_c$ psi	$f'_{bu}$ psi	$\frac{f'_{bu}}{f'_c}$	Type Failure
121	3970	4790	1.21	S	138	3870	1720	0.44	FT	155	4120	810	0.20	CE
122	3380	1140	0.34	DS	139	4420	2700	0.61	FT	156	3670	880	0.24	CE
123	4240	1410	0.33	DS	140	3890	2500	0.64	FT	157	4150	1040	0.25	CE
124	4240	1670	0.39	FT	141	4000	2710	0.68	S	158	4300	1090	0.25	CE
125	3250	1260	0.39	CE	142	4270	3540	0.83	S	159	4540	1080	0.24	DS
126	4480	1840	0.41	DS	143	4110	3850	0.94	S	160	4200	1040	0.25	CE
127	3300	1810	0.55	CE	144	4250	4260	1.00	S	161	4090	1140	0.28	CE
128	3610	1960	0.54	CE	145	3720	1140	0.31	FT	162	4470	1250	0.28	CE
129	4120	2710	0.66	S	146	4300	1120	0.26	FT	163	4350	1460	0.34	B
130	3950	1740	0.44	CE	147	4040	1980	0.49	FT	164	4080	1460	0.36	CE
131	4220	2290	0.54	DS	148	4250	2080	0.49	FT	165	4520	1810	0.40	DS
132	4120	1880	0.46	CE	149	4320	2340	0.54	FT	166	4110	1420	0.34	CE
133	4180	2240	0.54	CE	150	4050	2500	0.62	FT	167	4440	1940	0.35	CE
134	4290	3680	0.86	DS	151	4230	2920	0.69	S	168	4550	2370	0.52	DS
135	6430	1960	0.31	S	152	4130	3440	0.83	S	169	6650	1980	0.30	S
136	3870	1350	0.35	FT	153	3960	3250	0.82	DS					
137	4610	1510	0.33	FT	154	4750	690	0.14	CE					

H/V = 0 for 1 to 103

H/V =  $\frac{1}{2}$  for 104 to 134H/V =  $\frac{3}{4}$  for 135H/V =  $\frac{1}{2}$  for 136 to 168H/V =  $\frac{5}{4}$  for 169

**Table A7—Values of  $C_2 = 0.8 (10)^{a/d}$**   
(H/V = 0 and no stirrups)

a/d	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.80	0.81	0.81	0.82	0.82	0.83	0.84	0.84	0.85	0.86
0.1	0.86	0.87	0.88	0.88	0.89	0.90	0.90	0.91	0.92	0.92
0.2	0.93	0.94	0.95	0.95	0.96	0.97	0.98	0.98	0.99	1.00
0.3	1.01	1.01	1.02	1.03	1.04	1.05	1.05	1.06	1.07	1.08
0.4	1.09	1.10	1.10	1.11	1.12	1.13	1.14	1.15	1.16	1.16
0.5	1.17	1.18	1.19	1.20	1.21	1.22	1.23	1.24	1.25	1.26
0.6	1.27	1.28	1.29	1.30	1.31	1.32	1.33	1.34	1.35	1.36
0.7	1.37	1.38	1.39	1.40	1.41	1.42	1.43	1.44	1.45	1.47
0.8	1.48	1.49	1.50	1.51	1.52	1.54	1.55	1.56	1.57	1.58
0.9	1.60	1.61	1.62	1.63	1.64	1.66	1.67	1.68	1.70	1.71

**Table A8—Values of  $C_2 = 0.25 (10)^{a/d}$**   
(H/V = 0 and stirrups)

a/d	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.25	0.26	0.26	0.27	0.27	0.28	0.29	0.29	0.30	0.31
0.1	0.31	0.32	0.33	0.34	0.34	0.35	0.36	0.37	0.38	0.39
0.2	0.40	0.40	0.41	0.42	0.43	0.44	0.45	0.46	0.48	0.49
0.3	0.50	0.51	0.52	0.53	0.55	0.56	0.57	0.59	0.60	0.61
0.4	0.63	0.64	0.66	0.67	0.69	0.70	0.72	0.74	0.75	0.77
0.5	0.79	0.81	0.83	0.85	0.87	0.89	0.91	0.93	0.95	0.97
0.6	1.00	1.02	1.04	1.07	1.09	1.12	1.14	1.17	1.20	1.22
0.7	1.25	1.28	1.31	1.34	1.37	1.40	1.44	1.47	1.51	1.54
0.8	1.58	1.61	1.65	1.69	1.73	1.77	1.81	1.85	1.90	1.94
0.9	1.98	2.03	2.08	2.13	2.18	2.23	2.28	2.33	2.39	2.44

**Table A9—Values of  $C_1 = 1.5 (a/d)^{2/3}$**   
(H/V does not equal 0)

a/d	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00	0.07	0.11	0.14	0.18	0.20	0.23	0.25	0.28	0.30
0.1	0.34	0.36	0.38	0.40	0.42	0.44	0.46	0.48	0.50	0.51
0.2	0.53	0.55	0.56	0.58	0.60	0.61	0.63	0.64	0.66	0.67
0.3	0.69	0.70	0.72	0.73	0.74	0.76	0.77	0.79	0.80	0.81
0.4	0.83	0.84	0.85	0.87	0.88	0.89	0.91	0.92	0.93	0.94
0.5	0.96	0.97	0.98	0.99	1.01	1.02	1.03	1.04	1.06	1.07
0.6	1.08	1.09	1.10	1.11	1.12	1.14	1.15	1.16	1.17	1.18
0.7	1.19	1.20	1.22	1.23	1.24	1.25	1.26	1.27	1.28	1.29
0.8	1.30	1.31	1.32	1.34	1.35	1.36	1.37	1.38	1.39	1.40
0.9	1.41	1.42	1.43	1.44	1.45	1.46	1.47	1.48	1.49	1.50

**Table A10—Values of  $C_2 = 0.7 (10)^{a/2d}$**   
(H/V does not equal 0)

a/d	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.70	0.71	0.72	0.72	0.73	0.74	0.75	0.76	0.77	0.78
0.1	0.78	0.79	0.80	0.81	0.82	0.83	0.84	0.85	0.86	0.87
0.2	0.88	0.89	0.90	0.91	0.92	0.93	0.94	0.96	0.97	0.98
0.3	0.99	1.00	1.01	1.02	1.03	1.05	1.06	1.07	1.08	1.10
0.4	1.11	1.12	1.13	1.15	1.16	1.18	1.19	1.20	1.22	1.23
0.5	1.24	1.26	1.27	1.29	1.30	1.32	1.33	1.35	1.36	1.38
0.6	1.40	1.41	1.43	1.44	1.46	1.48	1.50	1.51	1.53	1.55
0.7	1.57	1.58	1.60	1.62	1.64	1.66	1.68	1.70	1.72	1.74
0.8	1.76	1.78	1.80	1.82	1.84	1.86	1.88	1.90	1.93	1.95
0.9	1.97	2.00	2.02	2.04	2.06	2.09	2.11	2.14	2.16	2.19